Introduction
In this section, the lessons focus on estimating, measuring, and classifying angles and triangles, identifying congruence and noncongruence, investigating complimentary, supplementary, and vertical angles.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the ARI Curriculum Companion. Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

Standards of Learning
The following Standards of Learning are addressed in this section:

5.11 The student will measure right, acute, obtuse, and straight angles.
5.12 The student will classify
   a) angles as right, acute, obtuse, or straight;
   b) triangles as right, acute, obtuse, equilateral, scalene, or isosceles.
6.12 The student will determine congruence of segments, angles, and polygons.
8.6 The student will
   a) verify by measuring and describe the relationships among vertical angles, adjacent angles, supplementary angles, and complementary angles; and
   b) measure angles of less than 360°.

Table of Contents
Lesson plans pertaining to the following Standards of Learning are found in this section. Click (or CTRL+click) on each to jump to that lesson.

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**SOL 5.12a**

**Lesson Summary**
Students define the term angle and classify a given angle as right, obtuse, acute, or straight. (45 minutes)

**Materials**
- “What I Know About ______” worksheets
- “Index Card Angles” worksheets
- Craft sticks
- “Station Answers” worksheets
- Colored paper
- “Reflection” worksheets
- 3 x 5 inch index cards

**Vocabulary**
- **ray.** Part of a line that has one endpoint and continues infinitely in one direction.
- **angle.** Two rays that have the same endpoint. This endpoint is called the **vertex.**
- **right angle.** An angle that measures exactly 90°.
- **acute angle.** An angle that measures greater than 0° but less than 90°.
- **obtuse angle.** An angle that measures greater than 90° but less than 180°.
- **straight angle.** An angle that measures exactly 180°.

**Warm-up**
Distribute the “What I Know About ______” worksheets. Write the word angle on the board, and have students fill in the blank at the top of the page with the word angles. Give students three to five minutes to write down everything they know or think they know about angles. When time is up, put students in pairs to share and expand their personal lists. Have the pairs share their ideas with the whole class, and discuss what the students know about angles. Record their responses on the board for everyone to see and add to their lists.

**Lesson**
1. Distribute an “Index Card Angles” worksheet, an index card, and two craft sticks to every student.
2. Explain to students that there are three types of angles and that they are going to create them, using the craft sticks and index cards.
3. Have students place their two craft sticks along the outside of the corners of the index card. This forms a right angle. Students may refer to the “Index Card Angles” worksheet for assistance. To help students remember what a right angle looks like, tell them the story of Roy and Rachel Right. In class, Roy and Rachel Right always have the right answer so they always raise their hand to answer the questions. Have students raise their hand to show the vertical ray of the right angle.
4. Have students keep the horizontal craft stick in place and slant the vertical craft stick to the right. Students may use the “Index Card Angles” worksheet for assistance. Ask students if this angle is larger or smaller than the right angle. (smaller) Tell students the story of Annette and Andrew Acute. They were always worried about their appearance. They were so cute that they always had their hand in their hair making sure that they looked just perfect. Have students pretend to primp: this forms an acute angle with the elbow as the vertex.
5. Have students keep the horizontal craft stick in place and slant the vertical craft stick far to the left so that the angle takes up the index card and much more. Students may use the “Index Card Angles” worksheet for assistance. Ask if this angle is larger or smaller than the right angle. (larger) Tell students the story of Oliver and Olga Obtuse. They could never keep their hands to themselves. They were always busy bothering the student beside them. They would always reach over to tap that student on the shoulder. Have students reach out as if they were going to tap the person beside them on the shoulder: this forms an obtuse angle with the elbow as the vertex.
6. Have students keep the horizontal craft stick in place and place the other craft stick so the ends meet and form a straight line along the long edge of the index card.
7. Before class, print each “Station” sheet on a different colored sheet of paper, and place each sheet at a certain spot (“station”) in the room. Give a “Station Angles” worksheet to each student. Have
students take turns going to each station and classifying each angle as right, acute, obtuse or straight. Show them how to do this by placing the corner of their index card on the vertex of the angle and comparing whether the angle is smaller than, larger than, or exactly the same as a right angle.

8. Check answers as each student completes the task.

Reflection
Have students complete the “Reflection” worksheet.
Name: __________________________

What I Know About ________

Here’s what I know about ________________:

1. __________________________________________________________
2. __________________________________________________________
3. __________________________________________________________
4. __________________________________________________________
5. __________________________________________________________
6. __________________________________________________________
7. __________________________________________________________
8. __________________________________________________________
9. __________________________________________________________
10. _________________________________________________________
11. _________________________________________________________
12. _________________________________________________________
13. _________________________________________________________
14. _________________________________________________________
15. _________________________________________________________
16. _________________________________________________________
17. _________________________________________________________
18. _________________________________________________________
19. _________________________________________________________
20. _________________________________________________________
Name: ______________________

**Index Card Angles**

- **Acute angle**
- **Right angle**
- **Obtuse angle**
straight angle
Station 1
Station 2
Station 3
Station 5
Station Angles

At each station, measure the angle, and decide if it is an acute, obtuse, a right, or straight angle. Circle the answer that best describes the angle. Be prepared to explain your choice.

Station 1
A acute
B obtuse
C right
D straight

Station 2
A acute
B obtuse
C right
D straight

Station 3
A acute
B obtuse
C right
D straight

Station 4
A acute
B obtuse
C right
D straight

Station 5
A acute
B obtuse
C right
D straight
Name: ANSWER KEY

Station Angles

At each station, measure the angle, and decide if it is an acute, obtuse, a right, or straight angle. Circle the answer that best describes the angle. Be prepared to explain your choice.

Station 1
A acute
B obtuse
C right
D straight

Station 2
A acute
B obtuse
C right
D straight

Station 3
A acute
B obtuse
C right
D straight

Station 4
A acute
B obtuse
C right
D straight

Station 5
A acute
B obtuse
C right
D straight
Name: __________________________

Reflection

1. To the right is a practice SOL question. Circle your answer.

2. Explain why you chose that answer.

________________________________________

________________________________________

3. Give another example of a time at which the hour and minute hands of the clock form this type of angle.

________________________________________

________________________________________

4. Look around the room for angles. List four examples below.

1. __________________________

2. __________________________

3. __________________________

4. __________________________

What type of angle is formed between the hands of the clock shown below?

A  Right
B  Acute
C  Obtuse
D  Straight
Name: **ANSWER KEY**

**Reflection**

1. To the right is a practice SOL question. Circle your answer.

   A

2. Explain why you chose that answer.

   *Because both hands of the clock meet together to form what looks like a corner.*

3. Give another example of a time at which the hour and minute hands of the clock form this type of angle.

   Some possible answers: 3:30, 9:00, 9:30

4. Look around the room for angles. List four examples below.

   Answers will vary, e.g., “corners of the wall, edges of furniture, posters, pencils, floor tiles.”

What type of angle is formed between the hands of the clock shown below?

- A Right
- B Acute
- C Obtuse
- D Straight
Lesson Summary
Students name, measure, and classify angles. (40 minutes)

Materials
“Warm-up” worksheets
“Naming and Measuring Angles” worksheets
“Types of Angles Vocabulary” handouts
Protractors
“Reflection” worksheets

Vocabulary
ray. Part of a line that has one endpoint and continues infinitely in one direction.
angle. Two rays that have the same endpoint. This endpoint is called the vertex.
right angle. An angle that measures exactly 90°.
acute angle. An angle that measures greater than 0° but less than 90°.
obtuse angle. An angle that measures greater than 90° but less than 180°.
straight angle. An angle that measures exactly 180°.

Warm-up
Distribute the “Warm-up” worksheets, and have students classify each angle by writing the correct term in the space provided. After students have completed the worksheet, go over the answers.

Lesson
1. Distribute copies of the “Naming and Measuring Angles” worksheet and the “Types of Angles Vocabulary” handout.
2. Have students compare the angles from the warm-up worksheet to the angles on the “Naming and Measuring Angles” worksheet.
3. Demonstrate the two methods of naming an angle—by its vertex only, or by three points on the angle with the vertex listed between the other two points. Then, ask the students to name each angle on the worksheet three ways (e.g., \( \angle PBK, \angle B, \angle KBP \)). (Teacher Note: Students often confuse “naming” and “classifying” an angle. An effective way for them to remember the difference between the two is for them to equate this to their initials, which are most often comprised of three letters—the first letter of their first, middle, and last names. We name angles in the same manner—with letters. We classify angles with the words—acute, obtuse, and right.)
4. Pass out protractors, and teach students how to use the protractor if they are not familiar with the tool. Place the protractor on the angle so that the center is on the vertex and one ray goes through 0° on the protractor. The point where the other ray of the angle meets the protractor is the measure of the angle. It is normal to have a difference of two to three degrees over or under. Once students know how to measure, have them measure each angle on the worksheet. (Teacher Note: Many students have difficulty using a protractor. They have problems knowing what numbers to use—top or bottom. Model for students the strategy of first asking, “Is this angle acute or obtuse?” before they measure. This will help them know which numbers to use. Also remind students that if the rays of the angle are short they may use a straightedge to extend them to reach the protractor measurements.)
5. After an exact measure is given, the student can use the “Types of Angles Vocabulary” handout, if necessary, to classify the angle as acute, obtuse, or right, and record the answers on the line.

Reflection
Have students complete the “Reflection” worksheet.
Warm-up

Measure and classify each of the following angles as acute, obtuse, right, or straight.

1. __________

2. __________

3. __________

4. __________

5. __________
Name: ANSWER KEY

Warm-up

Measure and classify each of the following angles as acute, obtuse, right, or straight.

1. obtuse

2. acute

3. right

4. obtuse

5. straight
Types of Angles Vocabulary

acute angle
An angle that measures more than 0° but less than 90°.

right angle
An angle that measures exactly 90°.

obtuse angle
An angle that measures more than 90° but less than 180°.

straight angle
An angle that measures exactly 180°.
**Name:** ____________________

**Naming, Measuring, and Classifying Angles**

Name, measure, and classify the following angles.

1. Name: ______________
   Measure: _______
   Classification: ______________

2. Name: ______________
   Measure: _______
   Classification: ______________

3. Name: ______________
   Measure: _______
   Classification: ______________

4. Name: ______________
   Measure: _______
   Classification: ______________

5. Name: ______________
   Measure: _______
   Classification: ______________
**Name: ANSWER KEY**

### Naming, Measuring, and Classifying Angles

Name, measure, and classify the following angles.

1. **Name:** ∠PBK, ∠B, ∠KBP  
   **Measure:** 34°  
   **Classification:** acute

2. **Name:** ∠TRX, ∠R, ∠XRT  
   **Measure:** 127°  
   **Classification:** obtuse

3. **Name:** ∠WYH, ∠Y, ∠HYW  
   **Measure:** 150°  
   **Classification:** obtuse

4. **Name:** ∠JLC, ∠L, ∠CLJ  
   **Measure:** 90°  
   **Classification:** right

5. **Name:** ∠BCA, ∠C, ∠ACB  
   **Measure:** 180°  
   **Classification:** straight
Name: ______________________

Reflection

1. To the right is a practice SOL question. Circle your answer.
2. Explain why you chose that answer.

Laura connected points Y and Z to make one side of an angle. Which other point should she connect to point Y in order to make a $28^\circ$ angle?

3. Name Laura’s angle three ways.

4. Classify Laura’s angle.

5. Using a straightedge, draw an angle of your own at the bottom of this page. Label the vertex with a point and letter and include two other points. Name your angle three ways. Use a protractor to measure your angle. Classify your angle.
Name: ANSWER KEY

Reflection

1. To the right is a practice SOL question. Circle your answer.
   
   D

2. Explain why you chose that answer.
   
   I put the center of the protractor on point Y and lined ray YZ on the 0° line. I looked at the mark for the protractor for 28°. Point N was on that mark, which is answer D.

3. Name Laura’s angle three ways.
   
   ∠NYZ, ∠Y, ∠ZYN

4. Classify Laura’s angle.
   
   acute

5. Using a straightedge, draw an angle of your own at the bottom of this page. Label the vertex with a point and letter and include two other points. Name your angle three ways. Use a protractor to measure your angle. Classify your angle.
   
   Answers will vary.
**SOL 5.11, 5.12a**

**Lesson Summary**
Students use a guideline protractor in intervals of 45° to estimate the measure of angles before finding the exact measures. (45 minutes)

**Materials**
- Wax paper, patty paper, or tracing paper
- Permanent markers
- Protractors
- “Warm-up” worksheets
- “Estimating and Measuring Angles” worksheets
- “Reflection” worksheets

**Vocabulary**
- ray. Part of a line that has one endpoint and continues infinitely in one direction.
- angle. Two rays that have the same endpoint. This endpoint is called the vertex.
- right angle. An angle that measures exactly 90°.
- acute angle. An angle that measures greater than 0° but less than 90°.
- obtuse angle. An angle that measures greater than 90° but less than 180°.
- straight angle. An angle that measures exactly 180°.

**Warm-up**
Distribute the “Warm-up” worksheets, and have the students answer the questions. Go over the answers before going on to the lesson.

**Lesson**
1. Distribute a piece of wax paper (or tracing paper or patty paper), a protractor, and a permanent marker to each student.
2. Have students draw a straight line on the wax paper with their permanent marker. The line should be 5 to 6 inches in length.
3. Direct the students to place their protractor so that both ends of the line are on the 0° marks, and have them mark a point at the center mark, the 45° mark, the 90° mark, and the 135° mark.
4. Have students connect each point to the center point with a straight line. (See drawing on next page.) This creates a “guideline protractor” marked in intervals of 45°—a great tool for estimating the measurement of angles.
5. Direct students to label the guideline protractor with the inside and outside measurements so that it looks like a regular protractor.
6. Explain to students how they can use their guideline protractor just like a regular protractor as a tool to estimate angles. Students place the guideline protractor on top of the angle they want to measure, making sure that the center is on the vertex and one ray is on the 0° mark of the protractor. They then estimate the measurement of the angle by seeing between which two measurements the angle lies.
7. Distribute copies of the “Estimating and Measuring Angles” worksheet. Have students use their guideline protractor to estimate the measurement of each angle and to record their estimates. Remind students to extend the rays, if needed, and to decide whether the angle is acute or obtuse before measuring.
8. After all angle measurement have been estimated, have students use a regular protractor to find the actual measurement of each angle and record these actual measurements.
9. Ask the students to compare their estimated measurements to their actual ones. Were they close? Have a discussion about how close an estimate needs to be for it to be useful. In what situations is an estimate useful?

**Reflection**
Have students complete the questions on the “Reflection” worksheet.
Name: _______________________

**Warm-up**

Name, measure, and classify the following angles.

1. Name: __________
   Measure: ______
   Classification: __________

2. Name: __________
   Measure: ______
   Classification: __________

3. Name: __________
   Measure: ______
   Classification: __________

4. Name: __________
   Measure: ______
   Classification: __________

5. What does the word *estimation* mean to you?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Name: **ANSWER KEY**

**Warm-up**

Name, measure, and classify the following angles.

1. Name: $\angle ZQK$, $\angle Q$, $\angle KQZ$
   - Measure: 12°
   - Classification: acute

2. Name: $\angle VAT$, $\angle A$, $\angle TAV$
   - Measure: 90°
   - Classification: right

3. Name: $\angle UHE$, $\angle H$, $\angle EHU$
   - Measure: 101°
   - Classification: obtuse

4. Name: $\angle NGR$, $\angle G$, $\angle RGN$
   - Measure: 45°
   - Classification: acute

5. What does the word *estimation* mean to you?
   
   Some possible responses: “A guess at the answer.” “Trying to get close to the real answer.” “An approximation.”
Name: __________________________

**Estimating and Measuring Angles**

Use your guideline protractor to estimate the measurement of each angle. Use your regular protractor to get the exact measurement.

1. Estimation: ____________________________
   Actual: _______

2. Estimation: ____________________________
   Actual: _______

3. Estimation: ____________________________
   Actual: _______

4. Estimation: ____________________________
   Actual: _______

5. Estimation: ____________________________
   Actual: _______

6. Estimation: ____________________________
   Actual: _______
Name: ANSWER KEY

Estimating and Measuring Angles

Use your guideline protractor to estimate the measurement of each angle. Use your regular protractor to get the exact measurement.

1. Estimation: between 135° and 180°, or about 150°
   Actual: 148°

2. Estimation: between 45° and 90°, or about 47°
   Actual: 46°

3. Estimation: between 45° and 90°, or about 68°
   Actual: 62°

4. Estimation: between 90° and 135°, or about 115°
   Actual: 110°

5. Estimation: exactly on 135°
   Actual: 135°

6. Estimation: exactly on 45°
   Actual: 45°
Name: _____________________________

Reflection

1. To the right is a practice SOL question. Use your guideline protractor only to find the correct answer. Circle your answer.

2. Explain how you used your guideline protractor to find the correct answer.

Which of the following angles measures closest to 170°?

A

B

C

D

3. How could you figure out the answer without a protractor or a guideline protractor?
Reflection

1. To the right is a practice SOL question. Use your guideline protractor only to find the correct answer. Circle your answer.

D

2. Explain how you used your guideline protractor to find the correct answer.

I placed the center of the guideline protractor on the vertex of the angle. I made sure that one ray was on the 0° mark. I looked at the other ray for the measurement. The angle was between 135° and 180°. It was closest to 180°. Therefore, that was the correct answer.

3. How could you figure out the answer without a protractor or a guideline protractor?

I know that a straight line is 180°. 170° is only 10° away from a straight line. The angle is answer choice D because it is almost a straight line.
**SOL 5.11, 5.12a,b**

**Lesson Summary**
Students measure the angles inside a triangle and then classify the triangle as acute, obtuse, or right. (60 minutes)

**Materials**
- Protractors
- “Warm-up” worksheets
- “Triangle Vocabulary” handouts
- “Triangle Classification” worksheets
- “Triangle Classification Table” worksheets
- “Reflection” worksheets

**Vocabulary**
- **polygon.** A closed plane figure in which all sides are line segments.
- **triangle.** A three-sided polygon.
- **acute triangle.** A triangle that has three acute angles.
- **obtuse triangle.** A triangle that has one obtuse angle.
- **right triangle.** A triangle that has one right angle.

**Warm-up**
Distribute the “Warm-up” worksheets, and remind students that the sum of the measures of the three angles in a triangle equals 180°. You may wish to give them an example to help them remember this fact. Review how to find the measure of the third angle in a triangle when the other two angle measures are known. Have students complete the worksheet, and go over the answers before going on to the lesson.

**Lesson**
1. Distribute the “Triangle Vocabulary” handouts, and explain to or review with students that a triangle is classified by its largest angle. Go over the terms on the handout.
2. As a class, classify the four triangles on the “Warm-up” worksheet.
3. Explain and demonstrate how to extend the sides of a triangle to measure the angles and how to measure each angle inside the triangle to get all three angle measurements.
4. Distribute copies of the “Triangle Classification” and the “Triangle Classification Table” worksheets. The “Triangle Classification” worksheet is two pages and was designed to give the students space to extend the sides of the triangles for more accurate measuring.
5. Have students measure all three angles of each triangle and then classify each triangle. Have them record their answers on the “Triangle Classification Table” worksheet. Make sure students check to see that all three angles do add up to 180°.

**Reflection**
Have students complete the “Reflection” worksheet.
Name: ____________________

**Warm-up**

Find the measure of the missing angle in each triangle below. Do not use a protractor. *Remember: The measures of the three angles in a triangle add up to 180°.*

1. Measure of $\angle X = ______$

![Triangle with angles 60° and 90°](image1)

2. Measure of $\angle B = ______$

![Triangle with angles 56° and 56°](image2)

3. Measure of $\angle M = ______$

![Triangle with angles 35° and 119°](image3)

4. Measure of $\angle K = ______$

![Triangle with angles 47° and 48°](image4)
Name: ANSWER KEY

Warm-up

Find the measure of the missing angle in each triangle below. Do not use a protractor. 
*Remember: The measures of the three angles in a triangle add up to 180°.*

1. Measure of $\angle X = 30^\circ$

   \[
   \begin{tikzpicture}
   \draw (0,0) -- (0,60) -- (90,0) -- cycle;
   \draw (0,0) -- (0,15) node [above] {$X$};
   \draw (0,0) -- (15,0) node [right] {$90^\circ$};
   \draw (0,0) -- (15,15) node [left] {$60^\circ$};
   \end{tikzpicture}
   \]

2. Measure of $\angle B = 68^\circ$

   \[
   \begin{tikzpicture}
   \draw (0,0) -- (56,56) -- (56,0) -- cycle;
   \draw (0,0) -- (11,11) node [above] {$B$};
   \draw (0,0) -- (28,28) node [left] {$56^\circ$};
   \draw (0,0) -- (84,0) node [right] {$56^\circ$};
   \end{tikzpicture}
   \]

3. Measure of $\angle M = 26^\circ$

   \[
   \begin{tikzpicture}
   \draw (0,0) -- (119,35) -- (119,119) -- cycle;
   \draw (0,0) -- (19.2,21) node [above] {$M$};
   \draw (0,0) -- (59,59) node [left] {$119^\circ$};
   \draw (0,0) -- (119,64) node [right] {$35^\circ$};
   \end{tikzpicture}
   \]

4. Measure of $\angle K = 85^\circ$

   \[
   \begin{tikzpicture}
   \draw (0,0) -- (47,48) -- (48,47) -- cycle;
   \draw (0,0) -- (23.5,24) node [above] {$K$};
   \draw (0,0) -- (23.5,24) node [left] {$47^\circ$};
   \draw (0,0) -- (72,72) node [right] {$48^\circ$};
   \end{tikzpicture}
   \]
Triangle Vocabulary

**polygon**
A closed plane figure in which all the sides are line segments.

**triangle**
A three-sided polygon. The sum of the measures of the three angles of a triangle equals 180°.

**acute triangle**
A triangle that has three acute angles.

**obtuse triangle**
A triangle that has one obtuse angle.

**right triangle**
A triangle that has one right angle.

Remember: A triangle is classified by the largest of the three angles that form the triangle.
**Triangle Classification**

Measure each of the angles in each triangle using a protractor. Extend the sides if necessary. Make sure that the measures of the three angles add up to 180°. Record your answers in the Triangle Classification Table. Once you have figured out the measurement of each angle in the triangle, classify the triangle as acute, obtuse, or right.

1. 

2. 

3.
Name: ______________________

Triangle Classification

4.

5.

6.
### Triangle Classification Table

<table>
<thead>
<tr>
<th>Triangle Number</th>
<th>Angle 1</th>
<th>Angle 2</th>
<th>Angle 3</th>
<th>Sum of the three angles</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<tr>
<td>3</td>
<td></td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
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<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Name: ANSWER KEY

Triangle Classification Table

<table>
<thead>
<tr>
<th>Triangle Number</th>
<th>Angle 1</th>
<th>Angle 2</th>
<th>Angle 3</th>
<th>Sum of the three angles</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>103°</td>
<td>49°</td>
<td>28°</td>
<td>180°</td>
<td>obtuse</td>
</tr>
<tr>
<td>2</td>
<td>30°</td>
<td>125°</td>
<td>25°</td>
<td>180°</td>
<td>obtuse</td>
</tr>
<tr>
<td>3</td>
<td>45°</td>
<td>90°</td>
<td>45°</td>
<td>180°</td>
<td>right</td>
</tr>
<tr>
<td>4</td>
<td>45°</td>
<td>65°</td>
<td>70°</td>
<td>180°</td>
<td>acute</td>
</tr>
<tr>
<td>5</td>
<td>75°</td>
<td>85°</td>
<td>25°</td>
<td>180°</td>
<td>acute</td>
</tr>
<tr>
<td>6</td>
<td>37°</td>
<td>97°</td>
<td>46°</td>
<td>180°</td>
<td>obtuse</td>
</tr>
</tbody>
</table>
Name: ______________________

Reflection

1. To the right are two practice SOL questions. Circle your answers.

2. Explain why you chose that answer to the first question.

____________________________________________________________________
____________________________________________________________________
____________________________________________________________________

3. Explain why you chose that answer to the second question.

____________________________________________________________________
____________________________________________________________________
____________________________________________________________________

4. Can a triangle have more than one right angle? _________ Explain.

____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
Name: **ANSWER KEY**

**Reflection**

1. To the right are two practice SOL questions. Circle your answers.
   
   B, G

2. Explain why you chose that answer to the first question.
   
   A triangle is classified by its largest angle. A 104° angle is an obtuse angle. Therefore, the triangle is an obtuse triangle.

3. Explain why you chose that answer to the second question.
   
   All the angles in the triangle are acute. Therefore, the triangle is an acute triangle.

4. Can a triangle have more than one right angle? **No**
   
   Explain.
   
   Two right angles add up to be 180°, and because the *three* angles of a triangle must add up to be 180°, there could not be another angle.
**SOL 6.12**

**Lesson Summary**
Students compare line segments, angles, and polygons for congruence, using tracing and direct measurement. (40 minutes)

**Materials**
- Protractors
- Centimeter rulers
- Permanent markers
- Patty paper or tracing paper

**Vocabulary**
- line segment. Part of a line. It has two endpoints and includes all the points between those endpoints.
- angle. Two rays that have the same endpoint. This endpoint is called the vertex.
- ray. Part of a line having one endpoint and continuing infinitely in one direction.
- congruent figures. Figures having exactly the same size and same shape.
- polygon. A closed plane geometric figure composed of at least three line segments that do not cross.

**Warm-up**
Distribute the “Warm-up” worksheets, and if necessary, explain to students or remind them how to measure using a centimeter ruler. After they have completed the worksheet, go over the answers before going on to the lesson.

**Lesson**
1. Distribute the “Determining Congruence” and “Determining Congruence Table” worksheets.
2. Part A: Have students measure each pair of line segments, using a centimeter ruler, and record their measurements on the “Determining Congruence Table.” Have them compare the two measurements of the pairs of segments. If the two measurements are the same, the two line segments are congruent; if the two measurements are different, the two line segments are noncongruent. (Teacher Note: It is important for students to fully understand that line segments and other shapes may be congruent even if they look different because they are oriented differently. If students need reinforcement with this concept, have them practice reorienting pairs of congruent line segments and pairs of congruent figures.
3. Part B: Have students measure each angle in degrees, using a protractor, and record the measures of the angles in the “Determining Congruence Table.” Have students compare the measures of the two angles: if the two angles have the same measure, then the two angles are congruent; if the two angles have different measures, then the two angles are not congruent.
4. Part C: Distribute patty paper or tracing paper and permanent markers. Have students trace on the paper one of the polygons in each pair, using a permanent marker, and compare the pair of polygons by placing the traced polygon on top of the other polygon in the pair. If they are an exact match in size and shape, then the two polygons are congruent; if the two polygons differ in size and/or shape, then the two polygons are noncongruent.

**Reflection**
Have students answer the questions on the “Reflection” sheet. Review the responses as you walk around and check the students’ work.
Warm-up

Measure each line segment, using a centimeter ruler, and record each measurement.

1. __________________________ Measure = __________________________

2. __________________________ Measure = __________________________

3. __________________________ Measure = __________________________

4. __________________________ Measure = __________________________

Measure each angle in degrees, using a protractor, and record each measure.

5. Measure = ______

6. Measure = ______

7. Measure = ______

8. Define congruent: ____________________________________________________
**Warm-up**

Measure each line segment, using a centimeter ruler, and record each measurement.

1. ________________  Measure = 4 1/2 cm or 4.5 cm

2. ____________________________  Measure = 7 9/10 cm or 7.9 cm

3. ______  Measure = 2 9/10 cm or 2.9 cm

4. _____  Measure = 1 6/10 = 1 3/5 cm or 1.6 cm

Measure each angle in degrees, using a protractor, and record each measure.

5. Measure = 139°

6. Measure = 22°

7. Measure = 44°

8. Define congruent: Answers will vary, e.g., “Having the same size and shape.”
Determine Congruence

Part A

Measure each line segment in each pair, using a centimeter ruler, and record each measurement in the table. Then, determine whether the two line segments in each pair are congruent or noncongruent. Record your answers in the table.

1. V
   J

2. S
   Q
   O

3. X
   U
   O
   H
Determining Congruence

Part B

Measure each angle in each pair, using a protractor, and record each measurement in the table. Then, determine whether the two angles in each pair are congruent or noncongruent. Record your answers in the table.

4.

5.

6.
Name: __________________________

**Determining Congruence**

**Part C**

Trace one of the polygons in each pair, using patty paper or tracing paper and a marker. Then, place the tracing over the other polygon in the pair to determine whether the two polygons are congruent or noncongruent. Record your answers in the table.

7.

8.

9.
**Determining Congruence Tables**

### Part A

<table>
<thead>
<tr>
<th>Number</th>
<th>Measure Line Segment 1</th>
<th>Measure Line Segment 2</th>
<th>Congruent or Noncongruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Part B

<table>
<thead>
<tr>
<th>Number</th>
<th>Measure Angle</th>
<th>Measure Angle 2</th>
<th>Congruent or Noncongruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Part C

<table>
<thead>
<tr>
<th>Number</th>
<th>Congruent or Noncongruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
## Determining Congruence Tables

### Part A

<table>
<thead>
<tr>
<th>Number</th>
<th>Measure Line Segment 1</th>
<th>Measure Line Segment 2</th>
<th>Congruent or Noncongruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6 7/10 cm or 6.7 cm</td>
<td>6 7/10 cm or 6.7 cm</td>
<td>Congruent</td>
</tr>
<tr>
<td>2</td>
<td>5 4/10 = 5 2/5 cm or 5.4 cm</td>
<td>5 7/10 cm or 5.7 cm</td>
<td>Noncongruent</td>
</tr>
<tr>
<td>3</td>
<td>4 4/10 = 4 2/5 cm or 4.4 cm</td>
<td>4 1/10 cm or 4.1</td>
<td>Noncongruent</td>
</tr>
</tbody>
</table>

### Part B

<table>
<thead>
<tr>
<th>Number</th>
<th>Measure Angle</th>
<th>Measure Angle 2</th>
<th>Congruent or Noncongruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>45°</td>
<td>74°</td>
<td>Noncongruent</td>
</tr>
<tr>
<td>5</td>
<td>134°</td>
<td>134°</td>
<td>Congruent</td>
</tr>
<tr>
<td>6</td>
<td>33°</td>
<td>27°</td>
<td>Noncongruent</td>
</tr>
</tbody>
</table>

### Part C

<table>
<thead>
<tr>
<th>Number</th>
<th>Congruent or Noncongruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Congruent</td>
</tr>
<tr>
<td>8</td>
<td>Noncongruent</td>
</tr>
<tr>
<td>9</td>
<td>Congruent</td>
</tr>
</tbody>
</table>
Name: __________________________

Reflection

1. To the right is a practice SOL question. Circle your answer.

Which figures appear to be congruent?

A  A and B
B  B and D
C  C and D
D  D and A

2. Explain why you chose that answer.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

3. Draw two congruent line segments.

4. Draw two congruent angles.
Name: ANSWER KEY

Reflection

1. To the right is a practice SOL question. Circle your answer.
   
   D

2. Explain why you chose that answer.
   
   Answers will vary, e.g. “A and D are the same size.” “A and D are made up of the same number of grid boxes.”

Which figures appear to be congruent?

3. Draw two congruent line segments.

4. Draw two congruent angles.
**SOL 8.6a**

**Lesson Summary**
Students review complementary and supplementary angles and are introduced to vertical angles. 
(40 minutes)

**Materials**
- “Warm-up” worksheets
- “Angle Vocabulary” handouts
- “Angle Relationships Formed by Two Intersecting Lines” worksheets
- “Reflection” worksheets
- Protractors
- Colored pencils

**Vocabulary**
- **intersecting lines.** Lines that have one point in common.
- **complementary angles.** Any two angles such that the sum of their measures is 90°.
- **supplementary angles.** Any two angles such that the sum of their measures is 180°.
- **vertical angles.** Nonadjacent angles formed by two intersecting lines. Vertical angles are congruent and share a common vertex.

**Warm-up**
Distribute the “Warm-up” worksheets, and if necessary, review the definitions of complementary angles and supplementary angles. Show students how to find a complement or supplement from the information given, and have them complete the worksheet. If they need extra practice in finding complementary and supplementary angles, use the additional-practice worksheet included. Go over the answers before going on to the lesson.

**Lesson**
1. Ask, “What does intersect mean?” List responses, and make sure that they clearly understand the term before moving on.
2. Distribute the “Angle Relationships Formed by Two Intersecting Lines” worksheet. Ask students, “How many angles are formed when two lines intersect?”
3. Have students use a protractor to measure each of the four numbered angles formed from the two intersecting lines. Have them record their information in the measurement box. Make sure that students realize that an angle may also be named by a number, if given. Also, students need to realize that the notation (m∠1) means “measure of angle 1.”
4. Go over the answers. Ask students, “Which pairs of angles have the same measure?” (∠1 and ∠4, ∠2 and ∠3). Have students color ∠1 and ∠4 the same color and ∠2 and ∠3 a second color. Explain that the angle pairs that they have just colored are vertical angles. Vertical angles are always congruent, meaning that they have the same measure.
5. Distribute copies of the “Angle Vocabulary” handout. Have students fill in the notes box with this information.
6. Finish the notes box with the students, explaining the supplementary angle pairs. It may help students to cover ∠1 and ∠2 with their hands or another piece of paper to understand that ∠3 and ∠4 are supplementary, equal 180° when added together, and form a straight line.

**Reflection**
Have students complete the “Reflection” worksheet.
Warm-up

Use the following definitions to find the complement and supplement of the given angles.

**complementary angles.** Any two angles such that the sum of their measures is 90°.

**supplementary angles.** Any two angles such that the sum of their measures is 180°.

1. \( \angle 70° \)
   
   Complement: _______  Supplement: _______

2. \( \angle 37° \)
   
   Complement: _______  Supplement: _______

3. \( \angle 143° \)
   
   Complement: _______  Supplement: _______

4. \( \angle 160° \)
   
   Complement: _______  Supplement: _______
Warm-up

Use the following definitions to find the complement and supplement of the given angles.

**complementary angles.** Any two angles such that the sum of their measures is 90°.

**supplementary angles.** Any two angles such that the sum of their measures is 180°.

1. \( \text{Complement: } 20° \quad \text{Supplement: } 110° \)

2. \( \text{Complement: } 53° \quad \text{Supplement: } 143° \)

3. \( \text{Complement: } \text{none} \quad \text{Supplement: } 37° \)

4. \( \text{Complement: } \text{none} \quad \text{Supplement: } 20° \)
**Angle Vocabulary**

**complementary angles**
Any two angles such that the sum of their measures is 90°.

**supplementary angles**
Any two angles such that the sum of their measures is 180°.

**vertical angles**
Nonadjacent angles formed by two intersecting lines. Vertical angles are congruent and share a common vertex.

**intersecting lines**
Lines that have one point in common.
Angle Relationships Formed by Two Intersecting Lines

\[ \angle 1 = \ldots \]
\[ \angle 2 = \ldots \]
\[ \angle 3 = \ldots \]
\[ \angle 4 = \ldots \]

\( \angle 1 \) and \( \angle 4 \) are ________ angles.
\( \angle 2 \) and \( \angle 3 \) are ________ angles.
\( \angle 1 \) and \( \angle 3 \) are ________ angles.
\( \angle 1 \) and \( \angle 2 \) are ________ angles.
\( \angle 2 \) and \( \angle 4 \) are ________ angles.
\( \angle 3 \) and \( \angle 4 \) are ________ angles.
Name: ANSWER KEY

Angle Relationships Formed by Two Intersecting Lines

\[
\begin{align*}
\angle 1 &= 118^\circ \\
\angle 2 &= 62^\circ \\
\angle 3 &= 62^\circ \\
\angle 4 &= 118^\circ \\
\end{align*}
\]

\angle 1 and \angle 4 are \textit{vertical (congruent)} angles.

\angle 2 and \angle 3 are \textit{vertical (congruent)} angles.

\angle 1 and \angle 3 are \textit{supplementary} angles.

\angle 1 and \angle 2 are \textit{supplementary} angles.

\angle 2 and \angle 4 are \textit{supplementary} angles.

\angle 3 and \angle 4 are \textit{supplementary} angles.
Additional Practice

Part A: Use Figure 1 to answer the following questions:
1. \( \angle 9 \) and \( \angle 6 \) are ___________________ angles.
2. \( \angle 4 \) and \( \angle 6 \) are ___________________ angles.
3. If \( m\angle 4 = 87^\circ \), then \( m\angle 8 = \) ______.
4. If \( m\angle 4 = 87^\circ \), then \( m\angle 9 = \) ______.

Part B: Use Figure 2 to answer the following questions:
5. \( \angle 1 \) and \( \angle 7 \) are ___________________ angles.
6. \( m\angle 1 + m\angle 7 = \) ______.
7. If \( m\angle 1 = 115^\circ \), then \( m\angle 7 = \) ______.

Part C: Use Figure 3 to answer the following questions:
8. \( \angle 2 \) and \( \angle 3 \) are ___________________ angles.
9. \( m\angle 2 + m\angle 3 = \) ______.
10. If \( m\angle 3 = 40^\circ \), then \( m\angle 2 = \) ______.
Name: ANSWER KEY

Additional Practice

Part A: Use Figure 1 to answer the following questions:
1. \( \angle 9 \) and \( \angle 6 \) are vertical (congruent) angles.
2. \( \angle 4 \) and \( \angle 6 \) are supplementary angles.
3. If \( m\angle 4 = 87^\circ \), then \( m\angle 8 = 87^\circ \).
4. If \( m\angle 4 = 87^\circ \), then \( m\angle 9 = 93^\circ \).

Part B: Use Figure 2 to answer the following questions:
5. \( \angle 1 \) and \( \angle 7 \) are supplementary angles.
6. \( m\angle 1 + m\angle 7 = 180^\circ \).
7. If \( m\angle 1 = 115^\circ \), then \( m\angle 7 = 65^\circ \).

Part C: Use Figure 3 to answer the following questions:
8. \( \angle 2 \) and \( \angle 3 \) are complementary angles.
9. \( m\angle 2 + m\angle 3 = 90^\circ \).
10. If \( m\angle 3 = 40^\circ \), then \( m\angle 2 = 50^\circ \).
Reflection

1. To the right is a practice SOL question. Circle your answer.

2. Explain why you chose that answer.
   
   ____________________________________________
   ____________________________________________
   ____________________________________________
   ____________________________________________

3. Find the m\(\angle d\). Explain how you came up with your answer.
   
   ____________________________________________
   ____________________________________________
   ____________________________________________
   ____________________________________________

4. Find the m\(\angle c\). Explain how you came up with your answer.
   
   ____________________________________________
   ____________________________________________
   ____________________________________________
   ____________________________________________
Name: ANSWER KEY

Reflection

1. To the right is a practice SOL question. Circle your answer.
   \[ \text{H} \]

2. Explain why you chose that answer.
   \( \angle b \) is supplementary to \( \angle a \) because together they form a straight line. This means that together they are 180°. 109° subtracted from 180° is 71°.

3. Find the \( m\angle d \). Explain how you came up with your answer.
   \[ 109° \]
   \( \angle d \) is vertical to \( \angle a \). Vertical angles are congruent. If \( \angle a \) is 109°, then so is \( \angle d \).

4. Find the \( m\angle c \). Explain how you came up with your answer.
   \[ 71° \]
   \( \angle c \) is vertical to \( \angle b \). Vertical angles are congruent. If \( \angle b \) is 71°, then so is \( \angle c \).