Virginia's College and Career Ready Initiative
Grade 12 Mathematics Capstone Course Content and Performance Expectations

Course Purpose and Description

Virginia's College and Career Ready Mathematics Performance Expectations grade 12 capstone course contains high-interest contextualized content designed to give certain students an additional boost for competent and successful entry into college and careers. The course will add to students' preparation for college and the workplace by 1) enhancing skills in number and quantity, functions and algebra, geometry, and statistics and probability; and 2) simultaneously reinforcing readiness skills and dispositions in adaptability and flexibility, creativity and innovation, leadership, team work, collaboration, and work ethic.

The course will augment skills in applied mathematical concepts through mathematical investigations targeting outcomes defined in *Virginia's College and Career Ready Mathematics Performance Expectations* (MPE). Students will research, collect, and analyze data; develop and support ideas and conjectures; investigate, evaluate, and incorporate appropriate resources; and determine appropriate problem-solving approaches and decision-making algorithms in a variety of real-world contexts and applied settings.

The mathematics capstone course is designed for high school seniors who

- have satisfactorily completed the required mathematics courses based on the Standards of Learning including Algebra, Functions, and Data Analysis or Algebra II;
- have earned at least two verified credits in mathematics; and
- are college intending, but may not be fully college ready. The course may also support students who meet the same academic requirements but plan to enter the work force (prepared for further work force training) directly after graduating from high school.

General Content Goals for the Mathematics Capstone Course

The grade 12 mathematics capstone course will meet the following goals:

- Students will apply algebraic, geometric, and statistical concepts and the relationships among them to solve problems, model relations, and make decisions using data and situations within and outside of mathematics.
- Students will recognize, use, and interpret various functions and their representations, including verbal descriptions, tables, equations, and graphs to make predictions and analyze relationships in solving complex, real-world mathematical problems.
- Students will perform and justify steps in mathematical procedures and calculations and graph and solve a range of equations types.
- Students will reason from a variety of representations such as graphs, tables, and charts and will use displays of univariate data to identify and interpret patterns.
- Students will be able to calculate probabilities and analyze distributions of data to make decisions.
- Students will recognize verification and proof as fundamental aspects of mathematical reasoning. They will integrate and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

Mathematics Capstone Course Program Objectives

The grade 12 mathematics capstone course program will:

- Integrate the College and Career Ready Mathematics Performance Expectations into a skill-enhancing program of intensive mathematical investigation, problem solving, decision making, and presentation through student collaboration;
- Build upon topics both provided by the teacher and generated by students;
- Use and reinforce content and data from a spectrum of real-world data sources (e.g., natural resource, environmental, weather, agricultural, energy-use and production, economic, labor, population and demographic, scientific, media, sports, entertainment, and health data);
- Use presentation and other communication technologies to develop, refine, and share solutions, ideas, and problems;
- Require research using technology, interviews, and traditional print resources;
- Require high-interest, high-level problem solving, decision making, analysis, and critical thinking, and evaluation in content and applied contexts;
- Enhance students' desire to use mathematics as a systematic decision-making tool; and
- Align with the division/school curriculum.

Capstone Course Content and Delivery

The grade 12 mathematics capstone course will:

- Comprise an intensive course designed to augment certain twelfth-grade students' skills in mathematics necessary for college and career readiness;
- Use *Virginia's College and Career Ready Mathematics Performance Expectations* and pertinent Virginia college and career readiness documents to develop a detailed core syllabus for a full-year course;
- Be based on a range of task modules requiring intensive mathematical analysis and problem solving, research, use of technology, individual and group performances and presentations, and other modern college and career skills while applying academic content;
- Utilize real-world mathematics problems, tasks, and decision-making scenarios that will:
    - ✓ meet *Virginia's College and Career Ready Mathematics Performance Expectations*;
    - ✓ be based on high interest topics;
    - ✓ use practical applications from public domain data sites;
    - ✓ draw additional content from a full range of general and applied academic disciplines and professional communities;
    - ✓ demonstrate spiraling of content and increasing skill complexity;
    - ✓ provide opportunities for students' individual and small- and large-group work;
    - ✓ require systematic research;
    - ✓ utilize word processing, presentation software, and graphics applications;
    - ✓ require the creation and analysis of images, graphs, charts, and tables; and
    - ✓ require recognized standards for source documentation in final products.

The grade 12 mathematics capstone course will **NOT**:

- focus on discrete mathematics skills in isolation of meaningful content or purpose; and
- serve as a program to remediate below-proficient skill attainment.

Virginia's College and Career Ready
Mathematics Performance Expectations and the Capstone Course Focus

Virginia's College and Career Ready Mathematics Performance Expectations (MPE) define the content and level of achievement students must reach to be academically prepared for success in entry-level, credit-bearing mathematics courses in college. A subset of the 2009 Mathematics Standards of Learning, the MPE were selected through a process that involved faculty from Virginia's two- and four-year colleges and universities and high school mathematics educators. The MPE are organized into four interacting strands that include content in the areas of algebra and functions, statistics, geometry, mathematical analysis, and trigonometry. This particular strand structure is one of several ways the performance expectations can be organized. The structure is not intended to be a curriculum organizer, as each expectation interacts with many others in a range of problem-solving, modeling, and decision-making situations.

Virginia's mathematics capstone course is driven by a high-priority subset of the *Mathematics College and Career Ready Performance Expectations*. The core focus expectations were identified by the MPE validation committee as high-priority expectations for success in college and further career training. It is not intended that the capstone course introduce or explicitly re-teach the full set of performance expectations. Students should have a strong level of proficiency with most of the expectations when they complete their grade 11 mathematics coursework. The core-focus expectations for the capstone course are indicated by the capstone symbol ▲ in the attached list, but the course should meet additional mathematics objectives and enhance readiness skills and dispositions as defined by the local curriculum.

Problem Solving, Decision Making, and Integration

Students will apply algebraic, geometric, and statistical concepts and the relationships among them to solve problems, model relations, and make decisions using data and situations within and outside of mathematics. In accomplishing this goal, students will develop and enhance a repertoire of skills and strategies for solving a variety of problem types.

▲ 1) Solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

▲ 2) Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

▲ 3) Use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include

   a) investigating and using formulas for finding distance, midpoint, and slope;
   b) applying slope to verify and determine whether lines are parallel or perpendicular;
   c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
   d) determining whether a figure has been translated, reflected, rotated, or

dilated, using coordinate methods.

- 4) Verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.
- 5) Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.
- 6) Use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
- 7) Use similar geometric objects in two- or three-dimensions to
    a) compare ratios between side lengths, perimeters, areas, and volumes;
    b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
    c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
    d) solve real-world problems about similar geometric objects.
- 8) Compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.
- 9) Design and conduct an experiment/survey. Key concepts include
    a) sample size;
    b) sampling technique;
    c) controlling sources of bias and experimental error;
    d) data collection; and
    e) data analysis and reporting.
- 10) Investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the *n*th term, and evaluating summation formulas. Notation will include Σ and a_n.
- 11) Use angles, arcs, chords, tangents, and secants to
    a) investigate, verify, and apply properties of circles;
    b) solve real-world problems involving properties of circles; and
    c) find arc lengths and areas of sectors in circles.

Understanding and Applying Functions

Students will be able to recognize, use, and interpret various functions and their representations, including verbal descriptions, tables, equations, and graphs to make predictions and analyze relationships in solving complex, real-world mathematical problems.

- 12) Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.
- 13) Investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.
- 14) Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the

shapes and behaviors of these functions.

- 15) Use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).
- 16) Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include
    a) continuity;
    b) local and absolute maxima and minima;
    c) domain and range, including limited and discontinuous domains and ranges;
    d) zeros;
    e) x- and y-intercepts;
    f) intervals in which a function is increasing or decreasing;
    g) asymptotes;
    h) end behavior;
    i) inverse of a function;
    j) composition of multiple functions;
    k) finding the values of a function for elements in its domain; and
    l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.
- 17) Determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Procedure and Calculation

Students will be able to perform and justify steps in mathematical procedures and calculations and graph and solve a range of equations types. Students will reason from a variety of representations such as graphs, tables, and charts and will use displays of univariate data to identify and interpret patterns. Students will be able to calculate probabilities and analyze distributions of data to make decisions.

- 18) Given rational, radical, or polynomial expressions,
    a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
    b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
    c) write radical expressions as expressions containing rational exponents and vice versa; and
    d) factor polynomials completely.
- 19) Graph linear equations and linear inequalities in two variables, including
    a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describing slope as rate of change and determine if it is positive, negative, zero, or undefined; and
    b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.
- 20) Given a point other than the origin on the terminal side of an angle, use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Relate trigonometric functions defined on the unit circle to trigonometric functions

defined in right triangles.

21) Given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.
22) Analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Use appropriate technology to create graphical displays.
23) Analyze the normal distribution. Key concepts include
    a) characteristics of normally distributed data;
    b) percentiles;
    c) normalizing data, using z-scores; and
    d) area under the standard normal curve and probability.
24) Describe orally and in writing the relationships between the subsets of the real number system.
25) Perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.
26) Solve, algebraically and graphically,
    a) absolute value equations and inequalities;
    b) quadratic equations over the set of complex numbers;
    c) equations containing rational algebraic expressions; and
    d) equations containing radical expressions. Use graphing calculators for solving and for confirming the algebraic solutions.

27) Given one of the six trigonometric functions in standard form,
    a) state the domain and the range of the function;
    b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
    c) sketch the graph of the function by using transformations for at least a two-period interval; and
    d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.
28) Find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This includes converting angle measures from radians to degrees and vice versa.
29) Investigate and identify the characteristics of conic section equations in (h, k) and standard forms. Use transformations in the coordinate plane to graph conic sections.
30) Using two-way tables, analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.
31) Calculate probabilities. Key concepts include
    a) conditional probability;
    b) dependent and independent events;
    c) addition and multiplication rules;
    d) counting techniques (permutations and combinations); and
    e) Law of Large Numbers.

Verification and Proof

Students will recognize verification and proof as fundamental aspects of mathematical reasoning. Students will integrate and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements. This applies equally through simple mathematical calculations, in geometric applications, and more abstract statistical and algebraic processes. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

32) Use the relationships between angles formed by two lines cut by a transversal to
    a) determine whether two lines are parallel;
    b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
    c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.
33) Given information in the form of a figure or statement, prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.
34) Given information in the form of a figure or statement, prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.
35) Construct and justify the constructions of
    a) a line segment congruent to a given line segment;
    b) the perpendicular bisector of a line segment;
    c) a perpendicular to a given line from a point not on the line;
    d) a perpendicular to a given line at a given point on the line;
    e) the bisector of a given angle,
    f) an angle congruent to a given angle; and
    g) a line parallel to a given line through a point not on the given line.
36) Verify basic trigonometric identities and make substitutions, using the basic identities.