Board of Education Agenda Item

Item: _____D._____ **Date:** February 19, 2009

Topic: Final Review of Proposed Revised *Mathematics Standards of Learning*

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Origin:

____ Topic presented for information only (no board action required)

__X__ Board review required by
- __X__ State or federal law or regulation
- ____ Board of Education regulation
- ____ Other: _____

__X__ Action requested at this meeting __Action requested at future meeting: _____

Previous Review/Action:

____ No previous board review/action

__X__ Previous review/action
- Date February 21, 2008
- Action Board of Education approved the timeline to proceed with the review process.

- Date October 23, 2008
- Action Board of Education accepted the Proposed Revised *Mathematics Standards of Learning* for first review.

Background Information:

The *Mathematics Standards of Learning* were developed in 1995 and revised in 2001. The *Standards of Quality* require the Board of Education to review the Standards of Learning on a regular schedule. The *Mathematics Standards of Learning* are scheduled for review in 2009. As a result, on February 21, 2008, the Board approved a plan to review these standards during the 2008-2009 academic year. In accordance with the plan, the Department of Education took the following steps to produce a draft of the proposed revised *Mathematics Standards of Learning* for the Board's first review:

- Received online comments from stakeholders, including teachers, parents, and administrators;
- Met with a teacher review committee that consisted of recommended individuals solicited from school divisions on August 5, 6, and 7, 2008, to review the public comment and consider recommendations and reports from Achieve, the College Board, ACT, as well as the

National Assessment of Educational Progress (NAEP) Frameworks, the Curriculum Focal Points from the National Council of Teachers of Mathematics (NCTM), Principles and Standards for School Mathematics from NCTM, the Singapore Curricula, and the Report of the President's National Mathematics Advisory Panel;

- Solicited a postsecondary review committee comprised of mathematics and mathematics education faculty and met with the review committee on August 20, 2008;
- Solicited business leaders review committee and sent a summary of the public comment with the current *Mathematics Standards of Learning*, requesting comments; and
- Developed a draft of the proposed revised *Mathematics Standards of Learning*.

On October 23, 2008, the Virginia Board of Education accepted the proposed revised standards for first review. The Board held two public hearings on Monday, December 1, 2008, and three public hearings on Wednesday, December 3, 2008, to solicit comments on the proposed revised *Mathematics Standards of Learning*. The public hearings were held at Pulaski County High School, Pulaski County; Hermitage High School, Henrico County; Robinson Secondary School, Fairfax County; Joliff Middle School, Chesapeake City; and Linkhorne Middle School, Lynchburg City. There were a total of 46 speakers. In addition to comments received at the public hearings, 224 comments were received either online or as letters and faxes.

The proposed revised *Mathematics Standards of Learning* in Attachment A contain changes made as a result of public comment. These changes are indicated by double underlines and strikethroughs. A more detailed review of public comments is contained in Attachment B.

Summary of Major Elements:
The attached draft of the proposed revised *Mathematics Standards of Learning* consists of the following elements:

Introduction
The Standards of Learning for mathematics identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands or topics are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement; Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning for each strand progress in complexity at each grade level and throughout the high school courses.

Goals
The *Mathematics Standards of Learning* address all students' needs today for stronger mathematical knowledge and skills to pursue higher education, to compete in a technologically-oriented work force, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and develop proficiency in mathematical skills. In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. The content of the

mathematics standards is intended to support the following five goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Strands/Reporting Categories
The *Mathematics Standards of Learning* for each grade level kindergarten through Grade 8, Algebra I, Geometry, and Algebra II are grouped into categories that address related content and skills.

Standards
The *Mathematics Standards of Learning* for Virginia public schools describe the Commonwealth's expectations for student learning and achievement in grades K-12.

Summary of the Proposed Revised *Mathematics Standards of Learning*
The major elements of the attached proposed revised *Mathematics Standards of Learning* include:

- Edits to enhance clarity, specificity, rigor, alignment of skills and content, and a reflection of the current academic research and practice;
- Emphasis on vertical alignment in grades K-7 to prepare students for Algebra I;
- Increased alignment of Algebra I and Algebra II; and
- Increase of focus at each grade level.

Superintendent's Recommendation:
The Superintendent of Public Instruction recommends that the Board of Education adopt the proposed revised *Mathematics Standards of Learning*.

Impact on Resources:
This responsibility can be absorbed by the agency's existing resources at this time. If the agency is required to absorb additional responsibilities related to this activity, other services may be impacted.

Timetable for Further Review/Action:
Upon approval of the *Mathematics Standards of Learning*, the Department of Education will post the document on the Department's Standards of Learning Web site.

Proposed Revised Mathematics Standards of Learning

for Virginia Public Schools

Final Review
February 19, 2009

Foreword

The Standards of Learning in this publication represent a ~~major~~ significant development in public education in Virginia. These standards focus on the mathematical knowledge and skills all students need for the future, and they have been aligned with national expectations for postsecondary success. The Standards of Learning provide a framework for instructional programs designed to raise the academic achievement of all students in Virginia and are an important part of Virginia's efforts to provide challenging educational programs in the public schools.

The Standards of Learning set reasonable targets and expectations for what teachers need to teach and students need to learn. The standards are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught; the standards are to be incorporated into a broader, locally designed curriculum. Teachers are encouraged to go beyond the standards and select instructional strategies and assessment methods appropriate for their students.

The Standards of Learning are recognized as a model for other states. They were developed through a series of public hearings and the efforts of parents, teachers, representatives from higher education ~~officials~~, and ~~representatives of~~ business and industry leaders. The standards set clear, concise, and measurable academic expectations for young people. Parents are encouraged to work with their children to help them achieve these academic standards.

A major objective of Virginia's educational agenda is to give the citizens of the ~~c~~Commonwealth a program of public education that is among the best in the nation and that meets the needs of all young people in the ~~c~~Commonwealth. These Standards of Learning chart the course for achieving that objective.

Mathematics Standards of Learning

Introduction

The Standards of Learning for mathematics identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Recommendations and reports from Achieve, the College Board, and ACT, as well as the National Assessment of Educational Progress (NAEP) Frameworks, the *Curriculum Focal Points* from the National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics* from NCTM, the Singapore Curricula, the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report* from the American Statistical Association, and the *Report of the President's National Mathematics Advisory Panel* were considered in identifying mathematics content necessary for success for all students in postsecondary pursuits.

Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands or topics are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement; Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning for each strand progress in complexity at each grade level and throughout the high school courses.

The *Mathematics Standards of Learning Curriculum Framework* is a companion document to the *Mathematics Standards of Learning* that amplifies the *Mathematics Standards of Learning* and defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers as they plan their lessons by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn.

The Standards of Learning are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught. Teachers are encouraged to go beyond the standards and to select instructional strategies and assessment methods appropriate for their students.

Goals

Students today require stronger mathematical knowledge and skills to pursue higher education, to compete in a technologically ~~oriented~~ sophisticated work force, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and develop proficiency in mathematical skills. In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving in science,

engineering, business and industry, government, and practical affairs. Hence, the use of technology must be an integral part of teaching, ~~and~~ learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skills should be the shared responsibility of teachers of all disciplines.

The content of the mathematics standards is intended to support the following five goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-life data and situations within and outside mathematics and then apply appropriate strategies to find an acceptable solution. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students become competent mathematical problem solvers.

Mathematical Communication

Students will use the language of mathematics, including specialized vocabulary and symbols, to express mathematical ideas precisely. Representing, discussing, reading, writing, and listening to mathematics will help students ~~to~~ clarify their thinking and deepen their understanding of the mathematics being studied.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will learn to apply proportional and spatial reasoning and to reason from a variety of representations such as graphs, tables, and charts.

Mathematical Connections

Students will relate concepts and procedures from different topics in mathematics to one another and see mathematics as an integrated field of study. Through the application of content and process skills, students will make connections between different areas of mathematics and between mathematics and other disciplines, especially science. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships with a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should move easily among different representations—graphical, numerical, algebraic, verbal, and physical—and recognize that representation is both a process and a product.

Kindergarten

The kindergarten standards place emphasis on developing the concept of number by counting; combining, sorting, and comparing sets of objects; recognizing and describing simple repeating patterns; and recognizing shapes and sizes of figures and objects. Students will investigate nonstandard and standard measurement, collect data, and create graphs. The idea of fractions is introduced.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
(Focus: Whole Number Concepts)

K.1 The student, given two sets each containing ~~10~~ ~~20~~ 10 or fewer concrete objects, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of one-to-one correspondence.

K.2 The student, given a set containing ~~10~~ ~~20~~ 15 or fewer concrete items, will
a) tell how many are in the set by counting the number of items orally;
~~c)~~ b) write the numeral to tell how many are in the set; and
~~b)~~ c) select the corresponding numeral from a given set of numerals~~; and~~.

K.3 The student, given an ordered set of ~~three~~ ten objects and/or pictures, will indicate the ordinal position of each item, first through ~~third~~ tenth, and the ordered position of each item from left-to-right, right-to-left, top-to-bottom, and/or bottom-to-top.

~~K.4~~ ~~The student will investigate and recognize patterns from counting by fives and tens to 30, using concrete objects and a calculator.~~ [Moved to new SOL K.4 c]

~~K.5~~ K.4 The student will ~~count~~

a) count forward to ~~30~~ 100 and backward from ~~10~~ ~~30~~ 10;
b) identify one more than and one less than a number; and
c) count by fives and tens to ~~30~~ 100 ~~using concrete objects and a calculator~~. [Move to Curriculum Framework]

K.5 The student will identify the part of a set and/or region that represents a fraction for halves and fourths.

Computation and Estimation
(Focus: Whole Number Operations)

K.6 The student will model adding and subtracting whole numbers, using up to 10 concrete items ~~using up to 10 concrete items~~ ~~using whole numbers up to ten~~ .

Measurement
(Focus: Instruments and Attributes)

K.7 The student will recognize a penny, nickel, dime, and quarter and will determine the value of a collection of pennies and/or nickels whose total value is 10 cents or less.

K.8 The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer).

K.9 The student will tell time to the hour, using an analog ~~or~~ and digital clock.

K.10 The student will compare two objects or events, using direct comparisons or nonstandard units of measure, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder). Examples of nonstandard units include foot length, hand span, new pencil, paper clip, block.

Geometry
(Focus: ~~Identify~~ Plane Shapes)

K.11 The student will

a) identify~~,~~ ~~and~~ describe~~,~~ and trace ~~and draw~~ ~~two-dimensional~~ ~~(~~plane~~)~~ geometric figures (circle, triangle, square, and rectangle)~~.~~ ; and
b) compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

K.12 The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their position and orientation in space.

~~K.13~~ ~~The student will compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).~~ [Moved to new SOL K.11 b]

Probability and Statistics
(Focus: Data Collection and Display)

~~K.14~~ K.13 The student will gather data ~~relating to familiar experiences~~ by counting and tallying.

~~K.15~~ K.14 The student will display gathered data in ~~objects and information, using~~ object graphs, ~~pictorial~~ picture graphs, and tables, and will answer questions related to the data.

~~K.16~~ ~~K.15~~ ~~The student will investigate and describe the results of dropping a two-colored counter or using a multicolored spinner.~~

Patterns, Functions, and Algebra
(Focus: Attributes and Patterning)

~~K.17~~ ~~K.16~~ K.15 The student will sort and classify objects according to ~~similar~~ attributes ~~(size, shape, and color)~~. [Move to Curriculum Framework]

~~K.18~~ ~~K.17~~ K.16 The student will identify, describe, and extend ~~a~~ repeating ~~and growing~~ ~~relationship~~ patterns ~~found in common objects, sounds, and movements~~. [Move to Curriculum Framework]

Grade One

The first-grade standards place emphasis on counting, sorting, and comparing sets of up to 100 objects; recognizing and describing simple repeating and growing patterns; and ~~drawing,~~ tracing, describing and sorting ~~two-dimensional~~ plane geometric figures. Students' understanding of number is expanded through learning and applying the basic addition facts through the ~~fives~~ nines table and the corresponding subtraction facts; using nonstandard and standard units to measure; and organizing and interpreting data. ~~The idea of fractions is introduced.~~ [Moved to Kindergarten.] Fractional concepts are expanded.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
(Focus: Place Value and Fraction Concepts)

1.1 The student will
- a) count ~~objects in a given set containing between 1 and~~ from 0 to 100 ~~objects~~ and write the corresponding numeral~~.~~; and
- b) group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.

~~1.2~~ ~~The student will group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.~~ [Moved to new SOL 1.1 b]

~~1.3~~ 1.2 The student will count forward by ones, twos, fives, and tens to 100~~, by twos to 20~~ and backward by ones from ~~20~~ 30.

~~1.4~~ ~~The student will recognize and write numerals 0 through 100.~~ [Moved to new SOL 1.1 a]

~~1.5~~ ~~The student will identify the ordinal positions first through tenth, using an ordered set of objects.~~ [Moved to new SOL K.3]

~~1.6~~ 1.3 The student will identify ~~and represent the concepts of one-half and one-fourth, using appropriate materials or a drawing.~~ the part of a set and/or region that represents a fraction for halves, thirds, and fourths and write the fraction.

Computation and Estimation
(Focus: Whole Number Operations)

~~1.7~~ 1.4 The student, given a familiar problem situation involving magnitude, will
a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, and 500); and
b) explain the reasonableness of ~~his/her~~ the choice.

~~1.8~~ 1.5 The student will recall basic ~~basic~~ addition facts ~~– i.e.,~~ with sums to ~~10~~ 18 or less and the corresponding subtraction facts.

~~1.9~~ 1.6 The student will create and solve one-step story and picture problems ~~involving one-step solutions,~~ using basic ~~basic~~ addition facts with sums to 18 or less and the corresponding subtraction facts.

Measurement
(Focus: Time and Nonstandard Measurement)

~~1.10~~ 1.7 The student will
a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and
b) determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.

~~1.11~~ 1.8 The student will tell time to the half-hour, using an analog ~~or~~ and digital clock.

~~1.12~~ 1.9 The student will use nonstandard units to measure length, ~~and~~ weight/mass and volume.

~~1.13~~ 1.10 The student will compare using the concepts of *more, less*, and *equivalent*,
a) the volumes of two given containers ~~by using concrete materials (e.g., jelly beans, sand, water, rice)~~ [Move to Curriculum Framework] ~~.~~; and
b) the weight/mass of two objects, using a balance scale.

~~1.14~~ ~~The student will compare the weights of two objects, using a balance scale.~~ [Moved to new SOL 1.10 b]

1.11 The student will use calendar language appropriately (e.g., *months*, *today, yesterday, next week, last week*).

Geometry
(Focus: Characteristics of Plane Figures)

~~1.15~~ ~~The student will describe the proximity of objects in space (*near, far, close by, below, above, up, down, beside,* and *next to).*~~ [Move to Curriculum Framework]

~~1.16~~ 1.12 The student will ~~draw, describe, and sort plane geometric figures)~~ identify and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, ~~corners~~ vertices, and ~~square corners~~ right angles. ~~[Move to Curriculum Framework]~~

~~1.17~~ 1.13 The student will ~~identify~~ construct, model, and describe objects in ~~his/her~~ the environment ~~using shapes and spatial reasoning~~ as geometric shapes ~~that depict plane geometric figures~~ (triangle, rectangle, square, and circle) ~~(triangle, rectangle, square, and circle)~~ and explain the reasonableness of the choice. ~~[Move to Curriculum Framework]~~

Probability and Statistics
(Focus: Data Collection and Interpretation)

~~1.18~~ 1.14 The student will investigate, identify, and describe various forms of data collection in ~~his/her~~ the world (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream), using tables, picture graphs, and object graphs.

~~1.19~~ 1.15 The student will interpret information displayed in a picture or object graph, using the vocabulary *more, less, fewer, greater than, less than,* and *equal to*.

Patterns, Functions, and Algebra
(Focus: Patterning and Equivalence)

~~1.20~~ 1.16 The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.

~~1.21~~ 1.17 The student will recognize, describe, extend, and create a wide variety of patterns including ~~rhythmic, color, shape, and numerical. Patterns will include both~~ [Move to Curriculum Framework] growing and repeating patterns. ~~Concrete materials and calculators will be used by students~~ [Move to Curriculum Framework]

1.18 The student will ~~recognize~~ demonstrate an understanding of equality through the use of the equal sign ~~as a representation of equivalency~~.

Grade Two

The second-grade standards extend the study of number and spatial sense to include three-digit whole numbers and solid geometric ~~(three-dimensional)~~ figures. Students will continue to learn, use, and gain proficiency in the basic addition facts through the ~~nines~~ tens table and the corresponding subtraction facts. Students will begin to use ~~standard~~ U.S. Customary and metric units of measure~~ment~~; predict, using simple probability; and create and interpret picture and bar graphs. Students will work with a variety of patterns and will develop knowledge of equality by identifying missing numbers in addition and subtraction facts.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
(Focus: Place Value, Number Patterns, and Fraction Concepts)

2.1 The student will
- a) read, write, and identify the place value of each digit in a three-digit numeral, using numeration models; ~~and~~
- b) round two-digit numbers to the nearest ten~~.~~; and
- c) compare two whole numbers between 0 and 999, using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).

~~2.2~~ ~~The student will compare two whole numbers between 0 and 999, using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).~~ [Moved to new SOL 2.1 c]

~~2.3~~ 2.2 The student will
- a) identify the ordinal positions first through twentieth, using an ordered set of objects; and~~.~~
- b) write the ordinal numbers.

~~2.4~~ 2.3 The student will
- a) identify the part of a set and/or region that represents a fraction~~s~~ for ~~one-half, one-third, one-fourth, one-eighth, and one-tenth~~ halves, thirds, fourths, sixths, eighths, and tenths;
- b) write the ~~corresponding~~ fraction; and~~.~~
- c) compare the unit fractions for halves, thirds, fourths, sixths, eighths, and tenths.

~~2.5~~ 2.4 The student will
- a) count forward by twos, fives, and tens to 100, starting at various multiples of 2, 5, or 10; ~~using mental mathematics, paper and pencil, hundred chart, calculators, and/or concrete objects, as appropriate~~ [Move to Curriculum Framework] ;
- b) count backward by tens from 100; and
- ~~c) group objects by threes and fours; and~~
- ~~d)~~ c) recognize even and odd numbers~~, using objects~~. [Move to Curriculum Framework]

Computation and Estimation
(Focus: Number Relationships and Operations)

~~2.6~~ 2.5 The student will recall ~~basic~~ addition facts~~, i.e.,~~ with sums to ~~18~~ 20 or less and the corresponding subtraction facts.

~~2.7~~ 2.6 The student, given two whole numbers whose sum is 99 or less, will
- a) estimate the sum; and
- b) find the sum, using various methods of calculation. ~~(mental computation, concrete materials, and paper and pencil)~~. [Move to Curriculum Framework]

~~2.8~~ 2.7 The student, given two whole numbers, each of which is 99 or less, will
- a) estimate the difference; and
- b) find the difference, using various methods of calculation. ~~(mental computation, concrete materials, and paper and pencil.)~~ [Move to Curriculum Framework]

~~2.9~~ 2.8 The student will create and solve one- or two-step addition and subtraction problems using data from simple tables, picture graphs, and bar graphs~~, and practical situations~~. [Move to Curriculum Framework]

~~2.10~~ 2.9 The student~~, given a simple addition or subtraction fact,~~ will recognize and describe the related facts which represent and describe the inverse relationship between addition and subtraction (e.g., 3 + __ = 7, ___ + 3 = 7; 7 – 3 = __, and 7 – __ = 3).

Measurement
(Focus: Money, Linear Measurement, Weight/mass, Volume)

~~2.11~~ 2.10 The student will
- a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and
- b) ~~identify the correct usage of~~ correctly use the cent symbol (¢), dollar symbol ($), and decimal point (.).

~~2.12~~ 2.11 The student will estimate and ~~then use a ruler to make linear measurements to~~ measure
- a) length to the nearest centimeter and inch~~, including measuring the distance around a polygon in order to determine perimeter.~~ ; [Moved to new SOL 3.10 a]
- b) weight/mass of objects using a scale in pounds, ounces/kilograms, grams; and
- c) liquid volume in ~~(~~cups, pints, quarts, gallons, and liters~~), using the concepts of *more, less*, and *equivalent*~~. [Moved to new SOL 1.10]

~~2.13 The student, given grid paper, will estimate and then count the number of square units needed to cover a given surface in order to determine area.~~ [Moved to new SOL 5.8 a]

~~2.14 The student will estimate and then count the number of cubes in a rectangular box in order to determine volume.~~ [Moved to new SOL 5.8 a]

~~2.15 The student will estimate and then determine weight/mass of familiar objects in pounds and/or kilograms.~~ [Moved to new SOL 2.11 b]

~~2.16~~ 2.12 The student will tell and write time to the ~~quarter hour~~ nearest five minutes, using analog and digital clocks.

~~2.17 The student will use actual measuring devices to compare metric and U.S. Customary units (cups, pints, quarts, gallons, and liters) for measuring liquid volume, using the concepts of *more, less*, and *equivalent*.~~ [Moved to new SOL 2.11 c]

~~2.18~~ 2.13 The student will
- ~~a) use calendar language appropriately (e.g., *months*, *today, yesterday, next week, last week*);~~ [Moved to new SOL 1.11]
- ~~b)~~ a) determine past and future days of the week; and
- ~~c)~~ b) identify specific days and dates on a given calendar.

~~2.19~~ 2.14 The student will read the temperature on a Celsius and/or Fahrenheit thermometer to the nearest 10 degrees.

Geometry
(Focus: Symmetry and Plane and Solid Figures)

~~2.20~~ ~~The student will identify, describe, and sort three-dimensional (solid) concrete figures,~~ [Moved to new SOL 3.14] ~~including a cube, rectangular solid (prism), square pyramid, sphere, cylinder, and cone, according to the number and shape of the solid's faces, edges, and corners.~~ [Move to Curriculum Framework in support of new SOL 3.14]

~~2.21~~ 2.15 The student will
a) draw a line of symmetry in a figure; and
b) identify and create figures ~~symmetric along a line,~~ with at least one line of symmetry ~~using various concrete materials~~. [Move to Curriculum Framework]

~~2.22~~ 2.16 The student will identify, describe, compare, and contrast plane and solid geometric shapes (circle/sphere, square/cube, and rectangle/rectangular ~~solid~~ prism).

Probability and Statistics
(Focus: Applications of Data)

~~2.23~~ 2.17 The student will use data from experiments to ~~read,~~ construct~~, and interpret a simple~~ picture graphs, pictographs, and bar graphs.

~~2.24~~ 2.18 The student will ~~record~~ use data from experiments~~, using spinners and colored tiles/cubes, and use the data~~ to predict outcomes ~~which of two events is more likely to occur~~ [Move to Curriculum Framework] if the experiment is repeated.

2.19 The student will analyze data displayed in ~~a~~ picture graphs, pictographs, and bar graphs.

Patterns, Functions, and Algebra
(Focus: Patterning and Numerical Sentences)

~~2.25~~ 2.20 The student will identify, create, and extend a wide variety of patterns~~, using numbers, concrete objects and pictures.~~ [Move to Curriculum Framework]

~~2.26~~ 2.21 The student will solve problems by completing a numerical sentence involving the basic facts for addition and subtraction. ~~Examples include: 3 + __ = 7, or 9 ____ 4 = 2.~~ [Move to Curriculum Framework] Students will create story problems, using the numerical sentences.

2.22 The student will demonstrate an understanding of equality by recognizing that the symbol, =, in an equation indicates equivalent quantities and the symbol, ≠, indicates that quantities are not equivalent.

Grade Three

The third-grade standards place emphasis on learning multiplication and division facts through the nines table. Students will be fluent in the basic addition facts through the ~~nines~~ tens table and the corresponding subtraction facts. Concrete materials and two-dimensional representations will be used to introduce addition and subtraction with fractions and decimals and the concept of probability as chance. Students will use standard units (U.S. Customary and metric) ~~for~~ to measure temperature, length, liquid volume, and weight and identify relevant properties of shapes, points, line segments, rays, ~~and~~ angles, vertices, and lines. Students will investigate and describe the identity and commutative properties for addition and multiplication.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
(Focus: Place Value and Fractions)

3.1 The student will

a) read and write six-digit numerals and identify the place value and value for each digit~~.~~;

b) round a whole number, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*).

~~3.2~~ ~~The student will round a whole number, 9,999 or less, to the nearest ten, hundred, and thousand.~~ [Moved to new SOL 3.1 b]

~~3.3~~ ~~The student will compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*).~~ [Moved to new SOL 3.1 c]

~~3.4~~ 3.2 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. Students will use these relationships to solve problems~~:~~ ~~such as 5 + 3 = 8 and 8 – 3 = ____~~. [Move to Curriculum Framework]

~~3.5~~ 3.3 The student will

~~a)~~ ~~divide regions and sets to represent a fraction; and~~ [Move to Curriculum Framework]

~~b)~~ ~~name and write the fractions represented by a given model (area/region, length/measurement, and set). Fractions (including mixed numbers) will include halves, thirds, fourths, eighths, and tenths.~~ [Moved to new SOL 3.3 a]

a) name and write fractions (including mixed numbers) represented by a model ~~Fractions (including mixed numbers);~~

b) model fractions (including mixed numbers) and write the fractions' names; and

c) compare ~~the numerical value of two~~ fractions having like and unlike denominators, using words and symbols for <, >, and =. ~~using concrete or pictorial models involving areas/regions and lengths/measurements~~. [Move to Curriculum Framework]

~~3.6~~ ~~The student will compare the numerical value of two fractions having like and unlike denominators, using concrete or pictorial models involving areas/regions, lengths/measurements, and sets.~~ [Moved to new SOL 3.3 c]

~~3.7~~ ~~The student will read and write decimals expressed as tenths and hundredths, using concrete materials and models.~~ [Moved to new SOL 4.3 a]

Computation and Estimation
(Focus: Computation and Fraction Operations)

~~3.8~~ 3.4 The student will estimate solutions to and solve single and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping ~~using various computational methods, including~~ ~~calculators, paper~~ ~~and pencil, mental computation, and estimation~~. [Move to Curriculum Framework]

~~3.9~~ 3.5 The student will

~~a)~~ recall ~~the~~ multiplication ~~and division~~ facts through the ~~nines~~ twelves table, and the corresponding division facts.

~~b)~~ ~~find the related quotients.~~

~~3.10~~ 3.6 The student will represent multiplication and division, using area ~~and~~ , set, and number line models; and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.

~~3.11~~ 3.7 The student will add and subtract ~~with~~ proper fractions having like denominators of ~~10~~ 12 or less~~, using concrete materials and pictorial models representing areas/regions, lengths/measurements, and sets~~. [Move to Curriculum Framework]

~~3.12~~ ~~The student will add and subtract with decimals expressed as tenths, using concrete materials, pictorial representations, and paper and pencil.~~ [Moved to new SOL 4.5 c]

Measurement
(Focus: U.S. Customary and Metric Units, Area and Perimeter, Time)

~~3.13~~ 3.8 The student will determine by counting the value of a collection of bills and coins whose total value is $5.00 or less, compare the value of the coins or bills, and make change.

~~3.14~~ 3. 9 The student will estimate and ~~then~~ use ~~actual measuring devices with metric and U.S. Customary~~ U.S. Customary and metric units to measure:

a) length - including part of an inch (1/2), inches, feet, yards, centimeters, and meters;
b) liquid volume - cups, pints, quarts, gallons, and liters; ~~and~~
c) weight/mass - ounces, pounds, grams, and kilograms~~.~~; and
d) area and perimeter.

3.10 The student will

a) measure the distance around a polygon in order to determine perimeter; and
b) count the number of square units needed to cover a given surface in order to determine area.

~~3.15~~ 3.11 The student will

a) tell time to ~~the nearest five-minute interval and to~~ [Moved to new SOL 2.12] the nearest minute, using analog and digital clocks~~.~~; and
b) determine elapsed time in one-hour increments over a 12-hour period.

~~3.16~~ 3.12 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.

~~3.17~~ 3.13 The student will read temperature to the nearest degree from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used.

Geometry
(Focus: ~~Properties and Congruence~~ Characteristics of Plane and Solid Figures)

~~3.18~~ 3.14 The student will ~~analyze~~ identify~~,~~ describe, compare, and contrast ~~analyze~~ characteristics of ~~two-dimensional (~~plane~~)~~ and ~~three-dimensional (~~solid~~)~~ geometric figures (circle, square, rectangle, triangle, cube, rectangular ~~solid [~~prism~~]~~, square pyramid, sphere, cone, and cylinder) ~~and identify~~ by identifying relevant ~~properties.~~ characteristics, including the number of ~~corners, square corners~~ angles, vertices, edges, and the number and shape of faces, using concrete models.

~~3.19~~ 3.15 The student will identify and draw representations of points, line segments, rays, angles, and lines ~~using a ruler or straightedge~~. [Move to Curriculum Framework]

~~3.20~~ 3.16 The student~~, given appropriate drawings or models,~~ [Move to Curriculum Framework] will identify and describe congruent and ~~symmetrical,~~ noncongruent, ~~two-dimensional (~~plane~~)~~ figures~~, using tracing procedures~~. [Move to Curriculum Framework]

Probability and Statistics
(Focus: Applications of Data and Chance)

~~3.21~~ 3.17 The student~~, given grid paper,~~ will

a) collect and organize data ~~on a given topic of his/her choice,~~ using observations, measurements, surveys, or experiments; ~~and~~

~~b)~~ construct a line plot, a picture graph, or a bar graph to represent ~~the results.~~ the data ~~Each graph will include an appropriate title and key.~~ [Move to Curriculum Framework]; and

c) read and interpret the data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.

~~3.22~~ ~~The student will read and interpret data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.~~ [Moved to new SOL 3.17 c]

~~3.23~~ 3.18 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

Patterns, Functions, and Algebra
(Focus: Patterns and Property Concepts)

~~3.24~~ 3.19 The student will recognize and describe a variety of patterns formed using ~~concrete objects,~~ numbers, tables, and pictures, and extend the pattern, using the same or different forms.

~~3.25~~ 3.20 The student will

a) investigate ~~and create patterns involving numbers,~~ [Move to Curriculum Framework] ~~operations (addition and multiplication), and relations that model~~ the identity and the commutative properties for addition and multiplication; ~~and~~

b) identify examples of the identity and commutative properties for addition and multiplication; and

~~b)~~c) demonstrate an understanding of equality ~~by recognizing that the equals sign (=) in an equation links equivalent quantities,~~ [Moved to new SOL 2.22] ~~such as 4 • 3 = 2 • 6.~~ [Move to Curriculum Framework] through the use of the symbols, = and ≠.

Grade Four

The fourth-grade standards place emphasis on multiplication and division with whole numbers and solving problems involving addition and subtraction of fractions and decimals by finding common multiples and factors, . ~~Students will continue to learn and use the basic multiplication facts.~~ Students will be fluent in the basic multiplication facts through the twelves ~~nines~~ table and the corresponding division facts as they become proficient in multiplying larger numbers. Students also will refine their estimation skills for computations and measurements ~~and investigate relationships between and among simple two-dimensional (plane) figures and three-dimensional (solid) figures~~. Students will identify and ~~draw~~ describe representations of points, lines, line segments, ~~and~~ rays, and angles, including endpoints and vertices. ~~Students will graph points in the first quadrant in the coordinate plane and extend and duplicate patterns.~~ Concrete materials and two-dimensional representations will be used to solve problems involving perimeter, patterns, probability, and equivalence of fractions and decimals. Students will recognize images of figures resulting from ~~a~~ geometric transformations, such as reflection ~~(flip)~~, translation ~~(slide)~~, and rotation ~~(turn)~~. Students will investigate and describe the associative property for addition and multiplication.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
(Focus: Place Value, Fractions, and Decimals)

4.1 The student will
- a) identify (orally and in writing) the place value for each digit in a whole number expressed through millions;
- b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and
- c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

4.2 The student will
a) ~~identify, model, and~~ compare and order ~~rational numbers~~ fractions and mixed numbers; ~~using concrete objects and pictures~~ [Move to Curriculum Framework]
b) represent equivalent fractions; and
~~c)~~ ~~relate fractions to decimals, using concrete objects.~~ [Moved to new SOL 4.3 d]
c) identify the ~~fraction~~ division statement that represents ~~division~~ a fraction.

~~4.3~~ ~~The student will compare the numerical value of fractions~~ [Moved to new SOL 4.2 a] ~~(with like and unlike denominators) having denominators of 12 or less, using concrete materials.~~ [Move to Curriculum Framework]

~~4.4~~ 4.3 The student will
a) read, write, represent, and identify decimals expressed ~~as tenths and hundredths~~ through thousandths;
b) round decimals to the nearest whole number, tenth, and hundredth; ~~and~~
c) compare ~~the value of two~~ and order decimals~~, using symbols (<, >, or =), concrete materials, drawings, and calculators.~~ [Move to Curriculum Framework] ; and
d) given a model, write the decimal and fraction equivalents.

Computation and Estimation
(Focus: Factors and Multiples, and Fraction and Decimal Operations)

~~4.5~~ 4.4 The student will ~~estimate whole-number sums and differences and describe the method of estimation. Students will refine estimates, using terms such as *closer to, between*, and *a little more than*.~~ [Move to Curriculum Framework]
a) estimate sums, differences, products, and quotients ~~for~~ of whole numbers;
b) add, subtract, and multiply whole numbers;
c) divide whole numbers, finding quotients with and without remainders; and
d) solve single and multistep addition, subtraction, and multiplication problems with whole numbers.

~~4.6~~ ~~The student will add and subtract whole numbers written in vertical and horizontal form, choosing appropriately between paper and pencil methods and calculators.~~ [Move to Curriculum Framework]

~~4.7~~ ~~The student will find the product of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer,~~ [Moved to new SOL 4.4 a] ~~using estimation and paper and pencil. For larger products (a two-digit numeral times a three-digit numeral), estimation and calculators will be used.~~ [Move to Curriculum Framework]

~~4.8~~ ~~The student will estimate and find the quotient of two whole numbers~~ [Moved to new SOL 4.4 c], ~~given a one-digit divisor~~. [Move to Curriculum Framework]

~~4.9~~ 4.5 The student will

a) ~~add and subtract with fractions having like and unlike denominators of 12 [Moved to new SOL 4.5 b]~~ ~~or less, using concrete materials, pictorial representations, and paper and pencil;~~ ~~simplify fractions;~~ [Moved to new SOL 4.5 b] determine common multiples and factors, including least common multiple and greatest common factor [Moved from new SOL 5.3a.] of up to two fractions;

b) ~~add and subtract with decimals~~ [Moved to new SOL 4.5 c] ~~through thousandths, using concrete materials, pictorial representations, and paper and pencil;~~ ~~and~~ [Move to Curriculum Framework]add and subtract ~~with~~ fractions having like and unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors;

c) ~~solve problems involving addition and subtraction with fractions [Moved to new SOL 4.5 d]~~ ~~having like and unlike denominators of 12 or less~~ [Move to Curriculum Framework] ~~and with decimals~~ [Moved to new SOL 4.5 d] ~~expressed through thousandths, using various computational methods, including calculators, paper and pencil, mental computation, and estimation~~ [Move to Curriculum Framework] add and subtract with decimals; and

d) solve single and multistep practical problems involving addition and subtraction with fractions and with decimals.

Measurement
(Focus: Equivalence between U.S. Customary and Metric Units)

~~4.10~~ 4.6 The student will

a) estimate and measure weight/mass~~, using actual measuring devices,~~ [Move to Curriculum Framework] and describe the results in U.S. Customary/metric units as appropriate~~, including ounces, pounds, tons, grams, and kilograms~~ [Move to Curriculum Framework]; and

b) identify equivalent measurements between units within the U.S. Customary system (ounces, ~~and~~ pounds, and tons, and between units within the metric system (grams and kilograms)~~.~~; ~~and~~

~~c) estimate the conversion of ounces and grams and pounds and kilograms, using approximate comparisons (1 ounce is about 28 grams, or 1 gram is about the weight of a paper clip; 1 kilogram is a little more than 2 pounds). *~~ [Moved to new SOL 6.9]

~~** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.*~~

~~4.11~~ 4.7 The student will

a) estimate and measure length~~, using actual measuring devices,~~ and determine the result in both metric and U.S. Customary ~~units including part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, miles, millimeters, centimeters, and meters~~ [Move to Curriculum Framework]; and

b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters). ~~; and~~

~~c) estimate the conversion of inches and centimeters, yards and meters, and miles and kilometers, using approximate comparisons (1 inch is about 2.5 centimeters, 1 meter is a little longer than 1 yard, 1 mile is slightly farther than 1.5 kilometers, or 1 kilometer is slightly farther than half a mile). *~~
[Moved to new SOL 6.9]

~~** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.*~~

~~4.12~~ 4.8 The student will

a) estimate and measure liquid volume~~, using actual measuring devices~~ [Move to Curriculum Framework] and ~~using metric and~~ describe the results in U.S. Customary; and

b) identify equivalent measurements between units within the U.S. Customary system (cups, pints, quarts, and gallons). ~~and between units within the metric system (milliliters and liters); and~~

~~c) estimate the conversion of quarts and liters, using approximate comparisons (1 quart is a little less than 1 liter, 1 liter is a little more than 1 quart).*~~ [Moved to new SOL 6.9]

~~** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U. S. Customary and metric units.*~~

~~4.13 The student will~~

~~a) identify and describe situations representing the use of perimeter and area;~~ [Moved to new SOL ~~5.9~~ 5.8a] ~~and~~

~~b) use measuring devices to find perimeter in both standard and nonstandard units of measure.~~ [Moved to new SOL ~~5.9~~ 5.8a]

4.9 The student will determine elapsed time in hours and minutes within a 12-hour period.

Geometry
(Focus: Representations and Polygons)

~~4.14~~ 4.10 The student will ~~investigate and describe the relationships between and among points, lines, line segments, and rays.~~
- a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and
- b) identify representations of lines that illustrate intersection, parallelism, and perpendicularity. ~~; and~~
- ~~c) describe the path of shortest distance between two points on a plane surface.~~ [Move to Curriculum Framework to support new SOL 4.10a]

~~4.15~~ ~~The student will~~
- ~~a) identify and draw representations of points, lines, line segments, rays, and angles, using a straightedge or ruler; and~~ [Moved to new SOL 4.10 a]
- ~~b) describe the path of shortest distance between two points on a flat surface.~~ [Move to ~~new SOL 4.10 c~~ Curriculum Framework in support of new SOL 4.10 a]

~~4.16~~ ~~The student will identify and draw representations of lines that illustrate intersection, parallelism, and perpendicularity.~~ [Moved to new SOL 4.10 b]

~~4.17~~ 4.11 The student will
- ~~a) analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]);~~ [Moved to new SOL 3.14]
- ~~b) identify congruent and noncongruent shapes; and~~ [Moved to new SOL 3.16]
- a) investigate congruence of plane figures after geometric transformations such as reflection ~~(flip)~~, translation ~~(slide),~~ and rotation ~~(turn)~~, using mirrors, paper folding, and tracing; and
- b) recognize the images of figures resulting from a geometric transformation~~s~~ such as translation ~~(slide)~~, reflection ~~(flip)~~, ~~or~~ and rotation ~~(turn)~~.

~~4.18~~ ~~The student will identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane.~~ [Moved to new SOL 6.11]

4.12 The student will
- a) define polygon; and
- b) identify polygons with 10 or fewer sides.

Probability and Statistics
(Focus: Outcomes and Congruent and Noncongruent Regions)

~~4.19~~ 4.13 The student will
- a) predict the likelihood of an outcome~~s~~ of a simple event~~,~~ ; and ~~using the terms *certain, likely, unlikely, impossible*~~ [Move to new SOL 2.18]
- ~~b) determine the probability~~ probabilities ~~of a given simple event, using concrete materials. geometric representations with congruent and noncongruent regions; and~~
- ~~c)~~ b) represent probability as a number between 0 and 1, inclusive.

~~4.20~~ 4.14 The student will collect, organize, ~~and~~ display, and interpret data ~~in line and bar graphs with scale increments of one or greater than one~~ [Move to Curriculum Framework] ~~and use the display to interpret the results, draw conclusions, and make predictions~~ from a variety of graphs .

Patterns, Functions, and Algebra
(Focus: Geometric Patterns, Equality, Properties)

~~4.21~~ 4.15 The student will recognize, create, and extend numerical and geometric patterns~~,~~. ~~using concrete materials, number lines, symbols, tables, and words~~. [Move to Curriculum Framework]

~~4.22~~ 4.16 The student will
- a) recognize and demonstrate the meaning of equality in an equation~~, using symbols representing numbers, operations, and relations [e.g., 3 + 5 = 5 +3 and 15 + (35 + 16) = (15 + 35) + 16]~~ [Move to Curriculum Framework]; and
- b) investigate and describe the associative property for addition and multiplication.

Grade Five

The fifth-grade standards place emphasis on ~~developing proficiency in using whole numbers~~ number sense with whole numbers, fractions, and decimals. ~~including~~ This focus includes concepts of prime and composite numbers, ~~finding common multiples and factors,~~ identifying even and odd numbers ~~with and without remainders~~ and solving problems using order of operations for positive whole numbers. Students will develop proficiency in the use of fractions~~,~~ and decimals to solve problems. Students will collect, display, and analyze data in a variety of ways and solve probability problems, using a sample space or tree diagram. Students also will solve problems involving volume, area, and perimeter. Students will be introduced to variable expressions and open sentences, and will model one-step linear equations in one variable using addition and subtraction. Students will investigate and recognize the distributive property. All of these skills ~~will~~ assist ~~to~~ in the development of the algebraic concepts needed for success in ~~pre-algebra at~~ the middle ~~level~~ grades.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
(Focus: Prime and Composite Numbers and Rounding Decimals)

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

~~a) read, write, and identify the place values of decimals through thousandths;~~

~~b) round decimal numbers to the nearest tenth or hundredth; and~~

~~c) compare the values of two decimals through thousandths, using the symbols >, <, or =.~~ [Moved to new SOL 4.3]

5.2 The student will

a) recognize and name ~~commonly used~~ fractions ~~(halves, fourths, fifths, eighths, and tenths)~~ [Move to Curriculum Framework] in their equivalent decimal form and vice versa; and

b) compare and order a given set of fractions and decimals from least to greatest and greatest to least. ~~Fractions will include like and unlike denominators limited to 12 or less, and mixed numbers.~~ [Move to Curriculum Framework]

5.3 The student will

~~a) find common multiples and factors, including least common multiple and greatest common factor;~~ [Moved to SOL 4.5a].

~~b)~~ a) identify and describe the characteristics of prime and composite numbers; and

~~c)~~ b) identify and describe the characteristics of even ~~numbers as having no remainder when divided by two~~ and odd numbers ~~as having a remainder of one when divided by two~~.

Computation and Estimation
(Focus: Multistep Applications and Order of Operations)

~~5.3~~ 5.4 The student will

~~a)~~ create and solve single and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers~~, using paper and pencil, estimation, mental computation, and calculators.~~[Move to Curriculum Framework]~~; and~~.

~~b) find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths.~~ [Move to new SOL 5.5 a]

~~5.4 The student will find the sum, difference, and product of two numbers expressed as decimals through thousandths,~~ [Moved to new SOL ~~5.4 b~~ 5.5 a] ~~using an appropriate method of calculation, including paper and pencil, estimation, mental computation, and calculators.~~ [Move to Curriculum Framework]

~~5.5 The student, given a dividend of four digits or fewer and a divisor of two digits or fewer, will find the quotient and remainder~~.

~~5.6~~ 5.5 The student~~,~~ will

a) find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths (divisors with only one nonzero digit); and

b) create and solve single and multistep practical problems involving decimals ~~given a dividend expressed as a decimal through thousandths and a single-digit divisor, will find the quotient~~.

~~5.7~~ 5.6 The student will solve single and multistep practical problems involving addition and subtraction with fractions and mixed numbers~~, with and without regrouping,~~ and express answers in simplest form. ~~Problems will include like and unlike denominators limited to 12 or less.~~ [Move to Curriculum Framework]

~~5.8~~ 5.7 The student will evaluate whole number numerical expressions ~~apply~~ using the ~~rules for the~~ order of operations limited to ~~positive whole numbers including~~ parentheses, addition, subtraction, multiplication, and division ~~to solve problems~~.

Measurement
(Focus: Perimeter, Area, Volume, and Equivalent Measures)

~~5.8~~ ~~5.9~~ 5.8 The student will

a) ~~describe and determine the~~ find perimeter, ~~of a polygon and the~~ area, and volume ~~of a square, rectangle, and right triangle, given the appropriate measures.~~ in standard units of measure;

b) differentiate between perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;

c) identify equivalent measurements within the metric system;

d) estimate and then measure to solve problems using U.S. Customary and metric units; and

e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.

~~5.9~~ ~~5.10~~ 5.9 The student will identify and describe the diameter, radius, chord, and circumference of a circle.

~~5.10~~ ~~The student will differentiate between perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.~~ [Moved to new SOL ~~5.9~~ 5.8b]

~~5.11~~ ~~The student will choose an appropriate measuring device and unit of measure to solve problems involving measurement of~~ [Moved to new SOL ~~5.9~~ 5.8e]

~~a) length part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;~~ [Moved to new SOL ~~5.9~~ 5.8d]

~~b) weight/mass ounces, pounds, tons, grams, and kilograms;~~ [Moved to new SOL ~~5.9~~ 5.8d]

~~c) liquid volume cups, pints, quarts, gallons, milliliters, and liters;~~ [Moved to new SOL ~~5.9~~ 5.8 d]

~~d) area square units; and~~ [Moved to new SOL ~~5.9~~ 5.8d]

~~e) temperature Celsius and Fahrenheit units.~~ [Moved to new SOL ~~5.9~~ 5.8d]

~~Problems also will include estimating the conversion of Celsius and Fahrenheit units relative to familiar situations (water freezes at 0°C and 32°F, water boils at 100°C and 212°F, normal body temperature is about 37°C and 98.6°F).~~ [Move to Curriculum Framework]

~~5.12~~ ~~5.11~~ 5.10 The student will determine an amount of elapsed time in hours and minutes within a 24-hour period.

~~5.13~~ ~~5.12~~ 5.11 The student will measure ~~and draw~~ right, acute, ~~and~~ obtuse, and straight angles ~~and triangles, using appropriate tools~~. [Move to Curriculum Framework]

Geometry
(Focus: Classification and Subdividing)

~~5.14~~ ~~5.13~~ 5.12 The student will classify
a) angles ~~and triangles~~ as right, acute, ~~or~~ obtuse, or straight; and
b) triangles as right, acute, obtuse, equilateral, scalene, or isosceles.

~~5.15~~ ~~5.14~~ 5.13 The student, using ~~two-dimensional (~~plane~~)~~ figures, (square, rectangle, triangle, parallelogram, rhombus, ~~kite,~~ and trapezoid) will
a) re~~cognize, identify, describe, and analyze their properties in order to~~ develop definitions of these plane figures; and
~~b)~~ ~~identify and explore congruent, noncongruent,~~ [Moved to new SOL 3.16] ~~and similar figures;~~ [Moved to new SOL 7.6]
~~c)~~ b) investigate and describe the results of combining and subdividing ~~shapes~~ plane figures~~;~~.
~~d)~~ ~~identify and describe a line of symmetry; and~~ [Moved to new SOL 2.15]
~~e)~~ ~~recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn).~~ [Moved to new SOL 4.11 b]

~~5.16~~ ~~The student will identify, compare, and analyze properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism).~~ [Moved to new SOL 3.14]

Probability and Statistics
(Focus: Outcomes and Measures of Center)

~~5.17~~ ~~5.15~~ 5.14 The student will
~~a)~~ make predictions and ~~solve problems involving~~ determine the probability of ~~a single event~~ an outcome ~~by using tree diagrams or~~ by constructing a sample space ~~representing all possible results; and~~.
~~b)~~ ~~predict the probability of outcomes of simple experiments, representing it with fractions or decimals from 0 to 1, and test the prediction;~~ [Moved to new SOL 4.13 c] ~~and~~
~~c)~~ ~~b)~~ ~~create a problem probability statement involving probability and based on information from a given problem situation. Students will not be required to solve the created problem statement. [Move to Curriculum Framework]~~

~~5.18~~ ~~5.16~~ 5.15 The student will, given a problem situation, collect, organize, and ~~display a set of numerical~~ interpret data in a variety of forms:, using ~~bar graphs,~~ stem-and-leaf plots~~,~~and line graphs~~, to draw conclusions and make predictions~~.

~~5.19~~ ~~5.17~~ 5.16 The student will
a) describe mean, median, and mode as measures of center;
~~d)~~ b) describe mean as fair share~~;~~;
~~b)~~ c) find the mean, median, mode, and range of a set of data; and
~~c)~~ d) describe the range of a set of data as a measure of variation.

Patterns, Functions, and Algebra
(Focus: Equations and Properties)

~~5.20~~ ~~5.18~~ 5.17 The student will ~~analyze the structure of numerical and geometric patterns (how they change or grow),~~ describe the relationship found in a number pattern~~s~~ ~~(how they change or grow)~~ and express the relationship~~,~~. ~~using words, tables, graphs, or a mathematical sentence~~. ~~Concrete materials and calculators will be used.~~ [Move to Curriculum Framework]

~~5.21~~ ~~5.19~~ 5.18 The student will

a) investigate and describe the concept of variable;

~~b) use a variable expression to represent a given verbal quantitative expression involving one operation; and~~

~~c)~~ b) write an open sentence to represent a given mathematical relationship, using a variable~~.~~;

c) model one-step linear equations in one variable using addition and subtraction; and

d) create a problem situation based on a given open sentence using a single variable.

~~5.22 The student will create a problem situation based on a given open sentence using a single variable.~~ [Moved to new SOL ~~5.19~~ 5.18 d]

~~5.20~~ 5.19 The student will investigate and recognize the distributive property of multiplication over addition.

Grade Six

~~The sixth-grade standards place continued emphasis on the study of whole numbers, decimals, and rational numbers (fractions). Students will use ratios to compare data sets; make conversions within a given measurement system; classify three-dimensional figures; collect, analyze, display, and interpret data, using a variety of graphical and statistical methods; begin using integers and percents; find the probability of an event; and investigate numerical and geometric patterns. Students will be introduced to algebraic terms and solving algebraic equations in one variable.~~

The sixth-grade standards are a transition from the emphasis placed on whole number arithmetic in the elementary grades to foundations of algebra. The standards emphasize rational numbers. Students will use ratios to compare data sets; recognize decimals, fractions and percents as ratios; solve single and multistep problems using rational numbers; and gain a foundation in the understanding of integers. Students will solve linear equations and use algebraic terminology. Students will solve problems involving area, perimeter and surface area, work with π (pi), and focus on the relationships among the properties of quadrilaterals. In addition, students will focus on applications of probability and statistics.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technolog~~ies~~y such as ~~fraction~~ calculators, computers, and spreadsheets~~, laser discs, and videos~~. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

(Focus: Relationships among Fractions, Decimals and Percents)

~~6.1~~ ~~The student will identify representations of a given percent and describe orally and in writing the equivalence relationships among fractions, decimals, and percents.~~
[Moved to new SOL 6.2 b and c]

~~6.2~~ 6.1 The student will describe and compare ~~two sets of~~ data, using ratios, and will use appropriate notations, such as ~~*a/b*~~ $\frac{a}{b}$, *a to b*, and *a:b*.

6.2 The student will
a) investigate and describe fractions, decimals and percents as ratios;
b) identify a given fraction, decimal or percent from a representation;
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
d) compare and order fractions, decimals, and percents.

~~6.3 The student will~~
~~a) find common multiples and factors, including least common multiple and greatest common factor;~~ [Moved to new SOL 4.5 a]
~~b) identify and describe prime and composite numbers; and identify and describe the characteristics of even and odd integers.~~ [Moved to new SOL 5.3]

~~6.4 The student will compare and order whole numbers,~~ [Moved to new SOL 3.1 c] ~~fractions, and decimals,~~ [Moved to new SOL 5.2 and new SOL 6.2] ~~using concrete materials, drawings or pictures, and mathematical symbols.~~ [Move to Curriculum Framework]

~~6.5~~ 6.3 The student will
a) identify~~,~~ and represent integers~~,~~ ;
b) order~~,~~ and compare integers~~.~~; and
c) identify and describe absolute value of integers.

6.4 The student will demonstrate multiple representations of multiplication and division of fractions.

6.5 The student will investigate and describe concepts of positive exponents and perfect squares.

Computation and Estimation
(Focus: Applications of Operations with Rational Numbers)

6.6 The student will:
a) ~~solve problems that involve addition, subtraction, multiplication,~~ multiply and~~/or division~~ divide fractions and mixed numbers~~, with and without regrouping, that include like and unlike denominators of 12 or less, and express their answers in simplest form~~ [Rewritten and moved to new SOL 6.6 b]; and
b) ~~find the quotient, given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit~~ estimate solutions and then solve single and multistep practical problems that involve addition, subtraction, multiplication and division of fractions.

6.7 The student will ~~use estimation strategies to~~ solve single and multistep practical problems involving ~~whole numbers, decimals, and fractions (rational numbers).~~ addition, subtraction, multiplication and division of decimals. [Fractions moved to new SOL 6.6 a; whole numbers moved to new SOL 5.4]

~~6.8~~ ~~The student will solve multistep consumer-application problems involving fractions and decimals~~ [Moved to new SOL 6.7] ~~and present data and conclusions in paragraphs, tables, or graphs. Planning a budget will be included~~. [Move to Curriculum Framework]

6.8 The student will ~~apply order of operations in solving equations.~~ evaluate whole number numerical expressions using the order of operations .

Measurement
(Focus: Problem Solving with Area, Perimeter, Volume and Surface Area)

6.9 The student will ~~compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system and estimate conversions between units in each system:~~

~~a) length — part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;~~
~~b) weight/mass — ounces, pounds, tons, grams, and kilograms;~~
~~c) liquid volume — cups, pints, quarts, gallons, milliliters, and liters; and~~
~~d) area — square units. *~~ [Move to Curriculum Framework]

~~* The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units~~. make ballpark comparisons between the U.S. Customary System of measurement and the metric system.

~~6.10~~ ~~The student will estimate and then determine length, weight/mass, area, and liquid volume/capacity, using standard and nonstandard units of measure.~~ [Moved to new SOL 5.8]

6.10 The student will

a) define pi (π) as the ratio of the circumference of a circle to its diameter;
b) solve practical problems involving circumference and area of a circle given the diameter or radius;
c) solve practical problems involving area and perimeter; and
d) describe and determine the volume and surface area of a rectangular prism.

~~6.11~~ ~~The student will determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area and apply the appropriate formula.~~ [Moved to new SOL 5.8 b]

~~6.12~~ ~~The student will~~

~~a) solve problems involving the circumference and/or area of a circle when given the diameter or radius; and~~ [Moved to new SOL 6.10 b]
~~b) derive approximations for pi (π) from measurements for circumference and diameter,~~ [Moved to new SOL 6.10 a] ~~using concrete materials or computer models.~~ [Move to Curriculum Framework]

~~6.13~~ ~~The student will~~
- ~~a) estimate angle measures, using 45°, 90°, and 180° as referents, and use the appropriate tools to measure the given angles; and~~
- ~~b) measure and draw right, acute, and obtuse angles and triangles.~~ [Moved to new SOL 5.11 and new SOL 5.12.]

Geometry
(Focus: Properties and Relationships)

6.11 The student will
- a) identify the coordinates of a point in a coordinate plane; and
- b) graph ordered pairs in a coordinate plane.

~~6.14~~ ~~The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties~~. [Moved to new SOL 6.13]

~~6.15~~ 6.12 The student will determine congruence of segments, angles, and polygons ~~by direct comparison, given their attributes. Examples of noncongruent and congruent figures will be included~~. [Move to Curriculum Framework]

~~6.16~~ ~~The student will construct the perpendicular bisector of a line segment and an angle bisector.~~

~~6.17~~ ~~The student will sketch, construct models of, and classify solid figures (rectangular prism, cone, cylinder, and pyramid).~~

6.13 The student will describe and identify properties of quadrilaterals.

Probability and Statistics
(Focus: Practical Applications of Statistics)

~~6.18~~ 6.14 The student, given a problem situation, will ~~collect, analyze, display, and interpret data in a variety of graphical methods, including~~
- a) ~~line,~~ ~~bar, and~~ [Moved to new SOL 5.15] construct circle graphs; ~~and histograms~~
- b) ~~stem-and-leaf plots;~~ [Moved to new SOL 5.15] ~~and~~ draw conclusions and make predictions using circle graphs ~~and histograms~~; and
- c) ~~box-and-whisker plots.~~ [Moved to A.10] compare and contrast graphs which present ~~the same~~ information from the same data set.

Circle graphs will be limited to halves, fourths, and eighths. [Move to Curriculum Framework]

~~6.19~~ 6.15 The student will
- a) describe ~~the~~ mean~~, median, and mode~~ as ~~measures of central tendency~~ [Moved to new SOL 5.16]~~,~~ balance point; and
- b) ~~describe the range, and determine their meaning for a set of data.~~ [Moved to new SOL 5.16] decide which measure of center is appropriate for a given ~~situation~~ purpose.

~~6.20~~ 6.16 The student will

a) ~~make a sample space for selected experiments and represent it in the form of a list, chart, picture, or tree diagram;~~ [Moved to new SOL 5.14] ~~distinguish between~~ compare and contrast dependent and independent events; and

b) determine ~~and interpret the probability of an event occurring from a given sample space and represent the probability as a ratio, decimal or percent, as appropriate for the given situation.~~ probabilities for dependent and independent events.

Patterns, Functions, and Algebra
(Focus: Variable Equations and Properties)

~~6.21~~ 6.17 The student will ~~investigate, describe,~~ identify and extend ~~numerical and~~ geometric ~~patterns, including triangular numbers, patterns formed by powers of 10,~~ and arithmetic sequences. [Move to Curriculum Framework]

~~6.22~~ ~~The student will investigate and describe concepts of positive exponents, perfect squares,~~ [Moved to new SOL 6.5] ~~square roots, and, for numbers greater than 10, scientific notation.~~ [Moved to new SOL 7.1] ~~Calculators will be used to develop exponential patterns~~. [Move to Curriculum Framework]

~~6.23~~ 6.18 The student will

~~a)~~ ~~model and solve algebraic equations, using concrete materials;~~ [Moved to new SOL 5.18]

~~b)~~ solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions~~; and~~ .

~~c)~~ ~~use the following algebraic terms appropriately: *variable, coefficient, term*, and *equation*.~~

6.19 The student will investigate and recognize

a) the identity properties for addition and multiplication;

b) the multiplicative property of zero; and

c) the inverse ~~properties~~ property for ~~addition and~~ multiplication.

6.20 The student will graph inequalities on a number line.

Grade Seven

~~The seventh-grade standards place emphasis on solving problems involving consumer applications, using proportional reasoning, and gaining proficiency in computations with integers. The students will gain an understanding of the properties of real numbers, solve one-step linear equations and inequalities, and use data analysis techniques to make inferences, conjectures, and predictions. Two- and three-dimensional representations, graphing transformations in the coordinate plane, and probability will be extended.~~

The seventh-grade standards continue to emphasize the foundations of algebra. Students who successfully complete the seventh-grade standards should be prepared to study Algebra I in grade eight. Topics in grade seven include proportional reasoning, integer computation, solving two-step linear equations, and recognizing different representations for relationships. Students will apply the properties of real numbers in solving equations, solve inequalities, and use data analysis techniques to make inferences, conjectures, and predictions.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technology~~ies~~ such as ~~fraction~~ calculators, computers, and spreadsheets~~, laser discs, and videos~~. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
(Focus: ~~Scientific Notation and Square Roots~~ Proportional Reasoning)

7.1 The student will
- a) investigate and describe the concept of negative exponents for powers of ten; ~~compare, order, and determine equivalent relationships between fractions, decimals, and percents,~~
- b) ~~including use of~~ determine scientific notation for numbers greater than ~~10~~ zero;
- c) compare and order fractions, decimals, percents and numbers written in scientific notation;
- d) determine square roots; and
- e) identify and describe absolute value for rational numbers.

~~7.2~~ ~~The student will simplify expressions that contain rational numbers (whole numbers, fractions, and decimals) and positive exponents, using order of operations,~~ [Moved to new SOL 6.8] ~~mental mathematics, and appropriate tools.~~ [Move to Curriculum Framework]

~~7.3~~ ~~The student will identify and apply the following properties of operations with real numbers:~~
- ~~a) the commutative and associative properties for addition and multiplication;~~ [Moved to new SOL 3.20 a and new SOL 4.16]
- ~~b) the distributive property;~~ [Moved to new SOL 5.19]
- ~~c) the additive and multiplicative identity properties;~~ [Moved to new SOL 6.19 a]
- ~~d) the additive and multiplicative inverse properties; and~~ [Moved to new SOL 6.19 c]
- ~~e) the multiplicative property of zero~~. [Moved to new SOL 6.19 b]

7.2 The student will describe and represent arithmetic and geometric sequences using variable expressions.

Computation and Estimation
(Focus: ~~Application of Rational Number~~ Integer Operations and Proportional Reasoning)

~~7.4~~ ~~The student will~~
- ~~a) solve practical problems using rational numbers (whole numbers, fractions, decimals) and percents; and~~ [Moved to new SOL 7.4]
- ~~b) solve consumer application problems involving tips, discounts, sales tax, and simple interest.~~ [Move to Curriculum Framework]

~~7.5~~ ~~7.2~~ 7.3 The student will

a) ~~formulate rules for~~ model addition, subtraction, multiplication and division of integers; and

b) ~~solve practical problems involving basic operations (addition, subtraction, multiplication, and division) with integers.~~ [Moved to new SOL 7.4] add, subtract, multiply, and divide integers.

~~7.6~~ ~~The student will use proportions to solve practical problems, which may include scale drawings, that contain rational numbers (whole numbers, fractions, and decimals) and percents~~. [Move to Curriculum Framework to support new SOL 7.4]

~~7.3~~ 7.4 The student will solve single and multistep practical problems using ~~rational numbers~~ proportional reasoning.

Measurement
(Focus: ~~Volume and Surface Area~~ Proportional Reasoning)

~~7.7~~ ~~The student, given appropriate dimensions, will~~

~~a)~~ ~~estimate and find the area of polygons by subdividing them into rectangles and right triangles;~~ and [Moved to new SOL 8.11]

~~b)~~ ~~apply perimeter and area formulas in practical situations.~~ [Moved to new SOL 6.10 c]

~~7.8~~ ~~7.4~~ 7.5 The student will

a) ~~investigate and~~ describe volume and surface area of cylinders; ~~and~~

b) solve practical problems involving the volume and surface area of ~~rectangular prisms and~~ rectangular prisms and [Moved to SOL 6.10 d] cylinders~~,~~ ; and ~~using concrete materials and practical situations to develop formulas~~. [Move to Curriculum Framework]

c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.

7.6 The student will determine if plane figures – quadrilaterals and triangles – are similar and write proportions to express the relationships between corresponding parts of similar figures.

Geometry
(Focus: Relationships Between Figures)

~~7.9~~ ~~7.5~~ 7.7 The student will compare and contrast the following quadrilaterals based on properties: parallelogram, rectangle, square, rhombus, and trapezoid. ~~Deductive reasoning and inference will be used to classify quadrilaterals.~~ [Move to Curriculum Framework]

~~7.10~~ ~~The student will identify and draw the following polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon.~~ [Moved to new SOL 4.12]

~~7.11~~ ~~7.6~~ ~~The student will determine if geometric~~ plane figures (quadrilaterals and ~~triangles) are similar and write proportions to express the relationships between corresponding parts of similar figures.~~ [Moved to new SOL 7.6]

~~7.12~~ ~~The student will identify and graph ordered pairs in the four quadrants of a coordinate plane.~~ [Moved to new SOL 6.11]

~~7.13~~ ~~7.7~~ 7.8 The student, given a polygon in the coordinate plane, will represent transformations (reflections, dilations, rotations, and translations) by graphing ~~the coordinates of the vertices of the transformed polygon and sketching the resulting figure~~ in the coordinate plane.

Probability and Statistics
(Focus: Applications of Statistics and Probability)

~~7.14~~ ~~7.8~~ 7.9 The student will investigate and describe the difference between the ~~probability of an event found through simulation versus~~ experimental and theoretical probability of ~~that same~~ an event.

~~7.15~~ ~~7.9~~ 7.10 The student will ~~identify and describe the number of possible arrangements of several objects, using a tree diagram or~~ [Moved to new SOL 5.14] determine the probability of compound events using the Fundamental (Basic) Counting Principle.

~~7.16~~ ~~The student will create and solve problems involving the measures of central tendency (mean, median, mode) and the range of a set of data.~~

~~7.17~~ ~~7.10~~ 7.11 The student, given data in a ~~problem~~ practical situation, will ~~collect, analyze, display, and interpret data, using a variety of graphical methods, including~~

a) ~~frequency distributions;~~ construct and analyze ~~box-and-whisker plots~~ histograms; and
b) ~~line plots;~~ compare and contrast ~~data presented in box-and-whisker plots~~ histograms ~~and~~ with other types of graphs presenting information from the same data set.
~~c)~~ ~~histograms;~~ [Moved to new SOL 7.11 a, b]
~~d)~~ ~~stem-and-leaf plots;~~ [Moved to new SOL 5.15]
~~e)~~ ~~box-and-whisker plots;~~ ~~and~~ [Moved to new SOL A.10]
~~f)~~ ~~scattergrams~~ [Move to Curriculum Framework to support new SOL A.11]

~~7.18~~ ~~The student will make inferences, conjectures, and predictions based on analysis of a set of data.~~ [Moved to new SOL 7.11]

Patterns, Functions, and Algebra
(Focus: Linear Equations)

~~7.19~~ ~~7.11~~ 7.12 The student will represent~~, analyze, and generalize a variety of patterns, including arithmetic sequences and geometric sequences,~~ relationships with tables, graphs, rules, and words. ~~in order to investigate and describe functional relationships.~~

~~7.20~~ ~~7.12~~ 7.13 The student will
- a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and
- b) evaluate algebraic expressions for given replacement values of the variables.

~~7.21~~ ~~The student will use the following algebraic terms appropriately: *equation, inequality,* and *expression.*~~

~~7.22~~ ~~7.13~~ 7.14 The student will
- a) solve one- ~~step~~ and two-step linear equations ~~and inequalities~~ [Moved to new SOL 7.15] in one variable ~~with strategies involving inverse operations and integers, using concrete materials, pictorial representations, and paper and pencil~~ [Move to Curriculum Framework]; and
- b) solve practical problems requiring the solution of ~~a~~one- ~~or~~ and two-step linear equations.

7.14 7.15 The student will
- a) solve one-step inequalities in one variable; and
- b) graph solutions to inequalities on the number line.

~~7.15~~ 7.16 The student will ~~identify and~~ apply the following properties of operations with real numbers:
- a) the commutative and associative properties for addition and multiplication;
- b) the distributive property;
- c) the additive and multiplicative identity properties;
- d) the additive and multiplicative inverse properties; and
- e) the multiplicative property of zero.

Grade Eight

The eighth-grade standards are intended to serve two purposes. First, the standards contain ~~both~~ content that reviews or extends concepts and skills learned in previous grades. ~~and~~ Second, they contain new content that prepares students for more abstract concepts in algebra and geometry. The eighth-grade standards provide students additional instruction and time to acquire the concepts and skills necessary for success in Algebra I. Students will gain proficiency in computation with rational numbers ~~(positive and negative fractions, positive and negative decimals, whole numbers, and integers)~~ and use proportions to solve a variety of problems. New concepts include solving ~~two-step~~ multistep equations and inequalities, graphing linear equations, visualizing three-dimensional shapes represented in two-dimensional drawings, and applying transformations to geometric shapes in the coordinate plane~~, and using matrices to organize and interpret data~~. Students will verify and apply the Pythagorean Theorem and represent relations and functions using tables, graphs, and rules. The eighth-grade standards provide a more solid foundation in Algebra I for those students not ready for Algebra I in grade 8.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies ~~such as fraction calculators, computers, spreadsheets, laser discs, and videos~~. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense
(Focus: Relationships within the Real Number System)

8.1 The student will
a) simplify numerical expressions involving positive ~~and negative~~ exponents, using rational numbers, order of operations, and properties of operations with real numbers; and
~~b)~~ ~~recognize, represent, compare, and order numbers expressed in scientific notation; and~~ [Moved to new SOL 7.1 c]
~~c)~~ b) compare and order decimals, fractions, percents, and numbers written in scientific notation.

8.2 The student will describe orally and in writing the relationship between the subsets of the real number system.

Computation and Estimation
(Focus: Practical Applications of Operations with Real Numbers)

8.3 The student will
a) solve practical problems involving rational numbers, percents, ratios, and proportions~~.~~; and ~~Problems will be of varying complexities and will involve real-life data, such as finding a discount and discount prices and balancing a checkbook.~~ [Move to Curriculum Framework]
b) determine the percent increase or decrease for a given situation.

~~8.4~~ ~~The student will determine the percent increase or decrease for a given situation.~~ [Moved to new SOL 8.3]

~~8.4~~ ~~8.5~~ 8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables. ~~Problems will be limited to positive exponents.~~ [Move to Curriculum Framework]

~~8.5~~ ~~8.6~~ 8.5 The student~~, given a whole number from 0 to 100,~~ will
a) ~~identify~~ determine whether a given ~~whole~~ number ~~it~~ ~~as~~ is a perfect square; and
b) ~~or~~ find the two consecutive whole numbers between which ~~the~~ a square root lies.

Measurement
(Focus: Problem Solving)

~~8.6~~ ~~8.7~~ 8.6 The student will
a) verify by measuring and describe the relationships among vertical angles, adjacent angles, supplementary angles, and complementary angles; and
b) ~~will~~ measure ~~and draw~~ angles of less than 360°.

~~8.7~~ ~~8.8~~ 8.7 The student will

a) investigate and solve practical problems involving volume and surface area of ~~rectangular solids (~~prisms~~)~~, cylinders, cones, and pyramids~~.~~ ; and

b) describe how changing one measured attribute of the figure affects the volume and surface area.

Geometry
(Focus: Problem Solving with 2- and 3-Dimensional Figures)

~~8.8~~ ~~8.9~~ 8.8 The student will

a) apply transformations ~~(rotate or turn, reflect or flip, translate or slide, and dilate or scale)~~ [Move to Curriculum Framework] to ~~geometric~~ plane figures; and ~~represented on graph paper. The student will~~

b) identify applications of transformations~~, such as tiling, fabric design, art, and scaling~~. [Move to Curriculum Framework]

~~8.9~~ ~~8.10~~ 8.9 The student will construct a three-dimensional model, given the top~~, side, and/~~or bottom ~~views.~~ , side and front views.

~~8.10~~ ~~8.11~~ 8.10 The student will

a) verify the Pythagorean Theorem~~, using diagrams, concrete materials, and measurement~~; and [Move to Curriculum Framework]

b) apply the Pythagorean Theorem ~~to find the missing length of a side of a right triangle when given the lengths of the other two sides~~. [Move to Curriculum Framework]

~~8.12~~ 8.11 The student will solve practical area and perimeter problems involving composite, plane figures.

Probability and Statistics
(Focus: Statistical Analysis of Graphs and Problem Situations)

~~8.11~~ ~~8.13~~ 8.12 The student will ~~analyze problem situations, including games of chance, board games, or grading scales,~~ [Move to Curriculum Framework] ~~and make predictions, using knowledge of probability.~~ determine the probability of independent and dependent events with and without replacement.

~~8.12~~ ~~8.14~~ 8.13 The student will

a) make comparisons, predictions, and inferences, using information displayed in ~~frequency distributions; box-and-whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms.~~ graphs~~.~~ [Move to Curriculum Framework] ; and

b) construct and analyze scatterplots.

~~8.13 The student will use a matrix to organize and describe data.~~

Patterns, Functions, and Algebra
(Focus: Linear Relationships)

~~8.14~~ ~~8.15~~ 8.14 The student will

a) ~~describe and represent a given relationship~~ ~~relations and functions, using in tables, graphs, word and rules~~ form; and

b) ~~relate and compare tables, graphs, and rules as different forms of representation for relationships.~~ make connections between any two forms (tables, graphs, word, and rules) of a given relationship.

~~8.15~~ ~~8.16~~ 8.15 The student will

a) solve ~~two-~~ multistep linear equations ~~and inequalities in one with~~ in one variable~~s,~~ on one ~~or~~ and two sides of the equation ~~using concrete materials, pictorial representations, and paper and pencil.~~ ; [Move to Curriculum Framework]

b) solve two-step linear inequalities and graph the results on a number line; and

c) identify properties of operations used to solve an equation.

~~8.16~~ ~~8.17~~ 8.16 The student will graph a linear equation in two variables~~, in the coordinate plane, using a table of ordered pairs~~.

~~8.17~~ ~~The student will create and solve problems, using proportions, formulas, and functions.~~ [Moved to new SOL 8.3]

~~8.18~~ 8.17 The student will ~~use the following algebraic terms appropriately: *domain, range, independent variable,* and *dependent variable*.~~ identify the domain, range, independent variable or dependent variable in a given situation.

Algebra I

The standards below outline the content for a one-year course in Algebra I. All students are expected to achieve the Algebra I standards. When planning for instruction, consideration will be given to the sequential development of concepts and skills by using concrete materials to assist students in making the transition from the arithmetic to the symbolic. Students should be helped to make connections and build relationships between algebra and arithmetic, geometry, and probability and statistics. Connections also should be made to other subject areas through practical applications. This approach to teaching algebra should help students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze functions behavior. ~~Matrices will be used to organize and manipulate data.~~

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of functions; they provide a powerful tool for solving and verifying solutions to equations and inequalities.

Throughout the course, students should be encouraged to ~~talk~~ engage in discourse about mathematics with teachers and other students, use the language and symbols of mathematics in representations and communication, discuss problems and problem solving, and develop ~~their~~ confidence in themselves as mathematics students.

Expressions and Operations

A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.

~~A.2~~ ~~The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.~~ [Moved to new SOL A.1] ~~Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.~~ [Move to Curriculum Framework]

A.2 The student will perform operations on polynomials, including

a) applying the laws of exponents to perform operations on expressions;
b) adding, subtracting, multiplying, and dividing polynomials; and
c) factoring completely first- and second-degree binomials and trinomials in one or two variables. The graphing calculator will be used as a tool for factoring and for confirming algebraic factorizations.

~~A.10~~ ~~The student will apply the laws of exponents to perform operations on expressions~~ [Moved to new SOL A.2 a] ~~with integral exponents, using scientific notation when appropriate.~~ [Move to Curriculum Framework]

~~A.11~~ ~~The student will add, subtract, and multiply polynomials and divide polynomials~~ [Moved to new SOL A.2 b] ~~with monomial divisors using concrete objects, pictorial and area representations, and algebraic manipulations.~~ [Move to Curriculum Framework]

~~A.12~~ ~~The student will factor completely first- and second-degree binomials and trinomials in one or two variables.~~ [Moved to new SOL A.2 c] ~~The graphing calculator will be used as a tool for factoring and for confirming algebraic factorizations.~~ ~~[Move to Curriculum Framework]~~ [Moved to new SOL A.2 c]

~~A.13~~ A.3 The student will express the square root and cube root of whole numbers and the square root of a monomial algebraic expression in simplest radical form. ~~and approximate square roots to the nearest tenth.~~

Equations and Inequalities

A.4 The student will solve multistep linear and quadratic equations in ~~and inequalities in one~~ two variables, including

a) solving literal equations (formulas) for a given variable~~, and~~ ;
b) justifying steps used in simplifying expressions and solving equations and inequalities using field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets;
c) solving quadratic equations ~~in one variable both~~ algebraically and graphically;
d) solving multistep linear equations algebraically and graphically;
e) solving systems of two linear equations in two variables ~~both~~ algebraically and graphically; and
f) ~~apply~~ solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.5 The student will solve multistep linear ~~equations and~~ inequalities in ~~one~~ two variables, including

a) solving multistep linear inequalities algebraically and graphically; ~~and~~
b) solving real-world problems involving inequalities; and
c) solving systems of inequalities.

~~A.1~~ ~~The student will solve multistep linear equations~~ [Moved to new SOL A.4 d] ~~and inequalities in one variable,~~ [Moved to new SOL A.5] ~~solve literal equations (formulas) for a given variable,~~ [Moved to new SOL A.4 a] ~~and apply these skills to solve practical problems.~~ [Moved to new SOL A.4 f] ~~Graphing calculators will be used to confirm algebraic solutions.~~ ~~[Move to Curriculum Framework]~~ [Moved to new SOL A.4]

~~A.3~~ ~~The student will justify steps used in simplifying expressions and solving equations and inequalities.~~ [Moved to new SOL A.4 b] ~~Justifications will include the use of concrete objects; pictorial representations; and the properties of real numbers, equality, and inequality.~~ [Move to Curriculum Framework]

~~A.9~~ ~~The student will solve systems of two linear equations in two variables both algebraically and graphically and apply these techniques to solve practical problems.~~ [Moved to new SOL A.4 e] ~~Graphing calculators will be used both as a primary tool for solution and to confirm an algebraic solution. [Move to Curriculum Framework]~~ [Moved to new SOL A.4]

~~A.14~~ ~~The student will solve quadratic equations in one variable both algebraically and graphically.~~ [Moved to new SOL A.4 c] ~~Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions. [Move to Curriculum Framework]~~ [Moved to new SOL A.4]

A.6 The student will ~~select, justify, and apply an appropriate technique to~~ graph linear ~~functions~~ equations and linear inequalities in two variables~~. Techniques will include slope-intercept, *x*- and *y*-intercepts, graphing by transformation, and the use of the graphing calculator~~.[Move to Curriculum Framework], including

a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined;
b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and
c) graphing linear functions and inequalities in two variables.

~~A.6~~ ~~The student will~~ [Moved to new SOL A.6], ~~select justify, and apply an appropriate technique to~~ [Move to Curriculum Framework] ~~graph linear functions and linear inequalities in two variables.~~ [Moved to new SOL A.6 c] ~~Techniques will include slope-intercept, *x*- and *y*-intercepts, graphing by transformation, and the use of the graphing calculator.~~ [Move to Curriculum Framework]

~~A.7~~ ~~The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined.~~ [Moved to new SOL A.6 a] ~~The graphing calculator will be used to investigate the effect of changes in the slope on the graph of the line.~~ [Move to Curriculum Framework]

~~A.8~~ ~~The student will write an equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.~~ [Moved to new SOL A.6 b]

Functions

A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including
- a) determining whether a relation is a function;
- b) domain and range;
- c) zeros of a function;
- d) x- and y-intercepts;
- ~~e) intervals in which the function is increasing/decreasing;~~
- ~~f)~~ e) finding the values of a function for elements in its domain; and
- ~~g)~~ f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

~~A.5~~ ~~The student will create and use tabular, symbolic, graphical, verbal, and physical representations~~ [Moved to new SOL A.7 ~~g~~ f] ~~to analyze a given set of data for the existence of a pattern, determine the domain and range of relations,~~ [Moved to new SOL A.7 b.] ~~and identify the relations that are functions.~~ [Moved to new SOL A.7 a.]

~~A.15~~ ~~The student will, given a rule, find the values of a function for elements in its domain~~ [Moved to new SOL A.7 f.] ~~and locate the zeros of the function~~ [Moved to new SOL A.7 c.] ~~both algebraically and with a graphing calculator. The value of *f(x)* will be related to the ordinate on the graph.~~ [Move to Curriculum Framework]

~~A.18~~ A.8 The student will, given a situation in a real-world context, analyze a relation to determine whether a direct or inverse variation exists, ~~and~~ represent a direct variation ~~it~~ algebraically and graphically~~. if possible.~~, and an inverse variation algebraically.

Statistics

~~A.4~~ ~~The student will use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication. Data will arise from business, industrial, and consumer situations.~~

~~A.17~~ A.9 The student will, given a set of data, ~~compare and contrast multiple one-variable data sets, using statistical techniques that include measures of central tendency and range~~, interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.

A.10 The student will compare and contrast multiple univariate data sets using box-and-whisker plots.

~~A.16~~ A.11 The student will~~, given a set of data points, write an equation for a line~~ collect and analyze data, determine the equation of the curve of best fit ~~use the equation~~ in order to make predictions~~.~~, and solve real-world problems using mathematical models. Mathematical models will include linear and quadratic functions.

Geometry

This course is designed for students who have successfully completed the standards for Algebra I. All students are expected to achieve the Geometry standards. The course includes, among other things, properties of geometric figures, trigonometric relationships, and reasoning to justify conclusions. Methods of justification will include paragraph proofs, two-column proofs, indirect proofs, coordinate proofs, algebraic methods, and verbal arguments. A gradual development of formal proof is encouraged. Inductive and intuitive approaches to proof as well as deductive axiomatic methods should be used.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques, including algebraic skills, should be used to implement these standards. Calculators, computers, graphing utilities (graphing calculators or computer graphing simulators), dynamic geometry software, and other appropriate technology tools will be used to assist in teaching and learning. Any technology that will enhance student learning should be used.

Reasoning, Lines, and Transformations

G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
- a) identifying the converse, inverse, and contrapositive of a conditional statement;
- b) translating a short verbal argument into symbolic form;
- c) using Venn diagrams to represent set relationships; and
- d) using deductive reasoning~~, including the law of syllogism.~~ [Move to Curriculum Framework]

~~G.4~~ G.2 The student will use the relationships between angles formed by two lines cut by a transversal to
- a) determine if two lines are parallel; ~~and~~
- b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs~~.~~; and
- c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

~~G.3 The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles,~~ [Moved to new SOL 8.7 a] ~~angles formed when parallel lines are cut by a transversal,~~ [Moved to new SOL G.2] ~~and angles in polygons.~~ [Moved to new SOL G.10]

~~G.2~~ G.3 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include

a) investigating and using formulas for finding distance, midpoint, and slope;
b) applying slope to verify and determine if lines are parallel or perpendicular;
~~b)~~ c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
~~c)~~ d) determining whether a figure has been translated, reflected, ~~or~~ rotated, or dilated using coordinate methods.

~~G.11~~ G.4 The student will construct and justify the constructions of

a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line ~~at~~ through a given point on the line;
e) the bisector of a given angle~~, and~~;
f) an angle congruent to a given angle; and
g) a line parallel to a given line through a point not on the given line.

Triangles

~~G.6~~ G.5 The student ~~will~~, given information concerning the lengths of sides and/or measures of angles in triangles, will ~~apply the triangle inequality properties to determine whether a triangle exists and to order sides and angles.~~

a) order the sides by length, given the angle measures;
b) order the angles by degree measure, given the side lengths;
c) determine whether a triangle exists; and
d) determine the range in which the length of the third side must lie.

These concepts will be considered in the context of ~~practical~~ real-world situations.

~~G.5~~ G.6 The student will

~~a) investigate and identify congruence and similarity relationships between triangles; and~~
~~b)~~ prove two triangles are congruent ~~or similar,~~ [Moved to new SOL G.7] given information in the form of a figure or statement, using algebraic and coordinate methods as well as deductive proofs.

G.7 The student will prove two triangles are similar given information in the form of a figure or statement, using algebraic and coordinate methods as well as deductive proofs.

~~G.7~~ G.8 The student will solve ~~practical~~ real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. ~~Solutions will be expressed in radical form or as decimal approximations.~~ [Move to Curriculum Framework]

Polygons and Circles

~~G.8~~ G.9 The student will
- ~~a)~~ ~~investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;~~
- ~~b)~~ ~~prove~~ verify ~~these~~ ~~properties~~ characteristics of quadrilaterals~~, using algebraic and coordinate methods as well as deductive reasoning;~~ [Move to Curriculum Framework] and
- ~~c)~~ use properties of quadrilaterals to solve ~~practical~~ real-world problems.

G.10 The student will solve real-world problems involving angles of polygons.

~~G.9~~ ~~The student will use measures of interior and exterior angles of polygons to solve problems.~~ [Moved to new SOL G.10] ~~Tessellations and tiling problems will be used to make connections to art, construction, and nature.~~ [Move to Curriculum Framework]

G.11 The student will use angles, arcs, chords, tangents, and secants to
- a) investigate, ~~prove~~ verify, and apply properties of circles;
- b) solve real-world problems involving properties of circles; and
- c) find arc length and areas of sectors in circles.

G.12 The student will, given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.

~~G.10~~ ~~The student will investigate and solve~~ ~~practical~~ ~~problems using properties of angles, arcs, chords, tangents, and secants~~. ~~Problems will include finding arc length and the area of a sector,~~ ~~and may be drawn from applications of architecture, art, and construction.~~ [Move to Curriculum Framework]

Three-Dimensional Figures

~~G.12~~ ~~The student will make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object. Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations.~~

~~G.13~~ ~~G.12~~ G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve ~~practical~~ real-world problems. ~~Calculators will be used to find decimal approximations for results.~~ [Move to Curriculum Framework]

~~G.14~~ ~~G.13~~ G.14 The student will use similar geometric objects in two- or three-dimensions to

a) ~~use proportional reasoning to solve practical problems, given similar geometric objects; and~~ compare ratios between side lengths, perimeters, areas, and volumes;

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object; ~~and~~

c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and

~~c)~~ d) solve real-world problems about similar geometric objects.

Algebra, Functions, and Data Analysis

[Adopted by the Virginia Board of Education on June 28, 2007]

The following standards outline the content for a one-year course in Algebra, Functions, and Data Analysis. All students who are pursuing a technical field are expected to achieve the Algebra, Functions, and Data Analysis or Algebra II standards. This course is designed for students who have successfully completed the standards for Algebra I. Within the context of mathematical modeling and data analysis, students will study functions and their behaviors, systems of inequalities, probability, experimental design and implementation, and analysis of data. Data will be generated by practical applications arising from science, business, and finance. Students will solve problems that require the formulation of linear, quadratic, exponential, or logarithmic equations or a system of equations.

Through the investigation of mathematical models and interpretation/analysis of data from real-life situations, students will strengthen conceptual understandings in mathematics and further develop connections between algebra and statistics. Students should use the language and symbols of mathematics in representations and communication throughout the course.

These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

The infusion of technology (graphing calculator and/or computer software) in this course will assist in modeling and investigating functions and data analysis.

Algebra and Functions

AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:

a) continuity;
b) local and absolute maxima and minima;
c) domain and range;
d) zeros;
e) intercepts;
f) intervals in which the function is increasing/decreasing;
g) end behaviors; and
h) asymptotes.

AFDA.2 The student will use knowledge of transformations to write an equation given the graph of a function (linear, quadratic, exponential, and logarithmic).

AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions including algebraic formulae, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

AFDA.5 The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Data Analysis

AFDA.6 The student will calculate probabilities. Key concepts include:
a) conditional probability;
b) dependent and independent events;
c) addition and multiplication rules;
d) counting techniques (permutations and combinations); and
e) Law of Large Numbers.

AFDA.7 The student will analyze the normal distribution. Key concepts include:
a) characteristics of normally distributed data;
b) percentiles;
c) normalizing data using z-scores; and
d) area under the standard normal curve and probability.

AFDA.8 The student will design and conduct an experiment/survey. Key concepts include:
a) sample size;
b) sampling technique;
c) controlling sources of bias and experimental error;
d) data collection; and
e) data analysis and reporting.

Algebra II

The standards below outline the content for a one-year course in Algebra II. Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. All students preparing for postsecondary and advanced technical studies are expected to achieve the Algebra II standards. A thorough treatment of advanced algebraic concepts is provided through the study of functions, "families of functions," equations, inequalities, systems of equations and inequalities, polynomials, rational and radical ~~expressions~~ equations, complex numbers, ~~matrices,~~ and sequences and series. Emphasis will be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results also should permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a "family of graphs" from a given graph and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (graphing calculators or computer graphing simulators), computers, spreadsheets, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions. They also provide an effective tool for solving/verifying equations and inequalities. Any other available technology that will enhance student learning should be used.

Expressions and Operations

~~AII.1~~ ~~The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets,~~ [Moved to new SOL A.4 b] ~~complex numbers,~~ [Moved to new SOL AII.3] ~~and matrices~~.

~~AII.2~~ AII.1 The student will~~, given rational, radical, or polynomial expressions,~~

a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
b) add, subtract, multiply, divide, and simplify radical expressions containing ~~positive~~ rational numbers and variables, and expressions containing rational exponents;
c) write radical expressions as expressions containing rational exponents and vice versa; and
d) factor ~~completely~~ polynomials completely.

~~AII.2~~ ~~The student will add, subtract, multiply, divide, and simplify rational expressions,~~ [Moved to new SOL AII.1 a] ~~including complex fractions.~~ [Move to Curriculum Framework]

~~AII.3~~ ~~The student will~~
~~a) add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents;~~ [Moved to new SOL AII.1 b] ~~and~~
~~b) write radical expressions as expressions containing rational exponents and vice versa.~~ [Moved to new SOL AII.1 c]

~~AII.5~~ ~~The student will identify and factor completely polynomials~~ [Moved to new SOL AII.1 d] ~~representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials~~. [Move to Curriculum Framework]

~~AII.16~~ AII.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve ~~practical~~ real-world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n.

~~AII.17~~ AII.3 The student will perform operations on complex numbers ~~and~~ , express the results in simplest form~~. Simplifying results will involve~~ using patterns of the powers of i~~.~~ , and identify field properties that are valid for the complex numbers.

Equations and Inequalities

AII.4 The student will solve, algebraically and graphically,
a) absolute value equations and inequalities;
b) quadratic equations over the set of complex numbers~~.~~;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.
Graphing calculators will be used for solving and for confirming the algebraic solutions.

~~AII.4~~ ~~The student will solve absolute value equations and inequalities~~ [Moved to new SOL AII.4 a] ~~graphically and algebraically~~. [Moved to new SOL AII.4] ~~Graphing calculators will be used as a primary method of solution and to verify algebraic solutions.~~ [Move to Curriculum Framework]

~~AII.6~~ ~~The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers.~~ [Moved to new SOL AII.4 b] ~~Graphing calculators will be used for solving and for confirming the algebraic solutions. [Move to Curriculum Framework]~~ [Moved to new SOL A.4]

~~AII.7~~ ~~The student will solve equations containing rational expressions~~ [Moved to new SOL AII.4 c] ~~and equations containing radical expressions algebraically and graphically.~~ [Moved to new SOL AII.4 d] ~~Graphing calculators will be used for solving and for confirming the algebraic solutions. [Move to Curriculum Framework]~~ [Moved to new SOL AII.4]

~~AII.14~~ AII.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. ~~The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions [Move to Curriculum Framework]~~ The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.

~~AII.13~~ ~~The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.~~

Functions

~~AII.8~~ ~~The student will recognize multiple representations of functions (absolute value, step, and exponential, functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.~~ [Moved to new SOL AII.6]

AII.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.

AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include

a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) x- and y-intercepts;
d) intervals in which a function is increasing/decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.

The graphing calculator will be used as a tool to assist in investigation of functions.

~~AII.9~~ ~~The student will find the domain,~~ [Moved to new SOL AII.7 a] ~~range,~~ [Moved to new SOL AII.7 a] ~~zeros,~~ [Moved to new SOL AII.7 b] ~~and inverse of a function~~ [Moved to new SOL AII.7 g]; ~~the value of a function for a given element in its domain;~~ [Moved to new SOL A.6 ~~f~~e] ~~and the composition of multiple functions~~. [Moved to new SOL AII.7 h] ~~Functions will include exponential, logarithmic,~~ [Move to Curriculum Framework] ~~and those that have domains and ranges that are limited and/or discontinuous.~~ [Moved to new SOL AII.7 a] ~~The graphing calculator will be used as a tool to assist in investigation of functions. [Move to Curriculum Framework]~~ [Moved to new SOL AII.7]

~~AII.15~~ ~~The student will recognize the general shape of polynomial, exponential, and logarithmic functions.~~ [Moved to new SOL AII.6] ~~The graphing calculator will be used as a tool to investigate the shape and behavior of these functions. [Move to Curriculum Framework]~~ [Moved to new SOL AII.6]

AII.8 The student will investigate and describe ~~through the use of graphs~~ the relationships between the solution of an equation, zero of a function, *x*-intercept of a graph, and factors of a polynomial expression.

~~AII.18~~ ~~The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations and graphs. Given the equations in *(h, k)* form, the student will sketch graphs of conic sections, using transformations.~~

~~AII.9~~ ~~The student will, given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.~~ [Move to new SOL G.12]

Statistics

~~AII.19~~ AII.~~10~~ 9 The student will collect and analyze data, determine the equation of the curve of best fit, ~~to~~ make predictions, and solve ~~practical~~ real-world problems using mathematical models. ~~Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit.~~ [Move to Curriculum Framework] Mathematical ~~M~~models will include ~~linear, quadratic~~, polynomial, exponential, and logarithmic functions.

~~AII.20~~ AII.~~11~~ 10 The student will identify, create, and solve ~~practical~~ real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII.~~12~~ 11 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

AII.~~13~~ 12 The student will compute and distinguish between permutations and combinations and use technology for applications.

~~**Analytical Geometry**~~

~~AII.10~~ ~~The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, *x*-intercept of a graph, and factors of a polynomial expression.~~ [Move to Curriculum Framework in support of new SOL AII.7]

~~Systems of Equations and Inequalities~~

~~AII.11~~ ~~The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.~~

~~AII.12~~ ~~The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.~~

~~AII.14~~ ~~The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.~~ [Moved to new SOL AII.5]

Trigonometry

The standards below outline the content for a one-semester course in trigonometry. Students enrolled in trigonometry are assumed to have mastered those concepts outlined in the Algebra II standards. A thorough treatment of trigonometry is provided through the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should also be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Emphasis should also be placed on oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of trigonometric functions and their inverses. They also provide a powerful tool for solving/verifying trigonometric equations and inequalities.

T.1 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. ~~Circular function definitions will be connected with trigonometric function definitions.~~ Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

T.2 The student will, given the value of one trigonometric function, find the values of the other trigonometric functions~~. Properties of the unit circle and~~ , using the definitions and properties of ~~circular~~ the trigonometric functions. ~~will be applied.~~

T.3 The student will find without the aid of a ~~calculating utility~~ calculator the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.

T.4 The student will find with the aid of a calculator the value of any trigonometric function and inverse trigonometric function.

T.5 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

T.6 The student, given one of the six trigonometric functions in standard form ~~[e.g., y = *A* sin (*Bx* + *C*) + *D*, where *A, B, C,* and *D* are real numbers],~~ will
a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, ~~and~~ vertical shift, and asymptotes;
c) sketch the graph of the function by using transformations for at least a ~~one~~ two-period interval~~.~~; and
d) ~~The graphing calculator will be used to~~ investigate the effect of changing ~~*A, B, C,* and *D*~~ the parameters in a trigonometric function on the graph of ~~a trigonometric~~ the function.

T.7 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

T.8 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. ~~Graphing utilities will be used to solve equations, check for reasonableness of results, and verify algebraic solutions.~~ [Move to Curriculum Framework]

T.9 The student will identify, create, and solve ~~practical~~ real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Algebra II and Trigonometry

The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.

Expressions and Operations

~~AII/T.1~~ ~~The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets,~~ [Moved to new SOL A.4 b] ~~complex numbers,~~ [Moved to new SOL AII/T.3] ~~and matrices~~.

~~AII/T.2~~ AII/T.1 The student will, given rational, radical, or polynomial expressions,

a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
b) add, subtract, multiply, divide, and simplify radical expressions containing ~~positive~~ rational numbers and variables, and expressions containing rational exponents;
c) write radical expressions as expressions containing rational exponents and vice versa; and
d) factor ~~completely~~ polynomials completely.

~~AII/T.2~~ ~~The student will add, subtract, multiply, divide, and simplify rational expressions,~~ [Moved to new SOL AII/T.1 a] ~~including complex fractions.~~ [Move to Curriculum Framework]

~~AII/T.3~~ ~~The student will~~

~~a)~~ ~~add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents;~~ [Moved to new SOL AII/T.1 b] ~~and~~
~~b)~~ ~~write radical expressions as expressions containing rational exponents and vice versa.~~ [Moved to new SOL AII/T.1 c]

~~AII/T.5~~ ~~The student will identify and factor completely polynomials~~ [Moved to new SOL AII/T.1 d] ~~representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials~~. [Move to Curriculum Framework]

~~AII/T.16~~ AII/T.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve ~~practical~~ real-world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n.

~~AII/T.17~~ AII/T.3 The student will perform operations on complex numbers ~~and~~ , express the results in simplest form~~. Simplifying results will involve~~ using patterns of the powers of *i*~~.~~ , and identify field properties that are valid for the complex numbers.

Equations and Inequalities

AII/T.4 The student will solve, algebraically and graphically,
a) absolute value equations and inequalities;
b) quadratic equations over the set of complex numbers~~.~~ ;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.
Graphing calculators will be used for solving and for confirming the algebraic solutions.

~~AII/T.4~~ ~~The student will solve absolute value equations and inequalities~~ [Moved to new SOL AII/T.4 a] ~~graphically and algebraically~~. [Moved to new SOL AII/T.4] ~~Graphing calculators will be used as a primary method of solution and to verify algebraic solutions.~~ [Move to Curriculum Framework]

~~AII/T.6~~ ~~The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers.~~ [Moved to new SOL AII/T.4 b] ~~Graphing calculators will be used for solving and for confirming the algebraic solutions.~~ ~~[Move to Curriculum Framework]~~ [Moved to new SOL AII/T.4]

~~AII/T.7~~ ~~The student will solve equations containing rational expressions~~ [Moved to new SOL AII/T.4 c] ~~and equations containing radical expressions algebraically and graphically.~~ [Moved to new SOL AII/T.4 d] ~~Graphing calculators will be used for solving and for confirming the algebraic solutions.~~ [Move to Curriculum Framework]

~~AII/T.14~~ AII/T.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. ~~The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.~~ ~~[Move to Curriculum Framework]~~ The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.

~~AII/T.13~~ ~~The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.~~

Functions

~~AII/T.8~~ ~~The student will recognize multiple representations of functions (absolute value, step, and exponential, functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.~~ [Moved to new SOL AII/T.6]

AII/T.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.

AII/T.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include

a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) x- and y-intercepts;
d) intervals in which a function is increasing/decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.

The graphing calculator will be used as a tool to assist in the investigation of functions.

~~AII/T.9~~ ~~The student will find the domain,~~ [Moved to new SOL AII/T.7 a] ~~range,~~ [Moved to new SOL AII/T.7 a] ~~zeros,~~ [Moved to new SOL AII/T.7 b] ~~and inverse of a function~~ [Moved to new SOL AII/T.7 g]; ~~the value of a function for a given element in its domain; and the composition of multiple functions~~. [Moved to new SOL AII/T.7 h] ~~Functions will include exponential, logarithmic,~~ [Move to Curriculum Framework] ~~and those that have domains and ranges that are limited and/or discontinuous.~~ [Moved to new SOL AII/T.7 a] ~~The graphing calculator will be used as a tool to assist in investigation of functions. [Move to Curriculum Framework]~~ [Moved to new SOL AII/T.7]

~~AII/T.15~~ ~~The student will recognize the general shape of polynomial, exponential, and logarithmic functions.~~ [Moved to new SOL AII/T.6] ~~The graphing calculator will be used as a tool to investigate the shape and behavior of these functions. [Move to Curriculum Framework]~~ [Moved to new SOL AII/T.6]

AII/T.8 The student will investigate and describe ~~through the use of graphs~~ the relationships between the solution of an equation, zero of a function, *x*-intercept of a graph, and factors of a polynomial expression.

~~AII/T.18~~ ~~The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations and graphs. Given the equations in *(h, k)* form, the student will sketch graphs of conic sections, using transformations.~~

~~AII/T.9~~ ~~The student will, given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.~~ [Move to new SOL G.12]

Statistics

~~AII/T.19~~ AII/T.~~10~~ 9 The student will collect and analyze data, determine the equation of the curve of best fit, ~~to~~ make predictions, and solve ~~practical~~ real-world problems using mathematical models. ~~Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit.~~ [Move to Curriculum Framework] Mathematical ~~M~~models will include ~~linear, quadratic~~, polynomial, exponential, and logarithmic functions.

~~AII/T.20~~ AII/T.~~11~~ 10 The student will identify, create, and solve ~~practical~~ real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII/T.~~12~~ 11 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

AII/T.~~13~~ 12 The student will compute and distinguish between permutations and combinations and use technology for applications.

~~Analytical Geometry~~

~~AII/T.10~~ ~~The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, *x*-intercept of a graph, and factors of a polynomial expression.~~ [Move to Curriculum Framework to support new SOL AII/T.7]

~~Systems of Equations and Inequalities~~

~~AII/T.11~~ ~~The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.~~

~~AII/T.12~~ ~~The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.~~

~~AII/T.13~~ ~~The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.~~

~~AII/T.14~~ ~~The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.~~ [Moved to new SOL AII/T.5]

Trigonometry

AII/T.~~14~~ 13 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. ~~Circular function definitions will be connected with trigonometric function definitions.~~ Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

AII/T.~~15~~ 14 The student will, given the value of one trigonometric function, find the values of the other trigonometric functions~~.~~ ~~Properties of the unit circle and~~ using the definitions and properties of ~~circular~~ the trigonometric functions. ~~will be applied.~~

AII/T.~~16~~ 15 The student will find without the aid of a ~~calculating utility~~ calculator the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.

AII/T.~~17~~ 16 The student will find with the aid of a calculator the value of any trigonometric function and inverse trigonometric function.

AII/T.~~18~~ 17 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

AII/T.~~19~~ 18 The student, given one of the six trigonometric functions in standard form ~~[e.g., $y = A \sin (Bx + C) + D$, where A, B, C, and D are real numbers],~~ will

a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, ~~and~~ vertical shift, and asymptotes;
c) sketch the graph of the function by using transformations for at least a ~~one~~ two-period interval~~.~~; and
d) ~~The graphing calculator will be used to~~ investigate the effect of changing ~~A, B, C, and D~~ the parameters in a trigonometric function on the graph of ~~a trigonometric~~ the function.

AII/T.~~20~~ 19 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

AII/T.~~21~~ 20 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. ~~Graphing utilities will be used to solve equations, check for reasonableness of results, and verify algebraic solutions.~~ [Move to Curriculum Framework]

AII/T.~~22~~ 21 The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Computer Mathematics

This course is intended to provide students with experiences in using computer programming techniques and skills to solve problems that can be set up as mathematical models. Students enrolled in Computer Mathematics are assumed to have studied the concepts and skills in Algebra I and beginning geometry. Students who successfully complete the standards for this course may earn credit toward meeting the mathematics graduation requirement. It is recognized that many students will gain computer skills in other mathematics courses or in a separate curriculum outside of mathematics and prior to high school. In such cases, the standards indicated by an asterisk (*) should be included in the student's course of study and treated as a review.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving per se should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course and may include programming a graphing calculator or scripting a problem solution in a database or spreadsheet. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

These standards identify fundamental principles and concepts in the field of computer science that will be used within the context of mathematical problem solving in a variety of applications. As students develop and refine skills in logic, organization, and precise expression, they will apply those skills to enhance learning in all disciplines.

COM.1 The student will apply programming techniques and skills to solve ~~practical~~ real-world problems in mathematics arising from consumer, business, and other applications in mathematics. Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.

*COM.2 The student will design, write, test, debug, and document a program. Programming documentation will include preconditions and postconditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.

*COM.3 The student will write program specifications that define the constraints of a given problem. These specifications will include descriptions of preconditions, postconditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions.

*COM.4 The student will design a step-by-step plan (algorithm) to solve a given problem. The plan will be in the form of a program flowchart, pseudo code, hierarchy chart, and/or data-flow diagram.

*COM.5 The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined function, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.

*COM.6 The student will design and implement the input phase of a program, which will include designing screen layout and getting information into the program by way of user interaction, data statements, and/or file input. The input phase also will include methods of filtering out invalid data (error trapping).

*COM.7 The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of output devices, using output statements, and labeling results.

COM.8 The student will design and implement computer graphics, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.

COM.9 The student will define simple variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean.

COM.10 The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.

COM.11 The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables addresses, pointers, parameter passing, scope of variables, and local versus global variables.

*COM.12 The student will translate a mathematical expression into a computer statement, which involves writing assignment statements and using the order of operations.

COM.13 The student will select and implement built-in (library) functions in processing data.

COM.14 The student will implement conditional statements that include "if/then" statements, "if/then/else" statements, case statements, and Boolean logic.

COM.15 The student will implement loops, including iterative loops. Other topics will include single entry point, single exit point, preconditions, and postconditions.

COM.16 The student will select and implement appropriate data structures, including arrays (one-dimensional and/or multidimensional), files, and records. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.

*COM.17 The student will implement pre-existing algorithms, including sort routines, search routines, and simple animation routines.

COM.18 The student will test a program, using an appropriate set of data. The set of test data should be appropriate and complete for the type of program being tested.

COM.19 The student will debug a program, using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference between syntax errors and logic errors.

COM.20 The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards.

Probability and Statistics

The following standards outline the content of a one-year course in Probability and Statistics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in this course are assumed to have mastered the concepts identified in the Standards of Learning for Algebra II. The purpose of the course is to present basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions.

A graphing calculator is essential for every student taking the Probability and Statistics course and is required for the Advanced Placement Statistics Examination. The calculator may not fully substitute for a computer, however. In the absence of a computer for student use, teachers may provide students with examples of computer output generated by a statistical software package.

*PS.1 The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

*PS.2 The student will analyze numerical characteristics of univariate data sets to describe patterns and departure from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers. ~~Appropriate technology will be used to calculate statistics.~~ [Move to Curriculum Framework]

*PS.3 The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features. ~~Appropriate technology will be used to generate graphical displays.~~[Move to Curriculum Framework]

*PS.4 The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points. ~~Appropriate technology will be used to generate scatterplots and identify outliers and influential points.~~ [Move to Curriculum Framework]

PS.5 The student will find and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity. ~~Appropriate technology will be used to compute correlation coefficients and residual plots.~~

PS.6 The student will make logarithmic and power transformations to achieve linearity. ~~Appropriate technology will be used.~~ [Move to Curriculum Framework]

PS.7 The student, using two-way tables, will analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

*PS.8 The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.

*PS.9 The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random and stratified) and methods to reduce bias.

PS.10 The student will plan and conduct an experiment. The plan will address control, randomization, and measurement of experimental error.

~~*PS.11 The student will compute and distinguish between permutations and combinations and use technology for applications.~~ [Moved to new SOL AII.~~13~~ 12 and new SOL AII/T.~~13~~ 12]

*PS.11 The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

*PS.12 The student will find probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the "law of large numbers" concept, the addition rule, and the multiplication rule.

*PS.13 The student will develop, interpret, and apply the binomial probability distribution for discrete random variables, including computing the mean and standard deviation for the binomial variable.

PS.14 The student will simulate probability distributions, including binomial and geometric.

PS.15 The student will identify random variables as independent or dependent and find the mean and standard deviations for sums and differences of independent random variables.

*PS.~~17~~ 16 The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator.

*PS.~~16~~ 17 The student, given data from a large sample, will find and interpret point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired).

PS.~~17~~ 18 The student will apply and interpret the logic of a hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, and difference between two means (independent and paired) and Chi-squared test for goodness of fit, homogeneity of proportions, and independence.

PS.~~18~~ 19 The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

PS.~~19~~ 20 The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures, using tables or graphing calculators.

Discrete Mathematics

The following standards outline the content of a one-year course in Discrete Mathematics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in Discrete Mathematics are assumed to have mastered the concepts outlined in the Standards of Learning for Algebra II.

Discrete mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems, they will analyze and determine whether or not a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines. The importance of discrete mathematics has been influenced by computers. ~~Modern~~ Technology (graphing calculators and/or computers) will be an integral component of this course.

*DM.1 The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).

*DM.2 The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

*DM.3 The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.

*DM.4 The student will apply algorithms, such as Kruskal's, Prim's, or Dijkstra's, relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

*DM.5 The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

*DM.6 The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.

*DM.7 The student will analyze and describe the issue of fair division (e.g., cake cutting, estate division). Algorithms for continuous and discrete cases will be applied.

DM.8 The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

DM.9 The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

DM.10 The student will use the recursive process and difference equations with the aid of appropriate technology to generate
a) compound interest;
b) sequences and series;
c) fractals;
d) population growth models; and
e) the Fibonacci sequence.

DM.11 The student will describe and apply sorting algorithms and coding algorithms used in storing, processing, and communicating information. These will include
a) bubble sort, merge sort, and network sort; and
b) ISBN, UPC, Zip, and banking codes.

DM.12 The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.

DM.13 The student will apply the formulas of combinatorics in the areas of
a) the Fundamental (Basic) Counting Principle;
b) knapsack and bin-packing problems;
c) permutations and combinations; and
d) the pigeonhole principle.

Mathematical Analysis

The standards below outline the content for a one-year course in Mathematical Analysis. Students enrolled in Mathematical Analysis are assumed to have mastered Algebra II concepts and have some exposure to trigonometry. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying equations and inequalities.

MA.1 The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.

MA.2 The student will ~~find~~ apply compositions of functions and inverses of functions to real-world situations. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions.

MA.3 The student will investigate and describe the continuity of functions using graphs and algebraic methods. ~~The functions will include piecewise and step functions.~~ [Move to Curriculum Framework]

MA.4 The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle.

MA.5 The student will find the sum (sigma notation included) of finite and infinite convergent series that will lead to an intuitive approach to a limit.

MA.6 The student will use mathematical induction to prove formulas/statements.

MA.7 The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.

MA.8 The student will investigate and identify the characteristics of conic section equations in *(h, k)* and standard forms. The techniques of translations in the coordinate plane will be used to graph conic sections.

MA.9 The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and ~~practical~~ real-world problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. ~~Graphing utilities will be used to investigate and verify the graphs and solutions.~~ [Move to Curriculum Framework]

MA.10 The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

MA.11 The student will perform operations with vectors in the coordinate plane and solve ~~practical~~ real-world problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

MA.12 The student will use parametric equations to model and solve application problems. ~~Graphing utilities will be used to develop an understanding of the graph of parametric equations.~~ [Move to Curriculum Framework]

MA.13 The student will identify, create, and solve ~~practical~~ real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

MA.14 The student will use matrices to organize data and will add, subtract, multiply matrices, multiply matrices by a scalar and use matrices to solve systems of equations.

Advanced Placement Calculus

[The College Board publishes the curricula for all Advanced Placement courses and updates these curricula biennially. Four comments from the public suggested that the course code for Advanced Placement Calculus be maintained but the Standards of Learning be deleted since teachers use the materials published by The College Board to guide instruction. The deletion of these Standards of Learning would be consistent with AP Statistics and AP Computer Science in the list of mathematics courses approved by the Board of Education to be used for high school graduation in Virginia.]

~~APC.1 The student will define and apply the properties of elementary functions, including algebraic, trigonometric, exponential, and composite functions and their inverses, and graph these functions, using a graphing calculator. Properties of functions will include domains, ranges, combinations, odd, even, periodicity, symmetry, asymptotes, zeros, upper and lower bounds, and intervals where the function is increasing or decreasing.~~

~~APC.2 The student will define and apply the properties of limits of functions. Limits will be evaluated graphically and algebraically. This will include~~

~~a) limits of a constant;~~
~~b) limits of a sum, product, and quotient;~~
~~c) one-sided limits; and~~
~~d) limits at infinity, infinite limits, and non-existent limits. *~~

~~*AP Calculus BC will include l'Hopital's Rule, which will be used to find the limit of functions whose limits yield the indeterminate forms: 0/0 and ∞/∞.~~

~~APC.3 The student will use limits to define continuity and determine where a function is continuous or discontinuous. This will include~~

~~a) continuity in terms of limits;~~
~~b) continuity at a point and over a closed interval;~~
~~c) application of the Intermediate Value Theorem and the Extreme Value Theorem; and~~
~~d) geometric understanding and interpretation of continuity and discontinuity.~~

~~APC.4 The student will investigate asymptotic and unbounded behavior in functions. This will include~~

~~a) describing and understanding asymptotes in terms of graphical behavior and limits involving infinity; and~~
~~b) comparing relative magnitudes of functions and their rates of change.~~

~~APC.5 The student will investigate derivatives presented in graphic, numerical, and analytic contexts and the relationship between continuity and differentiability. The derivative will be defined as the limit of the difference quotient and interpreted as an instantaneous rate of change.~~

~~APC.6 The student will investigate the derivative at a point on a curve. This will include~~

~~a) finding the slope of a curve at a point, including points at which the tangent is vertical and points at which there are no tangents;~~

~~b) using local linear approximation to find the slope of a tangent line to a curve at the point;~~

~~c) defining instantaneous rate of change as the limit of average rate of change; and~~

~~d) approximating rate of change from graphs and tables of values.~~

~~APC.7 The student will analyze the derivative of a function as a function in itself. This will include~~

~~a) comparing corresponding characteristics of the graphs of *f, f '*, and *f ''*;~~

~~b) defining the relationship between the increasing and decreasing behavior of *f* and the sign of *f '*;~~

~~c) translating verbal descriptions into equations involving derivatives and vice versa;~~

~~d) analyzing the geometric consequences of the Mean Value Theorem;~~

~~e) defining the relationship between the concavity of *f* and the sign of *f ''*; and~~

~~f) identifying points of inflection as places where concavity changes and finding points of inflection.~~

~~APC.8 The student will apply the derivative to solve problems. This will include~~

~~a) analysis of curves and the ideas of concavity and monotonicity;~~

~~b) optimization involving global and local extrema;~~

~~c) modeling of rates of change and related rates;~~

~~d) use of implicit differentiation to find the derivative of an inverse function;~~

~~e) interpretation of the derivative as a rate of change in applied contexts,~~ including velocity, speed, and acceleration; and

~~f) differentiation of nonlogarithmic functions, using the technique of logarithmic differentiation. *~~

~~* AP Calculus BC will also apply the derivative to solve problems. This will include~~

~~a) analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors;~~

~~b) numericalal solution of differential equations, using Euler's method;~~

~~c) l'Hopital's Rule to test the convergence of improper integrals and series; and~~

~~d) geometric interpretation of differential equations via slope fields and the relationship between slope fields and the solution curves for the differential equations.~~

~~APC.9 The student will apply formulas to find derivatives. This will include~~
~~a) derivatives of algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions;~~
~~b) derivations of sums, products, quotients, inverses, and composites (chain rule) of elementary functions;~~
~~c) derivatives of implicitly defined functions; and~~
~~d) higher order derivatives of algebraic, trigonometric, exponential, and logarithmic, functions. *~~
~~* AP Calculus BC will also include finding derivatives of parametric, polar, and vector functions.~~

~~APC.10 The student will use Riemann sums and the Trapezoidal Rule to approximate definite integrals of functions represented algebraically, graphically, and by a table of values and will interpret the definite integral as the accumulated rate of change of a quantity over an interval interpreted as the change of the quantity over the interval~~
~~$\int_a^b f'(x)\,dx = f(b) - f(a)$.~~
~~Riemann sums will use left, right, and midpoint evaluation points over equal subdivisions.~~

~~APC.11 The student will find antiderivatives directly from derivatives of basic functions and by substitution of variables (including change of limits for definite integrals). *~~
~~* AP Calculus BC will also include finding antiderivatives by parts and simple partial fractions (nonrepeating linear factors only), and finding improper integrals as limits of definite integrals.~~
~~* AP Calculus BC will also solve logistic differential equations and use them in modeling.~~

~~APC.12 The student will identify the properties of the definite integral. This will include additivity and linearity, the definite integral as an area, and the definite integral as a limit of a sum as well as the fundamental theorem:~~
~~$\frac{d}{dx}\int_a^x f(t)\,d(t) = f(x)$.~~

~~APC.13 The student will use the Fundamental Theorem of Calculus to evaluate definite integrals, represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.~~

~~APC.14 The student will find specific antiderivatives, using initial conditions (including applications to motion along a line). Separable differential equations will be solved and used in modeling (in particular, the equation $y' = ky$ and exponential growth).~~

~~APC.15 The student will use integration techniques and appropriate integrals to model physical, biological, and economic situations. The emphasis will be on using the integral of a rate of change to give accumulated change or on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. Specific applications will include~~

~~a) the area of a region;~~
~~b) the volume of a solid with known cross-section;~~
~~c) the average value of a function; and~~
~~d) the distance traveled by a particle along a line. *~~

~~* AP Calculus BC will include finding the area of a region (including a region bounded by polar curves) and finding the length of a curve (including a curve given in parametric form).~~

~~APC.16 The student will define a series and test for convergence of a series in terms of the limit of the sequence of partial sums. This will include~~

~~a) geometric series with applications;~~
~~b) harmonic series;~~
~~c) alternating series with error bound;~~
~~d) terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series; and~~
~~e) ratio test for convergence and divergence. *~~

~~* For those students who are enrolled in AP Calculus BC.~~

~~APC.17 The student will define, restate, and apply Taylor series. This will include~~

~~a) Taylor polynomial approximations with graphical demonstration of convergence;~~
~~b) Maclaurin series and the general Taylor series centered at $x = a$;~~
~~c) Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $1/(1 - x)$;~~
~~d) formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series;~~
~~e) functions defined by power series;~~
~~f) radius and interval of convergence of power series; and~~
~~g) Lagrange error bound of a Taylor polynomial. *~~

~~* For those students who are enrolled in AP Calculus BC.~~

Summary of Comments on the
Proposed Revised *Mathematics Standards of Learning*
October 24, 2008 through December 3, 2008

A total of 224 comments was received for the proposed revised *Mathematics Standards of Learning* during the public comment period and the public hearings. The public comment period was October 24, 2008 through November 22, 2008. Two public hearings were held on December 1, 2008, in Pulaski County and Henrico County. Three public hearings were held on December 3, 2008, in Chesapeake City, Lynchburg City, and Fairfax County. Public comments were received electronically, in written letters and faxes, and during the public hearings.

Summary of Attendance at the Public Hearings

Public Hearing Site and Date of Hearing	Number of Attendees	Number of Persons Commenting
Hermitage High School Henrico County December 1, 2008	11	8
Pulaski High School Pulaski County December 1, 2008	10	6
Joliff Middle School Chesapeake City December 3, 2008	21	14
Linkhorne Middle School Lynchburg City December 3, 2008	12	3
Robinson Secondary School Fairfax County December 3, 2008	15	4

Summary of Comments

Kindergarten through Grade 5	127 comments
Grades 6, 7, and 8	58 comments
High School	39 comments

Kindergarten-Grade 5:

- The K-5 revisions do not seem to reflect the intent of NCTM's Currriculum Focal Points. There still seems to be an abundance of content at each grade level rather than a few big ideas.
- The proposed revised *Mathematics Standards of Learning* have a greatly improved vertical articulation between elementary and middle grades and gaps have been removed.
- The Curriculum Framework needs to be noted in the opening paragraphs of the standards document so teachers understand that this document is an extension of the standards document and its use is critical to understanding the details of the standards.

Grades 6-8:

- Overall the middle school comments were positive about the vertical articulation with the elementary grades.
- There were conflicting opinions about whether or not the amount of content in grades 6-8 was reduced.
- There were some comments that the statement in the introduction to grade 7 stating "students should be ready for Algebra I after successful completion of the grade 7 standards" should be rewritten to state, "may be ready for Algebra I in grade 8."
- There was concern that students who take Algebra I in grade 8 would miss some crucial content such as scatterplots and multistep linear equations which appear in the grade 8 standards.
- There were some suggestions to increase the focus in grade 7 on proportional reasoning.
- The move to include more challenging topics in the middle grades, such as compound events and negative exponents, was questioned as being too abstract for students in this age group and developmentally inappropriate.
- There were conflicting opinions about the addition of inequalities to the grade six standards.

High School:

- The revisions enhance the rigor of the standards and represent the knowledge and skills required for successful entry into credit-bearing college courses and quality jobs.
- Added expectations about the normal distribution and the associated statistics in Algebra I and Algebra II will better prepare students for college and the workplace. However, variation should be addressed in Algebra II instead of Algebra I.
- The organization and focus of the proposed standards for high school courses add clarity. Additionally, the standards meet the criteria of coherence, specificity, accessibility, and measurability.
- The cognitive demand of the standards is increased.
- The actual and perceived connection between the Standards of Learning and the Curriculum Framework must be strengthened in order to clarify and support the idea of the Curriculum Framework as an expansion of the standards themselves.
- Consider tabling the implementation of the *Mathematics Standards of Learning* until the current economic situation is alleviated.
- Return at least some references to the graphing calculators to the standards instead of moving the references to the Curriculum Framework.

Both Achieve and The College Board sent letters evaluating the alignment between the proposed revised *Mathematics Standards of Learning* and their respective standards and/or benchmarks. Both letters follow.

November 21, 2008

Dr. Patricia Wright
Superintendent of Public Instruction
Virginia Department of Education
PO Box 2120
Richmond, VA 23218

Dear Dr. Wright:

Achieve has completed the final Quality Review of the alignment of Virginia's proposed revised *Mathematics Standards of Learning* (accepted by the Board of Education on October 23, 2008 for first review) during this period of time provided for public comment. While the standards have not yet been finalized, the primary purpose of this review is to ensure that the state's proposed revised academic standards for exiting high school align with the expectations for success in college and career. The American Diploma Project (ADP) Benchmarks to which these proposed revised Virginia standards were compared represent the knowledge and skills required for successful entry into credit-bearing college courses and quality jobs. A secondary purpose of this review is to ensure that these proposed revised standards meet the criteria of high quality standards that include rigor, coherence, focus, specificity, clarity/accessibility, and measurability.

The Virginia proposed revised *Mathematics Standards of Learning (SOL)* present student learning expectations that are intellectually demanding and generally well aligned with the ADP Benchmarks. If Virginia's students master the state standards, they will likely be well prepared for both workplace and college success.

Summary of Findings

- *Virginia's proposed revised Mathematics SOL are generally well aligned to the ADP Benchmarks in mathematics.* The revisions enhance the rigor of the standards and their alignment with the ADP Benchmarks in mathematics, particularly in the area of data, statistics and probability. Virginia proposes adding expectations about the normal distribution and its associated statistics in both Algebra I and Algebra II that will better prepare students for college and the workplace. In addition, the state has added explicit language to the introduction about the importance of technology, but has removed explicit references to technology from the standards themselves. The

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state should ensure that technology is comprehensively treated in the revisions to the *Mathematics SOL* Curriculum Framework that include the Essential Knowledge and Skills and Essential Understandings as proposed.

- *Virginia's proposed revised Mathematics SOL for Algebra, Functions, and Data Analysis provide the scaffold for a strong and innovative option to Algebra II.* The proposed revised Algebra, Functions, and Data Analysis has the potential to provide students with rigorous mathematics at or approaching the level of Algebra II in a more applied and contextualized format — one well suited to students not intending to pursue mathematics intensive STEM majors and careers. The state is encouraged to develop the *Mathematics SOL* Curriculum Framework that includes the Essential Knowledge and Skills and Essential Understandings to support and further develop the standards articulated for this course.

- *Virginia has enhanced the organization and increased the focus of its proposed revised high school standards while reducing the actual number of standards.* Virginia is to be commended for a number of steps taken to add clarity and focus to its high school expectations including the development of an organizational framework for each course that consists of headings under which standards are categorized. Topics have been streamlined as well. These proposed revisions should ensure that the standards are more useful to teachers and curriculum developers and more understandable to educators and non-educators alike.

- *While alignment between the proposed revised Mathematics SOL and the ADP Benchmarks is generally strong, there is a difference between the level of preparation students will receive if they follow a course of study that culminates with Algebra, Functions and Data Analysis rather than Algebra II, which the state has clarified for students and teachers.* Students who successfully complete a sequence culminating in Algebra II should be very well prepared for college level mathematics, including those courses required for STEM majors and careers. On the other hand, students who successfully complete a course sequence that culminates with Algebra, Functions and Data Analysis will be prepared to take non-remedial credit bearing courses such as College Algebra, but will not have the level of preparation required to successfully pursue mathematics intensive programs of study in college. The proposed revisions now make this distinction clear to students and teachers.

Overall, this review found that as expectations for graduating students, the proposed revised *Mathematics SOL* are intellectually challenging. However, as noted in prior Achieve reviews, it is still true that the high school graduation requirements are problematic in not requiring mathematics beyond Geometry. Given Virginia's graduation requirements for a Standard Diploma—at least three standard credits and one verified credit in mathematics to include at least two courses from among Algebra I, Geometry, Algebra II, or other mathematics courses above the level of algebra and geometry—it is possible that a student may not advance beyond the successful completion of Geometry. By not completing either Algebra II or Algebra, Functions, and Data Analysis, these students will not have had the opportunity to learn a substantial portion

of content that the ADP Benchmarks define as critical preparation for success in postsecondary education or 21st century careers.

In conclusion, with these proposed revisions to the *Mathematics SOL*, Virginia has taken an important step to better prepare young people for success in postsecondary education and in their careers. My Achieve colleagues and I look forward to continuing to support your efforts to ensure that Virginia's students are prepared for the real world demands they will face upon graduation.

Regards,

Laura M. Slover

Laura Slover
Vice President for Content & Policy Research,
Achieve

LMS:ms

Enclosure

cc: Dr. Thomas Morris
Secretary of Education, Commonwealth of Virginia

Dr. Mark Emblidge
President, Virginia Board of Education

Achieve Quality Review II

Feedback on Virginia's Proposed Revised *Mathematics Standards of Learning* (accepted by the Board of Education on October 23, 2008 for first review)

Documents Reviewed

In this review, Achieve focuses on Virginia's proposed revised *Mathematics Standards of Learning* (dated October 23, 2008) and how these standards align with the Achieve ADP Benchmarks for Mathematics. In preparing these proposed revised standards, the Virginia Department of Education took into consideration comments from a variety of groups, both within and outside of the Commonwealth. Achieve and Virginia's American Diploma Project (ADP) Alignment Team provided important feedback at formative junctures in this work, as part of the Commonwealth's participation in three ADP Alignment Institute meetings held since March 2007. Specifically, in March 2007 Achieve provided its initial analyses of Virginia's Algebra I, Geometry, and Algebra II standards—supplemented by selected standards from the middle grades, as needed, to align with the ADP Benchmarks. Virginia's ADP Alignment Team took Achieve's initial analysis into consideration when making proposed revisions to the standards, which were then re-submitted for a comprehensive "Quality Review I" analysis completed in the spring of 2008. For this Quality I Review, Standards of Learning for Algebra, Functions and Data Analysis were also available so Achieve was able to incorporate them into its analysis.

For this final review, a "Quality Review II," Achieve is focusing its analysis on the proposed revised *Mathematics Standards of Learning* (dated October 23, 2008, as accepted by the Board of Education for first review) for Algebra I, Geometry, Algebra II, and Algebra, Functions, and Data Analysis. Achieve also included proposed revised standards for earlier grades, as needed, since the ADP Benchmarks to which the Commonwealth's proposed revised standards were being aligned are cumulative in nature and contain content that may be addressed in middle school grades.

Achieve Review Panel Comments

- **Virginia's proposed revised *Mathematics Standards of Learning (SOL)* are generally well aligned with the ADP Benchmarks in mathematics.**

Virginia has taken its work to align its state standards with the ADP Benchmarks quite seriously. The feedback provided as part of earlier Achieve reviews has been taken into consideration and the Commonwealth's responses are reflected in the proposed revised standards. Since proposed revisions to the Commonwealth's Curriculum Framework documents were not provided for Achieve's review, it is assumed that none of the content from these supporting documents which further develop the Standards of Learning through the delineation of Essential Knowledge and Skills and Essential Understandings has been deleted. It is apparent from the review of the proposed revised *Mathematics SOL*, however, that substance from the current *Mathematics SOL* has been proposed to be

moved to the Curriculum Framework documents. Achieve reviewers viewed this as an organizational revision, not a substantive deletion from the proposed revised *Mathematics SOL*, and hence credited Virginia with including such content in its expectations.

The state has responded to Achieve's earlier feedback by making edits and additions to its standards to more closely align with the ADP Benchmarks. For example, Virginia's Algebra I, Geometry, and Algebra II standards had earlier been identified as having weak alignment with the ADP Benchmarks with respect to including an understanding of the normal distribution. The ADP Benchmarks clearly state that students should know the characteristics of the normal distribution. Virginia has responded by proposing to add expectations about the normal distribution and its associated statistics in both Algebra I and Algebra II. Coupled with the inclusion of the normal curve in Virginia's recently adopted Standards of Learning for Algebra, Functions and Data Analysis, the proposed revised standards will ensure that all students in Virginia have the opportunity to learn about a facet of data analysis and statistics that will better prepare them for college and the workplace. In those select instances where the state has chosen not to take Achieve's suggestions (for example, an even more extensive treatment of data, statistics and probability in Algebra I or Algebra II), the gaps are not significant enough to impact the overall strong alignment of the proposed revised *Mathematics SOL* with the ADP Benchmarks.

While technology is still apparent in the proposed revised *Mathematics SOL,* it is not as clearly visible within the proposed standards as it is in the existing standards. The Goals that accompany the proposed revised standards make clear that Virginia strives to prepare students who can "compete in a technologically sophisticated work force." It is explicitly stated that "graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving in science, engineering, business and industry, government, and practical affairs" and that "the use of technology must be an integral part of teaching, learning, and assessment." In addition, the introductory paragraphs that precede each set of course standards emphasize that graphing calculators, computers, and other appropriate technology tools are to be used to assist in teaching and learning. It is somewhat curious, given this strong advocacy for technology in the introductory material accompanying the proposed revised standards, that explicit references to technology have been deleted from the standards themselves. In many instances, references to technology have been moved to the Curriculum Framework documents. Hopefully, teachers will attend to the references to technology in the introductory text and in the Curriculum Framework.

- **Virginia's proposed revised *Mathematics SOL* for Algebra, Functions, and Data Analysis provide the scaffold for a strong and innovative option—one well suited to students not intending to pursue mathematics intensive STEM majors and careers. The Commonwealth is encouraged to develop a Curriculum Framework to support and further develop the standards articulated for this course.**

States across the country are struggling to define mathematics courses that provide students with rigorous mathematics at or approaching the level of Algebra II in a more applied and contextualized format. Virginia's Algebra, Functions, and Data Analysis standards provide a strong framework for such a course but would benefit from the same level of supporting detail that is provided for the other high school courses (Algebra I, Geometry, and Algebra II). To provide districts and schools with the level of detail they need to implement such a course, it is important that the Commonwealth develop Essential Understandings and Essential Knowledge and Skills for this course, enlisting both postsecondary faculty and high school teachers to help ensure that the course adequately prepares students for postsecondary success in non-remedial credit-bearing mathematics courses. In addition, the Commonwealth could add credibility to this course by also developing an end-of-course exam so that students could earn a verified mathematics credit for successful completion of this course.

- **Virginia has enhanced the organization and increased the focus of its proposed revised high school standards while reducing the actual number of standards. This should ensure that the standards are more useful to teachers and curriculum developers and more understandable to educators and non-educators alike.**

In developing the proposed revised *Mathematics SOL*, Virginia is to be commended for a number of steps taken to add clarity and focus to its high school expectations. As discussed earlier, clarifying language has been added to the introductions of each course to clarify which students are expected to achieve each set of course standards. In addition to this, a number of other steps have been taken to enhance the standards' focus. First, an organizational framework has been developed for each course that consists of headings under which standards are categorized. For example, the headings for Algebra I consist of: Expressions and Operations, Equations and Inequalities, Functions, and Statistics. Standards for each course have been re-sequenced and re-worded so as to fit well into these categories. This is a nice addition that is missing from the existing Standards of Learning. Second, to increase the brevity and focus of the proposed revised *Mathematics SOL*, it is proposed that significant portions of the existing *SOL* be moved into the supporting Curriculum Framework document. This increases the clarity of the standards, without eliminating the level of detail that will be critical to successful implementation by teachers. And third, some topics (such as matrices in Algebra I and Algebra II) have been eliminated from the proposed revised *Mathematics SOL* in an attempt to focus on content important for all students. Another example of this paring back of topics is the replacement of conic sections, in general, in Algebra II with a more focused standard dealing with circles and their equations.

The end result of these steps to enhance the focus and clarity of the standards is a reduction in the number of proposed revised standards. In Algebra I, the decrease is from eighteen to ten standards, and the proposed decrease for Algebra II is from twenty to thirteen standards. In Geometry, the proposed reduction is from fourteen standards to thirteen standards.

- **While alignment between the proposed revised *Mathematics SOL* and the ADP Benchmarks is generally strong, there are distinctions, which the state has clarified for students and teachers, between the level of preparation students will receive if they follow a course of study that culminates with Algebra, Functions and Data Analysis, as opposed to Algebra II.**

Virginia's proposed revised *Mathematics SOL* clearly state in the introductions to the Algebra I and Geometry standards that all students are expected to achieve these standards. Following completion of Geometry, it is clear that students in Virginia have two options. All students who intend to pursue a technical field are expected to achieve the Algebra, Functions and Data Analysis or Algebra II standards. All students preparing for postsecondary and advanced technical studies are expected to achieve the Algebra II standards.

Both the Algebra, Functions, and Data Analysis standards and the Algebra II standards define rigorous courses—but they prepare students for different options. Students who successfully complete a sequence culminating in Algebra II should be very well prepared for college level mathematics, including those courses required for STEM majors and careers. On the other hand, students who successfully complete a course sequence that culminates with Algebra, Functions and Data Analysis will be prepared to take non-remedial credit bearing courses such as College Algebra, but will not have the level of preparation required to successfully pursue mathematics intensive programs of study in college.

Students taking a course sequence that culminates with Algebra II will receive a more rigorous algebra preparation than those students who take a course sequence culminating in Algebra, Functions, and Data Analysis. For example, Virginia's proposed revised Algebra II standards are clear that students are to be able to add, subtract, multiply, divide and simplify rational algebraic expressions. This is not addressed explicitly in the standards for Algebra, Functions, and Data Analysis. On the other hand, the Algebra, Functions, and Data Analysis standards include a deeper and more comprehensive treatment of a number of statistical concepts such as experimental design, probability, and the normal curve than the Algebra II standards. (The proposed revised Algebra II standards have been enhanced to better address some statistical concepts, such as the normal distribution, that students might not otherwise encounter until Pre-calculus.)

Finally, this review has found that as expectations for graduating students, the proposed revised *Mathematics SOL* are intellectually challenging. However, as noted in prior Achieve reviews, it is still true that the high school graduation requirements are problematic in not requiring mathematics that extends beyond Geometry. Achieve has focused on the proposed revised *Mathematics SOL* for Algebra I, Geometry, Algebra II, and Algebra, Functions, and Data Analysis in this alignment study, and the resulting alignment with the ADP Benchmarks is generally strong. Unfortunately, given Virginia's graduation requirements for a Standard Diploma—at least three standard credits and one verified credit in mathematics to include at least two courses from among Algebra I, Geometry, Algebra II, or other mathematics courses above the level of algebra

and geometry—it is possible that a student may not advance beyond the successful completion of Geometry. By not completing either Algebra II or Algebra, Functions, and Data Analysis, these students will not have had the opportunity to learn a substantial portion of content that the ADP Benchmarks define as critical preparation for success in postsecondary education or 21st century careers. Virginia should consider raising its graduation requirement to require students to take a rigorous core curriculum that includes mathematics at the level of Algebra II and/or Algebra, Functions, and Data Analysis.

In conclusion, the proposed revised *Mathematics Standards of Learning* are intellectually demanding and align well with the ADP Benchmarks, when the benchmarks for Algebra I, Geometry, Algebra II, and Algebra, Functions, and Data Analysis are taken into consideration. Students achieving proficiency in these standards will be well prepared for both workplace and college success.

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Dr. Patricia I. Wright
Superintendent of Public Instruction
Commonwealth of Virginia
Department of Education
P.O. Box 2120
Richmond, Virginia 23218-2120

November 21, 2008

Dear Dr. Wright:

The College Board, a not-for-profit organization committed to connecting all students to college success, provides this review of the revised *Mathematics Standards of Learning* to the Virginia Department of Education in support of the Department's goals to strengthen the alignment of the *Virginia Standards of Learning* to postsecondary and career readiness.

The College Board finds significant improvement in the revised *Mathematics Standards of Learning* in terms of both content emphasis and rigor, and the Virginia Department of Education has addressed the majority of the recommendations presented in the College Board's 2007 alignment study of the *Virginia Standards of Learning*. There is significant evidence of improvement in the following areas:

- **Content Emphasis**: The revised standards fully address topics or concepts essential to postsecondary and career readiness that were not addressed as sufficiently in the previous standards, such as:
  - absolute value
  - similarity of figures
  - multiple representations of numbers, functions, and data
  - deduction and induction as methods of reasoning
  - the fundamental principle of counting
  - bivariate data and their applications
  - dependent and independent events
  - sampling and inferential methods in statistics
- **Organization and Progression**: The most significant improvement in this area was in the middle school *Standards of Learning*, where there is evidence of an accelerated

progression, especially in Algebra. The learning progression of standards in the middle school years is critical to building the necessary pathway to college readiness in high school mathematics.

- **Rigor**: The cognitive demand inherent in the revised *Mathematics Standards of Learning* is clearly higher than in the previous version. There is more emphasis placed on mathematical reasoning and communicating, which is reflected in the more frequent use of cognitive actions such as *justify, prove, investigate, analyze*, and *explain*. The increased emphasis on proving and justifying results, especially in the Algebra and Geometry *Standards of Learning*, will prepare students well for the mathematical reasoning skills required in college and careers.

Although the improvements in the revised *Mathematics Standards of Learning* are clear and evident, there are some areas in which the standards can be further refined in terms of clarity and specificity. These refinements will ensure that the grain size of the standards remains consistent across all strands and that the standards do not allow for different interpretations to be made in terms of their intent.

Overall, it is the College Board's perspective that the proposed *Mathematics Standards of Learning* are aligned well to the *College Board Standards for College Success* and students who complete a course of study aligned to revised *Mathematics Standards of Learning* will be college and career ready.

In providing this information, the College Board views itself as a committed partner with Virginia in setting clear and high expectations for all students in order to prepare them for college and the workforce and welcomes future opportunities to collaborate with the Virginia Department of Education.

Sincerely,

Natasha Vasavada

Senior Director, Standards and Curriculum Alignment Services
Research and Development
The College Board
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