Introduction

In this section, the lessons focus on identifying, classifying, describing, and comparing plane figures. Students determine similarity and congruence of figures and work with transformations.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion*.

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

Standards of Learning

The following Standards of Learning are addressed in this section:

- 5.12 The student will classify
 - b) triangles as right, acute, obtuse, equilateral, scalene, or isosceles.
- 5.13 The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
 - a) develop definitions of these plane figures; and
 - b) investigate and describe the results of combining and subdividing plane figures.
- 6.12 The student will determine congruence of segments, angles, and polygons.
- 6.13 The student will describe and identify properties of quadrilaterals.
- 7.6 The student will determine whether plane figures—quadrilaterals and triangles—are similar and write proportions to express the relationships between corresponding sides of similar figures.
- 7.7 The student will compare and contrast the following quadrilaterals based on properties: parallelogram, rectangle, square, rhombus, and trapezoid.
- 7.8 The student, given a polygon in the coordinate plane, will represent transformations (reflections, dilations, rotations, and translations) by graphing in the coordinate plane.
- 8.8 The student will
 - a) apply transformations to plane figures; and
 - b) identify applications of transformations.

Table of Contents

Lesson plans pertaining to the following Standards of Learning are found in this section. Click (or CTRL+click) on each to jump to that lesson.

*	SOL 5.12b
*	SOL 5.12b
*	<u>SOL 5.13a</u>
*	<u>SOL 5.13b</u>
*	<u>SOL 6.12, 7.6</u>
*	<u>SOL 6.12, 7.6</u>
*	<u>SOL 6.12</u>
*	SOL 6.12
*	<u>SOL 6.13, 7.7</u>
*	<u>SOL 6.13, 7.7</u>
*	<u>SOL 7.6</u>
*	<u>SOL</u> 7.8, 8.8
*	<u>SOL 7.8, 8.8a</u>
*	<u>SOL 7.8, 8.8a</u>
*	<u>SOL 8.8a</u> 124
*	SOL 8.8a
*	SOL 8.8b Coming soon

***** SOL 5.12b

Lesson Summary

Students are presented with a real-world problem involving the side lengths of triangles. (60 minutes)

Materials

"Naming Triangles" worksheets "Tricky Triangles Recording Sheet" handouts Pipe cleaners Rulers

Vocabulary

scalene triangle. Has no congruent sides. isosceles triangle. Has two congruent sides. equilateral triangle. Has all sides congruent. acute triangle. Has three acute angles. right triangle. Has one right angle. obtuse triangle. Has one obtuse angle.

Warm-up

Hand out the "Naming Triangles" worksheets, and have students complete them. Discuss the answers as a class.

Lesson

- 1. Pass out the "Tricky Triangles Recording Sheet" handouts, pipe cleaners, and rulers. Tell students that they should try to construct triangles with the side lengths listed in the table. As the students find some of the triangles impossible, have them conjecture why some are possible and some are not. Guide them to see the special relationship between any two sides of a triangle and the third side.
- 2. Following their hands-on experimentation, give the students the triangle inequality theorem, which states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- 3. Once students are clear on this relationship between any two sides of a triangle and the third side, present the following scenario to the students: "If a triangle has a side length of 24 cm, and another side measures 15 cm, what lengths can the other side be?" (Anything less than 39 cm)
- 4. Ask the students if there is a way to write the answer to this question, using a variable. Possible answers could include "The letter *x* must represent a number less than the sum of 24 and 15." Once the students state the inequality in words, help them transfer this "math sentence" into an algebraic inequality: *x* < 24 + 15. Explain to students that this is referred to as an *inequality* and that the value of *x* has more than one correct solution.
- 5. Explain to students that solving an inequality is the same process as solving an equation. Have students solve this inequality by combining like terms (adding the two values): x < 39. Discuss what this means in relation to the triangle example. (The length of the third side must be *less than* 39 cm.)
- 6. Show the students a number line like the one shown below:



7. Demonstrate how to graph x < 39. The circle below the number line is open to denote that the inequality cannot equal the number. If the inequality had read $x \le 39$, the circle would be shaded.

Reflection

Have students explain in a written paragraph the difference between using an open circle and using a shaded circle in graphing an inequality. (An open circle is used to graph an inequality when the inequality

stands alone, while a shaded circle is used to graph an inequality when the inequality reads "less than or equal to" or "greater than or equal to.")

Name:_____

Naming Triangles

Name each triangle by its angles and sides.



5.

Naming Triangles

Name each triangle by its angles and sides.



Virginia Department of Education

Name: _____

Tricky Triangles Recording Sheet

Using the pipe cleaners, try to construct triangles with the given side lengths. If a triangle with the given side lengths cannot be made, answer "no."

Side Lengths	Can a triangle be made? Yes or No
5 cm, 5 cm, 8 cm	
5 in., 8 in., 8 in.	
8 cm, 5 cm, 15 cm	
5 in., 6 in., 10 in.	
11 cm, 12 cm, 14 cm	
8 in., 8 in., 8 in.	

Based on your observations, what must be true in order to construct a triangle?

Tricky Triangles Recording Sheet

Using the pipe cleaners, try to construct triangles with the given side lengths. If a triangle with the given side lengths cannot be made, answer "no."

Side Lengths	Can a triangle be made? Yes or No
5 cm, 5 cm, 8 cm	<u>yes</u>
5 in., 8 in., 8 in.	<u>yes</u>
8 cm, 5 cm, 15 cm	<u>no</u>
5 in., 6 in., 10 in.	<u>yes</u>
11 cm, 12 cm, 14 cm	<u>no</u>
8 in., 8 in., 8 in.	yes

Based on your observations, what must be true in order to construct a triangle?

The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle is at most greater than the length of the third side of that triangle.

***** SOL 5.12b

Lesson Summary

Students measure the angles inside a triangle and then classify the triangle as either acute, obtuse, or right. (60 minutes)

Materials

Protractors "Warm-up" worksheets "Triangle Vocabulary" handouts "Triangle Classification" worksheets "Triangle Classification Table" worksheets "Reflection" worksheetsVocabulary **polygon.** A closed plane figure in which all sides are line segments. **triangle.** A polygon with three msides. **acute triangle.** Has three acute angles. **obtuse triangle.** Has one obtuse angle. **right triangle.** Has one right angle.

Warm-up

Distribute the "Warm-up" worksheets, and remind students that the sum of the measures of the three angles in a triangle equals 180°. You may wish to give them an example to help them remember this fact. Review how to find the measure of the third angle in a triangle when the other two angle measures are known. Have students complete the worksheet, and go over the answers before going on to the lesson.

Lesson

- 1. Distribute the "Triangle Vocabulary" handouts, and explain to or review with students that a triangle is classified by its largest angle. Go over the terms on the handout.
- 2. As a class, classify the four triangles on the "Warm-up" worksheet.
- 3. Explain and demonstrate how to extend the sides of a triangle to measure the angles and how to measure each angle inside the triangle to get all three angle measurements.
- 4. Distribute copies of the "Triangle Classification" and the "Triangle Classification Table" worksheets. The "Triangle Classification" worksheet is two pages and was designed to give the students space to extend the sides of the triangles for more accurate measuring.
- 5. Have students measure all three angles of each triangle and then classify each triangle. Have them record their answers on the "Triangle Classification Table" worksheet. Make sure students check to see that all three angles do add up to 180°.

Reflection

Have students complete the "Reflection" worksheet.

Name: _____

Warm-up

Find the measure of the missing angle in each triangle below. Do not use a protractor. *Remember: The measures of the three angles in a triangle add up to 180°.*



Warm-up

Find the measure of the missing angle in each triangle below. Do not use a protractor. *Remember: The measures of the three angles in a triangle add up to 180°.*



Triangle Vocabulary

polygon

A closed plane geometric figure in which all the sides are line segments.

triangle

A three-sided polygon. The sum of the three angles of a triangle equals 180°.

acute triangle

A triangle with three acute angles.

obtuse triangle

A triangle with one obtuse angle.

right triangle

A triangle with one right angle.

Remember: A triangle is classified by the *largest* of the three angles that form the triangle.

Name:

Triangle Classification

Measure each of the angles in each triangle using a protractor. Extend the sides if necessary. Make sure that the measures of the three angles add up to 180°. Record your answers in the Triangle Classification Table. Once you have figured out the measurement of each angle in the triangle, classify the triangle as acute, obtuse, or right.

1.





2

3.



Name: _____

Triangle Number	Angle 1	Angle 2	Angle 3	Sum of the three angles	Classification
1					
2					
3					
4					
5					
6					

Triangle Classification Table

Triangle Number	Angle 1	Angle 2	Angle 3	Sum of the three angles	Classification
1	<u>103°</u>	<u>49°</u>	<u>28°</u>	<u>180°</u>	obtuse
2	<u>30°</u>	<u>125°</u>	<u>25°</u>	<u>180°</u>	obtuse
3	<u>45°</u>	<u>90°</u>	<u>45°</u>	<u>180°</u>	right
4	<u>45°</u>	<u>65°</u>	<u>70°</u>	<u>180°</u>	acute
5	<u>75°</u>	<u>85°</u>	<u>25°</u>	<u>180°</u>	acute
6	<u>37°</u>	<u>97°</u>	<u>46°</u>	<u>180°</u>	obtuse

Triangle Classification Table

Name: _____

Reflection

1. To the right are two practice SOL questions. Circle your answers.

2. Explain why you chose that answer to the first guestion.

The largest angle in $\triangle ABC$ measures 104°. What kind of triangle is $\triangle ABC$?

- A Equiangular
- B Obtuse
- c Right
- D Acute

The angles in $\triangle ABC$ measure 27°, 73°, and 80°. What kind of triangle is $\triangle ABC$?

- F Equiangular
- G Acute
- H Obtuse
- J Right
- 3. Explain why you chose that answer to the second question.

4. Can a triangle have more than one right angle? _____ Explain.

Reflection

1. To the right are two practice SOL questions. Circle your answers.

<u>B, G</u>

2. Explain why you chose that answer to the first question.

<u>A triangle is classified by its largest angle. A 104° angle is an obtuse angle. Therefore, the triangle is an obtuse triangle.</u>

3. Explain why you chose that answer to the second question.

The largest angle in $\triangle ABC$ measures 104°. What kind of triangle is $\triangle ABC$?

A Equiangular B Obtuse C Right D Acute

The angles in $\triangle ABC$ measure 27°, 73°, and 80°. What kind of triangle is $\triangle ABC$?

F Equiangular G Acute H Obtuse

J Right

All the angles in the triangle are acute. Therefore, the triangle is an acute triangle.

4. Can a triangle have more than one right angle? <u>No</u> Explain.

Two right angles add up to be 180°, and because the *three* angles of a triangle must add up to be 180°, there could not be another angle.

*** SOL 5.13**a

Lesson Summary

Students analyze properties of polygons to develop definitions of *square, rectangle, parallelogram, rhombus,* and *trapezoid,* using geoboards. (45 minutes)

Materials

"Quadrilateral Concept Card" Geoboards and an overhead geoboard Geoboard dot paper (attached) "Quadrilateral Study Guide" worksheets "Quadrilateral Table" worksheets

Vocabulary

polygon. A closed plane geometric figure composed of at least three line segments that do not cross. **quadrilateral.** A polygon with four sides.

parallelogram. A quadrilateral with both pairs of opposite sides parallel.

rectangle. A quadrilateral with four right angles.

rhombus. A quadrilateral with 4 congruent sides

square. A rectangle with four sides of equal length.

trapezoid. A quadrilateral with exactly one pair of parallel sides.

Warm-up

Give each student a "Quadrilateral Concept Card." Have students follow along as you read and discuss each of the statements. Based on the information given on the concept card and the class discussion, have students draw their own examples of a polygon, non-polygon, quadrilateral, and non-quadrilateral. Have students also write their own definitions, based on their understanding of the words in context. When students are finished, have them share their card with a partner. Bring the class together, and go over the students' responses. Read the definitions of *polygon* and *quadrilateral* (see above) to the class, and allow students to change their definitions to make them more accurate, as necessary.

Lesson

- 1. Display on the overhead geoboard several four-sided figures. Ask students to identify what they all have in common. (The number of sides) Instruct students that all four-sided figures are called *quadrilaterals*. Focus on the prefix *quad*, and brainstorm other words to help with meaning.
- 2. Hand out geoboards and rubber bands, and review class rules about working with these materials. Allow the students to explore shapes with the geoboards for a few minutes.
- 3. Have students make a shape on their geoboard that is a quadrilateral with both pairs of opposite sides parallel and equal in length. Make this shape on the overhead geoboard. Discuss the properties of this figure: it has four sides, both pairs of opposite sides are parallel, opposite sides are congruent, opposite angles are congruent, a diagonal divides the shape into two congruent triangles. Tell the students that the name for this figure is **parallelogram**. Hand out dot paper, and have students copy their figure (or yours, if theirs was incorrect) on the dot paper and label it "parallelogram."
- 4. Have students make a **rectangle** on their geoboard. When they finish, make one on the overhead geoboard. Discuss the properties of a rectangle: it is a parallelogram with four right angles. Have students copy their rectangle on dot paper and label it "rectangle."
- 5. Ask for a volunteer to make a **square** on the overhead geoboard. Discuss the properties of a square: it has all of the properties of a parallelogram, and it is also a rectangle with four congruent sides. Have students copy the square on dot paper and label it "square."
- 6. Ask students to make a **rhombus** on their geoboard, and replicate it on the overhead. Guide students to discover that a rhombus has the properties of a parallelogram, which it has four congruent sides, and that opposite angles of a rhombus are congruent. Have the students label their figure "rhombus."

- 7. Ask students to make a four-sided shape on their geoboard with only one pair of opposite sides parallel. Ask whether anyone knows the name of this figure. Tell students that it is a **trapezoid**—a four-sided figure with exactly one pair of parallel sides.
- 8. Have individual students come to the overhead geoboard and create different types of quadrilaterals—rectangle, square, parallelogram, rhombus, and trapezoid—to review the properties and definitions of these quadrilaterals.
- 9. Have pairs of students complete the "Quadrilateral Study Guide." Provide assistance as needed.

Reflection

Have students complete the "Quadrilateral Table" worksheet.

Name: _____

Quadrilateral Concept Card

Polygons

These figures are polygons:



These figures are <u>not</u> polygons:



Which of these figures are polygons? (circle)



Draw your own example of a polygon.

Draw your own example of a non-polygon.

<u>Quadrilaterals</u>

These figures are quadrilaterals:



These figures are <u>not</u> quadrilaterals:



Which of these figures are quadrilaterals? (circle)



Draw your own example of a quadrilateral.

Draw your own example of a non-quadrilateral.

What is a polygon?

A polygon is

What is a quadrilateral?

A quadrilateral is

Quadrilateral Concept Card

Polygons

These figures are polygons:



These figures are <u>not</u> polygons:



Which of these figures are polygons? (circle)



Draw your own example of a polygon.

Drawing will vary.

Draw your own example of a non-polygon.

Drawing will vary. Sample answer:



What is a polygon? Sample answer:

A polygon is a simple, closed, plane figure formed by three or more straight lines.

<u>Quadrilaterals</u>

These figures are quadrilaterals:



These figures are <u>not</u> quadrilaterals:



Which of these figures are quadrilaterals? (circle)



Draw your own example of a quadrilateral.

Drawing will vary.

Draw your own example of a non-quadrilateral.

Drawing will vary. Sample answer:

What is a quadrilateral? Sample answer:

A quadrilateral is a four-sided polygon.

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Dot Paper

Name: _____

Quadrilateral Study Guide

Fill in the blanks, and draw the figures as directed.

- 1. A ______ is a polygon with four sides. Draw several examples of this below.
- 2. A ______ is a quadrilateral in which both pairs of opposite sides are parallel.
- 3. Properties of a parallelogram include the following:
 - a. A diagonal divides a parallelogram into two congruent ______.
 - b. The opposite sides of a parallelogram are _____.
 - c. The opposite angles of a parallelogram are ______.

For questions 4–8, refer to the drawings on the right.

- 4. A ______ is a parallelogram with four right angles. Since a _______ is a parallelogram, it has the same properties as those of a parallelogram.
- 5. A ______ is a rectangle with four congruent sides. Since a ______ is a rectangle, it has all the properties of a rectangle and of a parallelogram.
- 6. A ______ is a parallelogram with four congruent sides. Opposite angles of a ______ are congruent. Since a ______ is a parallelogram, it has all the properties of a parallelogram.



7. A ______ is a quadrilateral with exactly one pair of parallel sides.

Quadrilateral Study Guide

Fill in the blanks, and draw the figures as directed.

A quadrilateral is a polygon with four sides. Draw several examples of this below. 1.



- 2. A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.
- 3. Properties of a parallelogram include the following:
 - A diagonal divides a parallelogram into two congruent triangles. a.
 - b. The opposite sides of a parallelogram are parallel/congruent.
 - C. The opposite angles of a parallelogram are congruent.

For questions 4–8, refer to the drawings on the right.

- A rectangle is a parallelogram with four right angles. Since a 4. rectangle is a parallelogram, it has the same properties as those of a parallelogram. (Discuss with students that square also fits this definition, since a square is a special type of rectangle.)
- A square is a rectangle with four congruent sides. Since a square is a rectangle, 5. it has all the properties of a rectangle and of a parallelogram.
- 6. A rhombus is a parallelogram with four congruent sides. Opposite angles of a rhombus are congruent. Since a rhombus is a parallelogram, it has all the properties of a parallelogram. (Discuss with students that a square is a special rhombus, so it also fits this definition, but a square must have four right angles.)
- 7. A trapezoid is a quadrilateral with exactly one pair of parallel sides.











Name: _____

Quadrilateral Table

Place a check mark in the appropriate boxes to show which figures have which properties. Then answer the questions that follow.

PROPERTY	TYPES OF POLYGONS									
OF FIGURE	Quadrilateral	Parallelogram	Rectangle	Rhombus	Square	Trapezoid				
Only one set of parallel sides										
Two sets of parallel sides										
Two sides of equal length										
Four sides of equal length										
Four angles of equal measure										
All four angles are right angles										
It may contain a right angle										

Is a square a rectangle? _____ How do you know this? _____

I have four sides and two sets of equal sides, but not all four of my sides are equal, and I have two sets of parallel sides. What

shape(s) can I be?

Name: <u>ANSWER KEY</u>

Quadrilateral Table

Place a check mark in the appropriate boxes to show which figures have which properties. Then answer the questions that follow.

PROPERTY	TYPES OF POLYGONS									
OF FIGURE	Quadrilateral	Parallelogram	Rectangle	Rhombus	Square	Trapezoid				
Only one set of parallel sides						\checkmark				
Two sets of parallel sides		\checkmark	\checkmark	\checkmark	\checkmark					
Two sides of equal length		\checkmark	\checkmark		\checkmark					
Four sides of equal length				\checkmark	\checkmark					
Four angles of equal measure			\checkmark		\checkmark					
All four angles are right angles			\checkmark		\checkmark					
It may contain a right angle	\checkmark		\checkmark		\checkmark	\checkmark				

Is a square a rectangle? Yes How do you know this? Because it has 4 sides, 4 right angles, and 2 sets of parallel sides

I have four sides and two sets of equal sides, but not all four of my sides are equal, and I have two sets of parallel sides. What shape(s) can I be? <u>quadrilateral, rectangle, or parallelogram</u>

***** SOL 5.13b

Lesson Summary

Students investigate and describe the results of combining and subdividing shapes, using tangram pieces. (30 minutes)

Materials

Sets of tangrams

"Tangrams Activity Chart"

"Tangram Puzzles" worksheets

"Assessment Questions" worksheetsWarm-up

Pass out the tangram pieces, or have the students cut them out from the "Tangram Template." Tell the students to write down the names of all seven pieces: A, B, C, D, E are right triangles, F is a square, and G is a parallelogram.

Lesson

- 1. Discuss with students the facts that two or more polygons can be combined to form a new shape and, conversely, that a polygon can be subdivided into two or more shapes.
- 2. Distribute the "Tangram Activity Chart". Have students follow the directions and then describe their solutions orally.
- 3. Pass out the Tangram Puzzle activity sheet. Have students make the figures using all seven tangram pieces and then draw a sketch of their solutions. Participants can work individually or in small groups to solve the tangram puzzles.

Reflection

Have students reflect on and write answers to the assessment questions.





Name:

Tangram Activity Chart

Use the number of tangram pieces specified in the first column to form each of the geometric figures shown in the top row. As you make each shape, list in the proper box the pieces you use to make it. Some problems may have more than one solution, while others may have no solution.

Number of pieces	Square	Rectangle	Triangle	Trapezoid	Parallel -ogram
2					
3					
4					
5					
6					
7					

Tangram Activity Chart

Use the number of tangram pieces specified in the first column to form each of the geometric figures shown in the top row. As you make each shape, list in the proper box the pieces you use to make it. Some problems may have more than one solution, while others may have no solution.

(Teacher's Note: For each problem with multiple solutions, only one is included here.)

Number of pieces	Square	Rectangle	Triangle	Trapezoid	Parallel -ogram
2	<u>D & E</u>	<u>Not</u> possible	<u>D & E</u>	<u>D & F</u>	<u>D & E</u>
3	<u>D, E, & C</u>	<u>D, C, & E</u>	<u>D, E, & C</u>	<u>А, В, & С</u>	<u>D, E, & C</u>
4	<u>D, E, C, &</u> <u>В</u>	<u>D, E, C, &</u> <u>В</u>	<u>D, E, C, &</u> <u>В</u>	<u>D, E, A, &</u> <u>B</u>	<u>D, E, C, &</u> <u>В</u>
5	<u>D, E, F, G,</u> <u>& C</u>	<u>D, E, F, G,</u> <u>& C</u>	<u>D, E, F, G,</u> <u>& C</u>	<u>D, E, F, G,</u> <u>& C</u>	<u>D, E, F, G,</u> <u>& C</u>
6	<u>Not</u> possible	<u>C, E, F, D,</u> <u>G, & A</u>	<u>Not</u> possible	<u>C, E, F, D,</u> <u>G, & A</u>	<u>C, E, F, D,</u> <u>G, & A</u>
7	All pieces	All pieces	All pieces	All pieces	All pieces

Name:

Tangram Puzzles

Can you make these figures, using all seven tangram pieces? Make a sketch of your solutions.



Name:

Assessment Questions

1. What strategy did you use to find the shapes?

2. Is there a basic shape that could be used to make all the figures?

3. Which pieces did you tend to use more than others? Why?

4. What are some other ways to construct each of the shapes?

5. Why is there no six-piece square?

***** SOL 6.12, 7.6

Lesson Summary

Students sort a set of triangles into pairs and discover the relationship between the figures that are similar or congruent. Students create similar and congruent figures on geoboards. (40 minutes)

Materials

Sets of "Similar and Congruent Triangle Sorting Pieces" Geoboards Rubber bands "Applying the Lesson" worksheets "Similar or Congruent?" worksheets

Vocabulary

congruent figures. Two figures that have exactly the same size and the same shape.

similar figures. Two polygons that have the same shape but not the same size. Two figures are similar if corresponding (matching) angles are congruent and the lengths of corresponding sides are proportional.

Warm-up

Copy the 12 "Similar and Congruent Triangle Sorting Pieces" onto card stock and cut them out, making enough sets of 12 pieces to give a complete set to each pair of students. Alternatively, have each pair of students cut out their own set. After distributing the sets, ask the partners to pair up the triangles that are related in some way, explaining that for each triangle, there is another that is like it in one way or another. Have students write down the triangle pairs they matched and an explanation of why they paired the triangles the way they did.

Lesson

- 1. Have student partners orally explain how they paired the triangles.
- 2. Talk about the triangles that are the same shape and same size. Give the students the definition of *congruent figures* (see above). Show example of congruent triangles.
- 3. Talk about the triangles that are the same shape but not the same size. Give the students the definition of *similar figures* (see above). Show example of similar triangles.
- 4. Pass out geoboards and rubber bands. Have students create similar and congruent figures according to your directions. For example, say, "Show me similar rectangles." Students should create two rectangles of the same shape but different size. Have students practice making these until they are familiar with the terms *similar* and *congruent*.
- 5. Have the students complete the "Applying the Lesson" worksheet, giving assistance as needed.

Reflection

Have students complete the "Similar or Congruent?" worksheet.

Similar and Congruent Triangle Sorting Pieces







Similar and Congruent Triangle Sorting Pieces



Name:

Applying the Lesson

In the first box in each row, draw a simple figure. In the second box in that row, draw a figure <u>similar</u> to the original. In the third box, draw a figure <u>congruent</u> to the original figure.

Original Figure

Similar Figure

Congruent Figure



Virginia Department of Education

Applying the Lesson

In the first box in each row, draw a simple figure. In the second box in that row, draw a figure <u>similar</u> to the original. In the third box, draw a figure <u>congruent</u> to the original figure.

Original Figure









Similar Figure





.

Congruent Figure









Virginia Department of Education

Name:_____

Similar or Congruent?

Label each pair of figures below as **similar**, **congruent**, or **neither**. Then, justify your answer with its mathematical definition.



Similar or Congruent?

Label each pair of figures below as **similar**, **congruent**, or **neither**. Then, justify your answer with its mathematical definition.



These figures are <u>neither</u> because they are neither the same shape nor the same size.

***** SOL 6.12, 7.6

Lesson Summary

Students sort triangles and quadrilaterals and discover similar and congruent relationships among the figures. (30–40 minutes)

Materials

"Similar and Congruent Figures Concept Card" Sets of "Similar and Congruent Sorting Pieces" "Similar or Congruent Figures Chart" "Similar or Congruent?" worksheets

Vocabulary

congruent figures. Two figures that have exactly the same size and the same shape.

similar figures. Two polygons that have the same shape but not the same size. Two figures are similar if corresponding (matching) angles are congruent and the lengths of corresponding sides are proportional.

Warm-up

Give each student a "Similar and Congruent Figures Concept Card." Based on the information given on the card, have students draw their own pairs of congruent, similar, noncongruent, and nonsimilar figures. Have students write their own definitions of these terms, based on their understanding of the words in context. When they are done, have them share their card with a partner. Bring the class together, and go over the students' responses.

Lesson

- 1. Copy the "Similar and Congruent Sorting Pieces" onto card stock and cut them out, making enough sets of pieces to give a complete set to each pair of students. After distributing the sets of sorting pieces and the "Similar or Congruent Figures Charts," ask the partners to sort the shapes into pairs according to whether they are similar or congruent. Instruct them to place (or write) the pairs on the correct side of the chart.
- 2. Have each group orally explain how they paired the figures.
- 3. Talk about the figures that are the same shape and same size. Review the definition of *congruent figures* (see above), stressing that all properties of the two figures are exactly the same and that the two figures would match perfectly if one were laid on top of the other. Show other examples of congruent figures.
- 4. Talk about the figures that are the same shape but not the same size. Review the definition of *similar figures* (see above). Ask students to compare two similar figures, noticing what is similar and what is different. They should observe that the angles are congruent but the sides are not. Show other examples of similar figures. Ask: "Are all congruent figures also similar figures?"
- 5. Have students go back and make any necessary changes to the definitions on their concept cards.

Reflection

Have students complete the "Similar or Congruent?" worksheet.

Name: _____

Similar and Congruent Figures Concept Card

Similar Figures These pairs of figures are similar. These pairs of figures are nonsimilar. Which of these pairs of figures are similar? Draw your own pair of similar figures. Draw your own pair of nonsimilar figures. What are similar figures? Similar figures are

Congruent Figures

These pairs of figures are <u>congruent</u>.



These pairs of figures are noncongruent.



Which of these pairs of figures are congruent?



Draw your own pair of congruent figures.

Draw your own pair of noncongruent figures.

What are congruent figures?

Congruent figures are

Virginia Department of Education

Similar and Congruent Figures Concept Card















Name: _____

Similar or Congruent Figures Chart

Sort the figures into pairs according to whether they are similar or congruent. Then, identify the pairs by writing their letters in the chart below.

SIMILAR FIGURES	CONGRUENT FIGURES

Similar or Congruent Figures Chart

Sort the figures into pairs according to whether they are similar or congruent. Then, identify the pairs by writing their letters in the chart below.

SIMILAR FIGURES	CONGRUENT FIGURES
<u>0 & S</u>	<u>P & N</u>
<u>R & T</u>	<u>M & V</u>
<u>U & Q</u>	<u>C & F</u>
<u>J & G</u>	<u>H & B</u>
<u>A & E</u>	<u>I&L</u>
	<u>D & K</u>

Name: _____

Similar or Congruent?

Determine whether each pair of figures below is **similar** or **congruent**, and give a brief explanation of your answer.



4. Draw a pair of quadrilaterals that are similar and another pair that are congruent. Label the pairs.

Answer True or False to the following questions. If false, explain why.

- 5. Congruent figures are always similar.
- 6. Similar figures are never congruent.

Similar or Congruent?

Determine whether each pair of figures below is **similar** or **congruent**, and give a brief explanation of your answer.



4. Draw a pair of quadrilaterals that are similar and another pair that are congruent. Label the pairs. <u>Sample answers:</u>



Answer True or False to the following questions. If false, explain why.

- 5. Congruent figures are always similar. <u>True</u>
- 6. Similar figures are never congruent. <u>False. They may be the same in size, and if they are, they are congruent.</u>

*** SOL 6.12**

Lesson Summary

Students identify and explore congruent and noncongruent polygons. (40 minutes)

Materials

"Congruent Figures Concept Card" Scrap paper "Rectangle Templates" worksheets "Congruent Figures" worksheets Patty paper or tracing paper "Reflecting on Congruent Figures" worksheets

Vocabulary

congruent figures. Two figures that have exactly the same size and the same shape.

Warm-up

Give each student a "Congruent Figures Concept Card." Based on the information given on the card, have students draw their own examples of congruent and noncongruent figures. Have students write their own definitions of these terms, based on their understanding of the words in context. When they are done, have them share their card with a partner. Bring the class together, and go over the students' responses.

Lesson

- 1. Give each student a "Rectangle Templates" worksheet, and ask students to cut out one of the rectangles and use it as a template to draw a rectangle on a sheet of scrap paper. Then, ask them to draw another rectangle, using the same template, at another place on the same sheet but to turn the template a different way so that the two rectangles appear to be different. Next, ask them to use another smaller or larger rectangular template to draw a third rectangle on the sheet. Ask students to describe the three rectangles on the sheet. This discussion should lead to an understanding of the word *congruent*, meaning two figures that are exactly the same in shape and size, as well as the meaning of the word *noncongruent*, meaning two figures that are not the same in shape and/or size.
- 2. Based on the discussion for congruent figures, have students make necessary changes to their definitions and examples on their concept cards.
- 3. Ask students to trace the original rectangle template at the very top of another sheet of scrap paper. Then, ask them to trace another identical rectangle below it but to turn and flip the rectangle template so that it is difficult to tell whether the two rectangles really are congruent. Have students trace a few more identical rectangles in different positions and a few non-identical rectangles on the sheet. When they are done, have them trade papers.
- 4. After the papers have been traded, give students a minute to determine which rectangles are congruent to the one at the top of the paper. After they have decided, hand out the patty paper or tracing paper, and have them trace the original rectangle at the top. Then have them turn and/or flip their tracing to identify the rectangles that are congruent. Ask the students to circle the congruent rectangles.
- 5. Have students use the tracing method to prove which figures are congruent on the "Congruent Figures" worksheet.
- 6. Next, have students use the original rectangle template to trace a rectangle in the center of another sheet of paper and label the drawing "Original."
- 7. Have them realign the template with the new drawing, carefully slide the template in any direction without changing its orientation, trace another rectangle, and label this second rectangle "Translation (Slide)."

- 8. Next, have them execute a translation in another direction from the original but then rotate the template on one point, trace it, and label this third rectangle "Rotation (Turn)."
- 9. Finally, have the students execute a translation in yet another direction from the original but then flip the template over, trace it, and label this fourth rectangle "Reflection (Flip)."
- 10. While these figures are being drawn, discuss the reasons for using these terms to label these rectangles.
- 11. After you have discussed the terms, have students go back to the "Congruent Figures" worksheet. For each figure that they said was congruent with the one at the left, have students label how the second figure was moved, using one of the three terms *translation, rotation,* or *reflection.*

Reflection

Have students complete the "Reflecting on Congruent Figures" worksheet.

Name:

Congruent Figures Concept Card

These pairs of figures are <u>congruent</u>.



These pairs of figures are noncongruent.



Which of these pairs of figures are congruent?



Draw your own pair of congruent figures.

Draw your own pair of noncongruent figures.

What are congruent figures?

Congruent figures are

Congruent Figures Concept Card

These pairs of figures are congruent.



These pairs of figures are noncongruent.



Which of these pairs of figures are congruent?



Draw your own pair of congruent figures. <u>Sample answer:</u>



Draw your own pair of noncongruent figures. <u>Sample answer:</u>



What are congruent figures? Sample answer:

Congruent figures are two figures that are exactly the same in shape and size.

Name: _____

Rectangle Templates

L		







Name:

Congruent Figures Worksheet

Look carefully each row of figures. Circle the figures in that row that are congruent to the first figure in the row, and label them "translation," "rotation," or "reflection." Some figures might be labeled with two terms.



Virginia Department of Education

Congruent Figures Worksheet

Look carefully each row of figures. Circle the figures in that row that are congruent to the first figure in the row, and label them "translation," "rotation," or "reflection." Some figures might be labeled with two terms.



Name: _____

Reflecting on Congruent Figures

In the space below, draw a congruent figure for each figure shown.



Reflecting on Congruent Figures

7. Which 2 shapes below are congruent?



- A L and P
- B M and N
- C N and L
- D P and N
- 9. Which figures appear to be congruent?



- A A and B
- B B and D
- C C and D
- D D and A

 Wayne cut this shape out of a piece of paper.



Which of the following is missing a piece exactly the same size and shape as the piece shown above?









Reflecting on Congruent Figures

In the space below, draw a congruent figure for each figure shown.



Reflecting on Congruent Figures

7. Which 2 shapes below are congruent?



- A L and P B M and N
- C N and L
- D P and N

 Wayne cut this shape out of a piece of paper.



Which of the following is missing a piece exactly the same size and shape as the piece shown above?

9. Which figures appear to be congruent?



- A A and B
- B B and D
- C C and D
- D D and A









***** SOL 6.12

Lesson Summary

Students compare line segments, angles, and polygons for congruence, using tracing and direct measurement. (40 minutes)

Materials

Protractors Centimeter rulers Permanent markers Patty paper or tracing paper "Warm-up" worksheets "Determining Congruence" worksheets "Determining Congruence Tables" worksheets "Reflecting" worksheetsVocabulary **congruent.** Congruent figures have exactly the same size and the same shape.

Warm-up

Distribute the "Warm-up" worksheets, and if necessary, explain to students or remind them how to measure using a centimeter ruler. After they have completed the worksheet, go over the answers before going on to the lesson.

Lesson

- 1. Explain to students that they are going to compare two geometric images in order to reach one of two conclusions: (1) they are congruent, or (2) they are noncongruent.
- 2. Distribute the "Determining Congruence" and the "Determining Congruence Tables" worksheets.
- 3. Part A: Have students measure each pair of line segments, using a centimeter ruler, and record their measurements on the "Determining Congruence Table." Have them compare the two measurements of the pairs of segments. If the two measurements are the same, the two line segments are congruent; if the two measurements are different, the two line segments are noncongruent. (Note: It is important for students to fully understand that line segments and other shapes may be congruent even if they are oriented differently. If students need reinforcement with this concept, have them practice reorienting pairs of congruent line segments and pairs of congruent figures.
- 4. Part B: Have students measure each angle in degrees, using a protractor, and record the measures of the angles in the "Determining Congruence Table." Have students compare the measures of the two angles: if the two angles have the same measure, then the two angles are congruent; if the two angles have different measures, then the two angles are noncongruent.
- 5. Part C. Distribute patty paper or tracing paper and permanent markers. Have students trace on the paper one of the polygons in each pair, using a permanent marker, and compare the pair of polygons by placing the traced polygon on top of the other polygon in the pair. If they are an exact match in size and shape, then the two polygons are congruent; if the two polygons differ in size and/or shape, then the two polygons are noncongruent.

Reflection

Have students answer the questions on the "Reflecting" sheet. Review the responses as you walk around and check the students' work.

Name: _____

Warm-up

Measure each line segment, using a centimeter ruler, and record each measurement.



Measure each angle in degrees, using a protractor, and record each measure.



8. Define *congruent*:_____

Warm-up

Measure each line segment, using a centimeter ruler, and record each measurement.



Measure each angle in degrees, using a protractor, and record each measure.



8. Define *congruent*: <u>Answers will vary, e.g., "Having the same size and shape."</u>

Name: ____

Determining Congruence

Part A

Measure each line segment in each pair, using a centimeter ruler, and record each measurement in the table. Then, determine whether the two line segments in each pair are congruent or noncongruent. Record your answers in the table.



Name:

Determining Congruence

Part B

Measure each angle in each pair, using a protractor, and record each measurement in the table. Then, determine whether the two angles in each pair are congruent or noncongruent. Record your answers in the table.



Name:

Determining Congruence

Part C

Trace one of the polygons in each pair, using patty paper or tracing paper and a marker. Then, place the tracing over the other polygon in the pair to determine whether the two polygons are congruent or noncongruent. Record your answers in the table.



Name: _____

Determining Congruence Tables

Part A			
Number	Measure Line Segment 1	Measure Line Segment 2	Congruent or Noncongruent
1			
2			
3			

Part B

Number	Measure Angle	Measure Angle 2	Congruent or Noncongruent
4			
5			
6			

Part C

Number	Congruent or Noncongruent
7	
8	
9	

Part A			
Number	Measure Line Segment 1	Measure Line Segment 2	Congruent or Noncongruent
1	<u>6 7/10 cm</u> <u>or</u> <u>6.7 cm</u>	<u>6 7/10 cm</u> <u>or</u> <u>6.7 cm</u>	<u>Congruent</u>
2	$\frac{5 \text{ 4/10} = 5 \text{ 2/5 cm}}{\frac{\text{or}}{5.4 \text{ cm}}}$	<u>5 7/10 cm</u> <u>or</u> <u>5.7 cm</u>	Noncongruent
3	$\frac{4 4/10 = 4 2/5 \text{ cm}}{\frac{\text{or}}{4.4 \text{ cm}}}$	<u>4 1/10 cm</u> <u>or</u> <u>4.1</u>	Noncongruent

Determining Congruence Tables

Part B

Number	Measure Angle	Measure Angle 2	Congruent or Noncongruent
4	<u>45°</u>	<u>74°</u>	Noncongruent
5	<u>134°</u>	<u>134°</u>	<u>Congruent</u>
6	<u>33°</u>	<u>27°</u>	Noncongruent

Part (С
--------	---

Number	Congruent or Noncongruent
7	<u>Congruent</u>
8	Noncongruent
9	Congruent

Name: _____

Reflecting

- 1. To the right is a practice SOL question. Circle your answer.
- 2. Explain why you chose that answer.



Which figures appear to be congruent?

- A A and B B B and D
- C C and D
- D D and A
- 3. Draw two congruent line segments.

4. Draw two congruent angles.
Reflecting

1. To the right is a practice SOL question. Circle your answer.

<u>D</u>

2. Explain why you chose that answer.

Answers will vary, e.g. "A and D are the same size." "A and D are made up of the same number of grid boxes."

Which figures appear to be congruent?



- A A and B B B and D C C and D D and A
- 3. Draw two congruent line segments.

4. Draw two congruent angles.

***** SOL 6.13, 7.7

Lesson Summary

Students construct parallelograms, rectangles, squares, and rhombuses, using D-stix, geo-strips, or toothpicks and marshmallows, and identify the properties of the constructed figures. (approximately 40 minutes)

Materials

Sets of D-stix[™], Geo-strips[™], or toothpicks and miniature marshmallows "Quadrilateral Concept Card" "Types of Quadrilaterals" handouts

Vocabulary

polygon. A closed plane geometric figure composed of at least three line segments that do not cross. None of the sides are curved.

quadrilateral. A closed plane (two-dimensional) figure with four sides that are line segments. **parallelogram.** A quadrilateral whose opposite sides are parallel and opposite angles are congruent. **rectangle.** A parallelogram with four right angles.

square. A rectangle with four congruent sides whose diagonals are perpendicular. A square is a rhombus with four right angles.

rhombus. A parallelogram with four congruent sides.

trapezoid. A quadrilateral with exactly one pair of parallel sides.

Warm-up

Distribute the "Quadrilateral Concept Cards," and instruct students to work in pairs to complete them. When students are finished, have a class discussion in which they share their responses. Elicit from the class the essential elements of a polygon:

- 1. Composed of three or more line segments
- 2. Simple (the segments do not cross)
- 3. Closed
- 4. Lies in a plane (is 2-dimensional)

(Note: You may wish to demonstrate the concept of a plane figure by taking a polygon made of wire, which is 2-dimensional, and twisting it so that it is no longer flat; now it is no longer a polygon because it is no longer a plane [2-dimensional] figure.) Have the class work together to agree on class definitions of the terms *polygon* and *quadrilateral*.

Lesson

- 1. Divide the students into small groups, and give each pair a set of D-stix, geo-strips, or toothpicks with marshmallows.
- Have the partners pick two pairs of congruent pieces of their materials and connect them as shown at right. Have them flex the figure into different positions. Model constructing the quadrilateral and flexing it, if necessary.



- 3. Ask the students the following questions:
 - What is another name for this quadrilateral?" (parallelogram) Review the definition of *parallelogram* with the class.
 - What stays the same when you flex the parallelogram to different positions? (the lengths of the sides; the sum of the measures of the angles; the perimeter; opposite sides stay parallel; opposite angles stay congruent.)
 - What changes when you flex the parallelogram ? (size of the angles; the area; the lengths of diagonals; the shape)
 - What do you notice about the opposite sides of the parallelogram? (Opposite sides remain parallel and congruent/equal.)

- What is the sum of the measures of the interior angles of the parallelogram? (360°)
- What do you notice about the opposite angles? (They are congruent—equal in measure.)
- If you make one of the angles a right angle, using the square corner of a book or table to check for accuracy, what happens to the other angles? (They become right angles too.)
- Will this always be true when you make one angle of a parallelogram a right angle? (yes)
- How do you know? (The sum of the measures of the angles in a parallelogram is 360°. If one angle measures 90°, its opposite angle, which is congruent, also measures 90°. Subtracting these two angles from 360° leaves 180° for the combined measure of the two remaining angles. These two angles are also congruent since they are opposite angles in a parallelogram; therefore, each angle must measure 90°.)
- Is it still a parallelogram when every angle is 90°? (yes)
- Is it still a quadrilateral when every angle is 90°? (yes)
- Is it still a polygon when every angle is 90°? (yes)
- What other name besides polygon, quadrilateral, or parallelogram can be given to it when every angle is 90°? (rectangle) Review the definition of *rectangle* with the class.
- 4. Have the partners pick four congruent pieces of their materials and connect them as shown at right. Have them flex the figure into different positions. Model constructing the quadrilateral and flexing it, if necessary.



- 5. Ask the students the following questions:
 - What is another name for this parallelogram?" (rhombus) Review the definition of *rhombus* with the class.
 - What stays the same when you flex the rhombus to different positions?" (the lengths of the sides; the sum of the measures of the angles; the perimeter; opposite sides stay parallel; opposite angles stay congruent.)
 - What is the sum of the measures of the interior angles of the rhombus? (360°)
 - What changes when you flex the rhombus? (sizes of the angles; the area; the lengths of diagonals; the shape)
 - What do you notice about the opposite angles of the rhombus? (Opposite angles remain congruent/equal.)
 - Is it still a quadrilateral when flexed? (yes)
 - Is it still a polygon when flexed? (yes)
 - If you make one of the angles of the rhombus a right angle, using the square corner of a book or table to check for accuracy, what happens to the other angles? (They become right angles too.)
 - Is it still a parallelogram? (yes)
 - What other name besides polygon, quadrilateral, parallelogram, and rhombus can be given to it when every angle is 90°? (square) Review the definition of *square* with the class.
 - Is it a rectangle? (yes)
 - How do you know? (It has four right angles.)
- 6. Distribute the "Types of Quadrilaterals" handout, and discuss the definitions of *quadrilateral, parallelogram, rectangle, rhombus,* and *square.* Discuss the examples of each, making sure the students notice their orientations and how each example fits the definition even though it is not necessarily the typical figure usually seen.

Reflection

Have students write a letter to a teacher or someone else, explaining an important concept they learned in class today.

Name:

Quadrilateral Concept Card

Polygons

These figures are polygons:



These figures are <u>not</u> polygons:



Which of these figures are polygons? (circle)



Draw your own example of a polygon.

Draw your own example of a non-polygon.

<u>Quadrilaterals</u>

These figures are quadrilaterals:



These figures are <u>not</u> quadrilaterals:



Which of these figures are quadrilaterals? (circle)



Draw your own example of a quadrilateral.

Draw your own example of a non-quadrilateral.

What is a polygon?

A polygon is

What is a quadrilateral?

A quadrilateral is

Quadrilateral Concept Card

Polygons

These figures are polygons:



These figures are <u>not</u> polygons:



Which of these figures are polygons? (circle)



Draw your own example of a polygon.

Drawing will vary.

Draw your own example of a non-polygon.

Drawing will vary. Sample answer:



What is a polygon? Sample answer:

A polygon is a simple, closed, plane figure formed by three or more straight lines.

<u>Quadrilaterals</u>

These figures are quadrilaterals:



These figures are <u>not</u> quadrilaterals:



Which of these figures are quadrilaterals? (circle)



Draw your own example of a quadrilateral.

Drawing will vary.

Draw your own example of a non-quadrilateral.

Drawing will vary. Sample answer:

What is a quadrilateral? Sample answer:

A quadrilateral is a four-sided polygon.





***** SOL 6.13, 7.7

Lesson Summary

Students match quadrilaterals with various descriptions listed on an activity sheet and a quadrilateral table, and they determine which sets are identical, attach labels to each category, and create a quadrilateral family tree. (approximately 40 minutes)

Materials

"Warm-up" worksheets Sets of "Quadrilateral Sorting Pieces" "Quadrilateral Activity" worksheets "Quadrilateral Table" worksheets "Quadrilateral Family Tree" worksheets "Quadrilateral Puzzle" worksheets

Vocabulary

polygon. A closed plane geometric figure composed of at least three line segments that do not cross. None of the sides are curved.

quadrilateral. A closed plane (two-dimensional) figure with four sides that are line segments. **parallelogram.** A quadrilateral whose opposite sides are parallel and opposite angles are congruent. **rectangle.** A parallelogram with four right angles.

square. A rectangle with four congruent sides whose diagonals are perpendicular. A square is a rhombus with four right angles.

rhombus. A parallelogram with four congruent sides.

trapezoid. A quadrilateral with exactly one pair of parallel sides.

Warm-up

Give students 10 minutes to complete the "Warm-up" worksheet. After time is up, have them share their drawings, discussing the shapes as a whole group.

Lesson

- 1. Copy the 12 "Quadrilateral Sorting Pieces" onto card stock and cut them out, making enough sets of 12 pieces to give a complete set to each student. Alternatively, have each student cut out his/her own set.
- 2. Distribute the sets of sorting pieces and the "Quadrilateral Activity," "Quadrilateral Table," and "Quadrilateral Family Tree" worksheets.
- 3. Divide the students into small groups, and direct each group to use their sorting pieces to answer the questions on the "Quadrilateral Activity" worksheet and to complete the "Quadrilateral Table."
- 4. After the students have completed the worksheet and table, have pairs of groups compare their answers and reconcile any discrepancies.
- 5. For the "Quadrilateral Family Tree," students can work in small groups or, if you feel they need guidance, as a large group.

Reflection

For the reflection, have students complete the "Quadrilateral Puzzle."

Name: _____

Warm-up

For each question, draw a geometric shape that has all of the properties listed. Write the name of the shape on the line provided.

- 1. Opposite sides are congruent. Opposite sides are parallel. Opposite angles are congruent.
- All sides are congruent. Opposite sides are parallel. Opposite angles are equal. No angles are equal to 90°.
- All angles are right angles. Opposite sides are congruent. Opposite sides are parallel.

Warm-up

For each question, draw a geometric shape that has all of the properties listed. Write the name of the shape on the line provided.

1. Opposite sides are congruent. Opposite sides are parallel. Opposite angles are congruent.



Sample answers: parallelogram, rectangle, square

 All sides are congruent. Opposite sides are parallel. Opposite angles are equal. No angles are equal to 90°.

Sample answers: rhombus, parallelogram

 All angles are right angles. Opposite sides are congruent. Opposite sides are parallel.



Sample answers: rectangle, square



Quadrilateral Sorting Pieces



Name: _____

Quadrilateral Activity

Spread out your quadrilateral pieces with the letters facing up so you can see them. Find all of the quadrilaterals that have four right angles, and list them by letter alphabetically on the Question #1 answer line. Then, consider all of the quadrilaterals again. Find all of the quadrilaterals that have exactly one pair of parallel sides, and list them by letter alphabetically in Question #2 answer line. Continue in this manner until you complete all the questions.

1.	Has four right angles:							
2.	Has exactly one pair of parallel sides:							
3.	Has two pairs of congruent opposite sides:							
4.	Has four congruent sides:							
5.	Has two pairs of parallel opposite sides:							
6.	Has two pairs of congruent adjacent sides, but not all sides are congruent:							
7.	Has congruent opposite angles:							
8.	Does not have four sides:							
9.	Has four congruent angles:							
10.	Which lists are the same?What name can be used to describe quadrilaterals with these properties?							

Quadrilateral Activity

Spread out your quadrilateral pieces with the letters facing up so you can see them. Find all of the quadrilaterals that have four right angles, and list them by letter alphabetically on the Question #1 answer line. Then, consider all of the quadrilaterals again. Find all of the quadrilaterals that have exactly one pair of parallel sides, and list them by letter alphabetically in Question #2 answer line. Continue in this manner until you complete all the questions.

- 1. Has four right angles: <u>A, D, E, K</u>
- 2. Has exactly one pair of parallel sides: F, H
- 3. Has two pairs of congruent opposite sides: A, B, D, E, G, K
- 4. Has four congruent sides: <u>A, B, E</u>
- 5. Has two pairs of parallel opposite sides: <u>G, K, A, B, D, E</u>
- 6. Has two pairs of congruent adjacent sides, but not all sides are congruent: <u>L, J</u>
- 7. Has congruent opposite angles: <u>A, B, D, E, G, K</u>
- 8. Does not have four sides: <u>no examples</u>
- 9. Has four congruent angles: <u>A, D, E, K</u>
- 10. Which lists are the same? <u>#3, 5, and 8</u> What name can be used to describe quadrilaterals with these properties? <u>parallelograms</u>

.

Name: _____

Quadrilateral Table

Using the results of the "Quadrilateral Activity" worksheet, place a check mark in the appropriate spaces to show which figures have which properties.

PROPERTY	TYPES OF POLYGONS										
OF FIGURE	Quadrilateral	Parallelogram	Rectangle	Rhombus	Square	Trapezoid					
All four angles are right angles.											
Opposite sides are parallel.											
Opposite angels are congruent.											
All sides are congruent.											
All angles are congruent.											
May contain a right angle.											
Opposite sides are congruent.											
Has exactly one pair of parallel sides.											
Has two pairs of congruent adjacent sides.											

Name: <u>ANSWER KEY</u>

Quadrilateral Table

Using the results of the "Quadrilateral Activity" worksheet, place a check mark in the appropriate spaces to show which figures have which properties.

PROPERTY	TYPES OF POLYGONS											
OF FIGURE	Quadrilateral	Parallelogram	Rectangle	Rhombus	Square	Trapezoid						
All four angles are right angles.			\checkmark		\checkmark							
Opposite sides are parallel.		\checkmark	\checkmark	\checkmark	\checkmark							
Opposite angels are congruent.		\checkmark	\checkmark	\checkmark	\checkmark							
All sides are congruent.				\checkmark	\checkmark							
All angles are congruent.			\checkmark		\checkmark							
May contain a right angle.	\checkmark		\checkmark		\checkmark	\checkmark						
Opposite sides are congruent.		\checkmark	\checkmark	\checkmark	\checkmark							
Has exactly one pair of parallel sides.						\checkmark						
Has two pairs of congruent adjacent sides.												

Name:

Quadrilateral Family Tree

Fill out the family tree by writing the names *quadrilateral, rectangle, square, rhombus, parallelogram,* and *trapezoid* into the appropriate blocks on the diagram.



Quadrilateral Family Tree

Fill out the family tree by writing the names *quadrilateral, rectangle, square, rhombus, parallelogram,* and *trapezoid* into the appropriate blocks on the diagram.



Name: _

Quadrilateral Puzzle

Match the numbered vocabulary word with the block in the puzzle that shows an example of that word. Place the letter or symbol from the puzzle block onto the numbered line below to solve the puzzle. Each block may be used only once even though the shapes in some blocks have more than one name.

- 1. square
- 2. rectangle
- 3. right angle
- 4. parallel lines
- 5. rhombus
- 6. trapezoid
- 7. parallelogram
- 8. quadrilateral
- 9. polygon





Quadrilateral Puzzle

Match the numbered vocabulary word with the block in the puzzle that shows an example of that word. Place the letter or symbol from the puzzle block onto the numbered line below to solve the puzzle. Each block may be used only once even though the shapes in some blocks have more than one name.

- 1. square
- 2. rectangle
- 3. right angle
- 4. parallel lines
- 5. rhombus
- 6. trapezoid
- 7. parallelogram
- 8. quadrilateral
- 9. polygon





***** SOL 7.6

Lesson Summary

Students sort a set of rectangles into three groups and determine the similarity relationships. (30 minutes)

Materials

Sets of rectangles (see below) "Alphabet Similarity" worksheets "Similar Figures Recording Sheet" Graph paper "Reflecting on Similar Figures" worksheets

Vocabulary

similar figures. Two polygons are similar if corresponding (matching) angles are congruent and the lengths of corresponding sides are proportional.

Warm-up

Review the meaning of the word *congruent*—having exactly the same size and shape. Illustrate the definition by writing two congruent capital *Q*s on the board. Then, write a large *Q* and a smaller-size *Q*. Explain to the students that the *Q*s in this second pair are *similar* but noncongruent. Once students can give you other examples of similar letters, pictures, etc., have them complete the "Alphabet Similarity" worksheet. Discuss the answers before going on to the lesson.

Lesson

- In advance of the lesson, make a set of 14 rectangles for each pair of students. Cut them from grid paper in the following sizes: 1 x 1, 1 x 2, 1 x 4, 2 x 4, 2 x 8, 3 x 3, 3 x 6, 3 x 12, 4 x 8, 4 x 16, 5 x 5, 5 x 10, 7 x 7,10 x 10. Draw all diagonals on the rectangles before photocopying them and cutting them out.
- 2. Distribute the sets of rectangles. Have student pairs divide the rectangles into three groups with the members of each group having the same shape but differing in size. Students should find that one group consists of five squares of different sizes (1 x 1, 3 x 3, 5 x 5, 7 x 7, and 10 x 10). Another group consists of the 1 x 2, 2 x 4, 3 x 6, 4 x 8, and 5 x 10 rectangles. The third group consists of the 1 x 4, 2 x 8, 3 x 12, and 4 x 16 rectangles.
- 3. Once students grouped the rectangles, have them arrange each group from smallest to largest and look at the relationships among members of the same group. Suggest to students that they stack the members of each group on top of each other, starting with the largest. This will allow them to line up the diagonals.
- 4. Once students have them arranged, lead a class discussion based on the following questions:
 - What patterns do you see within each family? (Have them focus on length and width.)
 - What do you notice about the diagonals in each group of rectangles?
 - How could you determine another member of the group?
- 5. Have students complete the recording sheet with the information from the rectangles in each group.
- 6. Discuss the following questions:
 - Within a group, what patterns do you notice going down the chart?
 - Within a group, what patterns do you notice going across, from width to length?
- 7. Have students create ratios of width to length for each group. Ask them what they notice about each of the fractions in each set. (They are equivalent.)
- 8. Based on this discussion, have students define similar figures.
- 9. To extend this activity, have students graph the data by graphing the length and width of each rectangle. It works best to use a different color for each set. Students should see that the coordinates associated with a given set form a line.

10. Have students discuss the following questions based on the graphs:

- How can you use the graph to find another member of a group?
- Given a rectangle, how can you tell if it is similar to the ones already graphed?

Reflection

Have students complete the "Reflecting on Similar Figures" worksheet.

Name: _____

Alphabet Similarity

Determine whether the pairs of upper and lower case letters are similar or not.

Aa	Bb	Сс	Dd	Ee
Ff	Gg	Hh	li	Jj
Kk	LI	Mm	Nn	Оо
Рр	Qq	Rr	Ss	Tt
Uu	Vv	Ww	Хх	Yy

Zz

Alphabet Similarity

Determine whether the pairs of upper and lower case letters are similar or not.

Aa	Bb	Cc Similar	Dd	Ee
Ff	Gg	Hh	li	Jj
Kk <u>Similar</u>	LI	Mm	Nn	Oo Similar
Pp Similar	Qq	Rr	Ss <u>Similar</u>	Tt
Uu <u>Similar</u>	Vv Similar	Ww <u>Similar</u>	Xx <u>Similar</u>	Yy

Zz Similar

Name: _____

Similar Figures Recording Sheet

	GROUP ONE		
Rectangle	Width		Length
		-	
		-	
		-	
		-	
	GROUP TWO		
Rectangle	Width		Length
		-	
		- -	
		-	
		-	
	GROUP THREE		
Rectangle	Width		Length
		-	
		-	
		-	

Name:_____

Reflecting on Similar Figures

Four triangles are shown on the grid below.



Which two triangles appear to be similar?

- $\mathbf{F} \quad M \text{ and } S$
- G M and N
- H N and S
- $\mathbf{J} \quad R \text{ and } N$

In the first column below, draw a simple figure. In the second column, draw a figure that is *similar* to the first figure.







Reflecting on Similar Figures

Four triangles are shown on the grid below.



Which two triangles appear to be similar?



- G M and N
- H N and S
- $\mathbf{J} \quad R \text{ and } N$

In the first column below, draw a simple figure. In the second column, draw a figure that is *similar* to the first figure.











***** SOL 7.8, 8.8

Lesson Summary

Students define and recognize transformations as movement of geometric figures through a translation (slide), reflection (flip), or rotation (turn). (40 minutes)

Materials

Overhead geoboard and student geoboards "Transformation Definitions Chart" "Translation, Reflection, Rotation" worksheets Patty paper or tracing paper "Move Those Shapes!" activity sheets "Reflecting on Transformations" worksheets Music

Vocabulary

- **translation.** A transformation in which an image is formed by moving every point on a plane figure the same distance in the same direction (a slide).
- **reflection.** A transformation in which a plane figure is reflected (flipped) over a line called the line of reflection. All corresponding points in the image and preimage are equidistant from the line of reflections.
- **rotation.** A transformation in which a plane figure is rotated (turned) around a fixed point. The point may or may not be on the figure. The fixed point is called the center of rotation.

pre-image. The plane figure before the transformation.

image. The resulting plane figure after the transformation. A transformation of preimage point A can be denoted as the image A' (read as "A prime").

Warm-up

Begin by asking students to act out the motion of going down a roller coaster. Ask them whether they still look the same when they reach the bottom. (Their shape and orientation have not changed, but their location has changed.) Now, tell them to pretend they are making lunch, flipping hamburgers. Ask them what has changed for the flipped hamburger. (The shape has not changed, but the orientation has changed: it's been reversed, as a mirror image). Next, tell students to pretend they are playing basketball and their coach tells them to pivot. Ask them what has changed. (This move is simply a turn. Their shape has not changed, but their orientation has changed.) Emphasize that the shape of objects does not change when the objects are slid from place to place, flipped over, or turned around in place. Explain that there are three types of mathematical movements or transformations: translation (slide), reflection (flip), and rotation (turn).

Lesson

- 1. Demonstrate a **translation** on the overhead geoboard. Explain that a translation is a transformation in which an image is formed by moving every point on a pre-image the same distance in the same direction. Instruct students to demonstrate a translation on their geoboard.
- 2. Demonstrate a **reflection** on the overhead geoboard, and discuss how a reflection shows a "mirror image." Explain that a reflection is a transformation in which corresponding points in the image and pre-image are equidistant from the line of reflection. Instruct students to demonstrate a reflection on their geoboard.
- 3. Demonstrate a **rotation** on the overhead geoboard. Explain that a rotation is a transformation in which an image is formed by rotating or turning its pre-image around a fixed center point. Instruct students to demonstrate a rotation on their geoboard.
- 4. Explain to students that each of these focus words indicates a specific movement. Play some music, and ask everyone to stand up and slide to the right with you, and then to slide to the left. Then, ask them to put their hands out in front of them and flip their hands repeatedly. Then, ask them to make quarter turns clockwise or counterclockwise. Have students practice slide steps, hand flips, and quarter turns to the beat of the music. Using the words *slide, flip,* and *turn,* call out various transformations for the students to demonstrate and gauge their understanding of these basic terms.

Then, substitute the words *translate, reflect,* and *rotate* in your chant. (Note: If some students are strongly opposed to participating in this part of the activity, you may choose to have just a few volunteers demonstrate it. A few may even want to create a "transformation" dance.)

- 5. Give each student a "Transformation Definitions Chart" illustrating the same types of movements with geometric figures. Connect the actual movements to the geometric movements through discussion.
- 6. Have students complete the "Translation, Reflection, Rotation" worksheets. If necessary, demonstrate how to use patty paper or tracing paper to copy one of the shapes and perform the transformations.
- 7. Pair up the students, and have them complete the "Move Those Shapes" activity by following the directions on the sheet.

Reflection

Have students complete the "Reflecting on Transformations" worksheet.

Transformation Definitions Chart



A transformation in which an image is formed by moving every point on a figure the same distance in the same direction. Rotation (turn)



A transformation in which an image is formed by turning its pre-image around a fixed point. Reflection (flip)



A transformation in which a figure is flipped over a line, called the line of reflection. All corresponding points in the image and pre-image are equidistant from the line of reflection.

Name: _____

Translation, Reflection, Rotation

Trace each original figure. Use the tracing to perform a translation, reflection, and rotation, and draw the result of each in the appropriate box.

Original Figure	Translation	Reflection	Rotation

Translation, Reflection, Rotation

Trace each original figure. Use the tracing to perform a translation, reflection, and rotation, and draw the result of each in the appropriate box. <u>Sample answers:</u>

Original Figure	Translation	Reflection	Rotation

Name: ____

Move Those Shapes!

Draw all three transformations discussed in this lesson: **translation, reflection,** and **rotation.** Draw a figure in the top left portion of the grid paper. The figure should be different from all the others used in this lesson. Perform the first transformation on the original figure, and label the new figure "Figure B." Perform the second transformation on Figure B, and label the new figure "Figure C." Perform the final transformation on Figure C, and label the new figure "Figure D." Draw arrows to show the direction of each transformation.

Transfo	ransformations: Fig. B.					 F	ig. C	•		_	Fig	j. D			
ſ															
		Or	iginal	figure	 										

Move Those Shapes!

Draw all three transformations discussed in this lesson: **translation, reflection,** and **rotation.** Draw a figure in the top left portion of the grid paper. The figure should be different from all the others used in this lesson. Perform the first transformation on the original figure, and label the new figure "Figure B." Perform the second transformation on Figure B, and label the new figure "Figure C." Perform the final transformation on Figure C, and label the new figure "Figure D." Draw arrows to show the direction of each transformation.



Name:_____

Reflecting on Transformations

For each change in position of the figures below, determine the type of transformation it is, and write the name of the transformation on the line provided.

1. The example below is an illustration of a ______.



2. The example below is an illustration of a ______.



3. The example below is an illustration of a ______.



4. The example below is an illustration of a _____.



5. Write directions about how to go from your bedroom to your kitchen, using the verbs *translate, reflect,* and *rotate.*

Reflecting on Transformations

For each change in position of the figures below, determine the type of transformation it is, and write the name of the transformation on the line provided.

1. The example below is an illustration of a <u>rotation</u>.



2. The example below is an illustration of a translation.



3. The example below is an illustration of a <u>reflection</u>.



4. The example below is an illustration of a <u>rotation</u>.



5. Write directions of how to go from your bedroom to your kitchen, using translations, reflections, and rotations.

Sample Answer:

Rotate to the right out my door, translate past two doors, rotate to the left, translate down the stairs, and reflect into the kitchen.

*** SOL 7.8**, 8.8a

Lesson Summary

Students draw a polygon on a coordinate plane and perform translations by using patty paper or tracing paper. (45 minutes)

Materials

"Warm-up" worksheets "Translations" worksheets Pencils Colored pencils 2 x 2 inch squares of patty paper or tracing paper Rulers Scientific calculators "Reflecting on Translations" worksheets

Vocabulary

- **translation.** A transformation in which all points of a geometric figure move the same distance in the same direction.
- **reflection.** A transformation in which all points of a geometric figure move across an axis. Each point of the reflected figure is the same distance from the axis as the corresponding point in the original figure.
- **rotation.** A rotation of a geometric figure is a clockwise or counterclockwise turn of the figure around a fixed point. The point may or may not be on the figure. The fixed point is called the *center of rotation*.

pre-image. The original polygon before the transformation.

image. The resulting polygon after a transformation. A transformation of preimage point *A* can be denoted as the image *A*' (read as "A prime").

Warm-up

Have the students complete the "Warm-up" worksheet, offering assistance as needed.

Lesson

- 1. Prepare a transparency of the "Translations" worksheet for demonstration purposes.
- 2. Distribute the "Translations" worksheet, and have students use a regular pencil to draw the pre-image ABCD by graphing the coordinates given and connecting the dots. Make sure they label points A, B, C, and D on the coordinate plane and label the figure "Pre-Image."
- 3. Give students patty paper or tracing paper, and ask them to trace the figure ABCD along with the labels.
- 4. Explain that a translation is a "slide" of a figure along an imaginary "train track." The figure travels along a straight line in any direction but never rotates as it moves. Demonstrate on the overhead by sliding the patty paper figure across the coordinate plane, and then have the students practice doing the same.
- 5. Ask students to line up their patty paper figure on top of their pre-image on the coordinate plane. Demonstrate what it means to "translate the figure 4 units to the right." You might recommend that they focus their eyes on one of the points as they count spaces across the coordinate plane. Then have students practice doing the translation.
- 6. Have students use a colored pencil to record the new coordinates for A, B, C, and D. Ask if they notice whether the new coordinates give any clue about the translation they just performed. (Some may notice that all *x*-coordinates increased by 4. If not, just continue with the next step, and encourage students to be on the lookout for any patterns that arise.)
- 7. Have students use the same colored pencil to connect the new points A, B, C, D on the coordinate plane. Have them label this figure "Image 1" and label the points A', B', C', and D'. (The prime sign is used to indicate that a point belongs to an image.)
- 8. Have the students use a regular pencil and a ruler to draw arrows from B to B' and from D to D'. Explain that this represents the "train tracks."
- 9. Direct students to line up their patty paper figure again on top of their pre-image on the coordinate plane. Demonstrate what it means to "translate the figure 7 units to the left." Ask students to predict what they think will happen to the coordinates. (The *x*-coordinates will decrease by 7.) Then have students perform the translation.
- 10. Have students use a different colored pencil to record the new coordinates for A, B, C, and D. Ask them to compare this new set of coordinates with the original coordinates. Ask whether their prediction was correct. Have them use calculators to verify that the *x*-coordinates decreased by 7.
- 11. Have students use the same colored pencil to plot the new coordinates for A, B, C, and D on the coordinate plane. Have them label this figure "Image 2" and again label the points A', B', C', and D'.
- 12. Have the students use a regular pencil and a ruler to draw again to draw "train tracks" from B to B', and from D to D'.
- 13. Have students repeat the process, using different colored pencils, for the "up" (Image 3) and "down" (Image 4) translations, and discuss what they notice about the change in coordinates with each translation. (Translations to the left cause the *x*-coordinates to decrease; translations to the right cause the *x*-coordinates to increase; translations up cause the *y*-coordinates to increase; translations down cause the *y*-coordinates to decrease.)
- 14. Ask students what they think would happen if a figure translated up *and* right (both *x* and *y* coordinates would increase). Have them perform the translation for Image 5.
- 15. Have students make up their own "double translation" (left or right *and* up or down) that would cause the figure to move to the third quadrant (lower left corner). Have them record this translation under "Image 6" in the table. Make sure they include the number of units (e.g., 10 units left *and* 4 units down). Have them write the new coordinates *first* and then plot the points. Walk around and ask students if the translation turned out the way they planned. If not, have them make adjustments.

Reflection

Have students complete the "Reflecting on Translations" worksheet.

Name:_____

Warm-up

- 1. Label and number the *x*-axis and *y*-axis.
- 2. Graph the following points, and connect the dots as you go: (-3, -1), (2, -1), (4, 1), (-1, 1)



3. What shape did you just draw? _____

Warm-up

- 1. Label and number the *x*-axis and *y*-axis.
- 2. Graph the following points, and connect the dots as you go: (-3, -1), (2, -1), (4, 1), (-1, 1)



3. What shape did you just draw? parallelogram





Enter the new coordinates after performing each translation shown below:

IMAGE		ORIGINAL CO	DORDINATES	
IMAGE	A (2, 3)	B (3, 4)	C (4, 3)	D (3, 1)
Image 1:				
4 units right				
Image 2:				
7 units left				
Image 3:				
5 units up				
Image 4:				
8 units down				
Image 5:				
5 units right <i>and</i> 4 units up				
Image 6				
(coordinates of choice in the 3 rd quadrant):				



Enter the new coordinates after performing each translation shown below:

IMAGE		ORIGINAL CO	DORDINATES	
IMAGE	A (2, 3)	B (3, 4)	C (4, 3)	D (3, 1)
Image 1: 4 units right	<u>(6, 3)</u>	<u>(7, 4)</u>	<u>(8, 3)</u>	<u>(7, 1)</u>
Image 2: 7 units left	<u>(-5, 3)</u>	<u>(-4, 4)</u>	<u>(-3, 3)</u>	<u>(-4, 1)</u>
Image 3: 5 units up	<u>(2, 8)</u>	<u>(3, 9)</u>	<u>(4, 8)</u>	<u>(3, 6)</u>
Image 4: 8 units down	<u>(2, -5)</u>	<u>(3, -4)</u>	<u>(4, -5)</u>	<u>(3, -7)</u>
Image 5: 5 units right <i>and</i> 4 units up	<u>(7, 7)</u>	<u>(8, 8)</u>	<u>(9, 7)</u>	<u>(8, 5)</u>
Image 6 (coordinates of choice in the 3 rd quadrant):				

Name:

Reflecting on Translations

Each figure below shows a transformation. The pre-image is shown with solid lines and the image is shown with dashed lines. Next to each figure, tell whether the transformation shown represents a translation, and then explain why or why not.



- When you translate a figure on the coordinate plane, explain how the x-coordinate and y-5. coordinate of each point is affected.
- 6. What happens to the graph of a figure (in what direction and how far does it move) when... _____
 - 3 is added to the *x*-coordinate?
 - 10 is added to the *y*-coordinate?
 - 7 is subtracted from the y-coordinate?
 - 6 is subtracted from the *x*-coordinate?
 - 2 is added to the x-coordinate and 3 is subtracted from the y-coordinate?

Name: <u>ANSWER KEY</u>

Reflecting on Translations

Each figure below shows a transformation. The pre-image is shown with solid lines and the image is shown with dashed lines. Next to each figure, tell whether the transformation shown represents a translation, and then explain why or why not.



- 5. When you translate a figure on the coordinate plane, explain how the x-coordinate and y-coordinate of each point is affected. <u>Moving a figure left or right causes the x-coordinate to change. Moving a figure up or down causes the y-coordinate to change. (Actual responses may vary.)</u>
- 6. What happens to the graph of a figure (in what direction and how far does it move) when...
 - 3 is added to the *x*-coordinate? It moves 3 units to the right.
 - 10 is added to the y-coordinate? It moves 10 units up.
 - 7 is subtracted from the *y*-coordinate? <u>It moves 7 units down.</u>
 - 6 is subtracted from the *x*-coordinate? It moves 6 units to the left.
 - 2 is added to the *x*-coordinate *and* 3 is subtracted from the *y*-coordinate? <u>It moves 2 units to the right and 3 units down.</u>

*** SOL 7.8**, 8.8a

Lesson Summary

Students draw a polygon on a coordinate plane and use patty paper and a protractor to perform 90degree rotations around origin. (45 minutes)

Materials

"Warm-up" worksheets "Rotations" worksheets Colored pencils Protractors Tape Patty paper or tracing paper, cut into approximately 5 x 5 inch squares "Reflecting on Rotations" worksheets

Vocabulary

rotation. A rotation of a geometric figure is a clockwise or counterclockwise turn of the figure around a fixed point. The point may or may not be on the figure. The fixed point is called the *center of rotation*.

pre-image. The original polygon before the transformation.

image. The resulting polygon after a transformation. A transformation of preimage point A can be denoted as the image A' (read as "A prime").

origin. The point (0, 0) on the coordinate plane.

Warm-up

Distribute the "Warm-up" worksheet, and allow students ample time to complete it. Review student responses before proceeding with the lesson.

Lesson

- 1. Distribute the "Rotations" worksheet, protractors, and tape. Have the students tape the worksheet to their desk so that it will not move.
- Instruct the student to use a colored pencil to graph the original coordinates listed in the table. Then, have them use the straightedge of the protractor to connect the coordinates, thus drawing a triangle. They should label the vertices of the triangle A, B, and C in accordance with the letters of the coordinates. Have them write "Pre-image" inside the triangle.
- 3. Have students use a different colored pencil to draw a dot on the origin of the coordinate plane and then draw a line connecting the origin to point A.
- 4. Explain to students that they will now rotate the triangle around the origin. To help them grasp this concept, they may find it helpful to think of the following analogies: (1) The origin is like the axle of a tire in that it stays fixed while the triangle rotates around it. (2) The line connecting the origin to point A is like a kite string, and the origin is like a person's hand holding the string.
- 5. Distribute patty paper, and ask students to place the patty paper over the worksheet so that it covers the origin and the triangle.
- 6. Have students trace their triangle, along with the A, B, and C labels, using the first colored pencil (same color as the triangle). Have them trace the "kite string," using the other colored pencil.
- 7. Have students practice rotating by placing their pencil point on the origin and turning the patty paper. Point out that after a rotation, the original kite string and the patty paper kite string form an angle and that this angle represents how "far" the figure has been rotated. Therefore, rotations are measured in degrees. Also, the pre-image can rotate in two directions—clockwise and counterclockwise.
- 8. Ask students to use estimation to perform a 45° clockwise rotation of the pre-image. You may want them to see how close they got by measuring the angle of rotation with the protractor. In like manner, have them perform a 90° counterclockwise rotation, and a 180° clockwise rotation.

- Ask students to return their tracing to its original position on top of the original triangle and then to slowly rotate the tracing *counterclockwise*. Tell them to stop rotating when vertex A is located at point (-3, 3).
- 10. Ask, "How many degrees has the triangle rotated?" (90°) Have students use the protractor to verify the measure of the angle of rotation formed by the two "kite strings."
- 11. Have students record in the table the new coordinates for the three points A, B, and C. Ask, "Do you notice any patterns when comparing the new coordinates with the original coordinates?" (For each pair, the new *x*-coordinate is the opposite of the original *y*-coordinate, and the new *y*-coordinate is the same as the original *x*-coordinate.)
- 12. Ask students to set the patty paper aside, graph the new coordinates on the coordinate plane, and connect the dots. Have them add prime marks to the vertex letters (A', B', and C') to indicate these are the vertices of an *image*. Have them write "90° image" inside the new triangle.
- 13. Have the students continue in the same manner to perform a 180° rotation [point A at (-3, -3)], a 270° rotation [point A at (3, -3)], and a 360° rotation [point A again at (3, 3)].

Reflection

Have students complete the "Reflecting on Rotations" worksheet.

Name: _____

		1			
					•

Warm-up

Part I

- 1. Label and number the *x*-axis and the *y*-axis.
- 2. Graph the following points, and connect them as you go: (2, 0), (−1, −1), (−3, 2) (0, 5), (3, 3).
- 3. What shape did you just draw?

Part II

- 4. Stand up, face a wall, and hold your arm straight in front of you. While standing in the same spot, turn your body to the left. How many degrees did you just rotate? _____ Did you rotate clockwise or counterclockwise? _____
- 5. Return to your original standing position with your arm straight in front of you. While standing in the same spot, turn your body to right all the way around until you are back in the position where you started. How many degrees did you just rotate? _____ Did you rotate clockwise or counterclockwise? _____
- 6. Write a list of things that ordinarily rotate or turn.

Name: <u>ANSWER KEY</u>



Part I

- 1. Label and number the *x*-axis and the *y*-axis.
- 2. Graph the following points, and connect them as you go: (2, 0), (−1, −1), (−3, 2) (0, 5), (3, 3).
- 3. What shape did you just draw? ____pentagon_____

Part II

- 4. Stand up, face a wall, and hold your arm straight in front of you. While standing in the same spot, turn your body to the left. How many degrees did you just rotate? <u>90°</u> Did you rotate clockwise or counterclockwise? <u>counterclockwise</u>
- 5. Return to your original standing position with your arm straight in front of you. While standing in the same spot, turn your body to right all the way around until you are back in the position where you started. How many degrees did you just rotate? <u>____360°__</u> Did you rotate clockwise or counterclockwise? <u>_____</u> clockwise___
- 6. Write a list of things that ordinarily rotate or turn.

Possible responses: tire, merry-go-round, ferris wheel, fan, propeller, a hand on a clock

Name:_____

Rotations

Follow the verbal directions of your teacher to complete this worksheet.

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Original Coordinates	90° Rotation	180° Rotation	270° Rotation	360° Rotation
A (3 , 3)				
B (8 , 4)				
C (5 , 7)				

Rotations

Follow the verbal directions of your teacher to complete this worksheet.



Original Coordinates	90° Rotation	180° Rotation	270° Rotation	360° Rotation
A (3 , 3)	(-3, 3)	(-3, -3)	(3, -3)	(3, 3)
B (8 , 4)	(-4, 8)	(-8, -4)	(4, -8)	(8, 4)
C (5 , 7)	(-7, 5)	(-5, -7)	(7, -5)	(5, 7)

Name: _

Reflecting on Rotations

- 1. When a figure is rotated on the coordinate plane, does its shape change? _____
- 2. At right is a sample SOL test question related to rotations. How could you use patty paper to help answer the question?

The diagram below shows a geometric figure on a coordinate plane.

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-8	-8	4	-0	-2	-1	0	1	2	3	4	5	8	-	ľ
						-1	1							

Which of the diagrams below shows a rotation of this geometric figure?

3. Which answer is correct? _____









Reflecting on Rotations

- 1. When a figure is rotated on the coordinate plane, does its shape change? <u>no</u>
- 2. At right is a sample SOL test question related to rotations. How could you use patty paper to help answer the question?

You could trace the original pre-image on the patty paper and then put the patty paper over each answer image, rotating the tracing to see if the answer image is a rotation.

3. Which answer is correct? ______

The diagram below shows a geometric figure on a coordinate plane.

	у													
						. /	λ.							
						- 6								
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-8	-8	-	-0	-2	-1	0	1	2	3	4	5	6	-	Ľ
						-1	1							

Which of the diagrams below shows a rotation of this geometric figure?











*** SOL** 8.8a

Lesson Summary

Students perform dilations of polygons on the coordinate plane. (30 minutes)

Materials

"Warm-up" worksheetsCalculators"Dilations" worksheets"Reflecting on Dilations" worksheetsPencils

Vocabulary

dilation. A transformation that changes the size of a figure by a scale factor to create a similar figure.

Warm-up

Have students complete the "Warm-up" worksheet. Give assistance as needed.

Lesson

- 1. Distribute the "Dilations, Part 1" worksheet. Have students graph the given coordinates A, B, C, and D and connect the dots. Ask, "What shape did you draw?" (rectangle) Have them determine how long sides AB and AD are in grid units and record these lengths in the chart.
- 2. Ask students to fill in the second row of coordinates by multiplying both coordinates of each point in the first row by 2.
- 3. Have students predict what they think this new shape will look like. Then, have them plot the new coordinates and label the new coordinates A', B', C', and D'. (The prime sign is used to indicate that a point belongs to an image.) Ask them to describe the new shape. (It is the same shape but larger.) Prompt them to describe exactly how much larger it is (twice as large) by comparing the length of the new side A'B' to the original side AB, and comparing the length of the new side A'D' to the original side AB, and comparing the length of the new side A'D' to the original side AD. Explain that they have just performed a dilation, using a scale factor of 2. This means that the figure gets twice as large.
- 4. Ask students how they think they will perform a dilation of the original figure, using a scale factor of 3. (Multiply both coordinates of each point by 3.) Have them perform the dilation in the same manner, labeling the points A", B", C", and D".
- 5. Have students complete the questions on the worksheet.
- 6. Distribute the "Dilations, Part 2" worksheet. Have students graph the given coordinates and connect the dots. Ask, "What shape did you draw?" (pentagon)
- 7. Explain that this time they will perform a dilation using a scale factor of ½. Ask them how they think they will accomplish this task. (Multiply both coordinates of each point by ½ or 0.5.) Have them perform the dilation in the same manner and complete the questions on the worksheet.

Reflection

Have students complete the "Reflecting on Dilations" worksheet.

Name:

Warm-up

1. Make a list of real-life situations in which you see "scaled up" or "scaled down" versions of objects.

2. Practice multiplying a fraction by a whole number:

a.
$$\frac{1}{2} \cdot 6 =$$

b.
$$\frac{1}{3} \cdot 12 =$$

c.
$$\frac{3}{5} \cdot 10 =$$

d.
$$\frac{1}{4} \bullet 0 =$$

Warm-up

1. Make a list of real-life situations in which you see "scaled up" or "scaled down" versions of objects.

Answers will vary. Possible responses: maps, blueprints, model airplanes, architectural models, toys that are scaled down versions of real objects, images taken from a microscope. photocopies that are enlargements or reductions of the original.

2. Practice multiplying a fraction by a whole number:

a.
$$\frac{1}{2} \cdot 6 = \underline{3}$$

b. $\frac{1}{3} \cdot 12 = \underline{4}$

c.
$$\frac{3}{5} \cdot 10 = \underline{6}$$

d.
$$\frac{1}{4} \bullet 0 = \underline{0}$$





SCALE FACTOR			Length of AB	Length of AD						
original	A (−1, -	-1)	B (2, -	·1)	C (2, 3)	D (−1,	3)		
2	Α' ()	В′ ()	C' ()	D' ()		
3	A"() B"() C"() D"				D" ()				

1. What happens to the size of the figure after dilating it, using a scale factor of 2?

2. What happens to the size of the figure after dilating it, using a scale factor of 3?

3. Does a dilation cause the shape to change?





SCALE FACTOR		COORDINATES										
original	A (0, 4)	(0, 4) B (4, 0) C (8, 4) D (10, 10) E (4, 8)										
1/2	Α' (() B'() C'() D'() E'())		

1. What happens to the size of the figure after dilating it, using a scale factor of ½?

2. Did the shape change?



SCALE FACTOR		COORDINATES							
(original)	A (−1, −1)	B (2, −1)	<u>3 units</u>	<u>4 units</u>					
2	A' (<u>−2, −2</u>)	B′ (<u>4, −2</u>)	C' (<u>4, 6</u>)	D' (<u>-2, 6</u>)	<u>6 units</u>	<u>8 units</u>			
3	A" (<u>-3, -3</u>)	B" (<u>6, 3</u>)	C″ (<u>6, 9</u>)	D'' (<u>-3, 9</u>)	<u>9 units</u>	<u>12 units</u>			

- What happens to the size of the figure after dilating it, using a scale factor of 2?
 It gets twice as large.
- What happens to the size of the figure after dilating it, using a scale factor of 3?
 It gets three times as large.
- 3. Does a dilation cause the shape to change? No, only the size changes.



SCALE FACTOR	COORDINATES									
(original)	A (0, 4)	A (0, 4) B (4, 0) C (8, 4) D (10, 10) E (4, 8)								
1/2	A' (<u>0, 2</u>)	B' (<u>2, 0</u>)	C' (<u>4, 2</u>)	D' (<u>5, 5</u>)	E' (<u>2, 4</u>)					

- What happens to the size of the figure after dilating it, using a scale factor of ½?
 It gets half as large.
- 2. Did the shape change? No, only the size changes.

Name:

Reflecting on Dilations

- 1. How would you describe a dilation?
- 2. If you dilate a figure using a scale factor greater than 1, what happens to the figure?
- 3. If you dilate a figure using a scale factor less than 1, what happens to the figure?
- 4. If you were to dilate a figure using a scale factor of 1, what do you think would happen to the figure?

Reflecting on Dilations

- How would you describe a dilation?
 It causes a figure to change in size, but its shape stays the same.
- If you dilate a figure using a scale factor greater than 1, what happens to the figure?
 <u>It gets bigger.</u>
- If you dilate a figure using a scale factor less than 1, what happens to the figure?
 <u>It gets smaller.</u>
- 4. If you were to dilate a figure using a scale factor of 1, what do you think would happen to the figure?

It would stay the same size.

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*** SOL** 8.8a

Lesson Summary

Students draw a polygon on a coordinate plane and use patty paper or tracing paper to perform reflections across the *x*-axis and *y*-axis. (45 minutes)

Materials

5 x 5 inch squares of patty or tracing paper "Reflections" worksheets Pencils Rulers Tape "Reflecting on Reflections" worksheets

Vocabulary

reflection. A transformation in which all points of a geometric figure move across an axis. Each point of the reflected figure is the same distance from the axis as the corresponding point in the original figure.

line of reflection. The line (axis) across which a figure is reflected.

Warm-up

Distribute patty paper or tracing paper, and instruct students to fold it in half and draw a picture or write their name on one side of the folded paper. Then, have them then flip the paper over and trace what they drew. Finally, have them unfold the paper. Lead students in describing what happened to their picture. (The second picture is a "backwards" or "flipped" version of the first. It "mirrors" the first on the fold line.

Lesson

- 1. Distribute the "Reflections" worksheet. Ask students to enter the coordinates of the figure, as directed in step 1.
- 2. Explain the procedure for reflecting the image across the *x*-axis—that the *x*-axis will be the line of reflection or mirror line. Ask students to predict what the image will look like.
- 3. Distribute patty paper. Ask students to trace the *x*-axis, the *y*-axis, and the figure, including the points and the letters.
- 4. Have students perform a reflection across the *x*-axis by flipping the patty paper over in a downward direction, making sure to keep the *x*-axis and *y*-axis aligned. Have them tape the patty paper in place.
- 5. Tell students to enter in the second row of the table the coordinates of the reflected image and then to write down what they notice about the these coordinates. (They should see that the *x*-coordinates stay the same, but the *y*-coordinates change signs. Discourage them from saying the *y*-coordinates "go negative," because if the coordinates had been negative to begin with, they would change to positive.)
- 6. Have students complete steps 3 and 4 of Part 1.
- 7. Have the students complete Part 2, this time reflecting the figure across the *y*-axis; the patty paper will flip to the left.

Reflection

Have students complete the "Reflecting on Reflections" worksheet.



1. In the table below, write the coordinates of the figure above and its image after reflecting it across the *x*-axis.

Α()	В ()	C ()	D()	Ε()	F()
Α' ()	В′ ()	C' ()	D' ()	Ε' ()	F' ()

2. Compare the sets of coordinates. What do you notice about these pairs?

3. Count how many blocks each point is from the *x*-axis, and list them below:

A <u>5</u>	B	С	D	E	F
A'	B′	C′	D'	E′	F′





1. In the table below, write the coordinates of the figure above and its image after reflecting it across the *y*-axis.

Α()	В ()	C ()	D()	Ε()	F()
Α' ()	В' ()	C' ()	D' ()	Ε' ()	F' ()

2. Compare the sets of coordinates. What do you notice about these pairs?

3. Count how many blocks each point is from the *y*-axis, and list them below:

A <u>2</u>	В	С	D	E	F
A′	B′	C′	D'	E′	F′



Reflections, Part 1

1. In the table below, write the coordinates of the figure above and its image after reflecting it across the *x*-axis.

A (<u>5, 5</u>)	B (<u>4, 3</u>)	C (<u>5, 2</u>)	D (<u>6, 2</u>)	E (<u>7, 3</u>)	F (<u>6, 5</u>)
A' (<u>5, −5</u>)	B′ (<u>4, −3</u>)	C′ (<u>5, −2</u>)	D′ (<u>6, −2</u>)	E′ (<u>7, −3</u>)	F′ (<u>6,</u> −5)

2. Compare the sets of coordinates. What do you notice about these pairs?

The x-coordinates stay the same, but the y-coordinates change signs.

3. Count how many blocks each point is from the *x*-axis, and list them below:

A <u>5</u>	B _ <u>3</u>	C _2_	D <u>2</u>	E <u>3</u>	F <u>5</u>
A' <u>5</u>	B′ _ <u>3</u>	C' <u>2</u>	D' <u>2</u>	E' <u>3</u>	F' <u>5</u>

4. The distance from the mirror line to any point on the figure is <u>equal to</u> the distance from the mirror line to its *reflected image*.



Reflections, Part 2

1. In the table below, write the coordinates of the figure above and its image after reflecting it across the *y*-axis.

A (<u>2, 4</u>)	B (<u>3, 5</u>)	C (<u>5, 4</u>)	D (<u>6, 5</u>)	E (<u>6, 3</u>)	F (<u>3, 3</u>)
A' (<u>-2, 4</u>)	B′ (<u>−3, 5</u>)	C' (<u>-5, 4</u>)	D' (<u>-6, 5</u>)	E' (<u>-6, 3</u>)	F′ (<u>−3, 3</u>)

2. Compare the sets of coordinates. What do you notice about these pairs?

The *x*-coordinates change signs, but the *y*-coordinates stay the same.

3. Count how many blocks each point is from the *y*-axis, and list them below:

A <u>2</u>	B _ <u>3</u>	C <u>5</u>	D <u>6</u>	E <u>6</u>	F <u>3</u>
A' <u>2</u>	B' _ <u>3</u>	C' <u>5</u>	D′ <u>6</u>	E′ <u>6</u>	F′ _ <u>3</u>

4. The distance from the mirror line to any point on the figure is <u>equal to</u> the distance from the mirror line to its *reflected image*.

Name: ____

Reflecting on Reflections

- 1. How do the coordinates change when an object is reflected across the *x-axis*?
- 2. How do the coordinates change when an object is reflected across the *y*-axis?

Use your conclusions above to find the new coordinates for each point:

- 3. (4, 3) reflected across the *x*-axis _____
- 4. (-2, 1) reflected across the y-axis _____
- 5. (0, 0) reflected across the *y*-axis _____
- 6. (-5, -6) reflected across the *x*-axis _____
- 7. (0, 9) reflected across the *y*-axis _____
- 8. (0, 9) reflected across the *x*-axis _____
- 9. (7, 0) reflected across the *y*-axis _____
- 10. (7, 0) reflected across the *x*-axis _____
- 11. Reflect the figures below across the *x*-axis.



12. Reflect the figures below across the *y*-axis.



Reflecting on Reflections

- How do the coordinates change when an object is reflected across the *x-axis*? <u>The *x*-coordinate stays the same, but the *y*-coordinate changes sign.
 </u>
- How do the coordinates change when an object is reflected across the *y-axis*? <u>The *x*-coordinate changes sign, but the *y*-coordinate stays the same.
 </u>

Use your conclusions above to find the new coordinates for each point:

- 3. (4, 3) reflected across the x-axis (4, -3)
- 4. (-2, 1) reflected across the *y*-axis (2, 1)
- 5. (0, 0) reflected across the y-axis (0, 0)
- 6. (-5, -6) reflected across the *x*-axis (-5, 6)
- 7. (0, 9) reflected across the y-axis (0, 9)
- 8. (0, 9) reflected across the x-axis (0, -9)
- 9. (7, 0) reflected across the y-axis (-7, 0)
- 10. (7, 0) reflected across the x-axis (7, 0)
- 11. Reflect the figures below across the *x*-axis.



12. Reflect the figures below across the *y*-axis.

