Board of Education Agenda Item

Item: M. **Date:** February 17, 2011

Topic: Report on Virginia's College and Career Readiness Initiative and *College and Career Ready Mathematics Performance Expectations*

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Origin:

X Topic presented for information only (no board action required)

____ Board review required by
- ____ State or federal law or regulation
- ____ Board of Education regulation
- ____ Other: _____

X Action requested at this meeting ____ Action requested at future meeting: ____ (date)

Previous Review/Action:

X No previous board review/action

___ Previous review/action
Date:
Action:

Background Information:

In January 2007, the Board of Education authorized the Virginia Department of Education (VDOE) to conduct studies to determine factors that contribute to success in postsecondary education. This critical component of Virginia's College and Career Readiness Initiative included an external analysis and validation of the *Standards of Learning* (*SOL*) in English and mathematics. As a result, Achieve, the College Board, and ACT conducted studies comparing their respective standards for postsecondary readiness to the Virginia *SOL* in English/Reading and Mathematics. In 2009 the Board adopted revised *SOL* in mathematics. The revised standards reflect the substantial input and recommended changes provided by college faculty and other experts from the College Board, ACT, Achieve (the American Diploma Project), and the business community.

In June 2010, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) released the *Common Core State Standards for Mathematics*. As Achieve, The College Board, and ACT were partners with NGA and CCSSO, their earlier work with states in the ADP Network provided a foundation upon which the *Common Core State Standards for Mathematics* were developed.

In the fall of 2010, Department staff and external reviewers conducted analyses to ensure Virginia's *Mathematics SOL* and *Curriculum Framework* met or exceeded the *Common Core State Standards for Mathematics*. The review identified some additional concepts that were recommended for incorporation into the *Mathematics SOL Curriculum Framework.* At its January 13, 2011, meeting, the Board of Education adopted a supplement to the revised *Mathematics SOL Curriculum Framework* and accepted the final Report of the Analysis of Virginia's 2009 *Mathematics Standards of Learning* compared to the *Common Core State Standards for Mathematics*.

Summary of Major Elements

The identification of college and career ready mathematics performance expectations has been a critical component of Virginia's ongoing College and Career Readiness Initiative to prepare all students for success in postsecondary education and careers. The expectations are intended to define the level of achievement students must reach to be academically prepared for entry-level, credit-bearing, college courses in mathematics and/or further career and technical training. To develop the performance expectations, VDOE worked in partnership with Virginia's higher education agencies, the Virginia Community College System (VCCS), and the State Council of Higher Education for Virginia (SCHEV).

To facilitate the collaborative work between VDOE, VCCS, and SCHEV partners, the Department identified preliminary college and career ready mathematics performance expectations using the *Mathematics Standards of Learning*, the *Mathematics SOL Curriculum Framework*, and other validated state and national college and career readiness standards, including the *Common Core State Standards for Mathematics*. It was found that certain *Mathematics Standards of Learning* from middle school grades and high school courses correlated highly with the national college and career ready standards.

With assistance from VCCS and SCHEV in recruitment, faculty from Virginia's two- and four-year institutions of higher education provided feedback on the preliminary college and career ready mathematics performance expectations. More than 100 higher education respondents participated in the survey.

A mathematics consensus/review team composed of faculty from two- and four-year higher education institutions and secondary content area experts analyzed the survey data and made recommendations to VDOE on which performance expectations reached the level of "important" or "critical" for college and career readiness. Virginia's *College and Career Ready Mathematics Performance Expectations*, included as Attachment A, represent the consensus/review team's recommendation to the Department. A correlation crosswalk between Virginia's *College and Career Ready Mathematics Performance Expectations* and the *Common Core State Standards for Mathematics* is provided as Attachment B.

The completion of the *Mathematics Performance Expectations* finalizes the work that is one of five components of Virginia's College and Career Readiness Initiative. In support of the initiative, the Department continues to conduct research to further understand associations between secondary outcomes and postsecondary success. The Department, in collaboration with its partners, has also made progress on several other components. Below is a list of each component and a status update of the ongoing work of Virginia's College and Career Readiness Initiative.

1) Defining college and career ready performance expectations aligned to national and international college and career ready standards.
    - VDOE, in collaboration with VCCS and SCHEV have collaboratively established Virginia's College and Career Ready Performance Expectations. The *English Performance Expectations* were completed in November 2010, and the *Mathematics Performance Expectations*, presented in this document, finalize this component of the work.

2) Developing elective "capstone courses" to support students who need additional instruction to meet college and career ready performance expectations before leaving high school
    - VDOE has drafted course descriptions, program objectives, sample teaching strategies, and delivery options to define the grade-12 capstone courses. Course codes have been identified.
    - In combination with technical assistance and professional development, the course development is intended to enable school divisions to implement the capstone courses in the fall of 2011. At least two school divisions are in the process of finalizing commitments with VDOE to pilot the courses with support from Virginia's institutions of higher education.

3) Providing technical assistance and professional development to Virginia's educators to support implementation of the revised English and mathematics standards and the college and career ready performance expectations.
    - The Department has commitments from four state universities to pilot the establishment of professional development centers to support schools in their efforts to improve students' preparation for college and careers. These centers will provide coursework and ongoing teacher support for the content of the college and career ready performance expectations. The work will be accomplished through federal teacher training funds. As part of their work, the centers will develop sample capstone course materials so that teachers can teach secondary courses more effectively and be ready to teach the capstone courses when their divisions implement the programs.
    - The State Council of Higher Education for Virginia has issued a request for proposals that establishes as a priority support for teacher professional development on the performance expectations. These grants require collaboration between the school divisions being served , and four-year universities, to include colleges of education and arts and sciences within the higher education institutions. The program is funded with federal funds from Title IIa, *Elementary and Secondary Education Act.*
4) Aligning the state assessments to measure student mastery of the more rigorous mathematics and English standards adopted in 2009 and 2010.
    - As new tests in mathematics and English are developed to align to Virginia's revised *Standards of Learning*, certain high school end-of-course tests are being designed to include quantitative indicators of whether students have met or exceeded the achievement levels needed to be successful in introductory mathematics and English courses in college.

5) Identifying accountability measures and incentives for schools to increase the percentage of students who graduate high school having demonstrated the academic and career skills needed to be successful in postsecondary education programs.

- Virginia's Virginia Index of Performance (VIP) is an incentive program that rewards schools and school divisions for exceeding minimum accountability requirements. From its inception, the program has included indicators of college and career readiness. The board took action on proposed revisions to the program at the February 2011 meeting to provide additional incentives for school divisions and schools to strengthen incentives to increase students' college and career readiness, as well as promote student achievement in science, technology, engineering, and mathematics (STEM) areas.
- The Department has started a crucial dialogue with its partners in the higher education community and policy makers to determine whether it is appropriate to provide additional incentives to schools that make gains in increasing students' preparation for college. As well, there might be incentives available directly to students who meet or exceed Virginia's CCR Performance Expectations, with a particular focus on student groups who have been underrepresented in postsecondary education and training programs. For example, the Southern Regional Education Board (SREB) has recommended that Virginia's public postsecondary institutions adopt a policy that would permit direct enrollment in entry-level, credit-bearing college courses for students who meet or exceed the readiness performance standards on the eleventh-grade English reading and writing assessments and the Algebra II end-of-course assessment. The policy would exempt these students from additional placement or readiness testing, thereby reducing the costs and time associated with such testing. Further, this policy would afford more students the opportunity to earn credits towards college graduation.

Superintendent's Recommendation:
The Superintendent of Public Instruction recommends that the Board of Education accept the Report on Virginia's College and Career Readiness Initiative and *College and Career Ready Mathematics Performance Expectations.*

Impact on Resources:
The work to date has been completed with existing Department resources and consulting staff support from the SREB through a grant from the Bill and Melinda Gates Foundation. Funding for the SREB grant will end February 28, 2011. Additional responsibilities related to this activity may affect other Department services.

Timetable for Further Review/Action:
The Department will update the Board of Education as additional components and materials for Virginia's *College and Career Readiness Initiative* are developed.

Virginia's College and Career Ready
Mathematics Performance Expectations

The Mathematics Performance Expectations (MPE) define the content and level of achievement students must reach to be academically prepared for success in entry-level, credit-bearing mathematics courses in college. They were developed through a process that involved faculty from Virginia's two- and four-year colleges and universities, members of the business community, and high school mathematics educators. The MPE are organized into four interacting and overlapping strands that include content in the areas of algebra and functions, statistics, geometry, mathematical analysis, and trigonometry. This particular strand structure is one of several ways the performance expectations can be organized. The structure is not intended to be a curriculum organizer, as each expectation interacts with many others in a range of problem-solving, modeling, and decision-making situations.

Problem Solving, Decision Making, and Integration

Students will apply algebraic, geometric, and statistical concepts and the relationships among them to solve problems, model relations, and make decisions using data and situations within and outside of mathematics. In accomplishing this goal, students will develop and enhance a repertoire of skills and strategies for solving a variety of problem types.

1) Solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
2) Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.
3) Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
    a) investigating and using formulas for finding distance, midpoint, and slope;
    b) applying slope to verify and determine whether lines are parallel or perpendicular;
    c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
    d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.
4) Verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.
5) Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.
6) Use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

7) Use similar geometric objects in two- or three-dimensions to
    a) compare ratios between side lengths, perimeters, areas, and volumes;
    b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
    c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
    d) solve real-world problems about similar geometric objects.

8) Compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

9) Design and conduct an experiment/survey. Key concepts include
    a) sample size;
    b) sampling technique;
    c) controlling sources of bias and experimental error;
    d) data collection; and
    e) data analysis and reporting.

10) Investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the nth term, and evaluating summation formulas. Notation will include Σ and a_n.

11) Use angles, arcs, chords, tangents, and secants to
    a) investigate, verify, and apply properties of circles;
    b) solve real-world problems involving properties of circles; and
    c) find arc lengths and areas of sectors in circles.

Understanding and Applying Functions

Students will be able to recognize, use, and interpret various functions and their representations, including verbal descriptions, tables, equations, and graphs to make predictions and analyze relationships in solving complex, real-world mathematical problems.

12) Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

13) Investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

14) Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.

15) Use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

16) Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include
    a) continuity;
    b) local and absolute maxima and minima;
    c) domain and range, including limited and discontinuous domains and ranges;
    d) zeros;
    e) x- and y-intercepts;
    f) intervals in which a function is increasing or decreasing;
    g) asymptotes;
    h) end behavior;
    i) inverse of a function;
    j) composition of multiple functions;
    k) finding the values of a function for elements in its domain; and
    l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

17) Determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Procedure and Calculation

Students will be able to perform and justify steps in mathematical procedures and calculations and graph and solve a range of equations types. Students will reason from a variety of representations such as graphs, tables, and charts and will use displays of univariate data to identify and interpret patterns. Students will be able to calculate probabilities and analyze distributions of data to make decisions.

18) Given rational, radical, or polynomial expressions,
    a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
    b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
    c) write radical expressions as expressions containing rational exponents and vice versa; and
    d) factor polynomials completely.

19) Graph linear equations and linear inequalities in two variables, including
    a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and
    b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

20) Given a point other than the origin on the terminal side of an angle, use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Relate trigonometric functions defined on the unit circle to trigonometric functions defined in right triangles.

21) Given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.

22) Analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Use appropriate technology to create graphical displays.

23) Analyze the normal distribution. Key concepts include
    a) characteristics of normally distributed data;
    b) percentiles;
    c) normalizing data, using z-scores; and
    d) area under the standard normal curve and probability.

24) Describe orally and in writing the relationships between the subsets of the real number system.

25) Perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.

26) Solve, algebraically and graphically,
    a) absolute value equations and inequalities;
    b) quadratic equations over the set of complex numbers;
    c) equations containing rational algebraic expressions; and
    d) equations containing radical expressions.

Use graphing calculators for solving and for confirming the algebraic solutions.

27) Given one of the six trigonometric functions in standard form,
    a) state the domain and the range of the function;
    b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
    c) sketch the graph of the function by using transformations for at least a two-period interval; and
    d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

28) Find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This includes converting angle measures from radians to degrees and vice versa.

29) Investigate and identify the characteristics of conic section equations in (h, k) and standard forms. Use transformations in the coordinate plane to graph conic sections.

30) Using two-way tables, analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

31) Calculate probabilities. Key concepts include
    a) conditional probability;
    b) dependent and independent events;
    c) addition and multiplication rules;
    d) counting techniques (permutations and combinations); and
    e) Law of Large Numbers.

Verification and Proof

Students will recognize verification and proof as fundamental aspects of mathematical reasoning. Students will integrate and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements. This applies equally through simple mathematical calculations, in geometric applications, and more abstract statistical and algebraic processes. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

32) Use the relationships between angles formed by two lines cut by a transversal to
    a) determine whether two lines are parallel;
    b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
    c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

33) Given information in the form of a figure or statement, prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

34) Given information in the form of a figure or statement, prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.

35) Construct and justify the constructions of
    a) a line segment congruent to a given line segment;
    b) the perpendicular bisector of a line segment;
    c) a perpendicular to a given line from a point not on the line;
    d) a perpendicular to a given line at a given point on the line;
    e) the bisector of a given angle,
    f) an angle congruent to a given angle; and
    g) a line parallel to a given line through a point not on the given line.

36) Verify basic trigonometric identities and make substitutions, using the basic identities.

Attachment B

Comparison of Virginia's College and Career Ready Mathematics Performance Expectations with the Common Core State Standards for Mathematics

February 17, 2010

Common Core State Standards for Mathematics Mathematics Standards for High School[1] Number and Quantity	Virginia's Mathematics Performance Expectation
The Real Number System	
Extend the properties of exponents to rational exponents	
1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
Use properties of rational and irrational numbers	
3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**MPE.24 Describe orally and in writing the relationships between the subsets of the real number system.** **(SOL 8.2)**

[1] According to the *Common Core State Standards (CCSS) for Mathematics*, the CCSS high school standards specify the mathematics that all students should study in order to be college and career ready. The *CCSS for Mathematics* also includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics, as indicated by "(+)". This document includes all CCSS high school standards and the CCSS additional mathematics standards that align with Virginia's *College and Career Ready Mathematics Performance Expectations*.

Common Core State Standards for Mathematics Mathematics Standards for High School[1] Number and Quantity	Virginia's Mathematics Performance Expectation
Quantities	
Reason quantitatively and use units to solve problems	
1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**MPE.1 Solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.** **(SOL 8.3, 8.1b)**
2. Define appropriate quantities for the purpose of descriptive modeling.	
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
The Complex Number System	
Perform arithmetic operations with complex numbers	
1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	**MPE.25 Perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.** **(SOL AII.3)**
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	
Use complex numbers in polynomial identities and equations	
7. Solve quadratic equations with real coefficients that have complex solutions.	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.	

Common Core State Standards for Mathematics Mathematics Standards for High School[1] Number and Quantity	Virginia's Mathematics Performance Expectation
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	**MPE.13 Investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.** **(SOL AII.8)**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
Seeing Structure in Expressions	
Interpret the structure of expressions	
1. Interpret expressions that represent a quantity in terms of its context.	
a. Interpret parts of an expression, such as terms, factors, and coefficients.	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
Write expressions in equivalent forms to solve problems	
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	
a. Factor a quadratic expression to reveal the zeros of the function it defines.	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	graphically. Key concepts include **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)** **MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	MPE.26 Solve, algebraically and graphically, **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	MPE.18 Given rational, radical, or polynomial expressions, **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.** **(SOL AII.9)** **MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	**MPE.10 Investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the *n*th term, and**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	evaluating summation formulas. Notation will include Σ and a_n. **(SOL AII.2)**
Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials	
1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
Understand the relationship between zeros and factors of polynomials	
2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	containing rational numbers and variables, and expressions containing rational exponents; **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**MPE.13 Investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.** **(SOL AII.8)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric,**

Common Core State Standards for Mathematics Mathematics Standards for High School Algebra	Virginia's Mathematics Performance Expectation
	graphic, and algebraic. (SOL A.7) **(SOL AII.7)**
Use polynomial identities to solve problems	
4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
Rewrite rational expressions	
6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;** **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)**
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	
Creating Equations	
Create equations that describe numbers or relationships	
1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear*	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and**

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and quadratic functions, and simple rational and exponential functions.	**graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
2. Create equations in two or more variables to represent relationships between quantities; graph equations on	**MPE.19 Graph linear equations and linear inequalities in two variables, including**

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coordinate axes with labels and scales.	**a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and** **b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.** **(SOL A.6)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**MPE.17 Determine optimal values in problem situations by identifying constraints and using linear programming techniques.** **(SOL AFDA.5)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example,*	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;**

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rearrange Ohm's law V = IR to highlight resistance R.	**b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning	
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
Solve equations and inequalities in one variable	
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**MPE.17 Determine optimal values in problem situations by identifying constraints and using linear programming techniques.** **(SOL AFDA.5)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.**

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	(SOL AII.4)
4. Solve quadratic equations in one variable.	
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 =$ q that has the same solutions. Derive the quadratic formula from this form.	**MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
Solve systems of equations	
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a	**MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;**

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multiple of the other produces a system with the same solutions.	**b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.	**MPE.17 Determine optimal values in problem situations by identifying constraints and using linear programming techniques.** **(SOL AFDA.5)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
Represent and solve equations and inequalities graphically	
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	**MPE.12 Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.** **(AFDA.4)** **MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and**

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	behaviors of these functions. **(SOL AII.6)** **MPE.19 Graph linear equations and linear inequalities in two variables, including** **a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and** **b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.** **(SOL A.6)**
11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	**MPE.17 Determine optimal values in problem situations by identifying constraints and using linear programming techniques.** **(SOL AFDA.5)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**MPE.19 Graph linear equations and linear inequalities in two variables, including** **a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and**

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	b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line. **(SOL A.6)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**

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Interpreting Functions	
Understand the concept of a function and use function notation	
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	**MPE.10 Investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the *n*th term, and evaluating summation formulas. Notation will include Σ and a_n. (SOL AII.2)**

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Interpret functions that arise in applications in terms of the context	
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	**MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.15 Use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).** **(SOL AFDA.2)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain;**

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	(SOL A.7) and **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	**MPE.19 Graph linear equations and linear inequalities in two variables, including** **a) determining the slope of a line when given an equation of the**

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	line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and **b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.** **(SOL A.6)**
Analyze functions using different representations	
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric,**

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	graphic, and algebraic. (SOL A.7) **(SOL AII.7)**
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	**MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;**

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	h) end behavior; **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	**MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;**

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	i) inverse of a function; **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	**MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.15 Use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).** **(SOL AFDA.2)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;**

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	f) intervals in which a function is increasing or decreasing; **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)** **MPE.27 Given one of the six trigonometric functions in standard form,** **a) state the domain and the range of the function;** **b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;** **c) sketch the graph of the function by using transformations for at least a two-period interval; and** **d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.** **(SOL T.6)**
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	**MPE.18 Given rational, radical, or polynomial expressions,** **a) add, subtract, multiply, divide, and simplify rational algebraic expressions;** **b) add, subtract, multiply, divide, and simplify radical expressions**

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	containing rational numbers and variables, and expressions containing rational exponents; **c) write radical expressions as expressions containing rational exponents and vice versa; and** **d) factor polynomials completely.** **(SOL AII.1)** **MPE.26 Solve, algebraically and graphically,** **a) absolute value equations and inequalities;** **b) quadratic equations over the set of complex numbers;** **c) equations containing rational algebraic expressions; and** **d) equations containing radical expressions.** **Use graphing calculators for solving and for confirming the algebraic solutions.** **(SOL AII.4)**
b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.** **(SOL AII.9)** **MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and**

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	graphically. Key concepts include **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	**MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include**

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	a) continuity; (SOL AFDA.1) **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
Building Functions	
Build a function that models a relationship between two quantities	
1. Write a function that describes a relationship between two quantities.	
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous**
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the*	

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model.	**domains and ranges;**
c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*	**d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	**MPE.10 Investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the *n*th term, and evaluating summation formulas. Notation will include Σ and a_n.** **(SOL AII.2)**
Build new functions from existing functions	
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	**MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.15 Use knowledge of transformations to write an equation,**

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	given the graph of a function (linear, quadratic, exponential, and logarithmic). **(SOL AFDA.2)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
4. Find inverse functions.	
a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ for $x > 0$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*	**MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)**
b. (+) Verify by composition that one function is the inverse of	

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another.	**b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	
d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	
Linear, Quadratic, and Exponential Models	
Construct and compare linear and exponential models and solve problems	
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.	
a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.** **(SOL AII.9)** **MPE.12 Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and**
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	

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	words. Select and use appropriate representations for analysis, interpretation, and prediction. **(AFDA.4)** **MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)**
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.** **(SOL AII.9)** **MPE.10 Investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the *n*th term, and evaluating summation formulas. Notation will include Σ and a_n.** **(SOL AII.2)** **MPE.12 Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.** **(AFDA.4)** **MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic**

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	forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions. **(SOL AII.6)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)** **MPE.19 Graph linear equations and linear inequalities in two variables, including** **a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing**

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	slope as rate of change and determine if it is positive, negative, zero, or undefined; and **b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.** **(SOL A.6)**
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.** **(SOL AII.9)** **MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.15 Use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).** **(SOL AFDA.2)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;**

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	d) zeros; **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	**MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.** **(SOL AII.6)** **MPE.15 Use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).** **(SOL AFDA.2)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)**

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	b) local and absolute maxima and minima; (SOL AFDA.1) **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**
Interpret expressions for functions in terms of the situation they model	
5. Interpret the parameters in a linear or exponential function in terms of a context.	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.** **(SOL AII.9)** **MPE.14 Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and**

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	behaviors of these functions. **(SOL AII.6)** **MPE.15 Use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).** **(SOL AFDA.2)** **MPE.16 Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include** **a) continuity; (SOL AFDA.1)** **b) local and absolute maxima and minima; (SOL AFDA.1)** **c) domain and range, including limited and discontinuous domains and ranges;** **d) zeros;** **e) x- and y-intercepts;** **f) intervals in which a function is increasing or decreasing;** **g) asymptotes;** **h) end behavior;** **i) inverse of a function;** **j) composition of multiple functions;** **k) finding the values of a function for elements in its domain; (SOL A.7) and** **l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic. (SOL A.7)** **(SOL AII.7)**

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Trigonometric Functions	
Extend the domain of trigonometric functions using the unit circle	
1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	**MPE.11 Use angles, arcs, chords, tangents, and secants to** **a) investigate, verify, and apply properties of circles;** **b) solve real-world problems involving properties of circles; and** **c) find arc lengths and areas of sectors in circles.** **(SOL G.11)** **MPE.28 Find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This includes converting angle measures from radians to degrees and vice versa.** **(SOL T.3)**
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	**MPE.20 Given a point other than the origin on the terminal side of an angle, use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Relate trigonometric functions defined on the unit circle to trigonometric functions defined in right triangles.** **(SOL T.1)**
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for pi/3, pi/4 and pi/6, and use the unit circle to express the values of sine, cosines, and tangent for *x*, pi+*x*, and 2pi–*x* in terms of their values for *x*, where *x* is any real number.	**MPE.5 Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.** **(SOL G.8)** **MPE.20 Given a point other than the origin on the terminal side of an angle, use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Relate trigonometric functions**

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	defined on the unit circle to trigonometric functions defined in right triangles. **(SOL T.1)**
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	**MPE.27 Given one of the six trigonometric functions in standard form,** **a) state the domain and the range of the function;** **b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;** **c) sketch the graph of the function by using transformations for at least a two-period interval; and** **d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.** **(SOL T.6)**
Model periodic phenomena with trigonometric functions	
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	**MPE.27 Given one of the six trigonometric functions in standard form,** **a) state the domain and the range of the function;** **b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;** **c) sketch the graph of the function by using transformations for at least a two-period interval; and** **d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.** **(SOL T.6)**
Prove and apply trigonometric identities	
8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	**MPE.36 Verify basic trigonometric identities and make substitutions, using the basic identities.** **(SOL T.5)**

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Congruence	
Experiment with transformations in the plane	
1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	**MPE.3 Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include** **a) investigating and using formulas for finding distance, midpoint, and slope;** **b) applying slope to verify and determine whether lines are parallel or perpendicular;** **c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and** **d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.** **(SOL G.3)** **MPE.11 Use angles, arcs, chords, tangents, and secants to** **a) investigate, verify, and apply properties of circles;** **b) solve real-world problems involving properties of circles; and** **c) find arc lengths and areas of sectors in circles.** **(SOL G.11)** **MPE.32 Use the relationships between angles formed by two lines cut by a transversal to** **a) determine whether two lines are parallel;** **b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and** **c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.** **(SOL G.2)**
2. Represent transformations in the plane using, e.g.,	**MPE.3 Use pictorial representations, including computer**

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transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	**software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include** **a) investigating and using formulas for finding distance, midpoint, and slope;** **b) applying slope to verify and determine whether lines are parallel or perpendicular;** **c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and** **d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.** **(SOL G.3)**
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	**MPE.3 Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include** **a) investigating and using formulas for finding distance, midpoint, and slope;** **b) applying slope to verify and determine whether lines are parallel or perpendicular;** **c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and** **d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.** **(SOL G.3)** **MPE.32 Use the relationships between angles formed by two lines cut by a transversal to** **a) determine whether two lines are parallel;** **b) verify the parallelism, using algebraic and coordinate methods**

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	as well as deductive proofs; and **c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.** **(SOL G.2)**
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**MPE.3 Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include** **a) investigating and using formulas for finding distance, midpoint, and slope;** **b) applying slope to verify and determine whether lines are parallel or perpendicular;** **c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and** **d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.** **(SOL G.3)**
Understand congruence in terms of rigid motions	
6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	**MPE.3 Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include** **a) investigating and using formulas for finding distance, midpoint, and slope;** **b) applying slope to verify and determine whether lines are parallel or perpendicular;** **c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and** **d) determining whether a figure has been translated, reflected,**

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	rotated, or dilated, using coordinate methods. **(SOL G.3)** **MPE.33 Given information in the form of a figure or statement, prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.** **(SOL G.6)**
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	**MPE.33 Given information in the form of a figure or statement, prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.** **(SOL G.6)**
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
Prove geometric theorems	
9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	**MPE.32 Use the relationships between angles formed by two lines cut by a transversal to** **a) determine whether two lines are parallel;** **b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and** **c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.** **(SOL G.2)**
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	**MPE.33 Given information in the form of a figure or statement, prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.** **(SOL G.6)**
11. Prove theorems about parallelograms. *Theorems include:*	**MPE.4 Verify characteristics of quadrilaterals and use properties**

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opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	**of quadrilaterals to solve real-world problems.** **(SOL G.9)**
Make geometric constructions	
12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	**MPE.35 Construct and justify the constructions of** **a) a line segment congruent to a given line segment;** **b) the perpendicular bisector of a line segment;** **c) a perpendicular to a given line from a point not on the line;** **d) a perpendicular to a given line at a given point on the line;** **e) the bisector of a given angle;** **f) an angle congruent to a given angle; and** **g) a line parallel to a given line through a point not on the given line.** **(SOL G.4)**
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
Similarity, Right Triangles, and Trigonometry	
Understand similarity in terms of similarity transformations	
1. Verify experimentally the properties of dilations given by a center and a scale factor:	
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	**MPE.3 Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include** **a) investigating and using formulas for finding distance, midpoint, and slope;** **b) applying slope to verify and determine whether lines are parallel or perpendicular;** **c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and**
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	

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	d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods. **(SOL G.3)**
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	**MPE.7 Use similar geometric objects in two- or three-dimensions to** **a) compare ratios between side lengths, perimeters, areas, and volumes;** **b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;** **c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and** **d) solve real-world problems about similar geometric objects.** **(SOL G.14)** **MPE.34 Given information in the form of a figure or statement, prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.** **(SOL G.7)**
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	**MPE.34 Given information in the form of a figure or statement, prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.** **(SOL G.7)**
Prove theorems involving similarity	
4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	**MPE.34 Given information in the form of a figure or statement, prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.** **(SOL G.7)**
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	**MPE.7 Use similar geometric objects in two- or three-dimensions to** **a) compare ratios between side lengths, perimeters, areas, and**

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	volumes; **b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;** **c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and** **d) solve real-world problems about similar geometric objects.** **(SOL G.14)** **MPE.33 Given information in the form of a figure or statement, prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.** **(SOL G.6)** **MPE.34 Given information in the form of a figure or statement, prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.** **(SOL G.7)**
Define trigonometric ratios and solve problems involving right triangles	
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	**MPE.5 Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.** **(SOL G.8)**
7. Explain and use the relationship between the sine and cosine of complementary angles.	
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
Apply trigonometry to general triangles	
9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	**MPE.5 Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.** **(SOL G.8)**

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Circles G-C	
Understand and apply theorems about circles	
1. Prove that all circles are similar.	**MPE.11 Use angles, arcs, chords, tangents, and secants to** **a) investigate, verify, and apply properties of circles;** **b) solve real-world problems involving properties of circles; and** **c) find arc lengths and areas of sectors in circles.** **(SOL G.11)**
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	**MPE.4 Verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.** **(SOL G.9)** **MPE.35 Construct and justify the constructions of** **a) a line segment congruent to a given line segment;** **b) the perpendicular bisector of a line segment;** **c) a perpendicular to a given line from a point not on the line;** **d) a perpendicular to a given line at a given point on the line;** **e) the bisector of a given angle;** **f) an angle congruent to a given angle; and** **g) a line parallel to a given line through a point not on the given line.** **(SOL G.4)**
4. (+) Construct a tangent line from a point outside a given circle to the circle.	**MPE.35 Construct and justify the constructions of** **a) a line segment congruent to a given line segment;** **b) the perpendicular bisector of a line segment;** **c) a perpendicular to a given line from a point not on the line;** **d) a perpendicular to a given line at a given point on the line;** **e) the bisector of a given angle;** **f) an angle congruent to a given angle; and**

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	g) a line parallel to a given line through a point not on the given line. **(SOL G.4)**
Find arc lengths and areas of sectors of circles	
5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	**MPE.11 Use angles, arcs, chords, tangents, and secants to** **a) investigate, verify, and apply properties of circles;** **b) solve real-world problems involving properties of circles; and** **c) find arc lengths and areas of sectors in circles.** **(SOL G.11)**
Expressing Geometric Properties with Equations	
Translate between the geometric description and the equation for a conic section	
1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	**MPE.21 Given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.** **(SOL G.12)**
2. Derive the equation of a parabola given a focus and directrix.	**MPE.29 Investigate and identify the characteristics of conic section equations in (*h, k*) and standard forms. Use transformations in the coordinate plane to graph conic sections.** **(SOL MA.8)**
3. (+) Derive the equations of ellipses and hyperbolas given foci and directrices.	
Use coordinates to prove simple geometric theorems algebraically	
4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	**MPE.3 Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include** **a) investigating and using formulas for finding distance, midpoint, and slope;** **b) applying slope to verify and determine whether lines are parallel or perpendicular;**

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	c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and **d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.** **(SOL G.3)** **MPE.21 Given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.** **(SOL G.12)** **MPE.32 Use the relationships between angles formed by two lines cut by a transversal to** **a) determine whether two lines are parallel;** **b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and** **c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.** **(SOL G.2)**
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	**MPE.3 Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include** **a) investigating and using formulas for finding distance, midpoint, and slope;** **b) applying slope to verify and determine whether lines are parallel or perpendicular;** **c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and** **d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.** **(SOL G.3)**

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	MPE.32 Use the relationships between angles formed by two lines cut by a transversal to **a) determine whether two lines are parallel;** **b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and** **c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.** **(SOL G.2)**
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**MPE.3 Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include** **a) investigating and using formulas for finding distance, midpoint, and slope;** **b) applying slope to verify and determine whether lines are parallel or perpendicular;** **c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and** **d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.** **(SOL G.3)**
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	
Geometric Measurement and Dimension	
Explain volume formulas and use them to solve problems	
1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	**MPE.6 Use formulas for surface area and volume of three-dimensional objects to solve real-world problems.** **(SOL G.13)** **MPE.7 Use similar geometric objects in two- or three-dimensions to**

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	a) compare ratios between side lengths, perimeters, areas, and volumes; **b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;** **c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and** **d) solve real-world problems about similar geometric objects.** **(SOL G.14)** **MPE.11 Use angles, arcs, chords, tangents, and secants to** **a) investigate, verify, and apply properties of circles;** **b) solve real-world problems involving properties of circles; and** **c) find arc lengths and areas of sectors in circles.** **(SOL G.11)**
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	**MPE.7 Use similar geometric objects in two- or three-dimensions to** **a) compare ratios between side lengths, perimeters, areas, and volumes;** **b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;** **c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and** **d) solve real-world problems about similar geometric objects.** **(SOL G.14)**
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	**MPE.6 Use formulas for surface area and volume of three-dimensional objects to solve real-world problems.** **(SOL G.13)**
Visualize relationships between two-dimensional and three-dimensional objects	
4. Identify the shapes of two-dimensional cross-sections of	**MPE.29 Investigate and identify the characteristics of conic**

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three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**section equations in (*h, k*) and standard forms. Use transformations in the coordinate plane to graph conic sections. (SOL MA.8)**
Modeling with Geometry	
Apply geometric concepts in modeling situations	
1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	**MPE.6 Use formulas for surface area and volume of three-dimensional objects to solve real-world problems. (SOL G.13)** **MPE.7 Use similar geometric objects in two- or three-dimensions to** **a) compare ratios between side lengths, perimeters, areas, and volumes;** **b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;** **c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and** **d) solve real-world problems about similar geometric objects. (SOL G.14)**
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	

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Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on a single count or measurement variable	
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	**MPE.22 Analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Use appropriate technology to create graphical displays.** **(SOL PS.1)**
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	**MPE.8 Compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.** **(SOL PS.3)** **MPE.22 Analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Use appropriate technology to create graphical displays.** **(SOL PS.1)** **MPE.23 Analyze the normal distribution. Key concepts include** **a) characteristics of normally distributed data;** **b) percentiles;** **c) normalizing data, using z-scores; and** **d) area under the standard normal curve and probability.** **(SOL AFDA.7)**
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	
Summarize, represent, and interpret data on two categorical and quantitative variable	
5. Summarize categorical data for two categories in two-way	**MPE.30 Using two-way tables, analyze categorical data to**

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frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	**describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.** **(SOL PS.7)**
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.*	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.** **(SOL AII.9)**
b. Informally assess the fit of a function by plotting and analyzing residuals.	
c. Fit a linear function for a scatter plot that suggests a linear association.	
Interpret linear models	
7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.** **(SOL AII.9)** **MPE.19 Graph linear equations and linear inequalities in two variables, including** **a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and** **b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.** **(SOL A.6)**

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8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions. (SOL AII.9)**
9. Distinguish between correlation and causation.	**MPE.2 Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions. (SOL AII.9)**
Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments	
1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	**MPE.9 Design and conduct an experiment/survey. Key concepts include** **a) sample size;** **b) sampling technique;** **c) controlling sources of bias and experimental error;** **d) data collection; and** **e) data analysis and reporting.** **(SOL AFDA.8)**
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	
Make inferences and justify conclusions from sample surveys, experiments, and observational studies	
3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	**MPE.9 Design and conduct an experiment/survey. Key concepts include** **a) sample size;** **b) sampling technique;** **c) controlling sources of bias and experimental error;** **d) data collection; and**
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	

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5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	**e) data analysis and reporting.** **(SOL AFDA.8)**
6. Evaluate reports based on data.	
Conditional Probability and the Rules of Probability	
Understand independence and conditional probability and use them to interpret data	
1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	**MPE.31 Calculate probabilities. Key concepts include** **a) conditional probability;** **b) dependent and independent events;** **c) addition and multiplication rules;** **d) counting techniques (permutations and combinations); and** **e) Law of Large Numbers.** **(SOL AFDA.6)**
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects*	**MPE.30 Using two-way tables, analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.** **(SOL PS.7)**

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and compare the results.	
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	**MPE.31 Calculate probabilities. Key concepts include** **a) conditional probability;** **b) dependent and independent events;** **c) addition and multiplication rules;** **d) counting techniques (permutations and combinations); and** **e) Law of Large Numbers.** **(SOL AFDA.6)**
Use the rules of probability to compute probabilities of compound events in a uniform probability model	
6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.	**MPE.31 Calculate probabilities. Key concepts include** **a) conditional probability;** **b) dependent and independent events;** **c) addition and multiplication rules;** **d) counting techniques (permutations and combinations); and** **e) Law of Large Numbers.** **(SOL AFDA.6)**
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.	
8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	
Using Probability to Make Decisions	
Calculate expected values and use them to solve problems	
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	**MPE.31 Calculate probabilities. Key concepts include** **a) conditional probability;** **b) dependent and independent events;** **c) addition and multiplication rules;** **d) counting techniques (permutations and combinations); and** **e) Law of Large Numbers.** **(SOL AFDA.6)**
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per*	

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household. How many TV sets would you expect to find in 100 randomly selected households?	
Use probability to evaluate outcomes of decisions	
5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.	
a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.*	**MPE.31 Calculate probabilities. Key concepts include** **a) conditional probability;** **b) dependent and independent events;** **c) addition and multiplication rules;** **d) counting techniques (permutations and combinations); and** **e) Law of Large Numbers.** **(SOL AFDA.6)**
b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*	
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	