Board of Education Agenda Item

Item: O. **Date:** October 18, 2007

Topic: First Review of Proposed *Curriculum Framework for Algebra, Functions, and Data Analysis Standards of Learning*

Presenter: James C. Firebaugh, Jr., Director, Office of Middle and High School Instruction

Telephone Number: (804) 225-2651 **E-Mail Address:** Jim.Firebaugh@doe.virginia.gov

Origin:

_____ Topic presented for information only (no board action required)
__X__ Board review required by
__X__ State or federal law or regulation
____ Board of Education regulation
____ Other: _____

_____Action requested at this meeting __X__ Action requested at future meeting: Following public comment

Previous Review/Action:

__X__ No previous board review/action
___ Previous review/action
date
action

Background Information:

In June 2007, the Board of Education approved *Standards of Learning* for a new, optional, high school course, *Algebra, Functions, and Data Analysis*. Thus, it was necessary to develop an entirely new curriculum framework for these recently-approved standards to provide school divisions with an aligned resource document. Over the last three months, selected mathematics teachers and specialists assisted Department staff in developing the new framework document.

Summary of Major Elements:

The Department of Education has developed a *Curriculum Framework for Algebra, Functions, and Data Analysis Standards of Learning*, provided as Attachment A. The framework includes a brief introduction and describes the context for the *Algebra, Functions, and Data Analysis* standards as focusing on real-world problems. Each standard is explicated with Essential Understandings and Essential Knowledge and Skills sections to provide school division mathematics educators with detailed information for curriculum development.

Superintendent's Recommendation:

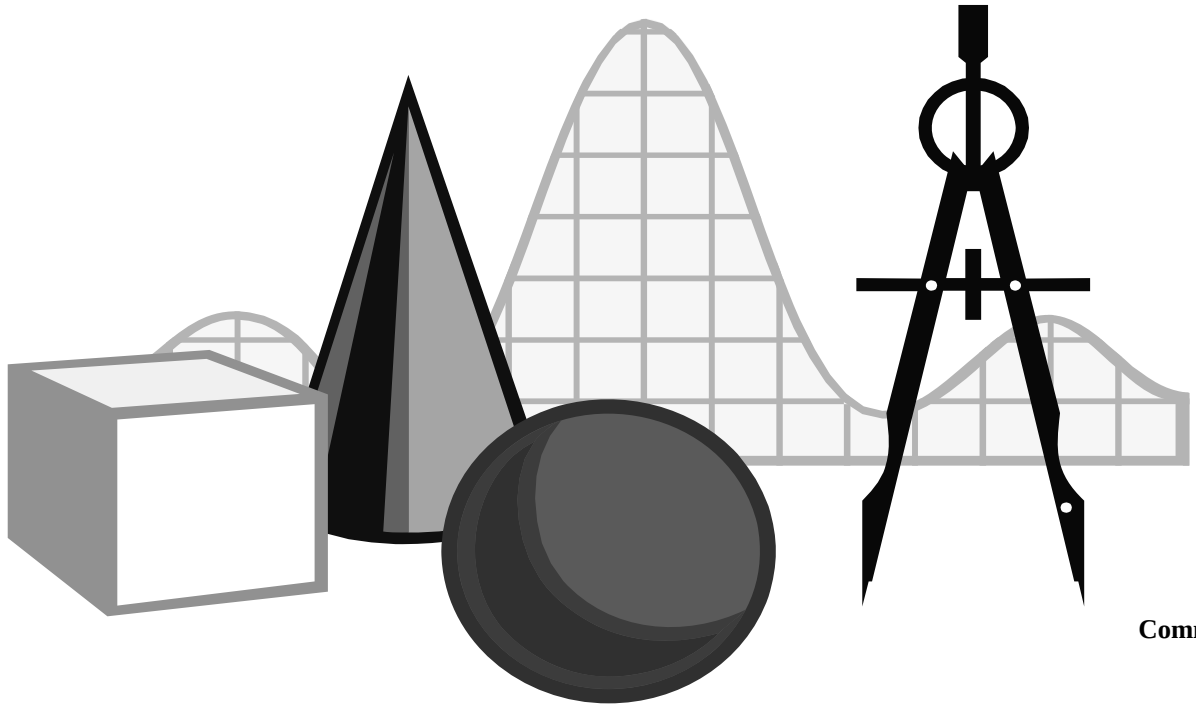
The Superintendent of Public Instruction recommends that the Board of Education accept for first review the proposed *Curriculum Framework for Algebra, Functions, and Data Analysis Standards of Learning.*

Impact on Resources:
There is minimal impact on DOE resources.

Timetable for Further Review/Action:
The *Code of Virginia* requires the Board to seek public comment on the *Curriculum Framework*. A public hearing will be scheduled following the Board meeting in November 2007.

PROPOSED
MATHEMATICS STANDARDS OF LEARNING
CURRICULUM FRAMEWORK

Algebra, Functions and Data Analysis



Commonwealth of Virginia
Board of Education
Richmond, Virginia
First Review - October 18, 2007

by the

Virginia Department of Education
P.O. Box 2120
Richmond, Virginia 23218-2120
http://www.doe.virginia.gov

Superintendent of Public Instruction
Billy K. Cannaday, Jr.

Chief Deputy Superintendent of Public Instruction
Patricia I. Wright

Assistant Superintendent for Instruction
Linda M. Wallinger

Office of Middle and High School Instructional Services
James C. Firebaugh, Jr., Director
Deborah Kiger Bliss, Mathematics Coordinator

NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov .

Introduction

The content of the *Mathematics Standards of Learning* supports five process goals for students:

- becoming mathematical problem solvers;
- communicating mathematically;
- reasoning mathematically;
- making mathematical connections; and
- using mathematical representations to model and interpret practical situations.

These goals provide a context within which to develop the knowledge and skills identified in the standards.

Mathematics in Context

Algebra, Functions and Data Analysis provides an opportunity for mathematical ideas to be developed in the context of real-world problems. Students will be asked to collect and analyze univariate and bivariate data using a variety of statistics and analytical tools. They will learn to attach functional algebra to statistics, allowing for the possibility of standardizing and analyzing data through the use of mathematical models. Students will use transformational graphing and the regression capabilities of graphing calculators to find regression equations, and they will use them to analyze the data and to predict the placement of data points between and beyond given data points.

Each topic in the ***Algebra, Functions and Data Analysis*** Curriculum Framework is developed around the *Standards of Learning*. Each standard is expanded in the Essential Knowledge and Skills column. The Essential Understandings column includes concepts, mathematical relationships, and ideas that are important to understanding and teaching the standards effectively.

Teachers should help students make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections should be made to other subject areas and fields of endeavor through applications. Using manipulatives, graphing calculators, and computer applications to develop concepts should help students develop and attach meaning to abstract ideas. Throughout the study of mathematics, students should be encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop their competence and their confidence in themselves as mathematics students.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS

AFDA.1

The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:

a) continuity
b) local and absolute maxima and minima
c) domain and range
d) zeros
e) intercepts
f) intervals in which the function is increasing/decreasing
g) end behaviors
h) asymptotes

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The domain of a function consists of the first coordinates of the ordered pairs that are elements of a function. Each element in the domain is an input into the independent variable of the function. • The range of a function consists of the second coordinates of the ordered pairs that are elements of a function. Each element in the range is an output in the dependent variable of a function. • For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $[x, f(x)]$ is a member of f. • A value x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$. • Functions describe the relationship between two variables where each input is paired to a unique output.	• Identify the domain and range for a relation, given a set of ordered pairs, a table, or a graph. • For each x in the domain of f, find $f(x)$. • Identify the zeros of the function algebraically and confirm them, using the graphing calculator. • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. • Recognize restricted/discontinuous domains and ranges. • Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Functions are used to model real-world phenomena. • A function is increasing on an interval if its graph, as read from left to right, is rising in that interval. • A function is decreasing on an interval if its graph, as read from left to right, is going down in that interval. • Exponential and logarithmic functions are either strictly increasing or strictly decreasing. • A function is continuous on an interval if the function is defined for every value in the interval and there are no breaks in the graph. A continuous function can be drawn without lifting the pencil. • A turning point is a point on a continuous interval where the graph changes from increasing to decreasing or from decreasing to increasing. • A function, *f*, has a local maximum in some interval at $x = a$ if $f(a)$ is the largest value of *f* in that interval. • A function, *f*, has a local minimum in some interval at $x = a$ if $f(a)$ is the smallest value of *f* in that interval. • Asymptotes can be used to describe local behavior and end behavior of graphs. They are lines or other curves that approximate the graphical behavior of a function.	• Identify *x*-intercepts (zeros), *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, points of discontinuity, end behavior, and maximum and minimum points, given a graph of a function. • Describe continuity of a function on its domain or at a point. • Express intervals using correct interval notation and/or a compound inequality.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The following statements are equivalent: – *k* is a zero of the polynomial function *f*; – *k* is a solution of the polynomial equation $f(x) = 0$; – *k* is an *x*-intercept for the graph of the polynomial; and – $(x - k)$ is a factor of the polynomial. • Continuous and discontinuous functions can be identified by their equations or graphs. The end behavior of a function refers to the graphical behavior of a function as x goes to positive and negative infinity.	

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS

AFDA.2

The student will use knowledge of transformations to write an equation given the graph of a function (linear, quadratic, exponential, and logarithmic).

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Knowledge of transformational graphing using parent functions can be used to generate a mathematical model from a scatter plot that approximates the data. • Transformations include: - Translations (horizontal and vertical shifting of a graph) - Reflections - Dilations (stretching and compressing graphs, and) - Rotations • The equation of a line can be determined by two points on the line or by the slope and a point on the line.	• Write an equation of a line when given the graph of a line. • Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions. • Write the equation of a linear, quadratic, exponential, or logarithmic function in *(h, k)* form given the graph of the parent function and transformation information. • Describe the transformation from the parent function given the equation written in *(h, k)* form or the graph of the function. • Given the equation of a function, recognize the parent function and transformation to graph the given function. • Recognize the vertex of a parabola given a quadratic equation in *(h, k)* form or graphed. • Describe the parent function represented by a scatter plot.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS

AFDA.3

The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The regression equation modeling a set of data points can be used to make predictions where appropriate. • Data and scatter plots may indicate patterns that can be modeled with a function. • Graphing calculators can be used to collect, organize, picture, and create an algebraic model of the data. • Data that fit linear, quadratic, exponential, and logarithmic models arise from practical situations • Two variables may be strongly associated without a cause-and-effect relationship existing between them. • Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured). • Residual = Actual – Fitted	• Write an equation for the line of best fit, given a set of data points in a table, on a graph, or from a practical situation. • Make predictions about unknown outcomes, using the equation of a line of best fit. • Collect and analyze data to make decisions and justify conclusions. • Investigate scatter plots to determine if patterns exist, and identify the patterns. • Find an equation for the curve of best fit for data, using a graphing calculator. Models will include linear, quadratic, exponential, and logarithmic functions. • Make predictions, using data, scatter plots, or equation of curve of best fit. • Given a set of data, determine the model that would best describe the data.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Least squares regression generates the equation of the line that minimizes the sum of the squared distances between the data points and the line. • A correlation coefficient measures the degree of association between two variables that are related linearly.	• Describe the errors inherent in extrapolation beyond the range of the data. • Estimate the correlation coefficient when given data and/or scatter plots.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS

AFDA.4

The student will transfer between and analyze multiple representations of functions including algebraic formulae, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The most appropriate representation of a function depends on the questions to be answered and/or the analysis to be done. • Given data may be represented as discrete points or as a continuous graph with respect to the real-world context. • Real-world data may best be represented as a table, a graph, or as a formula.	• Given an equation, graph a linear, quadratic, exponential or logarithmic function with the aid of a graphing calculator. • Make predictions given a table of values, a graph, or an algebraic formula. • Describe relationships between data represented in a table, in a scatter plot, and as elements of a function. • Determine the appropriate representation of data derived from real-world situations. • Analyze and interpret the data in context of the real-world situation.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS

AFDA.5

The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Linear programming models an optimization process. • A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized. • Any maximum or minimum value will occur at a corner point of a feasible region.	• Model practical problems with systems of linear inequalities. • Solve systems of linear inequalities with pencil and paper and using a graphing calculator. • Solve systems of equations algebraically and graphically. • Identify the feasibility region of a system of linear inequalities. • Identify the coordinates of the corner points of a feasibility region. • Find the maximum or minimum value for the function defined over the feasibility region. • Describe the meaning of the maximum or minimum value within its context.

TOPIC: DATA ANALYSIS

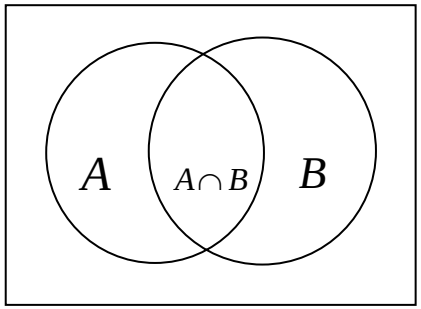
ALGEBRA, FUNCTIONS AND DATA ANALYSIS

AFDA.6

The student will calculate probabilities. Key concepts include:

a) conditional probability
b) dependent and independent events
c) addition and multiplication rules
d) counting techniques (permutations and combinations)
e) Law of Large Numbers

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The *Fundamental Counting Principle* states that if one decision can be made *n* ways and another can be made *m* ways, then the two decisions can be made *nm* ways. • *Permutations* are used to calculate the number of possible arrangements of objects. • *Combinations* are used to calculate the number of possible selections of objects without regard to the order selected. • A *sample space* is the set of all possible outcomes of a random experiment. • An *event* is a subset of the sample space. • *P(E)* is a way to represent the probability that the event E occurs • *Mutually exclusive events* are events that cannot both occur simultaneously. • If *A* and *B* are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.	• Compare and contrast permutations and combinations. • Calculate the number of permutations of *n* objects taken *r* at a time. • Calculate the number of combinations of *n* objects taken *r* at a time. • Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. • Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive. • Find conditional probabilities for dependent, independent, and mutually exclusive events. • Represent and calculate probabilities using Venn diagrams and probability trees.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The complement of event A consists of all outcomes in which event A does not occur. • *P(B|A)* is the probability that *B* will occur given that *A* has already occurred. *P(B|A*) is called *the conditional probability* of *B* given *A*. • Venn diagrams may be used to examine conditional probabilities.  $P(B\|A) = \frac{P(A \cap B)}{P(A)}$ $\Rightarrow P(A \cap B) = P(A)P(B\|A)$ • Two events, *A* and *B*, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If *A* and *B* are not independent, then they are said to be dependent. • If *A* and *B* are independent events, then $P(A \cap B) = P(A)P(B)$. • The *Law of Large Numbers* states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.	• Analyze, interpret and make predictions based on theoretical probability within real-world context. • Given a real-world situation, determine when to use permutations or combinations.

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS

AFDA.7

The student will analyze the normal distribution. Key concepts include:
a) characteristics of normally distributed data
b) percentiles
c) normalizing data using z-scores
d) area under the standard normal curve and probability

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Analysis of the descriptive statistical information generated by a univariate data set includes the relationships between central tendency, dispersion, and position. • The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation. • Areas under the curve represent probabilities associated with continuous distributions. • The normal curve is a probability distribution and the total area under the curve is 1. • The mean of the data in a standard normal density function is 0 and the standard deviation is 1. This allows for the comparison of unlike data. • The amount of data that falls within 1, 2, or 3 standard deviations of the mean is constant and the basis of *z*-score data normalization.	• Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem's context. • Explain the influence of outliers on a univariate data set. • Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation. • Identify the properties of a normal probability distribution. • Describe how the standard deviation and the mean affect the graph of the normal distribution. • Determine the probability of a given event, using the normal distribution.

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS

AFDA.8

The student will design and conduct an experiment/survey. Key concepts include:

a) sample size
b) sampling technique
c) controlling sources of bias and experimental error
d) data collection
e) data analysis and reporting

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of a sample statistic may vary from sample to sample, even if the simple random samples are taken repeatedly from the population of interest. • Poor data collection can lead to misleading and meaningless conclusions. • The purpose of sampling is to provide sufficient information so that population characteristics may be inferred. • Inherent bias diminishes as sample size increases. • Experiments must be carefully designed in order to detect a cause-and-effect relationship between variables. • Principles of experimental design include comparison with a control group, randomization, and blindness.	• Compare and contrast controlled experiments and observational studies and the conclusions one may draw from each. • Identify biased sampling methods. • Select a data collection method appropriate for a given context. • Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. • Determine which sampling technique is best, given a particular context. • Plan and conduct an experiment or survey. The experimental design should address control, randomization, and minimization of experimental error. • Design a survey instrument.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The precision, accuracy and reliability of data collection can be analyzed and described.	• Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. • Write a report describing the experiment/survey and the resulting data and analysis.