Board of Education Agenda Item

Item: H. **Date:** October 22, 2009

Board of Education Agenda Item

Topic: Final Review of Proposed Revised Curriculum Framework for 2009 *Mathematics Standards of Learning*

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Origin:

____ Topic presented for information only (no board action required)

_X_ Board review required by
- _X_ State or federal law or regulation
- ___ Board of Education regulation
- ___ Other: ____________
- _X_ Action requested at this meeting __Action requested at future meeting:

Previous Review/Action:

____ No previous board review/action

_X_ Previous review/action
Date July 23, 2009
Action Board of Education accepted the Proposed Revised Curriculum Framework for 2009 *Mathematics Standards of Learning* for first review and public comment.

Background Information:
New academic content *Mathematics Standards of Learning* were developed in 1995 and revised in 2001. The *Standards of Quality* require the Board of Education to review the *Standards of Learning* on a regular schedule. On February 19, 2009, the Board approved the 2009 *Mathematics Standards of Learning*. The Department of Education then took the following steps to produce a draft of the proposed revised Curriculum Framework for the 2009 *Mathematics Standards of Learning* for the Board's first review:
- Selected a review committee that consisted of individuals solicited from school divisions as well as other stakeholder groups to participate in the process;
- Met with the review committee during May 2009;
- Developed a draft of the proposed revised Curriculum Framework for the 2009 *Mathematics Standards of Learning*.

On July 23, 2009, the Virginia Board of Education accepted for first review the proposed revised Curriculum Framework for the 2009 *Mathematics Standards of Learning.* A public comment period was held from July 24, 2009, to September 18, 2009. A summarized review of public comment is contained in Attachment A.

Summary of Revisions:
The proposed revised Curriculum Framework for the 2009 *Mathematics Standards of Learning* in Attachment B contains minimal changes made as a result of public comment. These changes are indicated by double underlines and strikethroughs. Revisions include:

- corrections of typographical and formatting errors;
- revisions of language for clarification and/or specification of content and vocabulary;
- additions of essential knowledge and skills or background information for instruction, assessment, and/or vertical articulation;
- consolidation of content or background information;
- organization of topics to align with the sequence of the standard; and
- changes in word choice and limiters for instructional and assessment purposes.

Superintendent's Recommendation:
The Superintendent of Public Instruction recommends that the Board of Education accept for final review the proposed revised Curriculum Framework for the 2009 *Mathematics Standards of Learning* and permit the Department of Education to make technical edits as needed.

Impact on Resources:
This responsibility can be absorbed by the agency's existing resources at this time.

Timetable for Further Review/Action:
Following the Board of Education's approval of the Curriculum Framework for the 2009 *Mathematics Standards of Learning*, the Virginia Department of Education will post the document on the Department's Curriculum Framework Web site.

Attachment A

Summary of Comments on the Proposed Revised Curriculum Framework for 2009 *Mathematics Standards of Learning*

Summary of Online Comments
A total of 85 online comments were received for K-12 courses during the online public comment period from July 24, 2009, through September 18, 2009.

The number of online comments received by grade band, grade level, and course is as follows:

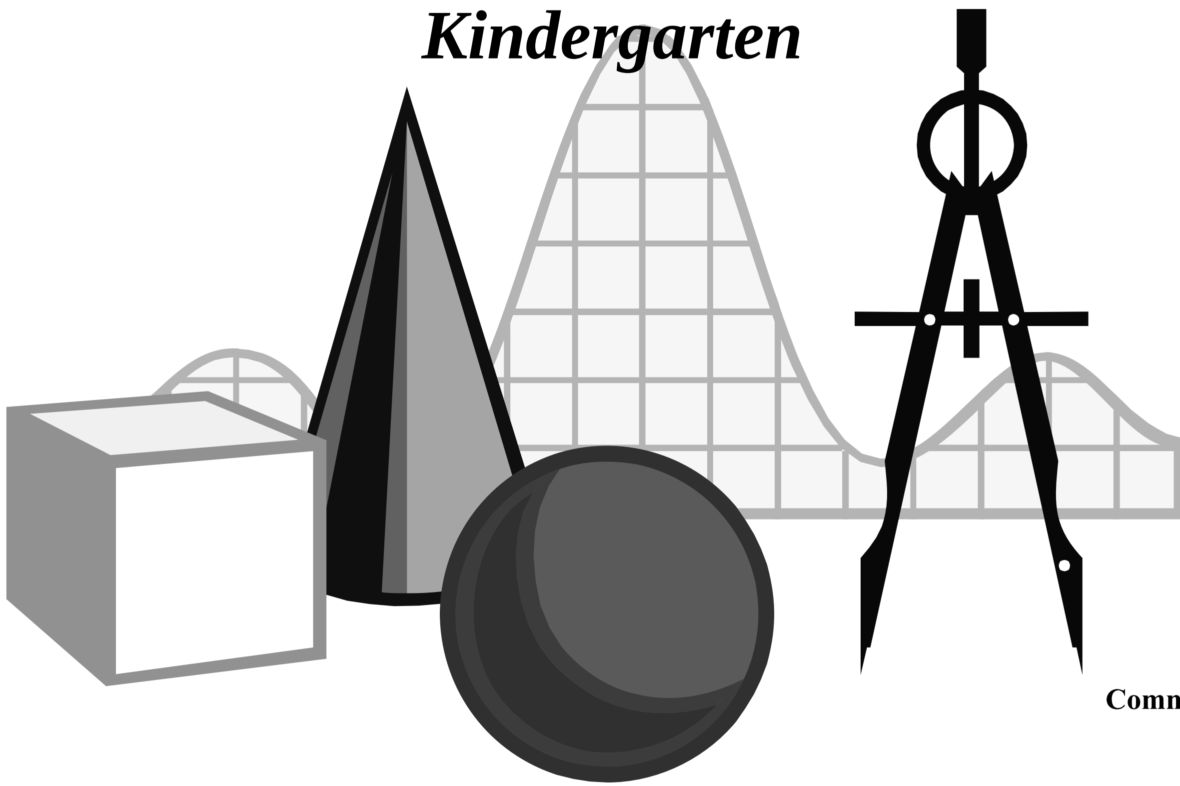
Grade Band, Grade Level, and Course	Number of Comments
K-12 General	48
1	7
2	6
3	5
4	0
5	6
6	4
7	1
8	1
Algebra I	1
Geometry	1
Algebra, Functions, and Data Analysis	0
Computer Mathematics	0
Algebra II	3
Trigonometry	1
Discrete Mathematics	0
Probability and Statistics	0
Mathematical Analysis	1
TOTAL	**85**

The majority of comments were positive regarding the revisions to the first draft of the Curriculum Framework. General, individual grade level, and course comments included suggestions for the following:

- alignment of essential knowledge and skills vertically among grade levels;
- clarification and/or specification of content;
- consideration of content for assessment purposes;
- consideration of the integration of fractal geometry;
- consideration of adding essential questions to the K-5 and end-of-course documents;
- consistency with vocabulary definitions and/or usage;
- correction of typographical errors;
- inclusion of additional essential knowledge and skills or background information;
- organization of topics to align with the sequence of the standards; and
- removal or consolidation of repeated content or background information.

PROPOSED REVISED MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Kindergarten



Commonwealth of Virginia
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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, ~~simple~~ fractions, number relationships (“more than,” “less than,” and “~~as many as~~ equal to”), and the effects of ~~simple~~ single-step and multistep computations ~~operations on numbers (fact families)~~. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

K.1 The student, given two sets, each containing 10 or fewer concrete objects, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of one-to-one correspondence.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A set is a collection of distinct elements or items. • A one-to-one correspondence exists when two sets have an equal number of items. • Strategies for developing the concept of one-to-one matching involve set comparisons without counting. Hands-on experiences in matching items between two sets by moving, touching, and aligning objects, using one-to-one correspondence, enable visual as well as kinesthetic comparisons of the number of items in the two sets. • Students can also ~~use the strategy of~~ count~~ing~~ to make comparisons between two sets without matching the sets, using one-to-one correspondence. • Students are generally familiar with the concept of *more*, but have had little experience with the term *less*. It is important to use the terms together to build an understanding of their relationship. For example, when asking which group has more, follow with which group has less and vice versa.	**All students should** • Understand how quantities relate to each other, which leads to an understanding of how numbers are related to each other.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Match each member of one set with each member of another set, using the concept of one-to-one correspondence to compare the number of members between sets, where each set contains 10 or fewer objects. • Compare and describe two sets of 10 or fewer objects, using the terms *more, fewer,* and *the same*. • Given a set of objects, construct a second set which has more, fewer or the same number of objects.

K.2 The student, given a set containing 15 or fewer concrete objects, will
a) tell how many are in the set by counting the number of objects orally;
b) write the numeral to tell how many are in the set; and
c) select the corresponding numeral from a given set of numerals.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Counting involves two separate skills: verbalizing the list of standard number words in order (“one, two, three, …”) and connecting this sequence with the objects in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken. Objects may be presented in random order or arranged for easy counting. • Kinesthetic involvement (e.g., tracing the numbers, using tactile materials, such as sand, sandpaper, carpeting, or finger paint) facilitates the writing of numerals. • Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals. • Zero (0) is both a number and a digit. As a number, it plays a central role in mathematics as the additive identity of the integers, real numbers, and many other algebraic structures. As a digit, zero is used as a placeholder in systems. • *Conservation of number* and *cardinality principle* are two important milestones in development to attaching meaning to counting. • The cardinality principle refers to the concept that the last counted number describes the total amount of the counted set. It is an extension of one-to-one correspondence.	**All students should** • Read and write numerals from 0 through ~~10~~ 15. • Understand that the total number of objects can be found by counting. • Understand that the last counted number describes the total amount in the set. • Understand that if the set is empty, it has 0 elements. • Understand that changing the spatial arrangement of a set of objects does not change the total amount of the set.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count orally the number of objects in a set containing ~~10~~ 15 or fewer concrete objects, using one-to-one correspondence, and identify the corresponding numeral. • Identify written numerals from 0 through ~~10~~ 15 represented in random order. • Select the numeral from a given set of numerals that corresponds to a set of ~~10~~ 15 or fewer concrete objects. • Write the numerals from 0 through ~~10~~ 15. • Write a numeral that corresponds to a set of ~~10~~ 15 or fewer concrete objects. • Construct a set of objects that corresponds to a given numeral, including an empty set.

K.2 The student, given a set containing 15 or fewer concrete objects, will
a) tell how many are in the set by counting the number of objects orally;
b) write the numeral to tell how many are in the set; and
c) select the corresponding numeral from a given set of numerals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Conservation of number is the understanding that the number of objects remains the same when they are rearranged spatially.		

K.3 The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Understanding the cardinal and ordinal meanings of numbers is necessary to quantify, measure, and identify the order of objects. • An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). *Ordered position*, *ordinal position*, and *ordinality* are terms that refer to the place or position of an object in a sequence or set. • The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects. • The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., the student's position in line when students are lined up alphabetically by first name).	**All students should** • Use ordinal numbers to describe the ~~order~~ position of objects in a sequence.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the ordinal positions first~~, second, and third~~ through tenth using ordered sets of ~~three~~ ten concrete objects and/or pictures of such sets presented from – left-to-right; – right-to-left; – top-to-bottom; and/or – bottom-to-top.

K.4 The student will
a) count forward to 100 and backward from 10;
b) identify one more than a number and one less than a number; and
c) count by fives and tens to 100.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Counting skills are essential components of the development of number ideas; however, they are only one of the indicators of the understanding of numbers. • Counting forward by rote advances the child's development of sequencing. The natural numbers are 1, 2, 3, 4…. The whole numbers are 0, 1, 2, 3, 4…. Students should count the ~~natural~~ whole numbers 0, 1, 2, 3, 4,…. ~~These are not to be confused with the whole numbers that begin with the integer zero.~~ • Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with 10, 9, 8,… through …3, 2, 1, 0. • Counting forward and backward leads to the development of counting on and counting back. • The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include, but are not limited to, counters, hundred chart, and calculators. • Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five. • Skip counting by tens is a precursor for use of place value, addition, counting money, and	**All students should** • Use the correct oral counting sequence in both forward and backward counting situations. • Understand that skip counting can be used to count a collection of objects. • Describe patterns in skip counting and use those patterns to predict the next number or numbers in the skip counting sequence. • Understand that numeric relationships include one more than, one less than, two more than, two less than, etc. • Understand benchmarks of five and ten.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count forward from ~~1~~ 0 to ~~30~~ 100. • Count backward from 10 to ~~1~~ 0. • Recognize the relationship of one more than and one less than a number using objects (i.e., five and one more is six; and one less than ten is nine). • Group ~~30~~ 100 or fewer objects together into sets of fives or tens and then count them by fives or by tens. ~~• Investigate and recognize the pattern of counting by fives and tens, using 30 or fewer concrete objects.~~ • Investigate and recognize the pattern of counting by fives ~~and tens~~ to ~~30~~ 100, using a ~~calculator~~ variety of tools. • Investigate and recognize the pattern of counting by ~~fives and~~ tens to ~~30~~ 100, using a ~~calculator~~ variety of tools.

K.4 The student will
a) count forward to 100 and backward from 10;
b) identify one more than a number and one less than a number; and
c) count by fives and tens to 100.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
multiplying by multiples of 10. • Calculators can be used to display the numeric patterns that result from skip counting. ~~Use the constant feature of the four-function calculator to display the numbers in the sequence when skip counting by that constant. For example, when skip counting by fives, press 5 + 5 = = =… to produce 5, 10, 15,….~~		

K.5 The student will identify the parts of a set and/or region that represent fractions for halves and fourths.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A fraction is a way of representing part of a whole (as in a region/area model) or part of a group (as in a set model). • In each fraction model, the parts must ~~be equal (i.e., each pie piece must~~ have the same area~~)~~. ~~In problems with fractions, a whole is broken into equal-size parts and reassembled into one whole.~~ • The fractional parts of a set model are subsets of an equal number. For example, in a set of ten cubes, each half would be a subset of five cubes. • Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions furthers this development (e.g., ~~*thirds*~~ *fourths* means "~~three~~ four equal parts of a whole" or $\frac{1}{4}$ represents one of ~~three~~ four ~~equal-size~~ parts of equal size when a pizza is shared among ~~three~~ four students).	**All students should** • Understand that fractional parts are equal shares of a whole region or a whole set. • Understand that the fraction name (*half*, *fourth*) tells the number of equal parts in the whole. • Understand that the fraction name (*half*, *fourth*) of the set model is a subset of the whole set with equal numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize fractions as representing parts of equal size of a whole. • Given a region, identify half and/or a fourth of the region. • Given a set, identify half and/or a fourth of the set.

A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling ~~and~~, discussing, and recording a variety of problem situations. This approach ~~to~~ helps students ~~move~~ transition from the concrete to the representation to the symbolic ~~the abstract~~ ~~and~~ in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, multiplication, division ~~facts~~ and related facts ~~families~~;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions ~~and decimals~~.

K.6 The student will model adding and subtracting whole numbers, using up to 10 concrete objects.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Whole numbers are 0, 1, 2, 3, 4, 5, 6, and so on. • Addition is the process of combining or joining sets. • Subtraction can be viewed as a "taking away" or "separating" process or as compare to find the difference between two sets. • Counting on from the larger set to determine the sum of the combined sets is a strategy for finding a sum. • Counting backward from the larger set to determine the difference between two sets is a strategy for subtraction. • Number relationships, including the following, help students develop strategies for adding and subtracting. – Instant recognition of the amount in a set of objects that are arranged in a familiar pattern such as the dots on number cubes – One more than, one less than, two more than, two less than	**All students should** • Understand that addition j~~oins~~ means putting things ~~objects~~ together and that subtraction is the inverse of addition and means ~~separates~~ to separate ~~objects~~ things out.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Combine two sets with known quantities in each set, and count the combined set using up to 10 concrete objects, to determine the sum, where the sum is not greater than 10 ~~concrete objects~~. • Given a set of 10 or fewer concrete objects, ~~Remove~~ remove, "take away," or separate part of ~~a~~ the set ~~from a given set to~~ and determine the result ~~of subtraction~~.

Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

K.7 The student will recognize a penny, nickel, dime, and quarter and will determine the value of a collection of pennies and/or nickels whose total value is 10 cents or less.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Involvement in varied activities such as physically manipulating coins and making comparisons about their sizes, colors, and values is prerequisite to the skills of coin recognition and valuation. • Counting money helps students gain an awareness of consumer skills and the use of money in everyday life. • A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to ensure competence with using money. • ~~Establishing a one-to-one correspondence between the number names and the objects in a set of coins (pennies and/or nickels) is essential for an accurate count.~~ • Students need experiences to develop the concept that a nickel has a value of five cents even though it is one object.	**All students should** • Develop common referents for identifying pennies, nickels, dimes, and quarters. • Understand the value of a collection of coins whose value is 10 cents or less.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the properties/characteristics (e.g., color, relative size) of a penny, nickel, dime, and quarter. • Identify a penny, nickel, dime, and quarter. • Identify that a nickel is the same value as five pennies. • Count a randomly placed collection of pennies and/or nickels (or models of pennies and/or nickels) whose value is 10 cents or less, and determine the value of the collection.

K.8 The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences in measuring physical objects, using nonstandard and standard units of measure, help to develop an intuitive understanding of measurement and will help students connect a tool with its purpose in measuring. • Selecting from among various measuring instruments and determining which can be used to solve various real-life problems are introduced at this level. • A precursor to connecting tools to a type of measurement is an introduction to the concepts of length, weight, time, and temperature.	**All students should** • Identify an appropriate measuring tool for a given unit of measure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a ruler as an instrument to measure length. • Identify different types of scales as instruments to measure weight. • Identify different types of clocks (analog and digital) as instruments to measure time. • Identify the components of a calendar, including days, months, and seasons. • Identify different types of thermometers as instruments used to measure temperature.

K.9 The student will tell time to the hour, using analog and digital clocks.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences in relating time on the hour to daily routines and school schedules (e.g., catching the bus, lunch time, recess time, and resource time) help students develop personal referents for time. • Making sense of telling time to the nearest hour is reinforced when students recognize the positions of the hands on an analog clock and identify the corresponding time to the hour.	**All students should** • Apply an appropriate technique, depending on the type of clock, to determine time to the nearest hour.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Tell time on an analog clock to the hour. • Tell time on a digital clock to the hour.

K.10 The student will compare two objects or events, using direct comparisons or nonstandard units of measure, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder). Examples of nonstandard units include foot length, hand span, new pencil, paper clip, and block.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Length is the distance ~~along a line or figure from one point to another~~ between two points. • Height is the vertical length of a perpendicular to its base. • Weight is a measure of the heaviness of an object. • Temperature is the degree of hotness or coldness of an object (e.g., a body) or environment. • Students need to identify the attribute that they are measuring (e.g., length, height, weight, temperature) before they begin to measure. • ~~Extensive opportunities~~ Multiple hands-on experiences are needed to gain the ability to compare the attributes of objects.	**All students should** • Compare and order objects according to their attributes. • Develop an understanding of measuring with nonstandard and standard units of measure. • Recognize attributes (length, height, weight, temperature) that can be measured.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and describe lengths of two objects (as shorter or longer), using direct comparison or nonstandard units of measure (e.g., foot length, hand span, new pencil, paper clip, block). • Compare and describe heights of two objects (as taller or shorter), using direct comparison or nonstandard units of measure (e.g., book, hand span, new pencil, paper clip, block). • Compare and describe weights of two objects (as heavier or lighter), using direct comparison or nonstandard units of measure (e.g., book, cubes, new pencil, paper clip, block). • Compare and describe temperatures of two objects or environment (as hotter or colder), using direct comparison.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of ~~shapes~~ figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent ~~two- and three-dimensional~~ plane and solid ~~shapes~~ figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on
- observing, identifying, describing, comparing, contrasting and investigating ~~three-dimensional~~ solid objects and their ~~two-dimensional~~ faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying ~~shapes~~ figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to ~~these shapes~~ figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.

K.11 The student will

a) identify, describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and

b) compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• An important part of the geometry strand in grades K through 2 is the naming and describing of ~~shapes~~ figures. Children move from their own vocabulary and begin to incorporate conventional terminology as the teacher uses geometric terms. • A plane geometric figure is any ~~two-dimensional~~ plane, closed ~~shape~~ figure. Circles and polygons are examples of plane geometric figures. • Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed. • The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. – **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. – **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., “I know it’s a rectangle because it looks like a door, and I know	**All students should** • Use their knowledge of ~~two-dimensional~~ plane figures to help them systematically represent and describe their world. • ~~Develop an understanding of the shapes of geometric figures by using various methods.~~ • Identify the characteristics of plane geometric figures (circle, triangle, square, and rectangle). • Compare the size and shape of plane geometric figures by using strategies to sort and/or group and begin to refine the vocabulary used to explain their strategies.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a circle, triangle, square, and rectangle. • Describe the ~~properties~~ characteristics of triangles, squares, and rectangles, including number of sides and number of ~~corners~~ angles. • Describe a circle using terms such as round and curved. • ~~Draw~~ Trace a circle, triangle, square, and rectangle. • Compare and group plane geometric figures (circle, triangle, square, and rectangle) according to their relative sizes (larger, smaller). • Compare and group plane geometric figures (circle, triangle, square, and rectangle) according to their shapes. • Distinguish between examples and nonexamples of identified geometric figures (circle, triangle, square, and rectangle).

K.11 The student will
a) identify, describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and
b) compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
that a door is a rectangle.") – **Level 2: Analysis.** Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it is closed; it has four sides and four right angles."). • A polygon is a geometric figure that – has sides that are ~~straight~~ line segments; – is simple (its sides do not cross); – is closed; and – is two-dimensional (~~it~~ lies in a plane). • A triangle is a polygon with three angles and three sides. Children should be shown different types of triangles such as equilateral, isosceles, scalene, right, acute, and obtuse; however, they are not expected to name the various types. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a rectangle with all four sides of equal length. • A circle is a closed curve with all points in one plane and the same distance from a fixed point (the center). • Early experiences with comparing and sorting ~~shapes~~ figures assist students in analyzing the characteristics ~~and properties of two-dimensional geometric shapes~~ of plane geometric figures. • Attribute blocks, relational attribute blocks, ~~power blocks~~, and tangrams are among the		

K.11 The student will
a) identify, describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and
b) compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
manipulatives that are particularly appropriate for sorting and comparing size. – Clay, straws, and paper and scissors are several manipulatives that are appropriate for constructing geometric figures.		

K.12 The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of ~~shape~~ figures those things represent. • Children are often confused when a ~~shape~~ figure such as a square is rotated: they frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — i.e., a square is a square regardless of its location in space; there is no such geometric ~~shape~~ figure as a diamond. • ~~Geometric manipulatives that can be used to combine plane geometric figures to create familiar shapes are~~ – ~~tangrams;~~ – ~~attribute blocks;~~ – ~~pattern blocks;~~ – ~~power blocks;~~ – ~~relational attribute blocks; and~~ – ~~transformations (slides, flips, turns) on shapes, which can be applied to change the orientation of the shape.~~	**All students should** • ~~Use a variety of skills that relate to direction, distance, and position in space in order to enhance their navigation skills.~~ • Understand that objects can have different orientations in space.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify pictorial representations of a circle, triangle, square, and rectangle, regardless of their position and orientation in space. • Describe the location of one object relative to another, using the terms *above*, *below*, and *next to*.

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

K.13 The student will gather data by counting and tallying.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are pieces of information collected about people or things. The primary purpose of collecting data is to answer questions. • Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence. • When data are presented in an organized manner, students can describe the results of their investigation (i.e., identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same number of responses). • In the process of gathering data, students make decisions about what is relevant to their investigation (e.g., when collecting data on their classmates' favorite pets, deciding to limit the categories to common pets). • When students begin to collect data, they recognize the need to categorize, which helps develop the understanding of "things that go together." Categorical data are used when constructing ~~pictorial~~ picture graphs and bar graphs.	**All students should** • Pose questions and gather data. • Understand how data are collected and presented in an organized manner by counting and tallying.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Gather~~ Use counting and tallying to gather data on ~~given~~ categories identified by ~~counting and tallying~~ the teacher and/or student (e.g., favorites, number of days of various types of weather during a given month, types of pets, types of shoes).

K.14 The student will display gathered data in object graphs, picture graphs, and tables, and will answer questions related to the data.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student). • ~~Pictorial~~ Picture graphs are graphs that use pictures to show and compare information. ~~A key is often used to indicate what each picture represents (e.g., one picture of a dog represents five dogs).~~ • Tables are an orderly arrangement of data in which the data are arranged in columns and rows in an essentially rectangular format. Tables may be used to display some type of numerical relationship or organized lists (e.g., input/output functions, tables showing one candy costs five cents and two candies cost 10 cents). • Students represent data to convey results of their investigations at a glance, using concrete objects, pictures, and numbers to give a "picture" of the organized data. • When data are displayed in an organized manner, children can describe the results of their investigations. • Graphs can be used to make connections between mathematics and social studies and/or science (e.g., job areas and the different people that work in these areas: health — doctors and nurses; education — teachers and principals). • Statements representing an analysis and interpretation of the characteristics of the data in	**All students should** • Understand that data can be represented using concrete objects, pictures, and graphs. • Understand that different types of representations emphasize different things about the same data. • Understand that picture graphs use pictures to show and compare information; object graphs use concrete materials to represent categorical data; and tables can be used to show an orderly arrangement of data in columns and rows. • Answer questions related to the gathered data from object graphs, picture graphs, and tables. • Relate their ideas about the data to concepts such as part-part-whole and number relationships.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Display data by arranging concrete objects into organized groups to form a simple object graph. • Display gathered data, using ~~pictorial representations of the data~~ pictures to form a simple ~~pictorial~~ picture graph (e.g., a picture graph of the types of shoes worn by students on a given day). • Display ~~information~~ gathered data in tables, either in rows or columns ~~(e.g., a table showing the number of bunnies in one column and the number of ears the bunnies have in another or a table showing the time schedule for classroom activities)~~. • Answer questions related to the gathered data displayed in object graphs, picture graphs, and tables by: – Describing the categories of data and the data as a whole (e.g., the total number of responses) and its parts. – Identifying parts of the data that represent numerical relationships, including categories with the greatest, the least, or the same.

K.14 The student will display gathered data in object graphs, picture graphs, and tables, and will answer questions related to the data.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be asked.		

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:
- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns ~~in the real world~~. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to ~~A~~algebra.

K.15 The student will sort and classify objects according to attributes.

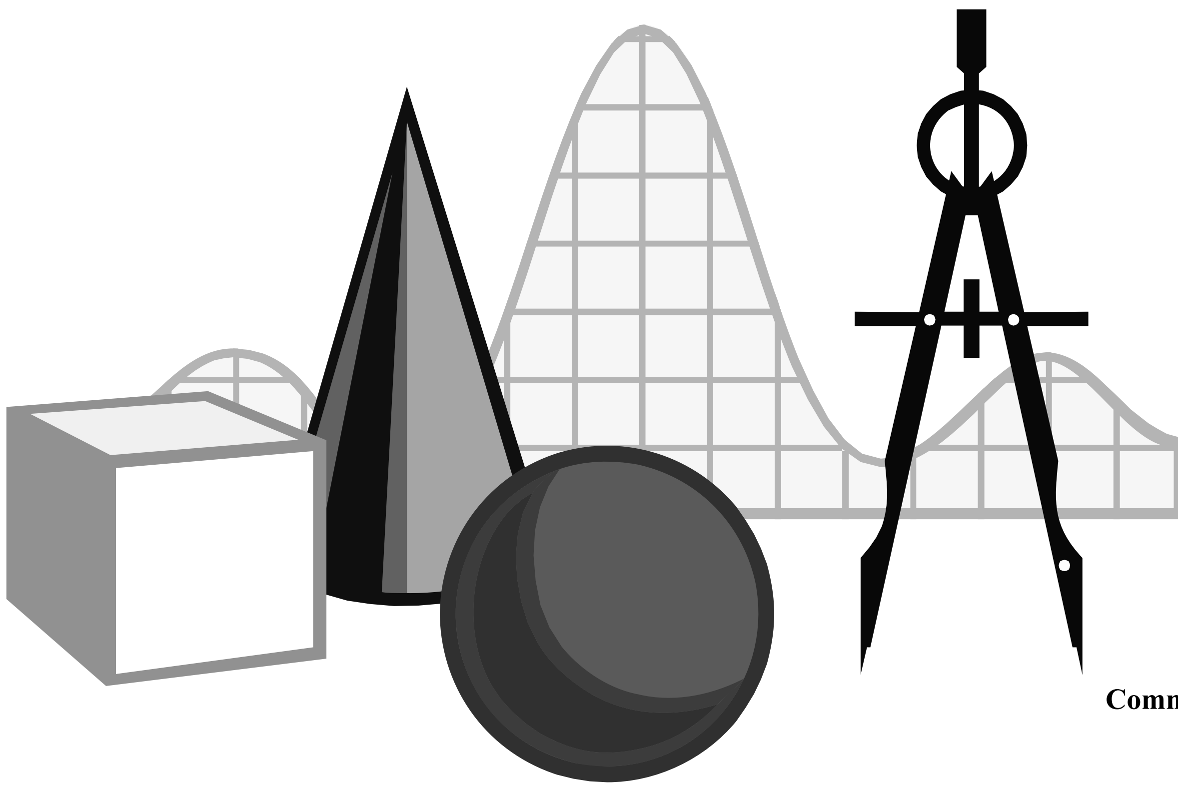
UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic ~~of a thing~~). An object has many attributes such as color, size. shape, thickness, etc. • General similarities and differences among objects are easily observed by children entering kindergarten, who are able to focus on any one attribute. The teacher's task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves with different colors and different ~~shapes~~ figures).	**All students should** • Understand that the same set of objects can be sorted and classified in different ways.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Sort objects into appropriate groups (categories) based on one attribute~~, such as size, shape, or color~~. • Classify sets of objects into ~~three~~ groups (categories) of one attribute ~~(e.g., for size — small, medium, and large)~~. • Label attributes of a set of objects that has been sorted. • Name multiple ways to sort a set of objects.

K.16 The student will identify, describe, and extend repeating patterns.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Pattern recognition is a fundamental cornerstone of mathematics, particularly algebra. • In a repeating pattern, the core of the pattern is the string of elements that repeats. By identifying the core, students demonstrate their understanding of the pattern. • Students should recognize that the sound pattern 'snap, clap, snap, clap' is the same in form as the color pattern 'red, blue, red, blue'. • Pattern recognition and the extension of the pattern allows students to make predictions. • The simplest types of patterns are repeating patterns. The patterns can be oral, such as the refrain in "Old MacDonald's Farm" ("e-i-e-i-o"), or physical with clapping and snapping patterns, or combinations of both, such as is found in songs like the "Hokey Pokey." In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. • Sample repeating patterns (repeating the ~~basic~~ core ~~unit)~~ are – ABABABAB; – ABCABC; – AABBAABBAABB; – AABAAB; – AABCAABC; and – ABACABAC.	**All students should** • Understand that patterns are a way to recognize order and organize their world and to predict what comes next in an arrangement. • Understand that the sound pattern 'snap, clap, snap, clap' is the same in form as the color pattern 'red, blue, red, blue'.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Observe and identify the basic repeating pattern (core) found in repeating patterns of common objects, sounds, and movements that occur in ~~real-life~~ practical situations~~, where there are four or fewer elements in the basic repeating pattern~~. • ~~Describe~~ Identify the ~~basic repeating pattern~~ core ~~found~~ in a repeating pattern~~, where there are four or fewer elements in the basic repeating pattern~~. • Extend a repeating pattern by adding at least two repetitions to the pattern. • Create a repeating pattern. • Compare similarities and differences between patterns.

MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 1



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The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, ~~simple~~ fractions, number relationships ("more than," "less than," and "~~as many as~~ equal to"), and the effects of ~~simple~~ single-step and multistep computations ~~operations on numbers (fact families)~~. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

1.1 The student will

a) count from 0 to 100 and write the corresponding numerals; and

b) group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• There are three developmental levels of counting: – rote sequence; – one-to-one correspondence; and – the cardinality of numbers. • Counting involves two separate skills: verbalizing the list of standard number words in order (“one, two, three, …”) and connecting this sequence with the items in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken. • The last number stated represents the number of objects in the set. This is known as the cardinality of the set. • Rote counting is a prerequisite skill for the understanding of addition, subtraction, and the ten-to-one concept of place value. • Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals. • The number system is based on a pattern of tens where each place has ten times the value of the place to its right. This is known as the ten-to-one concept of place value. • Opportunities to experience the relationships among tens and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of	**All students should** • Associate oral number names with the correct numeral and set of objects. • Understand that 1 and 10 are special units of numbers (e.g., 10 is 10 ones, but it is also 1 ten). • Understand the ten-to-one relationship of ones and tens (10 ones equals 1 ten). • Understand that numbers are written to show how many tens and how many ones are in the number. • Understand that groups of tens and ones can be used to tell how many.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count by rote from ~~1~~ 0 to 100, using the correct name for each numeral. • Use the correct oral counting sequence to tell how many objects are in a set. • Write numerals correctly. • Write each numerals ~~for the numbers 1 to 100~~ from 0 to 100. • Read two-digit numbers when shown a numeral, a Base-10 model of the number, or a pictorial representation of the number. • Identify the place value (ones, tens) of each digit in a two-digit numeral (e.g., The place value of the 2 in the number 23 is tens. The value of the 2 in the number 23 is 20). • Group a collection of objects into sets of tens and ones. Write the numeral that corresponds to the total number of objects in a given collection of objects that have been grouped into sets of tens and ones.

1.1 The student will

a) count from 0 to 100 and write the corresponding numerals; and

b) group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
our number system and to understanding the value of each digit in a two-digit number. Ten-to-one trading activities with manipulatives on place value mats provide excellent experiences for developing the understanding of the places in the ~~b~~Base-10 system. • Models that clearly illustrate the relationships among tens and ones are physically proportional (e.g., the tens piece is ten times larger than the ones piece). • Providing students with opportunities to model two-digit numbers expressed with groups of ones and tens will help students understand the ideas of trading, regrouping, and equality. • Recording the numeral when using physical and pictorial models leads to an understanding that the position of each digit in a numeral determines the size of the group it represents.		

1.2 The student will count forward by ones, twos, fives, and tens to 100 and backward by ones from 30.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include counters, number line, hundred chart, and calculators. • Skip counting by twos supports the development of the concept of even numbers. • Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five. • Skip counting by tens is a precursor for use of place value, addition, counting money, and multiplying by multiples of 10. • Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with ~~20, 19, 18~~, 30, 29, 28, … through …3, 2, 1, 0. • Calculators can be used to reinforce skip counting. ~~Use the constant feature of the four-function calculator to display the numbers in the sequence when skip counting by that constant. For example, when skip counting by twos, press 2 + 2 = = =… to produce 2, 4, 6, 8, 10,…; or when skip counting by fives, press 5 + 5 = = =… to produce 5, 10, 15,….~~	**All students should** • Understand that collections of objects can be grouped and skip counting can be used to count the collection. • Describe patterns in counting by ones (both forward and backward) and skip counting and use those patterns to predict the next number in the counting sequence.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count by ones, twos, fives, and tens to 100, using concrete objects, such as counters, connecting cubes, pennies, nickels, and dimes. • Demonstrate a one-to-one correspondence when counting by ones with concrete objects or representations. • Skip count orally by twos, fives and tens to 100 starting at various multiples of 2, 5, or 10. • Count backward by ones from ~~20~~ 30.

1.3 The student will identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A fraction is a way of representing part of a whole set or a whole region. • In a set fraction model, students need opportunities to make fair shares. For example, when sharing a set of 12 markers with three friends, each person would have one-third of the whole set. Also, each element of the set, no matter its size, is considered to be an equal share of the whole. • In an region/area fraction model, ~~In each fraction model, the parts must be equal (i.e., each pie piece must have the same area). In problems with fractions,~~ a whole is broken into equal-size parts and reassembled into one whole. • The words *denominator* and *numerator* are not required at this grade, but the concepts of part and whole are required for understanding of a fraction. • At this level, students ~~should not be~~ are expected to first understand the part-whole relationship (e.g., three out of four equal parts) before being expected to recognize or use symbolic representations for fractions (e.g., $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$)~~,~~ ~~until they understand the part-whole relationship~~. • Students should have opportunities to make connections and comparisons among fraction representations by connecting concrete or pictorial representations with spoken representations (e.g., "one-half," "one out of two equal parts", or "two-thirds", "two out of three equal parts" and "one half is more than one	**All students should** • Understand that a fraction represents a part of a whole. • Understand that fractional parts are equal shares of a whole. • Understand that the fraction name (*half*, *third*, *fourth*) tells the number of equal parts in the whole.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent a whole to show it having two equal parts and identify one-half ($\frac{1}{2}$), and two halves ($\frac{2}{2}$). • Represent a whole to show it having three equal parts and identify one-third ($\frac{1}{3}$), two-thirds ($\frac{2}{3}$) and three-thirds ($\frac{3}{3}$). • Represent a whole to show it having four equal parts and identify one-fourth ($\frac{1}{4}$), two-fourths ($\frac{2}{4}$), three-fourths ($\frac{3}{4}$) and four-fourths ($\frac{4}{4}$). • Identify and model ~~one-half and one-fourth~~ halves, thirds, and fourths of a whole, using the set model (e.g., connecting cubes and counters), and region/area models (e.g., pie pieces, pattern blocks, geoboards, paper folding, and drawings)~~; and measurement models (e.g., cuisenaire rods, connecting cubes, fraction strips, drawings)~~. • Name and write fractions represented by drawings or concrete materials for halves, thirds, and fourths. • Represent a given fraction using concrete materials, pictures, and symbols for halves, thirds, and fourths. For example, write the symbol for one-fourth, and represent it with concrete materials and pictures.

1.3 The student will identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
fourth of the same whole" ~~but not~~ $\cancel{\frac{1}{2}}$). • Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., *thirds* means "three equal parts of a whole" ~~or~~ $\cancel{\frac{1}{3}}$ ~~represents one of three equal-size parts when a pizza is shared among three students~~) furthers this development.		

A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from ~~real-life~~ practical experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as “How many books do we have altogether?” or “How many cookies are left if I eat two?” or “I have three more candies than you do.” Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling ~~and~~, discussing, and recording a variety of problem situations. This approach ~~to~~ helps students ~~move~~ transition from the concrete to the representation to the symbolic ~~the abstract~~ ~~and~~ in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, multiplication, division ~~facts~~ and related facts~~ families~~;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions ~~and decimals~~.

1.4 The student, given a familiar problem situation involving magnitude, will
a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and
b) explain the reasonableness of the choice.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Magnitude refers to the size of a set. • Exploring ways to estimate the number of objects in a set, based on appearance, (e.g., clustering, grouping, comparing) enhances the development of number sense. • To estimate means to find a number that is close to the exact amount. When asking for an estimate, teachers might ask, "*about* how much?" or "*about* how many?" or "Is this about 10 or about 50?" • Students should be provided opportunities to estimate a quantity, given a benchmark of 10 and/or 100 objects.	**All students should** • Develop an understanding of the order of magnitude (size) of whole numbers and use this knowledge to estimate quantities.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Select a reasonable order of magnitude for a given set from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, and 500 jelly beans in jars) in a familiar problem situation. • Given a familiar problem situation involving magnitude, explain why a particular estimate was chosen as the most reasonable from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral.

1.5 The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Associate the terms *addition*, *adding*, and *sum* with the concept of joining or combining. • Associate the terms *subtraction*, *subtracting*, *minus*, and *difference* with the process of ~~“~~taking away~~”~~ or separating (i.e., removing a set of objects from the given set of objects, finding the difference between two numbers, or comparing two numbers). • Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include – counting back; – “one-more-than,” “two-more-than” facts; – “one-less-than,” “two-less-than” facts; – “doubles” to recall addition facts (e.g., 2 + 2 =__; 3 + 3 =__); – “near doubles” [e.g., 3 + 4 = (3 + 3) + 1 = __]; – “make-ten” facts (e.g., at least one addend of 8 or 9); – “think addition for subtraction” (e.g., for 9 – 5 = __, think “5 and what number makes 9?”); – use of the commutative property, without naming the property (e.g., 4 +3 is the same as 3 + 4); – use of ~~fact families~~ related facts (e.g., 4 + 3 = 7 , 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4); – use of the ~~zero property~~ additive identity property (e.g., 4 + 0 = 4), without naming the property but saying, “When you add zero to a number, you always get the original number.” – use patterns to make sums (e.g., 0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.) • Manipulatives should be used ~~initially~~ to develop an understanding of addition and subtraction facts ~~and to engage students in meaningful~~ ~~memorization~~. ~~Rote~~	**All students should** • Develop an understanding of the addition and subtraction relationship. • Develop addition and subtraction strategies for fact recall. • Develop fluency with basic number combinations for addition and subtraction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify + as a symbol for addition, ~~and~~ – as a symbol for subtraction, and = as a symbol for equality. • Recall and state orally the basic addition facts for sums with two addends to ~~10~~ 18 or less and the corresponding subtraction facts. • Recall and write the basic addition facts for sums to ~~10~~ 18 or less and the corresponding subtraction facts, when addition or subtraction problems are presented in either horizontal or vertical written format.

1.5 The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~recall~~ Automaticity of ~~the~~ facts ~~is often~~ can be achieved through constant practice ~~and may come in a variety of formats, including presentation through flash cards, practice sheets, and/or games~~ which may include games, hands-on activities, flash cards, and paper and pencil. • Students should first master facts to 10 and then master facts to 18.		

1.6 The student will create and solve one-step story and picture problems using basic addition facts with sums to 18 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The problem solving process is enhanced when students – create their own story problems; and – model word problems, using manipulatives, ~~or drawings~~ representations, or number sentences/equations. • Students should ~~be exposed to~~ experience a variety of problem types related to addition and subtraction, including – join and separate problems (action involved); join (for example: Sam had 8 pennies. Tom gave him 3 more. How many pennies does Sam have now? separate (for example: Sam had 11 pennies. He gave 3 to Tom. How many pennies does Sam have now?) – part-part-whole problems (no action involved); missing part (for example: There are 8 marbles. Five are shown. How many are missing?) – classification problems (for example: Jane had 12 hats. Only ~~three~~ 3 of the hats are blue. How many are not blue?) – comparison problems (for example: Bill is 7 years old. Alice is 4 years old. How much younger is Alice than Bill?).	**All students should** • Understand various meanings of addition and subtraction in a variety of situations. • Understand that creating and solving problems involves the use of addition and/or subtraction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Interpret and solve oral or written story and picture problems involving one-step solutions, using basic addition and subtraction facts (sums to ~~10~~ 18 or less and the corresponding subtraction facts). • Identify a correct number sentence to solve an oral or written story ~~or~~ and picture problem, selecting from among basic addition and subtraction facts.

Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

1.7 The student will
a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and
b) determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences with coins help students develop an understanding of money, such as – drawing pennies to show the value of a given coin (e.g., a nickel, a dime, or a quarter); – playing store and purchasing classroom objects, using play money (pennies); – representing the value of a nickel, a dime, and a quarter, using pennies; and – trading the equivalent value of pennies for a nickel, a dime, and a quarter, using play money. • Counting money helps students gain an awareness of consumer skills and the use of money in everyday life. • A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to ensure competence with counting money. • Establishing a one-to-one correspondence between the number names and the items in a set of coins (pennies, nickels, or dimes) is essential for an accurate count. • The last number stated represents the value of a collection of coins being counted. This is known as the cardinality of the set.	**All students should** • Develop an understanding of exchanging the appropriate number of pennies for a nickel, a dime, or a quarter. • Develop an understanding of place value by skip counting a collection of coins by ones, fives, and tens.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the value of a nickel, a dime, and a quarter in terms of pennies. • Recognize the characteristics of pennies, nickels, and dimes (e.g., color, size). • Count by ones to determine the ~~total~~ value of a collection of pennies whose total value is 100 cents or less. • Count by fives to determine the ~~total~~ value of a collection of nickels whose total value is 100 cents or less. • Count by tens to determine the ~~total~~ value of a collection of dimes whose total value is 100 cents or less. • Count by ones, fives, and tens to determine the value of a collection of pennies and nickels, pennies and dimes, and nickels and dimes whose total value is 100 cents or less. • Count by ones, fives, and tens to determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less. ~~• Identify the value of a collection of pennies, nickels, and dimes, whose total value is 100 cents or less.~~

1.8 The student will tell time to the half-hour, using analog and digital clocks.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences using clocks help students develop an understanding of the telling of time to the half-hour, including – identifying the parts of an analog clock (minute and hour hands); – demonstrating a given time to the half-hour, using a model clock; – writing correct digital time to the half-hour; and – relating time on the half-hour to daily routines and school schedules (e.g., the times of TV programs, bedtime, resource time, lunch time, recess time).	**All students should** • Understand how to tell time to the half-hour, using an analog ~~or~~ and digital clock. • Understand the concepts of a.m., p.m., minutes, and hours. • Understand that there are sixty minutes in an hour.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Tell time shown on an analog clock to the half-hour. • Tell time shown on a digital clock to the half-hour. • Match a written time to the time shown on a digital ~~or~~ and analog clock to the half-hour.

1.9 The student will use nonstandard units to measure length, weight/mass, and volume.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The process of measurement involves selecting a unit of measure, comparing the ~~object to be measured to the~~ unit to the object to be measured, counting the number of times ~~the object can be measured by~~ the unit is used to measure the object, and arriving at an approximate total number of units. • Premature use of instruments or formulas leaves children without the understanding necessary for solving measurement problems. • When children's initial explorations of length, weight/mass, and volume involve the use of nonstandard units, they develop some understanding about the need for standard measurement units for length, weight/mass, and volume especially when they communicate about these measures.	**All students should** • Understand that measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book. The weight/mass of the cube measures the weight/mass of the book. The volume of the cube measures the volume of a book). • Understand how to measure length, weight/mass, and volume using various nonstandard units of measurement.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Measure the length of objects, using various nonstandard units (e.g., connecting cubes, paper clips, erasers). • Measure the weight/mass of objects, using a balance scale with various nonstandard units (e.g., paper clips, bean bags, cubes). • Measure the volume of objects, using various nonstandard units (e.g., connecting cubes, blocks, rice, water).

1.10 The student will compare, using the concepts of more, less, and equivalent,
a) the volumes of two given containers; and
b) the weight/mass of two objects, using a balance scale.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Estimation is a commonly used strategy to compare the volumes of two containers. • Determining the volume of a container by counting the number of nonstandard units (e.g., a spoonful, or scoopful~~, or teacupful~~ of concrete material, such as jelly beans, sand, water, or rice) that can be held by the container is a precursor to comparing volumes. • A variety of activities that focus on directly comparing the volume of objects leads to an understanding of volume. • The level of difficulty in measuring volume can be increased by varying and mixing the sizes of the containers (e.g., using short, wide containers as well as tall, narrow containers). • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • Balance scales are instruments used for ~~weighing~~ comparing weight/mass. A balance scale usually has a beam that is supported in the center. On each side of the beam are two identical trays. When the trays hold equal	**All students should** • Understand how to fill containers with objects to determine their volume and compare the volumes of two containers. • Understand that a balance ~~beam~~ scale can be used to compare the weights of two objects using the terms more, less, or equivalent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare the volumes of two containers to determine if the volume of one is ~~greater than~~ more, less ~~than~~, or ~~the same as~~ equivalent to the other, using nonstandard units of measure (e.g., a spoonful~~,~~ or scoopful~~, or teacupful~~). • Compare the volumes of two containers to determine if the volume of one is ~~greater than~~ more, less ~~than~~, or ~~the same as~~ equivalent to the other by pouring the contents of one container into the other. • Compare the weight/mass of two objects, using the terms *lighter, heavier*, or *the same*, using a balance scale. The pan containing less weight/mass will rise and the pan containing more weight/mass will fall. If the objects are of equivalent weight/mass, the two pans will balance.

1.10 The student will compare, using the concepts of more, less, and equivalent,
a) the volumes of two given containers; and
b) the weight/mass of two objects, using a balance scale.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
weights, the beam is level, and the scale is “balanced.” • Physically measuring the weights of objects, using a balance scale, helps students develop an intuitive idea of what it means to say something is “lighter,” “heavier,” or “the same.” • Experience estimating the weights of two objects (one in each hand) using the terms lighter, heavier, or the same promotes an understanding of the concept of balance.		

1.11 The student will use calendar language appropriately (e.g., names of the months, *today, yesterday, next week, last week*).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Time is a unit of measure.~~ • ~~Real-life~~ Practical situations are appropriate to develop a sense of the interval of time between events (e.g., Boy Scout meetings occur every week on Monday: there is a week between meetings). • The calendar is a way to represent ~~measure~~ units of time (e.g., days, weeks, and months). • Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset.	**All students should** • Understand how to use a calendar as a way to measure time.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Read a calendar to locate a given day or date. • Identify the months of the year. • Identify the seven days in a week. • Determine the days/dates before and after a given day/date (e.g., yesterday, today, tomorrow). • Determine the date that is a specific number of days or weeks in the past or in the future from a given date, using a calendar. • Identify specific dates (e.g., the third Monday in a given month).

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of ~~shapes~~ figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent ~~two-and three-dimensional~~ plane and solid ~~shapes~~ figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting and investigating ~~three-dimensional~~ solid objects and their ~~two-dimensional~~ faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying ~~shapes~~ figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these ~~shapes~~ figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

1.12 The student will identify and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles.

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(Background Information for Instructor Use Only)</th><th>ESSENTIAL UNDERSTANDINGS</th><th>ESSENTIAL KNOWLEDGE AND SKILLS</th></tr>
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• A plane geometric figure is any two-dimensional closed ~~shape~~ figure. Circles and polygons are examples of plane geometric figures.
• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.
– **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
– **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., “I know it’s a rectangle because it looks like a door, and I know that a door is a rectangle”).
– **Level 2: Analysis.** Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., “I know it’s a rectangle because it is closed; it has four sides and four right angles, and opposite sides are parallel.”).
• Students should have experiences with various plane geometric polygons.
– Triangles could be equilateral, right,
</td><td>

All students should
• Develop strategies to sort and/or group plane geometric figures and refine the vocabulary used to explain their strategies.
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The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
• Describe a circle.
• ~~Draw~~ Trace triangles, squares, rectangles, and circles.
• Describe triangles, squares, and rectangles by the number of sides, ~~corners, and square corners~~ vertices, and right angles.
• Sort plane geometric figures into appropriate subsets (categories) based on characteristics (number of sides, vertices, angles, curved, etc.).
• Identify the name of the ~~shape~~ geometric figure when given information about the number of sides, ~~corners, and/or square corners~~ vertices, and right angles.
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1.12 The student will identify and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
obtuse, acute, etc. – Quadrilaterals could be rectangles, squares, trapezoids, rhombi, etc. • A vertex is the point at which two line segments, lines, or rays meet to form an angle. • A polygon is a plane geometric figure which – has sides that are ~~straight~~ line segments; – is simple (its sides do not cross); – is closed; and – is two-dimensional (~~it~~ lies in a plane). • A triangle is a polygon with three angles and three sides. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a rectangle with four sides of equal length. • A circle is a closed curve with all points in one plane and the same distance from a fixed point (the center). • ~~Translations~~ Transformations (~~slides~~ translations, ~~flips~~ reflections, and ~~turns~~ rotation) can be used to change the location of objects. • Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed.		

1.13 The student will construct, model, and describe objects in the environment as geometric shapes (triangle, rectangle, square, and circle) and explain the reasonableness of each choice.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in the~~ir~~ environment by the type of ~~shape~~ figure those things represent. • ~~Children are often confused~~ A common misconception students have when a ~~shape~~ figure such as a square is rotated~~:~~ is they will frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — i.e., a square is a square regardless of its location in space; there is no such geometric ~~shape~~ figure as a diamond. • Building geometric and spatial capabilities fosters enthusiasm for mathematics while providing a context to develop spatial sense. • Polygons can be constructed using other polygons (e.g., six equilateral triangles can be used to construct a hexagon, a triangle can be added to a rectangle to create a pentagon, etc.).	**All students should** • Understand that geometric figures are integral parts of ~~his/her~~ the environment. • Use familiarity with the figure, structure, and location to develop spatial reasoning.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct plane geometric figures. • Identify models of representations of circles, squares, rectangles, and triangles in the environment at school and home and tell why they represent those ~~shapes~~ figures. • Describe representations of circles, squares, rectangles, and triangles in the environment and explain the reasonableness of the choice ~~(e.g., "I know it's a rectangle because it looks like a door, and I know that a door is a rectangle.")~~.

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

1.14 The student will investigate, identify, and describe various forms of data collection (e.g., recording daily temperature, lunch count, attendance, favorite ice cream), using tables, picture graphs, and object graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Students' questions about the physical world can often be answered by collecting data and observing the results. • Data are information collected about people or things. The primary purpose of collecting data is to answer questions. • After generating questions, students decide what information is needed and how it can be collected. • The collection of the data often leads to new questions to be investigated. • The entire process broadens children's views of mathematics and its usefulness. • Data collection could involve voting, informal surveys, tallying, and charts. • Surveys, which are data-collecting tools that list choices, should have a limited number of questions at the primary grades. • Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence.	**All students should** • Understand how data can be collected and presented in an organized manner. • Understand that data gathered and analyzed from observations and surveys can have an impact on our everyday lives.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate various forms of data collection, including counting and tallying, informal surveys, observations, and voting. • Identify and describe various forms of data collection in ~~his or her world~~ practical situations (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream.)

1.15 The student will interpret information displayed in a picture or object graph, using the vocabulary *more, less, fewer, greater than, less than,* and *equal to*.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Statistics is used to describe and interpret with numbers the world around us; it is a tool for problem solving. • Students' questions about everyday life can often be answered by collecting and interpreting data. • Organized data provides a clearer picture for interpretation. <u>Data may be described in object graphs and picture graphs.</u> • Picture graphs are graphs that use pictures to show and compare information. ~~A key is often used to indicate what each picture represents (e.g., one picture of a dog represents five dogs).~~ • Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student). • Interpretation of the data could lead to additional questions to be investigated.	**All students should** • Understand that picture graphs use pictures to represent and compare data while object graphs use concrete objects to represent and compare data. • Understand that data can be analyzed and interpreted, using the terms *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Demonstrate the meaning of the terms *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*, using concrete materials.~~ • Compare one category to another in a graph, indicating which has more or which has less, <u>or which is equal to.</u> • Interpret information displayed in object graphs and picture graphs, using the words *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*. • Find answers to questions, using graphs (e.g., "Which category has more?" ~~"Which category has less"~~, <u>"How many more?", and "How many in all?").</u>

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns ~~in the real world~~. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to ~~A~~algebra.

1.16 The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Sorting, classifying, and ordering objects facilitate work with patterns, geometric shapes, and data. • To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic ~~of a thing~~). • General similarities and differences among items are easily observed by primary students, who can begin to focus on more than one attribute at a time. During the primary grades, the teacher's task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves with different colors and different shapes).	**All students should** • Understand the same set of objects can be sorted and classified in different ways.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Sort and classify objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, or thickness.

1.17 The student will recognize, describe, extend, and create a wide variety of growing and repeating patterns.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Patterns allow children to recognize order, ~~to~~ generalize, and ~~to~~ predict. • Patterning should include – reproducing a given pattern, using manipulatives; – recording a pattern with pictures or symbols; – transferring a pattern into a different form or different representation (e.g., blue–blue–red to an AAB repeating pattern); and – analyzing patterns in ~~the real world~~ practical situations (e.g., calendar, seasons, days of the week). • The simplest types of patterns are repeating patterns. The part of the pattern that repeats is the core. The patterns can be oral, such as the refrain in "Old MacDonald's Farm" ("e-i-e-i-o"), or physical with clapping and snapping patterns, or combinations of both, such as is found in songs like the "Hokey Pokey." In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. • Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats. • Create an arithmetic number pattern. Sample numeric patterns include – 6, 9, 12, 15, 18, …(growing pattern);	**All students should** • Understand that patterns are a way to recognize order, to organize their world, and to predict what comes next in an arrangement. • Recognize and state the core of a pattern. • Analyze how both repeating and growing patterns are generated.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize the pattern in a given rhythmic, color, ~~shape~~ geometric figure, or numerical sequence. • Describe the pattern in a given rhythmic, color, ~~shape~~ geometric figure, or numerical sequence in terms of the core (the part of the sequence that repeats). • Extend a pattern, using manipulatives, geometric figures, numbers, or calculators. • Transfer a pattern from one form to another. • Create a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators (e.g., the growing patterns 2, 3, 2, 4, 2, 5, 2, 6, 2, …). • ~~Create an arithmetic number pattern, using a calculator (e.g., when skip counting by fives, use the constant feature on the calculator by pressing 5 + 5 = = = … to produce the pattern 5, 10, 15, 20, …).~~

1.17 The student will recognize, describe, extend, and create a wide variety of growing and repeating patterns.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
– ~~5, 7, 9, 11, 13~~ 20, 18, 16, 14, … (~~repeating numeric~~ growing pattern); ~~and~~ – 1, 2, 4, 7, 11, 16, … (growing ~~numeric~~ pattern)~~.~~; and – 1, 3, 5, 1, 3, 5, 1, 3, 5… (repeating pattern). • In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation ~~(turn)~~ is the result of turning a figure around a point or a vertex, and reflection ~~(flip)~~ is the result of flipping a figure over a line.		

1.18 The student will demonstrate an understanding of equality through the use of the equal sign.

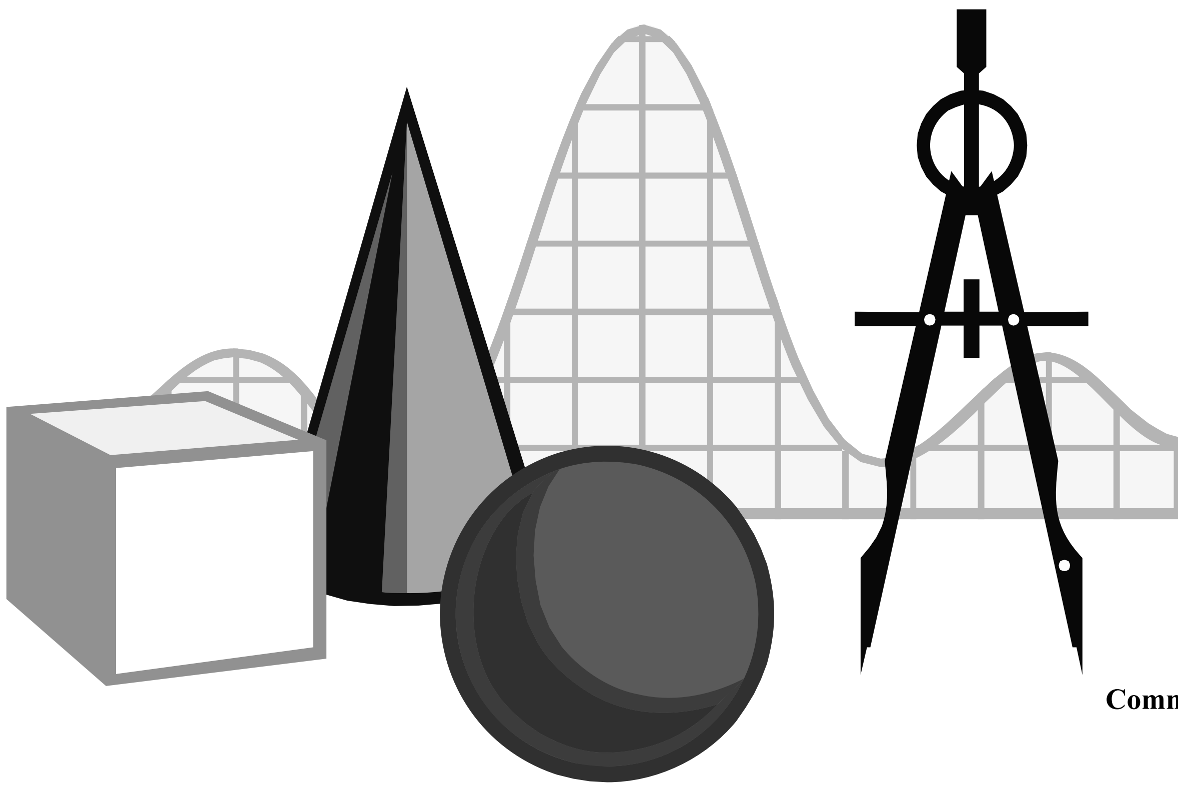
UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Equality can be shown by measuring with a balance scale or a number balance. • Manipulatives such as rods, connecting cubes, and counters can be used to model equations. • An expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal sign (e.g., x + 3). • An equation is a mathematical statement that two expressions are the same. Equations are written with an equal sign. • ~~Equations are not equal.~~ Equations have expressions of equal value on both sides (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2). • An equation can be represented using ~~must be~~ balance~~d~~ scales. There must be the same amount on each side of an equal sign (e.g., 5 + 3 = 3 + 5). • A common misunderstanding is that the equal sign always means the answer. The equal sign can represent an equality. • Equations should be shown in many different forms (e.g., 6 = 6, 4 + 2 = 6, 5 + 1 = 4 + 2, 6 = 4 + 2, 4 = 6 - 2). • Inequalities such as $5 < 4 + 3$ are not equations. Equations must have the equal sign. • It is important for children to understand that the expression 3 + 5 is another representation of eight. • The equal sign is used when two representations name the same number, 5 + 3 = 10 - 2. These	**All students should** • Understand that the equal sign means "is the same as" or "another name for" or "equal in value". • Understand that equality represents a balance concept. Both sides of the equation balance because they are equal (they have the same value).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the equality (=) symbol. • Recognize that the equations 4 + 2 = 2 + 4 and 6 + 1= 4 + 3 represent the relationship between two expressions of equal value. • Model an equation that represents the relationship of two ~~an~~ expressions of equal value. • Identify equivalent values (e.g., 3 = 3, 4 + 3 = 8 - 1, 7 = 2 + 5, etc.).

1.18 The student will demonstrate an understanding of equality through the use of the equal sign.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
two expressions in the equation represent the same number, eight. • Equations should be routinely modeled in conjunction with story problems. • Solving missing addend problems and stories helps with the understanding of equality and the equal sign (e.g., There are ~~four~~ 4 red birds in the tree. Some black birds fly to the tree. Now there are six birds in the tree. How many black birds flew to the tree? 4+__=6)		

MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 2



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, ~~simple~~ fractions, number relationships ("more than," "less than," and "~~as many as~~ equal to"), and the effects of ~~simple~~ single-step and multistep computations ~~operations on numbers (fact families)~~. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

2.1 The student will
- **a) read, write, and identify the place value of each digit in a three-digit numeral, using numeration models;**
- **b) round two-digit numbers to the nearest ten; and**
- **c) compare two whole numbers between 0 and 999, using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).**

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The number system is based on a simple pattern of tens where each place has ten times the value of the place to its right. • Opportunities to experience the relationships among hundreds, tens, and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a three-digit number. Ten-to-one trading activities with manipulatives on place value mats provide excellent experiences for developing the understanding of the places in the ~~b~~Base-10 system. • Models that clearly illustrate the relationships among hundreds, tens, and ones are physically proportional (e.g., the tens piece is ten times larger than the ones piece). • Students need to understand that 10 and 100 are special units of numbers (e.g., 10 is 10 ones, but it is also 1 ten). • Flexibility in thinking about numbers is critical. For example, 123 is 123 ones; or 1 hundred, 2 tens, and 3 ones; or 12 tens and 3 ones. • Rounding is finding the nearest easy-to-use number (e.g., the nearest 10) for the situation at hand. • Number lines are useful tools for developing the concept of rounding to the nearest ten. ~~Students can use the strategy of identifying a number on a~~	**All students should** • Understand the ten-to-one relationship of ones, tens, and hundreds (10 ones equals 1 ten; 10 tens equals 1 hundred). • Understand that numbers are written to show how many hundreds, tens, and ones are in the number. • Understand that rounding gives a close, easy-to-use number to use when an exact number is not needed for the situation at hand. • Understand that a knowledge of place value is essential when comparing numbers. • Understand the relative magnitude of numbers by comparing numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate the understanding of the ten-to-one relationships among ones, tens, and hundreds, using manipulatives (e.g., beans and cups, ~~b~~Base-10 blocks, bundles of 10 ~~Popsicle~~ sticks). • Determine the place value of each digit in a three-digit numeral presented as a pictorial representation (e.g., a picture of ~~b~~Base-10 blocks) or as a physical representation (e.g., actual ~~b~~Base-10 blocks). • Write numerals, using a ~~b~~Base-10 model or picture. • Read three-digit numbers when shown a numeral, a ~~b~~Base-10 model of the number, or a pictorial representation of the number. • Identify the place value (ones, tens, hundreds) of each digit in a three-digit numeral. • Determine the value of each digit in a three-digit numeral (e.g., in 352, the 5 represents 5 tens and its value is 50). • Round two-digit numbers to the nearest ten. • Compare two numbers between 0 and 999 represented pictorially or with concrete objects (e.g., ~~b~~Base-10 blocks), using the words *greater than, less than* or *equal to*.

2.1 The student will

a) read, write, and identify the place value of each digit in a three-digit numeral, using numeration models;

b) round two-digit numbers to the nearest ten; and

c) compare two whole numbers between 0 and 999, using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~number line and finding the multiple of ten that is closest to the identified number.~~ Rounding to the nearest ten using a number line is done as follows: – Locate the number on the number line. – Identify the two tens the number comes between. – Determine the closest ten. – If the number in the ones place is 5 (halfway between the two tens), round the number to the higher ten. • ~~A strategy~~ Once the concept for rounding numbers using a number line is developed, the procedure for rounding numbers to the nearest ten is as follows: – Look one place to the right of the digit in the place you wish to round to. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digit to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place, and change the digit to the right of the rounding place to zero. • A procedure for comparing two numbers by examining place value may include the following: – Line up the numbers by place value lining up the ones. – Beginning at the left, find the first place value where the digits are different. – Compare the digits in this place value to determine which number is greater (or which is less). – Use the appropriate symbol > or < or words *greater than* or *less than* to compare the numbers in the order in which they are		

2.1 The student will

a) read, write, and identify the place value of each digit in a three-digit numeral, using numeration models;

b) round two-digit numbers to the nearest ten; and

c) compare two whole numbers between 0 and 999, using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
presented. – If both numbers are the same, use the symbol = or the words *equal to*. • Mathematical symbols (>, <) used to compare two unequal numbers are called *inequality symbols*.		

2.2 The student will

a) identify the ordinal positions first through twentieth, using an ordered set of objects; and

b) write the ordinal numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Understanding the cardinal and ordinal meanings of numbers is necessary to quantify, measure, and identify the order of objects. • An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). *Ordered position*, *ordinal position*, and *ordinality* are terms that refer to the place or position of an object in a sequence or set. • The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects (e.g., left, right, top, bottom). • The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., a student's position in line when students are lined up alphabetically by first name). • Ordinal position can also be emphasized through sequencing events (e.g., months in a year or sequencing in a story). • Cardinality can be compared with ordinality when comparing the results of counting. There is obvious similarity between the ordinal number words *third* through *twentieth* and the cardinal number words *three* through *twenty*.	**All students should** • Use ordinal numbers to describe the position of an object in a sequence or set.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count an ordered set of objects, using the ordinal number words *first* through *twentieth*. • Identify the ordinal positions first through twentieth, using an ordered set of objects. • Identify the ordinal positions first through twentieth, using an ordered set of objects presented in lines or rows from – left to right; – right to left; – top to bottom; and – bottom to top. • Write 1^{st}, 2^{nd}, 3^{rd}, through 20^{th} in numerals.

2.3 The student will

a) identify the parts of a set and/or region that represent fractions for halves, thirds, fourths, sixths, eighths, and tenths;
b) write the fractions; and
c) compare the unit fractions for halves, thirds, fourths, sixths, eighths, and tenths.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The whole should be defined. • A fraction is a way of representing part of a whole (as in a region/area model) or part of a group (as in a set model). • In each fraction model, the parts must be equal (i.e., each pie piece must have the same area; the size of each chip in a set must be equal). In problems with fractions, a whole is broken into equal-size parts and reassembled into one whole. • Students should have experiences dividing a whole into additional parts. As the whole is divided into more parts, students understand that each part becomes smaller. • The denominator ~~(bottom number)~~ tells how many equal parts are in the whole or set. The numerator ~~(top number)~~ tells how many of those parts are being described. • Students should have opportunities to make connections among fraction representations by connecting concrete or pictorial representations with spoken or symbolic representations. • Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., *thirds* means "three equal parts of a whole" or $\frac{1}{3}$ represents one of three equal-size parts when a pizza is shared among three	**All students should** • Understand that fractional parts are equal shares of a whole or a whole set. • Understand that the fraction name (*half*, *fourth*) tells the number of equal parts in the whole. • Understand that when working with unit fractions, the larger the denominator, the smaller the part and therefore the smaller the fraction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize fractions as representing equal-size parts of a whole. • Identify the fractional parts of a whole or a set for $\cancel{\frac{1}{2}}$, $\cancel{\frac{1}{3}}$, $\cancel{\frac{1}{4}}$, $\cancel{\frac{1}{6}}$, $\cancel{\frac{1}{8}}$, ~~and~~ $\cancel{\frac{1}{10}}$ $\frac{2}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{6}$, $\frac{7}{8}$, $\frac{7}{10}$, etc. • Identify the fraction names (halves, thirds, fourths, sixths, eighths, tenths) for the fraction notations $\cancel{\frac{1}{2}}$, $\cancel{\frac{1}{3}}$, $\cancel{\frac{1}{4}}$, $\cancel{\frac{1}{6}}$, $\cancel{\frac{1}{8}}$, ~~and~~ $\cancel{\frac{1}{10}}$ $\frac{2}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{6}$, $\frac{7}{8}$, $\frac{7}{10}$, etc. • Represent fractional parts of a whole for $\cancel{\frac{1}{2}}$, $\cancel{\frac{1}{3}}$, $\cancel{\frac{1}{4}}$, $\cancel{\frac{1}{6}}$, $\cancel{\frac{1}{8}}$, ~~and~~ $\cancel{\frac{1}{10}}$ halves, thirds, fourths, sixths, eighths, tenths using – region/area models (e.g., pie pieces, pattern blocks, geoboards); – sets (e.g., chips, counters, cubes); and – measurement models (e.g., fraction strips, ~~cuisenaire~~ rods, connecting cubes). • Compare unit fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$) using the words *greater than*, *less than* or *equal to*

2.3 The student will

a) identify the parts of a set and/or region that represent fractions for halves, thirds, fourths, sixths, eighths, and tenths;

b) write the fractions; and

c) compare the unit fractions for halves, thirds, fourths, sixths, eighths, and tenths.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
students) will further this development. • A unit fraction is one in which the numerator is one. • Using models when comparing unit fractions will assist in developing the concept that the larger the denominator the smaller the piece; therefore, $\frac{1}{3} > \frac{1}{4}$.		and the symbols (>, <, =).

2.4 The student will

a) count forward by twos, fives, and tens to 100, starting at various multiples of 2, 5, or 10;
b) count backward by tens from 100; and
c) recognize even and odd numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The patterns developed as a result of grouping and/or skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, multiplication, and division. Powerful models for developing these concepts include counters, hundred chart, and calculators. • Skip counting by twos supports the development of the concept of even numbers. • Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five. • Skip counting by tens is a precursor for use of place value, addition, counting money, and multiplying by multiples of 10. • Calculators can be used to display the numeric patterns resulting from skip counting. Use the constant feature of the four-function calculator to display the numbers in the sequence when skip counting by that constant. ~~For example, when skip counting by twos, press 2 + 2 = = = … to produce 2, 4, 6, 8, 10, …; or when skip counting by fives, press 5 + 5 = = = … to produce 5, 10, 15, ….~~ • Odd and even numbers can be explored in different ways (e.g., dividing collections of objects into two equal groups or pairing objects).	**All students should** • Understand that collections of objects can be grouped and skip counting can be used to count the collection. • Describe patterns in skip counting and use those patterns to predict the next number in the counting sequence. • <u>Understand that the starting point for skip counting by 2 does not always begin at 2.</u> • <u>Understand that the starting point for skip counting by 5 does not always begin at 5.</u> • <u>Understand that the starting point for skip counting by 10 does not always begin at 10.</u> • <u>Understand that every counting number is either even or odd.</u>	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine patterns created by counting by twos, fives, and tens on a hundred chart. • Skip count by twos, fives, and tens to 100, using manipulatives, a hundred chart, mental mathematics, <u>a calculator,</u> and/or paper and pencil. • Skip count by twos, fives, and tens to 100~~, using the constant feature on the calculator~~. • Count backward by tens from 100. • ~~Group objects by threes.~~ • ~~Group objects by fours.~~ • Use objects to determine whether a number is odd or even.

A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as “How many books do we have altogether?” or “How many cookies are left if I eat two?” or “I have three more candies than you do.” Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling ~~and~~, discussing, and recording a variety of problem situations. This approach ~~to~~ helps students ~~move~~ transition from the concrete to the representation to the symbolic ~~the abstract~~ ~~and~~ in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, multiplication, division ~~facts~~ and related facts ~~families~~;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions ~~and decimals~~.

2.5 The student will recall addition facts with sums to 20 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Associate the terms *addition, adding,* and *sum* with the concept of joining or combining. • Associate the terms *subtraction, subtracting, minus*, and *difference* with the process of "taking away" or separating (i.e., removing a set of objects from the given set of objects, finding the difference between two numbers, or comparing two numbers). • Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include – counting on; – counting back; – "one-more-than," "two-more-than" facts; – "one-less-than," "two-less-than" facts; – "doubles" to recall addition facts (e.g., 2 + 2 =__; 3 + 3 =__); – "near doubles" [e.g., 3 + 4 = (3 + 3) + 1 = __]; – "make-ten" facts (e.g., at least one addend of 8 or 9); – "think addition for subtraction," (e.g., for 9 – 5 = __, think "5 and what number makes 9?"); – use of the commutative property, without naming the property (e.g., 4 +3 is the same as 3 + 4); – use of related facts ~~families~~ (e.g., 4 + 3 = 7 , 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4); – use of the ~~zero property~~ additive identity property (e.g., 4 + 0 = 4), without naming the property but saying, "When you add zero to a number, you always get the original number." • Manipulatives should be used initially to develop an understanding of addition and subtraction facts and	**All students should** • Understand that addition involves combining and subtraction involves separating. • Develop fluency in recalling ~~basic~~ facts for addition and subtraction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Recall and write the basic addition facts for sums to 18 or less and the corresponding subtraction facts.~~ • Recall and write the basic addition facts for sums to ~~18~~ 20 or less and the corresponding subtraction facts, when addition or subtraction problems are presented in either horizontal or vertical written format.

2.5 The student will recall addition facts with sums to 20 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
to engage students in meaningful memorization. Rote recall of the facts is often achieved through constant practice and may come from a variety of formats, including presentation through counting on, related facts, flash cards, practice sheets, and/or games.		

2.6 The student, given two whole numbers whose sum is 99 or less, will
a) estimate the sum; and
b) find the sum, using various methods of calculation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimation is a number sense skill used instead of finding an exact answer. When an actual computation is not necessary, an estimate will suffice. • Rounding is one strategy used to estimate. • Estimation is also used before solving a problem to check the reasonableness of the sum when an exact answer is required. • By estimating the result of an addition problem, a place value orientation for the answer is established. • Strategies for mentally adding two-digit numbers ~~mentally~~ include student-invented strategies, making-ten, partial sums and counting on, among others. – partial sums: 56 + 41 = __ 50 + 40 = 90 6 + 1 = 7 90 + 7 = 97 – counting on: 36 + 62 = __ 36 + 60 = 96 96 + 2 = 98 • Addition means to combine or join quantities. • The ~~standard algorithm for addition is~~ terms used in addition are 23 → *addend* + 46 → *addend* 69 → *sum* • Strategies for adding two-digit numbers can include, but are not limited to, using a hundreds chart, number line, and invented strategies.	**All students should** • Understand that estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed. • Understand that estimation skills are also valuable in determining the reasonableness of the sum when solving for the exact answer is needed. • Understand that addition is used to join groups in practical situations when exact answers are needed. • Develop flexible methods of adding whole numbers by combining numbers in a variety of ways to find the sum, most depending on place values.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Regroup 10 ones for 1 ten, using ~~b~~Base-10 models, when finding the sum of two whole numbers whose sum is 99 or less. • Estimate the sum of two whole numbers whose sum is 99 or less and recognize whether the estimation is reasonable. • ~~Determine~~ Find the sum of two whole numbers whose sum is 99 or less, using ~~b~~Base-10 models, such as ~~b~~Base-10 blocks and bundles of tens. • Solve problems presented vertically or horizontally that require finding the sum of two whole numbers whose sum is 99 or less, using paper and pencil. • Solve problems, using mental computation strategies, involving addition of two whole numbers whose sum is 99 or less.

2.6 The student, given two whole numbers whose sum is 99 or less, will
a) estimate the sum; and
b) find the sum, using various methods of calculation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Building an understanding of the algorithm by first using concrete materials and then a do-and-write approach connects it to the written form of the algorithm. • The traditional algorithm for two-digit numbers is contrary to the natural inclination to begin with the left-hand number. • Regrouping is used in addition when a sum in a particular place value is 10 or greater.		

2.7 The student, given two whole numbers, each of which is 99 or less, will
a) estimate the difference; and
b) find the difference, using various methods of calculation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimation is a number sense skill used instead of finding an exact answer. When an estimate is needed, the actual computation is not necessary. • Rounding is one strategy used to estimate. • Estimation is also used before solving a problem to check the reasonableness of the sum when an exact answer is required. • ~~Giving an estimate of~~ By estimat~~e~~ing the result of a subtraction problem, ~~establishes~~ a place value orientation for the answer is established. • Subtraction is the inverse operation of addition and is used for different reasons: – to remove one amount from another; – to compare one amount to another; and – to find the missing quantity when the whole quantity and part of the quantity are known. • Three terms often used ~~to discuss~~ in subtraction are *minuend* → 98 *subtrahend* → – 41 *difference* → 57 • Regrouping is a process of renaming a number to make subtraction easier. • An understanding of the subtraction algorithm should be built by first using concrete materials and then employing a do-and-write approach (i.e., use the manipulatives, then record what you have done). This connects the activity to the written form of the algorithm. • Mental computational strategies for subtracting two-	**All students should** • Understand that estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed. • Understand that estimation skills are also valuable in determining the reasonableness of the difference when solving for the exact answer is needed. • Understand that subtraction is used in practical situations when exact answers are needed. • Develop flexible methods of subtracting whole numbers to find the difference, by combining numbers in a variety of ways, most depending on place values.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Regroup 1 ten for 10 ones, using ~~b~~Base-10 models, such as ~~b~~Base-10 blocks and bundles of tens. • Estimate the difference of two whole numbers each 99 or less and recognize whether the estimation is reasonable. • ~~Determine~~ Find the difference of two whole numbers each 99 or less, using ~~b~~Base-10 models, such as ~~b~~Base-10 blocks and bundles of tens. • Solve problems presented vertically or horizontally that require finding the difference between two whole numbers each 99 or less, using paper and pencil. • Solve problems, using mental computation strategies, involving subtraction of two whole numbers each 99 or less.

2.7 The student, given two whole numbers, each of which is 99 or less, will
a) estimate the difference; and
b) find the difference, using various methods of calculation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
digit numbers might include – lead-digit or front-end strategy: 56 – 21 = __ 50 – 20 = 30 6 – 1 = 5 30 + 5 = 35 – counting up: 87 – 25 = __ 20 + 60 = 80 5 + 2 = 7 60 + 2 = 62 or 87 – 25 = __ 25 + 60 = 85 85 + 2 = 87 60 + 2 = 62 or 87 – 25 = __ 25 + 2 = 27 27 + 60 = 87 2 + 60 = 62 – partial differences: 98 – 41 = __ 90 – 40 = 50 8 – 1 = 7 50 + 7 = 57. • Strategies for subtracting two-digit numbers may include using a hundreds chart, number line, and invented strategies.		

2.8 The student will create and solve one- and two-step addition and subtraction problems, using data from simple tables, picture graphs, and bar graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Problem solving means engaging in a task for which a solution or a method of solution is not known in advance. Solving problems using data and graphs offers a natural way to connect mathematics to practical situations. • The ability to retrieve information from simple charts and picture graphs is a necessary prerequisite to solving problems. • An example of an approach to solving problems is Polya's four-step plan: – Understand: Retell the problem. – Plan: Decide what the operation is. – Solve: Write a number sentence. – Look back: Does the answer make sense? • The problem solving process is enhanced when students – create their own story problems; and – model word problems, using manipulatives or drawings.	**All students should** • Develop strategies for solving practical problems. • Enhance problem solving skills by creating their own problems.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the appropriate data and the operation needed to solve an addition or subtraction problem where the data are presented in a simple table, picture graph, or bar graph. • Solve addition and subtraction problems requiring a one- or two-step solution, using data from simple ~~charts~~ tables, picture graphs, bar graphs, and everyday life situations. • Create a one- or two-step addition or subtraction problem using data from simple tables, picture graphs, and bar graphs whose sum is 99 or less . ~~For subtraction, the difference will be between two whole numbers each 99 or less.~~

2.9 The student will recognize and describe the related facts that represent and describe the inverse relationship between addition and subtraction.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Addition and subtraction are inverse operations, that is, one undoes the other: 3 + 4 = 7 7 – 3 = 4 7 – 4 = 3 4 + 3 = 7 • For each addition fact, there is a related subtraction fact. • Developing strategies for solving missing addends problems and the missing part of subtraction facts ~~(e.g., 7 – __ = 3)~~ builds an understanding of the link between addition and subtraction. To solve 9 – 5 = __, think 5 + __ = 9. • Demonstrate joining and separating sets to investigate the relationship between addition and subtraction.	**All students should** • Understand how addition and subtraction relate to one another.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the missing number in a number sentence (e.g., 3 + __ = 5 or __+ 2 = 5; 5 – __ = 3 or 5 – 2 = __). • Write the related facts for a given addition or subtraction fact (e.g., given 3 + 4 = 7, write 7 – 4 = 3 and 7 – 3 = 4).

Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

2.10 The student will
a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and
b) correctly use the cent symbol (¢), dollar symbol ($), and decimal point (.).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The money system used in the United States consists of coins and bills based on ones, fives, and tens, making it easy to count money. • The dollar is the basic unit. • Emphasis is placed on the verbal expression of the symbols for cents and dollars (e.g., $0.35 and 35¢ are both read as "thirty-five cents"; $3.00 is read as "three dollars"). • Money can be counted by grouping coins and bills to determine the value of each group and then adding to determine the total value. • The most common way to add amounts of money is to "count on" the amount to be added.	**All students should** • Understand how to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less. • Understand the proper use of the cent symbol (¢), dollar sign ($), and decimal point (.).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify all coins and a one-dollar bill, recording the value, using the cent symbol (¢), dollar symbol ($), and decimal point (.).~~ • Determine the value of a collection of coins and one-dollar bills whose total value is $2.00 or less. • Compare the values of two sets of coins and one-dollar bills (each set having a total value of $2.00 or less), using the terms *greater than, less than*, or *equal to*. • Simulate everyday opportunities to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less. • <u>Use the cent (¢) and dollar ($) symbols and decimal point (.) to write a value of money which is $2.00 or less.</u>

2.11 The student will estimate and measure
- **a) length to the nearest centimeter and inch;**
- **b) weight/mass of objects in pounds/ounces and kilograms/grams, using a scale; and**
- **c) liquid volume in cups, pints, quarts, gallons, and liters.**

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A clear concept of the size of one unit ~~(one inch or one centimeter)~~ is necessary before one can measure to the nearest unit. • Knowledge of the exact relationships within the metric or U.S. Customary system of measurement for measuring liquid volume, such as 4 cups to a quart, is not required at this grade level. • Practical experiences measuring liquid volume, using a variety of actual measuring devices (e.g., containers for a cup, pint, quart, gallon, and liter), will help students build a foundation for estimating liquid volume with these measures. • The experience of making a ruler can lead to greater understanding of using one. • Proper placement of a ruler when measuring length (i.e., placing the end of the ruler at one end of the item to be measured) should be demonstrated. • ~~Perimeter is the distance around any two-dimensional shape and is found by adding the measures of the sides.~~ • ~~Linear metric measurement is based on the meter (a base-10 system) and includes centimeters.~~ • ~~Linear standard measurement is based on the foot and includes inches.~~ • ~~A polygon is a simple closed figure lying on a plane, such as a triangle or square. "Simple" means that the lines do not cross.~~ • Weight and mass are different. Mass is the amount	**All students should** • <u>Understand that centimeters/inches are units used to measure length.</u> • Understand how to estimate ~~linear measures and how to use a ruler to~~ <u>and measure to</u> determine a linear measure to the nearest centimeter and inch. • <u>Understand that pounds/ounces and kilograms/grams are units used to measure weight/mass.</u> • Understand how to use a scale to determine the weight/mass of an object and use the appropriate unit for measuring weight/mass. • <u>Understand that cups, pints, quarts, gallons, and liters are units used to measure liquid volume.</u> • Understand how to use measuring devices to determine liquid volume in both metric and customary units. • ~~Understand how to compare measures.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify an inch as a U.S. customary unit for measuring length.~~ • Estimate and measure the length of various line segments and objects to the nearest inch <u>and centimeter</u>. • ~~Identify a centimeter as a metric unit for measuring length.~~ • ~~Estimate and measure the length of various line segments and objects to the nearest centimeter.~~ • ~~Measure each side of a variety of concrete polygons and add them to determine the distance around the polygon (its perimeter).~~ • ~~Determine the distance around a polygon (its perimeter), given the measurements of the sides in centimeters or inches.~~ • ~~Identify pound as the U.S. customary units for measuring weight.~~ • ~~Identify kilogram as a metric unit for measuring mass.~~ • Estimate and then measure the <u>weight/</u>mass of f~~amiliar~~ objects to the nearest pound<u>s/ounces and kilograms/grams</u>, using a scale. • ~~Estimate and then measure the weight/mass of familiar objects to the nearest kilogram, using a scale.~~

2.11 The student will estimate and measure
a) length to the nearest centimeter and inch;
b) weight/mass of objects in pounds/ounces and kilograms/grams, using a scale; and
c) liquid volume in cups, pints, quarts, gallons, and liters.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • ~~Mass is more difficult for young children to understand than length.~~ • A balance is a scale for measuring mass. To determine the mass of an object by using a two-pan balance, first level both sides of the balance by putting standard units of mass on one side to counterbalance the object on the other; then find the sum of the standard units of mass required to level the balance. • Benchmarks of common objects need to be established for one pound and one kilogram ~~(e.g., the mass of a math book is about one kilogram).~~ Practical experience measuring the mass of familiar objects helps to establish benchmarks. • Pounds and kilograms are not compared at this level. • The terms *cups, pints, quarts, gallons*, and *liters* are introduced as terms used to describe the liquid volume of everyday containers. • The exact relationship between a quart and a liter is not expected at this level.		• ~~Identify the metric and U.S. customary units~~ for Estimate and measure ~~measuring~~ liquid volume in ~~(e.g.,~~ cups, pints, quarts, gallons, and liters~~)~~. • ~~Compare customary and metric units of liquid volume (e.g., cups to quarts, liters to quarts), using actual measuring devices and the concepts of *more*, *less*, and *equivalent*.~~

2.12 The student will tell and write time to the nearest five minutes, using analog and digital clocks.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Telling time requires reading a clock. The position of the two hands on an analog clock is read ~~with the little hand indicating the hour and the long hand indicating the number of minutes after or before an hour~~ to tell the time. A digital clock shows the time by displaying the time in numbers which are read as the hour and minutes. • ~~Time is a unit of measure.~~ • The use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise at all times. • The face of an analog clock can be divided into 4 equal parts, called *quarter hours*, of 15 minutes each.	**All students should** • Apply an appropriate technique to determine time to the ~~quarter hour~~ nearest five minutes, using analog and digital clocks. • Demonstrate an understanding of counting by fives to predict five minute intervals when telling time to the nearest five minutes.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Show, ~~and~~ tell, and write time to the ~~quarter hour~~ nearest five minutes, using an ~~model~~ analog and digital clock. • ~~Write the time indicated on a digital clock to the nearest quarter hour.~~ • ~~Write the time indicated on an analog clock to the nearest quarter hour.~~ • Match a written time to a time shown on a clock face to the ~~quarter hour~~ nearest five minutes.

2.13 The student will
a) determine past and future days of the week; and
b) identify specific days and dates on a given calendar.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The calendar is a way to represent ~~measure~~ units of time (e.g., days, weeks, and months). • Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset. • Practical situations are appropriate to develop a sense of the interval of time between events (e.g., Boy Scout meetings occur every week on Monday: there is a week between meetings).	**All students should** • Understand how to use a calendar as a way to measure time.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Read a calendar to locate a given day or date.~~ • ~~Identify the seven days in a week.~~ • Determine the days/dates before and after a given day/date. • Determine the ~~date~~ day that is a specific number of days or weeks in the past or in the future from a given date, using a calendar. • Identify specific days and dates (e.g., the third Monday in a given month or what day of the week does May 11 fall on).

2.14 The student will read the temperature on a Celsius and/or Fahrenheit thermometer to the nearest 10 degrees.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The symbols for degrees in Celsius (°C) and degrees in Fahrenheit (°F) should be used to write temperatures. • Celsius and Fahrenheit temperatures should be related to everyday occurrences by measuring the temperature of the classroom, the outside, liquids, body temperature, and other things found in the environment. • Estimating and measuring temperatures in the environment in Fahrenheit and Celsius require the use of real thermometers. • A physical model can be used to represent the temperature determined by a real thermometer.	**All students should** • Understand how to measure temperature in Celsius and Fahrenheit with a thermometer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Read temperature to the nearest 10 degrees from real Celsius and Fahrenheit thermometers and from physical models (including pictorial representations) of such thermometers.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of ~~shapes~~ figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent ~~two-and three-dimensional~~ plane and solid ~~shapes~~ figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting, and investigating ~~three-dimensional~~ solid objects and their ~~two-dimensional~~ faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying ~~shapes~~ figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these ~~shapes~~ figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

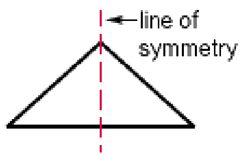
The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

2.15 The student will
a) draw a line of symmetry in a figure; and
b) identify and create figures with at least one line of symmetry.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- A figure ~~or shape~~ is symmetric along a line when one-half of the figure is the mirror image of the other half.
- A line of symmetry divides a symmetrical figure, object, or arrangement of objects into two parts that are congruent if one part is reflected ~~(flipped)~~ over the line of symmetry.



- Children learn about symmetry through hands-on experiences with geometric ~~shapes~~ figures and the creation of geometric pictures and patterns. ~~while playing with shapes.~~
- Guided explorations of the study of symmetry by using mirrors, miras, paper folding, and pattern blocks will enhance students' understanding of the attributes of symmetrical ~~shapes~~ figures.
- While investigating symmetry, children move ~~shapes~~ figures, such as pattern blocks, intuitively, thereby exploring transformations of those ~~shapes~~ figures. A transformation is the movement of a figure — either a translation, rotation, or reflection. A translation ~~(slide)~~ is the result of sliding a figure in any direction; rotation ~~(turn)~~ is the result of turning a figure around a point or a vertex; and reflection ~~(flip)~~ is the result of flipping a figure over a line.

ESSENTIAL UNDERSTANDINGS

All students should

- Develop strategies to determine whether or not a figure has at least one ~~a~~ line of symmetry.
- Develop strategies to create figures with at least one line of symmetry.
- Understand that some figures may have more than one line of symmetry.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify ~~and create~~ figures ~~that are symmetrical along a line~~ with at least one line of symmetry, using various concrete materials.
- Draw ~~the~~ a line~~(s)~~ of symmetry — horizontal, vertical, and diagonal — in a figure.
- ~~Identify and demonstrate a line of symmetry in an object or an arrangement of objects.~~
- ~~Investigate~~ Create figures with at least one line of symmetry using various concrete materials ~~using (e.g., paper folding, mirrors/miras, pattern blocks, wax paper, or tracing paper)~~.

2.16 The student will identify, describe, compare, and contrast plane and solid geometric figures (circle/sphere, square/cube, and rectangle/rectangular prism).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. – **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. – **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know that a door is a rectangle."). – **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it is closed; it has four sides and four right angles, and opposite sides are parallel."). • An important part of geometry is naming and describing ~~shapes~~ figures in two-dimensions (plane ~~shapes~~ figures) and three-dimensions (solid ~~shapes~~ figures). • A vertex is a point where two or more line segments, lines, or rays meet to form an angle. • An angle is two rays that share an endpoint. • Plane figures are two-dimensional figures formed by lines that are curved, straight, or a combination of both. They have angles and edges.	**All students should** • Understand the differences between plane and solid figures while recognizing the inter-relatedness of the two. • Understand that a solid figure is made up of a set of plane figures.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine similarities and differences between related plane and solid ~~shapes~~ figures (e.g., circle/sphere, square/cube, rectangle/rectangular ~~solid~~ prism, using models and cutouts. • Trace faces of solid ~~shapes~~ figures (e.g., cube and rectangular solid) to create the set of plane figures related to the solid ~~shape~~ figure. • Identify and describe plane and solid figures (e.g., circle/sphere, square/cube, and rectangle/rectangular ~~solid~~ prism), according to the number and shape of their faces, edges, and vertices using models. • Compare and contrast plane and solid geometric ~~shapes~~ figures (e.g., circle/sphere, square/cube, and rectangle/rectangular ~~solid~~ prism) according to the number and shape of their faces (sides, bases), edges, vertices, and ~~corners~~ angles.

2.16 The student will identify, describe, compare, and contrast plane and solid geometric figures (circle/sphere, square/cube, and rectangle/rectangular prism).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The identification of ~~two-~~plane and ~~three-dimensional~~ solid figures is accomplished by working with and handling objects. • Tracing faces of solid figures is valuable to understanding the set of plane figures related to the solid figure (e.g., cube and rectangular prism). • A circle is a closed curve in a plane with all its points the same distance from the center. • A sphere is a ~~three-dimensional (~~solid~~)~~ figure with all of its points the same distance from its center. • A square is a rectangle with four sides of equal length. • A rectangular prism is a solid in which all six faces are rectangles. A rectangular prism has 8 vertices and 12 edges. • A cube is a ~~three-dimensional (~~solid~~)~~ figure with six congruent, square faces. All edges are the same length. A cube has 8 vertices and 12 edges. It is a rectangular prism. • A rectangle is a plane figure with four right angles. A square is a rectangle. • ~~A square pyramid is a three-dimensional (solid) figure that has a square base and whose other faces are triangles that share a common vertex.~~ • ~~A cylinder is a three-dimensional (solid) figure with two congruent, parallel circular surfaces joined by a curved surface.~~ • ~~A cone is a three-dimensional (solid), pointed figure that has a flat, round base (usually a circle) that is joined to a vertex by a curved surface.~~		

2.16 The student will identify, describe, compare, and contrast plane and solid geometric figures (circle/sphere, square/cube, and rectangle/rectangular prism).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The edge is the line segment where two faces of a ~~three-dimensional~~ solid figure ~~meet~~ intersect. • A face is a polygon that serves as one side of a ~~three-dimensional~~ solid figure (e.g., a square is a face of a cube). • ~~A corner is the point at which three or more edges meet.~~ • A base is a special face of a ~~three-dimensional~~ ~~(~~solid~~)~~ figure. • The relationship between plane and solid geometric figures, such as the square and the cube or the rectangle and the rectangular ~~solid,~~ prism helps build the foundation for future geometric study of faces, edges, angles, and vertices.		

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

2.17 The student will use data from experiments to construct picture graphs, pictographs, and bar graphs.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The purpose of a graph is to represent data gathered to answer a question. • ~~Picture and bar graphs are created to communicate the data collected.~~ • Picture graphs are graphs that use pictures to show and compare information. ~~A key should be used to indicate what the picture symbol represents (e.g., one picture of a sneaker represents five sneakers in a graph of shoe types).~~ • Pictographs are graphs that use symbols/pictures to show and compare information. A student can be represented as a stick figure in a pictograph. A key should be used to indicate what the picture symbol represents (e.g., one picture of a sneaker represents five sneakers in a graph of shoe types). • Bar graphs are used to compare counts of different categories (categorical data). Using grid paper may ensure~~s~~ more accurate graphs. – A bar graph uses parallel, horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis displaying the scale that represents the count for the categories should extend one increment above the greatest recorded piece of data. Second grade students should be collecting data that are recorded in increments of whole numbers, usually multiples of 1, ~~or~~ 2, or 5. – Each axis should be labeled, and the graph should be given a title.	**All students should** • Understand that data may be generated from experiments. • Understand how data can be collected and organized in picture graphs, pictographs, ~~or~~ and bar graphs. • Understand that picture graphs use pictures to show and compare data. • Understand that pictographs use a symbol/representation of an object, person, etc. • Understand that bar graphs can be used to compare categorical data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Organize data from experiments, using lists, tables, objects, pictorial representations, tally marks, and charts, in order to construct a graph. • Read the information presented horizontally and vertically on ~~a simple~~ picture graphs, pictographs, ~~or~~ and bar graphs. • Collect no more than 16 pieces of data to answer a given question. • Represent data from experiments by constructing ~~a simple~~ picture graphs, pictographs, ~~or~~ and bar graphs. • Label the axes on a bar graph, limiting the number of categories (categorical data) to four and the increments to multiples of whole numbers (e.g., multiples of 1, 2, or 5). • Label the axes on a picture graph and pictograph, limiting the number of categories to four and including a key where appropriate. • ~~Interpret information from simple picture and bar graphs by writing at least one statement that covers one or both of the following:~~ ~~– Describe the categories of data and the data as a whole (e.g., the total number of responses).~~ ~~– Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same.~~ • ~~Select the best interpretation of a graph from a set of possible interpretations of the graph.~~

2.17 The student will use data from experiments to construct picture graphs, pictographs, and bar graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written.~~ • ~~When data is displayed in an organized manner, the results of investigations can be described, and the questions posed can be answered.~~		

2.18 The student will use data from experiments to predict outcomes when the experiment is repeated.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in investigations and have opportunities to use manipulatives. • Investigation of experimental probability is continued through informal activities, such as dropping a two-colored counter (usually a chip that has a different color on each side), using a multicolored spinner (a circular spinner that is divided equally into two, three, four or more equal "pie" parts where each part is filled with a different color), using spinners with numbers, or rolling random number generators (dice). • Probability is the chance of an event occurring (e.g., the probability of landing on a particular color when flipping a two-colored chip is $\frac{1}{2}$, representing one of two possible outcomes). • An event is a possible outcome in probability. Simple events include the possible outcomes when tossing a coin (heads or tails), when rolling a random number generator (a number cube or a die where there are six equally likely outcomes and the probability of one outcome is $\frac{1}{6}$), or when spinning a spinner. • If all the outcomes of an event are equally likely to occur, the probability of an event is equal to the number of favorable outcomes divided by the total number of possible outcomes: the probability of the event = $\dfrac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$.	**All students should** • Understand that data may be generated from experiments. • ~~Understand and apply basic concepts of probability.~~ • Understand that the likelihood of an event occurring is to ~~determine~~ predict the probability of it happening.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Conduct probability experiments, using multicolored spinners, colored tiles, or number cubes and use the data from the experiments to predict outcomes if the experiment is repeated. • Record the results of probability experiments, using tables, charts, and tally marks. • Interpret the results of probability experiments (e.g., the two-colored spinner landed on red 5 out of 10 times). • Predict which of two events is more likely to occur if an experiment is repeated.

2.18 The student will use data from experiments to predict outcomes when the experiment is repeated.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• At this level, students need to understand only this fractional representation of probability (e.g., the probability of getting heads when flipping a coin is $\frac{1}{2}$). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as*, ~~*unlikely*~~, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include practical examples.		

2.19 The student will analyze data displayed in picture graphs, pictographs, and bar graphs.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Statements that represent an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be discussed with students and written. • When data ~~is~~ <u>are</u> displayed in an organized manner, the results of investigations can be described, and the questions posed can be answered.	**All students should** • <u>Understand how to read the key used in a graph to assist in the analysis of the displayed data.</u> • <u>Understand how to interpret data in order to analyze it.</u> • <u>Understand how to analyze data in order to answer the questions posed, make predictions, and generalizations.</u>	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Interpret~~ Analyze information from simple picture graphs, pictographs, and bar graphs by writing at least one statement that covers one or both of the following: – Describe the categories of data and the data as a whole (e.g., the total number of responses). – Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same. • Select the best analysis of a graph from a set of possible analyses of the graph.

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns ~~in the real world~~. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to ~~A~~algebra.

2.20 The student will identify, create, and extend a wide variety of patterns.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Identifying and extending patterns is an important process in mathematical thinking. • Analysis of patterns in the real world (e.g., patterns on a butterfly's wings, patterns on a ladybug's shell) leads to the analysis of mathematical patterns such as number patterns and geometric patterns. • Reproduction of a given pattern in a different manifestation, using symbols and objects, lays the foundation for writing numbers symbolically or algebraically. • The simplest types of patterns are repeating patterns. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. • Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats. • In numeric patterns, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Create an arithmetic number pattern. Sample numeric patterns include – 6, 9, 12, 15, 18, …(growing pattern); – ~~5, 7, 9, 11, 13~~ 20, 18, 16, 14, … (~~repeating numeric~~ growing pattern); ~~and~~ – 1, 2, 4, 7, 11, 16, … (growing ~~numeric~~ pattern)~~.~~; and – 1, 3, 5, 1, 3, 5, 1, 3, 5… (repeating pattern).	**All students should** • Understand patterns are a way to recognize order and to predict what comes next in an arrangement. • Analyze how both repeating and growing patterns are generated.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a growing and/or repeating pattern from a given geometric or numeric sequence. • Predict the next number, geometric figure, symbol, picture, or object in a given pattern. • Extend a given pattern, using numbers, geometric figures, symbols, pictures, or objects. • Create a new pattern, using numbers, geometric figures, pictures, symbols, or objects. • Recognize the same pattern in different manifestations.

2.20 The student will identify, create, and extend a wide variety of patterns.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation ~~(turn)~~ is the result of turning a figure around a point or a vertex, and reflection ~~(flip)~~ is the result of flipping a figure over a line.		

2.21 The student will solve problems by completing numerical sentences involving the basic facts for addition and subtraction. The student will create story problems, using the numerical sentences.

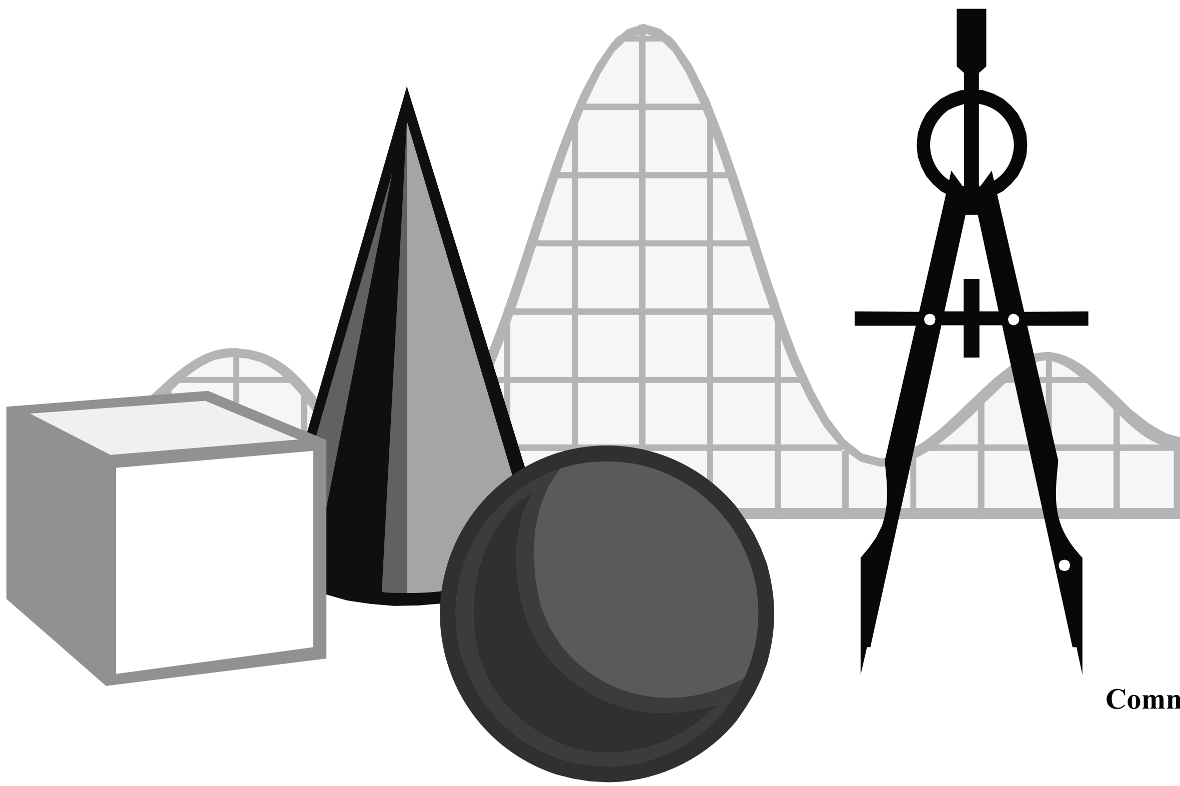
UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Recognizing and using patterns and learning to represent situations mathematically are important aspects of primary mathematics. • Discussing what a word problem is saying and writing a number sentence are precursors to solving word problems. • The patterns formed by related basic facts facilitate the solution of problems involving a missing addend in an addition sentence or a missing part (subtrahend) in a subtraction sentence. • Making mathematical models to represent simple addition and subtraction problems facilitates their solution. • By using story problems and numerical sentences, students begin to explore forming equations and representing quantities using variables. • Students can begin to understand the use of a symbol (e.g., __, ?, or ☐) to represent an unknown quantity.	**All students should** • Use mathematical models to represent and understand quantitative relationships. • Understand various meanings of addition and subtraction and the relationship between the two operations. • Understand how to write missing addend and missing subtrahend sentences.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve problems by completing a numerical sentence involving the basic facts for addition and subtraction (e.g., 3 + __ = 7, or 9 – __ = 2). • Create a story problem for a given numerical sentence.

2.22 The student will demonstrate an understanding of equality by recognizing that the symbol = in an equation indicates equivalent quantities and the symbol ≠ indicates that quantities are not equivalent.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The = symbol means that the values on either side are the same (balanced). • The ≠ symbol means that the values on either side are not the same (not balanced). • In order for students to develop the concept of equality, students need to see the = symbol used in various locations (e.g., 3 + 4 = 7 and 5 = 2 + 3). • A number sentence is an equation with numbers (e.g., 6 + 3 = 9; or 6 + 3 = 4 + 5).	**All students should** • Understand that the equal symbol means equivalent (same as) quantities. • The inequality symbol (≠) means not equivalent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the equality (=) and inequality (≠) symbols. • Identify equivalent values and equations. (e.g., 8 = 8 and 8 = 4 + 4) • Identify nonequivalent values and equations. (e.g., 8 ≠ 9 and 4 + 3 ≠ 8) • Identify and use the appropriate symbol to distinguish between equal and not equal quantities. (e.g., 8 + 2 = 7 + 3 and 1 + 4 ≠ 6 + 2)

MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 3



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, ~~simple~~ fractions, number relationships ("more than," "less than," and "~~as many as~~ equal to"), and the effects of ~~simple~~ single-step and multistep computations ~~operations on numbers (fact families)~~. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

3.1 The student will

a) read and write six-digit numerals and identify the place value and value of each digit;
b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and
c) compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the ~~b~~<u>B</u>ase-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. • The structure of the ~~b~~<u>B</u>ase-10 blocks is based on the ten-to-one place value relationship (e.g., 10 ~~cubes~~ units make a long, 10 longs make a flat, 10 flats make a ~~big~~ cube). • Place value refers to the value of each digit and depends upon the position of the digit in the number. In the number 7,864, the eight is in the hundreds place, and the value of the 8 is eight hundred. • Flexibility in thinking about numbers — or "decomposition" of numbers (e.g., 12,345 is 123 hundreds, 4 tens, and 5 ones) — is critical and supports understandings essential to multiplication and division. • Whole numbers may be written in a variety of formats: – Standard: 123,456; – Written: one hundred twenty-three thousand, four hundred fifty-six; and – Expanded: (1 × 100,000) + (2 × 10,000) + (3 × 1,000) + (4 × 100) + (5 × 10) + (6 × 1). • Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, and so on). Places within the periods repeat (hundreds,	**All students should** • Understand that knowledge of place value is essential when comparing numbers. • Understand the relationships in the place value system, where each place is ten times the value of the place to its right. • <u>Understand that rounding gives an estimate to use when exact numbers are not needed for the situation.</u> • Understand the relative magnitude of numbers by comparing numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and identify the place <u>and</u> value for each digit in a six-digit numeral, using ~~b~~<u>B</u>ase-10 manipulatives (e.g., ~~b~~<u>B</u>ase-10 blocks). • Use the patterns in the place value system to read and write numbers. • Read six-digit numerals orally. • Write six-digit numerals that are stated verbally or written in words. • <u>Round a given whole number, 9,999 or less, to the nearest ten, hundred, and thousand.</u> • <u>Solve problems, using rounding of numbers, 9,999 or less, to the nearest ten, hundred, and thousand.</u> • <u>Determine which of two whole numbers between 0 and 9,999 is greater.</u> • <u>Determine which of two whole numbers between 0 and 9,999 is less.</u> • <u>Compare two whole numbers between 0 and 9,999, using the symbols >, <, or =.</u> • <u>Use the terms *greater than, less than*, and *equal to* when comparing two whole numbers.</u>

3.1 The student will

a) read and write six-digit numerals and identify the place value and value of each digit;

b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*).

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
tens, ones). ~~The base-10 blocks of shapes repeat every period (e.g., cube – long – flat, big cube – big long – big flat).~~ Commas are used to separate the periods. Knowing the place value and period of a number helps students find the value of a digit~~s~~ in any number as well as read and write numbers. • To read a whole number through the hundred thousands place, – read the ~~number~~ digits to the first comma; – say the name of the period (e.g., “thousands”); ~~and~~ then – read the last three digits, but do not say the name of the ones period. • Reading and writing large numbers should be related to numbers that have meanings (e.g., numbers found in the students’ environment). Concrete materials, such as ~~b~~Base-10 blocks ~~and bundles of sticks,~~ may be used to represent whole numbers through thousands. Larger numbers may be represented ~~by symbol cards~~ on place value charts. • Rounding is one of the estimation strategies ~~and~~ that is often used to assess the reasonableness of a solution or to give ~~a rough idea~~ an estimate of an amount. • Students should explore reasons for estimation, using ~~real-life~~ practical experiences, and use rounding to solve ~~real-life~~ practical situations. • The concept of rounding may be introduced through the use of a number line. When given a number to round, locate it on the number line. Next, determine		

3.1 The student will
a) read and write six-digit numerals and identify the place value and value of each digit;
b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and
c) compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than, less than*, or *equal to*).

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
the multiple of ten, hundred, or thousand it is between. Then identify to which it is closer. • A procedure for rounding numbers to the nearest ten, hundred, or thousand is as follows: – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place, and change the digits to the right of the rounding place to zero. • A procedure for comparing two numbers by examining may include the following: – Line up the numbers by place value by lining up the ones. – Beginning at the left, find the first place value where the digits are different. – Compare the digits in this place value to determine which number is greater (or which is less). – Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented. – If both numbers are the same, use the symbol = or the words *equal to*.		

3.2 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. The student will use these relationships to solve problems.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition and subtraction are inverse operations, as are multiplication and division~~; that is, one undoes the other~~. • In building thinking strategies for subtraction, an emphasis is placed on connecting the subtraction fact to the related addition fact. The same is true for division, where the division fact is tied to the related multiplication fact. Building fact sentences helps strengthen this relationship. • Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • Multiplication and division should be taught concurrently in order to develop understanding of the inverse relationship.	**All students should** • Understand how addition and subtraction are related. • Understand how multiplication and division are related.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the inverse relationships between addition/subtraction and multiplication/division to solve related basic fact sentences. For example, $5 + 3 = 8$ and $8 - 3 = \_\_$; $4 \times 3 = 12$ and $12 \div 4 = \_\_$. • Write three related basic fact sentences when given one basic fact sentence for addition/subtraction and for multiplication/division. For example, given $3 \times 2 = 6$, ~~write~~ solve the related facts $\_\_ \times 3 = 6$, $6 \div 3 = \_\_$, and $6 \div \_\_ = 3$.

3.3 The student will
a) name and write fractions (including mixed numbers) represented by a model;
b) model fractions (including mixed numbers) and write the fractions' names; and
c) compare fractions having like and unlike denominators, using words and symbols (>, <, or =).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction is a way of representing part of a whole (as in a region/area model or a length/measurement model) or part of a group (as in a set model). Fractions are used to name a part of one thing or a part of a collection of things. Models can include pattern blocks, fraction bars, rulers, number line, etc. • In each ~~fraction~~ area/region and length/measurement model, the parts must be equal-sized (congruent) ~~(i.e., each pie piece must have the same area; the size of each chip in a set must be equal; the measures such as the red Cuisenaire rod or the connecting cube must be equal)~~. Wholes are ~~broken~~ divided or partitioned into equal-sized parts. ~~and reassembled into wholes.~~ In the set model, each member of the set is an equal part of the set. The members of the set do not have to be equal in size. • The denominator ~~(bottom number)~~ tells how many equal parts are in the whole or set. The numerator ~~(top number)~~ tells how many of those parts are being ~~described~~ considered. • ~~Students should have~~ Provide opportunities to make connections among fraction representations by connecting concrete or pictorial representations with oral language and symbolic representations. • Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the	**All students should** • Understand that the whole must be defined. • Understand that the denominator tells the number of equal parts ~~in~~ that represent a whole. • Understand that the numerator is a counting number that tells how many equal size parts are being considered. • Understand that the value of a fraction is dependent on both the number of parts in a whole (denominator) and the number of those parts being considered (numerator). • Understand that a proper fraction is a fraction whose numerator is smaller than its denominator. • Understand that an improper fraction is a fraction whose numerator is greater than or equal to the denominator and is one or greater than one. • Understand that an improper fraction can be expressed as a whole number or a mixed number. • Understand that a mixed number is written as a whole number and a proper fraction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Name and write fractions ~~and~~ (including mixed numbers) represented by ~~drawings or concrete materials~~ a model ~~for~~ to include halves, thirds, fourths, eighths, ~~and~~ tenths, and twelfths. • ~~Represent a given fraction or mixed number, and write the fractions' names using~~ Use concrete materials~~,~~ and pictures~~, and symbols~~ to model ~~for~~ at least halves, thirds, fourths, eighths, ~~and~~ tenths, and twelfths. ~~For example, write the symbol for one-fourth, and represent it with concrete materials and pictures.~~ • Compare fractions using the terms greater than, less than, or equal to and the symbols (<, >, and =). Comparisons are made between fractions with both like and unlike denominators, using models, concrete materials and pictures.

3.3 The student will
a) name and write fractions (including mixed numbers) represented by a model;
b) model fractions (including mixed numbers) and write the fractions' names; and
c) compare fractions having like and unlike denominators, using words and symbols (>, <, or =).

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
language of fractions (e.g., *thirds* means "three equal parts of a whole," $\frac{1}{3}$ represents one of three equal-size parts when a pizza is shared among three students, or *three-fourths* means "three of four equal parts of a whole") furthers this development. • Comparing unit fractions (a fraction in which the numerator is one) builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (e.g., $\frac{1}{5}$ of a bar is smaller than $\frac{1}{4}$ of a bar). • Comparing fractions to a benchmark on a number line (e.g., close to 0, less than $\frac{1}{2}$, exactly $\frac{1}{2}$, greater than $\frac{1}{2}$, or close to 1) facilitates the comparison of fractions when using concrete materials or pictorial models.		

A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling ~~and~~, discussing, and recording a variety of problem situations. This approach ~~to~~ helps students ~~move~~ transition from the concrete to the representation to the symbolic ~~the abstract~~ ~~and~~ in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, multiplication, division ~~facts~~ and related facts~~ families~~;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions ~~and decimals~~.

3.4 The student will estimate solutions to and solve single-step and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition is the combining of quantities; it uses the following terms: *addend* → 423 *addend* → + 246 *sum* → 669 • Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms: *minuend* → 7,698 *subtrahend* → – 5,341 *difference* → 2,357 • An algorithm is a step-by-step method for computing. • An example of an approach to solving problems is Polya's four-step plan: – Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new. – Plan: Decide what operation(s) and sequence of steps to use to solve the problem. – Solve: Follow the plan and work accurately. If the first attempt does not work, try another plan. – Look back: Does the answer make sense? • Knowing whether to find an exact answer or to make an estimate is learned through practical experiences in recognizing which is appropriate. • When an exact answer is required, opportunities to ~~practice~~ explore ~~recognizing~~ whether the answer can be determined mentally or must involve paper and pencil or calculators help students select the correct	**All students should** • Understand that estimation skills are valuable, time-saving tools particularly in ~~real-life~~ practical situations when exact answers are not required or needed. • Understand that estimation skills are also valuable in determining the reasonableness of the sum or difference when solving for the exact answer is needed. • Develop and use strategies to estimate whole number sums and differences ~~and~~ to ~~judge~~ determine the reasonableness of ~~such estimations~~ an exact answer. • Develop flexible methods of adding whole numbers by combining numbers in a variety of ways, most depending on place values.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether an estimate or an exact answer is an appropriate solution for practical addition and subtraction problem~~s~~ situations involving single-step and multistep problems. • Determine whether to add or subtract in practical problem situations. • Estimate the sum or difference of two whole numbers, each 9,999 or less when an exact answer is not required. • Add or subtract two whole numbers, each 9,999 or less. • ~~Estimate and find~~ Solve practical problems involving the sum of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation in practical problem situations. • ~~Estimate and~~ Solve practical problems involving the difference of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation in practical problem situations. • Solve single-step and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping.

3.4 The student will estimate solutions to and solve single-step and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
approach. • Determining whether an estimate is appropriate and using a variety of strategies to estimate requires experiences with problem situations involving estimation. • There are a variety of mental mathematics strategies for each basic operation, and opportunities to practice these strategies give students the tools to use them at appropriate times. For example, with addition, mental mathematics strategies include – Adding 9: add 10 and subtract 1; and – Making 10: for column addition, look for numbers that group together to make 10. • Using ~~b~~Base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms. ~~When adding, if the sum in a place is 10 or more, place value is used to regroup the sums so that there is just one digit in each place. When subtracting, if the number in a place is not enough to subtract from, regrouping is required.~~ • Conceptual understanding begins with concrete experiences. Next, the children must make connections that serve as a bridge to the symbolic. One strategy used to make connections is representations, such as drawings, diagrams, tally marks, graphs, or written comments. • ~~A certain amount of practice is necessary to develop fluency with computational strategies for multidigit numbers; however, the practice must be motivating and systematic if students are to develop fluency in~~		

3.4 The student will estimate solutions to and solve single-step and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~computation, whether mental, with manipulative materials, or with paper and pencil.~~		

3.5 The student will recall multiplication facts through the twelves table, and the corresponding division facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The development of computational fluency relies on quick access to ~~basic~~ number facts. • A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil. • Strategies to learn the multiplication facts through the ~~nines~~ twelves table include an understanding of multiples/skip counting, properties of zero and one as factors, ~~square numbers~~, pattern of nines, commutative property, and related facts ~~families (the two related multiplication and two division problems)~~. • In order to develop and use strategies to learn the multiplication facts through the ~~nines~~ twelves table, students should use concrete materials, hundred chart, and mental mathematics. • ~~Multiplication is a shortcut for adding same-size groups.~~ To extend the understanding of multiplication, three models may be used: – The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces repeated addition or skip counting. – The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array) helps build the commutative property. – The length model (e.g., a number line) also reinforces repeated addition or skip counting.	**All students should** • Develop fluency with ~~basic~~ number combinations for multiplication and division. • Understand that multiplication is repeated addition. • Understand that division is the inverse of multiplication. • Understand that patterns and relationships exist in the ~~basic~~ facts. • Understand that number relationships can be used to learn and retain the ~~basic~~ facts.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recall and state the multiplication and division facts through the ~~nines~~ twelves table. • Recall and write the multiplication and division facts through the ~~nines~~ twelves table.

3.6 The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The multiplication and division facts through the ~~nines~~ twelves tables should be modeled.
- Multiplication is a shortcut for repeated addition. The terms associated with multiplication are listed below:

  $$\begin{array}{rcr} \textit{factor} & \rightarrow & 54 \\ \textit{factor} & \rightarrow & \underline{\times 3} \\ \textit{product} & \rightarrow & 162 \end{array}$$

- Creating real-life problems and solving them facilitates the connection between mathematics and everyday experiences (e.g., area problems).
- The use of ~~b~~Base-10 blocks and repeated addition can serve as a model. For example, ~~17 × 4~~ 4 × 12 is read as four sets consisting of one rod and ~~seven~~ two units. The sum is renamed as ~~six~~ four rods and eight units or ~~68~~ 48. This can be thought of as

  ~~$(10 + 7) \times 4 = (10 \times 4) + (7 \times 4)$~~
  ~~$40 + 28 = 40 + (20 + 8)$~~
  ~~$(40 + 20) + 8 = 68$. (SET)~~
  $12 + 12 + 12 + 12 =$ (SET)

- The use of ~~b~~Base-10 blocks and the array model can be used to solve the same problem. A rectangle array that is one rod and ~~seven~~ two units long by four units wide is formed. The area of this array is represented by 4 ~~longs~~ rods and ~~2~~8 units. ~~and can be thought of as~~

  ~~$(10 + 7) \times 4 = (10 \times 4) + (7 \times 4)$~~
  ~~$40 + 28 = 40 + (20 + 8)$~~
  ~~$(40 + 20) + 8 = 68$.~~
  ~~(AREA)~~

- The number line model can be used to solve a

ESSENTIAL UNDERSTANDINGS

All students should

- Understand ~~various~~ the meanings of multiplication and division.
- Understand the ~~effects~~ models used to represent ~~of~~ multiplying and dividing whole numbers.

ESSENTIAL KNOWLEDGE AND SKILLS

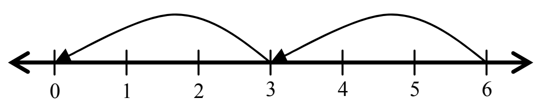
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Model multiplication, using area, ~~and~~ set, and number line models.
- Model division, using area, ~~and~~ set, and number line models.
- Solve multiplication problems, using the ~~standard~~ multiplication algorithm, where one factor is 99 or less and the second factor is 5 or less.
- Create and solve word problems involving multiplication, where one factor is 99 or less and the second factor is 5 or less.

3.6 The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.

<table>
<tr><th>UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)</th><th>ESSENTIAL UNDERSTANDINGS</th><th>ESSENTIAL KNOWLEDGE AND SKILLS</th></tr>
<tr><td>multiplication problem such as 3 × 4. This is represented on the number line by three jumps of four.

• The number line model can be used to solve a division problem such as 6 ÷ 3 and is represented on the number line by noting how many jumps of three go from 6 to 0.



The number of jumps (two) of a given length (three) is the answer to the question.

• An algorithm is a step-by-step method for computing.</td><td></td><td></td></tr>
</table>

3.7 The student will add and subtract proper fractions having like denominators of 12 or less.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A proper fraction is a fraction whose numerator is ~~smaller~~ less than ~~its~~ the denominator. A proper fraction is ~~another name for~~ a fraction ~~between zero and~~ that is always less than one. • An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one. • An improper fraction can be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction. • The ~~concepts~~ strategies of addition and subtraction applied to fractions are the same as ~~these concepts~~ the strategies applied to whole numbers. • Reasonable answers to problems involving addition and subtraction of fractions can be established by using benchmarks such as 0, $\frac{1}{2}$, and 1. For example, $\frac{3}{5}$ and $\frac{4}{5}$ are each greater than $\frac{1}{2}$, so their sum is greater than 1. • Concrete materials and pictorial models representing area/regions (circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters) can be used to add and subtract fractions having like denominators of ~~10~~ 12 or less.	**All students should** • Understand that a proper fraction is a fraction whose numerator is smaller than its denominator. • Understand that an improper fraction is a fraction whose numerator is greater than or equal to the denominator and is one or greater than one. • Understand that an improper fraction can be expressed as a whole number or a mixed number. • Understand that a mixed number is written as a whole number and a proper fraction. A mixed number is the sum of a whole number and the proper fraction. • Understand that computation with fractions uses the same ~~concepts~~ strategies as whole number computation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate a fractional part ~~(halves, thirds, fourths, eighths, and tenths)~~ of a whole, using – region/area models (e.g., pie pieces, pattern blocks, geoboards, drawings); – set models (e.g., chips, counters, cubes, drawings); and – length/measurement models (e.g., nonstandard units such as ~~cuisenaire~~ rods, connecting cubes, and drawings). • Name and write fractions and mixed numbers represented by drawings or concrete materials ~~for halves, thirds, fourths, eighths, and tenths~~. • Represent a given fraction or mixed number, using concrete materials, pictures, and symbols~~, for halves, thirds, fourths, eighths, and tenths~~. For example, write the symbol for one-fourth and represent it with concrete materials and/or pictures. • Add and subtract with proper fractions having denominators of ~~10~~ 12 or less, using concrete materials and pictorial models representing area/regions (circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters).

Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

3.8 The student will determine, by counting, the value of a collection of bills and coins whose total value is $5.00 or less, compare the value of the bills and coins, and make change.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills. • A variety of skills can be used to determine the change after a purchase, including – counting on, using coins and bills, i.e., starting with the amount to be paid (purchase price), counting forward to the next dollar, and then counting forward by dollar bills to reach the amount from which to make change; and – mentally calculating the difference.	**All students should** • Understand that a collection of coins and bills has a value that can be counted. • Understand how to make change from $5.00 or less.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count the value of collections of coins and bills up to $5.00. • Compare the values of two sets of coins or bills, up to $5.00, using the terms *greater than, less than*, and *equal to*. • Make change from $5.00 or less.

3.9 The student will estimate and use U.S. Customary and metric units to measure

a) length to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter;

b) liquid volume in cups, pints, quarts, gallons, and liters;

c) weight/mass in ounces, pounds, grams, and kilograms; and

d) area and perimeter.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • The concept of a standard measurement unit is one of the major ideas in understanding measurement. Familiarity with standard units is developed through hands-on experiences of comparing, estimating, measuring, and constructing. • Benchmarks of common objects need to be established for each of the specified units of measure (e.g., the mass of a mathematics book is about one kilogram). Practical experience measuring the mass of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate measures. • One unit of measure may be more appropriate than another to measure an object, depending on the size of the object and the degree of accuracy desired. • Correct use of measurement tools is essential to understanding the concepts of measurement. • Perimeter is the distance around any two-dimensional	**All students should** • Understand how to estimate measures of length, liquid volume, ~~and~~ weight/mass, area and perimeter. • Understand how to determine the actual measure of length, liquid volume, ~~and~~ weight/mass, area and perimeter. • Understand that perimeter is a measure of the distance around a polygon. • Understand that area is a measure of square units needed to cover a surface.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify and use the following units of length: centimeters, meters, part of an inch, inches, feet, and yards, centimeters, and meters.~~ • ~~Identify and use the following units of liquid volume: cups, pints, quarts, gallons, and liters.~~ • ~~Identify and use the following units of weight/mass: ounces, pounds, grams, and kilograms.~~ • Estimate and ~~then measure~~ use U.S. Customary and metric units to measure lengths of objects to the nearest ~~centimeter, and meter, and the nearest~~ $\frac{1}{2}$ of an inch, inch, foot, ~~and~~ yard, centimeter, and meter. • ~~Estimate and then measure lengths of~~ Determine the actual measure of length using U.S. Customary and metric units to measure objects to the nearest ~~centimeter and meter, and the nearest~~ $\frac{1}{2}$ of an inch, foot, ~~and~~ yard, centimeter, and meter. • Estimate and ~~then measure~~ use U.S. Customary and metric units to measure the weight/mass of

3.9 The student will estimate and use U.S. Customary and metric units to measure

a) length to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter;

b) liquid volume in cups, pints, quarts, gallons, and liters;

c) weight/mass in ounces, pounds, grams, and kilograms; and

d) area and perimeter.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~shape~~ figure and is found by adding the measures of the sides. • Area is a two-dimensional measure and is therefore measured in square units. • Area is the number of square units needed to cover a figure, or more precisely, it is the measure in square units of the interior region of a two-dimensional figure.		objects to the nearest ounce, ~~and~~ pound, ~~and the nearest~~ gram, and kilogram. • Determine the actual measure of weight/mass using U.S. Customary and metric units to measure the weight/mass of objects to the nearest ounce, ~~and~~ pound, ~~and the nearest~~ gram, and kilogram. • Estimate and ~~then measure~~ use U.S. Customary and metric units to measure liquid volume to the nearest cup, pint, quart, gallon, and liter. • Determine the actual measure of liquid volume using U.S. Customary and metric units to measure to the nearest cup, pint, quart, gallon, and liter. • Estimate and use U.S. Customary and metric units to measure area and perimeter. • Determine the actual measure of area or perimeter using U.S. Customary and metric units.

3.10 The student will
a) measure the distance around a polygon in order to determine perimeter; and
b) count the number of square units needed to cover a given surface in order to determine area.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A polygon is a closed plane figure composed of at least three line segments that do not cross. None of the sides are curved. • Perimeter is a measure of the distance around a polygon and is found by adding the measures of the sides. • ~~Area is a measure of square units needed to cover a surface. Area may be determined by counting the number of square units on a given surface.~~ Area is the number of iterations of a two-dimensional unit needed to cover a surface. The two-dimensional unit is usually a square, but it could also be another shape such as a rectangle or an equilateral triangle. • Opportunities to explore the concepts of perimeter and area should involve hands-on experiences (e.g., placing tiles (units) around a polygon and counting the number of tiles to determine its perimeter and filling or covering a polygon with cubes (square units) and counting the cubes to determine its area).	**All students should** • Understand the meaning of a polygon as a closed figure with at least three sides. None of the sides are curved and there are no intersecting lines. • Understand that perimeter is a measure of the distance around a polygon. • Understand how to determine the perimeter by counting the number of units around a polygon. • Understand that area is a measure of square units needed to cover a surface. • Understand how to determine the area by counting the number of square units.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Measure each side of a variety of ~~concrete~~ polygons and add ~~them~~ the measures of the sides to determine the ~~distance around~~ perimeter of ~~the~~ each polygon ~~(its perimeter)~~. • Determine the area of a given surface ~~on grid paper~~ by estimating and then counting the number of square units needed to cover the surface.

3.11 The student will
a) tell time to the nearest minute, using analog and digital clocks; and
b) determine elapsed time in one-hour increments over a 12-hour period.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Telling time is applying reading skills to a clock face.~~ • While digital clocks make reading time easy, it is necessary to ensure that students understand that there are sixty minutes in an hour. • Use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise when time is read**.** • Students need to understand that time has passed or will pass. • Elapsed time is the amount of time that has passed between two given times. • ~~Use of a~~ Elapsed time should be modeled and ~~demonstration~~ demonstrated using geared analog clocks ~~with gears ensures that the positions of the hour hand and the minute hand are precise when time is read~~ and timelines**.** • ~~While digital clocks make reading time easy, i~~ It is necessary to ensure that students understand that there are sixty minutes in an hour when using analog and digital clocks. • Elapsed time can be found by counting on from the beginning time to the finishing time. – Count the number of whole hours between the beginning time and the finishing time. For example, to find the elapsed time between 7 a.m. and 10 a.m., students can count on to find the difference between the times (7 and 10), so the total elapsed time is 3 hours.	**All students should** • Apply appropriate techniques to determine time to the nearest ~~five-minute interval~~ minute, using analog and digital clocks. • Understand how to determine elapsed time in one-hour increments over a 12-hour period.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Tell time to the ~~hour, half-hour, quarter-hour, nearest five-minute interval, and~~ nearest minute, using analog and digital clocks. • Match the times shown on analog and digital clocks to written times and to each other. • When given the beginning time and ending time, determine the elapsed time in one-hour increments~~.~~ ~~Determine the elapsed time in one-hour increments~~ within a 12-hour period (times do not cross between a.m. and p.m.). • Solve practical problems in relation to time that has elapsed.

3.12 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The knowledge that a year has $365\cancel{+}\frac{1}{4}$ days will help students understand the necessity of adding a full day every fourth year, called a *leap year*. • The use of a calendar facilitates the understanding of time relationships between days and months, days and weeks, days and years, and months and years. Recognize that students need to know the relationships, such as if there are 24 hours in one day, how many hours are in three days? If the date is January 6, what date would it be in two weeks? How many weeks are in March, April, and May? • The use of an analog clock facilitates the understanding of time relationships between minutes and hours and hours and days.	**All students should** • Understand the relationship that exists among periods of time, using calendars, and clocks.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify equivalent relationships observed in a calendar, including the number of days in a given month, the number of days in a week, the number of days in a year, and the number of months in a year. • Identify the number of minutes in an hour and the number of hours in a day.

3.13 The student will read temperature to the nearest degree from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimating and measuring temperatures in the environment in Fahrenheit and Celsius require the use of real thermometers. • A physical model can be used to represent the temperature determined by a real thermometer. • The symbols for degrees in Celsius (°C) and degrees in Fahrenheit (°F) should be used to write temperatures. • Celsius and Fahrenheit temperatures should be related to everyday occurrences by measuring the temperature of the classroom, the outside, liquids, body temperature, and other things found in the environment.	**All students should** • Understand how to measure temperature in Celsius and Fahrenheit with a thermometer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Read temperature to the nearest degree from real Celsius and Fahrenheit thermometers and from physical models (including pictorial representations) of such thermometers.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of ~~shapes~~ figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent ~~two- and three-dimensional~~ plane and solid ~~shapes~~ figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting and investigating ~~three-dimensional~~ solid objects and their ~~two-dimensional~~ faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying ~~shapes~~ figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these ~~shapes~~ figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.
  - **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
  - **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know that the door is a rectangle.").
  - **Level 2: Analysis**. Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it's closed, it has four sides and four right angles, and opposite sides are parallel.").
- A plane geometric figure is any two-dimensional closed ~~shape~~ figure. Circles and polygons are examples of plane geometric figures.
- Three-dimensional figures are called *solid figures* or simply *solids*. Solids enclose a region of space. The interior of both plane and solid figures are not part of the figure. Solids are classified by the types of surfaces they have. These surfaces may be flat,

ESSENTIAL UNDERSTANDINGS

All students should

- Understand how to identify and describe plane and solid geometric figures by using relevant characteristics.
- ~~Understand how to describe plane or solid using characteristics more precise ways to describe shapes by focusing on characteristics and the specialized vocabulary associated with these shapes and characteristics.~~
- Understand the similarities and differences between ~~two-dimensional (~~plane~~)~~ and ~~three-dimensional~~ ~~(~~solid~~)~~ figures ~~while recognizing the commonalties of the two~~.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify ~~by name,~~ models~~,~~ and pictures of plane geometric figures (circle, square, rectangle, and triangle) and solid geometric figures (cube, rectangular ~~solid~~ prism, square pyramid, sphere, cone, and cylinder) by name.
- Identify and describe plane geometric figures by counting the number of sides, ~~corners, and square corners, and~~ angles, the number and shape of faces, vertices, and edges.
- Identify and describe ~~geometric solids~~ solid geometric figures by counting the number of ~~corners, square corners~~ angles, vertices, and edges~~,~~ and by the number and shape~~s~~ of ~~the~~ faces.
- C~~lassify, compare~~ Compare and contrast characteristics of plane and solid geometric figures (e.g., circle/sphere, square/cube, triangle/square pyramid, and rectangle/rectangular ~~solid~~ prism), ~~using corners, square corners,~~ by counting the number of sides, angles, vertices, ~~faces, and~~ edges, and the number and shape of faces.
- Compare and contrast characteristics of ~~three-dimensional (~~solid~~)~~ ~~shapes~~ geometric figures (i.e., cube, rectangular ~~solid~~ prism, square pyramid, sphere, cylinder, and cone) to similar objects in everyday life (e.g., a party hat is like a cone).
- ~~Identify and name cubes, rectangular solids~~

3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
curved, or both. • The study of geometric figures must be active, using visual images and concrete materials. • Access to a variety of concrete tools such as graph paper, pattern blocks, geoboards, and geometric solids is greatly enhanced by computer software tools that support exploration. • Opportunity must be provided for building and using geometric vocabulary to describe ~~two- dimensional (~~plane~~)~~ and ~~three-dimensional (~~solid~~)~~ ~~shapes~~ figures. • ~~It is appropriate at this level to refer to "square corner." However, teachers should begin to introduce the term *right angle* for square corner as well.~~ • A cube is a ~~three-dimensional~~ solid figure with six congruent square faces and with every edge the same length. A cube has 8 ~~corners~~ vertices and 12 edges. • A cylinder is a ~~three-dimensional~~ solid figure formed by two congruent parallel circles joined by a curved surface. • A cone is a ~~three-dimensional,~~ solid, pointed figure that has a flat, round ~~base~~ face (usually a circle) that is joined to a vertex by a curved surface. • A rectangular ~~solid~~ prism is a ~~three-dimensional~~ solid figure in which all six faces are rectangles with three pair of parallel congruent opposite faces. • A sphere is a ~~three-dimensional~~ solid figure with all of its points the same distance from its center. • A square pyramid is a ~~three-dimensional~~ solid figure		~~(prisms), square pyramids, spheres, cylinders, and cones by their appearance.~~ • ~~Identify and describe cubes, rectangular solids (prisms), square pyramids, spheres, cylinders, and cones according to the number and shape of their faces (sides, bases), edges, and corners and the number of angles and vertices.~~ • Identify ~~properties~~ characteristics of ~~three-dimensional (~~solid~~)~~ geometric ~~shapes~~ figures (cylinder, cone, cube, square pyramid, and rectangular prism). • ~~Analyze and compare properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism).~~

3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~whose base is a~~ with one square face and ~~whose other faces are triangles~~ four triangular faces that share a common vertex. • A face is a polygon that serves as one side of a ~~three-dimensional~~ solid figure (e.g., a square is a face of a cube). • An angle is formed by two rays h~~aving~~ with a common endpoint. This endpoint is called the *vertex*. Angles are found wherever lines ~~and line segments~~ intersect. An angle can be named in three different ways by using – three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle. • An edge is the line segment where two faces of a ~~three-dimensional~~ solid figure ~~meet~~ intersect. • ~~A corner is the point at which three or more edges meet.~~ A vertex is the point at which two lines, line segments, or rays meet to form an angle. It is also the point on a three dimensional figure where three or more faces intersect. • Students should be reminded that a ~~concrete three-dimensional~~ solid geometric object~~, even when called a “solid,” may actually be~~ is hollow rather than solid. The “solid” indicates a three-dimensional figure.		

3.15 The student will identify and draw representations of points, line segments, rays, angles, and lines.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A point is an exact location in space. It has no length or width. Usually, a point is named with a capital letter. • A line is a collection of points going on and on in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. The line can also be named with a single, lower-case letter. Arrow~~heads~~s must be drawn to show that the line goes on in both directions infinitely. • A line segment is part of a line. It has two endpoints and includes all the points between those endpoints. The endpoints are used to name a line segment. • A ray is part of a line. It has one endpoint and continues on and on in one direction. • An angle is formed by two rays having a common endpoint. This endpoint is called the *vertex*. Angles are found wherever lines and line segments intersect. An angle can be named in three different ways by using – three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle. • Angle rulers may be particularly useful in developing the concept of an angle.	**All students should** • Understand that line segments and angles are fundamental components of ~~noncircular,~~ plane polygons. • Understand that a line segment is a part of a line, ~~having~~ has two end points, and ~~that it~~ contains all the points between those two endpoints. • Understand that points make up a line. • Understand that a line continues indefinitely in two opposite directions. • Understand that a ray is part of a line, ~~with~~ has one endpoint, and continues indefinitely in only one direction. • Understand that an angle is formed by two rays having a common endpoint.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify ~~and locate~~ examples of ~~a~~ points, line segments, rays, ~~and~~ angles, and lines. • Draw representations of points, line segments, rays, ~~and~~ angles, and lines, using a ruler or straightedge.

3.16 The student will identify and describe congruent and noncongruent plane figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Congruent plane figures are figures having exactly the same size and shape. Noncongruent plane figures are figures that are not exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models. • ~~A figure or shape is symmetrical when one-half of the figure is the mirror image of the other half. Opportunities for exploring symmetry in figures can best be accomplished by using physical models.~~ • ~~A line of symmetry divides a symmetrical figure, object, or arrangement of objects into two parts that are congruent if one part is reflected (flipped) over the line of symmetry.~~ • ~~While investigating symmetry, students move shapes, such as pattern blocks, intuitively, thereby exploring transformations of those shapes. A transformation is the movement of a figure — either a translation, rotation, or reflection. A translation (slide) is the result of sliding a figure in any direction; a rotation (turn) is the result of turning a figure around a point or a vertex; and a reflection (flip) is the result of flipping a figure over a line.~~ • Have students identify figures that are congruent or noncongruent by using direct comparisons and/or tracing procedures.	**All students should** • Understand that congruent ~~shapes~~ plane figures match exactly. • Understand that noncongruent plane figures do not match exactly. • Understand that congruent plane figures remain congruent even if they are in different spatial orientations. • Understand that noncongruent plane figures remain noncongruent even if they are in different spatial orientations. • ~~Understand that symmetrical figures can be divided into two halves that are the mirror image of each other.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Locate examples of symmetrical figures, and verify their symmetry by using tracing procedures.~~ • ~~Determine if given figures have a line or lines of symmetry (vertical, horizontal, diagonal), using tracing procedures.~~ • ~~Locate~~ Identify examples of congruent and noncongruent figures. ~~and verify~~ Verify their congruenc~~y~~e by laying one on top of the other using drawings or models. • Determine ~~if~~ ~~given~~ and explain why plane figures are congruent ~~and~~ or noncongruent, using tracing procedures.

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

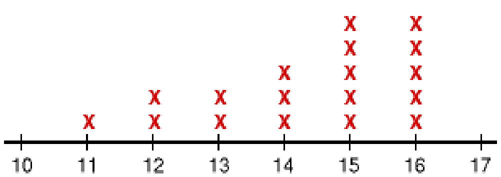
The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

3.17 The student will
a) collect and organize data, using observations, measurements, surveys, or experiments;
b) construct a line plot, a picture graph, or a bar graph to represent the data; and
c) read and interpret the data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Investigations involving data should occur frequently and relate to students' experiences, interests, and environment. • Formulating questions for investigations is student-generated at this level. For example: What is the ~~favorite~~ cafeteria lunch preferred by students in the class when four lunch menus are offered? • The purpose of a graph is to represent data gathered to answer a question. • Bar graphs are used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs. – A bar graph uses parallel, horizontal or vertical bars to represent counts for ~~several~~ categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis displaying the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Third grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, or 10. – Each axis should be labeled, and the graph should be given a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences,	**All students should** • Understand how ~~gathered~~ data can be collected and organized. • Understand that data can be displayed in different types of graphs depending on the ~~questions to be answered~~ data. • Understand how to construct a line plot, picture graph, or bar graph. • Understand that data sets can be interpreted and analyzed to ~~provide different kinds of information~~ draw conclusions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Formulate questions to investigate. • Design data investigations to answer formulated questions, limiting the number of categories for data collection to four. • Collect data, using surveys, polls, questionnaires, scientific experiments, and observations. • Organize data and construct a bar graph on grid paper representing 16 or fewer data points for no more than four categories. • Label bar graphs with a title, a description of each axis, and a key where appropriate. Limit increments on the numerical axis to whole numbers representing multiples of 1, 2, 5, or 10. • Read the information presented on a simple bar or picture graph (e.g., the title, the categories, the description of the two axes, the key). • Analyze and interpret information from picture and bar graphs, with up to 30 data points and up to 8 categories, by writing at least one sentence. • Construct a line plot with no more than 30 data points. • ~~Read information presented in line plots.~~ • ~~Analyze and interpret~~ Read, interpret and analyze

3.17 The student will
a) collect and organize data, using observations, measurements, surveys, or experiments;
b) construct a line plot, a picture graph, or a bar graph to represent the data; and
c) read and interpret the data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
least and greatest, the categories, and total number of responses) should be written. • ~~Line plots show the spread of data. To construct a line plot,~~ – ~~draw a number line on grid paper (The scale of numbers should include the greatest value and the least value in a set of numbers.);~~ – ~~for each piece of data, draw an *x* above the corresponding number; and~~ – ~~title the plot.~~ • A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range, mode, and any outliers. Frequency of Number of Books Read  • When data are displayed in an organized manner, the results of the investigations can be described and the posed question answered. • Recognition of appropriate and inappropriate statements begins at this level with graph interpretations.		information from line plots~~, with data points limited to 16,~~ by writing at least one statement. • Describe the categories of data and the data as a whole (e.g., data were collected on four ~~types~~ ways to cook or prepare ~~of~~ eggs — scrambled, fried, hard boiled, and egg salad — eaten by students). • Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs). • Select a correct interpretation of a graph from a set of interpretations of the graph, where one is correct and the remaining ~~three~~ are incorrect. For example, a bar graph containing data on four ~~types~~ ways to cook or prepare ~~of~~ eggs — eaten by students shows that more students prefer scrambled eggs. A correct answer response, if given, would be that more students prefer scrambled eggs than any other ~~type of eggs~~ way to cook or prepare eggs.

3.18 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives. • Investigation of experimental probability is continued at this level through informal activities using two-colored counters, spinners, and random number generators (number cubes). • Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is “impossible” if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is “certain” if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include real-life examples.	**All students should** • Investigate, ~~U~~understand, and apply basic concepts of probability. • Understand that probability is the chance of an event happening.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define probability as the chance that an event will happen. • List all possible outcomes for a given situation (e.g., heads and tails are the two possible outcomes of flipping a coin). • ~~Identify the possible outcomes for a common event, using terms such as *impossible*, *unlikely*, *equally likely*, *likely*, and *certain*.~~ • Identify the degree of likelihood of an outcome occurring using terms such as *impossible*, *unlikely*, *as likely as*, *equally likely*, *likely*, and *certain*.

3.18 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• For any event, such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. • A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, table, or chart.		

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns ~~in the real world~~. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to ~~A~~algebra.

3.19 The student will recognize and describe a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Exploring patterns requires active physical and mental involvement. • The use of materials to extend patterns permits experimentation or trial-and-error approaches that are almost impossible without them. • Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing numbers symbolically or algebraically. • The simplest types of patterns are repeating patterns. In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. • Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats. • Create an arithmetic number pattern. Sample numeric patterns include – 6, 9, 12, 15, 18, …(growing pattern); – 1, 2, 4, 7, 11, 16, …(growing pattern); – 20, 18, 16, 14,… (growing pattern); and – ~~2, 4, 8, 16, 32,~~ 1, 3, 5, 1, 3, 5, 1, 3, 5 … (repeating pattern)~~; and~~. ~~– 1, 5, 25, 125, 625, ….~~	**All students should** • Understand that numeric and geometric patterns can be expressed in words or symbols. • Understand the structure of a pattern and how it grows or changes. • Understand that mathematical relationships exist in patterns. • Understand that patterns can be translated from one representation to another.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize repeating and growing numeric and geometric patterns (e.g., skip counting, addition tables, and multiplication tables). • Describe repeating and growing numeric and geometric patterns formed using ~~concrete objects~~, numbers, tables, and/or pictures, using the same or different forms. • Extend repeating and growing ~~numeric and geometric~~ patterns of numbers or figures ~~formed~~ using concrete objects, numbers, tables, and/or pictures~~, using the same or different forms~~.

3.19 The student will recognize and describe a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation ~~(turn)~~ is the result of turning a figure around a point or a vertex, and reflection ~~(flip)~~ is the result of flipping a figure over a line. • Sample geometric patterns include – O Δ O O Δ Δ O O O Δ Δ Δ …; and – □□ □ □□ □ …. • A table of values can be analyzed to determine the pattern that has been used, and that pattern can then be used to find the next value.		

3.20 The student will
a) investigate the identity and the commutative properties for addition and multiplication; and
b) identify examples of the identity and commutative properties for addition and multiplication.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. • The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2). • The identity property for addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. • A number sentence is an equation with numbers (e.g., 6 + 3 = 9; or 6 + 3 = 4 + 5).	**All students should** • Understand that mathematical relationships can be expressed using number sentences. • Understand the identity property for addition. • Understand the identity property for multiplication. • Understand the commutative property of addition. • Understand the commutative property of multiplication. • Understand that quantities on both sides of an equal~~s~~ sign must be equal. • Understand that quantities on both sides of the not equal sign are not equal ~~and are not balanced~~ .	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate the identity property for addition and determine that when the number zero is added to another number or another number is added to the number zero, that number remains unchanged. Examples of the identity property for addition are 0 + 2 = 2; 5 + 0 = 5. • Investigate the identity property for multiplication and determine that when the number one is multiplied by another number or another number is multiplied by the number one, that number remains unchanged. Examples of the identity property for multiplication are 1 x 3 = 3; 6 x 1 = 6. • Recognize ~~T~~that the commutative property for addition is an order property. Changing the order of the addends does not change the sum (5 + 4 = 9 and 4 + 5 = 9). • Recognize ~~T~~that the commutative property for multiplication is an order property. Changing the order of the factors does not change the product (2 × 3 = 3 × 2) . • ~~Recognize that the equals sign (=) relates equivalent quantities.~~ • ~~Recognize that the not equal sign (≠) relates nonequivalent quantities.~~ • Write number sentences to represent equivalent

3.20 The student will

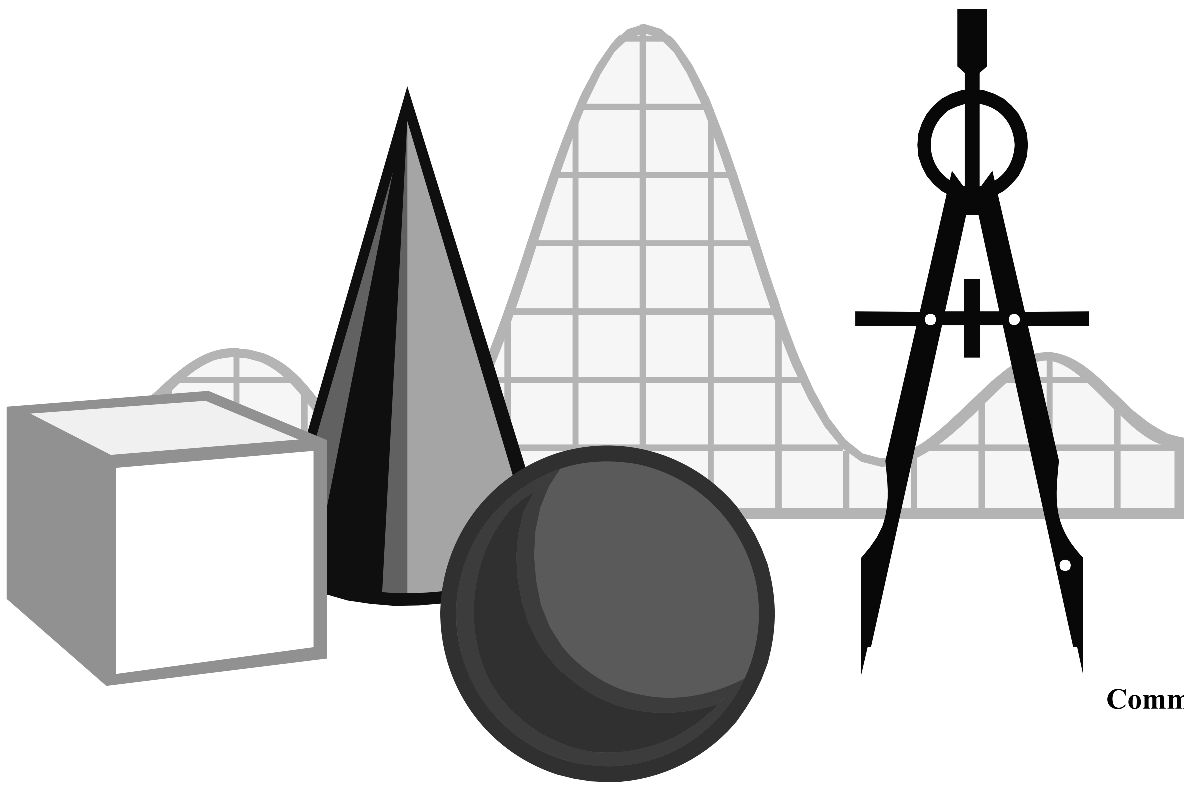
a) investigate the identity and the commutative properties for addition and multiplication; and

b) identify examples of the identity and commutative properties for addition and multiplication.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
		mathematical relationships (e.g., ~~4 · 3 = 2 · 6~~ 4 x 3 = 14 - 2). • Identify examples of the identity and commutative properties for addition and multiplication. • Identify number sentences that show appropriate use of the equals sign and the not equal sign. 6 × 8 ≠ 36 – 4 123 ≠ 132 18 + 4 + 2 ≠ 24 ÷ 6 36 – 14 = 11 × 2 49 – 0 = 7 × 7

MATHEMATICS STANDARDS OF LEARNING
CURRICULUM FRAMEWORK

Grade 4



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

Mathematics instruction in grades 4 and 5 should continue to foster the development of number sense, especially with decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction at grades 4 and 5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names). Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve problems.

4.1 The student will

a) identify orally and in writing the place value for each digit in a whole number expressed through millions;

b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The structure of the ~~b~~Base-10 number system is based upon a simple pattern of tens, in which the value of each place is ten times the value of the place to its right. • Place value refers to the value of each digit and depends upon the position of the digit in the number. For example, in the number 7,864,352, the eight is in the hundred thousands place, and the value of the 8 is eight hundred thousand or 800,000. • Whole numbers may be written in a variety of formats: – Standard: 1,234,567 – Written: one million, two hundred thirty-four thousand, five hundred sixty-seven – Expanded: (1 × 1,000,000) + (2 × 100,000) + (3 × 10,000) + (4 × 1,000) + (5 × 100) + (6 × 10) + (7 × 1) • Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, …). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students find values of digits in any number as well as read and write numbers. • Reading and writing large numbers should be ~~related to numbers that have~~ meaning~~s~~ful for students. Experiences can be provided that relate	**All students should** • Understand the relationships in the place value system in which the value of each place is ten times the value of the place to its right. • Use the patterns in the place value system to read and write numbers. • Understand that reading place value correctly is essential when comparing numbers. • Understand that rounding gives a close number to use when exact numbers are not needed for the situation at hand. • Develop strategies for rounding.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and communicate, both orally and in written form, the placed value for each digit in whole numbers expressed through the one millions place. • Read whole numbers through the one millions place that are presented in standard format, and select the matching number in written format. • Write whole numbers through the one millions place in standard format when the numbers are presented orally or in written format. • Identify and use the symbols for *greater than*, *less than*, and *equal to*. • Compare two whole numbers expressed through the one millions, using symbols >, <, or =. • Round whole numbers expressed through the one millions place to the nearest thousand, ten thousand, and hundred-thousand place.

4.1 The student will

a) identify orally and in writing the place value for each digit in a whole number expressed through millions;

b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
practical situations (e.g., numbers found in the students' environment including population, number of school lunches sold statewide in a day, etc.). Concrete materials such as ~~b~~Base-10 blocks and bundles of sticks may be used to represent whole numbers through thousands. Larger numbers may be represented by ~~symbol~~ digit cards and place value charts. • Mathematical symbols (>, <) used to compare two unequal numbers are called *inequality symbols*. • A procedure for comparing two numbers by examining place value may include the following: ~~– Line up the numbers by place value by lining up the ones.~~ ~~– Beginning at the left, find the first place value where the digits are different.~~ – Compare the digits in ~~this place value~~ the numbers to determine which number is greater (or which is less). – Use a number line to identify the appropriate placement of the numbers based on the place value of the digits. – Use the appropriate symbol > or < or words *greater than* or *less than* to compare the numbers in the order in which they are presented. – If both numbers ~~are the same~~ have the same value, use the symbol = or words *equal to*.		

4.1 The student will

a) identify orally and in writing the place value for each digit in a whole number expressed through millions;

b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A strategy for rounding numbers to the nearest thousand, ten thousand, and hundred thousand is as follows: – <u>Use a number line to determine the rounded number (e.g., when rounding 4,367,925 to the nearest thousand, identify the ‘thousands’ the number would fall between on the number line, then determine the thousand that the number is closest to):</u> 4,367**,000** ? 4,368**,000** – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero.		

4.2 The student will
a) compare and order fractions and mixed numbers;
b) represent equivalent fractions; and
c) identify the division statement that represents a fraction.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Rational numbers should be explained as any number that can be written as a fraction (e.g., $\frac{2}{1}$, $\frac{2}{3}$, $7\frac{1}{4}$).~~ • A fraction is a way of representing part of a whole (as in a region/area model or a measurement model) or part of a group (as in a set model). A fraction is used to name a part of one thing or a part of a collection of things. • In ~~each~~ the area/region and length/measurement fraction models, the parts must be equal ~~(i.e., each pie piece must have the same area; the size of each chip in a set must be equal; the measures such as the colored multi-length rods, or the parts in a fraction strip must be equal)~~. In the set model, the elements of the set do not have to be equal (i.e., "What fraction of the class is wearing the color red?"). • The denominator ~~(bottom number)~~ tells how many equal parts are in the whole or set. The numerator ~~(top number)~~ tells how many of those parts are being counted or described. • When fractions have the same denominator, they are said to have "common denominators" or "like denominators." Comparing fractions with like denominators involves comparing only the numerators.	**All students should** • Develop an understanding of fractions as parts of unit wholes, as parts of a collection, and as locations on a number line. • Understand that a mixed number is a fraction that has two parts: a whole number and a proper fraction. The mixed number is the sum of these two parts. • Use models, benchmarks, and equivalent forms to judge the size of fractions. • Recognize that a whole divided into nine equal parts has smaller parts than if the whole had been divided into five equal parts. • Recognize and generate equivalent forms of commonly used fractions and decimals. • Understand the division statement that represents a fraction. • Understand that the more parts the whole is divided into, the smaller the parts (e.g., $\frac{1}{5} < \frac{1}{3}$).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify, model, and compare fractions and mixed numbers through twelfths, using~~ ~~– region/area models (e.g., fraction circles, pattern blocks, geoboards, color tiles, graph paper);~~ ~~– set models (e.g., two-sided counters, chips); and~~ ~~– measurement models (e.g., cuisenaire rods, unifix cubes, fraction strips, number lines).~~ • Compare and order fractions having denominators of 12 or less, using manipulative models and drawings, such as region/area models ~~(e.g., fraction circles, pattern blocks, geoboards, color tiles, graph paper, drawings); set models (e.g., two-sided counters, chips, drawings); and measurement models (e.g., colored, multilength rods, linking cubes, fraction strips, rulers/number lines, drawings)~~. • Compare and order fractions with like denominators by comparing number of parts (numerators) (e.g., $\frac{1}{5} < \frac{3}{5}$). • Compare and order fractions with like numerators and unlike denominators by comparing the size of the parts (e.g., $\frac{3}{9} < \frac{3}{5}$). • Compare and order fractions having unlike denominators of 12 or less by comparing the fractions to benchmarks

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UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Strategies for comparing fractions having unlike denominators may include – comparing fractions to familiar benchmarks (e.g., 0, $\frac{1}{2}$, 1); – finding equivalent fractions, using manipulative models such as fraction strips, number lines, fraction circles, ~~cuisenaire~~ rods, pattern blocks, ~~unifix~~ cubes, ~~b~~Base-10 blocks, tangrams, ~~or~~ graph paper, or a multiplication chart and patterns; and – finding a common denominator by finding the least common multiple (LCM) of both denominators and then rewriting each fraction as an equivalent fraction, using the LCM as the denominator. • A variety of fraction models should be used to expand students' understanding of fractions and mixed numbers: – Region/area models: a surface or area is subdivided into smaller equal parts, and each part is compared with the whole (e.g., fraction circles, pattern blocks, geoboards, grid paper, color tiles). – Set models: the whole is understood to be a set of objects, and subsets of the whole make up fractional parts (e.g., counters, chips). – Measurement models: similar to area models but lengths instead of areas are compared		(e.g., 0, $\frac{1}{2}$ or 1) to determine their relationships to the benchmarks or by finding a common denominator. • Compare and order mixed numbers having denominators of 12 or less. • Use the symbols >, <, and = to compare the numerical value of fractions and mixed numbers having denominators of 12 or less. • ~~Represent fractions for halves, fourths, fifths, and tenths as decimals through thousandths, using concrete objects (e.g., demonstrate the relationship between the fraction~~ $\cancel{\frac{1}{4}}$ ~~and its decimal equivalent 0.25).~~ • ~~Relate fractions to decimals, using concrete objects (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money [coins]).~~ • ~~Identify and represent~~ Represent equivalent fractions through twelfths, using region/area models, set models, and measurement models. • Identify the division statement that represents a fraction (e.g., $\frac{3}{5}$ means the same as 3 divided by 5).

4.2 The student will
a) compare and order fractions and mixed numbers;
b) represent equivalent fractions; and
c) identify the division statement that represents a fraction.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
(e.g., fraction strips, ~~cuisenaire~~ rods, ~~unifix~~ cubes, number lines, rulers). • A mixed number has two parts: a whole number and a fraction. • Equivalent fractions name the same amount. Students should use a variety of models to identify different names for equivalent fractions. • Students should focus on finding equivalent fractions of familiar fractions such as halves, thirds, fourths, sixths, eighths, tenths, and twelfths. • Decimals and fractions represent the same relationships; however, they are presented in two different formats. The decimal 0.25 is written as $\frac{1}{4}$. ~~Decimal numbers are another way of writing fractions. The base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money).~~ When presented with the fraction $\frac{3}{5}$, the division expression representing a fraction is written as 3 divided by 5.		

4.3 The student will
- **a) read, write, represent, and identify decimals expressed through thousandths;**
- **b) round decimals to the nearest whole number, tenth, and hundredth;**
- **c) compare and order decimals; and**
- **d) given a model, write the decimal and fraction equivalents.**

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The structure of the ~~b~~Base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. • Understanding the system of tens means that ten tenths represents one whole, ten hundredths represents one tenth, ten thousandths represents one hundredth. • A decimal point separates the whole number places from the places that are less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*. • To read decimals, – read the whole number to the left of the decimal point, if there is one; – read the decimal point as "and"; – read the digits to the right of the decimal point just as you would read a whole number; and – say the name of the place value of the digit in the smallest place. • Any decimal less than 1 will include a leading zero (e.g., 0.125). • Decimals may be written in a variety of forms: – Standard: 26.537 – Written: twenty-six and five hundred thirty-seven thousandths – Expanded: (2 × 10) + (6 × 1) + (5 × 0.1) + (3 × 0.01) +	**All students should** • Understand the place value structure of decimals and use this structure to read, write, and compare decimals. • Understand that decimal numbers can be rounded to an estimate when exact numbers are not needed for the situation at hand. • Understand that decimals are rounded in a way that is similar to the way whole numbers are rounded. • Understand that decimals and fractions represent the same relationship; however, they are presented in two different formats. • Understand that models are used to show decimal and fraction equivalents.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate the ten-to-one place value relationship for decimals through thousandths, using ~~b~~Base-10 manipulatives (e.g., place value mats/charts, decimal squares, ~~b~~Base-10 blocks, money). • Represent and identify decimals expressed through thousandths, using ~~b~~Base-10 manipulatives, pictorial representations, ~~calculators~~, and numerical symbols (e.g., relate the appropriate drawing to 0.05). • Identify and communicate, both orally and in written form, the position and value of a decimal through thousandths. For example, in 0.385, the 8 is in the hundredths place and has a value of 0.08. • Read and write decimals expressed through thousandths, using ~~b~~Base-10 manipulatives, drawings, ~~calculators~~, and numerical symbols. ~~Any decimal less than 1 will include a leading zero (e.g., 0.125).~~ • Round decimals to the nearest whole number, tenth, and hundredth. • Compare ~~and order~~ ~~the value of~~ ~~two~~ decimals, using the symbols >, <, =. • Order a set of decimals from least to greatest or greatest to least.

4.3 The student will
- **a) read, write, represent, and identify decimals expressed through thousandths;**
- **b) round decimals to the nearest whole number, tenth, and hundredth;**
- **c) compare and order decimals; and**
- **d) given a model, write the decimal and fraction equivalents.**

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
(7 × 0.001). • Decimals and fractions represent the same relationships; however, they are presented in two different formats. The decimal 0.25 is written as $\frac{1}{4}$. Decimal numbers are another way of writing fractions. When presented with the fraction $\frac{3}{5}$, the division expression representing a fraction is written as 3 divided by 5. The ~~b~~Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money). • The procedure for rounding decimal numbers is similar to the procedure for rounding whole numbers. • A strategy for rounding decimal numbers to the nearest tenth and hundredth is as follows: – Look one place to the right of the digit you want to round to. – If the digit is 5 or greater, add 1 to the digit in the rounding place, and drop the digits to the right of the rounding place. – If the digit is less than 5, leave the digit in the rounding place as it is, and drop the digits to the right of the rounding place. • Different strategies for rounding decimals include: – Use a number line to locate a decimal between two numbers. For example, 18.83 is closer to 18.8 than to 18.9. – Compare the digits in ~~this~~ the numbers to determine which number is greater (or which is less).		• Represent fractions for halves, fourths, fifths, and tenths as decimals through ~~thousandths~~ hundredths, using concrete objects (e.g., demonstrate the relationship between the fraction $\frac{1}{4}$ and its decimal equivalent 0.25). • Relate fractions to decimals, using concrete objects (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money [coins]). • Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4}$ = 0.25 or 0.25 = $\frac{1}{4}$)

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a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number, tenth, and hundredth;
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
– Compare the value of decimals, using the symbols >, <, = (e.g., 0.83 > 0.8 or 0.19 < 0.2). – Order the value of decimals, from least to greatest and greatest to least (e.g., 0.83, 0.821, 0.8). • Decimal numbers are another way of writing fractions (halves, fourths, fifths, and tenths). The ~~b~~Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles money). • Provide a fraction model (halves, fourths, fifths, and tenths) and ask students for its decimal equivalent. • Provide a decimal model and ask students for its fraction equivalent (halves, fourths, fifths, and tenths).		

Computation and estimation in grades 4 and 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work~~ing~~ with ~~fractions and~~ decimals. Instruction should focus on computation activities that enable students to model, explain, and develop ~~reasonable~~ proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract with common fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability to identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use (not identify) properties of operations to solve problems [e.g., 7 × 28 is equivalent to (7 × 20) + (7 × 8)~~, or (7 × 30) – (7 × 2)~~].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their computations.

Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

4.4 The student will
a) estimate sums, differences, products, and quotients of whole numbers;
b) add, subtract, and multiply whole numbers;
c) divide whole numbers, finding quotients with and without remainders; and
d) solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A sum is the result of adding two or more numbers. • A difference is the amount that remains after one quantity is subtracted from another. • An estimate is a number close to an exact ~~amount~~ solution. An estimate tells about how much or about how many. • Different strategies for estimating include using compatible numbers to estimate sums and differences and using front-end estimation for sums and differences. – Compatible numbers are numbers that are easy to work with mentally. Number pairs that are easy to add or subtract are compatible. When estimating a sum, replace actual numbers with compatible numbers (e.g., 52 + 74 can be estimated by using the compatible numbers 50 + 75). When estimating a difference, use numbers that are close to the original numbers. Tens and hundreds are easy to subtract (e.g., 83 – 38 is close to 80 – 40). – The front-end strategy for estimating is computing with the front digits. Front-end estimation for addition can be used even when the addends have a different number	**All students should** • Develop and use strategies to estimate whole number sums and differences and to judge the reasonableness of such results. • Understand that addition and subtraction are inverse operations. • Understand that division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. • Understand that multiplication and division are inverse operations. • Understand various representations of division and the terms used in division are *dividend*, *divisor*, and *quotient.* $\textit{dividend} \div \textit{divisor} = \textit{quotient}$ $\begin{array}{r} \textit{quotient} \\ \textit{divisor}\;\overline{)\textit{dividend}} \end{array}$ • Understand how to solve single-step and multistep problems using whole number operations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate whole number sums, ~~and~~ differences, products, and quotients ~~using rounding, front-end strategies, and compatible number strategies. Describe the method of estimation used~~. • Refine estimates by adjusting the final amount, using terms such as *closer to, between*, and *a little more than*. • Determine the sum or difference of two whole numbers, each 999,999 or less, in vertical and horizontal form with or without regrouping, using paper and pencil, and using a calculator. • Estimate and find the products of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer, using paper and pencil and calculators. • Estimate and find the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend. • Solve single-step and multistep problems using whole number operations. • Verify the reasonableness of sums, differences, products, and quotients of whole numbers using estimation.

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UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
of digits. The procedure requires the addition of the values of the digits in the greatest of the smallest number. For example: 2367 → 2300 243 → 200 <u>+ 1186</u> → <u>+ 1100</u> 3600 • Front-end or leading-digit estimation always gives a sum less than the actual sum; however, the estimate can be adjusted or refined so that it is closer to the actual sum. • Addition is the combining of quantities; it uses the following terms: *addend* → 45,623 *addend* → <u>+ 37,846</u> *sum* → 83,469 • Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms: *minuend* → 45,698 *subtrahend* → <u>– 32,741</u> *difference* → 12,957 • Before adding or subtracting with paper and pencil, addition and subtraction problems in horizontal form should be rewritten in vertical		

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UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
form by lining up the places vertically. • Using ~~b~~Base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the pictorial, to the abstract. Regrouping is used in addition and subtraction algorithms. In addition, when the sum in a place is 10 or more, is used to regroup the sums so that there is only one digit in each place. In subtraction, when the number (minuend) in a place is not enough from which to subtract, regrouping is required. • A certain amount of practice is necessary to develop fluency with computational strategies for multidigit numbers; however, the practice must be meaningful, motivating, and systematic if students are to develop fluency in computation, whether mentally, with manipulative materials, or with paper and pencil. • Calculators are an appropriate tool for computing sums and differences of large numbers, particularly when mastery of the algorithm has been demonstrated.		

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- The terms associated with multiplication are

$$\begin{array}{lcr} \textit{factor} & \rightarrow & 376 \\ \textit{factor} & \rightarrow & \underline{\times 23} \\ \textit{product} & \rightarrow & 8{,}648 \end{array}$$

- One model of multiplication is repeated addition.
- Another model of multiplication is the "Partial Product" model.

$$\begin{array}{rl} 24 & \\ \underline{\times 3} & \\ 12 & \leftarrow \text{Multiply the ones: } 3 \times 4 = 12 \\ \underline{+\,60} & \leftarrow \text{Multiply the tens: } 3 \times 20 = 60 \\ 72 & \end{array}$$

- Another model of multiplication is the "Area Model" (which also represents partial products) and should be modeled first with Base-10 blocks. (e.g., 23 x 68)
- Students should continue to develop fluency with single-digit multiplication facts and their related division facts.
- Calculators should be used to solve problems that require tedious calculations.
- Estimation should be used to check the reasonableness of the product. Examples of estimation strategies include the following:

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UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
– The front-end method: multiply the front digits and then complete the product by recording the number of zeros found in the factors. It is important to develop understanding of this process before using the step-by-step procedure. $\begin{array}{rcr} 523 & \rightarrow & 500 \\ \underline{\times\ 31} & \rightarrow & \underline{\times\ 30} \\ & & 15{,}000 \end{array}$ – This is $3 \times 5 = 15$ with 3 zeros. – Compatible numbers: replace factors with compatible numbers, and then multiply. Opportunities for students to discover patterns with 10 and powers of 10 should be provided. $\begin{array}{rcr} 64 & \rightarrow & 64 \\ \underline{\times\ 11} & \rightarrow & \underline{\times\ 10} \end{array}$ • Division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. Both situations may be modeled with ~~b~~Base-10 manipulatives. • Multiplication and division are inverse operations. • Terms used in division are *dividend, divisor*, and *quotient.*		

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UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
dividend ÷ divisor = quotient $\text{divisor}\,\overset{\underline{\text{quotient}}}{\overline{)\text{dividend}}}$ • Opportunities to invent division algorithms help students make sense of the algorithm. Teachers should teach division by various methods such as repeated multiplication and subtraction (partial quotients) before teaching the traditional long division algorithm.		

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;

c) add and subtract with decimals; and

d) solve single-step and multistep practical problems involving addition and subtraction with fractions and with decimals.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A factor of a number is an integer that ~~divides evenly into that number~~ ~~has~~ divides evenly into that number with a remainder of zero. • A factor of a number is a divisor of the number. • A multiple of a number is the product of the number and any natural number. • A common factor of ~~two or more numbers~~ two or more numbers ~~a number~~ is a divisor that all of the numbers share. • The least common multiple of two or more numbers is the smallest common multiple of the given numbers. • The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share. • Students should investigate addition and subtraction ~~of~~ with fractions, using a variety of models (e.g., fraction circles, fraction strips, rulers, ~~unifix~~ linking cubes, pattern blocks). • When adding or subtracting with fractions having like denominators, add or subtract the numerators and use the same denominator. Write the answer in simplest form using common multiples and factors. • When adding or subtracting with fractions having unlike	**All students should** • Understand and use common multiples and common factors for simplifying fractions. • Develop and use strategies to estimate addition and subtraction involving fractions and decimals. • Use visual models to add and subtract with fractions and decimals.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find common multiples and common factors of ~~two~~ numbers. • Determine the least common multiple and greatest common factor of ~~two~~ numbers. • Use least common multiple and/or greatest common factor to find a common denominator for ~~two~~ fractions. • Add and subtract with fractions having like denominators ~~of 12 or less, using concrete materials, pictorial representations, and paper and pencil~~ whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. • Add and subtract with fractions having unlike denominators ~~of 12 or less, using concrete materials pictorial representations and paper and pencil~~ whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. • Solve problems that involve adding and subtracting with fractions having like and unlike denominators ~~of~~

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;
b) add and subtract fractions having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;
c) add and subtract with decimals; and
d) solve single-step and multistep practical problems involving addition and subtraction with fractions and with decimals.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
denominators, rewrite them as fractions with a common denominator. The least common multiple (LCM) of the unlike denominators is a common denominator (LCD). Write the answer in simplest form using common multiples and factors. • Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money). • For decimal computation, the same ideas developed for whole number computation may be used, and these ideas may be applied to decimals, giving careful attention to the placement of the decimal point in the solution. Lining up tenths to tenths, hundredths to hundredths, etc. helps to establish the correct placement of the decimal. • Fractions may be related to decimals by using models (e.g., 10-by-10 grids, decimal squares, money).		~~12 or less.~~ whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. • Add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. • Solve single-step and multistep problems that involve adding and subtracting with fractions and decimals through thousandths.

Students in grades 4 and 5 should be actively involved in measurement activities that require a dynamic interaction between students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects. ~~The intent is for students to make "ballpark" comparisons and *not* to memorize conversion factors between U.S. Customary and metric units. To fully understand these ballpark comparisons, students must be actively engaged in the process of measurement.~~

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure perimeter, weight/mass, liquid volume/capacity, area, temperature, and time. Students continue to enhance their understanding of measurement by using appropriate tools such as rulers, balances, clocks, and thermometers. The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.

4.6 The student will

a) estimate and measure weight/mass and describe the results in U.S. Customary and metric units as appropriate; and

b) identify equivalent measurements between units within the U.S. Customary system (ounces, pounds, and tons) and between units within the metric system (grams and kilograms).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • Balances are appropriate measuring devices to measure weight in U.S. Customary units (ounces, pounds) and mass in metric units (grams, kilograms). • Practical experience measuring the mass of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate <u>weight/</u>mass. ~~• Students may use familiar benchmarks to make ballpark comparisons, such as~~ ~~– 1 ounce is about 28 grams (a slice of bread weighs about 1 ounce);~~ ~~– 1 nickel has the mass of about 5 grams; and~~ ~~– 1 kilogram is a little more than 2 pounds (a textbook has a mass of about 1 kilogram).~~ • Students should estimate the mass/weight of everyday objects (e.g., foods, pencils, book bags, shoes), using appropriate metric or U.S. Customary units.	**All students should** • Use benchmarks to estimate and measure weight/mass. • Identify equivalent measures between <u>units within the</u> U.S. Customary and <u>between units within the</u> metric measurements.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (e.g., ounce, pound, <u>ton,</u> gram, kilogram) to use when measuring everyday objects in both metric and U.S. Customary units. • Measure objects in both metric and U.S. Customary units (e.g., ounce, pound, <u>ton,</u> gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. • Record the mass of an object including the appropriate unit of measure (e.g., 24 grams). ~~• Estimate conversions between U.S. Customary and metric units, using ballpark comparisons, such as~~ ~~– 1 ounce is about 28 grams;~~ ~~– 1 nickel has the mass of about 5 grams; and~~ ~~– 1 kilogram is a little more than 2 pounds.~~

4.7 The student will

a) estimate and measure length, and describe the result in both metric and U.S. Customary units; and

b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Length is the distance along a line or figure from one point to another. • U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler~~s~~, meter stick~~s~~, and tape~~s~~ measure. • Practical experience measuring the length of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate length. ~~• Students may use familiar benchmarks to make ballpark comparisons, such as~~ ~~– 1 inch is about 2.5 centimeters (the diameter of a quarter is about 1 inch);~~ ~~– 1 meter is a little longer than 1 yard (a doorway is about a yard in width); and~~ ~~– 1 mile is slightly farther than 1.5 kilometers, or 1 kilometer is slightly farther than half a mile.~~ • Students should estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure.	**All students should** • Use benchmarks to estimate and measure length. • Understand how to convert units of length between the U.S. Customary and metric systems, using ballpark comparisons. • Understand the relationship between U.S. Customary units and the relationship between metric units.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (e.g., inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring everyday objects in both metric and U.S. Customary units. • Estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure. • Measure the length~~s~~ of objects in both metric and U.S. Customary units, measuring to the nearest inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), foot, yard, mile, millimeter, centimeter, or meter, and record the length including the appropriate unit of measure (e.g., 24 inches). • Compare estimates of the length of objects with the actual measurement of the length of objects. • Identify equivalent measures of length between units within the U.S. Customary measurements and between units within the metric measurements. ~~Estimate conversions between the U.S. Customary and metric units, using ballpark comparisons, such as~~ ~~– 1 inch is about 2.5 centimeters;~~

4.7 The student will
a) estimate and measure length, and describe the result in both metric and U.S. Customary units; and
b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), inch, foot, or yard.		~~1 meter is a little longer than 1 yard;~~ ~~1 mile is slightly farther than 1.5 kilometers; and~~ ~~1 kilometer is slightly farther than half a mile.~~

4.8 The student will

a) estimate and measure liquid volume and describe the results in U.S. Customary units; and

b) identify equivalent measurements between units within the U.S. Customary system (cups, pints, quarts, and gallons).

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• U.S. Customary units for measurement of liquid volume include cups, pints, quarts, and gallons. ~~Metric units for measurement of liquid volume include milliliters and liters~~. • The measurement of the object must include the unit of measure along with the number of iterations. • Students should measure the liquid volume of everyday objects in ~~both metric and~~ U.S. Customary units, including cups, pints, quarts, gallons, ~~milliliters, and liters~~, and record the volume including the appropriate unit of measure (e.g., 24 gallons). • Practical experience measuring liquid volume of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate liquid volume. • ~~Students may use familiar benchmarks to make ballpark comparisons, such as 1 quart is a little less than 1 liter, and 1 liter is a little more than 1 quart.~~ • Students should estimate the liquid volume of containers in ~~both metric and~~ U.S. Customary units to the nearest cup, pint, quart, and gallon~~, milliliter, or liter~~.	**All students should** • Use benchmarks to estimate and measure volume. • ~~Understand how to convert units of volume~~ Identify equivalent measurements between units within the U.S. Customary ~~and metric~~ systems~~, using ballpark comparisons~~.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (cups, pints, quarts, gallons~~, milliliters, or liters~~) to use when measuring liquid volume in ~~both metric and~~ U.S. Customary units. • Estimate the liquid volume of containers in ~~both metric and~~ U.S. Customary units of measure to the nearest cup, pint, quart, and gallon~~, milliliter, or liter~~. • Measure the liquid volume of everyday objects in ~~both metric and~~ U.S. Customary units, including cups, pints, quarts, and gallons~~, milliliters, and liters~~, and record the volume including the appropriate unit of measure (e.g., 24 gallons). • Identify equivalent measures of volume between units within the U.S. Customary system ~~and metric measurements~~. • ~~Estimate conversion between U.S. Customary and metric units, using ballpark comparisons, such as 1 quart is a little less than 1 liter, and 1 liter is a little more than 1 quart.~~

4.9 The student will determine elapsed time in hours and minutes within a 12-hour period.

<table>
<tr><th>UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)</th><th>ESSENTIAL UNDERSTANDINGS</th><th>ESSENTIAL KNOWLEDGE AND SKILLS</th></tr>
<tr><td>
• Elapsed time is the amount of time that has passed between two given times.
• Elapsed time should be modeled and demonstrated using analog clocks and timelines.
• Elapsed time can be found by counting on from the beginning time to the finishing time.
– Count the number of whole hours between the beginning time and the finishing time.
– Count the remaining minutes.
– Add the hours and minutes.
For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count 10 minutes; and then, add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.
</td><td>
All students should
• Understanding the “counting on” strategy for determining elapsed time in hour and minute increments over a 12-hour period from a.m. to a.m. or p.m. to p.m.
</td><td>
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
• Determine the elapsed time in hours and minutes within a 12-hour period (times can cross between a.m. and p.m. ~~times~~).
• Solve practical problems in relation to time that has elapsed.
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The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric ~~shapes~~ figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among ~~shapes~~ figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric ~~shapes~~ figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare ~~shapes~~ figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. ~~Discussing ideas, conjecturing, and testing hypotheses precede the development of more formal summary statements.~~ In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles. ~~Students also explore coordinate geometry, using the coordinate plane to describe points in the first quadrant.~~

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking ~~A~~algebra.)

4.10 The student will

a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and

b) identify representations of lines that illustrate intersection, parallelism, and perpendicularity.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A point is ~~an exact~~ a location in space. It has no length~~,~~ ~~or~~ width, or height. A point is usually named with a capital letter. • A line is a collection of points going on and on infinitely in both directions. It has no endpoints. ~~A line is a collection of points going on and on infinitely in both directions. It has no endpoints.~~ When a line is drawn, at least two points on it can be marked and given capital letter names. ~~The line can also be named with a single, lower-case letter~~. ~~Arrowheads~~ Arrows must be drawn to show that the line goes on in both directions infinitely (e.g., $\overleftrightarrow{AB}$, read as "the line AB"). • A line segment is part of a line. It has two endpoints and includes all the points between those endpoints. To name a line segment, name the endpoints (e.g., $\overline{AB}$, read as "the line segment AB"). • A ray is part of a line. It has one endpoint and continues ~~on and on~~ infinitely in one direction. To name a ray, say the name of its endpoint first and then say the name of one other point on the ray (e.g., $\overrightarrow{AB}$, read as "the ray AB"). • Two rays that have the same endpoint form an angle. This endpoint is called the *vertex*. Angles are found wherever lines and line segments intersect. An angle can be named in three different ways by using	**All students should** • Understand that points, lines, line segments, ~~and~~ rays, and angles, including endpoints and vertices are fundamental components of noncircular geometric figures. • Understand that the shortest distance between two points on a flat surface is a line segment. • Understand that lines in a plane either intersect or are parallel. Perpendicularity is a special case of intersection. • Identify ~~real-world~~ practical situations that illustrate parallel, intersecting, and perpendicular lines.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Differentiate among a point, line, line segment, and ray by using the definitions to compare.~~ • ~~Investigate and describe~~ Identify and describe ~~the relationships between and among~~ representations of points, lines, line segments, ~~and~~ rays, and angles, including endpoints and vertices. • Understand that lines in a plane can intersect or are parallel. Perpendicularity is a special case of intersection. • Identify practical situations that illustrate parallel, intersecting, and perpendicular lines.

4.10 The student will
a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and
b) identify representations of lines that illustrate intersection, parallelism, and perpendicularity.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
– three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle. • Intersecting lines ~~are lines that cross and~~ have one point in common. • Perpendicular lines are special intersecting lines that form right angles ~~(square corners)~~ where they intersect. • Parallel lines are lines that lie ~~on the same flat~~ in the same place ~~surface (plane) and never cross~~ and do not intersect. Parallel lines are always the same distance apart and do not share any points. • Students should explore intersection, parallelism, and perpendicularity in both two and three dimensions. For example, students should analyze the relationships between the edges of a cube. Which ~~line segments~~ edges are parallel? Which are perpendicular? What plane contains the upper left edge and the lower right edge of the cube? Students can visualize this by using the classroom itself to notice the lines formed by the intersection of the ceiling and walls, of the floor and wall, and of two walls.		

4.11 The student will

a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and

b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. – **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. – **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.) – **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.) – **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusion are understood, but the role and significance of deduction is not	**All students should** • ~~Understand that two-dimensional (plane) figures are unique in their defining properties.~~ • ~~Understand that three-dimensional (solid) figures are unique in their defining properties~~. • Understand the meaning of the term *congruent*. • Understand how to identify congruent ~~shapes~~ figures. • Understand that the orientation of figures does not affect congruency or noncongruency.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify and describe the properties of squares, rectangles, triangles, parallelograms, rhombi, and circles.~~ • ~~Identify and describe the properties of spheres, cubes, and rectangular solids (prisms).~~ • ~~Identify congruent and noncongruent figures.~~ • ~~Analyze and compare the properties of circles and spheres; squares and cubes; and rectangles and rectangular solids (prisms).~~ • Recognize the congruence of plane figures resulting from geometric transformations such as translation, reflection, and rotation, using mirrors, paper folding and tracing.

4.11 The student will

a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and

b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.) • ~~A polygon is a two-dimensional geometric figure which has straight sides and can be described as simple and closed.~~ • ~~A triangle is a polygon with three angles and three sides.~~ • ~~A quadrilateral is a polygon with four sides.~~ • ~~A rectangle is a quadrilateral with four right angles.~~ • ~~A square is a rectangle with four sides of equal length.~~ • ~~A circle is a closed curve with all points in one plane and equidistant from a fixed point (the center).~~ • Congruent figures are figures having exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models. • ~~A cube is a three-dimensional solid with six congruent square faces and every edge the same length. A cube has 6 faces and 12 edges.~~ • ~~A cylinder is a three-dimensional object formed by two congruent parallel circles joined by a curved surface (usually a solid or hollow object~~		

4.11 The student will

a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and

b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~shaped like a can).~~ • ~~A cone is a solid, pointed figure having a flat, round base (usually a circle) that is joined to a vertex by a curved surface.~~ • ~~A rectangular solid (prism) is a three-dimensional figure in which all six faces are rectangles with three pair of parallel, congruent opposite faces.~~ • ~~A square pyramid is a solid whose base is a square and whose faces are triangles that share a common vertex.~~ • ~~A sphere is a three-dimensional object with all of its points the same distance from the center.~~ • A translation ~~(slide)~~ is a transformation in which an image is formed by moving every point on a figure the same distance in the same direction. • A reflection ~~(flip)~~ is a transformation in which a figure is flipped over a line called the *line of reflection*. All corresponding points in the image and preimage are equidistant from the line of reflection. • A rotation ~~(turn)~~ is a transformation in which an image is formed by turning its preimage about a point. • The resulting figure of a translation, reflection, or rotation is congruent to the original figure.		

4.12 The student will
a) define polygon; and
b) identify polygons with 10 or fewer sides.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a ~~two-dimensional~~ closed plane geometric figure ~~which has straight sides and can be described as simple and closed~~ composed of at least three line segments that do not cross. None of the sides are curved. • A triangle is a polygon with three angles and three sides. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a rectangle with four sides of equal length. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. • A parallelogram is a quadrilateral with both pairs of opposite sides parallel. • A rhombus is a quadrilateral with 4 congruent sides. • A pentagon is a 5-sided polygon. • A hexagon is a 6-sided polygon. • A heptagon is a 7-sided polygon. • An octagon is an 8-sided polygon. • A nonagon is a 9-sided polygon. • A decagon is a 10-sided polygon.	**All students should** • Identify polygons with 10 or fewer sides in everyday situations. • Identify polygons with 10 or fewer sides in multiple orientations (rotations, reflections, and translations of the polygons).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Define and identify properties of polygons with 10 or fewer sides. • Identify polygons by name with 10 or fewer sides in multiple orientations (rotations, reflections, and translations of the polygons)

Students entering grades 4 and 5 have explored the concepts of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students are able to predict whether the game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction at grades 4 and 5 is to deepen their understanding of the concepts of probability by

- ~~developing the continuum of terms to include *impossible*, *unlikely*, *equally likely*, *possible*, and *certain*;~~
- offering opportunities to set up models simulating ~~real-life~~ practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities ~~imbued~~ that instill ~~with~~ a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to ~~real-world~~ practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same?” “Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- writing a few sentences to communicate to others their ~~analysis and~~ interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

4.13 The student will
a) predict the likelihood of an outcome of a simple event; and
b) represent probability as a number between 0 and 1, inclusive.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives. • Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as unlikely*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring.	**All students should** • Understand and apply basic concepts of probability. • Describe events as likely or unlikely and discuss the degree of likelihood, using the terms *certain*, *likely*, *equally likely*, *unlikely*, and *impossible*. • Predict the likelihood of an outcome of a simple event and test the prediction. • Understand that the measure of the probability of an event can be represented by a number between 0 and 1, inclusive.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model and determine all possible outcomes of a given simple event where there are no more than ~~12~~ 24 possible outcomes, using a variety of manipulatives, such as coins, number cubes, and spinners. • Conduct experiments to determine the probability of an event occurring for a given number of trials (no more than ~~12~~ 25 trials), using manipulatives (e.g., the number of times "heads" occurs when flipping a coin 10 times; the chance that when the names of 12 classmates are put in a shoebox, a name that begins with *D* will be drawn). • Write the probability of a given simple event as a fraction, where the total number of possible outcomes is ~~12~~ 24 or fewer. • Identify the likelihood of an event occurring and relate it to its fractional representation (e.g., impossible/0; equally likely/$\frac{1}{2}$; certain/1). • ~~Set up a~~ Represent probability ~~number line showing that a number probability can be represented by~~ as a point between 0 and 1, inclusively, on a number line. • Determine the outcome of an event that is least likely to occur (less than half) or most likely to occur (greater than half) when the number of possible outcomes is 24 or less.

4.13 The student will
a) predict the likelihood of an outcome of a simple event; and
b) represent probability as a number between 0 and 1, inclusive.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
Activities should include ~~real-life~~ practical examples. • For ~~any~~ an event such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. • For another event such as spinning a spinner that is one-third red and two-thirds blue, the two outcomes, red and blue, are not equally likely. This is an unfair spinner (since it is not divided equally), therefore, the outcomes are not equally likely.		

4.14 The student will collect, organize, display, and interpret data from a variety of graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data analysis helps describe data, recognize patterns or trends, and make predictions. • Investigations involving ~~real-world~~ practical data should occur frequently, and data can be collected through brief class surveys or through more extended projects taking many days. • Students formulate questions, predict answers to questions under investigation, collect and represent initial data, and consider whether the data answer the questions. • Line graphs are used to show how two continuous variables are related. Line graphs ~~are~~ may be used to show how one variable changes over time. If this one variable is not continuous, then a broken line is used. By looking at a ~~single~~ line graph, it can be determined whether the variable is increasing, decreasing, or staying the same over time. – The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as "continuous data," as it represents data collected over a continuous period of time. – The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals, depending upon the data being collected. The scale should	**All students should** • Understand the difference between representing categorical data and representing numerical data. • Understand that line graphs show change over time (numerical data). • Understand that bar graphs should be used to compare counts of different categories (categorical data). • Understand how data displayed in bar and line graphs can be interpreted so that informed decisions can be made. • Understand that the title and labels of the graph provide the foundation for interpreting the data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect data, using, for example, observations, measurement, surveys, scientific experiments, polls, or questionnaires. • Organize data into a chart or table. • Construct and display data in bar graphs, labeling one axis with equal whole number increments of 1 or more (numerical data) (e.g., ~~multiples of~~ 2, 5, 10, or 100) and the other axis with categories related to the title of the graph (categorical data) (e.g., swimming, fishing, boating, and water skiing as the categories of "Favorite Summer Sports"). • Construct and display data in line graphs, labeling the vertical axis with equal whole number increments of 1 or more and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than ~~four~~ 10 identified points along a continuum for continuous data. For example, growth charts showing age versus height place age on the horizontal axis (e.g., 1 month, 2 months, 3 months, and 4 months). • Title ~~the given graph~~ or identify the title in a given graph and label or identify the axes. • ~~Analyze information~~ Interpret data from simple line and bar graphs by describing the characteristics of the data and the data as a whole (e.g., the category with the greatest/least, categories with the same number of responses, similarities and differences, the total number).

4.14 The student will collect, organize, display, and interpret data from a variety of graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
extend one increment above the greatest recorded piece of data. – Each axis should be labeled, and the graph should be given a title. – A line graph tells whether something has increased, decreased, or stayed the same with the passage of time. Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). • Bar graphs should be used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs. – A bar graph uses parallel, horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis that displays the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Fourth-grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, ~~or~~ 10, or 100. – Each axis should be labeled, and the graph should be given a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the		Data points will be limited to ~~20~~ 30 and categories to 4 8. • Interpret the data to answer the question posed, and compare the answer to the prediction (e.g., "The summer sport preferred by most is swimming, which is what I predicted before collecting the data."). • Write at least one sentence to describe the analysis and interpretation of the data, identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same. • ~~Select from among four choices a correct analysis of the data presented in a bar or line graph. For example, given a line graph showing the number of soccer players (in millions) in the U.S. over the time period 1980 to 2000 in five-year intervals, select the correct answer response that relates to the graphs, such as, "The greatest increase in number of soccer players occurred between 1985 and 1990."~~

4.14 The student will collect, organize, display, and interpret data from a variety of graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
categories, and total number of responses) should be written.		

Students entering grades 4 and 5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships in open number sentences~~, and they have begun to solve simple equations with one unknown~~.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, ~~graphic,~~ and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They ~~analyze~~ interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By ~~analyzing~~ interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write "rules" for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

4.15 The student will recognize, create, and extend numerical and geometric patterns.

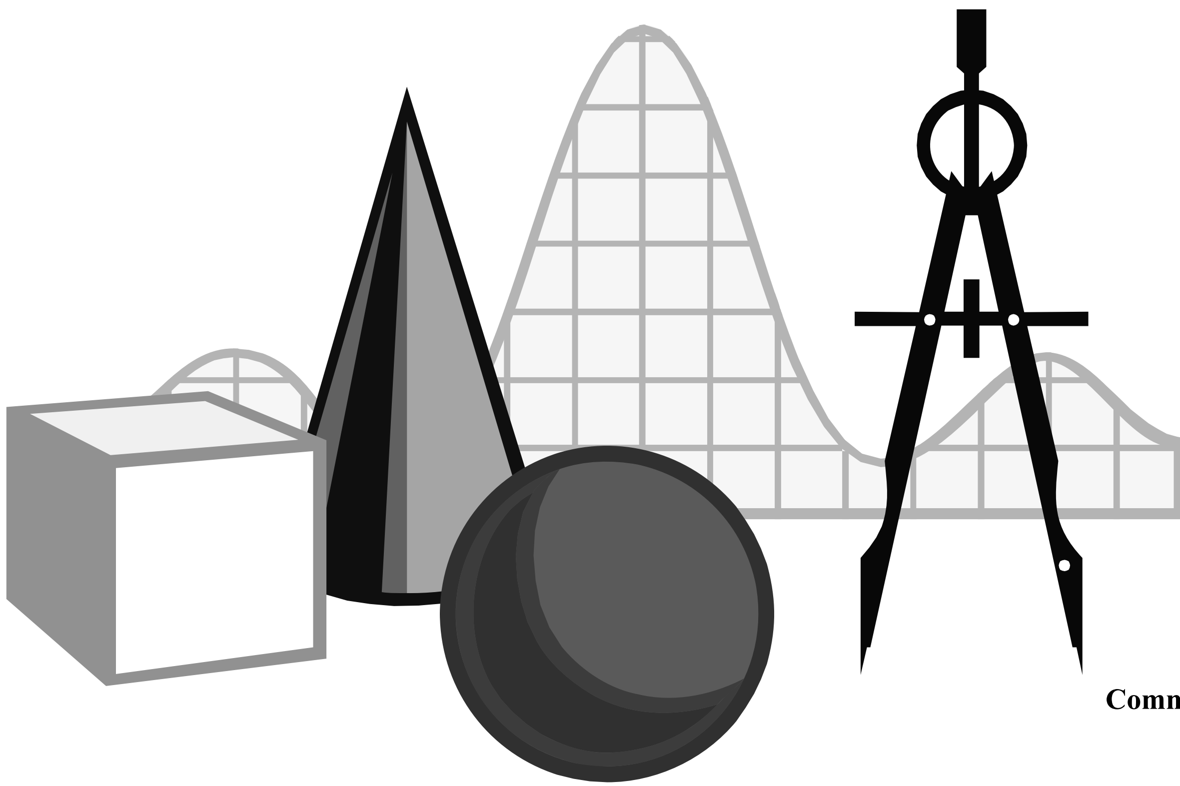
UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Most patterning activities should involve some form of concrete materials to make up a pattern. – Students will identify and extend a wide variety of patterns, including rhythmic, geometric, graphic, numerical, and algebraic. The patterns will include both growing and repeating patterns. • Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing the relationship symbolically or algebraically. • Tables of values should be analyzed for a pattern to determine ~~what element comes next~~ the next value.	**All students should** • Understand that patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe geometric and numerical patterns, using tables, symbols, or words. • Create geometric and numerical patterns, using concrete materials, number lines, tables, and words. • Extend geometric and numerical patterns, using concrete materials, number lines, tables, and words.

4.16 The student will
a) recognize and demonstrate the meaning of equality in an equation; and
b) investigate and describe the associative property for addition and multiplication.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. • The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2). • The associative property for addition states that the sum stays the same when the grouping of addends is changed [e.g., 15 + (35 + 16) = (15 + 35) + 16]. The associative property for multiplication states that the product stays the same when the grouping of factors is changed [e.g., 6 × (3 × 5) = (6 × 3) × 5].	**All students should** • Understand that mathematical relationships can be expressed using equations. • Understand that quantities on both sides of an equation must be equal. • Understand that the associative property for addition means you can change the groupings of three or more addends without changing the sum. • Understand that the associative property for multiplication means you can change the groupings of three or more factors without changing the product**.**	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and demonstrate that the equals sign (=) relates equivalent quantities in an equation. • Write ~~number sentences~~ an equation to represent equivalent mathematical relationships (e.g., 4 × 3 = 2 × 6). • ~~Identify number sentences~~ Recognize and demonstrate ~~that show~~ appropriate use of the equals sign in an equation. • Investigate and describe the associative property for addition as (6 + 2) + 3= 6 + (2 + 3). • Investigate and describe the associative property for multiplication as (3 x 2) x 4 = 3 x (2 x 4).

MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 5



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

Mathematics instruction in grades 4 and 5 should continue to foster the development of number sense, especially with decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction at grades 4 and 5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names). Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve problems.

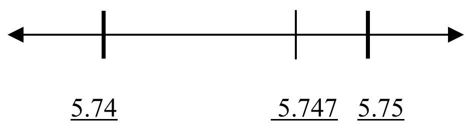
5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the ~~b~~Base-10 number system is based upon a simple pattern of tens in which each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. • A decimal point separates the whole number places from the places less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*. • To read decimals, – read the whole number to the left of the decimal point, if there is one; – read the decimal point as "and"; – read the digits to the right of the decimal point just as you would read a whole number; and – say the name of the place value of the digit in the smallest place. • Decimals may be written in a variety of forms: – Standard: 23.456 – Written: Twenty-three and four hundred fifty-six thousandths – Expanded: $(2 \times 10) + (3 \times 1) + (4 \times 0.1) + (5 \times 0.01) + (6 \times 0.001)$ • To help students identify the ten-to-one place value relationship for decimals through thousandths, use ~~b~~Base-10 manipulatives, such as place value mats/charts, decimal squares, ~~b~~Base-10 blocks, and money. • Decimals can be rounded to the nearest whole number, tenth or hundredth in situations when exact numbers are not needed.	**All students should** • ~~Understand the place-value structure of decimals and use this structure to read, write, and compare decimals.~~ • Understand that decimals are rounded in a way that is similar to the way whole numbers are rounded. • Understand that decimal numbers can be rounded to ~~an~~ estimate when exact numbers are not needed for the situation at hand.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify the place values for each digit in decimals through thousandths.~~ • ~~Read decimal numbers through thousandths from written words or place-value format.~~ • ~~Write decimal numbers through thousandths from written words or from decimal numbers presented orally.~~ • Round decimal numbers to the nearest whole number, tenth or hundredth. • ~~Identify the symbols for the terms *greater than*, *less than*, and *equal to*.~~ • ~~Compare the value of two decimal numbers through thousandths, using the symbols >, <, or =.~~

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~A~~ Strategies for rounding decimal numbers to the nearest whole number, tenth and hundredth ~~is~~ are as follows: – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero. – ~~If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero.~~ • ~~A procedure for comparing two decimals by examining place value may include the following:~~ – ~~Line up the decimal numbers at their decimal points.~~ – ~~Beginning at the left, find the first place value where the digits are different.~~ – ~~Compare the digits in this place value to determine which number is greater (or which is less).~~ – ~~Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented.~~ – ~~If both numbers are the same, use the symbol = or words *equal to*.~~ ~~Two numbers can be compared by examining place value and/or using a number line.~~ • Create a number line that shows the decimal that is to be rounded.		

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The position of the decimal will help children conceptualize the number's placement relative for rounding. An example is to round 5.747 to the nearest hundredth:  		

5.2 The student will

a) recognize and name fractions in their equivalent decimal form and vice versa; and

b) compare and order fractions and decimals in a given set from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Students should recognize, name, and focus on finding equivalent decimals of familiar fractions such as halves, fourths, fifths, eighths, and tenths. • Students should be able to determine equivalent relationships between decimals and fractions with denominators up to 12. • Students should have experience with fractions such as $\frac{1}{8}$, whose decimal representation is a terminating decimal (e. g., $\frac{1}{8} = 0.125$) and with fractions such as $\frac{2}{9}$, whose decimal representation does not end but continues to repeat (e. g., $\frac{2}{9} = 0.222\ldots$). The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$. • To help students compare the value of two decimals through thousandths, use manipulatives, such as place value mats/charts, 10-by-10 grids, decimal squares, ~~b~~Base-10 blocks, meter sticks, number lines, and money. • ~~A procedure for comparing two decimals by examining may include the following:~~ ~~– Line up the decimal numbers at their decimal points.~~	**All students should** • Understand the relationship between ~~commonly used~~ fractions and their decimal form and vice versa. • Understand that fractions and decimals can be compared and ordered from least to greatest and greatest to least.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent fractions (halves, fourths, fifths, eighths, ~~and~~ tenths, and twelfths) in their equivalent decimal form and vice versa. • ~~Represent decimals in their equivalent fraction form (halves, fourths, fifths, eighths, and tenths).~~ • ~~Determine~~ Recognize and name equivalent relationships between decimals and fractions with denominators up to 12. • ~~Order~~ Compare and order from least to greatest and greatest to least a given set of no more than five numbers written as decimals, ~~and as~~ fractions, and mixed numbers with denominators of 12 or less.

5.2 The student will
a) recognize and name fractions in their equivalent decimal form and vice versa; and
b) compare and order fractions and decimals in a given set from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~– Beginning at the left, find the first place value where the digits are different.~~ ~~– Compare the digits in this place value to determine which number is greater (or which is less).~~ ~~– Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented.~~ ~~– If both numbers are the same, use the symbol = or words *equal to*.~~ ~~Two numbers can be compared by examining place value and/or using a number line.~~ • Decimals and fractions represent the same relationships; however, they are presented in two different formats. Decimal numbers are another way of writing fractions. Base-10 models (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money) concretely relate fractions to decimals and vice versa.		

5.3 The student will
a) identify and describe the characteristics of prime and composite numbers; and
b) identify and describe the characteristics of even and odd numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A prime number is a natural number that has exactly two different factors, one and the number itself. • A composite number is a natural number that has more than two different factors. • The number 1 is neither prime nor composite because it has only one factor, itself. • The prime factorization of a number is a representation of the number as the product of its prime factors. For example, the prime factorization of 18 is $2 \times 3 \times 3$. • Prime factorization concepts can be developed by using factor trees. • ~~Zero is not a natural number. Although it has an infinite number of factors, it is neither prime nor composite.~~ • Prime or composite numbers can be represented by rectangular models or rectangular arrays on grid paper. A prime number can be represented by only one rectangular array (e.g., 7 can ~~only~~ be represented by a 7×1 and a 1 x 7). A composite number can always be represented by more than ~~one~~ two rectangular arrays (e.g., 9 can be represented by a 9×1, a 1 x 9, or a 3×3). • Divisibility rules are useful tools in identifying prime and composite numbers. • Students should use manipulatives (e.g., Base-10 blocks, cubes, tiles, hundreds board, etc.) to explore and categorize numbers into groups of	**All students should** • Understand and use the unique characteristics of certain sets of numbers, including prime, composite, even, and odd numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify ~~which numbers are~~ prime ~~for~~ numbers less than or equal to ~~50~~ 100. • Identify ~~which numbers are~~ composite ~~for~~ numbers less than or equal to ~~50~~ 100. • Explain orally and in writing why a number is prime or composite. • Identify which numbers are even or odd. • Explain and demonstrate with manipulatives, pictorial representations, oral language, or written language why a number is even or odd.

5.3 The student will
a) identify and describe the characteristics of prime and composite numbers; and
b) identify and describe the characteristics of even and odd numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
odd or even. • Students should use rules to categorize numbers into groups of odd or even. Rules can include: – An odd number does *not* have 2 as a factor or is *not* divisible by 2. – The sum of two even numbers is even. – The sum of two odd numbers is even. – The sum of an even and an odd is odd. – Even numbers have an even number or zero in the ones place. – Odd numbers have an odd number in the ones place. – An even number has 2 as a factor or is divisible by 2.		

Computation and estimation in grades 4 and 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work~~ing~~ with ~~fractions and~~ decimals. Instruction should focus on computation activities that enable students to model, explain, and develop ~~reasonable~~ proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract with common fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability not only to identify but to use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use (not identify) properties of operations to solve problems [e.g., 7 × 28 is equivalent to (7 × 20) + (7 × 8)~~, or (7 × 30) – (7 × 2)~~].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their computations.

Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental math<u>ematics</u>, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An example of an approach to solving problems is Polya's four-step plan: – Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new. – Plan: Decide what operation(s) to use and what sequence of steps to use to solve the problem. – Solve: Follow the plan and work accurately. If the first attempt doesn't work, try another plan. – Look back: Does the answer make sense? • Estimation gives a rough idea of an amount. Strategies such as front-end, rounding, and mental computation may be used to estimate addition, subtraction, multiplication, and division of whole numbers. • Examples of problems to be solved by using estimation strategies are encountered in shopping for groceries, buying school supplies, budgeting allowance, and sharing the cost of a pizza or the prize money from a contest. • Estimation can be used to check the reasonableness of the results. • ~~Terms used in multiplication are~~ ~~*factor* → 376~~ ~~*factor* → × 23~~ ~~*product* → 8,648~~	**All students should** • Understand the meaning of mathematical operations and how these operations relate to one another when creating and solving single-step and multistep word problems.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with whole numbers. • Create single-step and multistep problems involving the operations of addition, subtraction, multiplication, and~~/or~~ division with and without remainders of whole numbers, using ~~real-life~~ practical situations. • Estimate the sum, difference, product, and quotient of whole number computations. • Solve single-step and multistep problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers, using paper and pencil, mental computation, and calculators in which – sums, differences, and products will not exceed five digits; – multipliers will not exceed two digits; – divisors will not exceed two digits; or – dividends will not exceed four digits. • Use two or more operational steps to solve a multistep problem. Operations can be the same or different.

5.5 The student will

a) find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths (divisors with only one nonzero digit); and

b) create and solve single-step and multistep practical problems involving decimals.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Addition and subtraction ~~with~~ of decimals may be investigated using a variety of models (e.g., 10-by-10 grids, number lines, money).
- Decimal computation uses similar procedures as those developed for whole number computation and applies them to decimal place values, giving careful attention to the placement of the decimal point in the solution.
- Multiplication of decimals follows the same procedure as multiplication of whole numbers. The only difference is that a decimal point must be correctly placed in the product giving careful attention to the placement of the decimal point in the solution.
- The product of decimals is dependent upon the two factors being multiplied.

~~Factors~~		~~Products~~
~~tenths × tenths~~	~~=~~	~~hundredths~~
~~tenths × hundredths~~	~~=~~	~~thousandths~~
~~hundredths × hundredths~~	~~=~~	~~ten-thousandths~~
~~tenths × thousandths~~	~~=~~	~~ten-thousandths~~

- In cases where an exact product is not required, the product of decimals can be estimated using strategies for multiplying whole numbers, such as front-end and compatible numbers, or rounding. In each case, the student needs to determine where to place the decimal point to ensure that the product is reasonable.
- Division is the operation of making equal groups

ESSENTIAL UNDERSTANDINGS

All students should

- Use similar procedures as those developed for whole number computation and apply them to decimal place values, giving careful attention to the placement of the decimal point in the solution.
- Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with decimal numbers.
- Understand the various meanings of *division* and its effect on whole numbers.
- Understand various representations of division, i.e.,

$$\textit{dividend} \div \textit{divisor} = \textit{quotient}$$

$$\textit{divisor}\ \overset{\textit{quotient}}{\overline{\big)\,\textit{dividend}}}$$

$$\frac{\textit{dividend}}{\textit{divisor}} = \textit{quotient}.$$

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine an appropriate method of calculation to find the sum, difference, ~~and~~ product, and quotient of two numbers expressed as decimals through thousandths, selecting from among paper and pencil, estimation, mental computation, and calculators.
- Estimate to find the number that is closest to the sum, difference, and product of two numbers expressed as decimals through thousandths.
- Find the sum, difference, and product of two numbers expressed as decimals through thousandths, using paper and pencil, estimation, mental computation, and calculators.
- Determine the quotient, given a dividend expressed as a decimal through thousandths (and no ~~annexing~~ adding of zeros to the dividend during the division process) and a single-digit divisor. ~~All dividends should be evenly divisible by the divisor.~~ For example, 5.4 divided by 2 and 5.4 divided by 6.
- ~~Find the sum, difference, and product of two numbers expressed as decimals through thousandths, using mental computation.~~
- ~~Find the sum, difference, and product of two numbers expressed as decimals through thousandths, using calculators.~~
- Use estimation to check the reasonableness of a sum, difference, ~~and~~ product, and quotient.

5.5 The student will
a) find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths (divisors with only one nonzero digit); and
b) create and solve single-step and multistep practical problems involving decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
or shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. Both situations may be modeled with ~~b~~Base-10 manipulatives. • The fair-share concept of decimal division can be modeled, using manipulatives (e.g., ~~b~~Base-10 blocks). • Division ~~of~~ with decimals is performed the same way as division of whole numbers. The only difference is the placement of the decimal point in the quotient. • The quotient can be estimated, given a dividend expressed as a decimal through thousandths (and no ~~annexing~~ ~~of~~ adding of zeros to the dividend during the division process) and a single-digit divisor. • Estimation can be used to check the reasonableness of a quotient. • Division is the inverse of multiplication; therefore, multiplication and division are inverse operations. • Terms used in division are *dividend, divisor*, and *quotient.* dividend ÷ divisor = quotient quotient divisor)dividend • There are a variety of algorithms for division such	.	• Create and solve single-step and multistep problems. • A multistep problem needs to incorporate two or more operational steps (operations can be the same or different).

5.5 The student will

a) find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths (divisors with only one nonzero digit); and

b) create and solve single-step and multistep practical problems involving decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
as repeated multiplication and subtraction. Experience with these algorithms may enhance understanding of the traditional long division algorithm. • A multistep problem needs to incorporate no more than two operational steps (operations can be the same or different).		

5.6 The student will solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers and express answers in simplest form.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. • ~~A fraction is in simplest form when its~~ When the numerator and denominator have no common factors other than 1~~.~~, then the fraction is in simplest form. • Fractions having like denominators means the same as fractions having common denominators. • Equivalent fractions name the same amount. To find equivalent fractions, multiply or divide the numerator and denominator by the same nonzero number. • Addition and subtraction with fractions and mixed numbers can be modeled using a variety of concrete materials and pictorial representations as well as paper and pencil. • To add, subtract, and compare fractions and mixed numbers, it often helps to find the least common denominator. The least common denominator (LCD) of two or more fractions is the least common multiple (LCM) of the denominators. • To add or subtract with fractions having the same or like denominators, add or subtract the numerators and write in simplest form. • To add or subtract with fractions that do not have the same denominator, first find equivalent fractions with the least common denominator. Then add or subtract and write the answer in	**All students should** • Develop and use strategies to estimate and compute addition and subtraction of fractions. • Understand the concept of least common multiple and least common denominator as they are important when adding and subtracting fractions. • Understand that a fraction is in simplest form when its numerator and denominator have no common factors other than 1. The numerator can be greater than the denominator.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Add and subtract~~ Solve single-step and multistep practical problems involving addition and subtraction with fractions having like and unlike denominators. Denominators in the problems should be limited to 12 or less (e.g., $\frac{1}{5} + \frac{1}{4}$) and answers should be expressed in simplest form. • ~~Add and subtract~~ Solve single-step and multistep practical problems involving addition and subtraction with mixed numbers having like and unlike denominators, with and without regrouping. Denominators in the problems should be limited to 12 or less, and answers should be expressed in simplest form. • Use estimation to check the reasonableness of a sum or difference.

5.6 The student will solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers and express answers in simplest form.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
simplest form. • A mixed number has two parts: a whole number and a fraction. The value of a mixed number is the sum of its two parts. • To add or subtract with mixed numbers, students may use a number line, draw a picture, rewrite fractions with like denominators, or rewrite mixed numbers as fractions.		

5.7 The student will evaluate whole number numerical expressions, using the order of operations limited to parentheses, addition, subtraction, multiplication, and division.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An expression, like a phrase, has no equal sign. • Expressions are simplified by using the order of operations. • The order of operations defines the computation order to follow in simplifying an expression. • The order of operations is as follows: – First, complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right.	**All students should** • Understand that the order of operations describes the order to use to simplify expressions containing more than one operation. • ~~Select appropriate strategies and tools to simplify expressions.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify expressions by using the order of operations in a demonstrated step-by-step approach. • Find the value of numerical expressions, using <u>the</u> order of operations~~, mental mathematics, and appropriate tools. Exponents are limited to positive values. Fractions are limited to having denominators of 12 or less. Decimals are limited to the thousandth place~~. • <u>Given an expression involving more than one operation, describe which operation is completed first, which is second, etc.</u>

Students in grades 4 and 5 should be actively involved in measurement activities that require a dynamic interaction among students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects. ~~The intent is for students to make "ballpark" comparisons and *not* to memorize conversion factors between U.S. Customary and metric units. To fully understand these ballpark comparisons, students must be actively engaged in the process of measurement.~~

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure perimeter, weight/mass, liquid volume/capacity, area, temperature, and time. Students continue to enhance their understanding of measurement by using appropriate tools such as rulers, balances, clocks, and thermometers. The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.

5.8 The student will
a) find perimeter, area, and volume in standard units of measure;
b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;
c) identify equivalent measurements within the metric system;
d) estimate and then measure to solve problems, using U.S. Customary and metric units; and
e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Perimeter is the distance around an object. It is a measure of length. Area is the number of square units needed to cover a surface. Volume is a measure of capacity~~.~~ and is measured in cubic units. • To find the perimeter of any polygon, add the lengths of the sides. • Students should label the perimeter ~~or~~ , area, and volume with the appropriate unit of linear ~~or~~ , square or cubic measure. • Area is the number of square units needed to cover a surface or figure. • ~~Students should investigate the area of a square, rectangle, and triangle by using manipulatives (e.g., tiles, geoboards, graph paper).~~ • Students should investigate, using manipulatives, to discover the formulas for the area of a square, rectangle, and right triangle; and volume of a rectangular solid. – Area of a rectangle = Length × Width – Area of a square = Side × Side – Area of a right triangle = $\frac{1}{2}$ Base × Height – Volume of a rectangular solid = Length x Width x Height	**All students should** • Understand the concepts of perimeter, ~~and~~ area, and volume. • Understand and use appropriate units of measure for perimeter, ~~and~~ area, and volume. • Understand the difference between using perimeter, area, and volume in a given situation. • Understand how to select a measuring device and unit of measure to solve problems involving measurement.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the perimeter of a polygon, with or without diagrams, when – the lengths of all sides of a polygon that is not a rectangle or a square are given; – the length and width of a rectangle are given; or – the length of a side of a square is given. • Estimate and determine the perimeter of a polygon, and area of a square, rectangle, and right triangle following the parameters listed above, using only whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 24 square inches). • Estimate and determine the area of a square, with or without diagrams, when the length of a side is given. • Estimate and determine the area of a rectangle, with or without diagrams, when the length and width are given. • Estimate and determine the area of a right triangle, with or without diagrams, when the base and the height are given. • Differentiate ~~between~~ among the concepts of area, perimeter, and volume.

5.8 The student will

a) find perimeter, area, and volume in standard units of measure;
b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;
c) identify equivalent measurements within the metric system;
d) estimate and then measure to solve problems, using U.S. Customary and metric units; and
e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Length is the distance along a line or figure from one point to another. • U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter rulers, meter sticks, and tape~~s~~ measures. • When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), foot, or yard~~, or mile~~. • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight that an object changes is dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • Appropriate measuring devices to measure mass in U.S. Customary units (ounces, pounds) and metric units (grams, kilograms) are balances.		• Describe ~~real-life~~ practical situations where area, perimeter, and volume are appropriate measures to use, and justify their choices orally or in writing. • Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. • Develop a procedure for finding the volume of a rectangular solid. • Determine the volume of a rectangular solid using manipulatives (e.g., cubes) or pictorial representations. • Solve problems involving measurement by selecting an appropriate measuring device and a U.S. Customary or metric unit of measure for the following: – length: part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), inches, feet, yards, ~~miles,~~ millimeters, centimeters, meters, and kilometers; – weight: ounces, pounds, and tons; – mass: grams and kilograms; – liquid volume: cups, pints, quarts, gallons, milliliters, and liters; – area: square units; and – temperature: Celsius and Fahrenheit units. • Estimate the conversion of Celsius and Fahrenheit units relative to familiar situations: – Water freezes at 0°C and 32°F. – Water boils at 100°C and 212°F.

5.8 The student will

a) find perimeter, area, and volume in standard units of measure;

b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;

c) identify equivalent measurements within the metric system;

d) estimate and then measure to solve problems, using U.S. Customary and metric units; and

e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• U.S. Customary units to measure liquid volume (capacity) include cups, pints, quarts, and gallons. Metric units to measure liquid volume (capacity) include milliliters and liters. • Temperature is measured using a thermometer. The U.S. Customary unit of measure is degrees Fahrenheit; the metric unit of measure is degrees Celsius. • Practical experience measuring familiar objects helps students establish benchmarks and facilitates students' ability to use the units of measure to make estimates.		– Normal body temperature is about 37°C and 98.6°F.

5.9 The student will identify and describe the diameter, radius, chord, and circumference of a circle.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A circle is a set of points on a flat surface (plane) with every point equidistant from a given point called the *center*. • A chord is a line segment connecting any two points on a circle. Students will benefit from understanding that a chord goes from one side of the circle to the other, but does not need to pass through the center. • A diameter is a chord that goes through the center of a circle. The diameter is two times the radius. A radius is a segment from the center of a circle to any point on the circle. Two radii end-to-end form a diameter of a circle. • Circumference is the distance around or perimeter of a circle. The circumference is about 3 times larger than the diameter of a circle.	**All students should** • ~~Identify the parts of a circle.~~ • Understand that a chord is a line segment that extends between any two unique points of a circle. ~~The diameter is also a special chord that goes through the center of a circle.~~ • Understand that a diameter is also a special chord that goes through the center of a circle. • ~~Understand that a diameter is a line segment that goes through the center of the circle.~~ • Understand the relationship between the measures of diameter and radius and the relationship between the measures of radius and circumference. • Understand that a radius is a line segment that extends between the center and the circumference of the circle. • Understand that the circumference is the distance around the circle. Perimeter is the measure of the circumference.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe the diameter, radius, chord, and circumference of a circle. • Describe the relationship between – diameter and radius~~; and~~, – diameter and chord; ~~and~~ – radius and circumference; and~~.~~ – diameter and circumference. • ~~Identify the diameter, radius, chord, and circumference of a given circle.~~ • The length of the diameter of a circle is twice the length of the radius.

5.10 The student will determine an amount of elapsed time in hours and minutes within a 24-hour period.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Elapsed time is the amount of time that has passed between two given times. • Elapsed time can be found by counting on from the beginning time to the finishing time. – Count the number of whole hours between the beginning time and the finishing time. – Count the remaining minutes. – Add the hours and minutes. For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count on as follows: – from 10:15 a.m. to 1:15 p.m., count 3 hours; – from 1:15 p.m. to 1:25 p.m., count 10 minutes; and then – add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.	**All students should** • Understand that elapsed time can be found by counting on from the beginning time to the finishing time.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine elapsed time in hours and minutes within a 24-hour period.

5.11 The student will measure right, acute, obtuse, and straight angles.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Angles are measured in degrees. There are up to 360 degrees in an angle. A degree is $\frac{1}{360}$ of a complete rotation of a full circle. There are 360 degrees in a circle. • To measure the number of degrees in an angle, use a protractor or an angle ruler. • A right angle measures exactly 90°. • An acute angle measures less than 90°. • An obtuse angle measures greater than 90° but less than 180°. • A straight angle measures exactly 180°. • Before measuring an angle, students should first compare it to a right angle to determine whether the measure of the angle is less than or greater than 90°. • Students should understand how to work with a protractor or angle ruler as well as available computer software to measure and draw angles and triangles. • ~~A right triangle has one right angle.~~ • ~~An obtuse triangle has one obtuse angle and two acute angles.~~ • ~~An acute triangle has three acute angles.~~	**All students should** • Understand how to measure ~~and draw~~ acute, right, ~~and~~ obtuse, and straight angles. • ~~Understand how to identify a triangle as either acute, right, or obtuse.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the appropriate tools (e.g., protractor and straightedge or angle ruler as well as available software) used to measure and draw angles and triangles. • ~~Draw right, acute, and obtuse angles, using appropriate tools.~~ • Measure right, acute, straight, and obtuse angles, using appropriate tools, and identify their measures in degrees. • ~~Measure the angles of right, acute, and obtuse triangles, using appropriate tools, and identify their measures in degrees.~~

The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric ~~shapes~~ figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among ~~shapes~~ figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric ~~shapes~~ figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare ~~shapes~~ figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. ~~Discussing ideas, conjecturing, and testing hypotheses precede the development of more formal summary statements.~~ In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles. ~~Students also explore coordinate geometry, using the coordinate plane to describe points in the first quadrant.~~

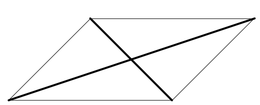
The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking ~~A~~algebra.)

5.12 The student will classify
a) angles as right, acute, obtuse, or straight; and
b) triangles as right, acute, obtuse, equilateral, scalene, or isosceles.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A right angle ~~is an angle that forms a square corner. A right angle~~ measures exactly 90°. • ~~An acute angle forms an angle less than a right angle.~~ An acute angle measures greater than 0° but less than 90°. • ~~An obtuse angle forms an angle greater than a right angle.~~ An obtuse angle measures greater than 90° but less than 180°. • A straight angle forms an angle that measures exactly 180°. • A right triangle has one right angle. • An obtuse triangle has one obtuse angle. • An acute triangle has three acute angles (or no angle measuring 90° or greater). • A scalene triangle has no congruent sides. • An isosceles triangle has two congruent sides. • To facilitate the exploration of relationships, ask students whether a right triangle can have an obtuse angle. Why or why not? Can an obtuse triangle have more than one obtuse angle? Why or why not? What type of angles are the two angles other than the right angle in a right triangle? What type of angles are the two angles other than the obtuse angle in an obtuse triangle?	**All students should** • Understand that ~~triangles~~ angles can be classified ~~by the measures of their angles.~~ as right, acute, obtuse, or straight according to their measures. • Understand ~~how to identify~~ that a triangle can be classified as either ~~acute,~~ right, acute, or obtuse according to the measure of its largest angle. • Understand that a triangle can be classified as equilateral, scalene, or isosceles according to the number of sides with equal length.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Classify angles as right, acute, straight, ~~and~~ or obtuse. • Classify triangles as right, acute, ~~and~~ or obtuse. • Classify triangles as equilateral, scalene, or isosceles.

5.13 The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
a) develop definitions of these plane figures; and
b) investigate and describe the results of combining and subdividing plane figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A triangle is a polygon with three sides. Triangles may be classified according to the measure of their angles, i.e., right, acute, or obtuse. Triangles may also be classified according to the measure of their sides, i.e., scalene (no sides congruent), isosceles (at least two sides congruent) and equilateral (all sides congruent). • A quadrilateral is a polygon with four sides. • A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. Properties of a parallelogram include the following: – A diagonal (a segment that connects two vertices of a polygon but is not a side) divides the parallelogram into two congruent triangles. – The opposite sides of a parallelogram are congruent. – The opposite angles of a parallelogram are congruent. – The diagonals of a parallelogram bisect each other. To bisect means to cut a geometric figure into two congruent halves. A bisector is a line segment, line, or plane that divides a geometric figure into two congruent halves. A sample of a bisected parallelogram is below. 	**All students should** • ~~Understand that the defining properties and symmetry of various plane figures are unique.~~ • Understand that simple plane figures can be combined to make more complicated figures and that complicated figures can be subdivided into simple plane figures.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • ~~Recognize and identify the properties of~~ Develop definitions for squares, rectangles, triangles, parallelograms, rhombi, ~~kites~~ and trapezoids. • ~~Describe the properties of squares, rectangles, triangles, parallelograms, rhombi, kites and trapezoids.~~ • ~~Analyze the properties of squares, rectangles, triangles, parallelograms, rhombi, kites and trapezoids.~~ • ~~Identify congruent, non-congruent, and similar figures~~. • Investigate and ~~Describe~~ describe the results of combining and subdividing ~~shapes~~ plane figures. • ~~Identify and describe a line of symmetry.~~ • ~~Recognize the images of figures resulting from geometric transformations such as translation, reflection, or rotation.~~

5.13 The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
a) develop definitions of these plane figures; and
b) investigate and describe the results of combining and subdividing plane figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A rectangle is a parallelogram with four right angles. Since a rectangle is a parallelogram, a rectangle has the same properties as those of a parallelogram. • A square is a rectangle with four congruent sides. Since a square is a rectangle, a square has all the properties of a rectangle and of a parallelogram. • A rhombus is a parallelogram with four congruent sides. Opposite angles of a rhombus are congruent. Since a rhombus is a parallelogram, the rhombus has all the properties of a parallelogram. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases,* and the nonparallel sides are called *legs*. If the legs have the same length, then the trapezoid is an isosceles trapezoid. • ~~A kite is a quadrilateral with two distinct pairs of adjacent congruent sides.~~ • Two or more figures can be combined to form a new ~~shape~~ figure. Students should be able to identify the figures that have been combined. • The region of ~~A~~ a polygon may be subdivided into two or more regions that represent figures. Students should understand how to divide the region of a polygon into familiar figures.		

Students entering grades 4 and 5 have explored the concepts of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students are able to predict whether the game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction at grades 4 and 5 is to deepen their understanding of the concepts of probability by

- ~~developing the continuum of terms to include *impossible*, *unlikely*, *equally likely, possible*, and *certain*;~~
- offering opportunities to set up models simulating ~~real-life~~ practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities ~~imbued~~ that instill ~~with~~ a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to ~~real-world~~ practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- writing a few sentences to communicate to others their ~~analysis and~~ interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

5.14 The student will make predictions and determine the probability of an outcome by constructing a sample space.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as unlikely*, *as likely as,* *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include ~~real-life~~ practical examples. • For any event such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. • A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, chart, or tree diagram. • Tree diagrams ~~are drawn to~~ show all ~~of the~~ possible	**All students should** • Understand ~~and apply~~ that the basic concepts of probability can be applied to make predictions of outcomes of simple experiments. • Understand that a sample space represents all possible outcomes of an experiment. • ~~Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a sample space, using a tree diagram to identify all possible outcomes of a single event ~~an outcome~~. • Construct a sample space, using a list or chart to represent all possible outcomes of a single event. • ~~Determine~~ Predict and determine the probability of ~~a single event~~ an outcome by constructing a sample space. The sample space ~~when the~~ will have a total ~~number~~ of ~~12~~ 24 or less possible outcomes ~~is 12 or less~~. • ~~Create a problem statement involving probability based on information from a given problem situation. Students will not be expected to solve the problem.~~

5.14 The student will make predictions and determine the probability of an outcome by constructing a sample space.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~combinations (~~outcomes~~)~~ in a sample space. The Fundamental ~~c~~Counting ~~p~~Principle ~~tells~~ describes how to find the number of outcomes when there ~~is more than one way to put things together~~ are multiple choices. For example, how many different outfit combinations can you make from 2 shirts (red and blue) and 3 pants (black, white, khaki)? The sample space displayed in a tree diagram would show that there are 2 × 3 = 6 (~~counting principle~~ Fundamental Counting Principle) outfit combinations: red-black; red-white; red-khaki; blue-black; blue-white; blue-khaki. • A spinner with eight equal-sized sections is equally likely to land on any one of the sections, three of which are red, three green, and two yellow. Have students write a problem statement involving probability, such as, "What is the probability that the spinner will land on green?"		

5.15 The student, given a problem situation, will collect, organize, and interpret data in a variety of forms, using stem-and-leaf plots and line graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The emphasis in all work with statistics should be on the analysis and the communication of the analysis, rather than on a single correct answer. Data analysis should include opportunities to describe the data, recognize patterns or trends, and make predictions. • Statistical investigations should be active, with students formulating questions about something in their environment and finding quantitative ways to answer the questions. • Investigations can be brief class surveys or more extended projects taking many days. • Through experiences displaying data in a variety of graphical representations, students learn to select an appropriate representation. • ~~Bar graphs are used to compare counts of different categories (categorical data).~~ – ~~A bar graph uses either horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category.~~ – ~~There is space before, between, and after the bars.~~ – ~~The axis displaying the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Fifth-grade students should collect data that is recorded in increments of multiples of whole numbers, decimals, and fractions.~~ – ~~Each axis should be labeled, and the graph should have a title.~~	**All students should** • ~~Understand that bar graphs compare categorical data, stem-and-leaf plots list data in a meaningful array, and line graphs show changes over time.~~ • Understand how to interpret ~~propose and justify conclusions and predictions that are based on~~ ~~displays of~~ collected and organized data. • Understand that ~~bar graphs compare categorical data,~~ stem-and-leaf plots list data in a meaningful array~~, and line graphs show changes over time~~. It helps in finding median, modes, minimum and maximum values, and ranges. • Understand that line graphs show changes over time.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Formulate the question that will guide the data collection. • Collect data, using observations (e.g., weather), measurement (e.g., shoe sizes), surveys (e.g., ~~favorite television shows~~ hours watching television), or experiments (e.g., plant growth). • Organize the data into a chart, ~~or~~ table, stem-and-leaf plots and line graphs. • ~~Construct bar graphs, labeling one axis with equal whole number or decimal increments and the other axis with attributes of the topic (categorical data) (e.g., skiing, basketball, ice hockey, skating, and sledding as the categories of "Favorite Winter Sports"). Bar graphs will have no more than six categories.~~ • Display data in line graphs~~, bar graphs,~~ and stem-and-leaf plots. • Construct line graphs, labeling the vertical axis with equal whole number, decimal, or fractional increments and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than six identified points along a continuum for continuous data (e.g., the decades: 1950s, 1960s, 1970s, 1980s, 1990s, and 2000s). • Construct a stem-and-leaf plot to organize and display data, where the stem is listed in ascending order and the leaves are in ascending order, with or without commas between leaves. • Title the given graph or identify the title.

5.15 The student, given a problem situation, will collect, organize, and interpret data in a variety of forms, using stem-and-leaf plots and line graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
– ~~Statements representing an analysis and interpretation of the characteristics of the data (e.g., similarities and differences, mode, and least and greatest) in the graph should be included.~~ • Line graphs are used to show how two continuous variables are related. Line graphs ~~are~~ may be used to show how one variable changes over time. If one variable is not continuous, then a broken line is used. By looking at a ~~single~~ line graph, it can be determined whether the variable is increasing, decreasing, or staying the same over time. – The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as "continuous data" because it represents data collected over a continuous period of time. – The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data. – Each axis should be labeled and the graph should have a title. – A line graph tells whether something has increased, decreased, or stayed the same with the passage of time. Statements representing an analysis and interpretation		• Interpret the data ~~to compare the answer to the prediction~~ in a variety of forms (e.g., orally or in written form.) • ~~Write a few sentences to describe the interpretation of the data.~~

5.15 The student, given a problem situation, will collect, organize, and interpret data in a variety of forms, using stem-and-leaf plots and line graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). A broken line is used if the data collected is not continuous data (such as test scores); a solid line is used if the data is continuous (such as height of a plant). • Stem-and-leaf plots allow the exact values of data to be listed in a meaningful array. Data covering a range of 25 numbers are best displayed in a stem-and-leaf plot and are utilized to organize numerical data from least to greatest, using the digits of the greatest to group data. – The data is organized from least to greatest. – Each value should be separated into a stem and a leaf [e.g., two-digit numbers are separated into stems (tens) and leaves (ones)]. – The stems are listed vertically from least to greatest with a line to their right. The leaves are listed horizontally, also from least to greatest, and can be separated by spaces or commas. Every value is recorded regardless of the number of repeats. – A key is often included to explain how to read the plot.		

5.16 The student will
a) describe mean, median, and mode as measures of center;
b) describe mean as fair share;
c) find the mean, median, mode, and range of a set of data; and
d) describe the range of a set of data as a measure of variation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Statistics is the science of conducting studies to collect, organize, summarize, analyze, and draw conclusions from data. • A measure of center is a value at the center or middle of a data set. Mean, median, and mode are measures of center. • The mean, median, and mode are three of the various ways that data can be analyzed. • ~~The mean is the numerical average of the data set and is found by adding all the values in the set and dividing the sum by the number of values.~~ Mean represents a fair share concept of the data. Dividing the data constitutes a fair share. This is done by equally dividing the data points. This should be demonstrated visually and with manipulatives. The arithmetic way is to add all of the data points then divide by the number of data points to determine the average or mean. • The median is the piece of data that lies in the middle of the set of data arranged in order. • The mode is the piece of data that occurs most frequently in the data set. There may be one, more than one, or no mode in a data set. Students should order the data from least to greatest so they can better find the mode. • The range is the spread of a set of data. The range of a set of data is the difference between the greatest and least values in the data set. It is	**All students should** • Understand that mean, median, and mode are described as measures of center. • Understand that mean, median, and mode are three of the various ways that data can be ~~measured~~ described or summarized. • Understand that mean as fair share is described as equally dividing the data set or the data set has already been divided equally. ~~• Understand that the mean is the numerical average of a data set; the median is the number in the middle of a set of data; the mode is the piece of data that occurs most often; and the range is the spread of a set of data.~~ • Understand how to ~~determine~~ find the mean, median, and mode~~, and range~~ of a set of data as measures of center. • Understand ~~what the number tells them about the data, and they need to see those~~ values in the context of other characteristics of the data in order to best describe ~~or analyze~~ the results.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Calculate~~ Describe and find the mean of a group of numbers representing data from a given context as a measure of center. • ~~Determine~~ Describe and find the median of a group of numbers representing data from a given context as a measure of center. • ~~Determine~~ Describe and find the mode of a group of numbers representing data from a given context as a measure of center. • Describe mean as fair share. • ~~Determine~~ Describe and find the range of a group of numbers representing data from a given context as a measure of variation.

5.16 The student will
a) describe mean, median, and mode as measures of center;
b) describe mean as fair share;
c) find the mean, median, mode, and range of a set of data; and
d) describe the range of a set of data as a measure of variation.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
determined by subtracting the s~~mallest~~ least number in the data set from the ~~largest~~ greatest number in the data set. An example is ordering test scores from least to greatest: 73, 77, 84, 87, 89, 91, 94. The greatest score in the data set is 94 and the least score is 73, so the least score is subtracted from the greatest score or 94 - 73 = 21. The range of these test scores is 21. • Students need to learn more than how to identify the mean, median, mode, and range of a set of data. They need to build an understanding of what the number tells them about the data, and they need to see those values in the context of other characteristics of the data in order to best describe ~~or analyze~~ the results.		

Students entering grades 4 and 5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships in open number sentences. ~~and they have begun to solve simple equations with one unknown.~~

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, ~~graphic~~ and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They ~~analyze~~ interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By ~~analyzing~~ interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write "rules" for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

5.17 The student will describe the relationship found in a number pattern and express the relationship.

<table>
<tr><th>UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)</th><th>ESSENTIAL UNDERSTANDINGS</th><th>ESSENTIAL KNOWLEDGE AND SKILLS</th></tr>
<tr><td>
• There are an infinite number of patterns.
• The simplest types of patterns are repeating patterns. In such patterns, students need to identify the basic unit of the pattern and repeat it.
• Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns ~~in both a numerical and geometric formats~~.
• Sample numerical patterns are
6, 9, 12, 15, 18, …;
5, 7, 9, 11, 13, …;
1, 2, 4, 7, 11, 16, …;
2, 4, 8, 16, 32, …; ~~and~~
32, 30, 28, 26, 24…; and
1, 5, 25, 125, 625, ….
• An expression, like a phrase, has no equal sign.
• When the pattern data are graphed in a T-table, an expression can represent that data. An example is:
<table><tr><th>X</th><th>Y</th></tr><tr><td>6</td><td>9</td></tr><tr><td>7</td><td>10</td></tr><tr><td>11</td><td>14</td></tr><tr><td>15</td><td>18</td></tr></table>
This example defines the relationship as $x + 3$.
• ~~In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation (turn) is the result~~
</td><td>
All students should
• Understand that patterns and functions can be represented in many ways and described using words, tables, ~~graphs,~~ and symbols.
• Understand the structure of a pattern and how it grows or changes using concrete materials and calculators.
• Understand that mathematical relationships exist in patterns.
• Understand that an expression uses symbols to define a relationship and shows how each number in the list, after the first number, is related to the preceding number.
• Understand that expressions can be numerical or variable or a combination of numbers and variables.
</td><td>
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
• Describe numerical and geometric patterns formed by using concrete materials and calculators.
• ~~Express~~ Describe the relationship found in ~~numerical and geometric~~ patterns, using words, tables, ~~graphs, or a mathematical sentence~~ and symbols to express the relationship.
</td></tr>
</table>

5.17 The student will describe the relationship found in a number pattern and express the relationship.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~of turning a figure around a point or a vertex, and reflection (flip) is the result of flipping a figure over a line.~~ • Expressions are simplified by using the order of operations. • A verbal quantitative expression involving one operation can be represented by a variable expression that describes what is going on. Numbers are used when they are known; variables are used when the numbers are unknown. For example, “a full box of cookies and four extra” can be represented by $b + 4$; “three full boxes of cookies” by $3b$; “a full box of cookies shared among four” by $\frac{b}{4}$. • A mathematical expression contains a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship. An expression cannot be solved.		

5.18 The student will
a) investigate and describe the concept of variable;
b) write an open sentence to represent a given mathematical relationship, using a variable;
c) model one-step linear equations in one variable, using addition and subtraction; and
d) create a problem situation based on a given open sentence, using a single variable.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A variable is a symbol that can stand for an~~y one of a~~ unknown ~~set of~~ number~~s~~ or ~~other~~ object~~s~~. ~~• A variable is a quantity that can have different values. Any letter can be used as a variable.~~ • A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an equals sign (=). • A verbal ~~quantitative~~ expression involving one operation can be represented by a variable expression that describes what is going on. Numbers are used when they are known; variables are used when the numbers are unknown. For example, "a full box of cookies and four extra" can be represented by $b + 4$; "three full boxes of cookies" by $3b$; "a full box of cookies shared among four" by $\frac{b}{4}$. • An open sentence ~~is a mathematical sentence~~ contains~~ing~~ a variable and an equals sign (=). For example, the sentence, "A full box of cookies and four extra equal 24 cookies." can be written as $b + 4 = 24$, where b stands for the number of cookies in one full box. "Three full boxes of cookies equal 60 cookies." can be written as $3b = 60$. • ~~An~~ Another example of an open sentence is ~~a mathematical sentence containing a variable. It also contains an equals sign (=). For example~~, $b + 3 = 23$ and represents the answer to the word problem, "How many cookies are in a box if the box plus three	**All students should** • Understand that a variable is a symbol that can stand for ~~any member of a set of numbers~~ an unknown number or object. • Understand that a variable expression is a variable or combination of variables, numbers, and symbols that represents a mathematical relationship. • Understand that verbal ~~quantitative~~ expressions can be translated to variable expressions. • Understand that an open sentence has ~~is a mathematical sentence with~~ a variable and an equal sign (=). • Understand that problem situations can be expressed as open sentences.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the concept of a variable (presented as boxes, letters, or other symbols) as a representation of an unknown quantity. ~~• Use a variable expression to represent a given verbal expression involving one operation (e.g., "5 more than a number" can be represented by $x + 5$).~~ • Write an open sentence with addition, subtraction, multiplication, or division, using a variable to represent a missing number. • Create and write a word problem to match a given open sentence with a single variable and one operation.

5.18 The student will

a) investigate and describe the concept of variable;
b) write an open sentence to represent a given mathematical relationship, using a variable;
c) model one-step linear equations in one variable, using addition and subtraction; and
d) create a problem situation based on a given open sentence, using a single variable.

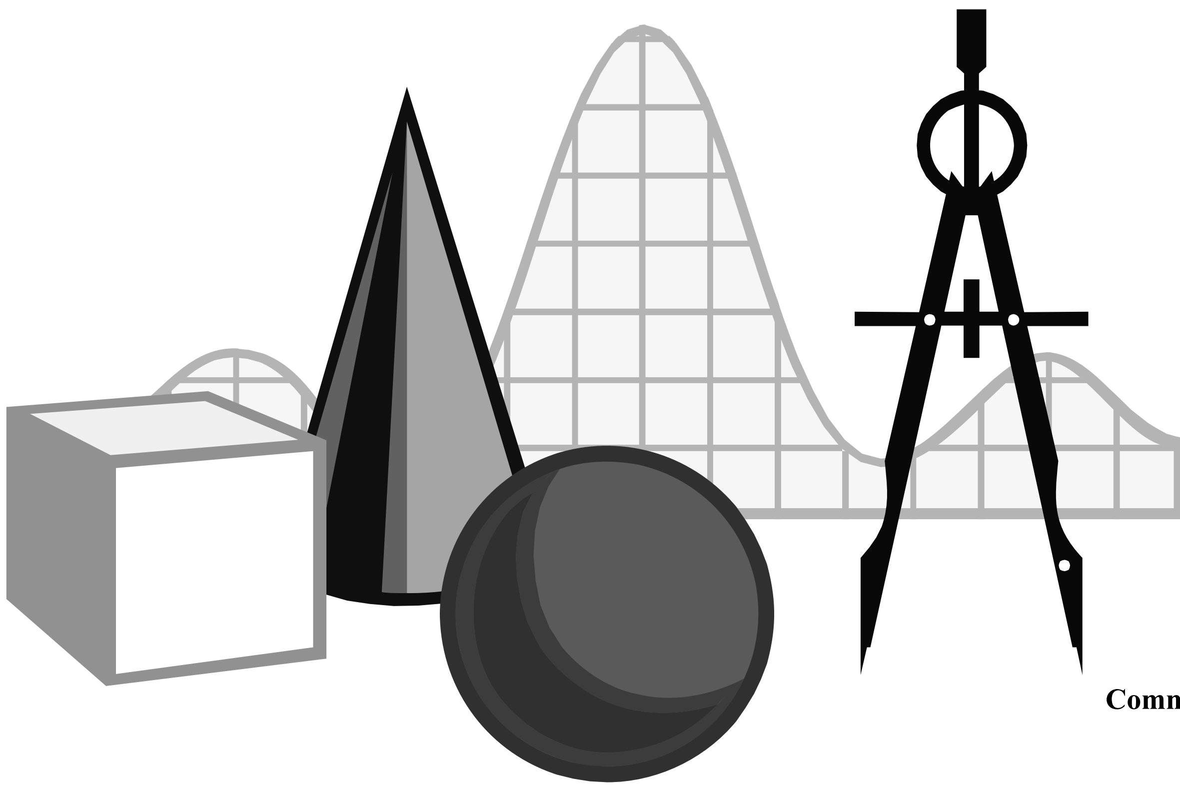
UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
more equals 23 cookies, where *b* stands for the number of cookies in the box? • At this level, discuss how the symbol × used to represent multiplication can often be confused with the variable *x*. Students can minimize this confusion by using parentheses [e.g., 4(*x*) = 20 or 4x = 20] or a small dot raised off the line to represent multiplication [4 • x = 20]. • By using story problems and numerical sentences, students begin to explore forming equations and representing quantities using variables. • An open sentence containing a variable is neither true nor false until the variable is replaced with a number.		

5.19 The student will investigate and recognize the distributive property of multiplication over addition.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products (e.g., 3(4 + 5) = 3 x 4 + 3 x 5, 5 x (3 + 7) = (5 x 3) + (5 x 7); or (2 x 3) + (2 x 5) = 2 x (3 + 5). • The distributive property can be used to simplify expressions (e.g., 9 x 23 = 9(20+3) =180+ 27 = 207; or 5 x 19 = 5(10 + 9) = 50 + 45 = 95).	**All students should** • Understand that the distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. • Understand that using the distributive property with whole numbers helps with understanding mathematical relationships. • Understand when and why the distributive property is used.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and recognize the distributive property of whole numbers, limited to multiplication over addition using diagrams and manipulatives. • Investigate and recognize an equation that represents the distributive property, when given several whole number equations, limited to multiplication over addition.

MATHEMATICS STANDARDS OF LEARNING
PROPOSED CURRICULUM FRAMEWORK

Grade 6



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.
- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.
- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.
- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

6.1 The student will describe and compare data, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A ratio is a comparison of any two quantities ~~and conveys an idea that cannot be expressed as a single number~~. A ratio is used to represent ~~a variety of~~ relationships within ~~a set~~ and between ~~two~~ sets. • A ratio can compare part of a set to the entire set (part-whole comparison). • A ratio can compare part of a set to another part of the same set (part-part comparison). • A ratio can compare part of a set to a corresponding part of another set (part-part comparison). • A ratio can compare all of a set to all of another set (whole-whole comparison). • The order of the quantities in a ratio is directly related to the order of the quantities expressed in the relationship. For example, if asked for the ratio of the number of cats to dogs in a park, the ratio must be expressed as the number of cats to the number of dogs, in that order. • A ratio is a multiplicative comparison of two numbers, measures, or quantities. • All fractions are ratios and vice versa. • Ratios may or may not be written in simplest form. • Ratios can compare two parts of a whole. • Rates can be expressed as ratios.	~~**All students should**~~ • ~~Understand that a ratio is a comparison of two quantities.~~ • ~~Understand that ratios can be represented in more than one way.~~ • What is a ratio? A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a set and between two sets. A ratio can be written using ~~a~~ fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe a relationship within a set by comparing part of the set to the entire set. • Describe a relationship between two sets by comparing part of one set to a corresponding part of the other set. • Describe a relationship between two sets by comparing all of one set to all of the other set. • Describe a relationship within a set by comparing one part of the set to another part of the same set. • Represent ~~the~~ a ~~verbal~~ relationship in words that makes a comparison by using the notations $\frac{a}{b}$, *a*:*b*, and *a to b*. • Create a relationship in words for a given ratio expressed symbolically.

6.1 The student will describe and compare data, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/		

6.2 The student will

a) investigate and describe fractions, decimals and percents as ratios;
b) identify a given fraction, decimal or percent from a representation;
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
d) compare and order fractions, decimals, and percents.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• *Percent* means "per 100" or how many "out of 100"; *percent* is another name for *hundredths*. • A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $30\% = \frac{30}{100} = \frac{3}{10} = 0.3$). • Percents can be expressed as fractions with a denominator of 100 (e.g., $75\% = \frac{75}{100} = \frac{3}{4}$). • Percents can be expressed as decimal (e.g., $38\% = \frac{38}{100} = 0.38$). • ~~A~~ Some fractions can be rewritten as ~~an~~ equivalent fractions with ~~a~~ denominators of ~~100~~ powers of 10, and~~, thus,~~ ~~as a~~ can be represented as decimals or percents (e.g., $\frac{3}{5} = \frac{6}{10} = \frac{60}{100} = 0.60 = 60\%$). • Decimals, fractions, and percents can be represented using concrete materials (e.g., ~~b~~Base-10 blocks, number lines, decimal squares, or grid paper). • Percents ~~should~~ can be represented by drawing ~~a~~ shaded regions on ~~a 10-by-10~~ grids or by finding a location on number lines ~~to represent a given percents~~. • Percents are used in real life for taxes, sales, data description, and data comparison.	~~**All students should**~~ • ~~Understand that percent is a way of representing fractions and decimals.~~ • ~~Understand that a number can be written as a fraction, decimal, or percent.~~ • ~~Understand that percent is a method of standardization that is efficient because each number is always based on 100ths hundredths.~~ • ~~Understand that percents are used in real-life applications to compare or describe data.~~ • What is the relationship among fractions, decimals and percents? Fractions, decimals, and percents are three different ways to express the same number. A ratio can be written using ~~a~~ fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3). Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Recognize that *percent* means "out of 100" or *hundredths*, using the percent symbol (%).~~ • Identify the decimal and percent equivalents for ~~halves, thirds, fourths, fifths, and tenths~~ numbers written in fraction form including repeating decimals. • Represent fractions, decimals, and percents on a number line. • Describe orally and in writing the equivalent relationships among decimals, percents, and fractions that have denominators that are factors of 100. • ~~Draw a shaded region on a 10-by-10 grid to represent a given fraction, decimal, and percent~~. • Represent, by shading a grid, a fraction, decimal, and percent. • Represent in ~~decimal,~~ fraction, decimal, and~~/or~~ percent form a given shaded region of a ~~10-by-10~~ grid. • Compare two decimals through thousandths ~~by representing the decimals with decimal~~ using manipulatives ~~or~~, ~~picture~~ pictorial representations, ~~or by using place-value charts~~ number lines, ~~or~~ and

6.2 The student will

a) investigate and describe fractions, decimals and percents as ratios;
b) identify a given fraction, decimal or percent from a representation;
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
d) compare and order fractions, decimals, and percents.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Fractions, decimals and percents are equivalent forms representing a given number. • The decimal point is a symbol that ~~indicates the location of the ones place and all other subsequent in the decimal system~~separates the whole number part from the fractional part of a number. • The decimal point separates the whole number amount from the part of a number that is less than one. • The symbol • can be used in Grade 6 in place of X to indicate multiplication. • ~~Decimals can be represented and compared, using decimal manipulatives, drawings, pictures, or symbols.~~ • ~~Fractions can be represented and compared by using fraction manipulatives, drawings, pictures, or symbols.~~ • Strategies using 0, $\frac{1}{2}$ and 1 as benchmarks can be used to compare fractions. • When comparing two fractions, use $\frac{1}{2}$ as a benchmark. Example: Which is greater, $\frac{4}{7}$ or $\frac{3}{9}$?		~~the~~ symbols ($<, \leq, \geq, .>,$ ~~or~~ $=$). • ~~Compare two whole numbers by representing the numbers with concrete objects or picture representations or by using the symbols $<, \leq, >, \geq$, or $=$.~~ • Compare two fractions with denominators of 12 or less ~~by representing the fractions with fraction~~ using manipulatives ~~or picture~~ , pictorial representations, number lines, ~~or~~ ~~by using the~~ and symbols ($<, \leq, \geq, .>,$ ~~or~~ $=$). • Compare two percents using pictorial representations and symbols ($<, \leq, \geq, .>, =$). • Order no more than 3 fractions, decimals, and~~/or~~ percents (decimals through thousandths, fractions with denominators of 12 or less), in ascending or descending order.

6.2 The student will

a) investigate and describe fractions, decimals and percents as ratios;
b) identify a given fraction, decimal or percent from a representation;
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
d) compare and order fractions, decimals, and percents.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
$\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. The denominator tells the number of parts that make the whole. $\frac{3}{9}$ is less than $\frac{1}{2}$ because 3, the numerator, is less than half of 9, the denominator, which tells the number of parts that make the whole. Therefore, $\frac{4}{7} > \frac{3}{9}$. • When comparing two fractions close to 1, use distance from 1 as your benchmark. Example: Which is greater, $\frac{6}{7}$ *or* $\frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{9}$ *is* $\frac{1}{9}$ away from 1 whole. Since $\frac{1}{7} > \frac{1}{9}$, then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$ so $\frac{8}{9} > \frac{6}{7}$. • Students should have experience with fractions such as $\frac{1}{8}$, whose decimal representation is a terminating		

6.2 The student will
- **a) investigate and describe fractions, decimals and percents as ratios;**
- **b) identify a given fraction, decimal or percent from a representation;**
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- **d) compare and order fractions, decimals, and percents.**

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
decimal (e. g., $\frac{1}{8} = 0.125$) and with fractions such as $\frac{2}{9}$, whose decimal representation does not end but continues to repeat (e. g., $\frac{2}{9} = 0.222\ldots$). The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/ under computation and estimation.		

6.3 The student will
a) identify and represent integers;
b) order and compare integers; and
c) identify and describe absolute value of integers.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Integers are the set of whole numbers, ~~and~~ their opposites, and zero. • Positive integers are greater than zero. • Negative integers are less than zero. • Zero is an integer that is neither positive nor negative. • A negative integer is always less than a positive integer. • When comparing two negative ~~numbers~~ integers, the negative ~~number~~ integer that is closer to zero is greater. • An integer and its opposite are the same distance from zero on a number line. For example, the opposite of 3 is -3. • The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented as $\lvert -6 \rvert = 6$. • On a conventional number line, a smaller number is always located to the left of a larger number (e.g., -7 lies to the left of -3, thus $-7 < -3$; 5 lies to the left of 8 thus 5 is less than 8).	~~**All students should**~~ • ~~Understand how to identify, represent, order, and compare integers.~~ • What role do negative integers play in practical situations? Some examples of the use of negative integers are found in temperature (below 0), finance (owing money), below sea level. There are many other examples. • How does the absolute value of an integer compare to the absolute value of its opposite? They are the same because an integer and its opposite are the same distance from zero on a number line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify an integer represented by a point on a number line. • Represent integers on a number line. • ~~Compare and order~~ Order and compare integers using a number line. • Compare integers, using ~~the~~ mathematical symbols (<, >, ~~and~~ =). • Identify and describe the absolute value of an integer.

6.4 The student will demonstrate multiple representations of multiplication and division of fractions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Using manipulatives to build conceptual understanding and using pictures and sketches to link concrete examples to the symbolic enhance students' understanding of operations with fractions and help students connect the meaning of whole number computation to fraction computation. • Multiplication and division of fractions can be represented with arrays, paper folding, repeated addition, repeated subtraction, fraction strips, pattern blocks and area models. • When multiplying a whole by a fraction such as 3 x $\frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. • When multiplying a fraction by a fraction such as $\frac{2}{3}\cdot\frac{3}{4}$, we are asking for part of a part. • When multiplying a fraction by a whole number such as $\frac{1}{2}$ x 6, we are trying to find a part of the whole. • ~~Using an area model assists with students' developing understanding of multiplication and division of fractions.~~ • For measurement division, the divisor is the number of groups. You want to know how many are in each of those groups . Division of fractions can be explained as how many of a given divisor are	• When multiplying fractions, what is the meaning of the operation? When multiplying a whole by a fraction such as 3 x $\frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. When multiplying a fraction by a fraction such as $\frac{2}{3}\bullet\frac{3}{4}$, we are asking for part of a part. When multiplying a fraction by a whole number such as $\frac{1}{2}$ x 6, we are trying to find a part of the whole. • What does it mean to divide with fractions? For measurement division, the divisor is the number of groups and the quotient will be the number of groups in the dividend. Division of fractions can be explained as how many of a given divisor are needed to equal the given dividend. In other words, for $\frac{1}{4}\div\frac{2}{3}$ the question is, "How many $\frac{2}{3}$ make $\frac{1}{4}$?" For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?"	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Demonstrate the concepts of multiplication and division of fractions.~~ • Demonstrate multiplication and division of fractions using multiple representations~~including, but not limited to, arrays, paper folding, fractions strips, pattern blocks and area models~~. • ~~Discover and then m~~Model algorithms for multiplying and dividing with fractions using appropriate representations.

6.4 The student will demonstrate multiple representations of multiplication and division of fractions.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
needed to equal the given dividend. In other words, for $\frac{1}{4} \div \frac{2}{3}$, the question is, "How many $\frac{2}{3}$ make $\frac{1}{4}$?" • For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?" Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#nns		

6.5 The student will investigate and describe concepts of positive exponents and perfect squares.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In 8^3, 8 is the base and 3 is the exponent. • A power of a number represents repeated multiplication of the number by itself (e.g., $8^3 = 8 \times 8 \times 8$ and is read "8 to the third power"). • Any real number other than zero raised to the zero power is 1. Zero to the zero power (0) is undefined. • ~~Patterns in place-value charts provide visual meaning of exponents: $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$.~~ • Perfect squares are the numbers that result from multiplying any whole number by itself (e.g., $36 = 6 \times 6 = 6^2$). • Perfect squares can be represented geometrically as the areas of squares the length of whose sides are whole numbers (e.g., 1×1, 2×2, or 3×3). This can be modeled with grid paper, tiles, geoboards and virtual manipulatives.	~~**All students should**~~ • ~~Understand that a power of a number is repeated multiplication of that number by itself.~~ • ~~Understand that squaring a number and taking a square root of a number are inverse operations.~~ • What does exponential form represent? Exponential form is a short way to write repeated multiplication of a common factor such as $5 \times 5 \times 5 \times 5 = 5^4$. • What is the relationship between perfect squares and a geometric square? A perfect square is the area of a geometric square ~~where~~ whose side length is a whole number.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and describe patterns with exponents that are natural numbers, by using a calculator. • Recognize and describe patterns of perfect squares not to exceed 20^2, by using grid paper, square tiles, tables, and calculators. • ~~Recognize and describe patterns with square roots and squares by using squares, grid paper, and calculators~~. • Recognize powers of ten by examining patterns in a place value chart: $10^4 = 10,000$, $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$, $10^0 = 1$.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.

6.6 The student will

a) multiply and divide fractions and mixed numbers; and

b) estimate solutions and then solve single-step and multistep practical problems that involving addition, subtraction, multiplication, and division of fractions.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Simplifying fractions to ~~their~~ simplest form assists with uniformity of answers ~~and concepts~~. • ~~Equivalent forms are needed to perform the operations of addition and subtraction with fractions.~~ • Addition and subtraction are inverse operations as are ~~M~~ multiplication and division. ~~are inverse operations.~~ • ~~Rewriting an improper fraction as a mixed numeral assists with uniformity of answers and concepts.~~ • ~~There is implied addition of the whole number part and the fractional part in mixed numerals.~~ • ~~Using manipulatives to build conceptual understanding and using pictures and sketches to link concrete examples to the symbolic enhance students' understanding of operations with fractions and help students connect the meaning of whole number computation to fraction computation.~~ • It is helpful to use estimation to develop computational strategies. For example, $2\frac{7}{8}\cdot\frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3. • When multiplying a whole by a fraction such as	~~**All students should**~~ • How are multiplication and division of fractions and multiplication and division of whole numbers alike? ~~Understand that f~~ Fraction computation ~~uses the same ideas~~ can be approached in the same way as whole number computation, applying those concepts to fractional parts. • What is the role of estimation in solving problems? ~~Use e~~ Estimation ~~to~~ helps determine the reasonableness of answers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Simplify fractional answers to simplest form.~~ • Multiply and divide with fractions and mixed numbers. Answers ~~may be~~ are expressed in simplest form. • Solve single and multistep practical problems that involve addition and~~/or~~ subtraction with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less ~~and express answers in simplest form~~. Answers ~~may be~~ are expressed in simplest form. • Solve single and multistep practical problems that involve multiplication and~~/or~~ division with fractions and mixed numbers that include denominators of 12 or less ~~and express answers in simplest form~~. Answers ~~may be~~ are expressed in simplest form.

6.6 The student will

a) multiply and divide fractions and mixed numbers; and

b) estimate solutions and then solve single-step and multistep practical problems that involving addition, subtraction, multiplication, and division of fractions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
$3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. • When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part. • When multiplying a fraction by a whole number such as $\frac{1}{2} \cdot 6$, we are trying to find a part of the whole. • ~~It is helpful for students to simplify before they multiply fractions, using the commutative property of multiplication to reduce fractions to simplest form before multiplying.~~		

6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Estimation is an essential skill used to make sense of the placement of the decimal point in performing operations on decimals and for checking the decimal point's correct placement.~~ • Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. For example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because 2 ÷ 1 = 2. • Understanding the placement of the decimal point is very important when finding quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, by 0.06, and by 0.006. • Solving multistep problems in the context of real-life situations enhances interconnectedness and proficiency with estimation strategies. • Examples of ~~real-life~~ practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, deciding what time to leave for school or the movies, and sharing a pizza or the prize money from a contest. • ~~A consumer application problem is defined as the type of problem that is normally encountered in daily living, such as problems related to money, travel, work, recreation, and home life.~~ • ~~A budget may be kept for short or long periods of~~	~~**All students should**~~ • ~~Be able to produce an approximate answer for a given problem.~~ • ~~Understand that an estimated answer helps validate the reasonableness of a computed answer.~~ • What is the role of estimation in solving problems? Estimation gives a reasonable solution to a problem when an exact answer is not required. If an exact answer is required, estimation allows you to know if the calculated answer is reasonable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Solve multistep practical problems involving decimals by using estimation strategies and checking for the reasonableness of results.~~ • ~~Given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit, find the quotient.~~ • ~~Given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with more than one non-zero digit, find the quotient by using a calculator.~~ • Solve single-step and multistep practical problems involving addition, subtraction, multiplication and division ~~of~~ with decimals expressed to thousandths with no more than two operations.

6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~time. Students may keep a short-term budget to enable the purchase of an expensive item or a long-term budget to facilitate a long-term spending plan.~~		

6.8 The student will evaluate whole number numerical expressions, using the order of operations.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The order of operations is a convention that defines the computation order to follow in simplifying an expression. • The order of operations is as follows: – First, complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right. * ~~Grouping symbols include p~~Parentheses ( ), brackets [ ], braces {}, and the division bar – as in $\frac{3+4}{5+6}$ should be treated as grouping symbols. • The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent. • Any number, except 0, raised to the zero power is 1. Zero to the zero power is undefined.	~~**All students should**~~ • What is the significance of the order of operations? ~~Understand that the~~ The order of operations ~~describes~~ prescribes the order to use to simplify expressions containing more than one operation. It ensures that there is only one correct answer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify expressions by using the order of operations in a demonstrated step-by-step approach. The expressions should be limited to positive values and not include ~~braces~~ parentheses ( ), brackets [ ], braces { }, or absolute value | |. • Find the value of numerical expressions, using order of operations, mental mathematics, and appropriate tools. Exponents are limited to positive values.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

6.9 The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Making sense of various units of measure is an essential life skill, requiring reasonable estimates of what measurements mean, particularly in relation to other units of measure. – 1 inch is about 2.5 centimeters. – 1 foot is about 30 centimeters. – 1 meter is a little longer than a yard, or about 40 inches. – 1 mile is slightly farther than 1.5 kilometers. – 1 kilometer is slightly farther than half a mile. – 1 ounce is about 28 grams. – 1 nickel has the mass of about 5 grams. – 1 kilogram is a little more than 2 pounds. – 1 quart is a little less than 1 liter. – 1 liter is a little more than 1 quart. – Water freezes at 0°C and 32°F. – Water boils at 100°C and 212°F. – Normal body temperature is about 37°C and 98°F. – Room temperature is about 20°C and 70°F. • ~~Multiple experiences with using nonstandard and standard units of measure to measure physical objects help students develop an intuitive understanding of size.~~ • Mass is the amount of matter in an object. Weight is the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight*, as shown by the questions: "How much	~~All students should~~ • ~~Understand that there is a structured relationship between and among units of measure for length, area, weight/mass, and volume in the metric and U.S. Customary systems.~~ • What is the difference between weight and mass? ~~Understand that w~~ Weight and mass are different. Mass is the amount of matter in an object. Weight is the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. • ~~Understand that measures are determined by quantitative comparison to a standard unit.~~ • How do you determine which units to use at different times? Units of measure are determined by the attributes of the object being measured. ~~Understand that m~~ Measures of length are expressed in linear units, measures of area are expressed in square units, and measures of volume are expressed in cubic units. • Why are there two different measurement systems? Measurement systems are conventions invented by different cultures to meet their needs. The U.S. Customary System is the preferred method in the United States. The metric system is the preferred system worldwide.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system.~~ • Estimate the conversion of units of length, ~~area,~~ weight/mass, ~~and~~ volume, and temperature between the U.S. Customary system and the metric system by using ballpark comparisons. Ex: 1 L ≈ 1qt. Ex: 4L ≈ 4 qts. • ~~Determine the most appropriate unit of measure for a given situation.~~ • Estimate measurements by comparing the object to be measured against a benchmark. • ~~Solve measurement problems by estimating and determining length, using standard and nonstandard units of measure.~~ • ~~Solve measurement problems by estimating and determining weight/mass, using standard and nonstandard units of measure~~. • ~~Solve measurement problems by estimating and determining area, using standard and nonstandard units of measure~~. • ~~Solve measurement problems by estimating and determining liquid volume/capacity, using standard and nonstandard units of measure.~~

6.9 The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
does it weigh?" versus "What is its mass?" • ~~Chunking or benchmarks are strategies used to make measurement estimates.~~ • ~~Chunks of length, such as a window's length, can be used to estimate the length of a classroom wall~~. • ~~Benchmarks, such as the two-meter height of a standard doorway, can be used to estimate height.~~ • The degree of accuracy of measurement required is determined by the situation. • Whether to use an underestimate or an overestimate is determined by the situation. • Physically measuring objects along with using visual and symbolic representations improves student understanding of both the concepts and processes of measurement. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#measurement		

6.10 The student will
a) define pi (π) as the ratio of the circumference of a circle to its diameter;
b) solve practical problems involving circumference and area of a circle, given the diameter or radius;
c) solve practical problems involving area and perimeter; and
d) describe and determine the volume and surface area of a rectangular prism.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Experiences in deriving the formulas for area and perimeter, using manipulatives such as tiles, one-inch cubes, adding machine tape, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and facility in their use. • The perimeter of a polygon is the measure of the distance around the polygon. • Circumference is the distance around or perimeter of a circle. • The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve. • The perimeter of a square whose side measures s is 4 times s ($P = 4s$), and its area is side times side ($A = s^2$). • The perimeter of a rectangle is the sum of twice the length and twice the width [$P = 2l + 2w$, or $P = 2(l + w)$], and its area is the product of the length and the width ($A = l$ ~~×~~w). • ~~Experiences in using a variety of measuring devices and making real measurements promote an understanding of measurements and the formulas associated with measurements.~~ • The value of pi (π) is the ratio of the circumference	~~**All students should**~~ • ~~Understand the attributes of polygons and the use of measures to determine area and perimeter.~~ • ~~Understand the derivation of formulas related to area and perimeter of polygons and how to determine which is used in problem situations.~~ • ~~Select the appropriate approximation for pi (π) when solving problems.~~ • ~~Understand the derivation of pi and formulas for finding circumference and area of a circle.~~ • What is the relationship between the circumference and diameter of a circle? The circumference of a circle is about 3 times the measure of the diameter. • What is the difference between area and perimeter? Perimeter is the distance around the outside of a figure while area is the measure of the amount of space enclosed by the perimeter. • What is the relationship between area and surface area? Surface area is calculated for a three-dimensional figure. It is the sum of the areas of the two-dimensional surfaces that make up the three-dimensional figure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. • Find the circumference of a circle by substituting a value for the diameter or the radius into the formula $C = \pi d$ or $C = 2\pi r$. • Find the area of a circle by using the formula $A = \pi r^2$. • Apply formulas to solve practical problems involving area and perimeter of triangles and rectangles. • Create and solve problems that involve finding the circumference and~~/or~~ area of a circle when given the diameter or radius. • Solve problems that require finding the surface area of a rectangular prism, given a diagram of the prism with the necessary dimensions labeled. • Solve problems that require finding the volume of a rectangular prism given a diagram of the prism with the necessary dimensions labeled.

6.10 The student will
a) define pi (π) as the ratio of the circumference of a circle to its diameter;
b) solve practical problems involving circumference and area of a circle, given the diameter or radius;
c) solve practical problems involving area and perimeter; and
d) describe and determine the volume and surface area of a rectangular prism.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
of a circle to its diameter. • The ratio of the circumference to the diameter of a circle is a constant value, pi (π), which can be approximated by measuring various sizes of circles. • The fractional approximation of pi generally used is $\frac{22}{7}$. • The decimal approximation of pi generally used is 3.14. • The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle. • The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle. • The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$). • The volume of a rectangular prism is computed by multiplying the area of the base, B, (length x width) by the height of the prism ($V = lwh = Bh$ ~~or B=Bh~~). Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#measurement		• ~~Determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area.~~ • ~~Determine the circumference and/or area of a circle, using various tools.~~

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.
- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.
- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking ~~A~~algebra.)
- **Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking ~~G~~geometry.)

6.11 The student will
a) identify the coordinates of a point in a coordinate plane; and
b) graph ordered pairs in a coordinate plane.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (*x*, *y*), where *x* is the first coordinate and *y* is the second coordinate. However, any letters may be used to label the axes and the corresponding ordered pairs. • ~~The first coordinate of a point is its distance from the vertical number line along a horizontal line.~~ • ~~The second coordinate of a point is its distance from the horizontal number line along a vertical line.~~ • ~~In a plane, a point can be located by its distances from two intersecting perpendicular number lines. The distance from one line is measured along a line parallel to the other line.~~ • The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular number lines. Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II, (–,+); for quadrant III, (–, –); and for quadrant IV, (+,–). • In a coordinate plane, the origin is the point at the intersection of the x-axis and y-axis; the coordinates of this point are (0,0). • For all points on the x-axis, the y-coordinate is 0. For all points on the y-axis, the x-coordinate is 0. ~~When a point lies on the y-axis, the x-coordinate is always 0.~~ • The coordinates may be used to name the point. (e.g., the point (2,7)). It is not necessary to say "the point	~~**All students should**~~ • Can any given point be represented by more than one ordered pair? ~~Understand that t~~ The coordinates of a point define its unique location in a coordinate plane. Any given point is defined by only one ordered pair. • In naming a point in the plane, does the order of the two coordinates matter? Yes. The first coordinate tells the location of the point to the left or right of the y-axis and the second point tells the location of the point above or below the x-axis. Point (0, 0) is at the origin.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and label the axes of a coordinate plane. • Identify and label the quadrants of a coordinate plane. • Identify the quadrant ~~or axis~~ ~~in which~~ or the axis on which a ~~an ordered pair~~ point is positioned by examining the ~~ordered pair~~ coordinates (ordered pair) of the point. • Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. • Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane.

6.11 The student will
a) identify the coordinates of a point in a coordinate plane; and
b) graph ordered pairs in a coordinate plane.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
whose coordinates are (2,7)".		

6.12 The student will determine congruence of segments, angles, and polygons.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Congruent figures have exactly the same size and the same shape. • Noncongruent figures may have the same shape but not the same size. • The symbol for congruency is $\cong$. • The ~~matching or~~ corresponding angles of congruent polygons have the same measure, and the ~~matching or~~ corresponding sides of congruent polygons have the same measure. • The ~~direct comparison~~ determination of ~~congruent or noncongruent~~ the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of ~~measuring~~ all sides and angles. • Construction of congruent line segments, angles, and polygons helps students understand congruency.	~~**All students should**~~ • ~~Understand the meaning of congruence.~~ • Given two congruent figures, what inferences can be drawn about how the figures are related? The congruent figures will have exactly the same size and shape. • Given two congruent polygons, what inferences can be drawn about how the polygons are related? Corresponding angles of congruent polygons will have the same measure. ~~and~~ ~~c~~Corresponding sides of congruent polygons will have the same ~~length~~ measure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Characterize polygons as congruent and noncongruent according to the measures of their sides and angles. • Determine the congruence of segments, angles, and polygons ~~by direct comparison,~~ given their attributes.

6.13 The student will describe and identify properties of quadrilaterals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A quadrilateral is a closed planar (two-dimensional) figure with four sides that are line segments. • A parallelogram is a quadrilateral whose opposite sides are parallel and opposite angles are congruent. • ~~For all parallelograms, both pairs of opposite sides and both pairs of opposite angles are congruent.~~ • ~~Parallelograms have special characteristics (such as both pairs of opposite sides are parallel and congruent) that are true for any parallelogram.~~ • A rectangle is a parallelogram with four right angles. • Rectangles have special characteristics (such as diagonals are perpendicular bisectors) that are true for any rectangle. • To bisect means to divide into two equal parts. • A square is a rectangle with four congruent sides or a rhombus with four right angles. • A rhombus is a parallelogram with four congruent sides. • ~~A trapezoid is a quadrilateral with exactly one pair of parallel sides.~~ • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases*, and the nonparallel sides are called *legs*. If the legs have the same length, then the trapezoid is an isosceles trapezoid. • ~~A trapezoid with congruent, nonparallel sides is called an isosceles trapezoid.~~	~~**All students should**~~ • Can a figure belong to more than one subset of quadrilaterals? Any figure that has the attributes of more than one subset of quadrilaterals can belong to more than one subset. For example, rectangles have opposite sides of equal length. Squares have all 4 sides of equal length thereby meeting the attributes of both subsets.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Sort and classify polygons as quadrilaterals, parallelograms, rectangles, trapezoids, kites, rhombi, and squares based on their properties. Properties include number of parallel sides, angle measures and number of congruent sides. • Identify the sum of the measures of the angles of a quadrilateral as 360°.

6.13 The student will describe and identify properties of quadrilaterals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A kite is a quadrilateral with two pairs of adjacent congruent sides. One pair of opposite angles is congruent. • Quadrilaterals can be sorted according to common attributes, using a variety of materials. • Quadrilaterals can be classified by the number of parallel sides: a parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has only one pair of parallel sides; other quadrilaterals have no parallel sides. • Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a trapezoid may have none, one, or two 90° angles. • Quadrilaterals can be classified by the number of congruent sides: a rhombus has four congruent sides; a square, which is a rhombus with four right angles, also has four congruent sides; a parallelogram and a rectangle each have two pairs of congruent sides. • A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals. • The sum of the measures of the angles of a quadrilateral is 360°. • A chart ~~or~~ , graphic organizer or Venn Diagram can be made to organize quadrilaterals according to attributes such as sides and/or angles. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#geometry		

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- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of ~~central tendency~~ center and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

6.14 The student, given a problem situation, will
a) construct circle graphs;
b) draw conclusions and make predictions, using circle graphs; and
c) compare and contrast graphs that present information from the same data set.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to answer the problem. • ~~Data can be discrete or continuous.~~ • Different types of graphs are used to display different types of data. – Bar graphs use categorical (discrete) data (e.g., months or eye color). – Line graphs use continuous data (e.g., temperature and time). – Circle graphs show a relationship of the parts to a whole. • All graphs include a title, and data categories should have labels. • A scale should be chosen that is appropriate for the data. • A key is essential to explain how to read the graph. • A title is essential to explain what the graph represents. • Data are analyzed by describing the various features and elements of a graph. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#statistics	~~**All students should**~~ • ~~Understand that data can be displayed in a variety of graphical representations.~~ • ~~Select and use appropriate statistical methods to analyze data.~~ • ~~Understand that different types of representations can tell different things about the same data.~~ • What type of data are best presented in a circle graph? Circle graphs are best used for data showing a relationship of the parts to the whole.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Collect data sets of no more than 20 items by using tally sheets, surveys, observations, questionnaires, interviews, and polls.~~ • ~~Organize data by using lists, charts, and tables.~~ • ~~Organize and display data in bar and line graphs, displaying the information as clearly as possible by using increments of whole numbers, fractions, and decimals rounded to the nearest tenth.~~ • Collect, ~~O~~organize and display data in circle graphs by depicting information as fractional. ~~parts that are limited to halves, fourths, and eighths.~~ • Draw conclusions and make predictions about data presented in a circle graph. • Compare and contrast data presented in a circle graph with the same data represented in other graphical forms~~ studied in the previous grade level~~. • ~~Decide which type of graph is appropriate for a given situation.~~ ~~– Bar graphs are used to display categorical (discrete) data.~~ ~~– Line graphs are used to display continuous data.~~

6.15 The student will
a) describe mean as balance point; and
b) decide which measure of center is appropriate for a given purpose.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Measures of ~~central tendency~~ center are types of averages for a data set. They represent numbers that ~~best~~ describe a data set. Mean, median, and mode are measures of ~~central tendency~~ center that are useful for describing the average for different situations. – Mean works well for sets of data with no very high or low numbers. – Median is a good choice when data sets have a couple of values much higher or lower than most of the others. – Mode is a good descriptor to use when the set of data has some identical values or when data are not conducive to computation of other measures of central tendency, as when working with data in a yes or no survey. • The mean is the numerical average of the data set and is found by adding the numbers in the data set together and dividing the sum by the number of data pieces in the set. • In grade 5 mathematics, mean is defined as fair-share. • Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point. • Defining mean as balance point is a prerequisite for understanding standard deviation.	~~**All students should**~~ • ~~Understand that measures of central tendency are types of averages for a data set.~~ • ~~Understand that mean, median, and mode are measures of central tendency that are useful for describing data in different situations.~~ • ~~Understand that the range describes the spread of a set of data.~~ • What does the phrase "measure of center" mean? This is a collective term for the 3 types of averages for a set of data – mean, median and mode. • What is meant by mean as balance point? Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the mean for a set of data. • Describe the three measures of ~~central tendency~~ center and a situation in which each would best represent a set of data. • Identify and draw a number line that demonstrates the concept of mean as balance point for a set of data.

6.15 The student will
a) describe mean as balance point; and
b) decide which measure of center is appropriate for a given purpose.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values. • The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal. ~~– For 2, 3, 4, 5, 5, 6, 7, 8, 8, 8, 9, 11, the mode is 8.~~ ~~– For 2, 3, 4, 5, 5, 5, 7, 8, 8, 8, 9, 11, the modes are 5 and 8 (bimodal).~~ ~~– For 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 17, there is no mode.~~ • ~~The range is the difference between the greatest and least values in a set of data and shows the spread in a set of data.~~		

6.16 The student will
a) compare and contrast dependent and independent events; and
b) determine probabilities for dependent and independent events.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes (sample space). • The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent. • The probability of an event occurring is a ratio between 0 and 1. – A probability of 0 means the event will never occur. – A probability of 1 means the event will always occur. • A simple event is one event (e.g., pulling one sock out of a drawer and examining the probability of getting one color). • Events are independent when the outcome of one has no effect on the outcome of the other. For example, rolling a number cube and flipping a coin are independent events. • The probability of two independent events is found by using the following formula: $P(A\,and\,B) = P(A)\cdot P(B)$ Ex: When rolling two number cubes simultaneously, what is the probability of rolling a 3 on one cube and a 4 on the other? $P(3\,and\,4) = P(3)\cdot P(4) = \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36}$ • Events are dependent when the outcome of one	~~**All students should**~~ • ~~Understand that a probability can be expressed as a ratio, decimal, or percent~~. • How can you determine if a situation involves dependent or independent events? Events are independent when the outcome of one has no effect on the outcome of the other. Events are dependent when the outcome of one event is influenced by the outcome of the other.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether two events are dependent or independent. • Compare and contrast dependent and independent events. • Determine the probability of two dependent events. • Determine the probability of two independent events.

6.16 The student will

a) compare and contrast dependent and independent events; and

b) determine probabilities for dependent and independent events.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
event is influenced by the outcome of the other. For example, when drawing two marbles from a bag, *not* replacing the first after it is drawn affects the outcome of the second draw. • The probability of two dependent events is found by using the following formula: $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$ Ex: You have a bag holding a blue ball, a red ball, and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick and then *without* replacing the blue ball in the bag, picking a red ball on the second pick? $P(\text{blue } \textit{and} \text{ red}) = P(\text{blue}) \cdot P(\text{red } \textit{after} \text{ blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$		

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- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

6.17 The student will identify and extend geometric and arithmetic sequences.

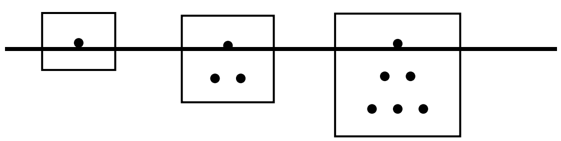
UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Numerical patterns may include linear and exponential growth, perfect squares, triangular and other polygonal numbers, or Fibonacci numbers. • Arithmetic and geometric sequences are types of numerical patterns. • In the numerical pattern of an arithmetic sequence, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Sample numerical patterns are 6, 9, 12, 15, 18, …; and 5, 7, 9, 11, 13, …. • In geometric number patterns, students must determine what each number is multiplied by to obtain the next number in the geometric sequence. This multiplier is called the *common ratio*. Sample geometric number patterns include 2, 4, 8, 16, 32, …; 1, 5, 25, 125, 625, …; and 80, 20, 5, 1.25, … • Strategies to recognize and describe the differences between terms in numerical patterns include, but are not limited to, examining the change between consecutive terms~~, looking for prime numbers,~~ and finding common factors. An example is the pattern 1, 2, 4, 7, 11, 16,… • ~~Strategies to recognize and describe geometric patterns include, but are not limited to, examining flips, slides, turns, growth, and symmetry. Rotation (turn) is the result of turning a figure around a point or a vertex. Translation (slide) is the result of sliding a figure in any direction within a plane. Dilatation (scale increase or decrease) is the result of enlarging or shrinking a figure by a scale amount. Reflection~~	~~**All students should**~~ • ~~Understand that mathematical patterns can be represented in various forms, geometrically or numerically.~~ • ~~Understand that patterns regularly occur in everyday life.~~ • ~~Understand that patterns can be recognized, extended, or generalized~~. • ~~Understand that numerical patterns may involve adding or multiplying by the same number.~~ • ~~Understand that geometric patterns may involve shape, size, angles, transformations of shapes, and growth.~~ • What is the difference between an arithmetic and a geometric sequence? While both are numerical patterns, arithmetic sequences are additive and geometric sequences are multiplicative.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and apply strategies to recognize and describe the change between terms in ~~numerical~~ arithmetic patterns. • Investigate and apply strategies to recognize and describe geometric patterns. • Describe verbally and in writing the relationships between consecutive terms in an ~~numerical~~ arithmetic or geometric ~~pattern~~ sequence. • Extend and apply ~~numerical~~ arithmetic and geometric ~~patterns~~ sequences to similar situations. • ~~Create~~ ~~Using a table as an organizing tool, c~~Extend ~~numerical~~ arithmetic and geometric ~~patterns~~ sequences in a table by using a given rule or mathematical relationship. • ~~Describe numerical and geometric patterns, including triangular numbers.~~ • Compare and contrast arithmetic and geometric sequences. • Identify the common difference for a given arithmetic sequence. • Identify the common ratio for a given geometric sequence.

6.17 The student will identify and extend geometric and arithmetic sequences.

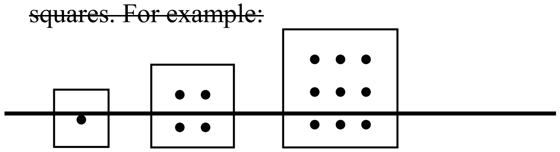
UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS

~~(flip) is the result of flipping a figure over a line.~~

- ~~The first five triangular numbers are 1, 3, 6, 10, and 15. A triangular number can be represented geometrically as a certain number of dots arranged in a triangle, with one dot in the first (top) row and each succeeding lower row having one more dot that the row above it. To find the next triangular number, a new row is added to an existing triangle, and total number of dots counted. Students should make the connection between the number of new dots in the triangle and the corresponding triangular number. Triangular numbers can be represented as a growing pattern of triangles.~~



- ~~A square number can be represented geometrically as the number of dots in a square array. Square numbers are perfect squares and are the numbers that result from multiplying any whole number by itself (e.g., 36 = 6 × 6). Square numbers (1, 4, 9, 16, …) can be represented as a growing pattern of squares. For example:~~



- ~~The possible number of patterns is infinite.~~
- ~~The simplest types of patterns are repeating patterns. In such patterns, students need to identify the basic unit of the pattern and repeat it.~~
- ~~Growing patterns are more difficult for students to~~

6.17 The student will identify and extend geometric and arithmetic sequences.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns both in numerical and geometric formats.~~		

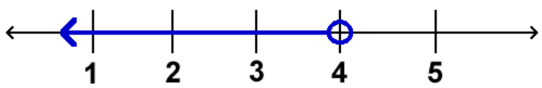
6.18 The student will solve one-step linear equations in one variable involving whole number coefficients and positive rational solutions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A one-step linear equation is an equation that requires one operation to solve. • A mathematical expression contains a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship. An expression cannot be solved. • A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3. • A coefficient is the numerical factor in a term. For example, in the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient. • Positive rational solutions are limited to whole numbers and positive fractions and decimals. • An equation is a mathematical sentence stating that two expressions are equal. • A variable is a symbol (placeholder) used to represent an unspecified member of a set.	~~**All students should**~~ • ~~Understand that physical objects can be used to represent and solve algebraic equations.~~ • ~~Understand that in an equation, the equal sign indicates that the value on the left side of the sign is the same as the value on the right side.~~ • When solving an equation, why is it necessary to perform the same operation on both sides of an equal sign? ~~Understand that to~~ To maintain equality, an operation performed on one side of an equation must be performed on the other side.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Represent and solve a one-step equation, using a variety of concrete materials such as colored chips ~~on an equation mat~~, ~~algebra tiles~~, algeblocks or weights on a balance scale. • Solve a one-step equation by demonstrating the steps algebraically. • Identify and ~~U~~use the following algebraic terms appropriately: *equation*, *variable, expression, term,* and *coefficient*. • ~~Identify examples of equations, variables, terms, and coefficients.~~

6.19 The student will investigate and recognize
a) the identity properties for addition and multiplication;
b) the multiplicative property of zero; and
c) the inverse property for multiplication.

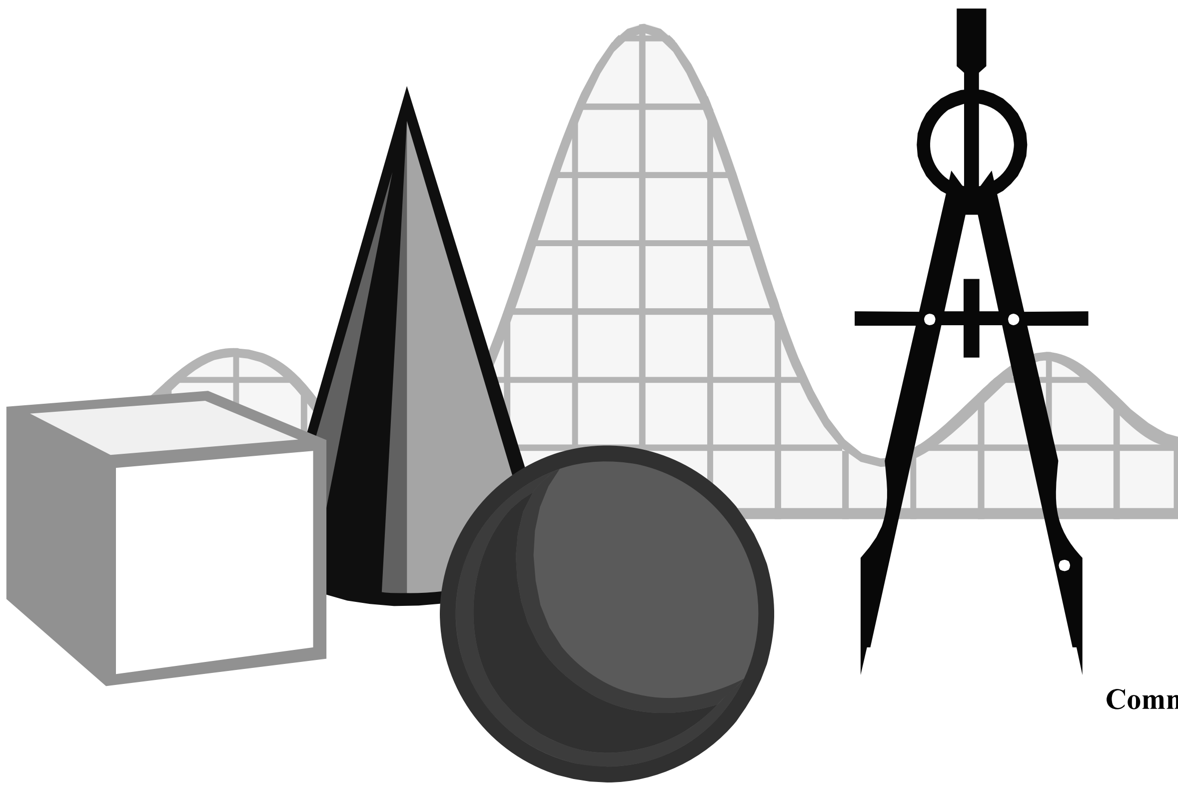
UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division. • The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., 5 + 0 = 5). • The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$). • Inverses are numbers that combine with other numbers and result in identity elements. • The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$). • Zero has no multiplicative inverse. • The multiplicative property of zero states that the product of any real number and zero is zero. • Division by zero is not a possible arithmetic operation. Division by zero is undefined. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#nns	~~**All students should**~~ • ~~Understand that using the properties of operations with real numbers helps with understanding mathematical relationships.~~ • ~~Understand how to use these properties when computing.~~ • How are the identity properties for multiplication and addition the same? Different? For each operation the identity elements are numbers that combine with other numbers without changing the value of the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). • What is the result of multiplying any real number by zero? The product is always zero. • Do all real numbers have a multiplicative inverse? No. Zero has no multiplicative inverse because there is no real number that can be multiplied by zero resulting in a product of one.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the real number equation that represents each property of operations with real numbers, when given several real number equations. • ~~Explore the properties of real numbers, using diagrams and manipulatives~~. • Test the validity of properties by using examples of the properties of operations on real numbers. • Identify the property of operations with real numbers that is illustrated by a real number equation. NOTE: The commutative, associative and distributive properties are taught in previous grades.

6.20 The student will graph inequalities on a number line.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Inequalities using the < or > symbols are represented on a number line with an open circle on the number and a shaded line over the solution set. Ex: x < 4  • When graphing x ≤ 4 fill in the circle above the 4 to indicate that the 4 is included. • Inequalities using the ≤ *or* ≥ symbols are represented on a number line with a closed circle on the number and shaded line in the direction of the solution set. • The solution set to an inequality is the set of all numbers that make the inequality true. • It is important for students to see inequalities written with the variable before the inequality symbol and after. For example x > -6 and 7 > y.	~~**All students should**~~ • ~~Does the order of the elements i~~In an inequality, does the order of the elements matter? Yes, the order does matter. For example, x > 5 is not the same relationship as 5 > x. However, x > 5 is the same relationship as 5 < x.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Given a simple inequality with integers, graph the relationship on a number line. • Given the graph of a simple inequality with integers, represent the inequality two different ways using the symbols <, >, ≤ *and* ≥.

MATHEMATICS STANDARDS OF LEARNING
PROPOSED CURRICULUM FRAMEWORK

Grade 7



Commonwealth of Virginia
Board of Education
Richmond, Virginia
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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.

- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.

- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.

- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) determine scientific notation for numbers greater than zero;
c) compare and order fractions, decimals, percents and numbers written in scientific notation;
d) determine square roots; and
e) identify and describe absolute value for rational numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Negative exponents for powers of 10 are used to represent numbers between 0 and 1. (e.g., $10^{-3} = \frac{1}{10^3} = 0.001$). • Negative exponents for powers of 10 can be investigated through patterns such as: $10^2 = 100$ $10^1 = 10$ $10^0 = 1$ $10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$ • ~~Fractions, decimals, and percents are three different representations of the same number. Some numbers also can be represented in scientific notation.~~ • A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$). • Scientific notation is used to represent very large or very small numbers. • A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10	~~**All students should**~~ • ~~Understand that a number can be represented as a decimal, fraction, percent, and/or in scientific notation.~~ • ~~Develop strategies to compare, order, and determine equivalency among fractions, decimals, and percents.~~ • ~~Understand real-life uses of scientific notation.~~ • When should scientific notation be used? Scientific notation should be used whenever the situation calls for use of very large or very small numbers. • How are fractions, decimals and percents related? Any rational number can be represented in fraction, decimal and percent form. • What does a negative exponent mean when the base is 10? A base of 10 raised to a negative exponent represents a number between 0 and 1. • How is taking a square root different from squaring a number? Squaring a number and taking a square root are inverse operations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize powers of 10 with negative exponents by examining patterns. • Write a power of 10 with a negative exponent in fraction and decimal form. • Write a number greater than ~~10~~ 0 in scientific notation. • Recognize a number greater than 0 in scientific notation. • Compare~~, order,~~ and determine equivalent relationships between numbers larger than ~~10~~ 0 written in scientific notation. • Represent a number in fraction, decimal, and percent forms. ~~Fractions will have denominators of 12 or less.~~ • Compare, order, and determine equivalent relationships among fractions, decimals, and percents. Decimals are limited to the thousandths place, and percents are limited to the tenths place. Ordering is limited to no more than 4 numbers. • Order no more than 3 numbers greater than 0 written in scientific notation.

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) determine scientific notation for numbers greater than zero;
c) compare and order fractions, decimals, percents and numbers written in scientific notation;
d) determine square roots; and
e) identify and describe absolute value for rational numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
(e.g., $3.1 \times 10^5 = 310{,}000$ and $2.85 \times 10^{-4} = 0.000285$). • ~~Decimals, fractions, and percents can be compared using concrete materials (e.g., base ten blocks, decimal squares, and grid paper).~~ • Equivalent relationships among fractions, decimals, and percents can be determined by using manipulatives (e.g., fraction bars, ~~base-ten~~ Base-10 blocks, fraction circles, graph paper, number lines and calculators). • A square root of a number is a number which, when multiplied by itself, produces the given number (e.g., $\sqrt{121}$ is 11 since 11 x 11 = 121). • The square of a number can be represented geometrically as the length of a side of the square. • The absolute value of a number is the distance from 0 on the number line regardless of direction. (e.g., $\left\lvert \frac{-1}{2} \right\rvert = \frac{1}{2}$).	~~**All students should**~~ • Why is the absolute value of a number positive? The absolute value of a number represents distance from zero on a number line regardless of direction. Distance is positive.	• ~~Compare very large numbers, using scientific notation.~~ ~~**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**~~ • ~~Order no more than five numbers written as fractions, decimals, percents, and numbers larger than 10 written in scientific notation in ascending (least to greatest) or descending (greatest to least) order.~~ • Determine the square root of a perfect square less than or equal to 400. • Demonstrate absolute value using a number line. • Determine the absolute value of a rational number.

7.2 The student will describe and represent arithmetic and geometric sequences using variable expressions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In the numeric pattern of an arithmetic sequence, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. • In geometric sequences, students must determine what each number is multiplied by in order to obtain the next number in the geometric sequence. This multiplier is called the *common ratio*. Sample geometric sequences include – 2, 4, 8, 16, 32, …; 1, 5, 25, 125, 625, …; and – 80, 20, 5, 1.25, …. • A variable expression can be written to express the relationship between two consecutive terms of a sequence (e.g.: 3, 6, 9, 12…). If n represents a number in this sequence, the next term in the sequence can be determined using the variable expression: n + 3 (e.g.: 1, 5, 25, 125…). If n represents a number in the sequence, the next term in the sequence can be determined by using the variable expression 5n.)	~~**All students should**~~ • When are variable expressions used? Variable expressions can express the relationship between two consecutive terms in a sequence.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Analyze situations to discover a variety of patterns~~. • Analyze numeric and geometric sequences to discover a variety of patterns. • Identify the common difference in an arithmetic sequence. • Identify the common ratio in a geometric sequence. • Given an arithmetic or geometric sequence, write a variable expression to describe the relationship between two consecutive terms in the sequence.

In the middle grades, the focus of mathematics learning is to
- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.

7.3 The student will
a) model addition, subtraction, multiplication and division of integers; and
b) add, subtract, multiply, and divide integers.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The set of integers is the set of whole numbers and their opposites (e.g., … –3, –2, –1, 0, 1, 2, 3, …). • Integers are used in ~~real-life~~ practical situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), and changes in altitude (above/below sea level). • Concrete experiences in formulating rules for adding and subtracting integers should be explored by examining patterns using calculators, along a number line and using manipulatives, such as two-color counters, or by using algeblocks. • Concrete experiences in formulating rules for multiplying and dividing integers should be explored by examining patterns with calculators, along a number line and using manipulatives, such as two-color counters, or by using algeblocks. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/# under computation. For additional information on using Algeblocks, go to http://www.vdoe.whro.org/A_Blocks05/index.html	~~**All students should**~~ • ~~Develop and apply strategies involving mathematical operations with integers.~~ • ~~Understand how problems in daily life can be represented and solved by using integers.~~ • The sums, differences, products and quotients of integers are either positive, zero, or negative. How can this be demonstrated? This can be demonstrated through the use of patterns and models.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model addition, subtraction, multiplication and division of integers using pictorial representations of concrete manipulatives. • Add, subtract, multiply and divide integers . • ~~Formulate rules for adding integers.~~ • ~~Formulate rules for subtracting integers.~~ • ~~Formulate rules for multiplying integers.~~ • ~~Formulate rules for dividing integers.~~ • Simplify numerical expressions involving addition, subtraction, multiplication and~~/or~~ division of integers using order of operations. • Solve practical problems involving addition, subtraction, multiplication, and division with integers.

7.4 The student will solve single-step and multistep practical problems, using proportional reasoning.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A proportion is a statement of equality between two ratios. • ~~A common property relates the numerators of the two ratios, and another common property relates the denominators of the two ratios. For example, both numerators relate to one property, such as length, while both denominators relate to another property, such as width. Alternatively, both numerators could relate to scale lengths, while both denominators relate to actual lengths.~~ • ~~The dimensions of a scale model are proportional to the corresponding dimensions of the object (e.g., a blueprint of a house floor plan is proportional to the actual dimensions of the floor).~~ • A proportion can be written as $\frac{a}{b} = \frac{c}{d}$, a:b = c:d, or a is to b as c is to d. • A proportion can be solved by finding the product of the means and the product of the extremes. For example, in the proportion a:b = c:d, a and d are the extremes and b and c are the means. If values are substituted for a, b, c, and d such as 5:12 = 10:24, then the product of extremes (5×24) is equal to the product of the means (12×10). • In a proportional situation, both quantities increase or decrease together. • In a proportional situation, two quantities increase multiplicatively. Both are multiplied by the same factor. • A proportion can be solved by finding equivalent fractions.	~~**All students should**~~ • ~~Understand that a proportion is an equation showing that two ratios are equal.~~ • ~~Understand how to set up a proportion, given the relationship between two items.~~ • What makes two quantities proportional? ~~Understand that when t~~Two quantities are proportional, when a change in one quantity corresponds to a predictable change in the other. • ~~Understand that proportions are useful in solving many types of problems.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write proportions that represent equivalent relationships between two sets. • Solve a proportion to find a missing term. • ~~Apply proportions to solve problems that involve percents.~~ • ~~Apply proportions to solve practical problems. Calculators may be used.~~ • Apply proportions to convert units of measurement between the U.S. Customary System and the metric system. Calculators may be used. • Apply proportions to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and~~/or~~ decimals no less than tenths. Calculators may be used. • Using 10% as a benchmark, mentally compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax and discounts. • Solve problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem.

7.4 The student will solve single-step and multistep practical problems, using proportional reasoning.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~• There is a distinction between a proportion and the idea of equivalent fractions. Equivalent fractions are symbols for the same quantity or amount and they represent the same rational number in different forms.~~ • ~~A rate is a special ratio that always has a denominator of 1~~. A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/minute. ~~A rate compares measures of different types.~~ • Proportions are used in everyday contexts, such as speed, recipe conversions, scale drawings, map reading, reducing and enlarging, comparison shopping, and monetary conversions. • Proportions can be used to convert between measurement systems. For example: if 2 inches is about 5 cm, how many inches are in 16 cm? – $\frac{2 inches}{x} = \frac{5 cm}{16 cm}$ • A percent is a special ratio in which the denominator is 100. • Proportions can be used to represent percent problems as follows: – $\frac{percent}{100} = \frac{part}{whole}$		

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- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

7.5 The student will
a) describe volume and surface area of cylinders;
b) solve practical problems involving the volume and surface area of rectangular prisms and cylinders; and
c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The area of a rectangle is computed by multiplying the lengths of two adjacent sides. • The area of a circle is computed by squaring the radius and multiplying that product by π ($A = \pi r^2$, where $\pi \approx 3.14$ or $\frac{22}{7}$). • A rectangular prism can be represented on a flat surface as a net that contains six rectangles — two that have measures of the length and width of the base, two others that have measures of the length and height, and two others that have measures of the width and height. The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$). • A cylinder can be represented on a flat surface as a net that contains two circles (bases for the cylinder) and one rectangular region whose length is the circumference of the circular base and whose width is the height of the cylinder. The surface area of the cylinder is the area of the two circles and the rectangle ($SA = 2\pi r^2 + 2\pi rh$). • The volume of a rectangular prism is computed by multiplying the area of the base, B, (length times width) by the height of the prism ($V = lwh = Bh$). • The volume of a cylinder is computed by multiplying the area of the base, B, (πr^2) by the height of the cylinder ($V = \pi r^2 h = Bh$). • There is a direct relationship between changing one	~~**All students should**~~ • ~~Understand how to apply volume and surface area in real-life situations.~~ • ~~Understand the derivation of formulas related to volume and surface area of polygons.~~ • How are volume and surface area related? Volume is a measure of the amount a container holds while surface area is the sum of the areas of the surfaces on the container. • How does the volume of a rectangular prism change when one of the attributes is increased? There is a direct relationship between the volume of a rectangular prism increasing when the length of one of the attributes of the prism is changed by a scale factor.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area. • ~~Develop a procedure and formula for finding~~ Find the surface area of a rectangular prism. • Solve practical problems that require finding the surface area of a rectangular prism. • ~~Develop a procedure and formula for finding~~ Find the surface area of a cylinder. • Solve practical problems that require finding the surface area of a cylinder. • ~~Develop a procedure and formula for finding~~ Find the volume of a rectangular prism. • Solve practical problems that require finding the volume of a rectangular prism. • Find the volume of a cylinder. • Solve practical problems that require finding the volume of a cylinder. • Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a scale factor. Problems will be limited to changing attributes by scale factors only. • Describe how the surface area of a rectangular prism

7.5 The student will

a) describe volume and surface area of cylinders;

b) solve practical problems involving the volume and surface area of rectangular prisms and cylinders; and

c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
measured attribute of a rectangular prism by a scale factor and its volume. For example, doubling the length of a prism will double its volume. This direct relationship does not hold true for surface area. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#geometry		is affected when one measured attribute is multiplied by a scale factor. Problems will be limited to changing attributes by scale factors only.

7.6 The student will determine whether plane figures – quadrilaterals and triangles – are similar and write proportions to express the relationships between corresponding sides of similar figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Two polygons are similar if corresponding (matching) angles are congruent and the lengths of corresponding sides are proportional. • Congruent polygons have the same size and shape. • Congruent polygons are similar polygons for which the ratio of the corresponding sides is 1:1. • Similarity statements can be used to determine corresponding parts of similar figures such as: $\Delta ABC \sim \Delta DEF$ $\angle$A corresponds to $\angle$D $\overline{AB}$ corresponds to $\overline{DE}$ • The traditional notation for marking ~~corresponding~~ congruent angles is to use a curve on each angle. Denote which angles ~~correspond~~ are congruent with the same number of curved lines. For example, if $\angle$A ~~corresponding~~ congruent to $\angle$B, then both angles will be marked with the same number of curved lines. • ~~Corresponding~~ Congruent sides are denoted with the same number of hatch ~~lines~~marks on each ~~corresponding~~ congruent side. For example, a side on a polygon with 2 hatch marks ~~corresponds~~ is congruent to the side with 2 hatch marks on a similar polygon. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#geometry	~~**All students should**~~ • ~~Understand that similar geometric figures have the same shape but may have different sizes.~~ • ~~Understand how ratios and proportions can be used to determine the length of something that cannot be measured directly~~ • How do polygons that are similar compare to polygons that are congruent? Congruent polygons have the same size and shape. Similar polygons have the same shape, and corresponding angles between the similar figures are congruent. However, the lengths of the corresponding sides are proportional. All congruent polygons are considered similar with the ratio of the corresponding sides being 1:1.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify corresponding sides and corresponding and congruent angles of similar figures using the traditional notation of curved lines for the angles and hatch marks on the sides. • Write proportions to express the relationships between the lengths of corresponding sides of similar figures. • ~~Examine congruence of corresponding angles and proportionality of corresponding sides to d~~ Determine if quadrilaterals or triangles are similar~~.~~ by examining congruence of corresponding angles and proportionality of corresponding sides. • Given two similar figures, write similarity statements using symbols such as $\Delta ABC \sim \Delta DEF$, $\angle$A corresponds to $\angle$D, and $\overline{AB}$ corresponds to $\overline{DE}$.

In the middle grades, the focus of mathematics learning is to

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- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.
- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.
- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking ~~A~~algebra.)
- **Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking ~~G~~geometry.)

7.7 The student will compare and contrast the following quadrilaterals based on properties: parallelogram, rectangle, square, rhombus, and trapezoid.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A quadrilateral is a closed plane (two-dimensional) figure with four sides that are line segments. • A parallelogram is a quadrilateral whose opposite sides are parallel and opposite angles are congruent. ~~Opposite angles of a parallelogram are congruent.~~ • A rectangle is a parallelogram with four right angles. The diagonals of a rectangle are the same length and bisect each other. • A square is a rectangle with four congruent sides whose diagonals are perpendicular. ~~or~~ A square is a rhombus with four right angles. • A rhombus is a parallelogram with four congruent sides whose diagonals bisect each other and intersect at right angles. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. • A trapezoid with congruent, nonparallel sides is called an *isosceles trapezoid*. • Quadrilaterals can be sorted according to common attributes, using a variety of materials. • A chart, ~~or~~ graphic organizer, or Venn diagram can be made to organize quadrilaterals according to attributes such as sides and/or angles. • ~~For all parallelograms, both pairs of opposite sides and both pairs of opposite angles are congruent~~. ~~• Parallelograms have special characteristics (such as both pairs of opposite sides are parallel and congruent) that are true for any parallelogram.~~ ~~• Rectangles, Squares have special characteristics~~	~~**All students should**~~ ~~• Understand that quadrilaterals can be classified according to the attributes of their sides and/or angles.~~ ~~• Understand that a quadrilateral can belong to one or more subsets of the set of quadrilaterals.~~ • Why can some quadrilaterals be classified in more than one category? ~~Understand that e~~ Every quadrilateral in a subset has all of the defining attributes of the subset. For example, if a quadrilateral is a rhombus, it has all the attributes of a rhombus. However, if that rhombus also has the additional property of 4 right angles, then that rhombus is also a square.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast attributes of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. • Identify the classification(s) to which a quadrilateral belongs, using deductive reasoning and inference. ~~• Classify quadrilaterals, using deductive reasoning and inference.~~

7.7 The student will compare and contrast the following quadrilaterals based on properties: parallelogram, rectangle, square, rhombus, and trapezoid.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~(such as diagonals are perpendicular bisectors) that are true for any rectangle~~ square. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#geometry		

7.8 The student, given a polygon in the coordinate plane, will represent transformations (reflections, dilations, rotations, and translations) by graphing in the coordinate plane.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A rotation of a geometric figure is a turn of the figure around a fixed point. The point may or may not be on the figure. The fixed point is called the *center of rotation*. • A translation of a geometric figure is a slide of the figure in which all the points on the figure move the same distance in the same direction. • A reflection is a transformation that reflects a figure across a line in the plane. • A dilation of a geometric figure is a transformation that changes the ~~shape~~ size of a figure by ~~changing the distance of all the points in the figure from a specific line. The distance is changed by~~ a ~~multiplicative~~ scale factor to create a similar figure. • The image of a polygon is the resulting polygon after the transformation. The preimage is the polygon before the transformation. • A transformation of preimage point A can be denoted as the image A' (read as “A prime”).	~~**All students should**~~ • ~~Understand that the size or shape of a figure does not change by a translation or rotation.~~ • How does the transformation of a figure affect the size, shape and position of that figure? Translations, rotations and reflections do not change the size or shape of a figure. A dilation of a figure and the original figure are similar. ~~Understand that~~ Reflections, translations and rotations usually change the position of the figure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the coordinates of the image of a ~~polygon~~ right triangle or rectangle that has been translated either vertically, ~~or~~ horizontally, or a combination of a vertical and horizontal translation. • Identify the coordinates of the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin. • Identify the coordinates of the image of a right triangle or a rectangle that has been reflected over the x- or y-axis. • Identify the coordinates of a right triangle or rectangle that has been dilated. The center of the dilation will be the origin. • Sketch the image of a ~~polygon~~ right triangle or rectangle translated vertically or horizontally. • Sketch the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin. • Sketch the image of a right triangle or rectangle that has been reflected over the x- or y-axis. • Sketch the image of a dilation of a right triangle or rectangle limited to a scale factor of $\frac{1}{4}$, $\frac{1}{2}$, 2, 3 or 4.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of ~~central tendency~~ center and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

7.9 The student will investigate and describe the difference between the experimental probability and theoretical probability of an event.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Theoretical probability of an event is the expected probability and can be found with a formula. • Theoretical probability of an event = $\frac{\text{number of possible favorable outcomes}}{\text{total number of possible outcomes}}$ • The experimental probability of an event is determined by carrying out a simulation or an experiment. • The experimental probability = $\frac{\text{number of times desired outcomes occur}}{\text{number of trials in the experiment}}$ • In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability (Law of Large Numbers).	~~**All students should**~~ • ~~Understand the meaning of theoretical probability.~~ • ~~Understand the difference between theoretical and experimental probability.~~ • What is the difference between the theoretical and experimental probability of an event? Theoretical probability of an event is the expected probability and can be found with a formula. ~~Understand that~~ The experimental probability of an event is determined by carrying out a simulation or an experiment. ~~Understand that i~~ In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the theoretical probability of an event. • Determine the experimental probability of an event. • Describe changes in the experimental probability as the number of trials increases. • Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event.

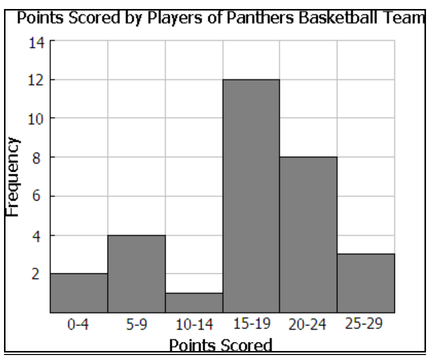
7.10 The student will determine the probability of compound events, using the Fundamental (Basic) Counting Principle.

UNDERSTANDING THE STANDARD (Teacher Notes)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The Fundamental (Basic) Counting Principle is a computational procedure to determine the number of possible ~~arrangements~~ outcomes of several ~~objects~~ events. It is the product of the number of ~~ways~~ outcomes for each ~~object~~ event that can be chosen individually (e.g., the possible ~~arrangements~~ outcomes or outfits of four shirts, two pants, and three shoes is 4 · 2 · 3 or 24). • Tree diagrams are used to illustrate possible outcomes of events. They can be used to support the Fundamental (Basic) Counting Principle. • A compound event combines two or more simple events. For example, a bag contains 4 red, 3 green and 2 blue marbles. What is the probability of selecting a green and then a blue marble?	~~**All students should**~~ • What is the Fundamental (Basic) Counting Principle? ~~Understand that t~~ The Fundamental (Basic) Counting Principle is a computational procedure used to determine the number of possible ~~arrangements (combinations or~~ outcomes~~)~~ of several ~~objects~~ events. • What is the role of the Fundamental (Basic) Counting Principle in determining the probability of compound events? The Fundamental (Basic) Counting Principle is used to determine the number of outcomes of each event. It is the product of the number of outcomes for each event that can be chosen individually.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compute the number of possible ~~arrangements~~ outcomes ~~of no more than three types of objects~~ by using the Fundamental (Basic) Counting Principle. • Determine the probability of a compound event containing no more than 2 events.

7.11 The student, given data in a practical situation, will
a) construct and analyze histograms; and
b) compare and contrast histograms with other types of graphs presenting information from the same data set.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- All graphs tell a story and include a title and labels that describe the data.
- A histogram is a form of bar graph in which the categories are consecutive and equal intervals. The length or height of each bar is determined by the number of data elements frequency falling into a particular interval.



- A frequency distribution shows how often an item, a number, or range of numbers occurs. It can be used to construct a histogram.

STUDENTS WHO READ GARFIELD				
Age Group	Tally	Frequency	Cumulative Frequency	
7-10	𝍸 II	7	7	
11-14	𝍸 II	7	14	← 7 + 7
15-18	III	3	17	← 14 + 3
19-22	III	3	20	← 17 + 3

- ~~Inferences, conjectures, and predictions are based on~~

ESSENTIAL UNDERSTANDINGS

~~**All students should**~~

- ~~Understand that graphs tell a story.~~
- ~~Understand that data can be displayed in a variety of graphical representations.~~
- ~~Select and use appropriate statistical methods to analyze data.~~
- ~~Understand that different types of graphs can be used to represent the same data in a variety of ways~~
- What type of data are most appropriate to display in a histogram?
  Numerical data that can be characterized using consecutive intervals are best displayed in a histogram.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect, analyze, display, and interpret a data set ~~of no more than 20 items,~~ using histograms. For collection and display of raw data, limit the data to 20 items.
- ~~Organize data not exceeding 20 items into tables and/or graphs that provide a clear representation of dispersion or convergence of the data.~~
- Determine patterns and relationships within data sets (e.g., trends).
- Make inferences, conjectures, and predictions based on analysis of a set of data. ~~Raw data displayed should not exceed 20 items.~~
- Compare and contrast histograms with line plots, circle graphs, and stem and leaf plots presenting information from the same data set.

7.11 The student, given data in a practical situation, will
a) construct and analyze histograms; and
b) compare and contrast histograms with other types of graphs presenting information from the same data set.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~careful data analysis.~~ • Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • The information displayed in different graphs may be examined to determine how data are or are not related, ascertaining differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inference). • ~~Methods of collecting and analyzing data are factors in determining the validity of any inferences or arguments based on the data.~~		

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through ~~life~~ practical experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

7.12 The student will represent relationships with tables, graphs, rules, and words.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~A function is a rule that pairs exactly one element of a set with one and only one element of another set. An example of a function is that which relates the sum, *s*, of the measures of the interior angles of a polygon to the number, *n*, of sides: $s = (n - 2) \cdot 180$.~~ • Rules that relate elements in two sets can be represented by word sentences, equations, tables of values, graphs, or illustrated pictorially. • A relation is any set of ordered pairs. For each first member, there may be many second members. • A function is a relation in which there is one and only one second member for each first member. • As a table of values, a function has a unique value assigned to the second variable for each value of the first variable. • As a graph, a function is any curve (including straight lines) such that any vertical line would pass through the curve only once. • Some relations are functions; all functions are relations.	~~**All students should**~~ • ~~Understand that patterns in mathematics are often represented by using a rule that relates elements in one set to elements in another set.~~ • What are the different ways to represent the relationship between two sets of numbers? ~~Understand that r~~ Rules that relate elements in two sets can be represented by word sentences, equations, tables of values, graphs or illustrated pictorially.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Represent a variety of patterns, using tables, graphs, rules, and words, in order to investigate and describe functional relationships.~~ • ~~Generalize a variety of patterns.~~ • Given a function in words, represent the function as an equation. • Given an equation that represents a relation, write a table of values. • Given a table of values, write an equation that represents a relation. • Graph in a coordinate plane ordered pairs that are represented on a table.

7.13 The student will
a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and
b) evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD **(Teacher Notes)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• An expression is a name for a number. • An expression that contains a variable is a variable expression. • An expression that contains only numbers is a numerical expression. • A verbal expression is a word phrase (e.g., “the sum of two consecutive integers”). • A verbal sentence is a complete word statement (e.g., “The sum of two consecutive integers is five.”). • An algebraic expression is a variable expression that contains at least one variable (e.g., $2x – 5$). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x + 1 = 5$). • ~~Key words in translating verbal expressions/ sentences to algebraic expressions/equations may include words and their translations such as: *is* to =, *of* to *multiplication*, *more than* to +, *less than* to –, *increased by* to +, and *decreased by* to –.~~ • To evaluate an algebraic expression, substitute a given replacement value for a variable and apply the order of operations. For example, if a = 3 and b = -2 then 5a + b can be evaluated as: 5(3) + (-2) = 15 + (-2) = 13.	~~**All students should**~~ • How can algebraic expressions and equations be written? ~~Understand that w~~ Word phrases and sentences can be used to represent~~ed as~~ algebraic expressions and equations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write verbal expressions as algebraic expressions. Expressions will be limited to no more than 2 operations. • Write verbal sentences as algebraic equations. Equations will contain no more than 1 variable term. • Translate algebraic expressions and equations to verbal expressions and sentences. Expressions will be limited to no more than 2 operations. • Identify examples of expressions and equations. • Apply the order of operations to evaluate expressions for given replacement values of the variables. Limit the number of replacements to no more than 3 per expression.

7.13 The student will

a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and

b) evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
Additional information on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#nns		

7.14 The student will

a) solve one- and two-step linear equations in one variable; and

b) solve practical problems requiring the solution of one- and two-step linear equations.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An equation is a mathematical sentence that states that two expressions are equal. • A one-step equation is defined as an equation that requires the use of one operation to solve (e.g., $x + 3 = -4$). • ~~A one-step inequality is defined as an inequality that requires the use one operation to solve (e.g., $x - 4 > 9$).~~ • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • A two-step equation is defined as an equation that requires the use of two operations to solve. E.g., $2x + 1 = -5$ $-5 = 2x + 1$ $\frac{x-7}{3} = 4$	~~**All students should**~~ • ~~Understand that different words have specific and different meanings to describe algebraic relationships.~~ • When solving an equation, why is it important to perform identical operations on each side of the equal sign? An operation that is performed on one side of an equation must be performed on the other side to maintain equality.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Apply the following algebraic terms appropriately: *equation*, *inequality*, and *expression*.~~ • ~~Identify examples of equations, inequalities, and expressions.~~ • Represent and demonstrate steps ~~in~~for solving one- and two-step equations in one variable using concrete materials, pictorial representations and algebraic sentences. • Solve practical problems that require the solution of a one- or two-step linear equation.

7.15 The student will
a) solve one-step inequalities in one variable; and
b) graph solutions to inequalities on the number line.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A one-step inequality is defined as an inequality that requires the use of one operation to solve (e.g., $x - 4 > 9$). • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • When both expressions of an inequality are multiplied or divided by a negative number, the inequality symbol reverses (e.g., $-3x < 15$ is equivalent to $x > -5$). • Solutions to inequalities can be represented using a number line.	~~**All students should**~~ • ~~Understand that an operation that is performed on one side of an equation must be performed on the other side to maintain equality.~~ • ~~Understand procedures for solving inequalities.~~ • ~~Understand that when both expressions are multiplied or divided by a negative number, the inequality symbol reverses~~. • How are the procedures for solving equations and inequalities the same? The procedures are the same except for the case when an inequality is multiplied or divided on both sides by a negative number. Then the inequality sign is changed from less than to greater than, or greater than to less than. • How is the solution to an inequality different from that of a linear equation? In an inequality, there can be more than one value for the variable that makes the inequality true.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Represent and demonstrate steps in solving equations in one variable, using concrete materials, pictorial representations, and algebraic sentences.~~ • Represent and demonstrate steps in solving inequalities in one variable, using concrete materials, pictorial representations, and algebraic sentences. • ~~Translate one-step word problems and practical problems into algebraic equations and solve them.~~ • Graph solutions to inequalities on the number line. • Identify a numerical value that satisfies the inequality.

7.16 The student will apply the following properties of operations with real numbers:
a) the commutative and associative properties for addition and multiplication;
b) the distributive property;
c) the additive and multiplicative identity properties;
d) the additive and multiplicative inverse properties; and
e) the multiplicative property of zero.

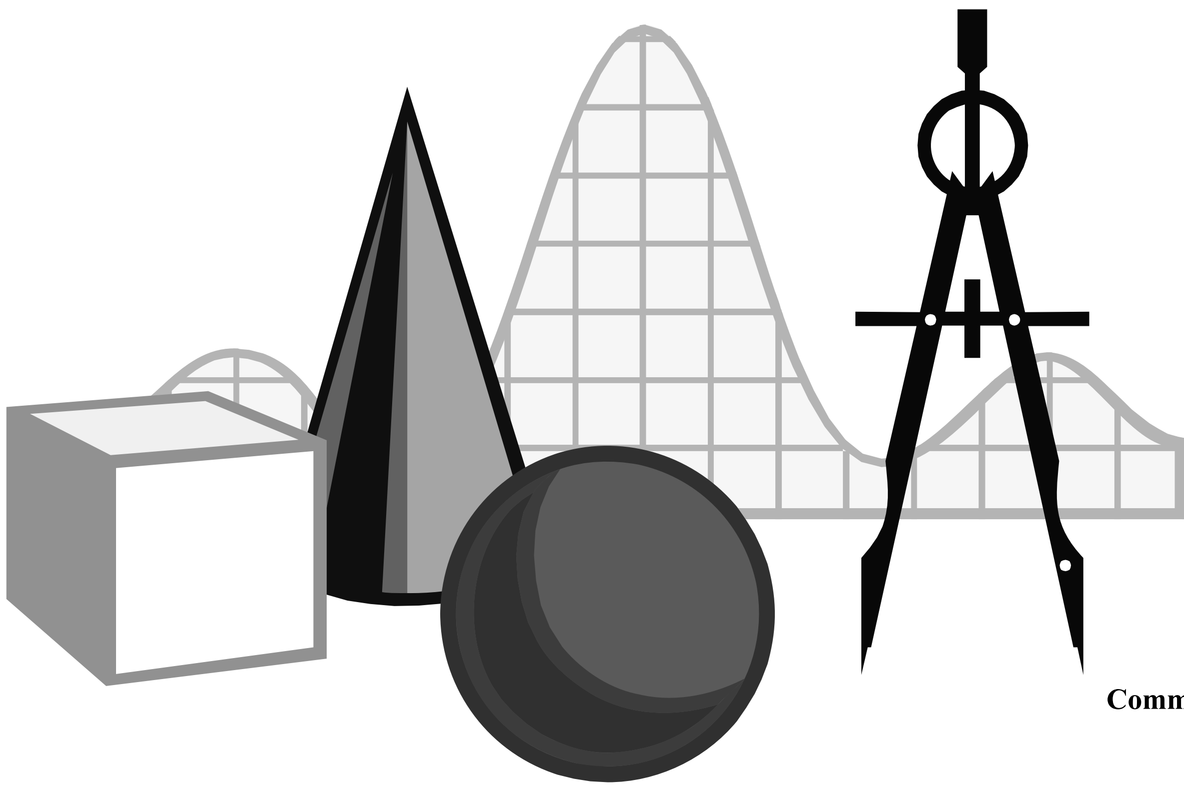
UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The commutative property for addition states that changing the order of the addends does not change the sum (e.g., 5 + 4 = 4 + 5). • The commutative property for multiplication states that changing the order of the factors does not change the product (e.g., 5 · 4 = 4 · 5). • The associative property of addition states that regrouping the addends does not change the sum [e.g., 5 + (4 + 3) = (5 + 4) + 3]. • The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5 · (4 · 3) = (5 · 4) · 3]. • Subtraction and division are neither commutative nor associative. • The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number [e.g., 5 · (3 + 7) = (5 · 3) + (5 · 7), or 5 · (3 – 7) = (5 · 3) – (5 · 7)]. • Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division. • The additive identity property states that the sum of	~~**All students should**~~ • Why is it important to apply properties of operations when simplifying expressions? ~~Understand that u~~ Using the properties of operations with real numbers helps with understanding mathematical relationships. ~~• Understand how to use these properties when computing.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Identify the real number equation that represents each property of operations with real numbers, when given several real number equations.~~ ~~• Explore the properties of real numbers, using diagrams and manipulatives.~~ ~~• Test the validity of properties by using examples of the properties of operations on real numbers.~~ • ~~Identify the property of operations with real numbers that is illustrated by a real number equation~~. • Identify properties of operations used in simplifying expressions. • Apply the properties of operations to simplify expressions.

7.16 The student will apply the following properties of operations with real numbers:
a) the commutative and associative properties for addition and multiplication;
b) the distributive property;
c) the additive and multiplicative identity properties;
d) the additive and multiplicative inverse properties; and
e) the multiplicative property of zero.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$). • The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$). • Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$]. • The additive inverse property states that the sum of a number and its additive inverse always equals zero [e.g., $5 + (-5) = 0$]. • The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$). • Zero has no multiplicative inverse. • The multiplicative property of zero states that the product of any real number and zero is zero. • Division by zero is not a possible arithmetic operation. Division by zero is undefined. Additional information on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#nns		

MATHEMATICS STANDARDS OF LEARNING
PROPOSED CURRICULUM FRAMEWORK

Grade 8



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.
- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational-number representations is encouraged when students solve problems.
- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.
- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

8.1 The student will

a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and

b) compare and order decimals, fractions, percents, and numbers written in scientific notation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• *Expression* is a word used to designate any symbolic mathematical phrase that may contain numbers and/or variables. Expressions do not contain ~~an~~ equal or inequality signs. • The set of rational numbers ~~is~~ includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero ($\sqrt{25}$, $\frac{1}{4}$ -2.3, 75%, ~~4.59~~ $4.\overline{59}$). • A rational number is any number that can be written in fraction form. • A numerical expression contains only numbers and the operations on those numbers. • Expressions are simplified using the order of operations and the properties for operations with real numbers, i.e., associative, commutative, and distributive and inverse properties. • The order of operations, a mathematical convention, is as follows: Complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right. *~~Grouping symbols include p~~Parentheses ( ), brackets [ ], braces { }, the absolute value │ │, division/fraction bar −,	**~~All students should~~** • ~~Understand that any real number can be shown represented on a number line~~. • What is the role of the order of operations when simplifying numerical expressions? The order of operations ~~describes~~ prescribes the order to use to ~~compute with rational numbers~~simplify a numerical expression. • How does the different ways rational numbers can be represented help us compare and order rational numbers? ~~Understand that n~~Numbers can be represented ~~—~~as decimals, fractions, percents, and in scientific notation. It is often useful to convert numbers to be compared and/or ordered to one representation (e.g., fractions, decimals or percents). • What is a rational number? A rational number is any number that can be written in fraction form. • When are numbers written in scientific notation? Scientific notation is used to represent very large and very small numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify numerical expressions containing: 1) exponents (where the base is a rational number and the exponent is a positive whole number); 2) fractions, decimals, integers and~~/or~~ square roots of perfect squares; and~~/or~~ 3) grouping symbols (no more than 2 embedded grouping symbols). Order of operations and properties of operations with real numbers should be used. • Compare and order no more than five fractions, decimals, percents, and numbers written in scientific notation using positive and~~/or~~ negative exponents. Ordering may be in ascending or descending order.

8.1 The student will

a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and

b) compare and order decimals, fractions, percents, and numbers written in scientific notation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
and the square root symbol $\sqrt{\ }$ should be treated as grouping symbols. • A power of a number represents repeated multiplication of the number. For example, $(-5)^4$ means $(-5)\cdot(-5)\cdot(-5)\cdot(-5)$. The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In this example, (–5) is the base, and 4 is the exponent. The product is 625. Notice that the base appears inside the grouping symbols. The meaning changes with the removal of the grouping symbols. For example, -5^4 means $5\cdot5\cdot5\cdot5$ negated which results in a product of -625. The expression $-(5)^4$ means to take the opposite of $5\cdot5\cdot5\cdot5$ which is -625. Students should be exposed to all three representations. • Scientific notation is used to represent very large or very small numbers. • A number written in scientific notation is the product of two factors: a decimal greater than or equal to one but less than 10 multiplied by a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $3.1 \times 10^{-5} = 0.000031$). • Any real number raised to the zero power is 1. The only exception to this rule is zero itself. Zero raised to the zero power is undefined. • All state approved scientific calculators use algebraic logic (follow the order of operations).		

8.2 The student will describe orally and in writing the relationships between the subsets of the real number system.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The set~~s~~ of real numbers includes natural numbers, ~~or~~ counting numbers, whole numbers, integers, rational and irrational numbers~~ are the subsets of the real number system~~. • The set of natural numbers is the set of counting numbers ~~(~~{1, 2, 3, 4, ...}~~)~~. • The set of whole numbers includes the set of all the natural numbers or counting numbers and zero ~~(~~{0, 1, 2, 3…}~~)~~. • The set of integers includes the set of whole numbers and their opposites ~~(~~{…-2, -1, 0, 1, 2…}~~)~~. • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero ($\sqrt{25}$, $\frac{1}{4}$, -2.3, 75%, ~~4.59~~ $4.\overline{59}$) . • The set of irrational numbers is the set of all non-repeating, nonterminating decimals. An irrational number cannot be written in fraction form ~~(~~{ π , $\sqrt{2}$, 2, 1.232332333…}~~).~~ • ~~The set of real numbers is includes the sets of all rational and irrational numbers. An irrational number cannot be expressed as an integer or the quotient of integers.~~	~~**All students should**~~ • ~~Understand the relationship between the subsets of the real number system.~~ • How are the real numbers related? Some numbers can appear in more than one subset, e.g., 4 is an integer, a whole number, a counting or natural number and a rational number. The attributes of one subset can be contained in whole or in part in another subset.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe orally and in writing the relationships among the sets of natural or counting numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers. • Illustrate the relationships among the subsets of the real number system by using graphic organizers such as Venn diagrams. Subsets include ~~real numbers,~~ rational numbers, irrational numbers, integers, whole numbers, and natural or counting numbers. • Identify the subsets of the real number system to which a given number belongs. • Determine whether a given number is a member of a particular subset of the real number system, and explain why. • Describe each subset of the set of real numbers and include examples and nonexamples.

In the middle grades, the focus of mathematics learning is to
- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.

8.3 The student will
a) solve practical problems involving rational numbers, percents, ratios, and proportions; and
b) determine the percent increase or decrease for a given situation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Practical problems may include, but not be limited to, those related to economics, sports, science, social sciences, transportation, and health. Some examples include problems involving the amount of a pay check per month, the discount price on a product, temperature, simple interest, sales tax and installment buying. • A percent is a special ratio with a denominator of 100. • A discount is a percent of the original price. The discount price is the original price minus the discount. • Simple interest for a number of years is determined by multiplying the principle by the rate of interest by the number of years of the loan or investment (I=prt). • The total value of an investment is equal to the sum of the original investment and the interest earned. • The total cost of a loan is equal to the sum of the original cost and the interest paid. • Percent increase and percent decrease are both percents of change. • Percent of change is the percent that a quantity increases or decreases.	~~**All students should**~~ • ~~Select an appropriate method or methods for computing with rational numbers and percents according to the context of the problem.~~ • ~~Understand how to set up a proportion given the relationship between two items.~~ • What is the difference between percent increase and percent decrease? Percent increase and percent decrease are both percents of change measuring the percent a quantity increases or decreases. Percent increase shows a growing change in the quantity while percent decrease shows a lessening change. • What is a percent? A percent is a special ratio with a denominator of 100.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write a proportion given the relationship of equality between two ratios. • Solve practical problems by using computation procedures for whole numbers, integers, ~~rational numbers~~ fractions, percents, ratios, and proportions. Some problems may require the application of a formula. • Maintain a checkbook and check registry for five or fewer transactions. • Compute a discount or markup and the resulting sale price for one discount or markup. • Compute the percent increase or decrease for a one-step equation found in a real life situation. • Compute the sales tax or tip and resulting total. • Substitute values for variables in given formulas. For example, use the simple interest formula I=prt to determine the value of any missing variable when given specific information. • Compute the simple interest and~~/or~~ new balance earned in an investment or on a loan for a given number of years.

8.3 The student will
a) solve practical problems involving rational numbers, percents, ratios, and proportions; and
b) determine the percent increase or decrease for a given situation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Percent increase determines the rate of growth and may be calculated using the following ratio: $\frac{\text{Change (new – original)}}{\text{original}}$. • For percent increase, ~~T~~the change will result in a positive number. • Percent decrease determines the rate of decline and may be calculated using the same ratio as percent increase. However, the change will result in a negative number.		

8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Algebraic expressions use operations with algebraic symbols (variables) and numbers. • Algebraic expressions are evaluated by ~~replacing the variables with numbers~~ substituting numbers for variables and applying the order of operations to simplify the resulting expression. ~~• The replacement values are the numbers that replace the variables in an algebraic expression.~~ • The order of operations is as follows: Complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols (embedded), do the innermost operation first. Evaluate all exponential expressions. Multiply and/or divide in order from left to right. Add and/or subtract in order from left to right. *~~Grouping symbols include~~ P~~p~~arentheses ( ), brackets [ ], braces { }, the absolute value $\lvert\ \rvert$, division/fraction bar −, and the square root symbol $\sqrt{\ }$ should be treated as grouping symbols.	~~**All students should**~~ ~~• Evaluate an algebraic expression by substituting a number for each variable and then simplifying the result.~~ ~~• Understand how to apply the order of operations after substituting given values for variables in algebraic expressions.~~ • What is the role of the order of operations when evaluating expressions? Using the order of operations assures only one correct answer for an expression.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Substitute numbers for variables in ~~an~~ algebraic expressions and simplify the expressions by using the order of operations. Exponents are positive and limited to ~~used are~~ whole numbers less than 4. Square roots are limited to perfect squares. • Apply the order of operations to evaluate formulas. Problems will be limited to positive exponents. Square roots may be included in the expressions but limited to perfect squares.

8.5 The student will

a) determine whether a given number is a perfect square; and

b) find the two consecutive whole numbers between which a square root lies.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A perfect square is a whole number whose square root is ~~a whole number~~ an integer (e.g., The square root of 25 is 5 and -5; thus, 25 is a perfect square). • The square root of a number is ~~that~~ any number which when multiplied by itself equals the number. • Whole numbers have both positive and negative roots. • Any whole number other than a perfect square has a square root that lies between two consecutive whole numbers. • The square root of a whole number that is not a perfect square is an irrational number (e.g., $\sqrt{2}$ is an irrational number). An irrational number cannot be expressed exactly as a ratio. • Students can use grid paper and estimation to determine what is needed to build a perfect square.	~~**All students should**~~ • ~~Understand that a perfect square is the product of a number multiplied by itself.~~ • ~~Develop strategies for finding the square root of a number.~~ • How does the area of a square relate to the square of a number? The area determines the perfect square number. If it is not a perfect square, the area provides a means for estimation. • Why do numbers have both positive and negative roots? The square root of a number is ~~that~~ any number which when multiplied by itself equals the number. A product, when multiplying two positive factors, is always the same as the product when multiplying their opposites. For example, if the number is 49, then $7 \cdot 7 = 49$ and $-7 \cdot -7 = 49$.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the perfect squares from 0 to ~~100~~ 400. • Identify the two consecutive whole numbers between which the square root of a given whole number from 0 to ~~100~~ 400 lies (e.g., $\sqrt{57}$ lies between 7 and 8 since $7^2 = 49$ and $8^2 = 64$). • Define a perfect square. • Find the positive or positive and negative square roots of a given whole number from 0 to 400. (Use the symbol $\sqrt{\ }$ to ask for the positive root and $-\sqrt{\ }$ when asking for the ~~positive and~~ negative root~~s~~.)

In the middle grades, the focus of mathematics learning is to
- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

8.6 The student will

a) verify by measuring and describe the relationships among vertical angles, adjacent angles, supplementary angles, and complementary angles; and

b) measure angles of less than 360°.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Vertical angles are ~~the opposite angles~~ (all nonadjacent angles) formed by two intersecting lines. ~~and have only a vertex in common~~. Vertical angles are congruent and share a common vertex. • Complementary angles are any two angles such that the sum of their measures is 90°. • Supplementary angles are any two angles such that the sum of their measures is 180°. • Reflex angles measure more than 180°. • ~~Vertical angles are the opposite angles formed by two intersecting lines. Vertical angles are congruent and share a common vertex.~~ • Adjacent angles are any two angles that share a common side and a common vertex.	~~**All students should**~~ • ~~Understand the meaning of the term *angle*.~~ • ~~Understand how to use angle-measuring tools.~~ • ~~Understand that pairs of angles are named by their defining attributes.~~ • How are vertical, adjacent, complementary and supplementary angles related? Adjacent angles are any two angles that share a common side and a common vertex. Vertical angles will always be nonadjacent angles. Supplementary and complementary angles may or may not be adjacent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Measure angles of less than 360° to the nearest degree, using appropriate tools. • Identify and describe the relationships among the ~~2~~ two types of angles formed by two intersecting lines. • Identify and describe pairs of vertical angles that are vertical. • Identify and describe pairs of angles that are supplementary. • Identify and describe pairs of angles that are complementary. • Identify and describe pairs of angles that are adjacent.

8.7 The student will
a) investigate and solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids; and
b) describe how changing one measured attribute of the figure affects the volume and surface area.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polyhedron is a solid figure whose faces are all polygons. • A pyramid is a polyhedron with a base that is a polygon and other faces that are triangles with a common vertex. • ~~The lateral area of a pyramid is the sum of the areas of the triangular faces.~~ • The area of the base of a pyramid is the area of the polygon which is the base. • The total surface area of a pyramid is the sum of the ~~lateral area~~ areas of the triangular faces and the area of the base. • The volume of a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height. • ~~A circular cone is a geometric solid whose base is a circle and whose side is a surface composed of line segments connecting points on the base to a fixed point (the vertex) not on the base.~~ • ~~The lateral area of a right circular cone is the area of the surface connecting the base with the vertex and is equal to πrl, where l is the slant height.~~ • The area of the base of a circular cone is πr^2. • The surface area of a right circular cone is $\pi r^2 + \pi rl$. l represents the slant height of the cone.	~~**All students should**~~ • ~~Understand the derivation of formulas for volume and surface area of prisms, cylinders, cones, and pyramids.~~ • ~~Understand the differences between volume and surface area.~~ • How does the volume of a three-dimensional figure differ from its surface area? Volume is the amount a container holds. Surface area of a figure is the sum of the area on surfaces of the figure. • How are the formulas for the volume of prisms and cylinders similar? For both formulas you are finding the area of the base and multiplying that by the height. • How are the formulas for the volume of cones and pyramids similar? For cones you are finding 1/3 of the volume of the cylinder with the same size base and height. For pyramids you are finding 1/3 of the volume of the prism with the same size base and height. • In general what effect does changing one attribute of a prism by a scale factor have on the volume of the prism? When you increase or decrease the length, width or height of a prism by a factor greater than 1, the volume of the prism is also increased by that factor.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Distinguish between situations that are applications of surface area and those that are applications of volume. • Investigate and ~~C~~ compute the surface area of a square or triangular pyramid by finding the sum of the areas of the triangular faces and the base using concrete objects, nets, diagrams and formulas. • Investigate and ~~C~~ compute the surface area of a cone by calculating the sum of the areas of the side and the base, using concrete objects, nets, diagrams and formulas. • Investigate and compute the surface area of a right cylinder using concrete objects, nets, diagrams and formulas. • Investigate and compute the surface area of a rectangular prism using concrete objects, nets, diagrams and formulas. • Investigate and ~~C~~compute the volume ~~and surface area~~ of ~~rectangular solids (~~prisms~~)~~, cylinders, cones, and ~~square~~ pyramids, using concrete objects, nets, diagrams and formulas. • ~~Investigate and s~~ Solve practical problems involving volume and surface area of ~~rectangular solids (~~prisms~~)~~, cylinders, cones and pyramids. • Compare and contrast the volume of a prism with

8.7 The student will

a) investigate and solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids; and

b) describe how changing one measured attribute of the figure affects the volume and surface area.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The volume of a cone is $\frac{1}{3}\pi r^2 h$, where h is the height and πr^2 is the area of the base. • The surface area of a right circular cylinder is $2\pi r^2 + 2\pi rh$. • The volume of a cylinder is the area of the base of the cylinder multiplied by the height. • The surface area of a rectangular prism is the sum of the areas of the ~~6~~six faces. • The volume of a rectangular prism is calculated by multiplying the length, width and height of the prism. • A prism is a solid figure that has a congruent pair of parallel bases and faces that are parallelograms. The surface area of a prism is the sum of the areas of the faces and bases. • When one attribute of a prism is changed through multiplication or division the volume increases by the same factor that the attribute increased by. For example, if a prism has a volume of 2 x 3 x 4, the volume is 24. However, if one of the attributes are doubled, the volume doubles. • The volume of a prism is *Bh*, where *B* is the area of the base and h is the height of the prism. • Nets are two-dimensional representations that can be folded into three-dimensional figures.		~~one~~a given set of attributes with the volume of a prism where one of the attributes has been increased by a factor of 2, 3, 5 or 10.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.
- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.
- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking ~~A~~algebra.)
- **Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking ~~G~~geometry.)

8.8 The student will
a) apply transformations to plane figures; and
b) identify applications of transformations.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A rotation of a geometric figure is a clockwise or counterclockwise turn of the figure around a fixed point. The point may or may not be on the figure. The fixed point is called the *center of rotation.* • A reflection of a geometric figure ~~is a flip of the figure~~ moves all of the points of the figure across ~~a line~~ an axis. Each point on the reflected figure is the same distance from the ~~line~~ axis as the corresponding point in the original figure. • A translation of a geometric figure ~~is a slide of the figure in which~~ moves all the points on the figure ~~move~~ the same distance in the same direction. • A dilation of a geometric figure is a transformation that changes the size of a figure by a scale factor to create a similar figure. • ~~Real-life~~ Practical applications may include, but are not limited to, the following: – A rotation of the hour hand of a clock from 2:00 to 3:00 shows a turn of 30° clockwise; – A reflection of a boat in water shows an image of the boat flipped upside down with the water line being the line of reflection~~.~~; – A translation of a ~~shape~~ figure on a wallpaper pattern shows the same ~~shape~~ figure slid the same distance in the same direction~~.~~; and – A dilation of a model airplane is the production model of the airplane. • The image of a polygon is the resulting polygon after a transformation. The preimage is the original polygon before the transformation.	~~**All students should**~~ • ~~Understand the relationship between transformations in a coordinate plane and their application in real-life.~~ • ~~Explain that rotations, reflections, translations and dilations are transformations.~~ • How does the transformation of a figure on the coordinate grid affect the congruency, orientation, location and symmetry of an image? Translations, rotations and reflections maintain congruence between the preimage and image but change location. Dilations by a scale factor other than 1 produce an image that is not congruent to the pre-image but is similar. Rotations and reflections change the orientation of the image.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate the reflection of a ~~figure~~ polygon over ~~a~~ the vertical or horizontal ~~line~~ axis on a coordinate grid. • Demonstrate 90°, 180°, 270°, and 360° clockwise and counterclockwise rotations of a figure on a coordinate grid. The center of rotation will be limited to the origin. • Demonstrate the translation of a ~~figure~~ polygon on a coordinate grid. • Demonstrate the dilation of a ~~figure~~ polygon from a fixed point on a coordinate grid. • Identify practical applications of transformations including, but not limited~~,~~ to, tiling, fabric, and wallpaper designs, art and scale drawings. • Identify the type of transformation in a given example.

8.8 The student will
a) apply transformations to plane figures; and
b) identify applications of transformations.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A transformation of preimage point A can be denoted as the image A' (read as "A prime").		

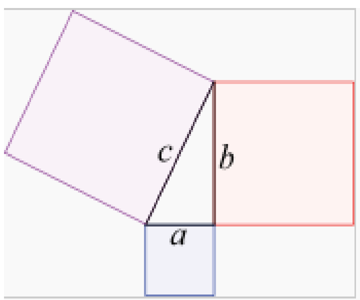
8.9 The student will construct a three-dimensional model, given the top or bottom, side, and front views.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Three-dimensional models of geometric solids can be used to understand perspective and provide tactile experiences in determining two-dimensional perspectives. • Three-dimensional models of geometric solids can be represented on isometric paper. • The top view is a mirror image of the bottom view.	~~**All students should**~~ • How does knowledge of two-dimensional figures inform work with three-dimensional objects? ~~Understand~~ It is important to know that a three-dimensional object can be represented as a two-dimensional model with views of the object from different perspectives.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct three-dimensional models, given ~~top, side, and bottom views.~~ the top or bottom, side, and front views. • Identify three-dimensional models given a two-dimensional perspective.

8.10 The student will
a) verify the Pythagorean Theorem; and
b) apply the Pythagorean Theorem.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Only)

- In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the legs (altitude and base). This relationship is known as the Pythagorean Theorem: $a^2 + b^2 = c^2$.



- The Pythagorean Theorem is used to find the measure of any one of the three sides of a right triangle if the measures of the other two sides are known.
- Whole number triples that are the measures of the sides of right triangles, such as (3,4,5), (6,8,10), (9,12,15), and (5,12,13), are commonly known as Pythagorean triples.
- The hypotenuse of a right triangle is the side opposite the right angle.
- The hypotenuse of a right triangle is always the longest side of the right triangle.
- The legs of a right triangle form the right angle.

ESSENTIAL UNDERSTANDINGS

~~**All students should**~~

- How can the area of squares generated by the legs and the hypotenuse of a right triangle be used to verify the Pythagorean theorem?
  ~~Understand that, f~~ For a right triangle, the area of a square with one side equal to ~~of~~ the measure of the hypotenuse equals the sum of the areas of the squares with one side each equal to the measures of the ~~base and altitude.~~legs of the triangle.
- ~~Understand that the Pythagorean Theorem is a tool to find the measure of any side of a right triangle, given the measures of the other two sides.~~
- ~~Demonstrate at least one model of the Pythagorean Theorem.~~

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify the parts of a right triangle (the hypotenuse and the legs).
- Verify a triangle is a right triangle given the measures of its three sides.
- Verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement.
- Find the measure of a side of a right triangle, given the measures of the other two sides. ~~The measures of the sides of the triangle may be whole numbers no larger than 15 or decimals in tenths.~~
- Solve ~~real-life~~ practical problems involving right triangles by using the Pythagorean Theorem.

8.11 The student will solve practical area and perimeter problems involving composite plane figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a simple, closed plane figure with sides that are line segments. • The perimeter of a polygon is the distance around the figure. • The area of any composite figure is based upon knowing how to find the area of the composite parts such as triangles and rectangles. • The area of a rectangle is computed by multiplying the lengths of two adjacent sides. ($A = lw$). • The area of a triangle is computed by multiplying the measure of its base by the measure of its height and dividing the product by 2. ($A = \frac{1}{2}bh$). • The area of a parallelogram is computed by multiplying the measure ~~if~~of its base by the measure of its height ($A = bh$). • The area of a trapezoid is computed by taking the average of the measures of the two bases and multiplying this average by the height. [$A = \frac{1}{2}(b_1 + b_2)h$] • The area of a circle is computed by multiplying Pi times the radius squared ($A = \pi r^2$). • The circumference of a circle is found by multiplying Pi by the diameter or multiplying Pi by 2 times the	~~**All students should**~~ • How does knowing the areas of polygons assist in calculating the areas of composite figures? The area of a composite figure can be found by subdividing the figure into triangles, rectangles, squares, trapezoids and semi-circles, calculating their areas, and adding the areas together.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Subdivide a figure into triangles, rectangles, squares, trapezoids and~~/or~~ semicircles. Estimate the area of subdivisions and combine to determine the area of the composite figure. ~~Figures will be limited to three subdivisions.~~ • Use the attributes of the subdivisions to determine the perimeter and~~/or~~ circumference of a figure. • Apply perimeter, circumference and~~/or~~ area formulas to solve real-life problems.

8.11 The student will solve practical area and perimeter problems involving composite plane figures.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
radius. ($C = \pi d$ or $C = 2\pi r$). • An estimate of the area of a composite figure can be made by subdividing the polygon into triangles, rectangles, squares, trapezoids and semicircles, estimating their areas, and adding the areas together.		

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of ~~central tendency~~ center and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

8.12 The student will determine the probability of independent and dependent events with and without replacement.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Two events are either dependent or independent. • If the outcome of one event does not influence the occurrence of the other event, they are called independent. If events are independent, then the second event occurs regardless of whether or not the first occurs. For example, the first roll of a number cube does not influence the second roll of the number cube. Other examples of independent events are, but not limited to: flipping two coins; spinning a spinner and rolling a number cube; flipping a coin and selecting a card; and choosing a card from a deck, replacing the card and selecting again. • ~~To find the probability of independent events, use the formula: P (A and B) = P(A) · P(B).~~ • The probability of three independent events is found by using the following formula: $P(A\,and\,B\,and\,C) = P(A)\cdot P(B)\cdot P(C)$ Ex: When rolling three number cubes simultaneously, what is the probability of rolling a 3 on one cube, a 4 on one cube, and a 5 on the third? $P(3\,and\,4\,and\,5) = P(3)\cdot P(4)\cdot P(5) = \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} = \frac{1}{216}$ • If the outcome of one event has an impact on the outcome of the other event, the events are called dependent. If events are dependent then the second event is considered only if the first event has already occurred. For example, if you are dealt a King from a deck of cards and you do not place the King back into the deck before selecting a second card, the chance of selecting a King the second time is	~~**All students should**~~ • How are the probabilities of dependent and independent events similar? Different? If events are dependent then the second event is considered only if the first event has already occurred. If events are independent, then the second event occurs regardless of whether or not the first occurs.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the probability of no more than ~~3~~ three independent events ~~with replacement~~. • Determine the probability of no more than ~~2~~ two dependent events without replacement. • Compare the outcomes of events with and without replacement.

8.12 The student will determine the probability of independent and dependent events with and without replacement.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
diminished because there are now only three Kings remaining in the deck. Other examples of dependent events are, but not limited to: choosing two marbles from a bag but not replacing the first after selecting it; and picking a sock out of a drawer and then picking a second sock without replacing the first. • The probability of two dependent events is found by using the following formula: $P(A\, and\, B) = P(A) \cdot P(B\, after\, A)$ Ex: You have a bag holding a blue ball, a red ball, and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick then *without* replacing the blue ball in the bag, picking a red ball on the second pick? $P(\text{blue}\, and\, \text{red}) = P(\text{blue}) \cdot P(\text{red}\, after\, \text{blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$ • ~~To find the probability of dependent events, use the formula: P (B/A) = P (A) · P (A/B).~~		

8.13 The student will
a) make comparisons, predictions, and inferences, using information displayed in graphs; and
b) construct and analyze scatterplots.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • The information displayed in different graphs may be examined to determine how data are or are not related, ascertaining differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences). • A scatterplot illustrates the relationship between two sets of data. A scatterplot consists of points. The coordinates of the point represent the measures of the two attributes of the point. • Scatterplots can be used to predict trends and estimate a line of best fit. • In a scatterplot, each point is represented by an independent and dependent variable. The independent variable is graphed on the horizontal axis and the dependent is graphed on the vertical axis.	~~**All students should**~~ • Why do we estimate a line of best fit for a scatterplot? A line of best fit helps in making interpretations and predictions about the situation modeled in the data set. • What are the inferences that can be drawn from sets of data points having a positive relationship, a negative relationship, and no relationship? Sets of data points with positive relationships demonstrate that the values of the two variables are increasing. A negative relationship indicates that as the value of the independent variable increases, the value of the dependent variable decreases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** ~~• Make comparisons, predictions, and inferences, given data sets of no more than 20 items that are displayed in frequency distributions; box-and-whisker plots; scatterplots; line, bar, circle, and picture graphs; and histograms.~~ • Collect, organize and interpret a data set of no more than 20 items using scatterplots. Predict from the trend an estimate of the line of best fit with a drawing. • Interpret the set of data points in a scatterplot as having a positive relationship, a negative relationship, or no relationship.

In the middle grades, the focus of mathematics learning is to
- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

8.14 The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A relation is any set of ordered pairs. For each first member ~~(domain)~~, there may be many second members ~~(range)~~. • A function is a relation in which there is one and only one second member ~~(range)~~ for each first member ~~(domain)~~. • As a table of values, a function has a unique value assigned to the second variable ~~(range)~~ for each value of the first variable ~~(domain)~~. • As a graph, a function is any curve (including straight lines) such that any vertical line would pass through the curve only once. • Some relations are functions; all functions are relations. • Graphs of functions can be discrete or continuous. • In a discrete function graph there are separate, distinct points. You would not use a line to connect these points on a graph. The points between the plotted points have no meaning and cannot be interpreted. • In a graph of continuous function every point in the domain can be interpreted therefore it is possible to connect the points on the graph with a continuous line as every point on the line answers the original question being asked. • Functions can be represented as tables, graphs, equations, physical models, or in words.	~~**All students should**~~ ~~• Understand the difference between functions and relations.~~ ~~• Understand that a function is a one-to-one relationship between the domain and range.~~ • What is the relationship among tables, graphs, words, and ~~equations~~rules in modeling a given situation? Any given ~~function~~relationship can be represented by all ~~three~~four.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Graph in a coordinate plane ordered pairs that represent a relation. ~~• Write a rule that represents a relation from a table of values.~~ ~~• Write a table of values from the rule that represents a relation.~~ ~~• Write a table of values from the graph of ordered pairs of a relation.~~ • Describe and represent relations and functions, using tables, graphs, words, and ~~rules~~rules ~~equations~~. Given one representation students will be able to represent the relation in another form. • Relate and compare different representations for the same relation.

8.15 The student will

a) solve multistep linear equations in one variable on one and two sides of the equation;

b) solve two-step linear inequalities and graph the results on a number line; and

c) identify properties of operations used to solve an equation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A multistep equation is an equation that requires more than one different mathematical operation to solve. • A two-step inequality is defined as an inequality that requires the use of two different operations to solve (e.g., $3x - 4 > 9$). • In an equation, the equal sign indicates that the value on the left is the same as the value on the right. • To maintain equality, an operation that is performed on one side of an equation must be performed on the other side. • When both expressions of an inequality are multiplied or divided by a negative number, the inequality sign reverses. • The commutative property for addition states that changing the order of the addends does not change the sum (e.g., $5 + 4 = 4 + 5$). • The commutative property for multiplication states that changing the order of the factors does not change the product (e.g., $5 \cdot 4 = 4 \cdot 5$). • The associative property of addition states that regrouping the addends does not change the sum [e.g., $5 + (4 + 3) = (5 + 4) + 3$]. • The associative property of multiplication states that regrouping the factors does not change the product [e.g., $5 \cdot (4 \cdot 3) = (5 \cdot 4) \cdot 3$].	~~**All students should**~~ • ~~Understand that an operation that is performed on one side of an equation must be performed on the other side to maintain equality.~~ • ~~Understand that when both expressions are multiplied or divided by a negative number, the inequality symbol reverses.~~ • How does the solution to an equation differ from the solution to an inequality? While a linear equation has only one replacement value for the variable that makes the equation true, an inequality can have more than one.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve two- to four- step linear equations in one variable ~~by showing the steps and using algebraic sentences.~~ using concrete materials, pictorial representations and paper and pencil illustrating the steps performed. • Solve two-step inequalities in one variable by showing the steps and using algebraic sentences. • Graph solutions to two-step linear inequalities on a number line. • Identify properties of operations used to solve an equation from among: - the commutative properties of addition and multiplication; - the associative properties of addition and multiplication; - the distributive property; - the identity properties of addition and multiplication; - the zero property of multiplication; - the additive inverse property; and - the multiplicative inverse property.

8.15 The student will

a) solve multistep linear equations in one variable on one and two sides of the equation;
b) solve two-step linear inequalities and graph the results on a number line; and
c) identify properties of operations used to solve an equation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Subtraction and division are neither commutative nor associative. • The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number [e.g., $5 \cdot (3 + 7) = (5 \cdot 3) + (5 \cdot 7)$, or $5 \cdot (3 - 7) = (5 \cdot 3) - (5 \cdot 7)$]. • Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division. • The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$). • The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$). Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$]. • The additive inverse property states that the sum of a number and its additive inverse always equals zero [e.g., $5 + (-5) = 0$]. • The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$).		

8.15 The student will
a) solve multistep linear equations in one variable on one and two sides of the equation;
b) solve two-step linear inequalities and graph the results on a number line; and
c) identify properties of operations used to solve an equation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Zero has no multiplicative inverse. • The multiplicative property of zero states that the product of any real number and zero is zero. • Division by zero is not a possible arithmetic operation. • Combining like terms means to combine terms that have the same variable and the same exponent (e.g., $8x + 11 – 3x$ can be $5x +11$ by combining the like terms of $8x$ and $-3x$).		

8.16 The student will graph a linear equation in two variables.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A linear equation is an equation in two variables whose graph is a straight line, a type of continuous function (see SOL 8.14). • A linear equation represents a situation with a constant rate. For example, when driving at a rate of 35 mph, the distance increases as the time increases, but the rate of speed remains the same. • Graphing a linear equation requires determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. • The axes of a coordinate plane are generally labeled *x* and *y*; however, any letters may be used that are appropriate for the function.	~~**All students should**~~ ~~• Understand that the graph of a linear equation in two variables is the set of all ordered pairs that satisfy the equation.~~ • What types of real life situations can be represented with linear equations? Any situation with a constant rate can be represented by a linear equation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a table of ordered pairs by substituting values for *x* in a linear equation to find values for *y*. • Plot in the coordinate plane ordered pairs (*x*, *y*) from a table. • Connect the ordered pairs to form a straight line (a continuous function).

8.17 The student will identify the domain, range, independent variable or dependent variable in a given situation.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Only)

- The domain is the ~~possible~~ set of all the input values for the independent variable in a given situation.
- The range is the ~~possible~~ set of all the output values for the dependent variable in a given situation.
- The independent variable is the input value.
- The dependent variable depends on the independent variable and is the output value.
- Below is a table of values for finding the circumference of circles, $C = \pi d$, where the value of π is approximated as 3.14.

Diameter	Circumference
1 in.	3.14 in.
2 in.	6.28 in.
3 in.	9.42 in.
4 in.	12.56 in.

- The independent variable, or input, is the diameter of the circle. The values for the diameter make up the domain.
- The dependent variable, or output, is the circumference of the circle. The set of values for the circumference makes up the range.

ESSENTIAL UNDERSTANDINGS

~~**All students should**~~

- ~~Understand that the domain represents all the values for the independent variables and the range represents all the values for the dependent variables.~~
- ~~Understand that the independent variable is the value that causes the change in the dependent variable and the dependent variable is affected by the independent variable.~~
- What are the similarities and differences among the terms domain, range, independent variable and dependent variable?
  The value of the dependent variable changes as the independent variable changes. The domain is the set of all input values for the independent variable. The range is the set of all possible values for the dependent variable.

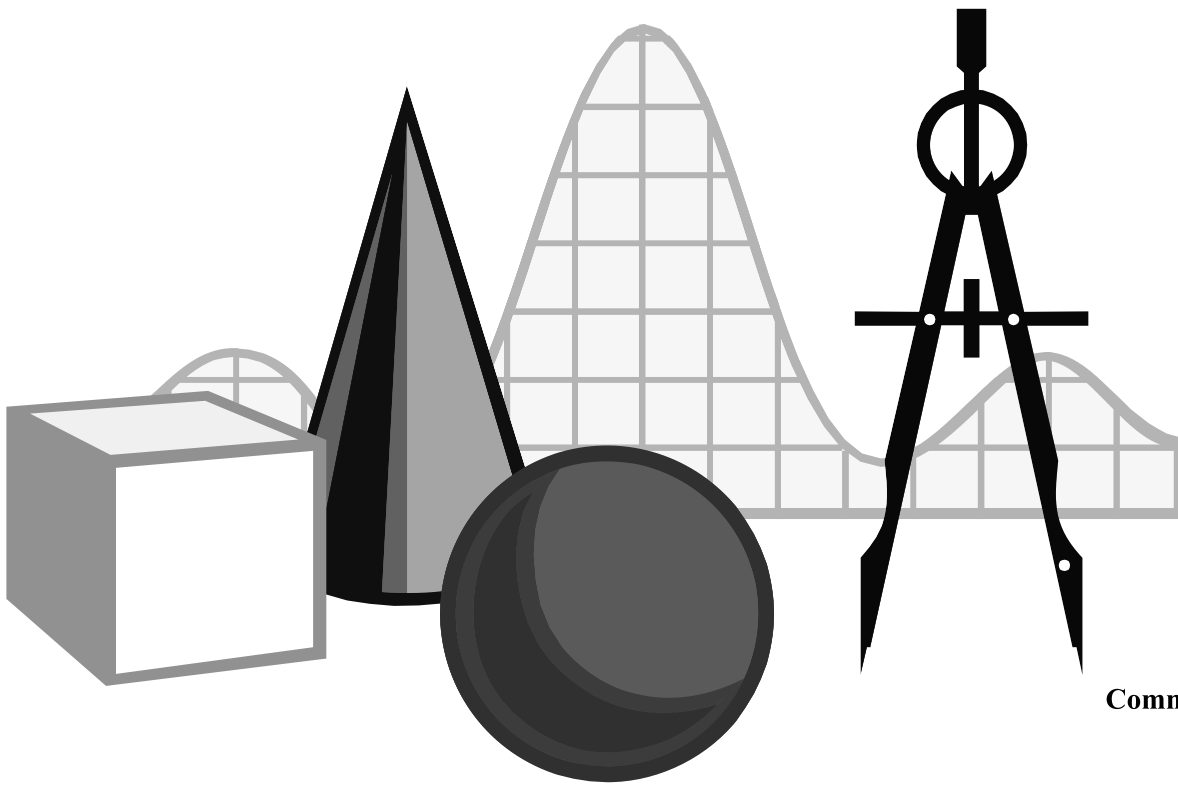
ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Apply the following algebraic terms appropriately: *domain*, *range*, *independent variable*, and *dependent variable*.
- Identify examples of domain, range, independent variable, and dependent variable.
- Determine the domain of a function.
- Determine the range of a function.
- Determine the independent variable of a relationship.
- Determine the dependent variable of a relationship.

MATHEMATICS STANDARDS OF LEARNING
PROPOSED CURRICULUM FRAMEWORK

Algebra I



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NOTICE TO THE READER

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills ~~and Essential Understandings~~. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.1

The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.

ESSENTIAL UNDERSTANDINGS

- Algebra is a tool for reasoning about quantitative situations so that relationships become apparent.
- Algebra is a tool for describing and representing patterns and relationships.
- Mathematical modeling involves creating algebraic representations of quantitative real-world situations.
- The numerical value of an expression is dependent upon the values of the replacement set for the variables.
- There are a variety of ways to compute the value of a numerical expression and evaluate an algebraic expression.
- The operations and the magnitude of the numbers in an expression impact the choice of an appropriate ~~method of computation~~ computational technique.
- An appropriate computational technique could be mental mathematics, calculator, or paper and pencil.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Translate verbal quantitative situations ~~expressions~~ into algebraic expressions and vice versa ~~with three or fewer terms~~.
- ~~Relate a polynomial expression with three or fewer terms to a verbal expression.~~
- Model real-world situations with algebraic expressions in a variety of representations (concrete, pictorial, symbolic, verbal).
- Evaluate algebraic expressions for a given replacement set to include ~~integers and~~ rational numbers.
- ~~Apply appropriate computational techniques to evaluate an algebraic expression.~~
- Evaluate expressions that contain absolute value, square roots, and cube roots.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.2

The student will perform operations on polynomials, including

a) applying the laws of exponents to perform operations on expressions;
b) adding, subtracting, multiplying, and dividing polynomials; and
c) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Repeated multiplication can be represented with exponents.~~ • The laws of exponents can be investigated using ~~patterns~~ inductive reasoning. • ~~The base and the exponent impact the magnitude of the expression.~~ • A relationship exists between the laws of exponents and scientific notation. • ~~A relationship exists between arithmetic operations and operations with polynomials.~~ • ~~Polynomials can be represented in a variety of forms.~~ • Operations with polynomials can be represented concretely, pictorially, and ~~algebraically~~ symbolically. • Polynomial expressions can be used to model real-~~life~~ world situations. • The distributive property is the unifying concept for polynomial	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify the base, exponent, and coefficient in a monomial expression.~~ • Simplify monomial expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents. • ~~Express numbers, using scientific notation, and perform operations, using the laws of exponents.~~ • Model sums, differences, products, and quotients of polynomials with concrete objects and their related pictorial representations. • Relate concrete and pictorial ~~representations for~~ manipulations that model polynomial operations to their corresponding ~~algebraic manipulations~~ symbolic representations. • Find sums and differences of polynomials. • Find products of polynomials. The factors will have no more than five total terms. (i.e. (4x+2)(3x+5) represents four terms and

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.2

The student will perform operations on polynomials, including

a) applying the laws of exponents to perform operations on expressions;
b) adding, subtracting, multiplying, and dividing polynomials; and
c) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
operations. • Factoring reverses polynomial multiplication. • Some polynomials are prime polynomials and cannot be factored over the set of real numbers. • Polynomial expressions ~~in a variable *x* and their factors~~ can be used to define functions ~~by setting *y* equal to the polynomial expression or *y* equal to a factor,~~ and these functions can be represented graphically. • There is a relationship between the factors of ~~a~~ any polynomial and the *x*-intercepts of ~~its related~~ the graph of its related function.	$(x+1)(2x^2+x+3)$ represents five terms) • ~~Multiply polynomials by monomials and binomials by binomials symbolically.~~ • Find the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor. • ~~Use the distributive property to "factor out" all common monomial factors.~~ • Factor completely first- and second-degree polynomials ~~and binomials~~ with integral coefficients ~~and a positive leading coefficient less than four~~. • Identify prime polynomials ~~that cannot be factored over the set of real numbers~~. • Use the *x*-intercepts from the graphical representation of the polynomial to determine and confirm its factors.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.3

The student will express the square roots and cube roots of whole numbers and the square root of a monomial algebraic expression in simplest radical form.

ESSENTIAL UNDERSTANDINGS

- ~~The square root of a perfect square is an integer.~~
- ~~The square root of a non-perfect square lies between two consecutive integers.~~
- ~~The inverse of squaring a number is determining the square root.~~
- A ~~radical~~ square root in simplest form is one in which the radicand (argument) has no perfect square factors other than one.
- A cube root in simplest form is one in which the argument has no perfect cube factors other than one.
- ~~The square root of a product is the product of the square roots.~~
- The cube root of a perfect cube is an integer.
- The cube root of a nonperfect cube lies between two consecutive integers.
- The inverse of cubing a number is determining the cube root.
- In the real number system, the argument of a square root must be nonnegative while the argument of a cube root may be any real number.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- ~~Estimate the square root of a non-perfect square to the nearest tenth by~~
  - ~~identifying the two perfect squares it lies between;~~
  - ~~finding the square root of those two perfect squares; and~~
  - ~~using those values to estimate the square root of the non-perfect square.~~
- ~~Find the square root of a number, and make a reasonable interpretation of the displayed value for a given situation, using a calculator.~~
- Express ~~the~~ square roots of a whole number ~~less than 1,000~~ in simplest ~~radical~~ form.
- Express the cube root of a whole number in simplest form.
- Express the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values.

ALGEBRA I
STANDARD A.4

The student will solve multistep linear and quadratic equations in two variables, including

a) solving literal equations (formulas) for a given variable;
b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
c) solving quadratic equations algebraically and graphically;
d) solving multistep linear equations algebraically and graphically;
e) solving systems of two linear equations in two variables algebraically and graphically; and
f) solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

ESSENTIAL UNDERSTANDINGS

- A solution to an equation is the value or set of values that can be substituted to make the equation true.
- ~~Equations can be solved in a variety of ways.~~
- The solution of an equation in one variable can be found by graphing the expression on each ~~side~~side ~~member~~ of the equation separately and finding the *x*-coordinate of the point of intersection.
- ~~Practical~~ Real-world problems can be interpreted, represented, and solved using linear and quadratic equations ~~and inequalities in one variable~~.
- The ~~representation and manipulation~~ process of solving ~~expressions,~~ linear and quadratic equations~~, and inequalities~~ can be modeled in a variety of ways, using concrete, pictorial, and symbolic representations.
- Properties of real numbers and properties of equality can be used

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- ~~Translate verbal sentences to algebraic equations in one variable.~~
- Solve a literal equation (formula) for a specified variable.
- Simplify expressions and solve equations, using the ~~commutative, associative, and distributive~~ field properties of the real numbers and properties of equality to justify simplification and solution.
- Solve quadratic equations. ~~algebraically or by using the graphing calculator~~. ~~When solutions are represented in radical form, the decimal approximation will also be given.~~
- Identify the ~~x-intercepts~~ roots or zeros of ~~the~~ a quadratic function over the real number system as the solution(s) to the quadratic equation that is formed by setting the given quadratic expression equal to zero.
- Solve multistep linear equations in one variable. ~~in one variable~~

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.4

The student will solve multistep linear and quadratic equations in two variables, including

a) solving literal equations (formulas) for a given variable;
b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
c) solving quadratic equations algebraically and graphically;
d) solving multistep linear equations algebraically and graphically;
e) solving systems of two linear equations in two variables algebraically and graphically; and
f) solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
to justify equation solutions and expression simplification. • The zeros or the *x*-intercepts of the quadratic function are the real root(s) or solution(s) of the quadratic equation that is formed by setting the given quadratic expression equal to zero. • ~~Justifications will include the use of concrete objects; pictorial representations.~~ • A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations. • ~~A point shared by two intersecting graphs and the ordered pair that satisfies the equations characterize a system of equations with only one solution.~~ • A system of two linear equations with no solution is characterized by the graphs of two lines that are parallel.	~~with the variable in both sides of the equation or inequality.~~ • ~~Solve multistep linear equations in one variable with grouping symbols in one or both sides of the equation or inequality.~~ • ~~Solve multistep equations in one variable with rational coefficients and constants.~~ • ~~Apply skills for solving linear equations to practical situations.~~ • Confirm algebraic solutions to linear and quadratic equations, using a graphing calculator. • ~~Simplify expressions and solve equations and inequalities, using the order of operations.~~ • Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to find the ordered pair which satisfies both equations. • Given a system of two linear equations in two variables that has a

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.4

The student will solve multistep linear and quadratic equations in two variables, including

a) solving literal equations (formulas) for a given variable;
b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
c) solving quadratic equations algebraically and graphically;
d) solving multistep linear equations algebraically and graphically;
e) solving systems of two linear equations in two variables algebraically and graphically; and
f) solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

ESSENTIAL UNDERSTANDINGS

- A system of two linear equations having infinite solutions is characterized by two graphs that coincide (the graphs will appear to be the graph of one line), and ~~all~~ the coordinates of all points on ~~this one~~ the line satisfy both equations.
- Systems of two linear equations can be used to model two real-world conditions that must be satisfied simultaneously.
- ~~Quadratic equations can be solved in a variety of ways.~~
- ~~A quadratic equation can have two solutions, one solution, or no solution.~~
- ~~A solution to a quadratic equation is the value or set of values that can be substituted to make the equation true.~~
- Equations and systems of equations can be used as mathematical models for real-world situations.
- Set builder notation may be used to represent solution sets of

ESSENTIAL KNOWLEDGE AND SKILLS

unique solution, solve the system graphically ~~to find~~ by identifying the point of intersection.

- Determine whether a system of two linear equations has one solution, no solution, or infinite solutions.
- Write a system of two linear equations that ~~describes~~ models a ~~practical~~ real-world situation.
- Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that ~~describes~~ models a ~~practical~~ real-world situation.
- ~~Verify algebraic solutions, using the graphing calculator.~~

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.4

The student will solve multistep linear and quadratic equations in two variables, including

- **a) solving literal equations (formulas) for a given variable;**
- **b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;**
- **c) solving quadratic equations algebraically and graphically;**
- **d) solving multistep linear equations algebraically and graphically;**
- **e) solving systems of two linear equations in two variables algebraically and graphically; and**
- **f) solving real-world problems involving equations and systems of equations.**

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
equations~~ and inequalities~~.	

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.5

The student will solve multistep linear inequalities in two variables, including

- **a) solving multistep linear inequalities algebraically and graphically;**
- **b) justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets;**
- **c) solving real-world problems involving inequalities; and**
- **d) solving systems of inequalities.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A solution to an inequality is the value or set of values that can be substituted to make the inequality true. • ~~Inequalities can be solved in a variety of ways.~~ • ~~Practical~~ Real-world problems can be ~~interpreted, represented,~~ modeled and solved using linear inequalities ~~in one variable~~. • Properties of inequality and order can be used to solve inequalities. • Set builder notation may be used to represent solution sets of inequalities.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Translate verbal sentences to algebraic inequalities in one variable.~~ • Solve multistep ~~noncompound~~ linear inequalities ~~in one variable~~ in one variable~~with the variable in both sides of the equation or inequality~~. • ~~Solve multistep linear inequalities in one variable with grouping symbols in one or both sides of the inequality.~~ • ~~Solve multistep inequalities in one variable with rational coefficients and constants.~~ • ~~Confirm algebraic solutions to linear inequalities, using a graphing calculator.~~ • Justify steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.5

The student will solve multistep linear inequalities in two variables, including

a) solving multistep linear inequalities algebraically and graphically;
b) justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets;
c) solving real-world problems involving inequalities; and
d) solving systems of inequalities.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
	• ~~Simplify expressions and solve inequalities, using the commutative, associative, and distributive properties.~~ • ~~Simplify expressions and solve inequalities, using the order of operations.~~ • ~~Create and interpret pictorial representations for simplifying expressions and solving equations and inequalities.~~ • Solve systems of linear inequalities algebraically and graphically.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.6

The student will graph linear equations and linear inequalities in two variables, including

- **a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and**
- **b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.**

ESSENTIAL UNDERSTANDINGS

- ~~Linear functions and inequalities can be written in a variety of forms.~~
- ~~Linear functions and inequalities can be graphed, using a variety of techniques.~~
- ~~An appropriate technique for graphing linear functions and inequalities can be determined by the given information and/or the tools available.~~
- Changes in slope may be described by dilations or reflections or both.
- Changes in the y-intercept may be described by translations.
- ~~Justification of an appropriate technique for graphing linear equations and inequalities is dependent upon the application of slope, *x*- and *y*-intercepts, and graphing by transformations.~~
- Linear equations can be graphed using slope, x- and y-intercepts, and/or transformations of the parent function.
- The slope of a ~~linear function~~ line represents a constant rate of change in the dependent variable when the independent variable

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Graph linear equations and inequalities in two variables, including those that arise from a variety of ~~practical~~ real-world situations.
- Use the ~~line~~ parent function $y = x$ ~~as a reference,~~ and ~~apply~~ describe transformations defined by changes in the slope or *y*-intercept.
- ~~Express linear functions or inequalities in slope-intercept form, and use the graphing calculator to display the relationship.~~
- ~~Explain why a given technique is appropriate for graphing a linear function.~~
- ~~Recognize that *m* represents the slope in the equation of the form *y = mx + b*.~~
- Find the slope of the line, given the equation of a linear function.
- ~~Calculate~~ Find the slope of a line, given the coordinates of two points on the line.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.6

The student will graph linear equations and linear inequalities in two variables, including

a) **determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and**

b) **writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
changes by a ~~fixed~~ constant amount. • ~~The slope of a line determines its relative steepness~~. • ~~The slope of a line can be determined in a variety of ways.~~ • ~~Changes in slope affect the graph of a line.~~ • The equation of a line defines the relationship between two variables. • The graph of a line represents the set of points that satisfies the equation of a line. • A line can be represented by its graph or by an equation. • ~~The equation of a line can be determined by two points on the line or by the slope and a point on the line.~~ • The graph of the solutions of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the boundary are included unless it is a strict inequality. • Parallel lines have equal slopes. • The product of the slopes of perpendicular lines is -1 unless one	• Find the slope of a line, given the graph of a line. • Recognize and describe a line with a slope that is positive, negative, zero, or undefined. • ~~Compare the slopes of graphs of linear functions, using the graphing calculator.~~ • ~~Describe slope as a constant rate of change between two variables.~~ • Use transformational graphing to investigate effects of changes in equation parameters on the graph of the equation. • ~~Recognize that equations of the form $y = mx + b$ and $Ax + By = C$ are equations of lines.~~ • Write an equation of a line when given the graph of a line. • Write an equation of a line when given two points on the line whose coordinates are integers. • Write an equation of a line when given the slope and a point on the line whose coordinates are integers.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.6

The student will graph linear equations and linear inequalities in two variables, including

a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and

b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
of the lines has an undefined slope.	• Write an equation of a vertical line as x =a. • Write the equation of a horizontal line as $y = c$.

TOPIC: FUNCTIONS

ALGEBRA I
STANDARD A.7

The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including

a) **determining whether a relation is a function;**
b) **domain and range;**
c) **zeros of a function;**
d) ***x*- and *y*-intercepts;**
e) **finding the values of a function for elements in its domain; and**
f) **making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A set of data may be characterized by patterns, and those patterns can be represented in multiple ways. • Graphs can be used as visual representations to investigate relationships between quantitative data. • ~~Algebra is a tool for describing patterns, making generalizations, and representing a relationship in which output is related to input.~~ • Inductive reasoning may be used to make conjectures about characteristics of function families. • ~~A function is a relation for which there is a unique output for each input.~~ • ~~A relation can be represented by a set of ordered pairs.~~ • Each element in the domain of a relation is the abscissa of a point of the graph of the relation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Analyze a table of ordered pairs for the existence of a pattern that defines the change relating input and output values.~~ • ~~Write a linear equation to represent a pattern in which there is a constant rate of change between variables.~~ • Determine ~~from~~whether a relation, represented by a set of ordered pairs, a table, or a graph~~ whether a relation~~ is a function. • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. • ~~Identify the domain and range for a relation, given a set of ordered pairs, a table, or a graph.~~ • ~~Use physical representations, such as algebra manipulatives, to~~

TOPIC: FUNCTIONS

ALGEBRA I
STANDARD A.7

The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including

- **a) determining whether a relation is a function;**
- **b) domain and range;**
- **c) zeros of a function;**
- **d) *x*- and *y*-intercepts;**
- **e) finding the values of a function for elements in its domain; and**
- **f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Each element in the range of a relation is the ordinate of a point of the graph of the relation. • ~~The domain consists of the first coordinates of the ordered pairs.~~ • ~~The range consists of the second coordinates of the ordered pairs.~~ • A relation is a function if and only if each element in the domain is paired with a unique element of the range. • ~~An equation represents the relationship between the independent and dependent variables.~~ • The values of $f(x)$ are the ordinates of the points of the graph of f. • The object $f(x)$ is the unique object in the range of the function f that is associated with the object x in the domain of f. • For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair	~~represent quantitative data.~~ • For each x in the domain of f, find $f(x)$. • ~~Identify the zeros of the function algebraically and confirm them, using the graphing calculator~~. • Detect patterns in data and represent arithmetic and geometric patterns algebraically. • ~~Detect departures from patterns in data.~~

TOPIC: FUNCTIONS

ALGEBRA I
STANDARD A.7

The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including

- **a) determining whether a relation is a function;**
- **b) domain and range;**
- **c) zeros of a function;**
- **d) *x*- and *y*-intercepts;**
- **e) finding the values of a function for elements in its domain; and**
- **f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
$[x, f(x)]$ is a member of f. • An object x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$. • Set builder notation may be used to represent domain and range of a relation.	

TOPIC: FUNCTIONS

ALGEBRA I
STANDARD A.8

The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

ESSENTIAL UNDERSTANDINGS

- The constant of proportionality ~~*k*~~ in a direct variation is represented by the ratio of ~~*y* to *x*~~ the dependent variable to the independent variable ~~$(k = \frac{y}{x})$, where *y* is the dependent variable~~.
- The constant of proportionality in an inverse variation is represented by the product of the dependent variable and the independent variable.
- ~~Direct variation is used to represent a constant rate of change in practical applications.~~
- A direct variation can be represented by a line passing through the origin.
- ~~In direct variation, equal changes in *x* result in proportional changes in *y*.~~
- Real-world problems may be modeled using direct and/or inverse variations.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Given a situation, including a real-world situation, ~~table of values,~~ determine whether a direct variation exists.
- Given a situation, including a real-world situation, determine whether an inverse variation exists.
- Write an equation for a direct variation, given a set of data.
- Write an equation for an inverse variation, given a set of data.
- ~~Graph a direct variation from a table of values or a practical situation.~~
- Graph an equation representing a direct variation, given a set of data.

TOPIC: STATISTICS

ALGEBRA I
STANDARD A.9

The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.

ESSENTIAL UNDERSTANDINGS

- Descriptive statistics may include measures of center and ~~variation~~dispersion.
- Variance, standard deviation, and mean absolute deviation ~~may be used to~~ measure the ~~variability~~dispersion of the data.
- The sum of the deviations of data points from the mean of a data set is 0.
- Standard deviation is expressed in the original units of measurement of the data.
- Standard deviation addresses the dispersion of data about the mean.
- Standard deviation is calculated by taking the square root of the variance.
- The greater the value of the standard deviation, the further the data tend to be dispersed from the mean.
- For a data distribution with outliers, the mean absolute deviation may be a better measure of dispersion than the standard deviation or variance.
- A z-score (standard score) is a measure of position derived from the mean and standard deviation of data.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Analyze descriptive statistics to determine the implications for the real-world situations from which the data derive.
- Given data, including data in a real-world context, calculate and interpret the mean absolute deviation of a data set.
- Given data, including data in a real-world context, calculate variance and standard deviation of a data set and interpret the ~~variance and~~ standard deviation.
- Given data, including data in a real-world context, calculate and interpret z-scores for a data set~~the data~~.
- Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.
- Compare and contrast mean absolute deviation and standard deviation in a real-world context.

TOPIC: STATISTICS

ALGEBRA I
STANDARD A.9

The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A z-score derived from a particular data value tells how many standard deviations that data value is above or below the mean of the data set. It is positive if the data value lies above the mean, and negative if the data value lies below the mean.	

TOPIC: STATISTICS

ALGEBRA I
STANDARD A.10

The student will compare and contrast multiple univariate data sets, using box-and-whisker plots.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Statistical techniques can be used to organize, display, and compare sets of data. • Box-and-whisker plots can be used to analyze data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare, contrast, and analyze ~~two sets of~~ data, including data from real-world situations ~~using~~ displayed in box-and-whisker plots.

TOPIC: STATISTICS

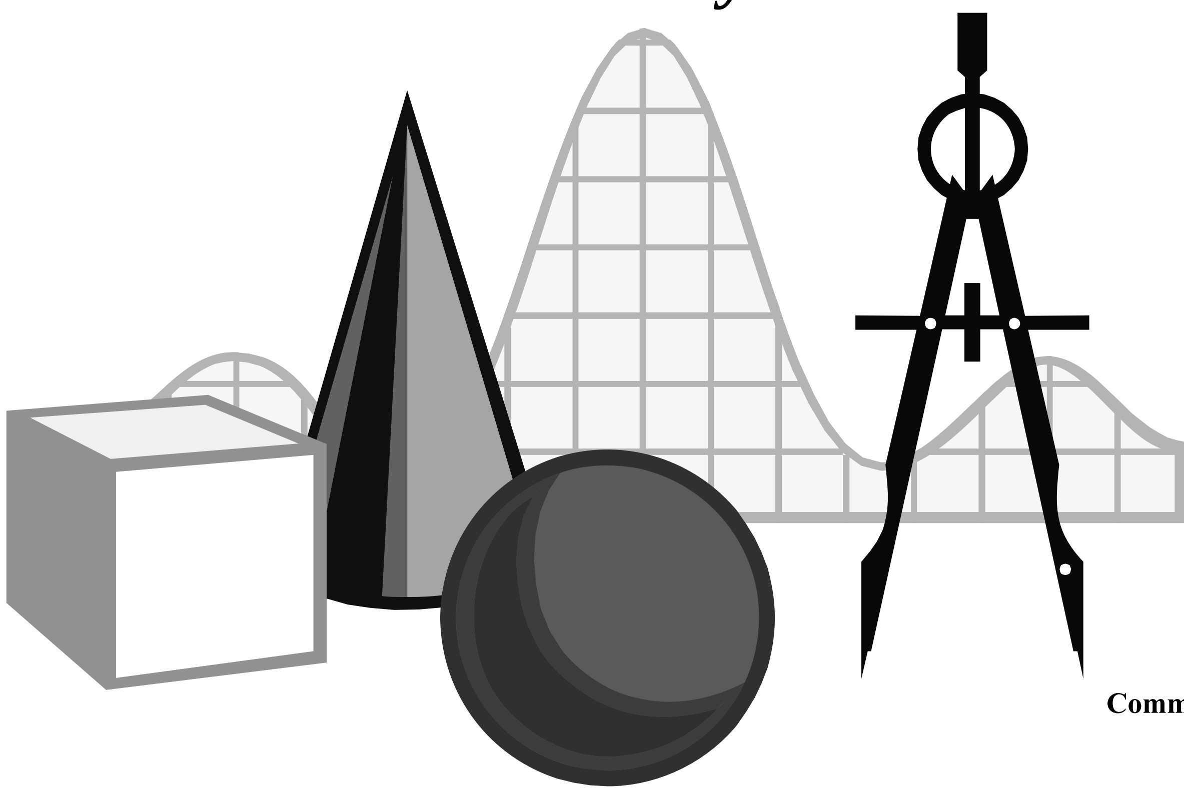
ALGEBRA I
STANDARD A.11

The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The graphing calculator can be used to determine the equation of a curve of best fit for a set of data. • The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. • Many problems can be solved by using a mathematical model as an interpretation of a real-world situation. The solution must then refer to the original real-world situation. • Considerations such as sample size, randomness, and bias should affect experimental design.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write an equation for ~~the line~~ a curve of best fit, given a set of ~~six to ten~~ no more than twenty data points in a table, ~~on~~ a graph, or ~~from~~ a ~~practical~~ real-world situation. • Make predictions about unknown outcomes, using the equation of the curve of best fit. • Design experiments and collect data to address specific, real-world questions. • Evaluate the ~~contextual validity~~reasonableness of a mathematical model of a real-world situation.

MATHEMATICS STANDARDS OF LEARNING PROPOSED CURRICULUM FRAMEWORK

Geometry



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills~~ and Essential Understandings~~. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.1

The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

a) identifying the converse, inverse, and contrapositive of a conditional statement;
b) translating a short verbal argument into symbolic form;
c) using Venn diagrams to represent set relationships; and
d) using deductive reasoning.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Reasoning~~ Inductive reasoning, deductive reasoning, and proof are critical in establishing general claims. • Deductive reasoning is the method that uses logic to draw conclusions based on definitions, postulates, and theorems. • Inductive reasoning is the method of drawing conclusions from a limited set of observations. • Proof is a justification that is logically valid and based on initial assumptions, definitions, postulates, and theorems. • Logical arguments consist of a set of premises or hypotheses and a conclusion. • Euclidean geometry is an axiomatic system based on undefined terms (point, line and plane), postulates, and theorems. • When a conditional and its converse are true, the statements can be written as a biconditional, i.e., *iff* or *if and only if*. • Logical arguments that are valid may not be true. Truth and	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the converse, inverse, and contrapositive of a conditional statement. • Translate ~~short~~ verbal arguments into symbolic form, such as $(p \rightarrow q)$ and $(\sim p \rightarrow \sim q)$. • Determine the validity of a logical argument. • Use valid forms of deductive reasoning, including the law of syllogism, the law of the contrapositive, the law of detachment, and counterexamples. • Select and use various types of reasoning and methods of proof, as appropriate. • ~~Use and interpret Venn diagrams.~~ • Use Venn diagrams to represent set relationships, such as intersection~~,~~ and union.

TOPIC: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.1

The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

a) identifying the converse, inverse, and contrapositive of a conditional statement;
b) translating a short verbal argument into symbolic form;
c) using Venn diagrams to represent set relationships; and
d) using deductive reasoning.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
validity are not synonymous.	• Interpret Venn diagrams. • Recognize and use the symbols of formal logic, which include $\rightarrow$, $\leftrightarrow$, $\sim$, $\therefore$, $\wedge$, and $\vee$.

TOPIC: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.2

The student will use the relationships between angles formed by two lines cut by a transversal to

a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Parallel lines ~~cut~~ intersected by a transversal form angles with specific relationships. • ~~A proof is a chain of logical statements starting with given information and leading to a conclusion.~~ • Some angle relationships may be used ~~to prove~~ when proving ~~that~~ two lines ~~cut~~ intersected by a transversal are parallel. • The Parallel Postulate differentiates Euclidean from non-Euclidean geometries such as spherical geometry and hyperbolic geometry.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Use properties, postulates, and theorems to determine whether two lines are parallel.~~ • Use algebraic~~,~~ and coordinate methods as well as deductive ~~methods~~ proofs to ~~determine~~ verify whether two lines are parallel. • Solve problems by using the relationships between pairs of angles formed by the intersection of two parallel lines and a transversal including corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles. • Solve ~~practical~~ real-world problems involving intersecting and parallel lines in a plane.

TOPIC: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.3

The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include

a) investigating and using formulas for finding distance, midpoint, and slope;
b) applying slope to verify and determine whether lines are parallel or perpendicular;
c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Transformations and combinations of transformations can be used to describe movement of objects in a plane. • The distance formula is an application of the Pythagorean Theorem. • Geometric figures can be represented in the coordinate plane. • Techniques for investigating symmetry may include paper folding, coordinate methods, and dynamic geometry software. • Parallel lines have the same slope. • <u>The product of the slopes of perpendicular lines is -1.</u> • <u>The image of an object or function graph after an isomorphic transformation is congruent to the preimage of the object.</u>	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the coordinates of the midpoint of a segment, using the midpoint formula. • <u>Compare the slopes to determine whether two lines are parallel, perpendicular, or neither.</u> • Determine whether a figure has point symmetry, line symmetry, <u>both,</u> or neither. • Given an image and preimage, identify the transformation that has taken place as a reflection, rotation, <u>dilation</u>, or translation. • Apply the distance formula to find the length of a line segment when given the coordinates of the endpoints.

TOPIC: REASONING, LINES, AND TRANSFORMATIONS

GEOMETRY
STANDARD G.4

The student will construct and justify the constructions of

- **a) a line segment congruent to a given line segment;**
- **b) the perpendicular bisector of a line segment;**
- **c) a perpendicular to a given line from a point not on the line;**
- **d) a perpendicular to a given line at a given point on the line;**
- **e) the bisector of a given angle;**
- **f) an angle congruent to a given angle; and**
- **g) a line parallel to a given line through a point not on the given line.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Construction techniques are used to solve ~~real-life~~ real-world problems in engineering, architectural design, and building construction. • Construction techniques may include using a straightedge and compass, paper folding, and dynamic geometry software.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct and justify the constructions of – a line segment congruent to a given line segment; – the perpendicular bisector of a line segment; – a perpendicular to a given line from a point not on the line; – a perpendicular to a given line at a point on the line; – the bisector of a given angle; – an angle congruent to a given angle; and – a line parallel to a given line through a point not on the given line.

TOPIC: TRIANGLES

GEOMETRY
STANDARD G.5

The student, given information concerning the lengths of sides and/or measures of angles in triangles, will

a) order the sides by length, given the angle measures;
b) order the angles by degree measure, given the side lengths;
c) determine whether a triangle exists; and
d) determine the range in which the length of the third side must lie.

These concepts will be considered in the context of real-world situations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The longest side of a triangle is opposite the largest angle of the triangle and the shortest side is opposite the smallest angle. • In a triangle, ~~the measure of an angle and the lengths of the adjacent sides~~ the length of two sides and the included angle determine the length of the side opposite the angle. • In order for a triangle to exist, the length of each side must be within a range that is determined by the lengths of the other two sides.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Arrange the sides of a triangle in order from smallest to largest when given the measures of the angles.~~ Order the sides of a triangle by their lengths when given the measures of the angles. • ~~Arrange the angles of a triangle in order from smallest to largest when given the lengths of the sides.~~ Order the angles of a triangle by their measures when given the lengths of the sides. • Given the lengths of three segments, determine whether a triangle could be formed. • Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie. • Solve real-world problems given information about the lengths of sides and/or measures of angles in triangles.

TOPIC: TRIANGLES

GEOMETRY
STANDARD G.6

The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Congruence has real-life applications in a variety of areas, including art, architecture, and the sciences. • Congruence does not depend on the position of the triangle. • Concepts of logic can demonstrate congruence or similarity. • Congruent figures are also similar, but similar figures are not necessarily congruent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use definitions, postulates, and theorems to ~~determine~~ prove ~~whether~~ triangles ~~are~~ congruent. • Use coordinate methods, such as the distance formula and the slope formula, to prove two triangles are congruent. • Use algebraic methods to prove two triangles are congruent.

TOPIC: TRIANGLES

GEOMETRY
STANDARD G.7

The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Similarity has real ~~life~~ world applications in a variety of areas, including art, architecture, and the sciences. • Similarity does not depend on the position of the triangle. • Congruent figures are also similar, but similar figures are not necessarily congruent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use definitions, postulates, and theorems to ~~determine~~ prove ~~whether~~ triangles ~~are~~ similar. • Use algebraic methods, such as properties of proportions, to prove that triangles are similar. • Use coordinate methods, such as the distance formula, to prove two triangles are similar.

TOPIC: TRIANGLES

GEOMETRY
STANDARD G.8

The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The Pythagorean Theorem is essential for solving problems involving right triangles. • <u>Many historical and algebraic proofs of the Pythagorean Theorem exist.</u> • The relationships between the sides and angles of right triangles are useful in many applied fields. • Some practical problems can be solved by choosing an efficient representation of the problem. • <u>Another formula for the area of a triangle is $A = \frac{1}{2}ab\sin C$.</u> • <u>The ratios of side lengths in similar right triangles (adjacent/hypotenuse or opposite/hypotenuse) are independent of the scale factor and depend only on the angle the hypotenuse makes with the adjacent side, thus justifying the definition and calculation of trigonometric functions using the ratios of side lengths for similar right triangles.</u>	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether a triangle formed with three given lengths is a right triangle. • Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles. • Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles. • Solve problems involving right triangles, using sine, cosine, and tangent ratios. • Solve <u>real-world</u> problems, using right triangle trigonometry and properties of right triangles.

TOPIC: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.9

The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The terms characteristics and properties can be used interchangeably to describe quadrilaterals. The term characteristics is used in elementary and middle school mathematics. • Quadrilaterals have a hierarchical nature based on the relationships between their sides, angles, and diagonals. • ~~Properties~~ Characteristics of quadrilaterals can be used to identify the quadrilateral and to find the measures of sides and angles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve ~~practical~~ problems, including real-world problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids. • Prove that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the distance formula, slope, and midpoint formula. • Prove the ~~properties~~ characteristics of quadrilaterals, using deductive reasoning, algebraic, and coordinate methods.

TOPIC: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.10

The student will solve real-world problems involving angles of polygons.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A regular polygon will tessellate the plane if the measure of an interior angle is a factor of 360. • Both regular and nonregular polygons can tessellate the plane. • Two intersecting lines form angles with specific relationships. • An exterior angle is formed by extending a side of a polygon. • The exterior angle and the corresponding interior angle form a linear pair. • The sum of the measures of the interior angles of a convex polygon may be found by dividing the interior of the polygon into nonoverlapping triangles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve <u>real-world</u> problems involving the measures of interior and exterior angles of polygons. • Identify tessellations in art, construction, and nature. • Find the sum of the measures of the interior and exterior angles of a convex polygon. • Find the measure of each interior and exterior angle of a regular polygon. • <u>Find the number of sides of a regular polygon, given the measures of interior or exterior angles of the polygon.</u>

TOPIC: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.11

The student will use angles, arcs, chords, tangents, and secants to

a) investigate, verify, and apply properties of circles;
b) solve real-world problems involving properties of circles; and
c) find arc lengths and areas of sectors in circles.

ESSENTIAL UNDERSTANDINGS

- Many relationships exist between and among angles, arcs, secants, chords, and tangents of a circle.
- All circles are similar.
- A chord is part of a secant.
- Real-world applications may be drawn from architecture, art, and construction.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Find lengths, angle measures, and arc measures associated with
  - two intersecting chords;
  - two intersecting secants;
  - an intersecting secant and tangent;
  - two intersecting tangents; and
  - central and inscribed angles.
- Calculate the area of a sector and the length of an arc of a circle, using proportions.
- Solve real-world problems associated with circles, using properties of angles, lines, and arcs.
- ~~Prove~~Verify properties of circles, using deductive reasoning, algebraic, and coordinate methods.

TOPIC: POLYGONS AND CIRCLES

GEOMETRY
STANDARD G.12

The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

ESSENTIAL UNDERSTANDINGS

- A circle is a locus of points equidistant from a given point, the center.
- Standard form for the equation of a circle is $(x-h)^2+(y-k)^2=r^2$, ~~(x – h)2 + (y – k)2 = r 2~~ where the coordinates of the center of the circle are (h, k) and r is the length of the radius.
- ~~The coordinates of the center of the circle are (h, k) and r is the length of the radius.~~
- The circle is a conic section.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify the center, radius, and diameter of a circle from a given standard equation.
- Use the distance formula to find the radius of a circle.
- Given the coordinates of the center and radius of the circle, identify a point on the circle.
- Given the equation of a circle in standard form, identify the coordinates of the center and find the radius of the circle.
- Given the coordinates of the endpoints of a diameter, find the equation of the circle.
- Given the coordinates of the center and a point on the circle, find the equation of the circle.

TOPIC: THREE-DIMENSIONAL FIGURES

GEOMETRY
STANDARD G.13

The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The surface area of a three-dimensional object is the sum of the areas of all its faces. • The volume of a three-dimensional object is the number of unit cubes that would fill the object.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the total surface area of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas. • Calculate the volume of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas. • Solve ~~practical~~ problems, including real-world problems, involving total surface area and volume of cylinders, prisms, pyramids, cones, and spheres as well as combinations of three-dimensional figures. • Calculators may be used to find decimal approximations for results.

TOPIC: THREE-DIMENSIONAL FIGURES

GEOMETRY
STANDARD G.14

The student will use similar geometric objects in two- or three-dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

ESSENTIAL UNDERSTANDINGS

- A change in one dimension of an object results in predictable changes in area and/or volume.
- A constant ratio exists between corresponding lengths of sides of similar figures.
- Proportional reasoning is integral to comparing attribute measures in similar objects.

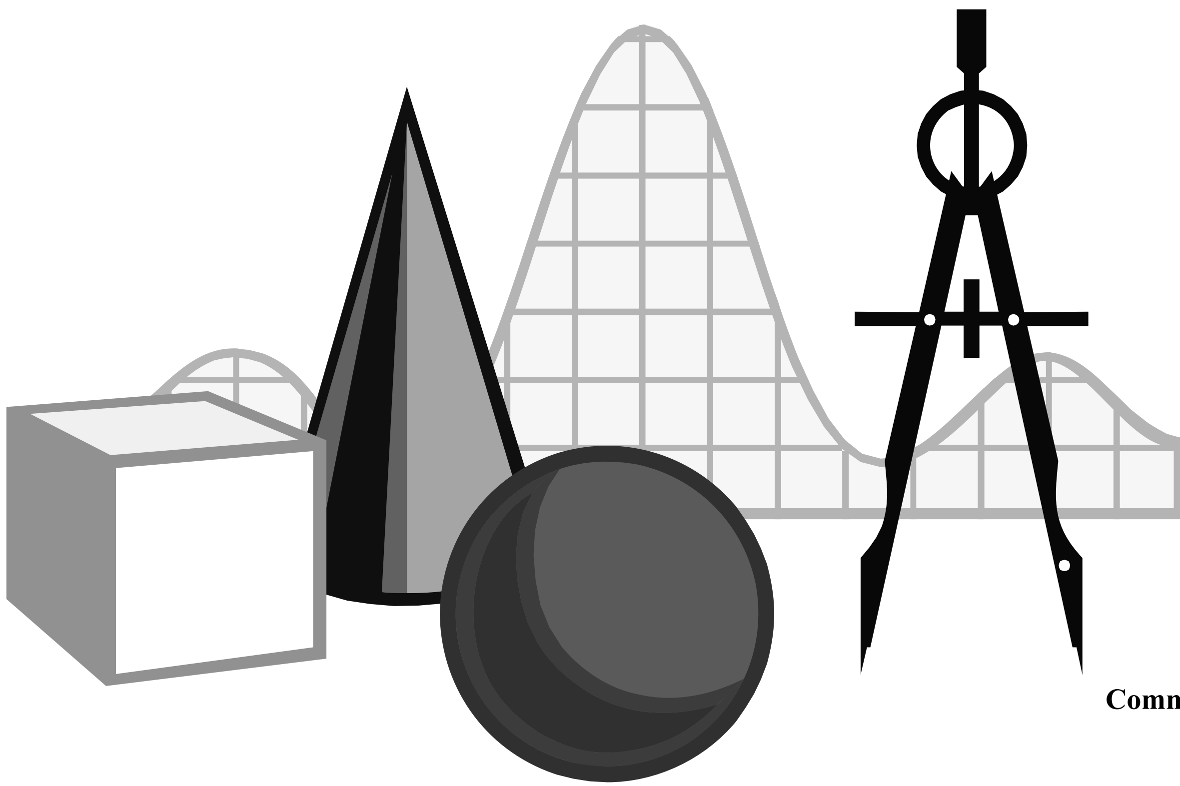
ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- ~~Compare perimeters and areas of similar two-dimensional figures, using proportions.~~
- Describe how ~~a~~ changes in one or more ~~measures~~ dimensions affect~~s~~ other derived measures (perimeter, area, total surface area, and volume) of an object.
- Describe how changes in one or more measures (perimeter, area, total surface area, and volume) affect other measures of an object.
- Solve ~~practical~~ real-world problems involving measured attributes of similar objects.

MATHEMATICS STANDARDS OF LEARNING PROPOSED CURRICULUM FRAMEWORK

Algebra, Functions, and Data Analysis



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills~~ and Essential Understandings~~. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1

The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:

- **a) continuity;**
- **b) local and absolute maxima and minima;**
- **c) domain and range;**
- **d) zeros;**
- **e) intercepts;**
- **f) intervals in which the function is increasing/decreasing;**
- **g) end behaviors; and**
- **h) asymptotes.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The domain of a function consists of the first coordinates of the ordered pairs that are elements of a function. Each element in the domain is an input into the independent variable of the function. • The range of a function consists of the second coordinates of the ordered pairs that are elements of a function. Each element in the range is an output in the dependent variable of a function. • For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $[x, f(x)]$ is a member of f. • A value x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$. • Functions describe the relationship between two variables where each input is paired to a unique output.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the domain and range for a relation, given a set of ordered pairs, a table, or a graph. • For each x in the domain of f, find $f(x)$. • Identify the zeros of the function algebraically and confirm them, using the graphing calculator. • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. • Recognize restricted/discontinuous domains and ranges. • Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
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- **c) domain and range;**
- **d) zeros;**
- **e) intercepts;**
- **f) intervals in which the function is increasing/decreasing;**
- **g) end behaviors; and**
- **h) asymptotes.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Functions are used to model real-world phenomena. • A function is increasing on an interval if its graph, as read from left to right, is rising in that interval. • A function is decreasing on an interval if its graph, as read from left to right, is going down in that interval. • Exponential and logarithmic functions are either strictly increasing or strictly decreasing. • A function is continuous on an interval if the function is defined for every value in the interval and there are no breaks in the graph. A continuous function can be drawn without lifting the pencil. • A turning point is a point on a continuous interval where the graph changes from increasing to decreasing or from decreasing to increasing.	• Identify *x*-intercepts (zeros), *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, points of discontinuity, end behavior, and maximum and minimum points, given a graph of a function. • Describe continuity of a function on its domain or at a point. • Express intervals using correct interval notation and/or a compound inequality.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1

The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:

- **a) continuity;**
- **b) local and absolute maxima and minima;**
- **c) domain and range;**
- **d) zeros;**
- **e) intercepts;**
- **f) intervals in which the function is increasing/decreasing;**
- **g) end behaviors; and**
- **h) asymptotes.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A function, f, has a local maximum in some interval at $x = a$ if $f(a)$ is the largest value of f in that interval. • A function, f, has a local minimum in some interval at $x = a$ if $f(a)$ is the smallest value of f in that interval. • Asymptotes can be used to describe local behavior and end behavior of graphs. They are lines or other curves that approximate the graphical behavior of a function. • The following statements are equivalent: – k is a zero of the polynomial function f; – k is a solution of the polynomial equation $f(x) = 0$; – k is an x-intercept for the graph of the polynomial; and – $(x - k)$ is a factor of the polynomial. • Continuous and discontinuous functions can be identified by their equations or graphs. The end behavior of a function refers to the	

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.1

The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:

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- **c) domain and range;**
- **d) zeros;**
- **e) intercepts;**
- **f) intervals in which the function is increasing/decreasing;**
- **g) end behaviors; and**
- **h) asymptotes.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
graphical behavior of a function as x goes to positive and negative infinity.	

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.2

The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Knowledge of transformational graphing using parent functions can be used to generate a mathematical model from a scatter plot that approximates the data. • Transformations include: - Translations (horizontal and vertical shifting of a graph) - Reflections - Dilations (stretching and compressing graphs) and - Rotations • The equation of a line can be determined by two points on the line or by the slope and a point on the line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write an equation of a line when given the graph of a line. • Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions. • Write the equation of a linear, quadratic, exponential, or logarithmic function in *(h, k)* form given the graph of the parent function and transformation information. • Describe the transformation from the parent function given the equation written in *(h, k)* form or the graph of the function. • Given the equation of a function, recognize the parent function and transformation to graph the given function. • Recognize the vertex of a parabola given a quadratic equation in *(h, k)* form or graphed. • Describe the parent function represented by a scatter plot.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.3

The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The regression equation modeling a set of data points can be used to make predictions where appropriate. • Data and scatter plots may indicate patterns that can be modeled with a function. • Graphing calculators can be used to collect, organize, picture, and create an algebraic model of the data. • Data that fit linear, quadratic, exponential, and logarithmic models arise from practical situations. • Two variables may be strongly associated without a cause-and-effect relationship existing between them. • Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured). • Residual = Actual – Fitted • Least squares regression generates the equation of the line that minimizes the sum of the squared distances between the data points and the line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write an equation for the line of best fit, given a set of data points in a table, on a graph, or from a practical situation. • Make predictions about unknown outcomes, using the equation of a line of best fit. • Collect and analyze data to make decisions and justify conclusions. • Investigate scatter plots to determine if patterns exist, and identify the patterns. • Find an equation for the curve of best fit for data, using a graphing calculator. Models will include linear, quadratic, exponential, and logarithmic functions. • Make predictions, using data, scatter plots, or equation of curve of best fit. • Given a set of data, determine the model that would best describe the data. • Describe the errors inherent in extrapolation beyond the range of

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.3

The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A correlation coefficient measures the degree of association between two variables that are related linearly.	the data. • Estimate the correlation coefficient when given data and/or scatter plots.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.4

The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The most appropriate representation of a function depends on the questions to be answered and/or the analysis to be done. • Given data may be represented as discrete points or as a continuous graph with respect to the real-world context. • Real-world data may best be represented as a table, a graph, or as a formula.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given an equation, graph a linear, quadratic, exponential or logarithmic function with the aid of a graphing calculator. • Make predictions given a table of values, a graph, or an algebraic formula. • Describe relationships between data represented in a table, in a scatter plot, and as elements of a function. • Determine the appropriate representation of data derived from real-world situations. • Analyze and interpret the data in context of the real-world situation.

TOPIC: ALGEBRA AND FUNCTIONS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.5

The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Linear programming models an optimization process. • A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized. • Any maximum or minimum value will occur at a corner point of a feasible region.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model practical problems with systems of linear inequalities. • Solve systems of linear inequalities with pencil and paper and using a graphing calculator. • Solve systems of equations algebraically and graphically. • Identify the feasibility region of a system of linear inequalities. • Identify the coordinates of the corner points of a feasibility region. • Find the maximum or minimum value for the function defined over the feasibility region. • Describe the meaning of the maximum or minimum value within its context.

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS
STANDARD AFDA.6

The student will calculate probabilities. Key concepts include:

a) **conditional probability;**
b) **dependent and independent events;**
c) **addition and multiplication rules;**
d) **counting techniques (permutations and combinations); and**
e) **Law of Large Numbers.**

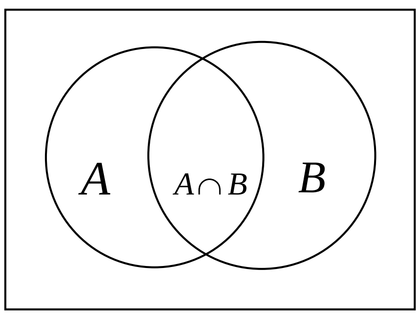
ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The *Fundamental Counting Principle* states that if one decision can be made *n* ways and another can be made *m* ways, then the two decisions can be made *nm* ways. • *Permutations* are used to calculate the number of possible arrangements of objects. • *Combinations* are used to calculate the number of possible selections of objects without regard to the order selected. • A *sample space* is the set of all possible outcomes of a random experiment. • An *event* is a subset of the sample space. • *P(E)* is a way to represent the probability that the event E occurs. • *Mutually exclusive events* are events that cannot both occur simultaneously. • If *A* and *B* are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast permutations and combinations. • Calculate the number of permutations of *n* objects taken *r* at a time. • Calculate the number of combinations of *n* objects taken *r* at a time. • Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. • Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive. • Find conditional probabilities for dependent, independent, and mutually exclusive events. • Represent and calculate probabilities using Venn diagrams and probability trees.

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.6

The student will calculate probabilities. Key concepts include:

a) conditional probability;
b) dependent and independent events;
c) addition and multiplication rules;
d) counting techniques (permutations and combinations); and
e) Law of Large Numbers.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The complement of event A consists of all outcomes in which event A does not occur. • *P(B|A)* is the probability that *B* will occur given that *A* has already occurred. *P(B|A)* is called *the conditional probability* of *B* given *A*. • Venn diagrams may be used to examine conditional probabilities.  $P(B\|A) = \frac{P(A \cap B)}{P(A)}$ $\Rightarrow P(A \cap B) = P(A)P(B\|A)$ • Two events, *A* and *B*, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If *A* and *B* are not independent, then they are said to be dependent.	• Analyze, interpret and make predictions based on theoretical probability within real-world context. • Given a real-world situation, determine when to use permutations or combinations.

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.6

The student will calculate probabilities. Key concepts include:

- **a) conditional probability;**
- **b) dependent and independent events;**
- **c) addition and multiplication rules;**
- **d) counting techniques (permutations and combinations); and**
- **e) Law of Large Numbers.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• If A and B are independent events, then $P(A \cap B) = P(A)P(B)$. • The Law of Large Numbers states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.	

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.7

The student will analyze the normal distribution. Key concepts include:

a) characteristics of normally distributed data;
b) percentiles;
c) normalizing data using z-scores; and
d) area under the standard normal curve and probability.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Analysis of the descriptive statistical information generated by a univariate data set includes the relationships between central tendency, dispersion, and position. • The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation. • Areas under the curve represent probabilities associated with continuous distributions. • The normal curve is a probability distribution and the total area under the curve is 1. • The mean of the data in a standard normal density function is 0 and the standard deviation is 1. This allows for the comparison of unlike data. • The amount of data that falls within 1, 2, or 3 standard deviations of the mean is constant and the basis of *z*-score data normalization.	• Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem's context. • Explain the influence of outliers on a univariate data set. • Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation. • Identify the properties of a normal probability distribution. • Describe how the standard deviation and the mean affect the graph of the normal distribution. • Determine the probability of a given event, using the normal distribution.

TOPIC: DATA ANALYSIS

ALGEBRA, FUNCTIONS AND DATA ANALYSIS STANDARD AFDA.8

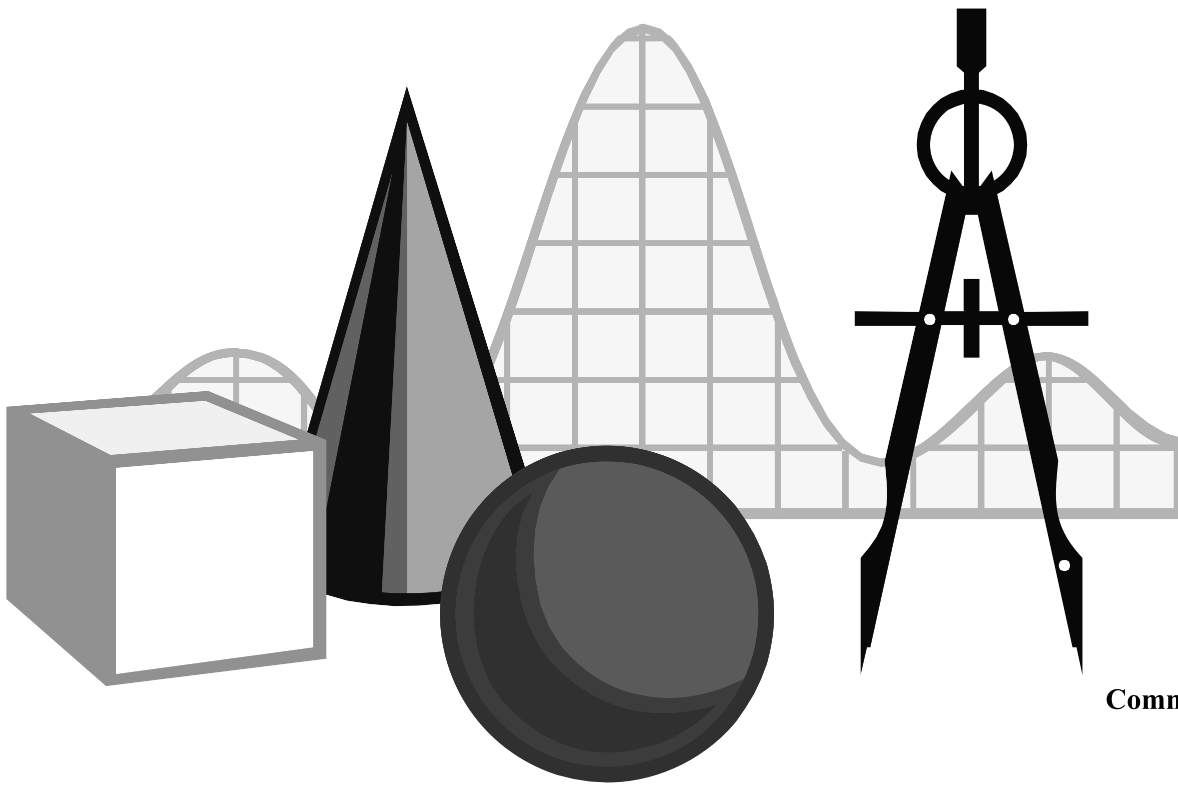
The student will design and conduct an experiment/survey. Key concepts include:

a) **sample size;**
b) **sampling technique;**
c) **controlling sources of bias and experimental error;**
d) **data collection; and**
e) **data analysis and reporting.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of a sample statistic may vary from sample to sample, even if the simple random samples are taken repeatedly from the population of interest. • Poor data collection can lead to misleading and meaningless conclusions. • The purpose of sampling is to provide sufficient information so that population characteristics may be inferred. • Inherent bias diminishes as sample size increases. • Experiments must be carefully designed in order to detect a cause-and-effect relationship between variables. • Principles of experimental design include comparison with a control group, randomization, and blindness.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast controlled experiments and observational studies and the conclusions one may draw from each. • Identify biased sampling methods. • Select a data collection method appropriate for a given context. • Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. • Determine which sampling technique is best, given a particular context. • Plan and conduct an experiment or survey. The experimental design should address control, randomization, and minimization of experimental error. • Design a survey instrument.

MATHEMATICS STANDARDS OF LEARNING
PROPOSED CURRICULUM FRAMEWORK

Computer Mathematics



Commonwealth of Virginia
Board of Education
Richmond, Virginia
Draft October 22, 2009

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Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills ~~and Essential Understandings~~. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: PROBLEM SOLVING

COMPUTER MATHEMATICS
STANDARD COM.1

The student will apply programming techniques and skills to solve practical real-world problems in mathematics arising from consumer, business, and other applications in mathematics. Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The computer is an essential tool for mathematical problem solving in consumer-related problems. • A subtask that has been solved previously may be used again and again. • Programming languages require the use of particular structures to express algorithms as programs. • Designing algorithms is the problem solving phase of computer programming. • Real-world problems that can be modeled mathematically can be solved with a computer program. • Data arising from probability and statistics applications can be displayed in tables and graphs and analyzed within the structure of a computer program.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design and implement computer programs to solve consumer problems. • Analyze and interpret graphs, charts, and tables in the design and implementation of a computer program. • Design and implement computer programs to – solve mathematical problems, using formulas; – solve mathematical problems, using equations; – solve mathematical problems, using functions; – solve problems related to geometry, business, and leisure; – solve probability problems; – solve data-analysis problems; and – solve statistical problems.

TOPIC: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.2

The student will design, write, test, debug, and document a program. Programming documentation will include preconditions and postconditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The successful completion of a structured program requires problem solving skills.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe what a computer program is. • List and describe the stages involved in writing a computer program. • Describe the function of an algorithm. • Describe the interplay between hardware and software in program execution. • Compare and contrast compiling and executing a program. • Determine what a given output statement will print. • Debug a program. • Provide required documentation for a program.

TOPIC: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.3

The student will write program specifications that define the constraints of a given problem. These specifications will include descriptions of preconditions, postconditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A programmer begins the programming process by analyzing the problem and developing a general solution (algorithm). • The successful completion of a structured program requires problem solving skills.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • For a given problem, describe the preconditions, postconditions, and desired output. • Determine whether or not a problem is solvable. • Write program specifications that define the constraints of a problem.

TOPIC: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.4

The student will design a step-by-step plan (algorithm) to solve a given problem. The plan will be in the form of a program flowchart, pseudo code, hierarchy chart, and/or data-flow diagram.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• All programs are implementations of algorithms.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design a step-by-step plan to solve a problem. • Utilize the following problem solving formats: – flowchart; – pseudo code; – hierarchy chart; and – data-flow diagram.

TOPIC: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.5

The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined function, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Functional decomposition is a way to develop a program in which the problem is divided into subproblems whose solutions comprise the solution to the original problem.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Subdivide a problem into modules by task. • Implement the solution of the problem. • Write task-oriented modules, including – a user-defined function; – subroutines; and – procedures. • Determine the need for a subroutine or user-defined function. • Determine the difference between and the need for internal and external subroutines and functions.

TOPIC: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.12

The student will translate a mathematical expression into a computer statement, which involves writing assignment statements and using the order of operations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A variable identifies a location in memory where a data value that can be changed is stored. • An assignment statement stores the value of an expression into a variable. • The order of operations is – parentheses; – exponents; – multiplication and division in order from left to right; and – addition and subtraction in order from left to right. • Variable assignment statements will differ depending upon the programming language used.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Translate a mathematical expression into a computer statement. • Use the order of operations to simplify expressions. • Write variable assignment statements. • Construct and evaluate expressions that include multiple arithmetic operations.

TOPIC: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.13

The student will select and implement built-in (library) functions in processing data.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The argument of a library function is a value or expression associated with the independent variable. • A library function is a subroutine.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use library functions in designing programs to process data. • Use library functions that are arithmetic or string operators. • Invoke a value-returning function.

TOPIC: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.14

The student will implement conditional statements that include "if/then" statements, "if/then/else" statements, case statements, and Boolean logic.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Boolean logic is a system using variables with only two values: TRUE and FALSE. • The "if" statement is the fundamental control structure that allows branches in the flow of control.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a simple logical (Boolean) expression to evaluate a given condition. • Construct an "if/then" statement to perform a specific task. • Construct an "if/then/else" statement to perform a specific task. • Use conditional statements to incorporate decision making into programs.

TOPIC: PROGRAM DESIGN

COMPUTER MATHEMATICS
STANDARD COM.17

The student will implement pre-existing algorithms, including sort routines, search routines, and simple animation routines.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Arranging values into an order is known as sorting. • A sequential search algorithm starts at the beginning of a list and examines each data value in sequence. • Implementation of animation routines will differ depending upon the programming language used.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Implement pre-existing algorithms into a program. • Implement a sort routine on a one-dimensional array. • Implement a sequential search routine on a one-dimensional array. • Implement a binary search routine on a one-dimensional array. • Implement a simple animation routine.

TOPIC: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.6

The student will design and implement the input phase of a program, which will include designing screen layout and getting information into the program by way of user interaction, data statements, and/or file input. The input phase will also include methods of filtering out invalid data (error trapping).

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A program needs data on which to operate. • A file is a named area in secondary storage that holds a collection of information.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design a screen layout to facilitate input. • Design program information input by – user interaction; – data statements (BASIC); and – file input. • Filter out invalid data, using a variety of methods (error trapping). • Construct input statements to read values into a program. • Determine the contents of variables that have been assigned values by input statements.

TOPIC: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.7

The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of output devices, using output statements, and labeling results.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Output is dependent on input. • Implementation of the output portion of a program includes designing the output and displaying it in the desired format.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design an output layout. • Access various output devices. • Use output statements. • Label results.

TOPIC: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.8

The student will design and implement computer graphics, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Work with computer graphics is specific to the computer operating system.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design computer graphics. • Implement computer graphics. • Plot points and areas. • Determine and set window or screen dimensions. • Determine and set screen and background colors. • Use box commands. • Describe the role of graphics in the computer environment.

TOPIC: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.15

The student will implement loops, including iterative loops. Other topics will include single entry point, single exit point, preconditions, and postconditions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A loop executes a sequence of statements repeatedly. • Nested loops contain other loops.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine when a loop is needed in a program. • Implement loops into programs. Include – iterative loops; – pretest loops; and – posttest loops. • Incorporate single entry point, single exit point, pre-conditions, and post-conditions into loops.

TOPIC: PROGRAM IMPLEMENTATION

COMPUTER MATHEMATICS
STANDARD COM.16

The student will select and implement appropriate data structures, including arrays (one-dimensional and/or multidimensional), files, and records. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Structured data types are collections of components that are given a single name and whose organization is characterized by the method used to access the individual components. • Multidimensional arrays may be viewed as arrays of one-dimensional arrays.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Declare a one-dimensional or two-dimensional array for a given problem. • Choose an appropriate component type for an array. • Assign a value to an array component. • Fill an array with data, and process the data in the array. • Access a particular component of a two-dimensional array. • Process a two-dimensional array by rows and by columns. • Retrieve data from an array.

TOPIC: DATA MANIPULATION

COMPUTER MATHEMATICS
STANDARD COM.9

The student will define simple variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean.

STANDARD COM.10

The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A data type is a set of values and a set of operations on the values. • Boolean data has only two literal constants, and they represent TRUE and FALSE. • A string is an array for which there exists an aggregate constant.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define and use variable data types, including – integer; – real (fixed and scientific notation); – character; – string; and – Boolean. • Write numeric and string variables, using valid names.

TOPIC: DATA MANIPULATION

COMPUTER MATHEMATICS
STANDARD COM.11

The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables' addresses, pointers, parameter passing, scope of variables, and local versus global variables.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Computers consist of hardware components that interact with software.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine when use of a variable is appropriate. • Describe how a computer stores, accesses, and processes variables. • Incorporate variable addresses, pointers, and parameter passing into programs. • Differentiate between local and global variables, and describe their appropriate use. • Compare and contrast variables and constants.

TOPIC: PROGRAM TESTING

COMPUTER MATHEMATICS
STANDARD COM.18

The student will test a program, using an appropriate set of data. The set of test data should be appropriate and complete for the type of program being tested.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A program test can reveal problems (bugs) in the program. • Testing a program for bugs is part of problem solving. • Various forms of data can be used to debug a program.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Test a program, using an appropriate and complete set of data.

TOPIC: PROGRAM TESTING

COMPUTER MATHEMATICS
STANDARD COM.19

The student will debug a program, using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference between syntax errors and logic errors.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Debugging a program is problem solving.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Debug a program, using controlled breaks, the printing of intermediate results, and other debugging tools. • Identify the differences among syntax errors, runtime errors, and logic errors.

TOPIC: PROGRAM TESTING

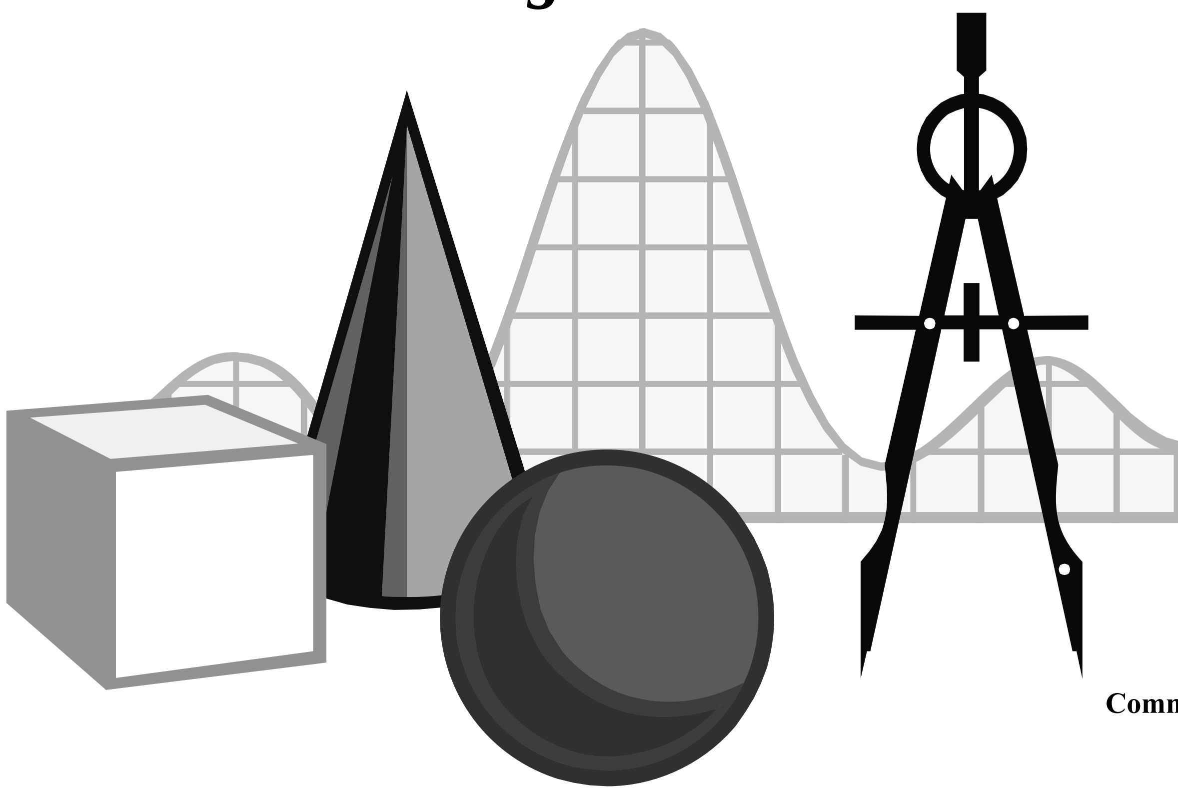
COMPUTER MATHEMATICS
STANDARD COM.20

The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The successful completion of a structured program requires problem solving skills.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design, write, test, debug, and document a complete structured program.

MATHEMATICS STANDARDS OF LEARNING PROPOSED CURRICULUM FRAMEWORK

Algebra II



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Board of Education
Richmond, Virginia
Draft October 22, 2009

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TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.1

The student, given rational, radical, or polynomial expressions, will

a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
c) write radical expressions as expressions containing rational exponents and vice versa; and
d) factor polynomials completely.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Computational skills applicable to numerical fractions also apply to rational expressions involving variables. • Radical expressions can be written and simplified using rational exponents. • Only radicals with a common radicand and index can be added or subtracted. • <u>A relationship exists among arithmetic complex fractions, algebraic complex fractions, and rational numbers.</u> • The complete factorization of polynomials has occurred when each factor is a prime polynomial. • Pattern recognition can be used to determine complete factorization of a polynomial.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Add, subtract, multiply, and divide rational <u>algebraic</u> expressions ~~whose denominators are monomials or polynomial expressions in completely factored form~~. • Simplify a rational <u>algebraic</u> expression with common monomial or binomial factors. • Recognize a complex <u>algebraic</u> fraction, and simplify it as a quotient or product of simple <u>algebraic</u> fractions. • Simplify radical expressions containing positive rational numbers and variables. • Convert from radical notation to exponential notation, and vice versa. • Add and subtract radical expressions ~~with like radicands~~. • Multiply and divide radical expressions not requiring rationalizing

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.1

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c) write radical expressions as expressions containing rational exponents and vice versa; and
d) factor polynomials completely.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
	the denominators. • ~~Determine the greatest monomial factor as a first step in complete factorization.~~ • ~~Recognize squares and cubes of positive integers.~~ • ~~Recognize examples of general patterns:~~ Factor polynomials by applying general patterns including difference of squares, sum and difference of cubes, and perfect square trinomials. • ~~Factor polynomials by applying general patterns.~~ • Factor polynomials completely over the integers

TOPIC: ~~RELATIONS AND~~ FUNCTIONS

ALGEBRA II
STANDARD AII.2

The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the n^{th} term, and evaluating summation formulas. Notation will include $\sum$ and a_n.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Sequences and series arise from real-world situations. • The study of sequences and series is an application of the investigation of patterns. • A sequence is a function whose domain is the set of natural numbers. • Sequences can be defined explicitly and recursively.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Distinguish between a sequence and a series. • ~~Recognize patterns in a sequence.~~ • Generalize patterns in a sequence using explicit and recursive formulas. • Use and interpret the notations $\sum$, n, n^{th} term, and a_n. • ~~Write the first *n* terms in an arithmetic or geometric sequence.~~ • Given the formula, find a_n (the n^{th} term) for an arithmetic or a geometric sequence. • Given formulas, find the sum, S_n, of the first n terms of an arithmetic or geometric series~~, including infinite series~~. • Given the formula, find the sum of a convergent infinite series. • ~~Use finite geometric series to m~~Model real-world situations using sequences and series.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.3

The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Complex numbers are organized into a hierarchy of subsets ~~with properties applicable to each subset~~. • ~~Complex numbers are a superset of real numbers and, as a system, contain solutions for equations that are not solvable over the set of real numbers.~~ • A complex number multiplied by its conjugate is a real number. • Equations having no real number solutions may have solutions in the set of complex numbers. • Field properties apply to complex numbers as well as real numbers. • All complex numbers can be written in the form a+b*i* where *a* and *b* are real numbers and *i* is $\sqrt{-1}$. • ~~Complex numbers are a superset of real numbers.~~	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Identify examples of field properties: commutative, associative, identity, inverse, and distributive.~~ • ~~Identify examples of axioms of equality: reflexive, symmetric, transitive, substitution, addition, and multiplication.~~ • ~~Identify examples of axioms of inequality and order: trichotomy, transitive, addition, and multiplication.~~ • Recognize that the square root of –1 is represented as *i*. • ~~Define and identify a complex number.~~ • ~~Apply the definition of *i* to simplify square roots of negative numbers.~~ • Determine which field properties apply to the complex number system. • Simplify radical expressions containing negative rational numbers and express in a+b*i* form. • Simplify powers of *i*.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.3

The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
	• Add, subtract, and multiply complex numbers. • Place the following sets of numbers in a hierarchy of subsets: complex, pure imaginary, real, rational, irrational, integers, whole, and natural. • Write a real number in a+b*i* form. • Write a pure imaginary number in a+b*i* form.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.4

The student will solve, algebraically and graphically,
- **a) absolute value equations and inequalities;**
- **b) quadratic equations over the set of complex numbers;**
- **c) equations containing rational algebraic expressions; and**
- **d) equations containing radical expressions.**

Graphing calculators will be used for solving and for confirming the algebraic solutions.

ESSENTIAL UNDERSTANDINGS

- ~~A solution of an equation makes the equation true.~~
- A quadratic function whose graph does not intersect the *x*-axis has roots with imaginary components.
- The quadratic formula can be used to solve any quadratic equation.
- The value of the discriminant of a quadratic equation can be used to describe the ~~nature~~number of ~~the roots~~real and complex solutions.
- ~~A quadratic equation whose graph does not intersect the *x*-axis has only complex solutions.~~
- ~~Complex solutions occur in pairs (conjugates).~~
- ~~Absolute value equations and inequalities can be used to model practical problems.~~
- The definition of absolute value (for any real numbers a and b, where b $\geq$ 0, if $|a| = b$, then a = b or ~~-a = b~~a = - b) is used in

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve absolute value equations and inequalities algebraically and graphically.
- ~~Solve absolute value equations in one variable algebraically and graphically, using a graphing calculator.~~
- ~~Solve absolute value inequalities in one variable algebraically and graphically.~~
- ~~Express the solutions to absolute value equations and inequalities in one variable graphically and as an algebraic inequality.~~
- ~~Graph absolute value equations in two variables.~~
- ~~Verify solutions to absolute value equations and inequalities in two variables, using a graphing calculator.~~
- ~~Recognize a quadratic equation~~.
- Solve a quadratic equation over the set of complex numbers using

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.4

The student will solve, algebraically and graphically,

a) absolute value equations and inequalities;
b) quadratic equations over the set of complex numbers;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
solving absolute value equations and inequalities. • Absolute value inequalities can be solved graphically or by using a compound statement. • Real-world problems can be interpreted, represented, and solved using equations and inequalities. • The process of solving radical or rational equations can lead to extraneous solutions. • Equations can be solved in a variety of ways. • ~~The solution of an equation in one variable can be found by graphing each side of the equation separately and finding the *x*-coordinate of the point of intersection~~. • ~~Practical problems can be interpreted, represented, and solved using equations.~~ • Set builder notation may be used to represent solution sets of equations and inequalities.	an appropriate strategy. • ~~Identify from a graph the real solutions to a quadratic equation.~~ • ~~Find the real roots of a quadratic equation, using a graphing calculator.~~ • Calculate the discriminant of a quadratic equation to determine the number of real and complex solutions. • Solve equations containing rational algebraic expressions with monomial or binomial denominators algebraically and graphically. • Solve an equation containing a radical expression algebraically and graphically. ~~The equation will contain a linear expression under the radical, and all terms outside the radical will be constants.~~ • Verify possible solutions to an equation containing rational or radical expressions. • Apply an appropriate equation to solve a real-world problem.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.4

The student will solve, algebraically and graphically,

a) absolute value equations and inequalities;
b) quadratic equations over the set of complex numbers;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
	• ~~Identify from a graph the solutions to an equation containing rational or radical expressions.~~ • ~~Solve an equation containing rational or radical expressions, using a graphing calculator.~~ • ~~Check possible solutions to an equation containing rational or radical expressions, using a graphing calculator.~~

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.5

The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Solutions of a nonlinear system of equations are numerical values that satisfy every equation in the system. • The coordinates of points of intersection in any system of equations are solutions to the system. • Real-world problems can be interpreted, represented, and solved using systems of equations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Predict the number of solutions to a nonlinear system of two equations. • ~~Identify nonlinear systems of equations as linear-quadratic or quadratic-quadratic.~~ • ~~Visualize a nonlinear system of two equations, and predict the number of solutions, using the graphing calculator.~~ • Solve a linear-quadratic system of two equations algebraically and graphically. • Solve a quadratic-quadratic system of two equations algebraically and graphically.

TOPIC: FUNCTIONS

ALGEBRA II
STANDARD AII.6

The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

ESSENTIAL UNDERSTANDINGS

- The graphs/equations for a family of functions can be determined using a transformational approach.
- Transformations of graphs include translations, reflections, and dilations.
- A parent graph is an anchor graph from which other graphs are derived with transformations.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Recognize graphs of parent functions.
- Given a transformation of a parent function, identify the graph of the transformed function.
- Given the equation and using a transformational approach, graph a function.
- Given the graph of a function, identify the parent function.
- Given the graph of a function, identify the transformations that map the preimage to the ~~pre~~image in order to determine the equation of the image.
- Using a transformational approach, write the equation of a function given its graph.
- ~~Recognize graphs of parent functions for linear, quadratic, absolute value, step, and exponential functions.~~
- ~~Given an equation of a function, identify the function as linear, quadratic, absolute value, step, or exponential.~~

TOPIC: FUNCTIONS

ALGEBRA II
STANDARD AII.6

The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
	• ~~Write the equation of a linear (slope-intercept form), quadratic ([*h*, *k*] form), absolute value, step, or exponential function, given the graph of the parent function or an integral translation of a parent function.~~ • ~~Given an equation, graph a linear, quadratic, absolute value, step, or exponential function with the aid of a graphing calculator.~~ • ~~Using the general shape of the graph of a function, identify the family of graphs to which a particular graph belongs. Characteristics of a graph may include the *x*- and *y*-intercepts, number and location of turning points, and end behaviors.~~

TOPIC: FUNCTIONS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze functions algebraically and graphically. Key concepts include

a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) *x*- and *y*-intercepts;
d) intervals in which a function is increasing or decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.

ESSENTIAL UNDERSTANDINGS

- Functions may be used to model real-world situations.
- The domain and range of a function may be restricted algebraically or by the real-world situation modeled by the function.
- A function can be described on an interval as increasing, decreasing, or constant.
- ~~Exponential and logarithmic functions are either strictly increasing or strictly decreasing.~~
- Asymptotes may describe both local and global behavior of functions.
- End behavior describes a function as *x* approaches positive and negative infinity.
- A zero of a function is a value of *x* that makes f(x) equal zero.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify the domain, range, zeros, and ~~inverse~~intercepts of a function presented algebraically or graphically.
- Describe restricted/discontinuous domains and ranges.
- ~~Distinguish between relations and functions that are expressed algebraically and graphically.~~
- ~~Use interchange of variables to find the inverse of a function~~.
- ~~Given the graphs, recognize that exponential and logarithmic functions are inverses of each other.~~
- Given the graph of a function, identify intervals on which the function is increasing and decreasing.

TOPIC: FUNCTIONS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze functions algebraically and graphically. Key concepts include

a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) *x*- and *y*-intercepts;
d) intervals in which a function is increasing or decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.

ESSENTIAL UNDERSTANDINGS

- ~~Functions describe the relationship between two variables.~~
- ~~Graphs of functions that are inverses of each other are reflections across the line *y* = *x*.~~
- ~~The composition of a function and its inverse is the identity function.~~
- ~~Functions arise from practical situations.~~
- If (*a*, *b*) is an element of a function, then (*b*, *a*) is an element of the inverse of the function.
- Exponential ($y = a^x$) and logarithmic ($y = \log_a x$) functions are inverses of each other.
- Functions can be combined using composition of functions.
- ~~Shapes and behavior of graphs of polynomials can be determined~~

ESSENTIAL KNOWLEDGE AND SKILLS

- Find the equations of vertical and horizontal asymptotes of functions.
- Describe the end behavior of a function.
- ~~Recognize~~
- Find the inverse of a function.
- ~~Find the value of a function for a given element from the domain.~~
- Graph the inverse of a function as a reflection across the line *y* = *x*.
- Investigate exponential and logarithmic functions, using the graphing calculator.
- Convert between logarithmic and exponential forms of an equation with bases consisting of natural numbers.

TOPIC: FUNCTIONS

ALGEBRA II
STANDARD AII.7

The student will investigate and analyze functions algebraically and graphically. Key concepts include

a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) *x*- and *y*-intercepts;
d) intervals in which a function is increasing or decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~by analyzing transformations of parent functions.~~ • ~~Using graphing calculators is a strategy for investigating the shape and behavior of polynomial functions.~~	• Find the composition of two functions. • <u>Use composition of functions to verify two functions are inverses.</u> • ~~Investigate the shape and behavior of linear, quadratic, and cubic functions. Behaviors will include intercepts, number of turning points, and end behavior.~~ • ~~Investigate the shape and behavior of exponential ($a^x = y$) and logarithmic ($\log_b x = y$) functions, including intercepts and end behavior.~~

TOPIC: FUNCTIONS

ALGEBRA II
STANDARD AII.8

The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The *Fundamental Theorem of Algebra* states that, including complex and repeated solutions, an n^{th} degree polynomial equation has exactly n roots (solutions). • The following statements are equivalent: – k is a zero of the polynomial function f; – $(x - k)$ is a factor of $f(x)$; – k is a solution of the polynomial equation $f(x) = 0$; and – k is an x-intercept for the graph of $y = f(x)$.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression. • Define a polynomial function, given its zeros. • Determine ~~the~~a factored form of a polynomial expression from the x-intercepts of the graph of its corresponding function. • For a function, identify zeros of multiplicity greater than 1 and describe the effect of those zeros on the graph of the function. • Given a polynomial equation, determine the number of real solutions and nonreal solutions.

TOPIC: STATISTICS

ALGEBRA II
STANDARD AII.9

The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

ESSENTIAL UNDERSTANDINGS

- Data and scatterplots may indicate patterns that can be modeled with an algebraic equation.
- Graphing calculators can be used to collect, organize, picture, and create an algebraic model of the data.
- Data that fit polynomial ($f(x) = a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$, where n is a nonnegative integer, and the coefficients are real numbers), exponential ($y = b^x$), and logarithmic ($y = \log_b x$) models arise from real-world situations.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect and analyze data.
- Investigate scatterplots to determine if patterns exist and then identify the patterns.
- Find an equation for the curve of best fit for data, using a graphing calculator. Models will include ~~linear, quadratic~~, polynomial, exponential, and logarithmic functions.
- Make predictions, using data, scatterplots, or the equation of the curve of best fit.
- Given a set of data, determine the model that would best describe the data.

TOPIC: STATISTICS

ALGEBRA II
STANDARD AII.10

The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Practical~~ Real-world problems can be modeled and solved by using inverse variation, joint variation, and a combination of direct and inverse variations. • Joint variation is a combination of direct variations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Translate "*y* varies jointly as *x* and *z*" as *y* = *kxz*.~~ • Translate "y varies jointly as x and z" as $y = kxz$. • ~~Translate "*y* is directly proportional to *x*" as *y* = *kx*.~~ • Translate "y is directly proportional to x" as $y = kx$. • ~~Translate "*y* is inversely proportional to *x*" as~~ $\cancel{y = \frac{k}{x}}$. • Translate "y is inversely proportional to x" as $y = \frac{k}{x}$. • Given a situation, determine the value of the constant of proportionality~~, *k*, given initial conditions for *x* and *y*~~. • Set up and solve ~~practical~~ problems, including real-world problems, ~~using~~ involving inverse variation, joint variation, and a combination of direct and inverse variations.

ALGEBRA II
STANDARD AII.11

The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~The~~A normal distribution curve is a ~~family of~~ symmetrical, bell-shaped curve~~s~~ defined by the mean and the standard deviation of a data set. The mean is located on the line of symmetry of the curve. • Areas under the curve represent probabilities associated with continuous distributions. • The normal curve is a probability distribution and the total area under the curve is 1. • For a normal distribution, approximately 68 percent of the data fall within one standard deviation of the mean, approximately 95 percent of the data fall within two standard deviations of the mean, and approximately 99.7 percent of the data fall within three standard deviations of the mean. • The mean of the data in a standard normal ~~density function~~ distribution is 0 and the standard deviation is 1. ~~This allows for the comparison of unlike data.~~ • The standard normal curve allows for the comparison of data from different normal distributions. • A z-score is a measure of position derived from the mean and standard deviation of data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a normal probability distribution. • Describe how the standard deviation and the mean affect the graph of the normal distribution. • Compare two sets of normally distributed data using a standard normal ~~curve~~distribution and z-scores. • Represent probability as area under the curve of a standard normal probability distribution. • Use the graphing calculator or a standard normal probability table to determine probabilities or percentiles based on z-scores.

TOPIC: STATISTICS

ALGEBRA II
STANDARD AII.11

The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• <u>A z-score expresses, in standard deviation units, how far an element falls from the mean of the data set.</u> • <u>A z-score is a derived score from a given normal distribution.</u> • <u>A standard normal distribution is the set of all z-scores.</u>	

TOPIC: STATISTICS

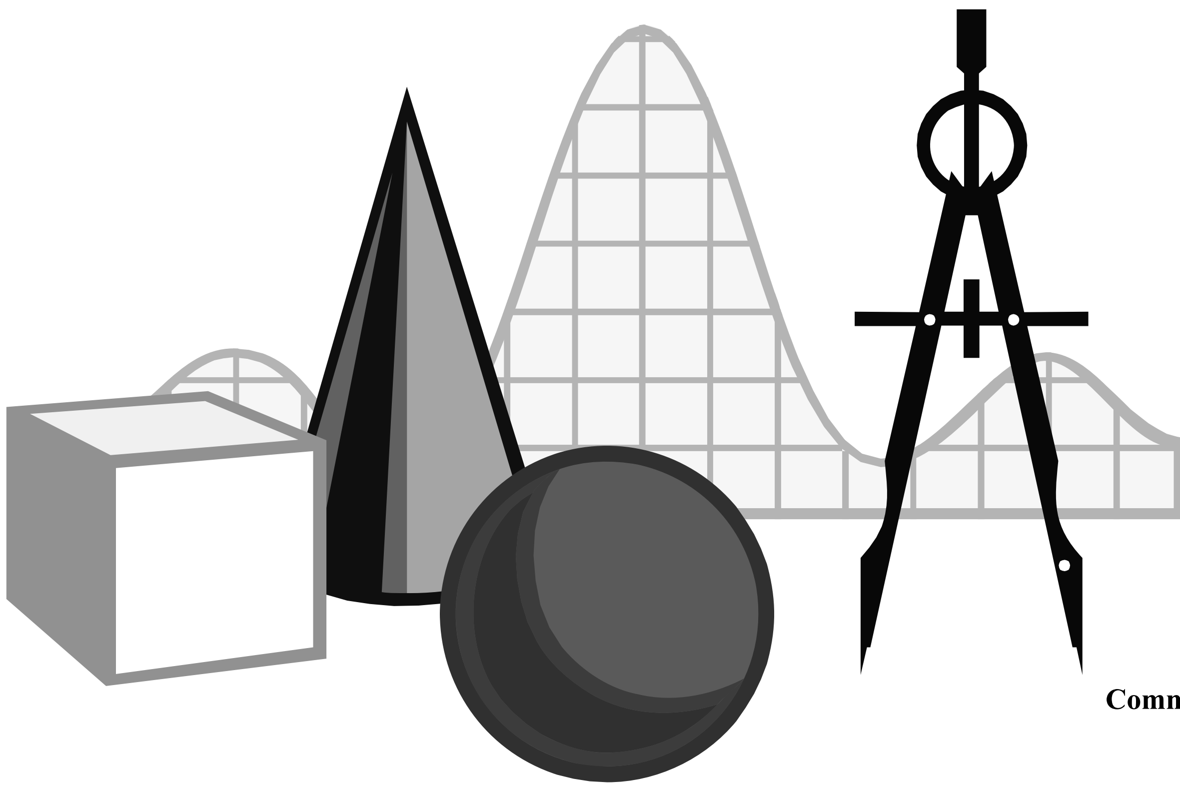
ALGEBRA II
STANDARD AII.12

The student will compute and distinguish between permutations and combinations and use technology for applications.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The *Fundamental Counting Principle* states that if one decision can be made *n* ways and another can be made *m* ways, then the two decisions can be made *nm* ways. • *Permutations* are used to calculate the number of possible arrangements of objects. • *Combinations* are used to calculate the number of possible selections of objects without regard to the order selected.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast permutations and combinations. • Calculate the number of permutations of *n* objects taken *r* at a time. • Calculate the number of combinations of *n* objects taken *r* at a time. • Use permutations and combinations as counting techniques to solve real-world problems.

MATHEMATICS STANDARDS OF LEARNING
PROPOSED CURRICULUM FRAMEWORK

Trigonometry



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills~~ and Essential Understandings~~. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY
STANDARD T.1

The student, given a point other than the origin on the terminal side of the angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Triangular trigonometric function definitions are related to circular trigonometric function definitions. • Both degrees and radians are units for measuring angles. • Drawing an angle in standard position will force the terminal side to lie in a specific quadrant. • A point on the terminal side of an angle determines a reference triangle from which the values of the six trigonometric functions may be derived.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define the six triangular trigonometric functions of an angle in a right triangle. • Define the six circular trigonometric functions of an angle in standard position. • Make the connection between the triangular and circular trigonometric functions. • Recognize and draw an angle in standard position. • Show how a point on the terminal side of an angle determines a reference triangle.

TOPIC: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY
STANDARD T.2

The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• If one trigonometric function value is known, then a triangle can be formed to use in finding the other five trigonometric function values. • Knowledge of the unit circle is a useful tool for finding all six trigonometric values for special angles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given one trigonometric function value, find the other five trigonometric function values. • Develop the unit circle, using both degrees and radians. • Solve problems, using the circular function definitions and the properties of the unit circle. • Recognize the connections between the coordinates of points on a unit circle and – coordinate geometry; – cosine and sine values; and – lengths of sides of special right triangles (30°-60°-90° and 45°-45°-90°).

TOPIC: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY
STANDARD T.3

The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Special angles are widely used in mathematics. • Unit circle properties will allow special angle and related angle trigonometric values to be found without the aid of a calculator. • Degrees and radians are units of angle measure. • A radian is the measure of the central angle that is determined by an arc whose length is the same as the radius of the circle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find trigonometric function values of special angles and their related angles in both degrees and radians. • Apply the properties of the unit circle without using a calculator. • Use a conversion factor to convert from radians to degrees and vice versa without using a calculator.

TOPIC: INVERSE TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY
STANDARD T.4

The student will find, with the aid of a calculator, the value of any trigonometric function and inverse trigonometric function.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The trigonometric function values of any angle can be found by using a calculator. • The inverse trigonometric functions can be used to find angle measures whose trigonometric function values are known. • Calculations of inverse trigonometric function values can be related to the triangular definitions of the trigonometric functions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use a calculator to find the trigonometric function values of any angle in either degrees or radians. • Define inverse trigonometric functions. • Find angle measures by using the inverse trigonometric functions when the trigonometric function values are given.

TOPIC: TRIGONOMETRIC IDENTITIES

TRIGONOMETRY
STANDARD T.5

The student will verify basic trigonometric identities and make substitutions, using the basic identities.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Trigonometric identities can be used to simplify trigonometric expressions, equations, or identities. • Trigonometric identity substitutions can help solve trigonometric equations, verify another identity, or simplify trigonometric expressions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include – reciprocal identities; – Pythagorean identities; – sum and difference identities; – double-angle identities; and – half-angle identities.

TOPIC: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

TRIGONOMETRY
STANDARD T.6

The student, given one of the six trigonometric functions in standard form, will

a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
c) sketch the graph of the function by using transformations for at least a two-period interval; and
d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The domain and range of a trigonometric function determine the scales of the axes for the graph of the trigonometric function. • The amplitude, period, phase shift, and vertical shift are important characteristics of the graph of a trigonometric function, and each has a specific purpose in applications using trigonometric equations. • The graph of a trigonometric function can be used to display information about the periodic behavior of a real-world situation, such as wave motion or the motion of a Ferris wheel.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the amplitude, period, phase shift, and vertical shift of a trigonometric function from the equation of the function and from the graph of the function. • Describe the effect of changing A, B, C, or D in the standard form of a trigonometric equation $\{$e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D\}$. • State the domain and the range of a function written in standard form $\{$e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D\}$. • Sketch the graph of a function written in standard form $\{$e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D\}$ by using transformations for at least one period or one cycle.

TOPIC: INVERSE TRIGONOMETRIC FUNCTIONS

TRIGONOMETRY
STANDARD T.7

The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Restrictions on the domains of some inverse trigonometric functions exist.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the domain and range of the inverse trigonometric functions. • Use the restrictions on the domains of the inverse trigonometric functions in finding the values of the inverse trigonometric functions. • Identify the graphs of the inverse trigonometric functions.

TOPIC: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

TRIGONOMETRY
STANDARD T.8

The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Solutions for trigonometric equations will depend on the domains. • A calculator can be used to find the solution of a trigonometric equation as the points of intersection of the graphs when one side of the equation is entered in the calculator as Y_1 and the other side is entered as Y_2.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve trigonometric equations with restricted domains algebraically and by using a graphing utility. • Solve trigonometric equations with infinite solutions algebraically and by using a graphing utility. • Check for reasonableness of results, and verify algebraic solutions, using a graphing utility.

TOPIC: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

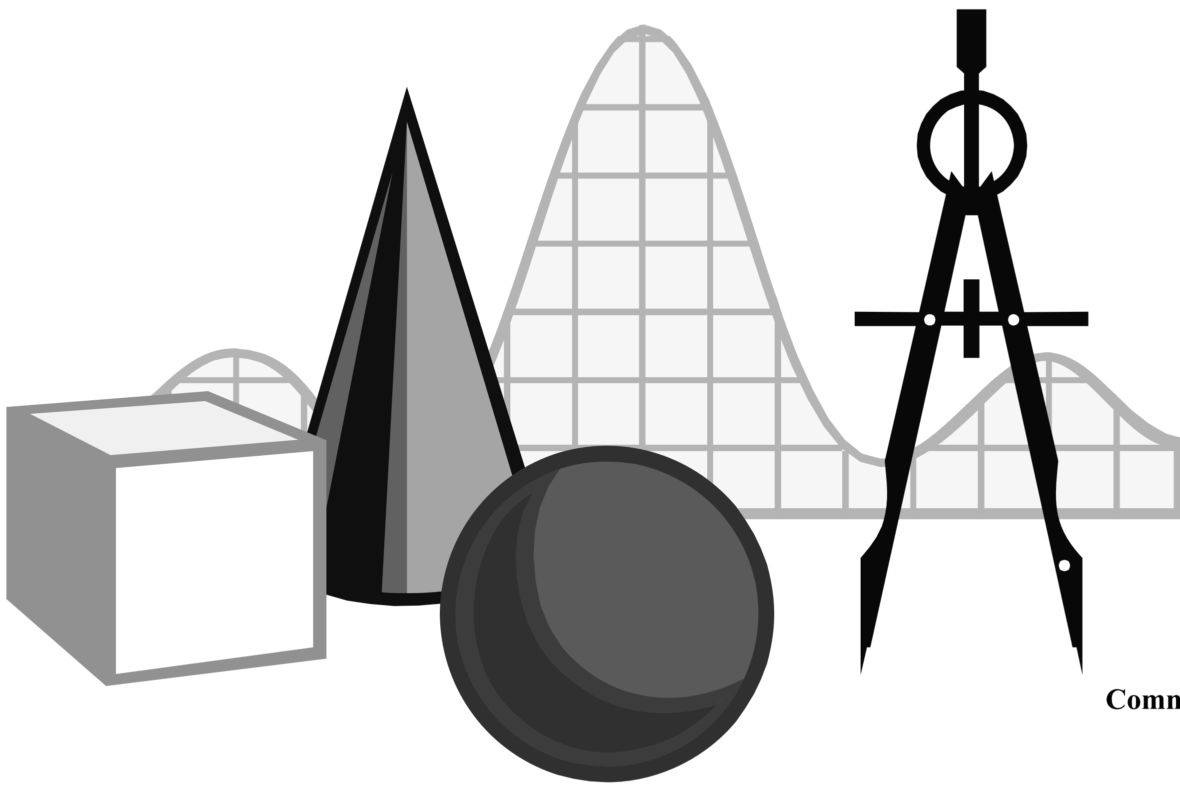
TRIGONOMETRY
STANDARD T.9

The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A ~~practical~~ real-world problem may be solved by using one of a variety of techniques associated with triangles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write a ~~practical~~ real-world problem involving triangles. • Solve ~~practical~~ real-world problems involving triangles. • Use the trigonometric functions, Pythagorean Theorem, Law of Sines, and Law of Cosines to solve ~~practical~~ real-world problems. • Use the trigonometric functions to model real-world situations. • Identify a solution technique that could be used with a given problem.

MATHEMATICS STANDARDS OF LEARNING PROPOSED CURRICULUM FRAMEWORK

Discrete Mathematics



Commonwealth of Virginia
Board of Education
Richmond, Virginia
Draft October 22, 2009

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Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills~~ and Essential Understandings~~. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.1

The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A tournament is a digraph that results from giving directions to the edges of a complete graph. • Adjacent vertices are connected by an edge. • In a connected graph, every pair of vertices is adjacent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the valence of each vertex in a graph. • Use graphs to model situations in which the vertices represent objects, and edges (drawn between vertices) represent a particular relationship between objects. • Represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems. • Investigate and describe valence and connectedness. • Determine whether a graph is planar or nonplanar. • Use directed graphs (digraphs) to represent situations with restrictions in traversal possibilities.

TOPIC: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.2

The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Euler's Theorem states: If G is a connected graph and all its valences are even, then G has an Euler Circuit. • Pairs of routes (circuits) correspond to the same Hamilton Circuit because one route can be obtained from the other by traversing the vertices in reverse order. There are $\frac{(n-1)!}{2}$ Hamilton Circuits. • A multigraph is connected if there is a path between every pair of vertices.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine if a graph has an Euler Circuit or Path, and find it. • Determine if a graph has a Hamilton Circuit or Path, and find it. • Count the number of Hamilton Circuits for a complete graph with n vertices. • Use the Euler Circuit algorithm to solve optimization problems.

TOPIC: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.3

The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Every planar graph has a chromatic number that is less than or equal to four (the four-color-map theorem). • A graph can be colored with two colors if and only if it contains no cycle of odd length. • The chromatic number of a graph cannot exceed one more than the maximum number of degrees of the vertices of the graph.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model projects consisting of several subtasks, using a graph. • Use graphs to resolve conflicts that arise in scheduling.

TOPIC: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.4

The student will apply algorithms, such as Kruskal's, Prim's, or Dijkstra's, relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A spanning tree of a connected graph *G* is a tree that is a subgraph of *G* and contains every vertex of *G*.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use Kruskal's Algorithm to find the shortest spanning tree of a connected graph. • Use Prim's Algorithm to find the shortest spanning tree of a connected graph. • Use Dijkstra's Algorithm to find the shortest spanning tree of a connected graph.

TOPIC: ELECTION THEORY AND FAIR DIVISION

DISCRETE MATHEMATICS
STANDARD DM.7

The student will analyze and describe the issue of fair division (e.g., cake cutting, estate division). Algorithms for continuous and discrete cases will be applied.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Group decision making combines the wishes of many to yield a single fair result. • A fair division problem may be discrete or continuous. • The success of the estate division algorithm requires that each heir be capable of placing a value on each object in the estate. • A fair division problem consists of *n* individuals (players) who must partition some set of goods, *s*, into *n* disjoint sets.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and describe situations involving discrete division (e.g., estate division). • Use an algorithm for fair division for a group of indivisible objects. • Investigate and describe situations involving continuous division of an infinitely divisible set (e.g., cake cutting). • Use an algorithm for fair division of an infinitely divisible set.

TOPIC: ELECTION THEORY AND FAIR DIVISION

DISCRETE MATHEMATICS
STANDARD DM.8

The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Historically, popular voting methods have often led to counterintuitive results. • A candidate who wins over every other candidate in a one-on-one ballot is a Condorcet winner. • A Borda count assigns points in descending order to each voter's subsequent ranking and then adds these points to arrive at a group's final ranking. • To select a voting system is to compromise between the shortcomings inherent in each system.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine in how many different ways a voter can rank choices. • Investigate and describe the following voting procedures: – weighted voting; – plurality; – majority; – sequential (winners run off); – sequential (losers are eliminated); – Borda count; and – Condorcet winner. • Compare and contrast different voting procedures. • Describe the possible effects of approval voting, insincere and sincere voting, a preference schedule, and strategic voting on the election outcome.

TOPIC: ELECTION THEORY AND FAIR DIVISION

DISCRETE MATHEMATICS
STANDARD DM.9

The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The apportionment of Congressional representatives is based on the latest census.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast the Hamilton and Jefferson methods of political apportionment with the Hill-Huntington method (currently in use in the U.S. House of Representatives) and the Webster-Willcox method. • Solve allocation problems, using apportionment methods. • Investigate and describe how salary caps affect apportionment.

TOPIC: COMPUTER MATHEMATICS

DISCRETE MATHEMATICS
STANDARD DM.11

The student will describe and apply sorting algorithms and coding algorithms used in sorting, processing, and communicating information. These will include

a) bubble sort, merge sort, and network sort; and
b) ISBN, UPC, Zip, and banking codes.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A bubble sort orders elements of an array by comparing adjacent elements. • A merge sort combines two sorted lists into a single sorted list. • Coding algorithms must account for the number of possible codes within the constraints of the coding system.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Select and apply a sorting algorithm, such as a – bubble sort; – merge sort; and – network sort. • Describe and apply a coding algorithm, such as – ISBN numbers; – UPC codes; – Zip codes; and – banking codes.

TOPIC: COMPUTER MATHEMATICS

DISCRETE MATHEMATICS
STANDARD DM.12

The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Two-valued (Boolean) algebra serves as a workable method for interpreting the logical truth and falsity of compound statements. • Venn diagrams provide pictures of topics in set theory, such as intersection and union, mutually exclusive sets, and the empty set.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Generate truth tables that encode the truth and falsity of two or more statements. • Use Venn diagrams to codify and solve logic problems. • Use matrices as arrays of data to solve logic problems.

TOPIC: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.5

The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Critical path scheduling sometimes yields optimal solutions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Specify in a digraph the order in which tests are to be performed. • Identify the critical path to determine the earliest completion time (minimum project time). • Use the list-processing algorithm to determine an optimal schedule. • Create and test scheduling algorithms.

TOPIC: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.6

The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Linear programming models an optimization process. • A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized. • Any maximum or minimum value for a system of inequalities will occur at a corner point of a feasible region.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model real-world problems with systems of linear inequalities. • Identify the feasibility region of a system of linear inequalities with no more than four constraints. • Identify the coordinates of the corner points of a feasibility region. • Find the maximum or minimum value of the system. • Describe the meaning of the maximum or minimum value in terms of the original problem.

TOPIC: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.10

The student will use the recursive process and difference equations with the aid of appropriate technology to generate

a) compound interest;
b) sequences and series;
c) fractals;
d) population growth models; and
e) the Fibonacci sequence.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Recursion is a process that creates new objects from existing objects that were created by the same process. • A fractal is a figure whose dimension is not a whole number. • Fractals are self-similar.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use finite differences and recursion to model compound interest and population growth situations. • Model arithmetic and geometric sequences and series recursively. • Compare and contrast the recursive process, and create fractals. • Compare and contrast the recursive process and the Fibonacci sequence. • Find a recursive relationship that generates the Fibonacci sequence.

TOPIC: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.13

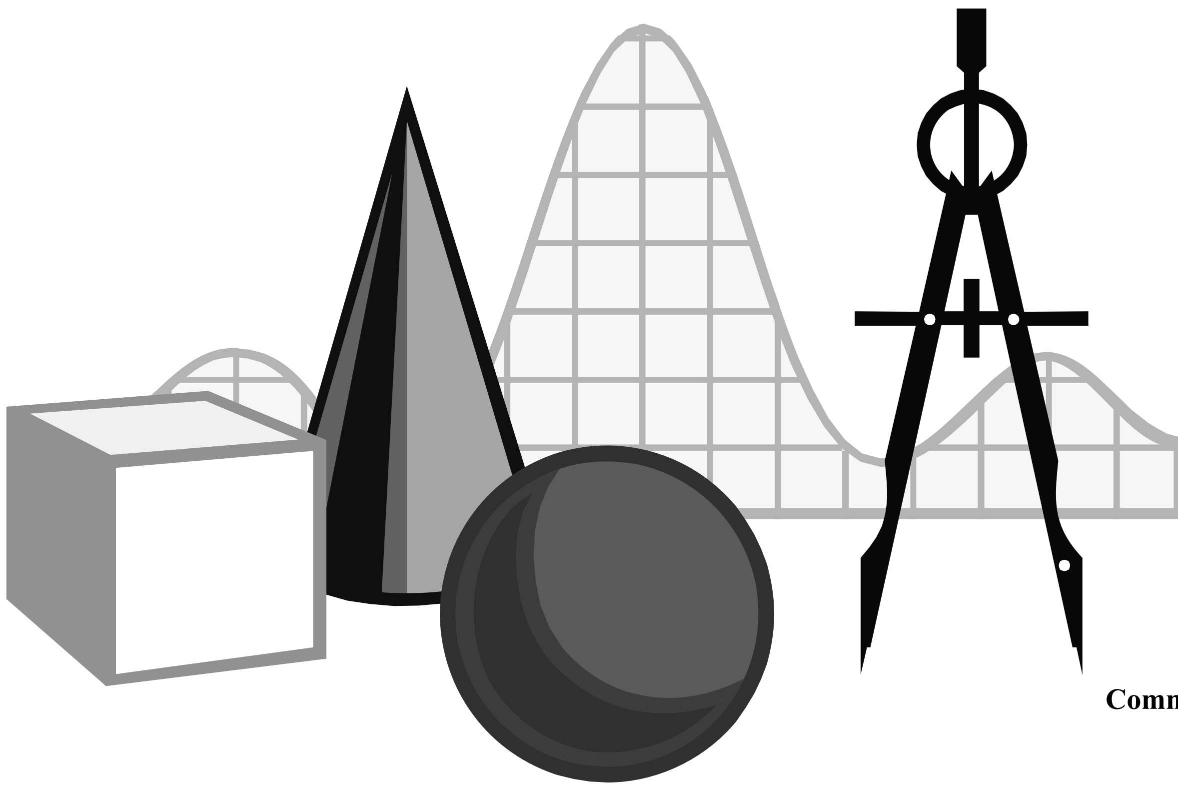
The student will apply the formulas of combinatorics in the areas of

a) the Fundamental (Basic) Counting Principle;
b) knapsack and bin-packing problems;
c) permutations and combinations; and
d) the pigeonhole principle.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The branch of mathematics that addresses the number of ways objects can be arranged or combined is combinatorics. • If n and r are positive integers and $n \geq r$, $n\,P\,r = \frac{n!}{(n-r)!}$ and $n\,C\,r = \frac{n!}{r!\,(n-r)!}$. • A bin-packing problem determines the minimum number of containers of fixed volume (bins) required to hold a set of objects. • A knapsack problem determines the most valuable set of objects that fit into a container (knapsack) of fixed volume. • Bin packing and knapsack packing are optimization techniques.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the number of combinations possible when subsets of r elements are selected from a set of n elements without regard to order. • Use the Fundamental (Basic) Counting Principle to determine the number of possible outcomes of an event. • Use the knapsack and bin-packing algorithms to solve real-world problems. • Find the number of permutations possible when r objects selected from n objects are ordered. • Use the pigeonhole principle to solve packing problems to facilitate proofs.

MATHEMATICS STANDARDS OF LEARNING
PROPOSED CURRICULUM FRAMEWORK

Probability and Statistics



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TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.1

The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Measures of central tendency describe how the data cluster or group. • Measures of dispersion describe how the data spread (disperse) around the center of the data. • Graphical displays of data may be analyzed informally. • Data analysis must take place within the context of the problem.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Create and interpret graphical displays of data, including dotplots, stem-and-leaf plots, and histograms. • Examine graphs of data for clusters and gaps, and relate those phenomena to the data in context. • Examine graphs of data for outliers, and explain the outlier(s) within the context of the data. • Examine graphs of data and identify the central tendency of the data as well as the spread. Explain the central tendency and the spread of the data within the context of the data.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.2

The student will analyze numerical characteristics of univariate data sets to describe patterns and departure from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning within a context. • Analysis of the descriptive statistical information generated by a univariate data set should include the interplay between central tendency and dispersion as well as among specific measures. • Data points identified algorithmically as outliers should not be excluded from the data unless sufficient evidence exists to show them to be in error.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem's context. • Identify possible outliers, using an algorithm. • Explain the influence of outliers on a univariate data set. • Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.3

The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Statistical tendency refers to typical cases but not necessarily to individual cases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast two or more univariate data sets by analyzing measures of center and spread within a contextual framework. • Describe any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data. • Analyze in context kurtosis and skewness in conjunction with other descriptive measures.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.4

The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A scatterplot serves two purposes: – to determine if there is a useful relationship between two variables, and – to determine the family of equations that describes the relationship. • Data are collected for a purpose and have meaning in a context. • Association between two variables considers both the direction and strength of the association. • The strength of an association between two variables reflects how accurately the value of one variable can be predicted based on the value of the other variable. • Outliers are observations with large residuals and do not follow the pattern apparent in the other data points.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Examine scatterplots of data, and describe skewness, kurtosis, and correlation within the context of the data. • Describe and explain any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data. • Identify influential data points (observations that have great effect on a line of best fit because of extreme *x*-values) and describe the effect of the influential points.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.5

The student will find and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Least squares regression generates the equation of the line that minimizes the sum of the squared distances from the data points to the line. • Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured). • Residual = Actual – Fitted • A correlation coefficient measures the degree of association between two variables that are related linearly. • Two variables may be strongly associated without a cause-and-effect relationship existing between them.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Calculate a correlation coefficient. • Explain how the correlation coefficient, *r*, measures association by looking at its formula. • Use regression lines to make predictions, and identify the limitations of the predictions. • Use residual plots to determine if a linear model is satisfactory for describing the relationship between two variables. • Describe the errors inherent in extrapolation beyond the range of the data. • Use least squares regression to find the equation of the line of best fit for a set of data. • Explain how least squares regression generates the equation of the line of best fit by examining the formulas used in computation.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.6

The student will make logarithmic and power transformations to achieve linearity.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A logarithmic transformation reduces positive skewness because it compresses the upper tail of the distribution while stretching the lower tail. • Nonlinear transformations do not preserve relative spacing between data points.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Apply a logarithmic transformation to data. • Explain how a logarithmic transformation works to achieve a linear relationship between variables. • Apply a power transformation to data. • Explain how a power transformation works to achieve a linear relationship between variables.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.7

The student, using two-way tables, will analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Simpson's paradox refers to the fact that aggregate proportions can reverse the direction of the relationship seen in the individual parts. • Two categorical variables are independent if the conditional frequencies of one variable are the same for every category of the other variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Produce a two-way table as a summary of the information obtained from two categorical variables. • Calculate marginal, relative, and conditional frequencies in a two-way table. • Use marginal, relative, and conditional frequencies to analyze data in two-way tables within the context of the data.

TOPIC: DATA COLLECTION

PROBABILITY AND STATISTICS
STANDARD PS.8

The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of a sample statistic varies from sample to sample if the simple random samples are taken repeatedly from the population of interest. • Poor data collection can lead to misleading and meaningless conclusions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast controlled experiments and observational studies and the conclusions one can draw from each. • Compare and contrast population and sample and parameter and statistic. • Identify biased sampling methods. • Describe simple random sampling. • Select a data collection method appropriate for a given context.

TOPIC: DATA COLLECTION

PROBABILITY AND STATISTICS
STANDARD PS.9

The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random and stratified) and methods to reduce bias.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The purpose of sampling is to provide sufficient information so that population characteristics may be inferred. • Inherent bias diminishes as sample size increases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. • Determine which sampling technique is best, given a particular context. • Plan a survey to answer a question or address an issue. • Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. • Design a survey instrument. • Conduct a survey.

TOPIC: DATA COLLECTION

PROBABILITY AND STATISTICS
STANDARD PS.10

The student will plan and conduct an experiment. The plan will address control, randomization, and measurement of experimental error.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Experiments must be carefully designed in order to detect a cause-and-effect relationship between variables. • Principles of experimental design include comparison with a control group, randomization, and blindness.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Plan and conduct an experiment. The experimental design should address control, randomization, and minimization of experimental error.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.11

The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The complement of event A consists of all outcomes in which event A does not occur. • Two events, A and B, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If A and B are not independent, then they are said to be dependent. • Events A and B are mutually exclusive if they cannot occur simultaneously.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. • Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.12

The student will find probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Venn diagrams may be used to find conditional probabilities. • The ~~“l~~Law of ~~l~~Large ~~n~~Numbers~~”~~ states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Calculate relative frequency and expected frequency. • Find conditional probabilities for dependent, independent, and mutually exclusive events.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.13

The student will develop, interpret, and apply the binomial probability distribution for discrete random variables, including computing the mean and standard deviation for the binomial variable.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A probability distribution is a complete listing of all possible outcomes of an experiment together with their probabilities. The procedure has a fixed number of independent trials. • A random variable assumes different values depending on the event outcome. • A probability distribution combines descriptive statistical techniques and probabilities to form a theoretical model of behavior.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Develop the binomial probability distribution within a real-world context. • Calculate the mean and standard deviation for the binomial variable. • Use the binomial distribution to calculate probabilities associated with experiments for which there are only two possible outcomes.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.14

The student will simulate probability distributions, including binomial and geometric.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A probability distribution combines descriptive methods and probabilities to form a theoretical model of behavior. • A probability distribution gives the probability for each value of the random variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design and conduct an experiment that simulates a binomial distribution. • Design and conduct an experiment that simulates a geometric distribution.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.15

The student will identify random variables as independent or dependent and find the mean and standard deviations for sums and differences of independent random variables.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A random variable is a variable that has a single numerical value, determined by chance, for each outcome of a procedure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast independent and dependent random variables. • Find the standard deviation for sums and differences of independent random variables.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.16

The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation. • Areas under the curve represent probabilities associated with continuous distributions. • The normal curve is a probability distribution and the total area under the curve is 1.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a normal probability distribution. • Describe how the standard deviation and the mean affect the graph of the normal distribution. • Determine the probability of a given event, using the normal distribution.

TOPIC: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.17

The student, given data from a large sample, will find and interpret point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired).

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A primary goal of sampling is to estimate the value of a parameter based on a statistic. • Confidence intervals use the sample statistic to construct an interval of values that one can be reasonably certain contains the true (unknown) parameter. • Confidence intervals and tests of significance are complementary procedures. • Paired comparisons experimental design allows control for possible effects of extraneous variables.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct confidence intervals to estimate a population parameter, such as a proportion or the difference between two proportions; or a mean or the difference between two means. • Select a value for alpha (Type I error) for a confidence interval. • Interpret confidence intervals in the context of the data. • Explain the importance of random sampling for confidence intervals. • Calculate point estimates for parameters and discuss the limitations of point estimates.

TOPIC: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.18

The student will apply and interpret the logic of a hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, and difference between two means (independent and paired) and Chi-squared tests for goodness of fit, homogeneity of proportions, and independence.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Confidence intervals and tests of significance are complementary procedures. • Paired comparisons experimental design allows control for possible effects of extraneous variables. • Tests of significance assess the extent to which sample data support a hypothesis about a population parameter. • The purpose of a goodness of fit test is to decide if the sample results are consistent with results that would have been obtained if a random sample had been selected from a population with a known distribution. • Practical significance and statistical significance are not necessarily congruent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the Chi-squared test for goodness of fit to decide if the population being analyzed fits a particular distribution pattern. • Use hypothesis-testing procedures to determine whether or not to reject the null hypothesis. The null hypothesis may address proportion, mean, difference between two proportions or two means, goodness of fit, homogeneity of proportions, and independence. • Compare and contrast Type I and Type II errors. • Explain how and why the hypothesis-testing procedure allows one to reach a statistical decision.

TOPIC: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.19

The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The Central Limit Theorem states: – The mean of the sampling distribution of means is equal to the population mean. – If the sample size is sufficiently large, the sampling distribution approximates the normal probability distribution. – If the population is normally distributed, the sampling distribution is normal regardless of sample size. • Sampling distributions have less variability with larger sample sizes.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the use of the Central Limit Theorem for drawing inferences about a population parameter based on a sample statistic. • Describe the effect of sample size on the sampling distribution and on related probabilities. • Use the normal approximation to calculate probabilities of sample statistics falling within a given interval. • Identify and describe the characteristics of a sampling distribution of a sample proportion, mean, difference between two sample proportions, or difference between two sample means.

TOPIC: INFERENTIAL STATISTICS

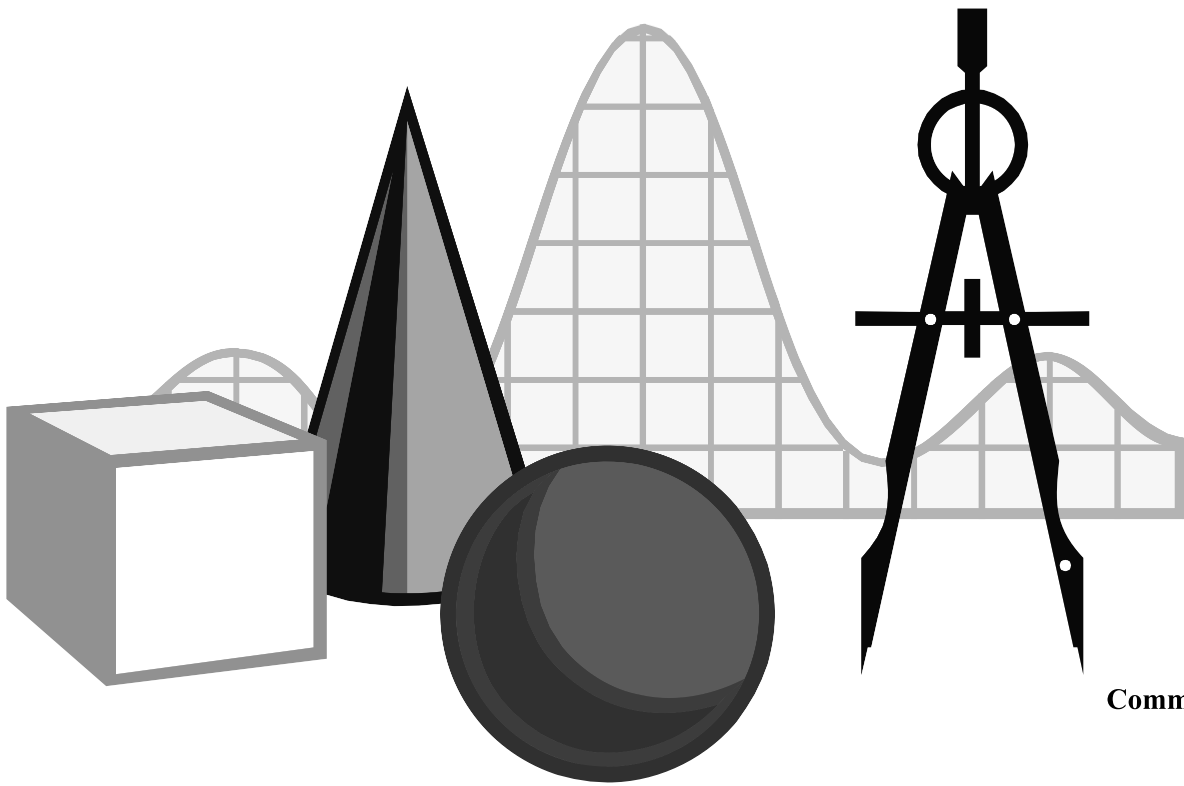
PROBABILITY AND STATISTICS
STANDARD PS.20

The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures, using tables or graphing calculators.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Paired comparisons experimental design allows control for possible effects of extraneous variables. • The sampling distribution of means with a small sample size follows a t-distribution.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a t-distribution. • Compare and contrast a t-distribution and a normal distribution. • Use a t-test for single-sample and two-sample data.

MATHEMATICS STANDARDS OF LEARNING
PROPOSED CURRICULUM FRAMEWORK

Mathematical Analysis



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills ~~and Essential Understandings~~. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.1

The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The graphs of polynomial and rational functions can be determined by exploring characteristics and components of the functions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a polynomial function, given an equation or graph. • Identify rational functions, given an equation or graph. • Identify domain, range, zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, points of discontinuity, end behavior, and maximum and minimum points, given a graph of a function. • Sketch the graph of a polynomial function. • Sketch the graph of a rational function. • Investigate and verify characteristics of a polynomial or rational function, using a graphing calculator.

TOPIC: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.2

The student will apply compositions of functions and inverses of functions to real-world situations. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In composition of functions, a function serves as input for another function. • A graph of a function and its inverse are symmetric about the line $y = x$. • $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the composition of functions. • Find the inverse of a function algebraically and graphically. • Determine the domain and range of the composite functions. • Determine the domain and range of the inverse of a function. • Verify the accuracy of sketches of functions, using a graphing utility.

TOPIC: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.3

The student will investigate and describe the continuity of functions, using graphs and algebraic methods.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Continuous and discontinuous functions can be identified by their equations or graphs.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe continuity of a function. • Investigate the continuity of absolute value, step, rational, and piece-wise-defined functions. • Use transformations to sketch absolute value, step, and rational functions. • Verify the accuracy of sketches of functions, using a graphing utility.

TOPIC: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.7

The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The limit of a function is the value approached by $f(x)$ as x approaches a given value or infinity.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Verify intuitive reasoning about the limit of a function, using a graphing utility. • Find the limit of a function algebraically, and verify with a graphing utility. • Find the limit of a function numerically, and verify with a graphing utility. • <u>Use limit notation when describing end behavior of a function.</u>

TOPIC: FUNCTIONS

MATHEMATICAL ANALYSIS
STANDARD MA.9

The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and real-world problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS

• Exponential and logarithmic functions are inverse functions.
• Some examples of appropriate models or situations for exponential and logarithmic functions are:
  • Population growth;
  • Compound interest;
  • Depreciation/appreciation;
  • Richter scale; and
  • Radioactive decay.
	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
	• Identify exponential functions from an equation or a graph.
	• Identify logarithmic functions from an equation or a graph.
	• Define e, and know its approximate value.
	• Write logarithmic equations in exponential form and vice versa.
	• Identify common and natural logarithms.
	• Use laws of exponents and logarithms to solve equations and simplify expressions.
	• Model real-world problems, using exponential and logarithmic functions.
	• Graph exponential and logarithmic functions, using a graphing utility, and identify asymptotes, intercepts, domain, and range.

TOPIC: DISCRETE MATHEMATICS

MATHEMATICAL ANALYSIS
STANDARD MA.4

The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The Binomial Theorem provides a formula for calculating the product $(a + b)^n$ for any positive integer n. • Pascal's Triangle is a triangular array of binomial coefficients.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Expand binomials having positive integral exponents. • Use the Binomial Theorem, the formula for combinations, and Pascal's Triangle to expand binomials.

TOPIC: DISCRETE MATHEMATICS

MATHEMATICAL ANALYSIS
STANDARD MA.5

The student will find the sum (sigma notation included) of finite and infinite convergent series, which will lead to an intuitive approach to a limit.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Examination of infinite sequences and series may lead to a limiting process. • Arithmetic sequences have a common difference between any two consecutive terms. • Geometric sequences have a common factor between any two consecutive terms.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use and interpret the notation: $\sum$, n, nth, and a_n. • Given the formula, find the nth term, a_n, for an arithmetic or geometric sequence. • Given the formula, find the sum, S_n, if it exists, of an arithmetic or geometric series. • Model and solve problems, using sequence and series information. • Distinguish between a convergent and divergent series. • Discuss convergent series in relation to the concept of a limit.

TOPIC: DISCRETE MATHEMATICS

MATHEMATICAL ANALYSIS
STANDARD MA.6

The student will use mathematical induction to prove formulas and mathematical statements.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Mathematical induction is a method of proof that depends on a recursive process. • Mathematical induction allows reasoning from specific true values of the variable to general values of the variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare inductive and deductive reasoning. • Prove formulas/statements, using mathematical induction.

TOPIC: EQUATIONS

MATHEMATICAL ANALYSIS
STANDARD MA.10

The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The real number system is represented geometrically on the number line, and the complex number system is represented geometrically on the plane where $a + bi$ corresponds to the point (a, b) in the plane.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize polar equations (rose, cardioid, limacon, lemniscate, spiral, and circle), given the graph or the equation. • Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility. • Convert complex numbers from rectangular form to polar form and vice versa. • Find the intersection of the graphs of two polar equations, using a graphing utility.

TOPIC: EQUATIONS

MATHEMATICAL ANALYSIS
STANDARD MA.12

The student will use parametric equations to model and solve application problems.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Parametric equations are used to express two dependent variables, x and y, in terms of an independent variable (parameter), t. • Some curves cannot be represented as a function, $f(x)$. Parametric graphing enables the representation of these curves in terms of functions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Graph parametric equations, using a graphing utility. • Use parametric equations to model motion over time. • Determine solutions to parametric equations, using a graphing utility. • Compare and contrast traditional solution methods with parametric methods.

TOPIC: EQUATIONS

MATHEMATICAL ANALYSIS
STANDARD MA.14

The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Matrices are a convenient shorthand for solving systems of equations. • Matrices can model a variety of linear systems. • Solutions of a linear system are values that satisfy every equation in the system. • Matrices can be used to model and solve real-world problems.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Add, subtract, and multiply matrices and multiply matrices by a scalar. • Model problems with a system of no more than three linear equations. • Express a system of linear equations as a matrix equation. • Solve a matrix equation. • Find the inverse of a matrix. • <u>Verify the commutative and associative properties for matrix addition and multiplication.</u>

TOPIC: ANALYTICAL GEOMETRY

MATHEMATICAL ANALYSIS
STANDARD MA.8

The student will investigate and identify the characteristics of conic section equations in (*h, k*) and standard forms. Transformations in the coordinate plane will be used to graph conic sections.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Matrices can be used to represent transformations of figures in the plane.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a translation or rotation matrix, find an equation for the transformed function or conic section. • Investigate and verify graphs of transformed conic sections, using a graphing utility.

TOPIC: ANALYTICAL GEOMETRY

MATHEMATICAL ANALYSIS
STANDARD MA.11

The student will perform operations with vectors in the coordinate plane and solve real-world problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Every vector has an equal vector that has its initial point at the origin. • The magnitude and direction of a vector with the origin as the initial point are completely determined by the coordinates of its terminal point.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use vector notation. • Perform the operations of addition, subtraction, scalar multiplication, and inner (dot) product on vectors. • Graph vectors and resultant vectors. • Express complex numbers in vector notation. • Define *unit vector*, and find the unit vector in the same direction as a given vector. • Identify properties of vector addition, scalar multiplication, and dot product. • Find vector components. • Find the norm (magnitude) of a vector. • Use vectors in simple geometric proofs.

TOPIC: ANALYTICAL GEOMETRY

MATHEMATICAL ANALYSIS
STANDARD MA.11

The student will perform operations with vectors in the coordinate plane and solve real-world problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
	• Solve real-world problems using vectors.

TOPIC: ANALYTIC GEOMETRY

MATHEMATICAL ANALYSIS
STANDARD MA.13

The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Real life~~ Real-world problems can be modeled using trigonometry and vectors.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve and create problems, using trigonometric functions. • Solve and create problems, using the Pythagorean Theorem. • Solve and create problems, using the Law of Sines and the Law of Cosines. • Solve real-world problems using vectors.