## Introduction

In this section, the lessons focus on comparing and ordering fractions, decimals, percents and numbers written in scientific notation.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion.*

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

## Standards of Learning

The following Standards of Learning are addressed in this section:

5.2     The student will
     b)   compare and order fractions and decimals in a given set from least to greatest and greatest to least.

6.2     The student will
     b)   identify a given fraction, decimal, or percent from a representation;
     c)   demonstrate equivalent relationships among fractions, decimals, and percents; and
     d)   compare and order fractions, decimals, and percents.

7.1     The student will
     c)   compare and order fractions, decimals, percents, and numbers written in scientific notation;

8.1     The student will
     b)   compare and order decimals, fractions, percents, and numbers written in scientific notation.

## Table of Contents

Lesson plans pertaining to the following Standards of Learning are found in this section. Click (or CTRL+click) on each to jump to that lesson.

# ✳ SOL 5.2b, 6.2d, 7.1c, 8.1b

## Lesson Summary
Students develop strategies for comparing fractions. (45 minutes)

## Materials
Fraction circles, fraction bars, or other fraction manipulatives
Overhead projector and transparencies
Copies of the attached "Strategies for Comparing Fractions" worksheet

## Warm-up
Display the "whole" circle from the fraction circles (or the comparable piece from any fraction-manipulatives set) on the overhead. Then, display two fractions for students to compare, such as $\frac{1}{4}$ and $\frac{3}{5}$.

Ask students which fraction is greater, and tell them to justify their responses. Repeat a few times.

## Lesson

1.  Put students into groups, and give each group a set of fraction circles or other fraction manipulatives. Give each student a copy of the "Strategies for Comparing Fractions" worksheet.

2.  Have the students in each group pull the representations from their set for fractions with the *same denominators.* For example, have one student in each group take all of the 4$^{ths}$, another student all the 5$^{ths}$, etc. Have each student write all the fractions that can be represented with his/her pieces, arrange the pieces in order from smallest to largest, and write these fractions in the same order on

    the worksheet under Observation #1. For example, the student who took 5$^{ths}$ would write: $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$.

    Have students in each group compare their results and discuss the similarities they notice. Then, have them complete Observation #1 on the worksheet. Next, ask groups to share their responses. Write the consensus on the overhead or chalk board, and make sure that every student has a similar response on his/her sheet. (Answer: When the denominators are the same, the fraction with the *largest* numerator is the largest fraction because the numerator indicates the number of same-sized pieces of the whole.)

3.  Have each group model the following fractions with their pieces: $\frac{2}{10}, \frac{2}{8}, \frac{2}{4}, \frac{2}{3}, \frac{2}{5}, \frac{2}{6}, \frac{2}{12}$, order the models

    from smallest to largest, and write these fractions in the same order on the worksheet under

    Observation #2. Then, have students display models of $\frac{3}{10}, \frac{3}{4}, \frac{3}{8}, \frac{3}{12}, \frac{3}{5}, \frac{3}{7}, \frac{3}{6}$ and repeat the process.

    Repeat this process for fractions that have a numerator of 4, and then repeat it once more for fractions with a numerator of 5. Have students discuss the similarities they notice in the results. Then, have them complete Observation #2 on the worksheet. Next, ask groups to share their responses. Write the consensus on the overhead or chalk board, and make sure that every student has a similar response on his/her sheet. (Answer: When the numerators are the same, the fraction with the *smallest* denominator is the largest fraction because the denominator indicates the total number of pieces that make up the whole, and the fewer the pieces, the larger they are.

4.  Have each group model the following fractions with their pieces: $\frac{2}{5}, \frac{3}{4}, \frac{1}{6}, \frac{2}{3}, \frac{7}{8}$, order the models from

    smallest to largest, and write these fractions in the same order on the worksheet under Observation #3. Then, allow student groups to mix and discuss strategies for comparing these fractions. After several minutes, ask the students to share their responses. Write the consensus on the overhead or chalk board, and make sure that every student has a similar response(s) on his/her sheet. There will likely be several different valid strategies, so display all of them. If no one suggests the strategy of

    comparing the fractions to $\frac{1}{2}$, offer the strategy, and demonstrate the reasoning for it.

### Reflection

Write the following numbers on the board: **2**, **4**, **6**, **7**, **9**, **10**. Ask the students the following questions:

1. What is the smallest fraction you can make from these numbers? Explain your answer.

   (Answer: $\frac{2}{10}$, because it has the smallest number as the numerator and the largest number as the denominator.)

2. What is the largest fraction less than 1 that you can make? Explain your answer.

   (Answer: $\frac{9}{10}$, because it is only $\frac{1}{10}$ less than 1.)

3. What fractions less than $\frac{3}{5}$ can you make from these numbers? Explain your answer.

   (Answer: $\frac{2}{4}, \frac{2}{6}, \frac{2}{7}, \frac{2}{9}, \frac{2}{10}, \frac{4}{9}$, and $\frac{4}{10}$, because each of them is less than or equal to $\frac{1}{2}$, and $\frac{3}{5}$ is greater than $\frac{1}{2}$.)

4. Which fractions greater than $\frac{3}{8}$ can you make from these numbers? Explain your answer.

   (Answer: $\frac{2}{4}, \frac{4}{6}, \frac{4}{7}, \frac{6}{7}, \frac{6}{9}, \frac{7}{9}, \frac{6}{10}, \frac{7}{10}$, and $\frac{9}{10}$, because each of them is greater than or equal to $\frac{1}{2}$, and $\frac{3}{8}$ is less than $\frac{1}{2}$.)

5. What fraction equal to $\frac{1}{2}$ can you make from these numbers? Explain your answer.

   (Answer: $\frac{2}{4}$, because 2 is half of 4.)

6. What strategy did you use to answer the last three questions above?

   (Answer: The strategy of mentally comparing fractions to one-half ($\frac{1}{2}$) or one whole (1).)

**Name:** _____

# Strategies for Comparing Fractions

**Observation #1 – Fractions with Like Denominators**

My fractions in order are:

**Conclusion:**

When ordering fractions with like denominators, the strategy I use is

**Observation #2 – Fractions with Like Numerators**

My fractions in order are:

**Conclusion:**

When ordering fractions with like numerators, the strategy I use is

**Observation #3 – Fractions with Different Numerators and Denominators**

My fractions in order are:

**Conclusion:**

When ordering fractions with different numerators and denominators, one strategy I use is

# ✱ SOL 5.2b, 6.2d, 7.1c, 8.1b

## Lesson Summary

Students develop fraction number sense by comparing fractions and learning how to decide whether one fraction is greater than another. (45 minutes)

## Materials

Fraction circles, fraction bars, or other fraction manipulatives
Scissors
Copies of the attached "Fraction-Sorting Activity Board" and "Fraction-Sorting Cards" worksheets

## Warm-up

Display the "whole" circle from the fraction circles (or the comparable piece from any fraction-manipulatives set) on the overhead. Then, display the $\frac{1}{3}$ piece, and ask students to name it. Ask a student who knew the answer to convince you that the answer is correct. If the student's argument does not center on explaining the roles of the numerator and denominator, review them. Repeat this procedure, using different fractional pieces.

## Lesson

1. Divide students into pairs or groups. Give the groups copies of the attached worksheets, and instruct them to cut out the fraction cards, shuffle them, and place them face down in the middle of the table. Each group turns one card over at a time, decides as a group whether the fraction is closer to zero, one-half, or one, and places it in the correct column on the activity board. At the completion of the activity, each group should have sorted all cards into three piles. If students have difficulty, use the fraction pieces from the warm-up to demonstrate.

2. Ask the groups to examine all the fractions sorted under zero. These should include $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{2}{9}, \frac{2}{10}$, and $\frac{2}{15}$. Ask what these fractions have in common. (Answer: The number of fractional pieces—the numerator—is much smaller than the total number of pieces that make up the whole—the denominator.)

3. Ask the groups to examine all the fractions sorted under one-half. These should include $\frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{4}{10}$, $\frac{6}{12}$, and $\frac{8}{15}$. Ask what these fractions have in common. (Answer: The number of fractional pieces—the numerator—is either half or close to half of the total number of pieces that make up the whole—the denominator.)

4. Ask the groups to examine all the fractions sorted under the one. These should include $\frac{3}{4}, \frac{4}{5}, \frac{7}{8}$, and $\frac{10}{12}$. Ask what these fractions have in common. (Answer: The number of fractional pieces—the numerator—is only a little less than the total number of pieces that make up the whole—the denominator.)

5. Sum up by explaining that being able to recognize whether a fraction is closer to zero, one-half, or one is very helpful when ordering fractions with like and unlike denominators.

### Reflection

Write the following numbers on the board: **1**, **3**, **5**, **8**, **10**, **12**. Ask the students the following questions:

1. What is the smallest fraction you can make from these numbers? Explain your answer.

   (Answer: $\frac{1}{12}$, because it has the smallest number as the numerator and the largest number as the denominator.)

2. What is the largest fraction less than 1 that you can make? Explain your answer.

   (Answer: $\frac{10}{12}$, because it is only $\frac{2}{12}$ less than 1.)

3. What fractions less than $\frac{3}{5}$ can you make from these numbers? Explain your answer.

   (Answer: $\frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{3}{8}, \frac{3}{10}, \frac{3}{12}, \frac{5}{10}$, and $\frac{5}{12}$, because each of them is less than or equal to $\frac{1}{2}$, and $\frac{3}{5}$ is greater than $\frac{1}{2}$.)

4. Which fractions greater than $\frac{3}{8}$ can you make from these numbers? Explain your answer.

   (Answer: $\frac{3}{5}, \frac{5}{8}, \frac{5}{10}, \frac{8}{10}, \frac{8}{12}$ and $\frac{10}{12}$, because each of them is greater than or equal to $\frac{1}{2}$, and $\frac{3}{8}$ is less than $\frac{1}{2}$.

5. What fraction equal to $\frac{1}{2}$ can you make from these numbers? Explain your answer.

   (Answer: $\frac{5}{10}$, because 5 is half of 10.)

6. What strategy did you use to answer the last three questions above?

   (Answer: The strategy of mentally comparing fractions to one-half ($\frac{1}{2}$) or one whole (1).

**Name:** _____

# Fraction-Sorting Activity Board

Working with a partner or group, cut out the fraction cards found on the other sheet. For each card, decide whether the fraction is closer to zero, one-half, or one. Then, place the card in the correct column of the activity board below.

| CLOSEST TO ZERO | CLOSEST TO ONE-HALF | CLOSEST TO ONE |
|---|---|---|
| | | |

1.    How did you decide that the fractions in column one are closest to zero?

2.    How did you decide that the fractions in column two are closest to one-half?

3.    How did you decide that the fractions in column three are closest to one?

# Fraction-Sorting Cards

| | | | | |
|---|---|---|---|---|
| $\dfrac{4}{8}$ | $\dfrac{1}{3}$ | $\dfrac{5}{8}$ | $\dfrac{1}{5}$ | $\dfrac{2}{10}$ |
| $\dfrac{1}{7}$ | $\dfrac{3}{4}$ | $\dfrac{3}{8}$ | $\dfrac{7}{8}$ | $\dfrac{4}{5}$ |
| $\dfrac{10}{12}$ | $\dfrac{2}{15}$ | $\dfrac{8}{15}$ | $\dfrac{6}{12}$ | $\dfrac{2}{9}$ |

# ✳ SOL 6.2b,c

## Lesson Summary

Students discuss and interpret common uses and meanings of fractions, decimals, and percents, reinforcing the concept that the same quantity can be expressed as a fraction, decimal, or percent. (45 minutes)

## Materials

Chart paper
Newspapers
Colored markers
Graph paper
Colored pencils
Blank dice
Small plastic bags

## Warm-up

1. Make three charts with the headings "Real-Life Uses of Fractions," "Real-Life Uses of Decimals," and "Real-Life Uses of Percents." Divide the students into three groups, and assign one group to each chart. Give each group a set of newspapers and markers, and ask the group to search through the newspapers to find at least five examples to write on their chart.

2. Display the completed charts in the front of the room, and conduct a class discussion comparing the ways fractions, decimals, and percents are alike and different when they are used in real-life.

3. Write the fraction, decimal, and percent equivalents of one-half on the board. Ask the students in what way these three numbers are the same. (Answer: They are equal or equivalent representations.) Explain that even though the three numbers are equal, in certain real-life uses, we may use one or two forms but not the other(s). For example, we say $\frac{1}{2}$ inch, not 50% of an inch. Sale prices are listed as 25% off or $\frac{1}{4}$ off, not 0.25 off. We say a baseball player has a batting average of 0.325, not 13/40.

## Lesson

1. Give each student a piece of graph paper, and have him/her draw six large squares, each of which is 10 blocks long and 10 blocks wide. Explain that each of the six squares represents a whole divided into 100 equal parts. An entire square is equivalent to 100%, while each of the blocks is 1% of the square or whole. Ask students to use a colored pencil to shade 1% (one block) of the first square and write "1%" in the shaded area. Ask students to identify the shaded area as a fraction ($\frac{1}{100}$) and write the fraction under the square. Ask them to identify the shaded area as a decimal (0.01) and write the decimal under the square.

2. In the second square, ask students to shade 10 blocks in a row and write "10%" in the shaded area. Ask students to identify the shaded area as a fraction ($\frac{10}{100}$) and write the fraction under the square. Ask them to identify the shaded area as a decimal (0.01) and write the decimal under the square.

3. Repeat this procedure for the remaining four squares, having students shade 25 blocks for 25%, 50 blocks for 50%, 75 blocks for 75%, and 99 blocks for 99%. Offer assistance to students having difficulty.

4. Next, create sets of 10 to 15 Hundred Squares like to the ones the students created previously. Shade in various fractional parts of the squares corresponding to the fractional parts that your students need to practice for equivalence. Put each set of squares in a small plastic bag. Prepare dice by writing on each unmarked die **F** (Fraction) on two opposite sides, **D** (Decimal) on two opposite

sides, and **P** (Percent) on the remaining opposite sides (i.e., each die will have two **F**s, two **D**s and two **P**s.)

5. Divide the students into teams of 2 to 4 players each, and give each team a set of Hundreds Squares and one marked die. Instruct the teams to shuffle their Hundreds Squares and place them face down in the center of the table. Each student takes a turn drawing a square and rolling the die. If the player rolls a **P**, he/she must name the percent that is shaded. If the player rolls a **D**, he/she must name the decimal, and if the player rolls an **F**, he/she must name the fraction. A correct response entitles the student to keep the Hundreds Square. If the response is incorrect, the opponent or the person sitting to the player's left gets to say the correct answer and steal the square. Once all the squares have been drawn, the player with the most squares wins.

## Reflection

Distribute copies of the "Picture Perfect" worksheet and colored pencils, and have students complete the worksheet individually.

**Name:** _____

# Picture Perfect

Use three different colored pencils to create a design in the Hundreds Square below by coloring individual blocks one of the three different colors. You must color at least 75% of your square. Once your design is finished, complete the information below.

Color 1: _____     Percent: _____     Fraction: _____     Decimal: _____

Color 2: _____     Percent: _____     Fraction: _____     Decimal: _____

Color 3: _____     Percent: _____     Fraction: _____     Decimal: _____

           Total Colored:     Percent: _____     Fraction: _____     Decimal: _____

<table>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

# ✱ SOL 6.2b,c

## Lesson Summary

Students will identify equivalent fractions, decimals, and percents, using concrete materials, drawings and/or pictures, and mathematical symbols. (45 minutes)

## Materials

Fraction circles, fraction bars, or other fraction manipulatives
Overhead projector and transparencies
Play money
Newspaper articles or ads containing fractions, decimals, or percents
Light-colored construction paper
Glue
Scissors
Colored markers or pencils

## Warm-up

Display the "whole" circle from the fraction circles (or the comparable piece from any fraction-manipulatives set) on the overhead. Then, display two fractions for students to compare, such as $\frac{1}{4}$ and $\frac{3}{5}$.

Ask students which fraction is greater, and have them justify their responses. Repeat this procedure a few times with different fractions.

## Lesson

1. Divide students into groups of four each. Give each student a set of play money—dollar bills, quarters, dimes, nickels, and pennies. Tell students to consider a dollar bill as the number 100, and ask them to create four separate piles of coins, each of which contains like coins whose total amount equals one dollar, i.e., 4 quarters, 10 dimes, 20 nickels, or 100 pennies.

2. Explain that the students are going to build on their knowledge of currency values to evaluate relationships among fractions, decimals, and percents. Have the groups take one quarter and write its value in fraction form ($\frac{1}{4}$), in decimal form (0.25), and as a percentage of a dollar (25%). Point out that they have just written the three equivalent representations of a quarter. Have the groups follow the same procedure for 2 quarters ($\frac{2}{4}$ or $\frac{1}{2}$ simplified, 0.50, 50%) and 3 quarters ($\frac{3}{4}$, 0.75, 75%).

3. Have students work in groups to determine the fraction, decimal, and percent values of a penny ($\frac{1}{100}$, 0.01, 1%); a nickel ($\frac{1}{20}$, 0.05, 5%); a dime ($\frac{1}{10}$, 0.10, 10%). Compare student responses on the board, and discuss any discrepancies.

4. Ask students to write the fraction, decimal, and percent values of 3 pennies, 3 nickels, and 3 dimes. Finally, have them write the values of 12 pennies, 12 nickels, and 12 dimes. Compare student responses on the board, and discuss any discrepancies. Ask students why 12 dimes is greater than 100%.

## Reflection

Give a newspaper article or ad containing fractions, decimals, or percents to each student. Have the students mount the articles on construction paper and then rewrite the articles below the originals, using the equivalent amount in a different form. For example, if an ad reads "50% Off!," the student might rewrite it to read "$\frac{1}{2}$ off!" Have students include a picture or graphic to explain the meaning of the number.

# ✳ SOL 6.2d, 7.1c, 8.1b

## Lesson Summary

Students practice ordering and comparing fractions, decimals, and percents. (Optional: If the lesson includes SOL 7.1c and 8.1b, scientific notation is included.) (45 minutes)

## Materials

Index cards

## Warm-up

Review student strategies for ordering and comparing fractions, decimals, and percents. (Optional: Include scientific notation if the lesson includes SOL 7.1c and 8.1b.)

## Lesson

1. Create sets of playing cards for each group by writing the following fractions, decimals, and percents on index cards:

   0.001, 0.005, 0.01, 0.05, 0.10, 0.15, 0.2, 0.25, 0.45, 0.5, 0.55, 0.6, 0.75, 0.9, 1, 2, 2.5, 5, .1%, .5%, 1%, 5%, 10%, 15%, 20%, 25%, 45%, 50%, 55%, 60%, 75%, 90%, 100%, 200%, 250%, 500%, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{5}{1000}$, $\frac{5}{100}$, $\frac{1}{10}$, $\frac{15}{100}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{45}{100}$, $\frac{1}{2}$, $\frac{55}{100}$, $\frac{3}{5}$, $\frac{3}{4}$, $\frac{9}{10}$, $\frac{5}{5}$, $\frac{10}{5}$, $2\frac{1}{2}$.

   (Optional: For SOL 8.1b, create cards with numbers shown in scientific notation. For SOL 7.1c and 8.1b, create cards with numbers greater than 0 shown in standard and scientific notation .)

2. Put students in groups of two to four each, and give each group a set of the cards.

3. Have students shuffle the cards and place them face down on the table. In turn, each player draws two cards, places them face up, and orders the numbers from lowest to highest. If the two numbers are equal, the cards are placed one on top of the other. Correct ordering earns a player one point.

4. For the next round, each player draws three cards and orders the numbers from lowest to highest. If two numbers are equal, the cards are again placed one on top of the other. In this round, correct ordering earns a player two points.

5. In the next round, each player draws four cards and follows the same procedure as in the first two rounds. Correct ordering earns a player three points.

6. Students may continue with additional rounds, adding one more card and one more point per round. The player with the most points at the end of play wins.

## Reflection

Give each student six blank index cards, and have each student create two sets of cards for use in the game. For each set, students must write an equivalent fraction, decimal, and percent. All the students' cards can be combined for use in playing the game again in a future lesson.