Introduction

In this section, the lessons focus on exploring the relationships among the radius, diameter, center, circumference, chord, and area of a circle and on using formulas involving pi.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion.*

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

Standards of Learning

The following Standards of Learning are addressed in this section:

5.9   The student will identify and describe the diameter, radius, chord, and circumference of a circle.

6.10   The student will
   a)   define π (pi) as the ratio of the circumference of a circle to its diameter;
   b)   solve practical problems involving circumference and area of a circle, given the diameter or radius;

Table of Contents

Lesson plans pertaining to the following Standards of Learning are found in this section. Click (or CTRL+click) on each to jump to that lesson.

✱ SOL 5.9

Lesson Summary

Students explore and define the radius, diameter, center, circumference, and chord of a circle.
(approximately 30 minutes)

Materials

"Circle" worksheets	Yarn	"Reflection" worksheets
Scissors	Rulers	
Colored pencils	"Circle Parts" worksheets	

Vocabulary

circle. A set of points on a flat surface (plane) with every point equidistant from a given point called the center.
chord. A line segment connecting any two points on the circle.
diameter. A chord that goes through the center of a circle.
radius. A segment from the center of a circle to any point on the circle.
circumference. The distance around or perimeter of a circle.

Warm-up

Have students brainstorm a list of "real-life" objects that are shaped like circles. If students mention objects that are spheres, remind them that circles are 2-dimensional and spheres are 3-dimensional.

Lesson

1. Pass out copies of the "Circle" worksheet, and have students cut out the circle.
2. Tell the students to fold the circle in half and then unfold it. Ask them to use a colored pencil and a ruler to trace the line segment formed by the crease they just made, and ask them to label this colored line "diameter" (see "Labeled Circle: Front" sample).
3. Have students fold the circle in half again, but not along the same fold as before. Again, tell them to unfold, and ask whether they notice anything special about where these two lines intersect. (It is the center of the circle.) Have them draw a dot there and label it "center."
4. Repeat step three twice more for a total of four diameters.
5. Have the students use a different colored pencil and a ruler to trace the line segment along any diameter *from the center to the edge of the circle.* Tell them to label this line segment "radius."
6. Have students make a final crease by making a "flap," i.e., making any fold that does *not* go through the center. Have them use yet a different colored pencil and a ruler to trace and label this line "chord."
7. Now, have the students turn the circle over and use a different colored pencil to trace around the periphery of the circle. Have them label this "circumference."
8. Ask students to use their labeled circle to verbally define *radius, diameter, chord,* and *circumference* (see vocabulary list above). Give guidance as necessary, and point out that the diameter is a special chord because it goes through the center.
9. Have students use the ruler and yarn to measure each circle part. Compare answers and discuss. Through discussion, be sure students notice that all diameters are the same length, that the radius is half the diameter and, conversely, that the diameter is twice the radius, and that chords can have many different lengths. Ask, "Are all radii the same length?"
10. Have students complete the "Circle Parts" worksheet.
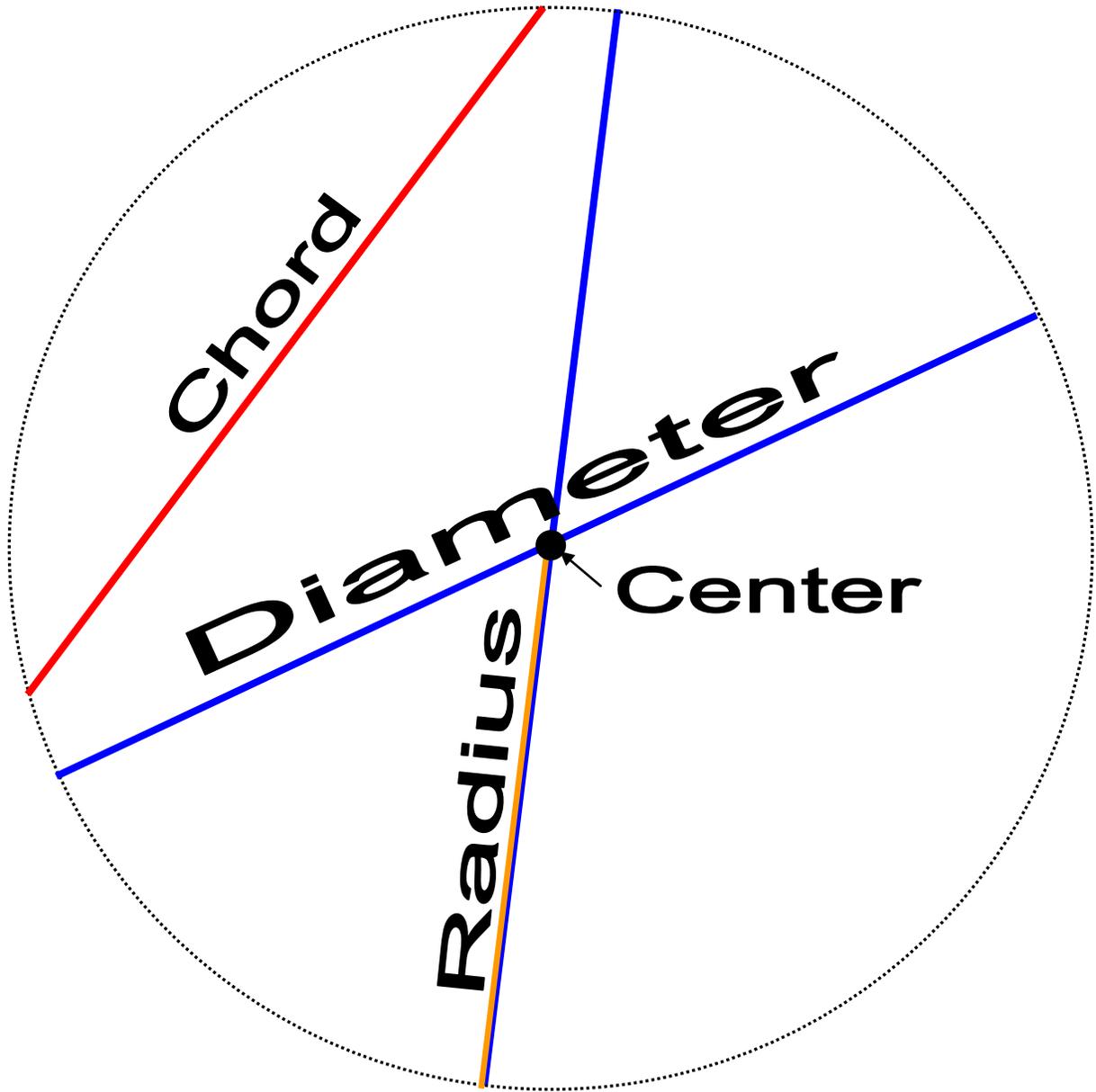
Reflection
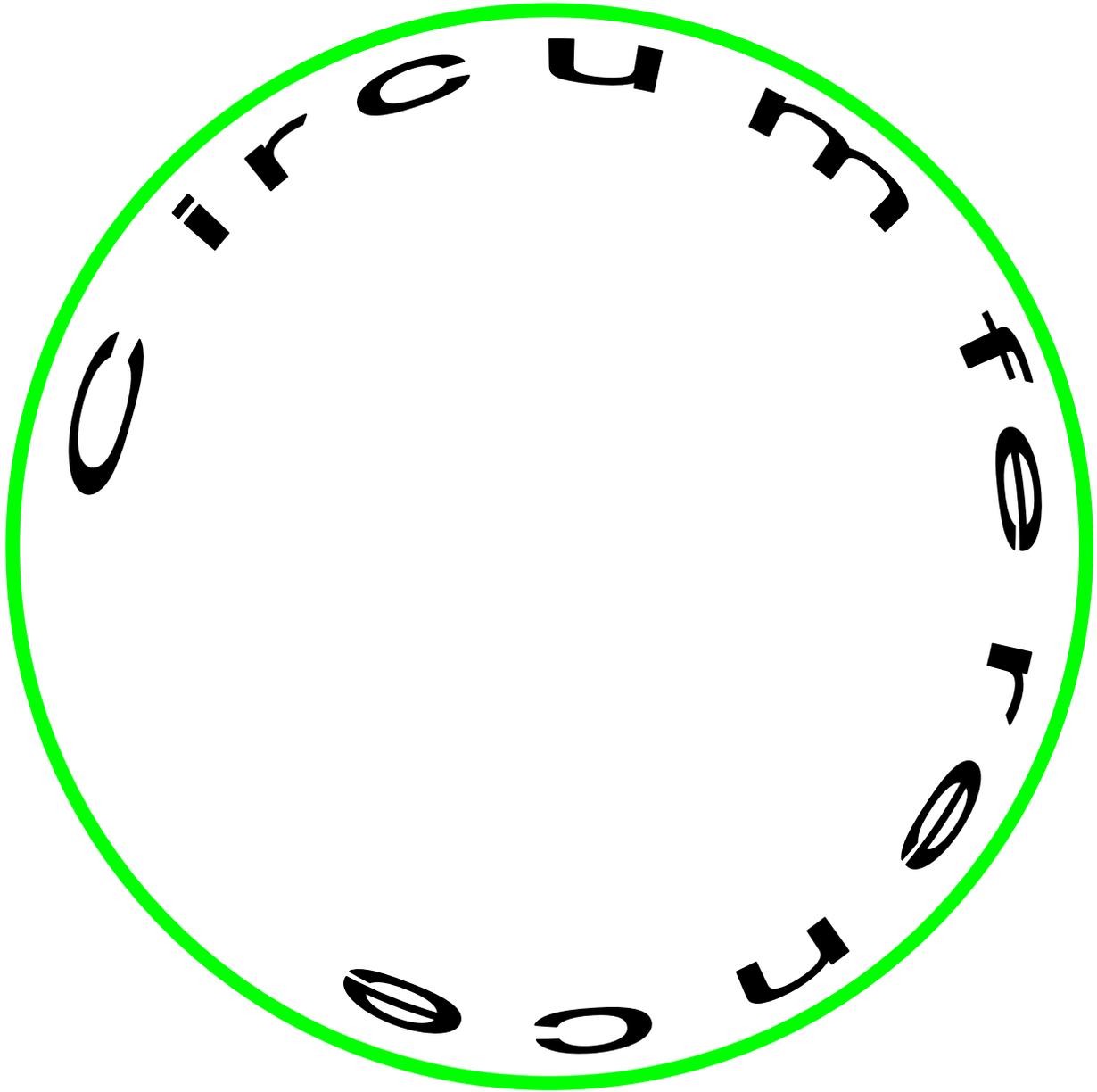
Have students complete the "Reflection" worksheet.

Circle

Cut out the circle carefully along
the dotted line.

_____
Your name
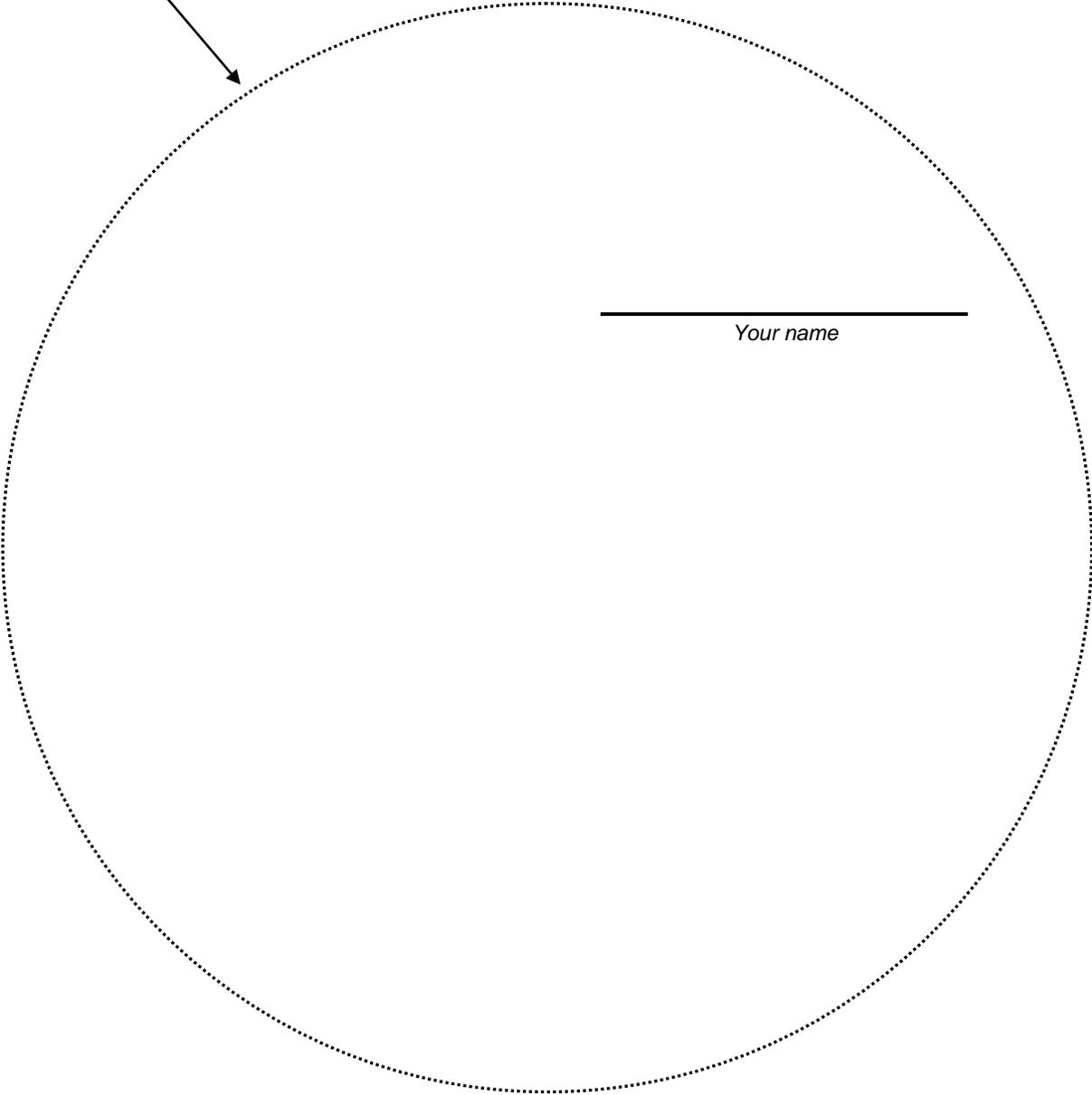
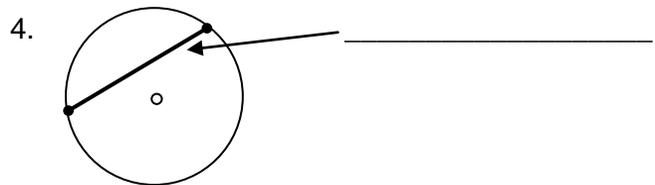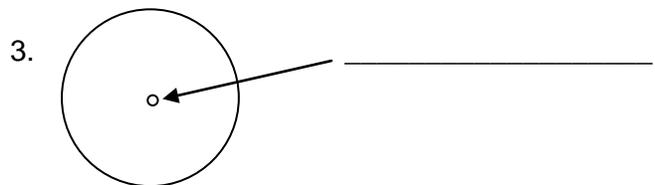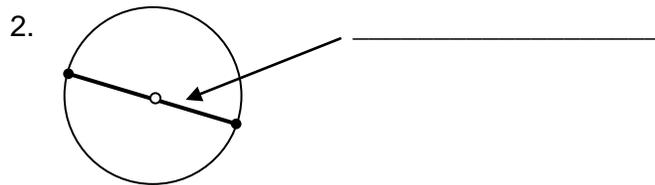## Labeled Circle: Front

Labeled Circle: Back

Circumference

Name: _____

Circle Parts

Use the words in the Word Bank to label each picture or description.
Words may be used more than once.

Word Bank
chord
diameter
circumference
radius
center

1. _____

2. _____

3. _____

4. _____

5.   Half as long as the diameter: _____

6.   The distance around a circle: _____

7.   A line segment connecting any two points on a circle: _____

8.   The line segment from the center of a circle to any point on the circle: _____

9.   Twice as long as the radius: _____

10.   The perimeter of a circle: _____

11.   A chord that passes through the center of a circle: _____
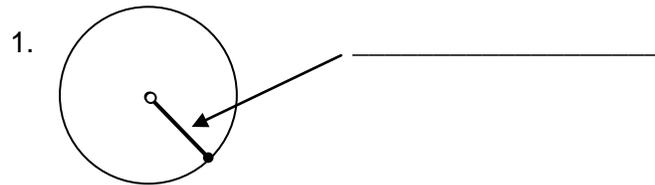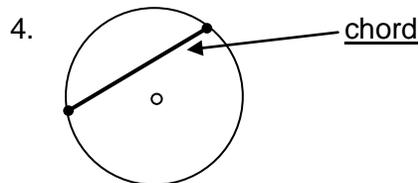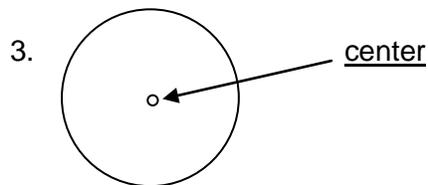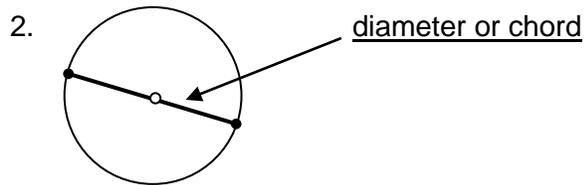
Name: <u>**ANSWER KEY**</u>

Circle Parts

Use the words in the Word Bank to label each picture or description. Words may be used more than once.

<u>**Word Bank**</u>
chord
diameter
circumference
radius
center

1. <u>radius</u>

2. <u>diameter or chord</u>

3. <u>center</u>

4. <u>chord</u>

5. Half as long as the diameter: <u>radius</u>

6. The distance around a circle: <u>circumference</u>

7. A line segment connecting any two points on a circle: <u>chord</u>

8. The line segment from the center of a circle to any point on the circle: <u>radius</u>

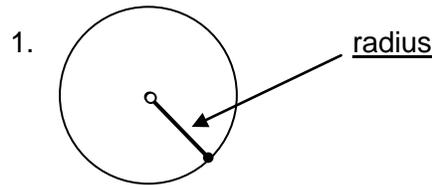9. Twice as long as the radius: <u>diameter</u>

10. The perimeter of a circle: <u>circumference</u>

11. A chord that passes through the center of a circle: <u>diameter</u>

Name: _____

Reflection

1. How is a circle different from other shapes?

    _____

    _____

    _____

    _____

2. Use the letters on the figure at right to name

    a. the center: _____

    b. a line segment that is a radius: _____

    c. a line segment that is a chord: _____

    d. a line segment that is a diameter: _____
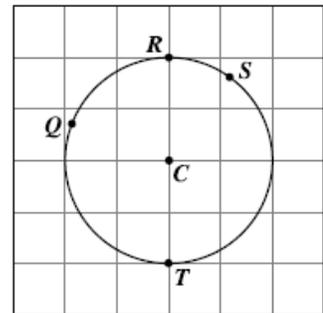
3. Compare diameter and chord. How are they the same? How are they different?

    _____

   _____

Name: <u>**ANSWER KEY**</u>

Reflection

1. How is a circle different from other shapes?

    <u>(Answers will vary.) A circle is different from other shapes in that it has no straight edges or corners. Also, it has certain parts that no other shapes have, such as diameters, radii, chords, and a circumference.</u>

2. Use the letters on the figure at right to name

    a. the center: <u>C</u>

    b. a line segment that is a radius: <u>CQ, CR, CS, and CT</u>

    c. a line segment that is a chord: <u>QT, QS, QR, RT, RS, and ST</u>

    d. a line segment that is a diameter: <u>RT</u>
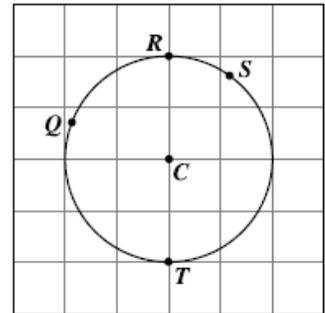
3. Compare diameter and chord. How are they the same? How are they different?

    <u>A diameter and a chord are the same in that both are line segments that have their end points on the circle. They are different in that a diameter is a special chord that goes through the center of the circle, while a chord may not go through the center.</u>

✱ SOL 6.10a,b

Lesson Summary
Students define the relationships among circumference, diameter, and pi through a measuring activity. (approximately 45 minutes)

Materials
Circular objects of varying sizes (lids, cans) Scientific calculators
Measuring tapes, or string and rulers "Reflection" worksheets
"Exploring Circles" worksheets

Vocabulary
diameter. A chord that goes through the center of a circle.
radius. A segment from the center of a circle to any point on the circle.
circumference. The distance around or perimeter of a circle.

Warm-up
Ask students to define orally or in writing *radius, diameter,* and *circumference,* and/or to draw a picture showing each of these.

Lesson
1. Pass out three or four circular objects to each group of students, along with a copy of the "Exploring Circles" worksheet for each student.
2. Have students measure to the nearest tenth of a centimeter the circumference and diameter of each object and fill in the first three columns of the table in Part I.
3. Have students fill in the last column of the table by dividing the circumference by the diameter for each circular object. Ask them what they notice. (Regardless of the size of the circle, the circumference divided by the diameter is a little more than 3.)
4. Have students find the mean of the data in the last column, using calculators if they wish. The mean should be close to 3.14. You may want to establish a class mean by having the students find the mean of the all the groups' means.
5. Explain to students that they have just discovered the value of pi, which is used for many calculations having to do with circles. Have students write the symbol for pi (π), the numerical value of pi, and a definition of pi.
6. Ask student groups to come up with a formula that would allow them to calculate the circumference of a circle if they knew only the diameter of the circle and the value of pi. Tell them that they must be prepared to prove that their formula works.
7. Once students agree that the formula for circumference is $C = \pi d$, have them calculate the circumference of each of their objects. Check on the students as they work, providing assistance as needed.
8. Discuss how closely their calculated results match their measurements listed in the table.
9. Share with students that another formula for circumference is $C = 2\pi r$, and ask them to explain why this is an equivalent formula. (Because 2r is the same as d)
10. Have students complete Part II of the "Exploring Circles" worksheet.

Reflection
Have the students complete the "Reflection" worksheet.

Name: _____

Exploring Circles

Part I

1. Carefully measure to the nearest tenth of a centimeter the circumference and diameter of several circular objects and fill in the first three columns of the table below.

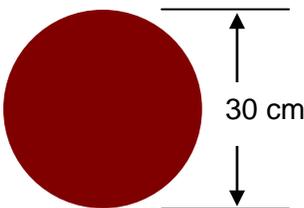| Object	Circumference (cm)	Diameter (cm)	Circumference ÷ Diameter
		Mean	

2. Use your calculator to divide each object's circumference by its diameter and round to the nearest hundredth. Use the values to fill in the last column.

3. Find the mean of the data in the last column.

4. Write the symbol for pi, the numerical value of pi, and a definition of pi.

_____ _____ _____

Part II

5. What is the formula for calculating the circumference, *C*, of a circle, if you know pi and the diameter, *d*?

    _____

6. What is the formula for calculating the circumference, *C*, of a circle, if you know pi and the radius, *r*?

    _____

7. Use a formula to find the circumference for each circle. Show your work!

    a. A pizza with a diameter of 10 inches: *C* = _____

    b. A pipe with a radius of 2 feet: *C* = _____

    c. 30 cm   *C* = _____
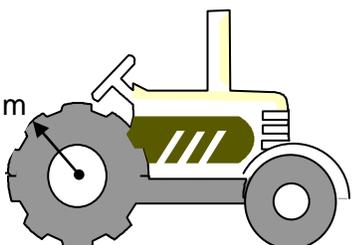
    d. 0.5 m   *C* = _____

Name: **ANSWER KEY**

Exploring Circles

Part I

1. Carefully measure to the nearest tenth of a centimeter the circumference and diameter of several circular objects and fill in the first three columns of the table below.

Object	Circumference (cm)	Diameter (cm)	Circumference ÷ Diameter
	(Values will vary.)		
		Mean	Close to 3.14

2. Use your calculator to divide each object's circumference by its diameter and round to the nearest hundredth. Use the values to fill in the last column.

3. Find the mean of the data in the last column.

4. Write the symbol for pi, the numerical value of pi, and a definition of pi.

   π,   3.14,   The ratio of the circumference of a circle to its diameter.

Part II

5. What is the formula for calculating the circumference, *C*, of a circle, if you know pi and the diameter, *d*?

   $C = \pi d$

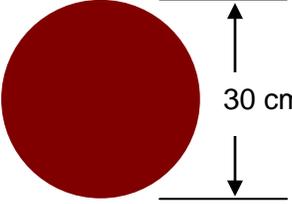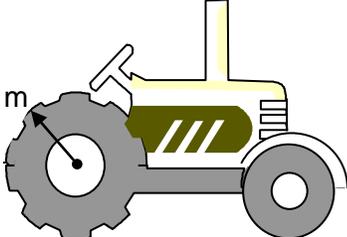6. What is the formula for calculating the circumference, *C*, of a circle, if you know pi and the radius, *r*?

   $C = 2\pi r$

7. Use a formula to find the circumference for each circle. Show your work!

   a. A pizza with a diameter of 10 inches: *C* = 3.14 in.

   b. A pipe with a radius of 2 feet: *C* = 12.56 ft.

   c.   30 cm   *C* = 94.2 cm

   d. 0.5 m   *C* = 3.14 m

Name: _____

Reflection

1. What is the approximate value of pi? _____

2. Explain what pi represents:

    _____

    _____

3. Explain how you would *calculate* the circumference of a circle:

    _____

    _____

Name: <u>**ANSWER KEY**</u>

Reflection

1. What is the approximate value of pi? <u>3.14</u>

2. Explain what pi represents:

    <u>The number you get when you divide the circumference of any circle by its diameter.</u>

3. Explain how you would *calculate* the circumference of a circle:

    <u>Multiply the diameter times pi, or multiply 2 times the radius times pi.</u>

✲ SOL 6.10b

Lesson Summary
Students determine the formula for finding the area of a circle. (approximately 45 minutes)

Materials
"Circle" worksheets   Scissors
"Circles and Radius Squares" worksheets   Scientific calculators
"Covering Circles" worksheets   "Reflection" worksheets

Vocabulary
diameter. A chord that goes through the center of a circle.
radius. A segment from the center of a circle to any point on the circle.

Warm-up
Ask students to define *area.* (The number of square units it takes to cover a 2-dimensional figure) Review how to find the area of a rectangle or square. (length × width) Pass out copies of the "Circle" worksheet, and have students draw and label a radius, chord, diameter, and the circumference. Walk around and check each drawing as the students work.

Lesson
1. Arrange students into small groups. Pass out a copy of the "Circles and Radius Squares" worksheet to each student. Explain that the squares are called "radius squares" because they have a side length equal to the length of the radius of the accompanying circle.

2. Have students cut out the circles and squares and then verify that the length of the side of the square is equal to the length of the radius of the accompanying circle. Instruct the students to put the four equal-size squares of each set together to make a larger square and then to place the accompanying circle on top of this larger square.

3. Pass out the "Covering Circles" worksheet, and ask students to fill in the first two columns in the table in Part I.

4. Explain that the students are to work together in groups to figure out how many radius squares it would take to cover the accompanying circle. They may cut the squares up into pieces to make them fit better. Encourage students to be as precise as possible. Once they have finished their work for each circle, they should fill in the last two columns of the table.

5. Discuss the results. Students should find that it takes a little more than 3 radius squares to cover each circle. Ask students to remember a special number they have seen before when working with circles—a number that is a little more than three. (pi = 3.14)

6. Remind students that the radius squares have an area of r^2.

7. Show students that by putting all the information together, they can come up with a formula for the area of a circle: $A = \pi r^2$. Discuss how to use the formula if the diameter of a circle is given instead of the radius.

8. Have students complete Part II of the "Covering Circles" worksheet.

Reflection
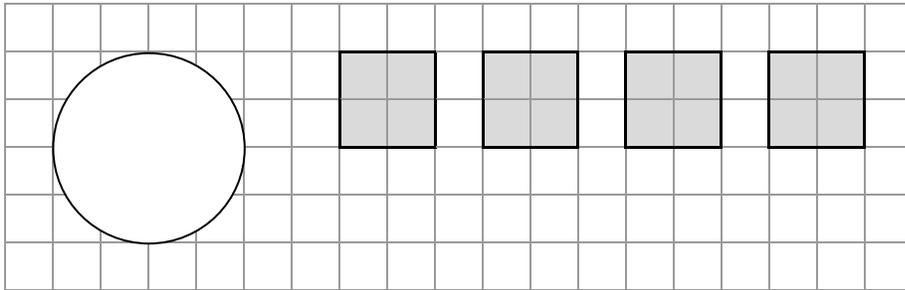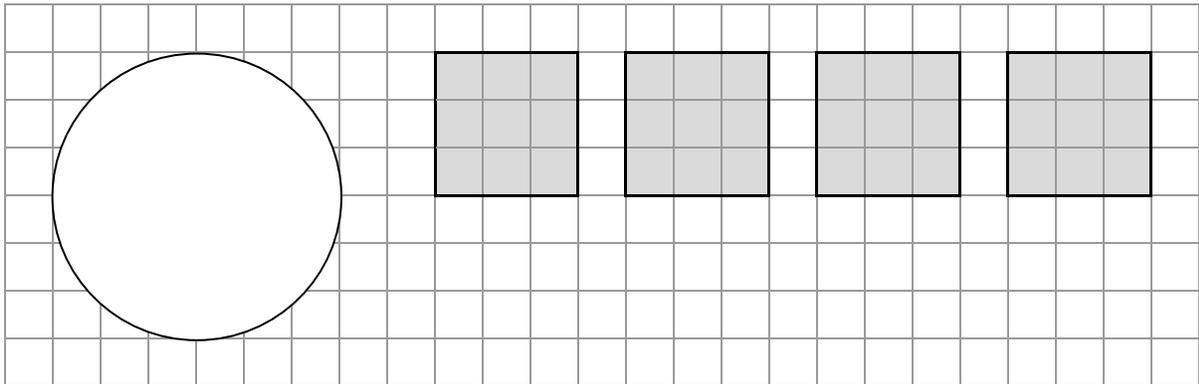Have students complete the "Reflection" worksheet.

Name: _____

Circle
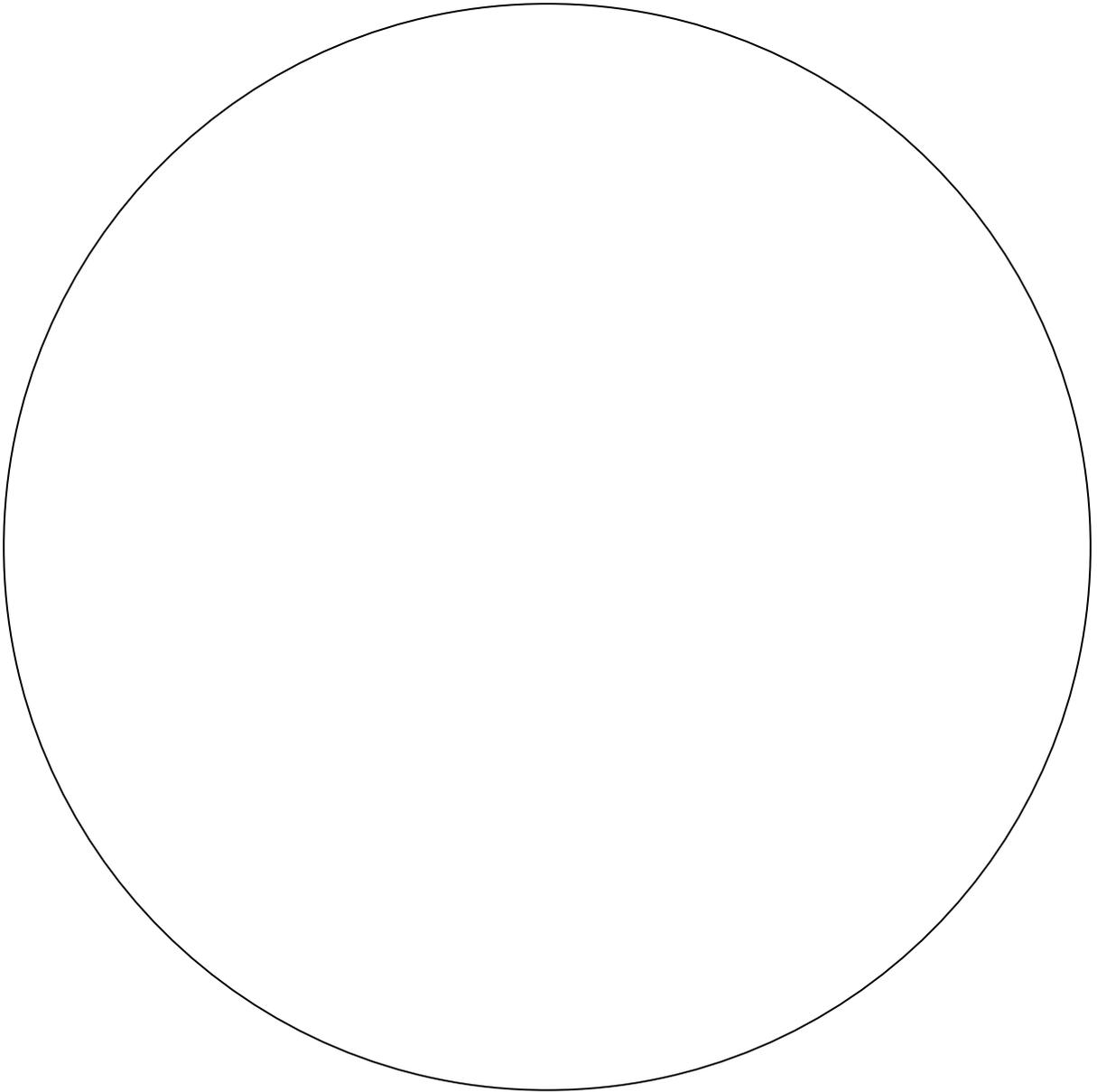
Name: _____

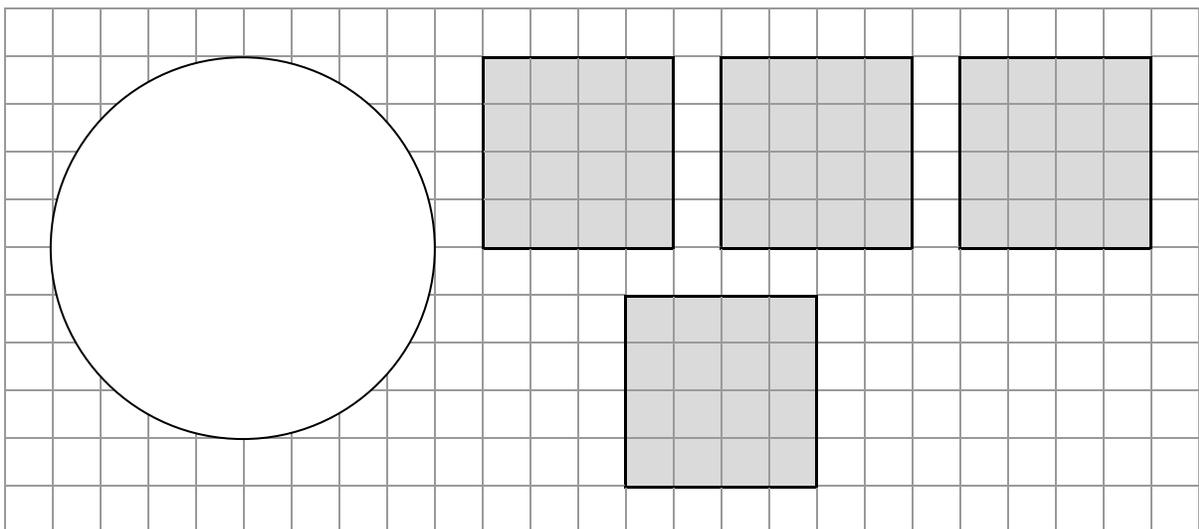# Circles and Radius Squares

Circle 1

Circle 2

Circle 3

Name: _____

Covering Circles

Part I

Fill in the information in the table below for each circle.

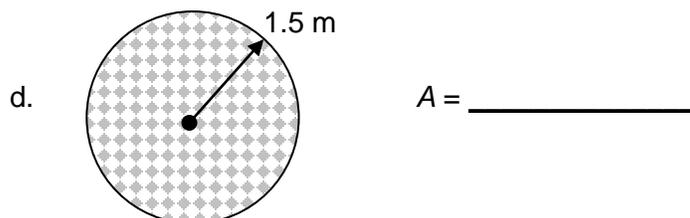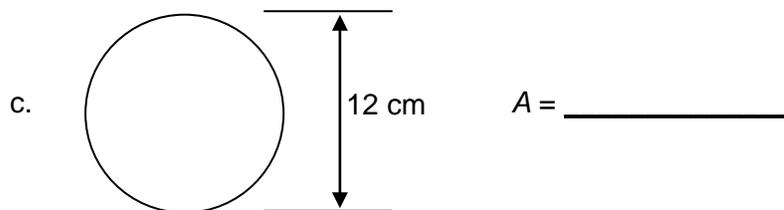| Circle	Length of radius (length of the side of a radius square)	Area of radius square (grid units)	Number of radius squares needed to cover circle	Estimated area of circle (grid units)
1				
2				
3				

Part II

4. What is the formula for calculating the area, *A*, of a circle, if you know the radius, *r*?

   _____

5. Use the formula to find the area of each circle. Show your work!

   5. Use the formula to find the area of each circle. Show your work!

   a. A pizza with a radius of 5 inches: $A =$ _____

   b. A table top with a diameter of 6 feet: $A =$ _____

   c. 12 cm $A =$ _____

   d. 1.5 m $A =$ _____

Name: <u>**ANSWER KEY**</u>

Covering Circles

Part I

Fill in the information in the table below for each circle.

| Circle	Length of radius (length of the side of a radius square)	Area of radius square (grid units)	Number of radius squares needed to cover circle	Estimated area of circle (grid units)
1	<u>2</u>	<u>4</u>	<u>A little more than 3</u>	<u>~12.5</u>
2	<u>3</u>	<u>9</u>	<u>A little more than 3</u>	<u>~28</u>
3	<u>4</u>	<u>16</u>	<u>A little more than 3</u>	<u>~50</u>

Part II

4. What is the formula for calculating the area, *A*, of a circle, if you know the radius, *r*?

    <u>$A = \pi r^2$</u>

5. Use the formula to find the area of each circle. Show your work!

    a. A pizza with a radius of 5 inches: <u>$A = 78.5$ in.^{2}</u>

    b. A table top with a diameter of 6 feet: <u>$A = 28.26$ ft.^{2}</u>

    c.   12 cm   <u>$A = 113.04$ cm^2</u>

    d.   1.5 m   <u>$A = 7.065$ m^2</u>
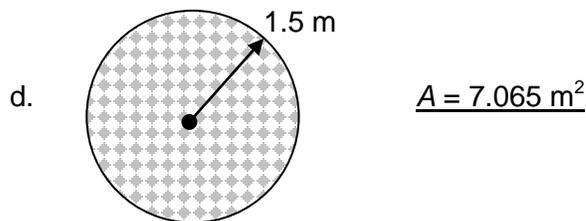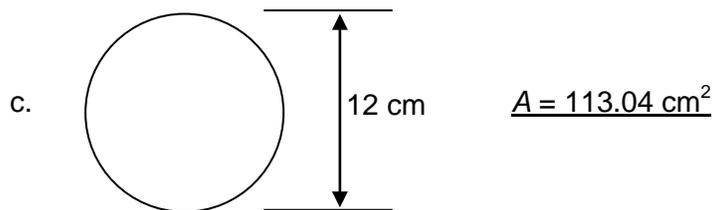
Name: _____

Reflection

1. Explain how you would solve this problem.

    How much more area does a 12-inch diameter pizza have than an 8-inch diameter pizza?

    _____

    _____

2. Draw a diagram of each pizza, label the radius of each, and solve the problem. Show your work!

    12-inch pizza   **8-inch pizza**

3. Express the answer in a complete sentence:

    _____

    _____

Name: <u>**ANSWER KEY**</u>
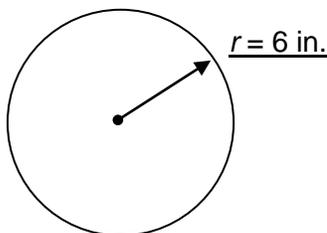
Reflection

1. Explain how you would solve this problem.

    How much more area does a 12-inch diameter pizza have than an 8-inch diameter pizza?

    <u>Find the area of each pizza, using the formula $A = \pi r^2$, then subtract.</u>
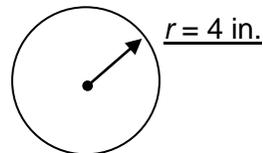
2. Draw a diagram of each pizza, label the radius of each, and solve the problem. Show your work!

    12-inch pizza   **8-inch pizza**

    <u>$r = 6$ in.</u>   <u>$r = 4$ in.</u>

                                                         <u>$A = \pi r^2$</u>

                                                         <u>$= (3.14)(4^2)$</u>

    <u>$A = \pi r^2$</u>   <u>$= (3.14)(16)$</u>

    <u>$= (3.14)(6^2)$</u>   <u>$= 50.24$ in.^{2}</u>

    <u>$= (3.14)(36)$</u>

    <u>$= 113.04$ in.^{2}</u>

    <u>$113.04 - 50.24 = 62.8$ in.^{2}</u>

3. Express the answer in a complete sentence:

    <u>The difference between the area of the 12-inch pizza and the 8-inch pizza is 62.8 in.^{2}.</u>