Introduction

In this section, the lessons focus on the geometry and measurement of three-dimensional figures including surface area and volume of prisms, cylinders, cones and pyramids. They will investigate and describe how changing the measure of one attribute affects the volume and surface area.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion.*

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

Standards of Learning

The following Standards of Learning are addressed in this section:

5.8 The student will
- a) find perimeter, area, and volume in standard units of measure;
- b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;

6.10 The student will
- d) describe and determine the volume and surface area of a rectangular prism.

7.5 The student will
- a) describe volume and surface area of cylinders;
- b) solve practical problems involving the volume and surface area of rectangular prisms and cylinders; and
- c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.

8.7 The student will
- a) investigate and solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids; and
- b) describe how changing one measured attribute of a figure affects the volume and surface area.

8.9 The student will construct a three-dimensional model, given the top or bottom, side, and front views.

Table of Contents

Lesson plans pertaining to the following Standards of Learning are found in this section. Click (or CTRL+click) on each to jump to that lesson.

✱ SOL 5.8b

Lesson Summary

Students differentiate among perimeter, area, and volume and identify which concept is appropriate for a given situation. (two 45-minute periods)

Materials

Pattern blocks or color tiles
Grid paper
Linking cubes
Copies of the attached worksheets
Card stock
Scissors
Overhead projector and transparencies or an LCD projector and drawing software (optional)

Warm-up

1. <u>Perimeter</u>: Demonstrate the concept of perimeter by tracing on grid paper the outline of a pattern block or a figure made of color tiles. You may wish to use an overhead projector and transparencies or an LCD projector and drawing software to do this. Describe perimeter as the “distance around a figure,” and demonstrate this by tracing around the figure again. Have students sketch the resulting polygon and estimate the perimeter, given the units on the grid paper.
2. <u>Area</u>: Ask students to estimate in square grid-paper units the area of the same figure. Have them share their strategies and estimates. Ask which strategies work and how they can write the area in square units.
3. <u>Volume</u>: Hold up a rectangular prism made of linking cubes, and put a similar model on each table for students to manipulate. Ask students how many cubic units fill this solid. Substitute cubic centimeters or cubic inches for cubic units, as appropriate. Ask how they can write the volume of this prism in cubic units?
4. Have students record definitions of *perimeter*, *area*, and *volume* in their math logs, and post these definitions on a chart or on the board.
5. Distribute copies of the attached “Banking Business” worksheet, and have students complete it. Have a few students share what they wrote.

Lesson

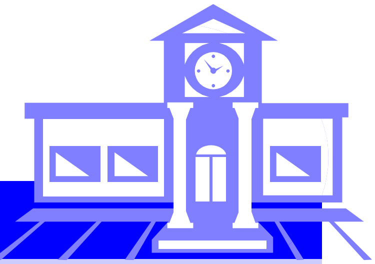
1. Distribute copies of the attached “Word Frames” worksheet, and have students complete each frame according to the printed directions. Give assistance as necessary.
2. Copy the attached Matching Cards on card stock, and cut them out. Have students use the cards to play a Matching Game by the rules listed below to practice measurement concepts.

   <u>Rules</u>: Turn all cards face down on the table in two rows—one row of situation cards and one row of word cards. Each player takes a turn simultaneously turning over two cards, one from each row, to see if they match. If they match, the player gets to keep them. If they do not match, they are returned face down to the table. Play continues until all cards are matched. The player with the most matches wins.

Reflection

Have students complete the “Perimeter, Area, or Volume?” worksheet and/or the “Sketch” worksheet.

Name: ____________________________



Banking Business

BANK of WORDS

paint	**water**	**bed**	**dirt**	**roof**	**grass**
carpeting	**cover**	**table**	**boxes**	**chalk**	**tile**
flower bed	**jar**	**barrel**	**shingles**	**fill**	**string**
wallpaper	**room**	**kite**	**sidewalk**	**rope**	**bucket**
swimming pool	**bowl**	**wood**	**cat**	**tractor**	**lawn**
fish tank	**cup**	**fence**	**dog**	**mow**	**garden**

1. Go to the Bank of Words above, and withdraw at least three words to use in a sentence about a perimeter-measurement situation. Scratch the words out of the bank to indicate they have been withdrawn. Write your sentence here:

   ____

   ____

2. Return to the bank for another withdrawal of three or more words to use in a sentence about an area-measurement situation. Scratch the words out of the bank to indicate they have been withdrawn. Write your sentence here:

   ____

   ____

3. Make a final withdrawal of three or more words to use in a sentence about a volume-measurement situation. Scratch the words out of the bank to indicate they have been withdrawn. Write your sentence here:

   ____

   ____

4. Now that you have spent at least nine words out of your bank account, prepare to show how you spent them by sharing your sentences with the class.

Name: ___________________________

Word Frames

Complete the words frames below for the words *perimeter*, *area*, and *volume*.

1. Write an example of the term in the lower left corner of the frame.
2. Draw a sketch illustrating the term in the upper right corner.
3. Write a definition of the term in the upper left corner.
4. Write a sentence using the term in the lower right corner.

Compare your frames with those of a partner. How are they similar and different? Explain:

____

____

Definition		Sketch
	perimeter	
Example		Non-example

Definition		Sketch
	area	
Example		Non-example

Definition		Sketch
	volume	
Example		Non-example

Matching Cards

perimeter	How many yards of fencing will you need to fence your back yard?
perimeter	How many feet of string will you need to make an outline of where your vegetable garden will be planted?
perimeter	How many meters will you walk when you walk around the outside of your school?
perimeter	How many tiles do you need to frame your picture?

perimeter	How many feet of fencing will you need to use to make a dog pen?
perimeter	Old MacDonald has 40 feet of fencing to build a pig pen. What are the dimensions of the pen that will give the pigs the most room?
perimeter	How many gallons of paint will it take to paint your bedroom walls?
perimeter and area	How many square feet of lawn do you have to mow?

area	How much of the floor does the rug cover?
area	How large of a book cover do you need to cover your math book?
area	How many square feet of tile will you need to cover the new bathroom floor?
area	How much water does the swimming pool hold?

area	Will that shipping crate hold the birthday present you bought to send to Grandma?
area	What is the distance of the dirt bike path around the park?
area	Will that tablecloth fit the table?
volume	How much trash will the trash can hold?

volume	How much water will you need to fill your fish tank?
volume	How much room do you need on the sidewalk to play hopscotch or 4-square?
volume	Will you be able to pack all of your school books in that box?
volume	How far is it to run twice around my block?

Name: ______________________

Perimeter, Area, or Volume?

Draw a line from each unit of measurement listed below to the name of the item it can measure.

1 inch	1 square inch	1 cubic inch
rug	water in a fish tank	border of a picture frame

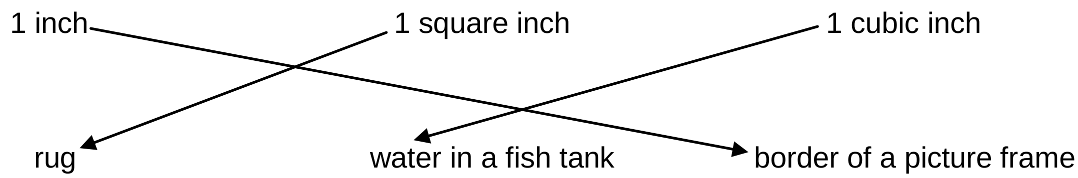
The questions below ask for a measurement of *perimeter* (P), *area* (A), or *volume* (V). Fill in the blank by each question with a letter (P, A, or V) to identify what kind of measurement it is asking for.

1. How much fencing is needed to enclose a garden? _______
2. How much air will fill your bike tire? _______
3. How much wallpaper do you need to cover one bedroom wall? _______
4. How much rainwater can be collected in the barrel? _______
5. How long is the yellow police tape around the crime scene? _______
6. How much material do you need to make a quilt? _______
7. What is the length around the track? _______
8. Which jewelry box will hold the most? _______
9. How much wrapping paper do you need to cover the present? _______
10. How long is the border around that bulletin board? _______

Name: ANSWER KEY

Perimeter, Area, or Volume?

Draw a line from each unit of measurement listed below to the name of the item it can measure.



The questions below ask for a measurement of *perimeter* (P), *area* (A), or *volume* (V). Fill in the blank by each question with a letter (P, A, or V) to identify what kind of measurement it is asking for.

1. How much fencing is needed to enclose a garden? ___P___
2. How much air will fill your bike tire? ___V___
3. How much wallpaper do you need to cover one bedroom wall? ___A___
4. How much rainwater can be collected in the barrel? ___V___
5. How long is the yellow police tape around the crime scene? ___P___
6. How much material do you need to make a quilt? ___A___
7. What is the length around the track? ___P___
8. Which jewelry box will hold the most? ___V___
9. How much wrapping paper do you need to cover the present? ___A___
10. How long is the border around that bulletin board? ___P___

Name: ________________________

Sketch

Draw a picture of a real-life scene that includes three examples each of perimeter, area, and volume. Record these objects or situations in the chart below your sketch. Add details and color to your sketch, and be prepared to share your examples of perimeter, area, and volume with a partner.

Title: ____

Object in My Picture	Perimeter, Area, or Volume?

Explain how perimeter, area, and volume are different.

✱ SOL 6.10d

Lesson Summary

Students measure the surface area and volume of solid figures in order to rank them least to greatest. Students practice measuring square and cubic units, comparing and contrasting the processes. Various strategies emerge for measuring area and volume, pointing to future work with formulas. (45 minutes)

Materials

Linking cubes
Labeled plastic zip-top bags
Copies of the attached "Volume and Surface Area" worksheet

Warm-up

1. Display two solids made from linking cubes. One might be a rectangular prism while the other might be an irregular solid. Ask the students to sketch and compare these two solid figures. Ask: "Which has the greater surface area? Which has the greater volume? How can you tell?" Allow students to manipulate the two solids you made. Have them write down strategies in their math logs for determining surface area and volume as well as for comparing the surface areas and volumes of the two solids.
2. Have students share their various strategies for determining surface area and volume. Ask them to decide which figure has the greater surface area and which has the greater volume. Reinforce that square units describe surface area and cubic units describe volume.

Lesson

1. Use linking cubes to create at least eight solids, including a cube, a rectangular prism that is not a cube, and several irregular solids. Place each one in a plastic zip-top bag, label the bags with letters, and place them in a prominent spot in the front of the room.
2. Give each student a copy of the attached "Volume and Surface Area" worksheet.
3. Ask pairs of students to discuss how to rank the solids in the bags from least to greatest by volume. Have each pair work together to form a hypothesis and rank the solids by writing down the bag numbers in order.
4. Repeat step 3 for surface area.
5. Tell the class they are going to check their hypotheses by measuring. Give each pair of students one of the bagged solids, and challenge them to figure out how to measure the surface area of the solid by counting the number of square units covering its surface. Then, have them figure out how to measure the volume of the solid by counting the number of cubic units, or cubes, it contains. Instruct students to record their answers on the worksheets.
6. Rotate the bags among the pairs of students every three minutes. After all solids have been measured by all pairs, have each pair share their measurement of the last solid they measured and explain their measurement strategy.
7. Have students arrange the bagged solids in order from least to greatest by volume. Next, have them rearrange the bags from least to greatest by surface area. Discuss what they notice about this process.

Reflection

Display a 2 x 3 x 4 rectangular prism made from linking cubes. Ask students to sketch this solid and explain in writing how to measure its surface area and volume. Also, ask them to explain in writing how measuring surface area and volume are similar and how they are different.

Name: ______________________

Volume and Surface Area

1. Rank the linking-cube solids from least to greatest by volume. (Keep in mind that volume is measured by the number of *cubic units* making up the solid.)

Least ———————————————————→ ***Greatest***
Solid: ______ ______ ______ ______ ______ ______ ______ ______

2. Rank the linking-cube solids from least to greatest by surface area. (Keep in mind that surface area is measured by the number of *square units* covering the surface of the solid.)

Least ———————————————————→ ***Greatest***
Solid: ______ ______ ______ ______ ______ ______ ______ ______

3. Measure each solid, and record your data in the chart below.

Solid	Volume (cubic units)	Surface area (square units)	Rank L to G by volume	Rank L to G by surface area
A				
B				
C				
D				
E				
F				
G				
H				

4. Explain your method for measuring the volume of each solid.

5. Explain your method for measuring the surface area of each solid.

6. What surprised you about the final rankings of the solids?

7. Share one thing you learned while working with your partner on this problem.

✱ SOL 6.10d, 7.5a

Lesson Summary

Students devise a method for measuring the volume and surface area of a gelatin box and a toilet paper roll. (two 45-minute periods)

Materials

Empty gelatin boxes
Toilet paper rolls
Grid paper
Construction paper
Scissors
Glue
Rice
Measuring cup
Copies of the attached worksheets

Warm-up

Display a gelatin box and toilet paper roll, and ask students what types of solids they represent (rectangular prism and cylinder). Sketch each solid on the board, and write their dimensions (length, width, and height of the gelatin box and radius and height of the toilet paper roll). Ask which solid has the greater volume, that is, if both were filled with rice, which one would hold more rice. Have students write down their hypotheses and their reasoning and then share these hypotheses and reasonings in a class discussion. Then, demonstrate the answer by filling each container with rice and measuring each amount of rice in a measuring cup.

Lesson

Measuring Volume

1. Give each pair of students a gelatin box and a toilet paper roll. Ask the class how they could measure the exact volume of each object without filling it? Discuss methods of figuring out how many cubic centimeters would fill each container. Students might suggest stacking the bases to fill the containers. Record all student-developed formulas, and circle those that would work.
2. Demonstrate how to find the volume of the each object by measuring the area of its base and multiplying this area by its height.
3. Have students compare each exact volume measurement found using a formula to the estimate obtained by filling each container with rice during the warm-up. Ask whether the class estimates were reasonable.
4. Ask how measuring the volume of a rectangular prism and measuring the volume of a cylinder are similar and different.

Measuring Surface Area

5. Ask the students to look at the *surface* of each object. Ask which object would take more paper to wrap it with no overlaps. Have students write a hypothesis and a rationale and then share their hypotheses and reasonings.
6. Ask the class how the amount of paper needed to cover each figure could be measured. Have students share their ideas.
7. Suggest that one method to determine the surface area of each solid is by constructing a net for the solid and counting how many square units make up the net.
8. Have students trace the faces of a gelatin box on centimeter grid paper, cut out the six tracings, arrange them on construction paper into a net that will fold up into a box, and glue them down on the construction paper.

9. Have students devise a method for calculating the number of square units that make up the net, and have them write down their calculations and methods in words in a box next to the net.
10. Have students trace the two circular bases of the toilet paper roll on grid paper. Then, have them cut open the lateral curved face of the roll, flatten it out, and trace it on the grid paper. Have them cut out these three tracings, arrange them on construction paper into a net, and glue them down on the construction paper.
11. Have students devise a method for calculating the number of square units that make up the net, and have them write down their calculations and methods in words in a box next to the net. If students have difficulty with this, prompt by asking how they might use their knowledge of calculating the area of a circle.
12. Ask students to explain how they calculated the surface area of each object, and write on the board the different methods used. Discuss the methods, asking whether each one would always work. Circle the ones that will always work, and show how to translate the methods expressed in words to formulas:

$$SA_{\text{surface area prism}} = 2lw + 2lh + 2wh$$

$$SA_{\text{surface area cylinder}} = 2\pi rh + 2\pi r^2$$

13. Show how to organize the formula for the surface area of a rectangular prism into a graphic organizer. Have students use the worksheet or their math logs to practice calculating the surface areas of several different rectangular prisms.
14. Use the attached worksheets for additional practice in follow-up lessons, as needed by your students.

Reflection

1. Have students answer the two SOL questions about volume and surface area on the attached “Reflection” worksheet. After they are finished, have students pair up to do a Think-Pair-Share. Then, ask for volunteers to share their solutions with calculations. If time permits, have each pair write another problem by changing the dimensions of the box, and have pairs exchange problems to solve.

Name: ______________________

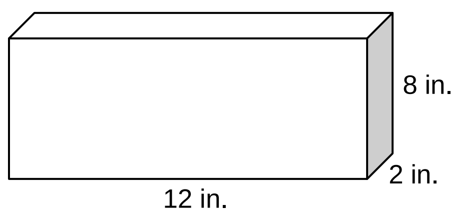
Reflection

Answer the following released SOL test questions about volume and surface area:

1. A cylinder-shaped barrel has a diameter of 3 feet and a height of 4.5 feet. If the barrel is empty, which measurement is closest to the minimum amount of water needed to completely fill the barrel?

   A 32 cu ft.
   B 49 cu ft.
   C 71 cu ft.
   D 98 cu ft.

2. Carl wants to covering this rectangular-prism-shaped box with wrapping paper. It has the dimensions shown:



   What is the minimum amount of wrapping paper Carl needs to cover the entire box?

   A 96 sq in.
   B 136 sq in.
   C 192 sq in.
   D 272 sq in.

Name: ________________________

Ranking Prisms and Cylinders by Volume

1. Given four solids (prisms and cylinders) lettered A through D, estimate the volume of each, and rank them from least to greatest by volume. Record your ranking:

Least ⟶ ***Greatest***				
Solid:	_______	_______	_______	_______

2. Sketch each of the four solids in the appropriate boxes in the chart below.
3. Measure the dimensions of the solids, and record the measurements in the chart.
4. Label the units you used to measure.
5. Find the area of the base of each solid, and record.
6. Multiply the area of the base by the height to find the volume (use $\pi \approx 3.14$), and record.
7. Rank the four solids from least to greatest by volume.

Sketch	Dimensions	Area of base (square units)	Height	Volume (cubic units)	Rank L to G by volume
Solid A					
Solid B					
Solid C					
Solid D					

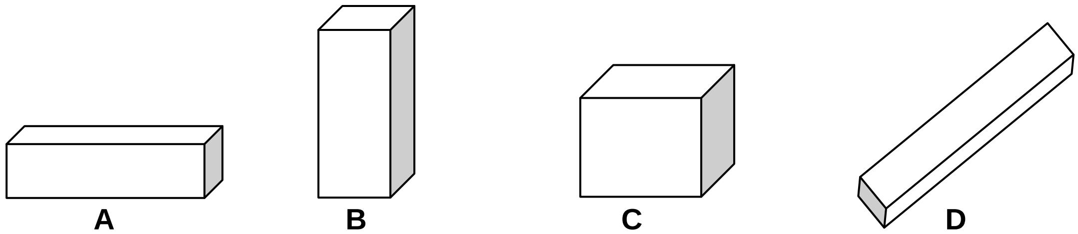
8. Did your ranking change after you found the volume of each solid? ______ Did any of the measurements surprise you? ______. Explain:

9. Explain how measuring the volume of a prism and measuring the volume of a cylinder are similar and different.

Name: ______________________

Which Prism Holds the Most?

1. The boxes shown below are examples of rectangular prisms. Note that they are *not* drawn to scale, so you must look at their dimensions in the chart. Circle the box that you think would hold the *most*. Explain your choice.



Rectangular Prism	Length (cm)	Width (cm)	Height (cm)
A	9	2	3
B	4	3	8
C	5	5	4
D	12	2	2

2. Rank of the prisms from *greatest to least* by estimated volume.

Greatest → ***Least***

Solid: _______ _______ _______ _______

3. How can you calculate the exact volume of each rectangular prism?

____

4. Test your estimates by calculating the volume of each prism. Record your calculations in the chart, including the correct unit of measurement for volume, and rank the prisms from greatest to least by actual volume.

Rectangular Prism	Actual volume	Rank G to L by actual volume
A		
B		
C		
D		

5. Which of the actual volumes surprised you? Explain.

____

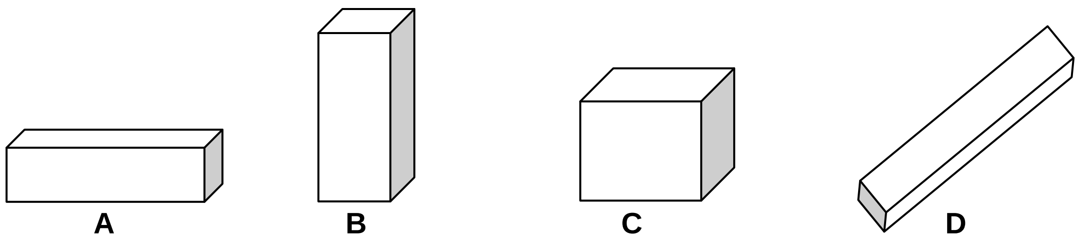
____

6. Which two prisms were closest in volume? _____________

Name: ANSWER KEY

Which Prism Holds the Most?

1. The boxes shown below are examples of rectangular prisms. Note that they are *not* drawn to scale, so you must look at their dimensions in the chart. Circle the box that you think would hold the *most*. Explain your choice.



Rectangular Prism	Length (cm)	Width (cm)	Height (cm)
A	9	2	3
B	4	3	8
C	5	5	4
D	12	2	2

2. Rank of the prisms from *greatest to least* by estimated volume.

Greatest → ***Least***

Solid: (Answers will vary.)

3. How can you calculate the exact volume of each rectangular prism?

Multiply its length × width × height

4. Test your estimates by calculating the volume of each prism. Record your calculations in the chart, including the correct unit of measurement for volume, and rank the prisms from greatest to least by actual volume.

Rectangular Prism	Actual volume	Rank G to L by actual volume
A	54 cm^3	3
B	96 cm^3	2
C	100 cm^3	1
D	48 cm^3	4

5. Which of the actual volumes surprised you? Explain.

(Answers will vary.)

6. Which two prisms were closest in volume? B and C

Name: ________________________

Finding the Volume of a Rectangular Prism

Find how many cubic units fill each of the following prisms. Label the units in each situation.

1. A tissue box that measures 9 inches by 5 inches by 3½ inches:

   ____

2. A CD case that measures 14 cm by 12 cm by 1 cm:

   ____

3. A box of cereal that measures 8 inches by 4 inches by 13.5 inches:

   ____

4. A shoe box that measures 12 inches by 6 inches by 5 inches:

   ____

5. A package of computer paper that measures 8½ inches x 11 inches x 4 inches:

   ____

6. A ring box that measures 6 cm by 5 cm by 3 cm:

   ____

7. Explain why the formula for volume of a rectangular prism ($l \times w \times h$) makes sense to you.

   ____

   ____

Name: <u>ANSWER KEY</u>

Finding the Volume of a Rectangular Prism

Find how many cubic units fill each of the following prisms. Label the units in each situation.

1. A tissue box that measures 9 inches by 5 inches by 3½ inches:

   9 in. (length) × 5 in. (width) × 3.5 in. (height) = 157.5 in^3 (volume)

2. A CD case that measures 14 cm by 12 cm by 1 cm:

   14 cm × 12 cm × 1 cm = 168 cm^3

3. A box of cereal that measures 8 inches by 4 inches by 13.5 inches:

   8 in. × 4 in. × 13.5 in. = 432 $in.^3$

4. A shoe box that measures 12 inches by 6 inches by 5 inches:

   12 in. × 6 in. × 5 in. = 360 $in.^3$

5. A package of computer paper that measures 8½ inches x 11 inches x 4 inches:

   8.5 in. × 11 in. × 4 in. = 374 $in.^3$

6. A ring box that measures 6 cm by 5 cm by 3 cm:

   6 cm × 5 cm × 3 cm = 90 cm^3

7. Explain why the formula for volume of a rectangular prism ($l \times w \times h$) makes sense to you.

   (Answers will vary.) ______________________________

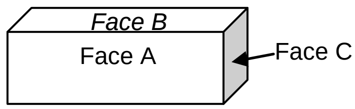
   ____

Name: ________________________

Finding the Surface Area of a Rectangular Prism I

Deconstruct each figure below to find the total surface area in square units.

1. The dimensions of the rectangular prism below are: l = 7 cm; w = 2 cm; h = 4 cm



Area of Face A = ____________________

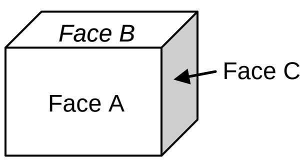
Area of Face B = ____________________

Area of Face C = ____________________

Because the opposite faces are congruent, the total surface area of all 6 faces is

____

2. The dimensions of the rectangular prism below are: l = 6 in.; w = 3 in.; h = 4 in.



Area of Face A = ____________________

Area of Face B = ____________________

Area of Face C = ____________________

Because the opposite faces are congruent, the total surface area of all 6 faces is

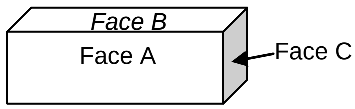
____

Name: ANSWER KEY

Finding the Surface Area of a Rectangular Prism I

Deconstruct each figure below to find the total surface area in square units.

1. The dimensions of the rectangular prism below are: l = 7 cm; w = 2 cm; h = 4 cm



Area of Face A = 7 × 4 = 28 cm^2

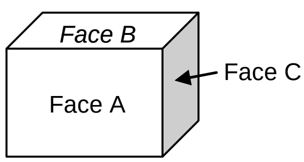
Area of Face B = 7 x 2 = 14 cm^2

Area of Face C = 2 x 4 = 8 cm^2

Because the opposite faces are congruent, the total surface area of all 6 faces is

28 + 28 + 14 + 14 + 8 + 8 = 100 cm^2

2. The dimensions of the rectangular prism below are: l = 6 in.; w = 3 in.; h = 4 in.



Area of Face A = 6 × 4 = 24 $in.^2$

Area of Face B = 6 x 3 = 18 $in.^2$

Area of Face C = 3 x 4 = 12 $in.^2$

Because the opposite faces are congruent, the total surface area of all 6 faces is

24 + 24 + 18 + 18 + 12 + 12 = 108 $in.^2$

Name: ________________________

Finding the Surface Area of a Rectangular Prism II

In each situation below, find the total surface area of the figure. Explain your strategy or formula in words, and show your calculations. You may include sketches as well as numbers in your work. Include appropriate units with your numbers!

1. What is the minimum amount of cardboard needed to make a cereal box with dimensions 8 inches by 4 inches by 13 inches?

   Strategy: ______________________________

   Calculations: ______________________________

   ____

   ____

   ____

2. How much wrapping paper is needed to cover a dress box that measures 20 cm x 15 cm x 5 cm?

   Strategy: ______________________________

   Calculations: ______________________________

   ____

   ____

   ____

3. A board game and a drawing kit need to be gift wrapped. The board game measures 12 inches by 10 inches by 4 inches, while the drawing kit measures 18 inches by 10 inches by 2 inches. Which box will take the most wrapping paper to cover?

   Strategy: ______________________________

   Calculations: ______________________________

   ____

   ____

   ____

   ____

   ____

4. A chocolate sheet cake measures 15 inches x 11 inches x 3 inches high, and a cherry sheet cake measures 12 inches x 12 inches x 4 inches high. Which cake will need the most icing to cover its top and sides but not its base?

   Strategy: ______________________________

   Calculations: ______________________________

   ____

   ____

   ____

   ____

   ____

5. Extra for Experts: A round cake has a radius of 4½ inches and a height of 8 inches, while a rectangular cake measures 13 inches by 9 inches by 4 inches high. Which cake will take the most icing to cover its top and sides but not its base? Use $\pi \approx 3.14$.

   Strategy: ______________________________

   Calculations: ______________________________

   ____

   ____

   ____

   ____

   ____

Name: <u>ANSWER KEY</u>

Finding the Surface Area of a Rectangular Prism II

In each situation below, find the total surface area of the figure. Explain your strategy or formula in words, and show your calculations. You may include sketches as well as numbers in your work. Include appropriate units with your numbers!

1. What is the minimum amount of cardboard needed to make a cereal box with dimensions 8 inches by 4 inches by 13 inches?

   Strategy: Answers will vary. Students may choose to sketch the rectangular prism.

   Calculations: 2(8 × 4) + 2(8 × 13) + 2(4 × 13) =

   2(32) + 2(104) +2(52) =

   64 + 208 + 104 = 376 $in.^2$

2. How much wrapping paper is needed to cover a dress box that measures 20 cm x 15 cm x 5 cm?

   Strategy: Answers will vary. Students may choose to sketch the rectangular prism.

   Calculations: 2(20 × 15) + 2(20 × 5) + 2(15 × 5) =

   2(300) + 2(100) +2(75) =

   600 + 200 + 150 = 950 cm^2

3. A board game and a drawing kit need to be gift wrapped. The board game measures 12 inches by 10 inches by 4 inches, while the drawing kit measures 18 inches by 10 inches by 2 inches. Which box will take the most wrapping paper to cover?

   Strategy: Answers will vary. Students may choose to sketch the two rectangular prisms.

   Calculations for board game: 2(12 × 10) + 2(12 × 4) + 2(10 × 4) =

   2(120) + 2(48) +2(40) =

   240 + 96 + 80 = 416 $in.^2$

   Calculations for drawing kit: 2(18 × 10) + 2(18 × 2) + 2(10 × 2) =

   2(180) + 2(36) +2(20) =

   360 + 72 + 40 = 472 $in.^2$

   The drawing kit will take more paper to wrap.

4. A chocolate sheet cake measures 15 inches x 11 inches x 3 inches high, and a cherry sheet cake measures 12 inches x 12 inches x 4 inches high. Which cake will need the most icing to cover its top and sides but not its base?

Calculations for chocolate cake: $(15 \times 11) + 2(15 \times 3) + 2(11 \times 3) =$

$165 + 2(45) + 2(33) =$

$165 + 90 + 66 = 321$ in.^{2}

Calculations for cherry cake: $(12 \times 12) + 2(12 \times 4) + 2(12 \times 4) =$

$144 + 2(48) + 2(48) =$

$144 + 96 + 96 = 336$ in.^{2}

The cherry cake will need the most icing.

5. Extra for Experts: A round cake has a radius of 4½ inches and a height of 8 inches, while a rectangular cake measures 13 inches by 9 inches by 4 inches high. Which cake will take the most icing to cover its top and sides but not its base? Use $\pi \approx 3.14$.

Surface area of top of round cake = πr^2

Surface area of side of round cake = $2\pi rh$

Calculations for round cake: $3.14(4.5^2) + 2 \times 3.14 \times 4.5 \times 8) =$

$3.14(20.25) + 226.08 =$

$63.585 + 226.08 = 289.665$ in.^{2}

Calculations for rectangular cake: $(13 \times 9) + 2(13 \times 4) + 2(9 \times 4) =$

$117 + 2(52) + 2(36) =$

$117 + 104 + 72 = 293$ in.^{2}

The rectangular cake will take more icing.

Name: ______________________

Covering a Surface

1. How much metal makes up a soup can with a height of 10 cm and a radius of 3 cm?

   Draw the surfaces that make up the can, and label each one. On each piece, write a formula or method of finding its surface area (*SA*).

Find the area of each piece, using $\pi \approx 3.14$, and add the totals together.

____

____

2. On the Grade 7 Mathematics Formula Sheet, find the formula for the surface area (*SA*) of a cylinder, and write it below.

   ____

   Explain how this formula makes sense to you. Which parts of the soup can match which parts of the formula?

   ____

3. Write the number that you would use for each variable in the formula.

   ____

   Using the formula for the surface area of a cylinder from step 2, write the formula for the surface area of the soup can in step 1.

   ____

   Enter this formula into your calculator to find the answer.

   Answer: ________________

   Does the measurement match what you got by adding the surfaces together in step 2?

   ____

4. Now try a different cylinder—a golf-ball can with a radius of 2 cm and a height of 19 cm. Do you think the surface area of this can is greater than or less than the surface area of the soup can? ______________________

   Sketch the golf-ball can in the space at right.

   Find the surface area in square centimeters.
   Show your work, and explain your formula in words.

   ____

   Using the formula for the surface area of a cylinder from
   step 2, write the formula for the surface area of the golf-ball can.

   ____

   Enter this formula into your calculator to find the answer.

   Answer: __________________

   Is the surface area of the golf-ball can greater than or less than the surface area of the soup can? _________________________

5. Explain how you use the formula for surface area if the radius changes or the height changes.

   ____

6. List key words that let you know that you are measuring surface area.

   ____

   ____

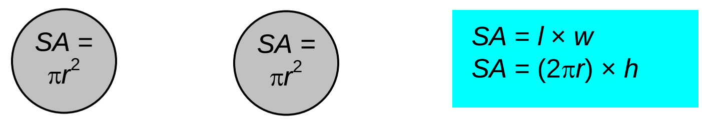
   ____

Name: ANSWER KEY

Covering a Surface

1. How much metal makes up a soup can with a height of 10 cm and a radius of 3 cm?

   Draw the surfaces that make up the can, and label each one. On each piece, write a formula or method of finding its surface area (SA).



Top of can   Base of can   Label

Find the area of each piece, using $\pi \approx 3.14$, and add the totals together.

(3.14 × 3 × 3) + (3.14 × 3 × 3) + (2 × 3.14 × 3 ×10) =

28.26 + 28.26 + 188.4 = 244.92 cm^2

2. On the Grade 7 Mathematics Formula Sheet, find the formula for the surface area (SA) of a cylinder, and write it below.

   $SA = 2\pi rh + 2\pi r^2$

   Explain how this formula makes sense to you. Which parts of the soup can match which parts of the formula?

   $2\pi rh$ = lateral area $2\pi r^2$ = 2 circular base areas

3. Write the number that you would use for each variable in the formula.

   $r = 3$ $h = 10$ $\pi = 3.14$

   Using the formula for the surface area of a cylinder from step 2, write the formula for the surface area of the soup can in step 1.

   $SA = 2 \times 3.14 \times 3 \times 10 + 2 \times 3.14 \times 3 \times 3$

   Enter this formula into your calculator to find the answer.

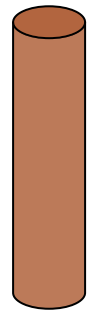
   Answer: 244.92 cm^2

   Does the measurement match what you got by adding the surfaces together in step 2?

   yes

4. Now try a different cylinder—a golf-ball can with a radius of 2 cm and a height of 19 cm. Do you think the surface area of this can is greater than or less than the surface area of the soup can? (Answers will vary.)

   Sketch the golf-ball can in the space at right.



   Find the surface area in square centimeters.
   Show your work, and explain your formula in words.

   $r = 2$ $h = 17$ $\pi = 3.14$

   Using the formula for the surface area of a cylinder from step 2, write the formula for the surface area of the golf-ball can.

   $SA = 2 \times 3.14 \times 2 \times 17 + 2 \times 3.14 \times 2 \times 2$

   Enter this formula into your calculator to find the answer.

   Answer: 238.64 cm^2

   Is the surface area of the golf-ball can greater than or less than the surface area of the soup can? less than

5. Explain how you use the formula for surface area if the radius changes or the height changes.

   Substitute the new height for *h* or the new radius for *r*.

6. List key words that let you know that you are measuring surface area.

   Answers will vary but may include *cover, covering, net, jacket, outside, surface, exterior, and shell.*

✱ SOL 6.10d, 7.5a,b

Lesson Summary

Students measure the volume and surface area of rectangular prisms and cylinders to solve practical problems. This lesson should occur only after students have developed formulas for the volume and surface area of a rectangular prism and a cylinder. (45 minutes)

Materials

Real-life examples of rectangular prisms and cylinders, such as cereal boxes, paint cans, gelatin boxes, soup cans, tissue boxes, potato chip canisters, and vegetable cans
Metric tape measures
Copies of the Grade 7 Mathematics Formula Sheet
Copies of the attached "Solid Situations Task Cards"

Warm-up

1. Give copies of the Grade 7 Mathematics Formula Sheet to the students, and write the formulas for the volume and surface area of a rectangular prism and a cylinder on the board.
2. Have each student select one rectangular prism and one cylinder from the items on display, for example, a tissue box and a soup can. Discuss volume and surface area as demonstrated by these items. Review measuring volume by multiplying the area of the base by the height. Review measuring surface area by adding up the areas of all of the shapes (faces) that make up the net.
3. Have students point to all of the shapes that make up the net of a rectangular prism (six rectangles). Have students identify the areas that make up the surface of the cylinder (two circular bases and one rectangle whose length is the circumference of the base and whose width is the height of the cylinder).
4. Follow with a matching game. Hold up each example of a rectangular prism and each example of a cylinder, and call out a surface area or volume situation. Ask students to match the situation to the correct measurement formula. For example, ask: "What is the minimum amount of wrapping paper needed to cover this box? Which formula would help you measure this?"

Lesson

1. Distribute copies of the "Solid Situations Task Cards" to the class. Have students cut out the cards and work individually to solve them.
2. When all students are finished, have them share their solutions and methods.

Reflection

Have students make a foldable two-tab concept map for measurement of surface area and volume. This is done as follows:

- Fold a sheet of paper in half.
- Cut a line down one of the halves to the center creating two flaps.
- Label one flap "surface area" and one flap "volume."
- Under the surface area flap, sketch the nets for a rectangular prism and cylinder with formulas and an example.
- Under the volume flap, create a 3-D sketch of a rectangular prism and cylinder with formulas and an example.

Name: ____________________________

Solid Situations Task Cards

Cut out the task cards below and solve. Record your solutions and strategies in your own words. Prepare to share your solutions and methods with the class.

Note: for pi, use $\pi \approx 3.14$.

You are entering a pancake-eating contest! Each plate has a stack of pancakes 9 cm high. Each pancake has a diameter of 5 cm. If your stomach can hold about 1 liter (1,000 cubic centimeters), about how many plates of pancakes can your stomach hold?

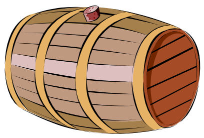
Solution in numbers:

____

____

____

Strategy in words:



How much apple cider can a barrel that is 30 inches high hold? The radius of the barrel is 16 inches.

Solution in numbers:

____

____

____

Strategy in words:

Which rectangular cake gives you more to eat—a vanilla cake measuring 8 in. x 11 in. x 3 in., or a chocolate cake measuring 9 in. x 12 in. x 2 in.?

Solution in numbers:

____

____

____

Strategy in words:

Which can of mixed nuts give you the most—a blue can with a radius of 5 cm and a height of 10 cm, or a red can with a radius of 4 cm and a height of 12 cm?

Solution in numbers:

____

____

____

Strategy in words:

Hannah is wrapping a gift whose dimensions are 17 inches wide by 5 inches wide by 4 inches high. What is the minimum amount of paper she will need to cover the entire box?

Solution in numbers:

____

____

____

Strategy in words:

Cole is planning to cover a drum in leather. The diameter of the drum is 10 inches, and its height is 16 inches. What is the minimum amount of leather Cole will need?

Solution in numbers:

____

____

____

Strategy in words:

Name: <u>ANSWER KEY</u>

Solid Situations Task Cards

Cut out the task cards below and solve. Record your solutions and strategies in your own words. Prepare to share your solutions and methods with the class.

Note: for pi, use $\pi \approx 3.14$.

You are entering a pancake-eating contest! Each plate has a stack of pancakes 9 cm high. Each pancake has a diameter of 5 cm. If your stomach can hold about 1 liter (1,000 cubic centimeters), about how many plates of pancakes can your stomach hold?

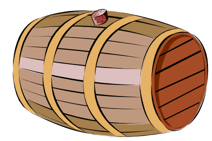
Solution in numbers:

$\underline{\pi r^2 h}$

$\underline{3.14 \times 2.5 \times 2.5 \times 9 = 176.63 \text{ cm}^3}$

$\underline{1{,}000 \div 176.63 = 5.66 \text{ plates}}$

Strategy in words:



How much apple cider can a barrel that is 30 inches high hold? The radius of the barrel is 16 inches.

Solution in numbers:

$\underline{\pi r^2 h}$

$\underline{3.14 \times 16 \times 16 \times 30 = 24{,}115.2 \text{ in.}^3}$

Strategy in words:

Which rectangular cake gives you more to eat—a vanilla cake measuring 8 in. x 11 in. x 3 in., or a chocolate cake measuring 9 in. x 12 in. x 2 in.?

Solution in numbers:

$\underline{\text{Vanilla cake: } 8 \times 11 \times 3 = 264 \text{ in}^3}$

$\underline{\text{Chocolate cake: } 9 \times 12 \times 2 = 216 \text{ in}^3}$

Strategy in words:

Which can of mixed nuts give you the most—a blue can with a radius of 5 cm and a height of 10 cm, or a red can with a radius of 4 cm and a height of 12 cm?

Solution in numbers:

$\underline{\text{Blue can: } 3.14(5)^2(10) = 785 \text{ cm}^3}$

$\underline{\text{Red can: } 3.14(4)^2(12) = 602.88 \text{ cm}^3}$

Strategy in words:

Hannah is wrapping a gift whose dimensions are 17 inches wide by 5 inches wide by 4 inches high. What is the minimum amount of paper she will need to cover the entire box?

Solution in numbers:

$\underline{2(17 \times 5) + 2(17 \times 4) + 2(5 \times 4) = 346 \text{ in}^2}$

Strategy in words:

Cole is planning to cover a drum in leather. The diameter of the drum is 10 inches, and its height is 16 inches. What is the minimum amount of leather Cole will need?

Solution in numbers:

$\underline{2\pi r^2 + 2\pi rh}$

$\underline{2 \times 3.14 \times 5 \times 5 + 2 \times 3.14 \times 5 \times 16 = 659.4 \text{ in.}^3}$

Strategy in words:

✱ SOL 8.7a

Lesson Summary

Students use mathematical reasoning to compute the volume of prisms, cylinders, cones and square pyramids, using formulas. (45 minutes)

Materials

Relational solids (having the same height and congruent bases) for a cylinder and a cone and for a prism and a square pyramid
Rice or water
Copies of the Grade 8 Mathematics Formula Sheet
Copies of the attached "Finding Volumes of Solids" worksheet

Warm-up

1. Put on display the relational solids listed above. Ask students how the prism and the square pyramid, which have the same height and congruent bases, compare in volume. Are they the same? Is the volume of one greater? If so, how much greater? Ask students how they could check the two volumes. They may suggest filling the solids with rice or water and measuring the two quantities in order to compare them. Ask students to write down an estimate of how many pyramids would fit into the prism. Test student hypotheses by filling the prism, using the pyramid as a measure.
2. Repeat this process with the cylinder and cone.
3. Have students write a summary statement about the relationship between the volumes of a prism and a square pyramid and between the volumes of a cylinder and a cone.

Lesson

1. Move from estimating such volumes to finding them exactly. Hand out copies of the Grade 8 Mathematics Formula Sheet. Review the formula for calculating the volume of a prism or cylinder. Model calculating the area of the base and multiplying by the height.
2. Do the same for a pyramid and cone, emphasizing that it is the same process of calculating the area of the base and multiplying by the height except that you divide the resulting volume by 3, which is the same as multiplying by $\frac{1}{3}$. Ask students what they learned in the warm-up that helps them rationalize dividing by 3 (or multiplying by $\frac{1}{3}$).
3. Hand out copies of the attached "Finding Volumes of Solids" worksheet, and have students work together in pairs to complete it.

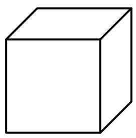
Reflection

Have students explain in writing how the formula for finding the volume of a pyramid is like and unlike the formula for finding the volume of a prism. Then, have them explain how the formula for finding the volume of a cone is like and unlike the formula for finding the volume of a cylinder.

Name: ________________________

Finding Volumes of Solids

Sketch each solid named in the chart below. Write a real-life question that involves finding the volume of the solid. Write the necessary dimensions that must be given in your question in order to answer it. Write the formula for finding the volume of the solid, and use the given dimensions to calculate the volume. Explain the process in words. The first example is done for you.

Sketch of Solid	Real-life volume question	Necessary dimensions	Formula for volume	Explanation of process
Cube 	What is the volume of an ice cube having a side length of 2 cm?	2 cm x 2 cm x 2 cm	$l \times w \times h$ 2 x 2 x 2 = 8 cm^3	I multiplied length and width to get the area of the base—4 cm^2. Then I multiplied the base area by the height to get the volume—8 cm^3.
Rectangular prism (not cube)				
Square pyramid				
Cylinder				
Cone				

Explain how the formula for finding the volume of a pyramid or cone is like the formula for finding the volume of a prism or cylinder.

____

Explain how it is different.

____

✱ SOL 8.7a

Lesson Summary

Students use mathematical reasoning and formulas to compute the surface area of prisms, cones, cylinders, and square pyramids. (45 minutes)

Materials

Square pyramids and cones
Nets of a cone and a square pyramid made out of grid paper
Rulers
Rice or water
Copies of the Grade 8 Mathematics Formula Sheet
Copies of the attached worksheets

Warm-up

Hold up the relational solid of a cone, and ask students to estimate the minimum amount of paper needed to make a cone for a birthday hat. Use this to lead into a discussion of the lateral area of a cone. Hold up the relational solid of the square pyramid, and ask how much paper it would take to reproduce the object in paper. Have students sketch the nets of a cone and a square pyramid. Then, have them share how to find the total surface area of a square pyramid and a cone. Discuss the meaning of "measuring surface area." Students may use terms such as "covering a surface" and "the outside," and they may name the shapes of the faces, such as triangles, squares, rectangles, and circles.

Lesson

1. Give each student a net of a cone made out of grid paper, and discuss how to talk about the "lateral area" of the cone by referring to the net. Define *lateral area* of a cone as a surface composed of line segments connecting all points on the circular base to the vertex. Tell students that this area is equal to pi times the radius times the slant height (l), and have them write this formula, πrl. Emphasize the difference between the *slant height* and the *height* of a cone, and ask whether the slant height is always greater than the height of a cone.
2. Have students measure the net of the cone and use these measurements to calculate the lateral area of the cone, using the formula πrl they just wrote.
3. Have students estimate the area of the circular base of the cone, using the grid on the net. Then, model calculating the area of the circular base by multiplying pi by the radius squared, or πr^2. Have students use the measure of the radius and the formula to calculate the area of the base and record the area of the base on the net.
4. Ask students how they can find the total surface area of the cone. If needed, guide them to suggest adding the area of the base and the lateral area of the cone. Show students the formula for surface area of a cone on the formula sheet, and discuss how the formula models the process they just finished. Have students work a new surface-area-of-a-cone problem with different dimensions. Measuring a real object, such as a slush cone or party hat, works well if you add a base.
5. Have students do a Think-Pair-Share, asking: "How would you find the total surface area of a square pyramid?"
6. Give each student a net of a square pyramid. Model finding the total surface area of the square pyramid while having the students record the areas of the square base and the triangular faces directly on the net. Have students find the total surface area by adding the base area to the lateral area of the pyramid. Have them record the steps of the process in sketches, numbers, and words.
7. Shift to the formula $SA = \frac{1}{2}lp + B$, where p equals the perimeter of the square base and B equals the area of the square base. Ask students to explain which part of the formula refers to the total lateral area ($\frac{1}{2}lp$) and which part to the area of the base (B). Link the formula for the area of one triangular

face ($\frac{1}{2}$ × length of side of base × slant height) to the sum of the four faces needed in surface area by using the perimeter of the square base as the sum of all four triangular bases.

8. Give students two new sets of dimensions for two new situations requiring measurement of square pyramids. Have them record these situations on the “Finding the Surface Area of a Square Pyramid” worksheet.
9. Have students work together in pairs to complete the “Finding Surface Area of Solids” worksheet, using their solids as a reference.
10. For additional practice, have students play the attached “Measure Up! Bingo” game.

Reflection

Ask students to respond in writing to the following questions:

- How is measuring surface area of a solid different from measuring volume?
- What do the lateral areas of a cone and a square pyramid look like? Sketch them, and explain how they are the same and how they are different.
- Explain the parts of the formula for measuring the surface area of a cone. What do the dimensions r, l, and h stand for? Use both sketches and words to explain.
- Explain the parts of the formula for measuring the surface area of a pyramid. What do the dimensions l, p, and B stand for? Use both sketches and words to explain.
- Write down a question that you still have about measuring surface area.

Name: ________________________

Finding the Surface Area of a Square Pyramid

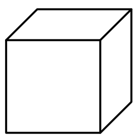
Draw a sketch for each word problem below. Label the dimensions from the word problem on the sketch. Write the formula in symbols and in words. Find the surface area in square units by using the formula and then explaining your steps in words.

Taylor is building a paper model of a square-based pyramid for his Egypt project. The square base measures 7 cm x 7 cm. The slant height is 8.5 cm. How much paper does Taylor need? Sketch: Formula in symbols: Formula in words: Surface area in square units: Explain your steps in words.	Jake is making a tent in the shape of a square pyramid, using canvas. Each side of the square base is 5 feet long, and the slant height is 7 feet. How much canvas does Jake need for the base and the lateral area of the tent? Sketch: Formula in symbols: Formula in words: Surface area in square units: Explain your steps in words.

Name: ________________________

Finding Surface Areas of Solids

Sketch each solid named in the chart below. Write a real-life question that involves finding the surface area of the solid. Write the necessary dimensions that must be given in your question in order to answer it. Write the formula for finding the surface area of the solid, and use the given dimensions to calculate the SA. Explain the process in words. The first example is done for you.

Sketch of Solid	Real-life SA situation	Given dimensions	Formula for SA	Explanation of process
Cube 	What is the minimum amount of cardboard needed to make a tissue cube with a side length of 6 cm?	6 cm x 6 cm x 6 cm	$SA = 6lw$ $SA = 6 \times 6 \times 6 = 216 \text{ cm}^2$	I found the area of one face, which is 6 x 6 or 36 cm^2. Then I multiplied the area of one face by 6 since there are 6 congruent faces in a cube: $36 \times 6 = 216 \text{ cm}^2$.
Rectangular prism (not cube)				
Square pyramid				
Cylinder				
Cone				

Sketch a pyramid having a different shaped base.

Explain how you would find the surface area of this or *any* pyramid.

____

Name: ________________________

Measure Up! Bingo Cards

Find the volume of a 4 meter by 4 meter by 2 meter rectangular prism.	How much container material is needed for a can of fruit juice having a diameter of 2 inches and a height of 5½ inches?	Find the volume of a juice box with dimensions of 2½ inches by 1.75 inches by 4 inches.
How much foil will you need to cover a cone-shaped chocolate kiss having a radius of 3 inches, a height of 8 inches, and a slant height of about 8.5 inches?	How much will a barrel of rainwater hold with a diameter of 3 feet and a height of 5 feet?	How much wax is in a candle shaped like a square-based pyramid if the height is 6 inches and the edge length of the base is 3 inches?
How much ice cream will fit into a waffle cone having a radius of 2½ inches and a height of about 6.3 inches?	How much popcorn will a prism-shaped box hold? The dimensions are 6 inches by 2 inches by 8 inches.	A hollow playground model of a pyramid in Egypt is a regular square pyramid. The sides of the base are about 7.5 feet, the height is about 4.8 feet, and the slant height is about 6.1 feet. How much plastic is needed to make the model?
A slush cone is sold in 6-inch cups filled to the rim. If the diameter is 3 inches, how much slush is in the cone?	An ice cube measures 1 inch by 1 inch by 1 inch. What is the surface area of the ice cube?	A candle mold shaped like a cylinder has a radius of 2 inches and a height of 7 inches. How much wax fills this candle mold?
How much cake do you get in a rectangular cake measuring 9 inches by 12 inches by 3½ inches?	How much water will it take to totally fill a fish tank measuring 30 inches by 17 inches by 20 inches?	A soup can has a height of 10 centimeters and a diameter of 6½ centimeters. How much soup will the can hold?

Name: ANSWER KEY

Measure Up! Bingo Cards

Find the volume of a 4 meter by 4 meter by 2 meter rectangular prism. = 32 m^3 (cubic meters)	How much container material is needed for a can of fruit juice having a diameter of 2 inches and a height of 5½ inches? ≈ 40.82 $in.^2$ (square inches)	Find the volume of a juice box with dimensions of 2½ inches by 1.75 inches by 4 inches. = 17.5 $in.^3$ (cubic inches)
How much foil will you need to cover a cone-shaped chocolate kiss having a radius of 3 inches, a height of 8 inches, and a slant height of about 8.5 inches? ≈ 108.33 $in.^2$ (square inches)	How much will a barrel of rainwater hold with a diameter of 3 feet and a height of 5 feet? ≈ 35.33 $ft.^3$ (cubic feet)	How much wax is in a candle shaped like a square-based pyramid if the height is 6 inches and the edge length of the base is 3 inches? = 18 $in.^3$ (cubic inches)
How much ice cream will fit into a waffle cone having a radius of 2½ inches and a height of about 6.3 inches? ≈ 41.21 $in.^3$ (cubic inches)	How much popcorn will a prism-shaped box hold? The dimensions are 6 inches by 2 inches by 8 inches. = 96 $in.^3$ (cubic inches)	A hollow playground model of a pyramid in Egypt is a regular square pyramid. The sides of the base are about 7.5 feet, the height is about 4.8 feet, and the slant height is about 6.1 feet. How much plastic is needed to make the model? = 147.75 $ft.^2$ (square feet)
A slush cone is sold in 6-inch cups filled to the rim. If the diameter is 3 inches, how much slush is in the cone? ≈ 14.13 $in.^3$ (cubic inches)	An ice cube measures 1 inch by 1 inch by 1 inch. What is the surface area of the ice cube? = 6 $in.^2$ (square inches)	A candle mold shaped like a cylinder has a radius of 2 inches and a height of 7 inches. How much wax fills this candle mold? ≈ 87.92 in^3 (cubic inches)
How much cake do you get in a rectangular cake measuring 9 inches by 12 inches by 3½ inches? = 378 $in.^3$ (cubic inches)	How much water will it take to totally fill a fish tank measuring 30 inches by 17 inches by 20 inches? = 10,200 $in.^3$ (cubic inches)	A soup can has a height of 10 centimeters and a diameter of 6½ centimeters. How much soup will the can hold? ≈ 331.66 cm^3 (cubic centimeters)

Name: ________________________

Measure Up! Bingo Game Board

4. Write “FREE SPACE” in one square of the game board below.

Write a number from the solutions bank in each of the other squares.

As each question is read aloud, decide which solid it involves—rectangular prism, cylinder, pyramid, or cone. Then decide which measurement it involves—surface area or volume.

Use your formula sheet to find the necessary formula.

Use your calculator and $\pi \approx 3.14$ to solve the problem. Cross out the solution with a large ✕ on your game board.

96 in.^{3}	32 m^3	17.5 in.^{3}
378 in.^{3}	≈14.13 in.^{3}	10,200 in.^{3}
≈87.92 in.^{3}	≈35.33 ft.^{3}	6 in.^{2}
≈ 40.82 in.^{2}	18 in.^{3}	147.75 ft.^{2}
≈331.66 in.^{3}	≈108.33 in.^{2}	≈ 41.21 in.^{3}

✱ SOL 8.9

Lesson Summary

Students construct a three-dimensional model, given the front, side, and top views. (45 minutes)

Materials

Cubes (linking or non-linking)
Isometric dot paper
Copies of the attached "Orthogonal Views: 2-D to 3-D" worksheet

Warm-up

Discuss the meaning of the work *orthogonal* in preface to having students practice drawing orthogonal views. Distribute isometric dot paper to students, and ask them to explain how this paper is different from regular graph paper. Have students practice drawing single cubes, and discuss sketching strategies. One method is to draw a Y in the center and circumscribe a hexagon around it. A method for sketching a cube on plain paper is to draw two squares and connect corresponding vertices of the squares.

Technology Alternative: Have students visit the Isometric Drawing Tool at http://illuminations.nctm.org/LessonDetail.aspx?ID=L607. Have them label the vertices of one cube with different capital letters. Then, have them use their cube sketch as a reference to name the following:

- a pair of parallel lines
- a pair of perpendicular lines
- a pair of skew lines
- a square
- a right angle
- a face diagonal
- a diagonal of the cube

Lesson

1. Before class, use eight cubes to create a 3-D model, draw the top view of it on the board, showing how many blocks are in each stack, and hide the model away.
2. Give each student eight cubes, and ask them to use their cubes to build a model of the object shown in your drawing, as seen from the top view.
3. Next, ask students to draw the top, side, and front views of their model.
4. Bring out your 3-D model, and check student models and sketches for accuracy. Discuss the term “perspective” and how the various views represent the same object from various perspectives. For students who have trouble visualizing, have them look at the model at eye level and at close range, focusing only on one view at a time. Ask students what the volume of the object would be in cubic units.
5. Distribute additional cubes and copies of the "Orthogonal Views: 2-D to 3-D” worksheet. Have students build the 3-D models from the orthogonal views and complete the worksheet. Have each student explain one model to the class.
6. Technology Alternative: Have students draw 3-D models on the computer with the Isometric Drawing Tool located on the Web site listed above.

Reflection

Think-Pair-Share: Have student pairs discuss which model was most difficult to construct from the views, explaining their reasoning and identifying the strategy that helped most in building the model. Have pairs join a second pair to share strategies for building their most challenging model. Have groups report strategies to the class. List these on the board, and have students list effective strategies in their math logs to use in future constructions.

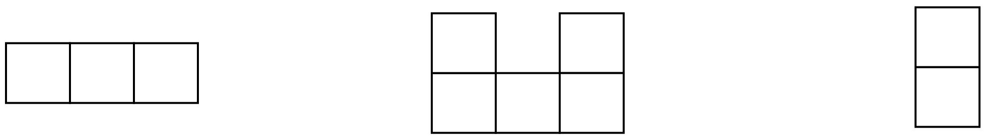
Name: ___________________________

Orthogonal Views: 2-D to 3-D

For each of the following questions, use the three orthogonal views to build each figure with cubes. Record the total number of cubes used for each figure and the volume of each cube.

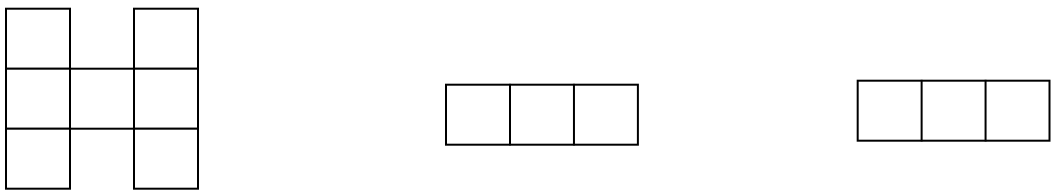
Top View **Front View** **Right Side View**

1.



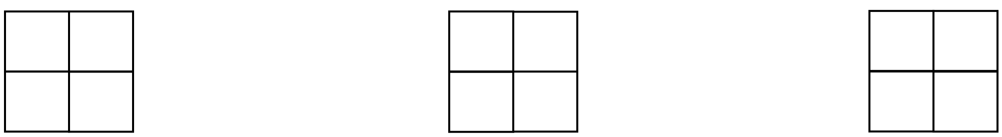
Total number of cubes: ______ Volume in cubic units: ____________

2.



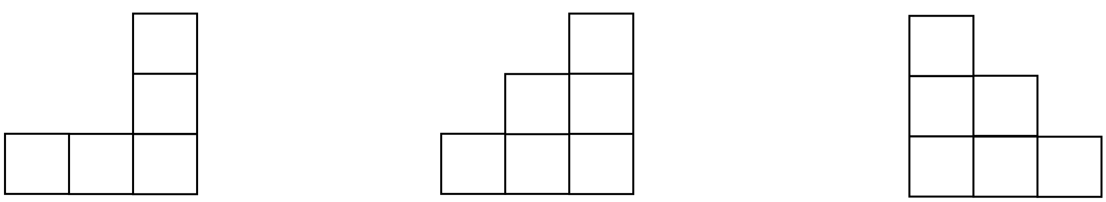
Total number of cubes: ______ Volume in cubic units: ____________

3.



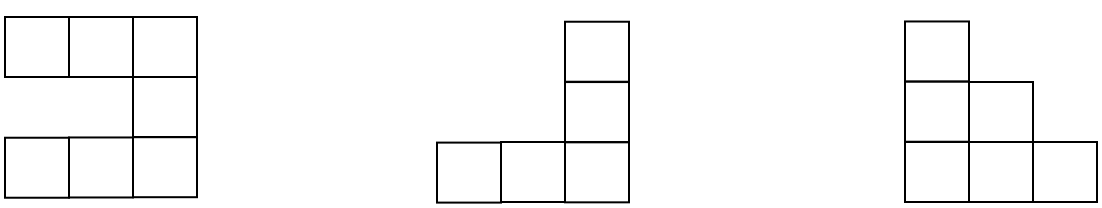
Total number of cubes: ______ Volume in cubic units: ____________

4.



Total number of cubes: ______ Volume in cubic units: ____________

5.



Total number of cubes: ______ Volume in cubic units: ____________