Introduction

In this section, the lessons focus on the meaning of ratios and proportions. Students practice writing ratios and build on their knowledge of ratios to work with proportions. Using proportional reasoning to solve practical problems is a common element in this section.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion.*

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

Standards of Learning

6.1 The student will describe and compare data, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

6.2 The student will

a) investigate and describe fractions, decimals, and percents as ratios;

7.4 The student will solve single-step and multistep practical problems, using proportional reasoning.

7.6 The student will determine whether plane figures—quadrilaterals and triangles—are similar and write proportions to express the relationships between corresponding sides of similar figures.

8.3 The student will

a) solve practical problems involving rational numbers, percents, ratios, and proportions;

Table of Contents

Lesson plans pertaining to the following Standards of Learning are found in this section. Click (or CTRL+click) on each to jump to that lesson.

✱ SOL 6.1

Lesson Summary

Students use color tiles (or construction paper squares) to describe and compare two sets of data, using ratios and appropriate notations. (30–45 minutes)

Materials

Sets of 20 red and 30 yellow color tiles or construction paper squares
Red and yellow colored pencils
Copies of the attached worksheets
Copies of the attached grid

Vocabulary

ratio. Comparison of any two quantities.

Warm-up

Ask students to compare the number of boys in the class to the number of girls. Then, ask them to compare the number of girls to the number of boys. Have them write each comparison in the three ratio notations. (For example, for 6 boys and 3 girls, the ratio of boys to girls is 6 to 3, 6:3, and $\frac{6}{3}$, while the ratio of girls to boys is 3 to 6, 3:6, and $\frac{3}{6}$.) You may wish to have students work in groups so that group members can confer. Next, ask the students to write the ratio of boys to all students in the class (6 to 9, 6:9, or $\frac{6}{9}$ for the above example) and the ratio of girls to all students (3 to 9, 3:9, or $\frac{3}{9}$). Ask the students to describe the relationship between the way the question is asked and the way the ratio is written. (The set named first usually goes first in the ratio.) Discuss this concept.

Lesson

1. Distribute the sets of red and yellow color tiles or squares, colored pencils, sheets of grid paper, and the "Understanding Ratios" worksheet. Give students a few minutes to experiment with putting together various sets of red and yellow tiles or squares.
2. Have students do number 1 on the worksheet. Make sure they color the set on the grid paper and number the picture, as well as write the ratios requested. It is very important that students use the squares or tiles, color the grid, and write the ratio. Review answers with the students.
3. Have students complete the worksheet. Monitor their work during the activity to make sure they are focusing on the relationships between the identified sets. Some students may begin to notice similarities in the sets: for example, when the red tiles double in number, so do the yellow tiles, as well as the total tiles in the set. These relationships will be discussed at the end of the lesson. When students are finished, discuss the answers.
4. Distribute the "Understanding Ratios, Extension" worksheet. Make sure students write complete answers to the statements. If students had noticed the relationships when doing the problems on the previous worksheet, ask them to explain how they "figured it out." Several explanations are valid, including, for example, "separating set 4 into two groups, each of which looks like set 1," or "the numbers were doubled."
5. Let students exchange their worksheets and then check the work of a fellow student.

Reflection

Have students write a definition of *ratio* and give several examples of a ratio.

Name: ______________________

Understanding Ratios

Use a set of color tiles or construction paper squares to model each of the following ratios. After you have modeled a set, use colored pencils to draw a picture of the set by coloring it on as sheet of grid paper. Write each ratio three different ways.

1. Color <u>2 red tiles and 3 yellow tiles</u> on a sheet of grid paper.
   The ratio of red tiles to yellow tiles is ______________________________.
   The ratio of yellow tiles to red tiles is ______________________________.
   The ratio of red tiles to all the tiles is ______________________________.
   The ratio of all the tiles to yellow tiles is ____________________________.

2. Color <u>5 red tiles and 4 yellow tiles</u> on a second sheet of grid paper.
   The ratio of red tiles to yellow tiles is ______________________________.
   The ratio of yellow tiles to red tiles is ______________________________.
   The ratio of yellow tiles to all the tiles is ____________________________.
   The ratio of all the tiles to red tiles is ______________________________.

3. Color <u>7 red tiles and 10 yellow tiles</u> on a third sheet of grid paper.
   The ratio of red tiles to yellow tiles is ______________________________.
   The ratio of yellow tiles to red tiles is ______________________________.
   The ratio of red tiles to all the tiles is ______________________________.
   The ratio of all the tiles to yellow tiles is ____________________________.

4. Color <u>4 red tiles and 6 yellow tiles</u> on a fourth sheet of grid paper.
   The ratio of red tiles to yellow tiles is ______________________________.
   The ratio of yellow tiles to red tiles is ______________________________.
   The ratio of yellow tiles to all the tiles is ____________________________.
   The ratio of all the tiles to red tiles is ______________________________.

5. Color <u>15 red tiles and 12 yellow tiles</u> on a fifth sheet of grid paper.
   The ratio of red tiles to yellow tiles is ______________________________.
   The ratio of yellow tiles to red tiles is ______________________________.
   The ratio of red tiles to all the tiles is ______________________________.
   The ratio of all the tiles to yellow tiles is ____________________________.

6. Color <u>14 red tiles and 20 yellow tiles</u> on a sixth sheet of grid paper.
   The ratio of red tiles to yellow tiles is ______________________________.
   The ratio of yellow tiles to red tiles is ______________________________.
   The ratio of yellow tiles to all the tiles is ____________________________.
   The ratio of all the tiles to red tiles is ______________________________.

Name: ANSWER KEY

Understanding Ratios

Use a set of color tiles or construction paper squares to model each of the following ratios. After you have modeled a set, use colored pencils to draw a picture of the set by coloring it on as sheet of grid paper. Write each ratio three different ways.

1. Color 2 red tiles and 3 yellow tiles on a sheet of grid paper.
   The ratio of red tiles to yellow tiles is 2 to 3, 2:3, 2/3.
   The ratio of yellow tiles to red tiles is 3 to 2, 3:2, 3/2.
   The ratio of red tiles to all the tiles is 2 to 5, 2:5, 2/5.
   The ratio of all the tiles to yellow tiles is 5 to 3, 5:3, 5/3.

2. Color 5 red tiles and 4 yellow tiles on a second sheet of grid paper.
   The ratio of red tiles to yellow tiles is 5 to 4, 5:4, 5/4.
   The ratio of yellow tiles to red tiles is 4 to 5, 4:5, 4/5.
   The ratio of yellow tiles to all the tiles is 4 to 9, 4:9, 4/9.
   The ratio of all the tiles to red tiles is 9 to 5, 9:5, 9/5.

3. Color 7 red tiles and 10 yellow tiles on a third sheet of grid paper.
   The ratio of red tiles to yellow tiles is 7 to 10, 7:10, 7/10.
   The ratio of yellow tiles to red tiles is 10 to 7, 10:7, 10/7.
   The ratio of red tiles to all the tiles is 7 to 17, 7:17, 7/17.
   The ratio of all the tiles to yellow tiles is 17 to 10, 17:10, 17/10.

4. Color 4 red tiles and 6 yellow tiles on a fourth sheet of grid paper.
   The ratio of red tiles to yellow tiles is 4 to 6, 4:6, 4/6.
   The ratio of yellow tiles to red tiles is 6 to 4, 6:4, 6/4.
   The ratio of yellow tiles to all the tiles is 6 to 10, 6:10, 6/10.
   The ratio of all the tiles to red tiles is 10 to 4, 10:4, 10/4.

5. Color 15 red tiles and 12 yellow tiles on a fifth sheet of grid paper.
   The ratio of red tiles to yellow tiles is 15 to 12, 15:12, 15/12.
   The ratio of yellow tiles to red tiles is 12 to 15, 12:15, 12/15.
   The ratio of red tiles to all the tiles is 15 to 27, 15:27, 15/27.
   The ratio of all the tiles to yellow tiles is 27 to 12, 27:12, 27/12.

6. Color 14 red tiles and 20 yellow tiles on a sixth sheet of grid paper.
   The ratio of red tiles to yellow tiles is 14 to 20, 14:20, 14/20.
   The ratio of yellow tiles to red tiles is 20 to 14, 20:14, 20/14.
   The ratio of yellow tiles to all the tiles is 20 to 34, 20:34, 20/34.
   The ratio of all the tiles to red tiles is 34 to 14, 34:14, 34/14.

Grid

Name: ________________________

Understanding Ratios, Extension

1. Compare the pictures of the sets in number 1 and number 4. Look closely at the ratios you wrote for number 1 and number 4 on the worksheet. What relationship do you notice between the ratios for the two sets?

   ____

2. Compare the pictures of the sets in number 2 and number 5. Look closely at the ratios you wrote for number 2 and number 5 on the worksheet. What relationship do you notice between the ratios for the two sets?

   ____

3. Compare the pictures of the sets in number 3 and number 6. Look closely at the ratios you wrote for number 3 and number 6 on the worksheet. What relationship do you notice between the ratios for the two sets?

   ____

4. Create your own problem using the red and yellow tiles. Exchange your problem with someone in your group, and have him/her identify the ratios for each set and relationship between the ratios for the two sets.

   ____

   ____

Name: ANSWER KEY

Understanding Ratios, Extension

1. Compare the pictures of the sets in number 1 and number 4. Look closely at the ratios you wrote for number 1 and number 4 on the worksheet. What relationship do you notice between the ratios for the two sets?

   The ratios in number 4 are two times the ratios in number 1.

2. Compare the pictures of the sets in number 2 and number 5. Look closely at the ratios you wrote for number 2 and number 5 on the worksheet. What relationship do you notice between the ratios for the two sets?

   The ratios in number 5 are three times the ratios in number 2.

3. Compare the pictures of the sets in number 3 and number 6. Look closely at the ratios you wrote for number 3 and number 6 on the worksheet. What relationship do you notice between the ratios for the two sets?

   The ratios in number 6 are two times the ratios in number 3.

4. Create your own problem using the red and yellow tiles. Exchange your problem with someone in your group, and have him/her identify the ratios for each set and relationship between the ratios for the two sets.

   Answers will vary.

✱ SOL 6.2a, 7.4

Lesson Summary

Students apply proportions to solve problems that involve percents. (45–60 minutes)

Materials

Colored pencils
Copies of the attached worksheets

Warm-up

Begin a class discussion to find out what kind of number sense the students have about percents. Ask them what "50 percent" means. What is the definition of *percent*? What does 50 percent look like as a ratio? What would a picture of 50 percent look like? Explain that a good example of what 50 percent looks like is a grid of 100 squares with 50 (or half) of the squares shaded.

Distribute the "Understanding Percents" worksheet. Have students shade in the three grids to represent the given percents. Ask students to explain the shadings. They should say that since "25 percent" means 25 out of 100 and since there were 100 boxes, they shaded 25 boxes.

Lesson

1. Discuss percent as a ratio, reminding students that a ratio is a comparison of two numbers and that with percents, one of the numbers is always 100. Therefore, the ratio for 25% is 25 to 100, or 25:100, or $\frac{25}{100}$. Have the students write 30% as a ratio. (30 to 100, 30:100, $\frac{30}{100}$) Have them write 80% as a ratio. Tell students that they will be using the fraction form of the ratio for solving percent problems.
2. Distribute the "Percents as Ratios" worksheet. Have students do problems 1 through 5, and check their answers. It is not important for students to simplify the ratios.
3. Have the students apply the concept of percent to solve some simple problems. Ask them what the grade for a test means, reminding them that 100% is the basis for most test grades. Show students that a grade of 90% that means that $\frac{90}{100}$ points were scored.
4. Give students the following problem: "Your friend tells you that he made a 90% on a test that had 20 questions. How many questions did your friend answer correctly?" Tell students that we know that if he had answered all 20 questions correctly, he would have made 100 percent. Have them use that information to set up a proportion, keeping in mind that a proportion means that two ratios are equal. Hence, 90% is $\frac{90}{100}$, and all 20 correct is 100%. Since the 20 corresponds to 100 and we do not know how many questions were correct, the proportion will be $\frac{90}{100} = \frac{n}{20}$. Make sure students understand that the total number on the test always corresponds to the 100 and that the number of correct answers always corresponds to the grade. Have students solve the proportion:

   $\frac{90}{100} = \frac{n}{20}$ $\quad$ $90 \times 20 = 100 \times n$ $\quad$ $1{,}800 = 100n$ $\quad$ $18 = n$
5. Ask students to determine how many questions a student got correct if the test had 25 questions and the students got a grade of 76%. Help them set up the proportion, if necessary:

   $\frac{76}{100} = \frac{n}{25}$ $\quad$ $76 \times 25 = 100 \times n$ $\quad$ $1{,}900 = 100n$ $\quad$ $19 = n$
6. Have students do problems 6 through 9 on the worksheet. Review the answers with the class.
7. Ask students how a teacher uses proportions to determine grades on the test. For example, Mrs. Jones gives a test with 25 questions. If Sarah answers 20 questions correctly, what grade does she earn on the test? Help students set up the proportion, if necessary:

$\frac{20}{25} = \frac{n}{100}$ $20 \times 100 = 25 \times n$ $2{,}000 = 25n$ $80 = n$ Sarah's grade is 80%.

Point out that the only difference from the proportion in step 5 is the location of the missing number (n) in the ratio.

8. Ask students to solve this problem: "Mrs. Jones gave a test with 20 problems, and Marcie got 15 correct. What is Marcie's grade?"

   $\frac{15}{20} = \frac{n}{100}$ $15 \times 100 = 20 \times n$ $1{,}500 = 20n$ $75 = n$ Marcie's grade is 75%.

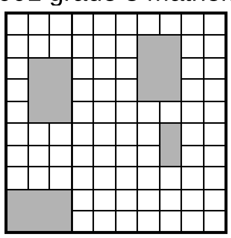
9. Assign problems 10 through12 on the worksheet. Review the answers with the class.

Reflection

Ask students to explain what the 100 in the ratio for percent corresponds to on a test. (The total number of questions on the test) Then, have them write how they can use that information to solve problems about test grades, using proportions.

Sample Assessment

Released 2001 grade 8 mathematics SOL test question



The shaded part of the square can be expressed by —

A 0.02

*B 20%

C $\frac{1}{4}$

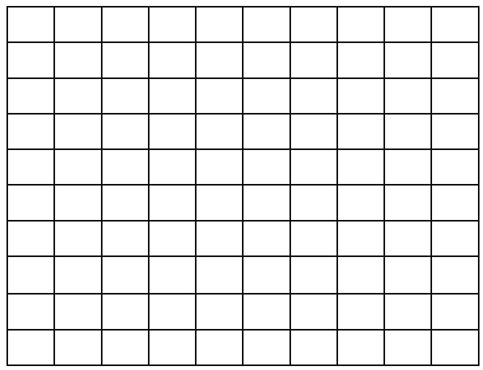
D $\frac{2}{5}$

Name: ________________________

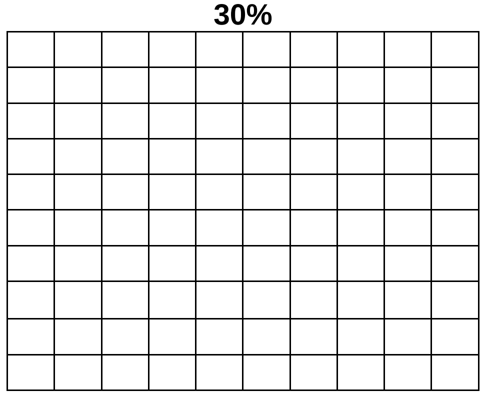
Understanding Percents

Each grid below has 100 squares. Shade in the appropriate number of squares to create a picture of the given percent.

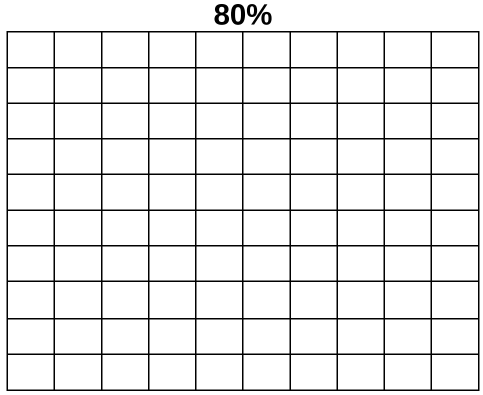
25%



30%



80%

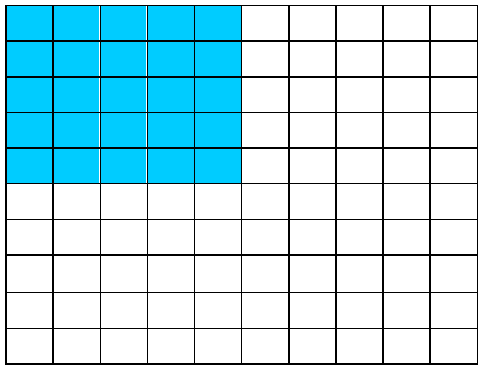


Name: <u>ANSWER KEY</u>

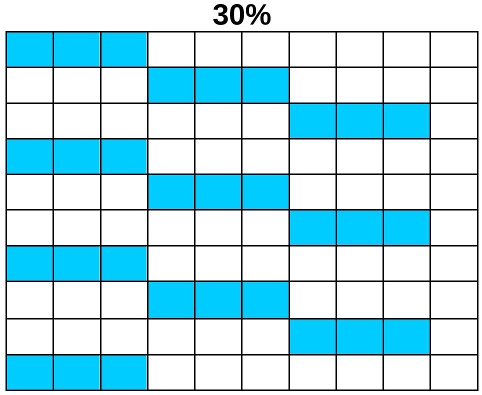
Understanding Percents

Each grid below has 100 squares. Shade in the appropriate number of squares to create a picture of the given percent.

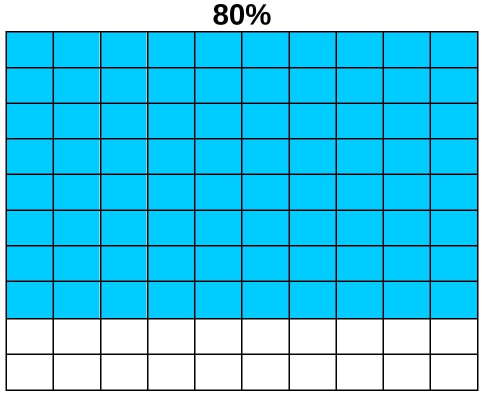
25%



30%



80%



Name: ________________________

Percents as Ratios

Write each percent as a ratio in fraction form. Remember the definition of percent. If you need a picture, use the grids provided.

1. 40% _______________
2. 75% _______________
3. 15% _______________
4. 33% _______________
5. 20% _______________

Set up a proportion for each problem below, and determine the number of test questions each student got correct for each test.

6. Sue receives a score of 75% on a test with 20 questions. How many did she get correct?

____

7. John received a score of 68% on a test with 50 questions. How many did he get correct?

____

8. Anne received a score of 84% on a test with 25 questions. How many did she get correct?

____

9. Adam received a score of 90% on a test with 40 questions. How many did he get correct?

____

Set up a proportion for each problem below, and determine the score earned by the student on each test.

10. Ms. Smith gave a test with 25 questions, and John got 22 correct. What is John's score?

____

11. Ms. Smith gave a test with 30 questions, and Jim got 21 correct. What is Jim's score?

____

12. Ms. Smith gave a test with 40 questions, and Mark got 32 correct. What is Mark's score?

____

Name: ANSWER KEY

Percents as Ratios

Write each percent as a ratio in fraction form. Remember the definition of percent. If you need a picture, use the grids provided.

1. 40% $\frac{40}{100}$
2. 75% $\frac{75}{100}$
3. 15% $\frac{15}{100}$
4. 33% $\frac{33}{100}$
5. 20% $\frac{20}{100}$

Set up a proportion for each problem below, and determine the number of test questions each student got correct for each test.

6. Sue receives a score of 75% on a test with 20 questions. How many did she get correct?

   $\frac{75}{100} = \frac{n}{20}$ $1{,}500 = 100n$ $n = 15$

7. John received a score of 68% on a test with 50 questions. How many did he get correct?

   $\frac{68}{100} = \frac{n}{50}$ $3{,}400 = 100n$ $n = 34$

8. Anne received a score of 84% on a test with 25 questions. How many did she get correct?

   $\frac{84}{100} = \frac{n}{25}$ $2{,}100 = 100n$ $n = 21$

9. Adam received a score of 90% on a test with 40 questions. How many did he get correct?

   $\frac{90}{100} = \frac{n}{40}$ $3{,}600 = 100n$ $n = 36$

Set up a proportion for each problem below, and determine the score earned by the student on each test.

10. Ms. Smith gave a test with 25 questions, and John got 22 correct. What is John's score?

    $\frac{22}{25} = \frac{n}{100}$ $2{,}200 = 25n$ $n = 85$

11. Ms. Smith gave a test with 30 questions, and Jim got 21 correct. What is Jim's score?

    $\frac{21}{30} = \frac{n}{100}$ $2{,}100 = 30n$ $n = 70$

12. Ms. Smith gave a test with 40 questions, and Mark got 32 correct. What is Mark's score?

    $\frac{32}{40} = \frac{n}{100}$ $3{,}200 = 40n$ $n = 80$

✱ SOL 7.4, 8.3a

Lesson Summary

Students set up proportions and solve for a missing term. This lesson covers the basics of proportions and sets the framework for the following lessons. (30–45 minutes)

Materials

Copies of the attached worksheets
Scientific calculators

Vocabulary

ratio. Comparison of any two quantities.

proportion. A statement of equality between two ratios.

Warm-up

Tell students that to make lemonade from a frozen concentrate, the directions say to add 3 cups of water to the concentrate and stir. Ask what would happen if you were to add 4 cups of water. How would the lemonade taste? (Adding more water means it would be overly diluted and would taste weak.) What if you were to add only 2 cups of water? (It would taste too strong.) Have the students talk about the relationship between the water and the concentrate. This is proportional reasoning.

Lesson

1. Distribute the "Ratio Tables" worksheet. Read the first problem with the class, clarifying any misunderstandings. Have students complete the first table.
2. Give students the correct answers for the table entries, and have them check and correct their entries as needed. Then, explain that the ratio of cases to bottles is 1 to 15, which is also written 1:15 or $\frac{1}{15}$. The table shows other ratios, e.g., 2:30, 3:45, and 4:60. Demonstrate that the relationship between any two ratios from the table can be written as a proportion, for example, $\frac{1}{15} = \frac{4}{60}$. Give the definition of *proportion*. Ask students to complete the proportion $\frac{1}{15} = \frac{6}{x}$ by using the data in the table. ($\frac{1}{15} = \frac{6}{90}$) If more examples are needed, use other data from the table.
3. Once students understand the above concept, point out that in a proportion, like quantities, i.e., those with a common property, are always placed in the *same position* of the ratios. In this example, number of cases is on the top (numerator) and number of bottles is on the bottom (denominator). Explain that once the proportion is written, a special relationship exists that enables you to find other values. For example, for the proportion $\frac{1}{15} = \frac{2}{30}$, the special relationship can be found by using a process called "cross multiplication." Cross multiplication is multiplying the numerator of one ratio by the denominator of the other ratio: $\frac{1}{15} = \frac{2}{30}$; 1 × 30 = 2 × 15 = 30.
4. Repeat step 3, using another proportion, e.g., $\frac{1}{15} = \frac{4}{60}$, from the table: $\frac{1}{15} = \frac{4}{60}$; 1 × 60 = 4 × 15 = 60. This relationship exists whenever there is a proportion.

5. Tell students that the table of values established the special relationship for this problem. Now that this relationship is known, it is possible to solve other problems. For example, a store owner wants to purchase 225 bottles of water from Camps, Inc., a supplier of mineral water. How many cases should she order? We know 1 case = 15 bottles, and we want to know *n* cases = 225 bottles. The proportion is: $\frac{1}{15} = \frac{n}{225}$. Using cross multiplication, $1 \times 225 = 15 \times n$, or $225 = 15n$. To solve this equation, divide both sides by 15, which is the number multiplied by *n*. The answer is 15 cases. Have the students check to make sure the multiplication works: substitute 15 for *n* in the proportion and cross multiply.
   ($\frac{1}{15} = \frac{15}{225}$; $1 \times 225 = 15 \times 15 = 225$. The answer is correct.)
6. Have students complete the "Movie Tickets" table and then answer the questions, using the data from the table. Review students' answers, and discuss them as a group.

Reflection

Have students write a problem that involves a proportion. Suggest some possible ratios, e.g., 3 candy bars cost $1.00, or a 6 pack of soda costs $2.40.

Sample Assessment

Released 2003 grade 8 mathematics SOL test question

The basketball concession stand sold 327 drinks in two games. Which proportion could be used to make the best estimate for the number of drinks that will be sold for 10 games?

A $\frac{327}{10} = \frac{2}{x}$

B $\frac{10}{2} = \frac{327}{x}$

C $\frac{2}{327} = \frac{x}{10}$

***D** $\frac{2}{327} = \frac{10}{x}$

Name: ________________________

Ratio Tables

Mineral Water
Camps, Inc. gets mineral water shipped in cases. Trudy Camp, the owner of Camps, Inc., uses a table to keep track of the number of bottles and the number of cases. Each case holds 15 bottles. Find out how many bottles there are in 2, 3, 4, 5, 6, and 7 cases, and complete the table below. What is the simplest way to find each number? ________________________________
Once the table is complete, your teacher will explain the special relationship that exists among these numbers.

Cases	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Bottles	**15**						

Movie Tickets
Mary Lou got a new job selling tickets at a movie theater. A ticket costs $5.50, but many people buy two or more tickets. Mary Lou decides to make a table to help her figure out the total price instantly. Complete the table.

Tickets	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Dollars	**$5.50**								

1. What is the ratio of 1 ticket to dollars?

2. Write the proportion for: 1 ticket is to $5.50 as 3 tickets are to $16.50. Use cross multiplication to check that your proportion is correct.

3. How much will 16 tickets cost? Use a proportion to determine the cost.

Name: ANSWER KEY

Ratio Tables

Mineral Water
Camps, Inc. gets mineral water shipped in cases. Trudy Camp, the owner of Camps, Inc., uses a table to keep track of the number of bottles and the number of cases. Each case holds 15 bottles. Find out how many bottles there are in 2, 3, 4, 5, 6, and 7 cases, and complete the table below. What is the simplest way to find each number? Multiply the number of cases by 15. Once the table is complete, your teacher will explain the special relationship that exists among these numbers.

Cases	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Bottles	**15**	30	45	60	75	90	105

Movie Tickets
Mary Lou got a new job selling tickets at a movie theater. A ticket costs $5.50, but many people buy two or more tickets. Mary Lou decides to make a table to help her figure out the total price instantly. Complete the table.

Tickets	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Dollars	**$5.50**	$11.00	$16.50	$22.00	$27.50	$33.00	$38.50	$44.00	$49.50

1. What is the ratio of 1 ticket to dollars?

   $\frac{1}{\$5.50}$

2. Write the proportion for the following: 1 ticket is to $5.50 as 3 tickets are to $16.50. Use cross multiplication to check that your proportion is correct.

   $\frac{1}{5.50} = \frac{3}{16.50}$

   $1 \times 16.50 = 3 \times 5.50 = 16.50$

3. How much will 16 tickets cost? Use a proportion to determine the cost.

   $\frac{1}{5.50} = \frac{16}{n}$; $1n = 16(5.50) = 88$. Therefore, 16 tickets cost $88.00.

✱ SOL 7.4, 8.3a

Lesson Summary

Students apply proportions to solve practical problems, including those involving scale drawings. (45–60 minutes)

Materials

Scientific calculators
Rulers
Maps of Virginia
String
Copies of the attached worksheet
Meter or yard sticks

Vocabulary

ratio. Comparison of any two quantities.

proportion. A statement of equality between two ratios.

Warm-up

Have students share the types of information that can be found on a map. Ask students if they have used a map to plan a trip, and explain that they are now going to use what they know about proportions to plan a trip, using a map. They are also going to apply what they learn about scale drawing to making a "blueprint" or map of the classroom. You may want to ask whether any students build models, which are always made to scale. If so, you might have those students explain what a "scale model" is.

Lesson

1. Display a map of Virginia, pointing out that the map is drawn to scale—i.e., that it is not the actual size of Virginia, of course, but that its dimensions are *proportional* or *in proportion* to the dimensions of the state of Virginia. Have students locate the key on the map, which gives the scale. Point out that the scale is a ratio.
2. Distribute maps, string, rulers, and worksheets to pairs of students. Have students do Part 1 of the worksheet. Monitor the students as they measure with the string, making sure they are carefully following a road between cities. Check on the proportions that students set up. You may have to remind them about placing like dimensions in the same positions in the ratios.
3. Before assigning Part 2, have the students share the trips they planned, including the number of miles they will travel. Review what it means to say that you travel "60 miles per hour." Show students that this is the same as a 60:1 ratio.
4. Have students complete Part 2. Discuss their results.
5. Discuss Part 3 with the students. Then, have them work in pairs to complete the drawing. This will allow students to work from large to small and apply the notion of scale drawing. You may want to use a metric scale, such as 1 cm to 3 m, or 1:3.

Reflection

Have students write about the ways that scale drawing is used in daily life and explain why it is important.

Sample Assessment

Released 2001 grade 8 mathematics SOL test question

A scale distance of 3.5 centimeters on a certain map represents an actual distance of 175 kilometers. What actual distance does 5.7 centimeters on the same map represent?

A 0.285 km

B 2.85 km

C 28.5 km

*D 285 km

Released 2002 grade 8 mathematics SOL test question

Harold was looking at a scale drawing of a city park. A reflection pool 27 meters long measured 1.8 centimeters on the drawing. Which is most likely the scale used to make the drawing?

A 1 cm represents 9 m

B 1 cm represents 12 m

*C 1 cm represents 15 m

D 1 cm represents 18 m

Released 2003 grade 8 mathematics SOL test question

On a certain map, $\frac{1}{2}$ inch represents 75 miles. If the distance between two towns on this map is $2\frac{1}{4}$ inches, what is the actual distance between the two towns?

F **15 mi.**

G **$16\frac{2}{3}$ mi.**

***H** **337.5 mi.**

J **375 mi.**

Released 2004 grade 8 mathematics SOL test question

On a map, the distance between Milvale and Dracut is $1\frac{1}{4}$ inches. If the map uses a scale of $\frac{1}{4}$ inch represents 1 mile, what is the actual distance in miles from Milvale to Dracut?

F 2

G 3

H $3\frac{1}{2}$

*J 5

Name: ______________________

Scale Drawing

The scale for the map is: __________

Part 1
With your partner, decide on a trip that you will make in Virginia. Determine where you want to start and where you want to finish. (For example, you may start in Richmond and travel to Abingdon to visit the Barter Theatre.) Place the end of your string on the start location, and carefully lay the string along the path of a road or roads that lead to the goal location. Mark the point on the string where it comes to the goal location. Then, straighten the string, and use a ruler to measure the string from start point to goal point.

To find out how many miles it is between the cities, set up a proportion. The first ratio is the scale of the map. The second ratio compares the length of the string to the distance between the start location and goal location. Be sure to put like units in the same positions in both ratios.

Part 2
Now that you know how far it is between the two locations, determine how much time it will take to travel that distance. Assume that you can average 60 miles per hour on the highway. How long will it take you to make this trip? Set up a proportion to find out. (A ratio table for number of hours and miles traveled might be helpful.)

Check your work with your teacher.

Part 3
Now, make a scale drawing. Using the ratio of 1 in. to 5 ft., make a drawing of your classroom. Decide with your partner if you will show the placement of desks and other furniture. Then, make your measurements, and make your scale drawing.

✱ SOL 7.4, 8.3a

Lesson Summary

Students apply proportions to solve practical problems. (45 minutes)

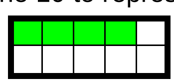
Materials

Copies of the attached worksheet
Grid sheets
Colored pencils
Color tiles or construction paper squares (red and yellow)
Scientific calculators

Warm-up

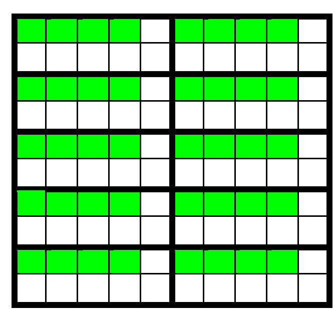
Give each student a set of four red and six yellow color tiles or paper squares. Ask students to state the ratio of red tiles to all the tiles in the set ($\frac{4}{10}$) and the ratio of yellow tiles to the whole set. ($\frac{6}{10}$) Ask: "If the ratio stays the same, how many red tiles would be in a set of 100 tiles?" Remind students that the numerator of the ratio represents the *part* of the set and the denominator represents the *whole* set. When setting up a proportion, the second ratio must have the same type of numerator and the same type of denominator. For this problem, $\frac{4}{10} = \frac{n}{100}$. Using cross-multiplication, $10n = 400$, so $n = 40$.

Distribute a sheet of grid paper to each student, and help students use it to see this relationship by outlining 10 squares and shading four out of the 10 to represent the original ratio, as shown below.



Then, have students outline 100 squares and shade four out of each set of 10, as shown at right. Encourage students to continue to use grid pictures to helps them visualize various ratios.

Lesson



1. Distribute the "Applications of Proportions" worksheet, and remind students that the most important aspect of solving proportions is setting them up correctly. Care must be used in determining what unit of measure will be placed in the numerator and what unit of measure will be placed in the denominator of the first ratio. Once that is done, the second ratio in the proportion must be set up with like units of measure in the numerator and denominator.
2. Go over the answers with the students. Check to see whether students set up the proportions correctly with like units placed in the same positions in the ratios. The answer key shows one of the possible answers for setting up the first ratio and then for setting up the proportion. Students may be surprised to learn that there are many ways to arrive at the "right" answer.

Reflection

Ask students to describe what a proportion is and how to solve a proportion.

Sample Assessment

Released 2002 grade 8 mathematics SOL test question

A recipe calls for $3\frac{1}{4}$ cups of flour and $1\frac{1}{2}$ cups of sugar. If the recipe is doubled, how much flour will be required?

F 3 cups

G $3\frac{1}{4}$ cups

H $4\frac{3}{4}$ cups

*J $6\frac{1}{2}$ cups

Released 2003 grade 8 mathematics SOL test question

At the same time that a 60-foot tall building casts a shadow that is 21.5 feet long, a nearby tree casts a shadow that is 18 feet long. Which measure is closest to the height of the tree?

F 56.5 ft

*G 50.2 ft

H 6.5 ft

J 3.3 ft

Name: ________________________

Applications

Solve each of the following proportions.

1. $\frac{n}{15} = \frac{4}{5}$ ________________________

2. $\frac{12}{21} = \frac{y}{14}$ ________________________

3. $\frac{16}{x} = \frac{2}{7}$ ________________________

4. $\frac{105}{35} = \frac{27}{n}$ ________________________

For each problem below, set up a proportion and solve it.

5. At the rate of 3 items for 16¢, how many items can you buy for 80¢?

____

6. A recipe calls for 4 cups of flour to 6 tablespoons of shortening. How many tablespoons of shortening are needed for 6 cups of flour?

____

7. A tree casts a shadow of 15 meters, while a 2-meter post nearby casts a shadow of 3 meters. How tall is the tree?

____

8. If a scale distance of 3.5 centimeters on a map represents an actual distance of 175 kilometers, what actual distance does a scale distance of 5.7 centimeters represent?

____

9. The girl's soccer team scored 60 points in 10 games. If this trend were to continue, what would be the points scored in 15 games?

____

10. A train travels 90 miles in 1.5 hours. How many miles will the train go in 6 hours if it continues to travel at the same rate of speed?

11. If 3 tickets to a certain show cost $13.20, what would 7 tickets cost?

12. A 40-acre farm yields 600 bushels of wheat. At the same rate, how much wheat would a 75-acre farm yield?

13. A workman received $110 for working 20 hours. At the same rate of pay, how many hours must he work to earn $187?

14. In a certain school, there are 4 girls for every 5 boys. If there are 560 girls in the school, how many boys attend the school?

15. To make concrete, a person mixes 1 bag of cement to 4 bags of sand. How many bags of cement should be used with 100 bags of sand?

Name: ANSWER KEY

Applications

Solve each of the following proportions.

1. $\frac{n}{15} = \frac{4}{5}$ $5n = 60$ $n = 12$

2. $\frac{12}{21} = \frac{y}{14}$ $168 = 21y$ $8 = y$

3. $\frac{16}{x} = \frac{2}{7}$ $112 = 2x$ $56 = x$

4. $\frac{105}{35} = \frac{27}{n}$ $105n = 945$ $n = 9$

For each problem below, set up a proportion, and solve it.

5. At the rate of 3 items for 16¢, how many items can you buy for 80¢?

   $\frac{\text{items}}{\text{cost}}$ $\frac{3}{16} = \frac{n}{80}$ $16n = 240$ $n = 15$ items

6. A recipe calls for 4 cups of flour to 6 tablespoons of shortening. How many tablespoons of shortening are needed for 6 cups of flour?

   $\frac{\text{flour}}{\text{shortening}}$ $\frac{4}{6} = \frac{6}{x}$ $4x = 36$ $x = 9$ tablespoons

7. A tree casts a shadow of 15 meters, while a 2-meter post nearby casts a shadow of 3 meters. How tall is the tree?

   $\frac{\text{tree height}}{\text{shadow}}$ $\frac{n}{15} = \frac{2}{3}$ $3n = 30$ $n = 10$ meters

8. If a scale distance of 3.5 centimeters on a map represents an actual distance of 175 kilometers, what actual distance does a map distance of 5.7 centimeters represent?

   $\frac{\text{scale}}{\text{actual}}$ $\frac{3.5}{175} = \frac{5.7}{x}$ $3.5x = 997.5$ $x = 285$ kilometers

9. The girl's soccer team scored 60 points in 10 games. If this trend were to continue, what would be the points scored in 15 games?

   $\frac{\text{points}}{\text{games}}$ $\frac{60}{10} = \frac{n}{15}$ $10n = 900$ $n = 90$ points

10. A train travels 90 miles in 1.5 hours. How many miles will the train go in 6 hours if it continues to travel at the same rate of speed?

$\frac{\text{miles}}{\text{hours}}$ $\frac{90}{1.5} = \frac{x}{6}$ $1.5x = 540$ $x = 360$ mi.

11. If 3 tickets to a certain show cost \$13.20, what would 7 tickets cost?

$\frac{\text{tickets}}{\text{cost}}$ $\frac{3}{13.20} = \frac{7}{x}$ $3x = 92.40$ $x = \$30.80$

12. A 40-acre farm yields 600 bushels of wheat. At the same rate, how much wheat would a 75-acre farm yield?

$\frac{\text{acres}}{\text{bushels}}$ $\frac{40}{600} = \frac{75}{n}$ $40n = 45{,}000$ $n = 1{,}125$ bushels

13. A workman received \$110 for working 20 hours. At the same rate of pay, how many hours must he work to earn \$187?

$\frac{\text{wages}}{\text{hours}}$ $\frac{110}{20} = \frac{187}{x}$ $110x = 3{,}740$ $x = 34$ hours

14. In a certain school, there are 4 girls for every 5 boys. If there are 560 girls in the school, how many boys attend the school?

$\frac{\text{girls}}{\text{boys}}$ $\frac{4}{5} = \frac{560}{n}$ $4n = 2{,}800$ $n = 700$ boys

15. To make concrete, a person mixes 1 bag of cement to 4 bags of sand. How many bags of cement should be used with 100 bags of sand?

$\frac{\text{cement}}{\text{sand}}$ $\frac{1}{4} = \frac{x}{100}$ $4x = 100$ $x = 25$ bags of cement

✱ SOL 7.4, 8.3a

Lesson Summary

Students solve practical problems, using computational procedures for percents, ratios, and proportions. (45–90 minutes)

Materials

Newspaper ads and/or brochures for items on sale
Copies of the attached worksheet
Scientific calculators

Warm-up

Ask students how they can “figure out” how much something will cost if it is advertised at 50% off, or 25% off, or 30% off. Tell them that they can do this by focusing on how much is saved per dollar. For example, 25% = $\frac{25}{100}$, so it is the same as *saving 25 cents on every dollar.* Therefore, to find the sale price of a $10 item on sale at 25% off, it may be easier to multiply 10 times 25 cents to find the amount of the discount ($10 × $.25 = $2.50) and then subtract the discount from the original price.

Ask students how they estimate Virginia sales tax when they are in a store. Students should be able to explain that it will be close to 5 cents for every dollar because the tax rate in Virginia is 4.5% on most items.

The purpose of this lesson is to allow students to “see” how percents are used in different ways each day.

Lesson

1. Remind students that proportions are one way to solve problems involving discounts and sales tax.
   - Display this proportion: $\frac{\text{\% of discount}}{100} = \frac{\text{amount saved}}{\text{original price}}$. Once the proportion has been solved, the sale price is found by subtracting the amount saved from the original price.
   - Display this proportion: $\frac{\text{\% of sales tax}}{100} = \frac{\text{amount of tax}}{\text{cost of item}}$. Once the proportion has been solved, the total spent is found by adding the amount of tax to the cost of the item.
2. Discuss the importance of being an informed consumer. Then distribute the “Shop until You Spend Most All of It” worksheet and newspaper ads and/or brochures advertising a sale. Alternatively, you could ask students to bring in ads. Review the instructions with the students before they begin the activity.
3. Have a class discussion about these tasks, asking students to compare them to the tasks involved in real-life shopping.

Reflection

Ask students to write a paragraph about how this lesson will help them become smarter consumers.

Sample Assessment

Released 2003 grade 8 mathematics SOL test question

Marti wants to buy a dress priced at $89.75. If the sales tax is 8%, what is the total amount she must pay for the dress?

A $71.80
B $82.57
*C $96.93
D $97.75

Released 2004 grade 8 mathematics SOL test question

For large parties, a restaurant adds a 15% service charge to the bill. How much would be added to a bill of $638.40?

A $27.68
B $63.84
*C $95.76
D $150.00

Name: ______________________

Shop Until You Spend Most All of It

(Adapted from a lesson in V-Quest)

You have $500 to spend at *one* store. You must buy at least six items and receive no more than $10.00 in change. You must also produce an itemized bill for your purchase. The items you buy should be chosen from an ad or sale brochure that you will include in your project.

1. Complete the table below, which will serve as your itemized bill. You must list at least six items. If you want to list more than 10 items, ask for an additional table.

2. Compute your total savings—the difference between sum of the original prices and the sum of the sale prices (before tax).

3. Compute the amount of Virginia sales tax (4.5%) on your total purchases, and record it.

4. Compute the amount you owe at the checkout, and record it.

4. Compute the amount of change you would receive from your $500.00. Remember, this can be no more than $10.00.

5. Include the ad or sale brochure you used for the items and prices.

Itemized Bill

Item Name	**Original Price**	**Sale Discount**		**Sale Price**
		%	**Amount**	
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
TOTAL				
Amount of Virginia sales tax on your total purchases:				
Amount you owe at the checkout:				
Amount of change received:				

✱ SOL 7.4, 8.3a

Lesson Summary

Students solve practical problems, using computational procedures for percents, ratios, and proportions. (45 minutes)

Materials

Copies of the attached worksheet
Calculators

Warm-up

Check for student understanding about the relationship established in a proportion. Review with the students the need to keep like units in the same positions in a proportion's ratios, i.e., either in the numerators or the denominators. Remind students that they may set up a proportion as they wish provided they keep like units in the same positions in the ratios. Have students identify the units and set up proportions for each of the following problems, but do not have them solve the proportions.

- A car travels 50 miles per hour. How many hours will it take to travel 400 miles? ($\frac{\text{miles}}{\text{hours}} = \frac{\text{miles}}{\text{hours}}$ *or* $\frac{\text{hours}}{\text{miles}} = \frac{\text{hours}}{\text{miles}}$)
- 3 oranges cost $.55. How much will 12 oranges cost? ($\frac{\text{oranges}}{\text{cost}} = \frac{\text{oranges}}{\text{cost}}$ *or* $\frac{\text{cost}}{\text{oranges}} = \frac{\text{cost}}{\text{oranges}}$)

Have students make up one problem like this to exchange with a classmate, who will solve it.

Lesson

1. Distribute the "Practical Applications for Proportions" worksheet. If there is any confusion about how to set up the proportions, assign one problem at a time, and check the setup with the students.
2. After students have finished the problems, make sure all students have an opportunity to explain how they set up the proportions, as not every student will approach the problems in the same way. Students should soon realize that the final answer to a problem will be the same even when the proportion used was set up differently. If you believe that some students could benefit from a more pictorial explanation, refer to Lesson 7.6.

Reflection

Have students list the ways that proportions are used in everyday life. (Students will most likely mention various rates, such as miles per hour, miles per gallon, number per cost, discounts, and tax.)

Sample Assessment

Released 2002 grade 8 mathematics SOL test question

The sonar system of a submarine receives an echo back from a ship 5,000 yards away after 6.1 seconds. It picks up an echo from a second ship after 8.4 seconds. Which proportion could be used to find the distance to the second ship?

*F $\frac{6.1}{5000} = \frac{8.4}{x}$

G $\frac{6.1}{8.4} = \frac{x}{5000}$

H $\frac{8.4 - 6.1}{8.4} = \frac{x}{5000}$

J $\frac{2.3}{5000} = \frac{6.1}{x}$

Released 2003 grade 8 mathematics SOL test questions

In a random survey, 25 out of 150 people were able to give a correct answer to the question asked. If 500 people are surveyed, about how many would be expected to answer the question correctly?

F 4

G 20

*H 83

J 325

The basketball concession stand sold 327 drinks in two games. Which proportion could be used to make the best estimate for the number of drinks that will be sold for 10 games?

A $\frac{327}{10} = \frac{2}{x}$

B $\frac{10}{2} = \frac{327}{x}$

C $\frac{2}{327} = \frac{x}{10}$

*D $\frac{2}{327} = \frac{10}{x}$

Released 2004 grade 8 mathematics SOL test question

Laurie used 5 yards of fabric to make 2 costumes for the school play. How many yards of fabric would she need to make 20 more of these costumes?

A 8

B 27

C 40

*D 50

Name: ______________________

Applications for Proportions

Use a proportion to solve each problem.

1. If 3 tickets to a show cost $13.20, what is the cost of 7 tickets?

____

2. How much would you pay for 5 apples if 12 apples cost $.96?

____

3. A 40-acre field yields 600 bushels of wheat. At that rate, what will a 75-acre field yield?

____

4. A train travels 90 miles in 1½ hours. How many miles will the train go in 6 hours when traveling at the same rate?

____

5. A recipe calls for 1½ cups of sugar for a 3-pound cake. How many cups of sugar would be used for a 5-pound cake?

____

6. The scale on a map is 1 inch = 500 miles. If two cities are 875 miles apart, how far apart are they on this map?

____

7. Two numbers are in the ratio 3:2. The smaller number is 36. What is the larger number?

____

8. In a school, the ratio of the number of boys to the number of girls is 5:4. If 500 girls attend the school, what is the number of boys attending the school?

____

9. The ratio of the length of a rectangle to its width is 10:7. If the width of the rectangle is 70 feet, what is its length?

____

10. There are 120 planes on an airfield. If 75% of the planes take off, how many planes take off?

____

11. Helen bought a coat at a “20%-off” sale and saved $12. What was the original price of the coat?

____

12. If a store advertises a 25%-off sale, what is the cost of an article marked $40?

____

13. During a sale, a dress with a price of $48 is discounted by $16. What is the percent of the discount?

____

14. If a coat has an original price of $165 and there is a 30% discount, how much will the customer save?

____

15. A store owner is required to collect a 5% sales tax. One day he collected $281 in taxes. What is the total amount of sales he made that day?

Name: <u>ANSWER KEY</u>

Applications for Proportions

Use a proportion to solve each problem.

1. If 3 tickets to a show cost \$13.20, what is the cost of 7 tickets?

   $\frac{3}{13.20} = \frac{7}{n}$ $3n = 92.40$ $n = \$30.80$

2. How much would you pay for 5 apples if 12 apples cost \$.96?

   $\frac{5}{n} = \frac{12}{.96}$ $12n = 4.80$ $n = \$.40$

3. A 40-acre field yields 600 bushels of wheat. At that rate, what will a 75-acre field yield?

   $\frac{40}{600} = \frac{75}{n}$ $40n = 45{,}000$ $n = 1{,}125$ bushels

4. A train travels 90 miles in 1½ hours. How many miles will the train go in 6 hours when traveling at the same rate?

   $\frac{90}{1\frac{1}{2}} = \frac{n}{6}$ $1\frac{1}{2}\,n = 540$ $n = 360$ mi.

5. A recipe calls for 1½ cups of sugar for a 3-pound cake. How many cups of sugar would be used for a 5-pound cake?

   $\frac{1\frac{1}{2}}{3} = \frac{n}{5}$ $3n = 7\frac{1}{2}$ $n = 2\frac{1}{2}$ cups

6. The scale on a map is 1 inch = 500 miles. If two cities are 875 miles apart, how far apart are they on this map?

   $\frac{1}{500} = \frac{n}{875}$ $500n = 875$ $n = 1.75$ in.

7. Two numbers are in the ratio 3:2. The smaller number is 36. What is the larger number?

   $\frac{3}{2} = \frac{n}{36}$ $2n = 108$ $n = 54$

8. In a school, the ratio of the number of boys to the number of girls is 5:4. If 500 girls attend the school, what is the number of boys attending the school?

   $\frac{5}{4} = \frac{n}{500}$ $4n = 2{,}500$ $n = 625$ boys

9. The ratio of the length of a rectangle to its width is 10:7. If the width of the rectangle is 70 feet, what is its length?

   $\frac{10}{7} = \frac{n}{70}$ $7n = 700$ $n = 100$ ft.

10. There are 120 planes on an airfield. If 75% of the planes take off, how many planes take off?

    $\frac{75}{100} = \frac{n}{120}$ $100n = 9{,}000$ $n = 90$ planes

11. Helen bought a coat at a "20%-Off" sale and saved \$12. What was the original price of the coat?

    $\frac{20}{100} = \frac{12}{n}$ $20n = 1{,}200$ $n = \$60$

12. If a store advertises a 25%-off sale, what is the cost of an article originally marked \$40?

    $\frac{25}{100} = \frac{n}{40}$ $100n = 1{,}000$ $n = 10$ $40 – 10 = \$30$

13. During a sale, a dress with a price of \$48 is discounted by \$16. What is the percent of the discount?

    $\frac{16}{48} = \frac{n}{100}$ $48n = 1{,}600$ $n = 33\frac{1}{3}\%$

14. If a coat has an original price of \$165 and there is a 30% discount, how much will the customer save?

    $\frac{30}{100} = \frac{n}{165}$ $100n = 4{,}950$ $n = \$49.50$

15. A store owner is required to collect a 5% sales tax. One day he collected \$281 in taxes. What is the total amount of sales he made that day?

    $\frac{5}{100} = \frac{281}{n}$ $5n = 28{,}100$ $n = \$5{,}620$

✱ SOL 7.6

Lesson Summary

Students write proportions to express the relationships between the lengths of corresponding sides of similar figures (quadrilaterals and triangles). (45 minutes)

Materials

Copies of the attached worksheets
Plain copy paper for tracing
Scissors

Vocabulary

similar figures. Two polygons in which the corresponding (matching) angles are congruent and the lengths of corresponding sides are proportional.

Warm-up

Review with students basic concepts about triangles and quadrilaterals:

- The sum of the measures of the angles in a triangle is always 180°.
- The sum of the measures of the angles in a quadrilateral is always 360°.

Hand out the "Warm-up" worksheet, and have students apply these basic concepts to answer the questions on the sheet. Students will need to be able to apply this information to determine the corresponding angles and thus the corresponding sides in similar figures.

Lesson

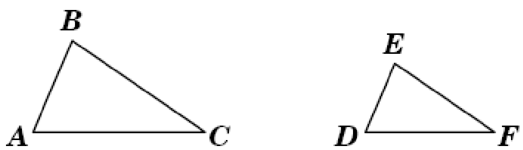
1. Distribute the "Similar Figures and Proportions, 1" worksheet. Give the students the definition of *similar figures* and discuss with the class the concepts on the worksheet.
2. It is quite helpful for students to physically place similar figures on top of each other in order to actually *see* the correspondence—i.e., that the corresponding angles are congruent and that the lengths of corresponding sides are proportional. Give students some copy paper to use for tracing, and have them trace and then cut out the smaller figure of each pair of figures on the worksheet.
3. Do the first pair of triangles on the worksheet together, step by step. Have students place triangle 1 on top of triangle 2 in order to match the corresponding angles and observe that they are congruent (identical). Then, have them verify that the corresponding sides are proportional by laying each corresponding side of triangle 1 *twice* along the corresponding side of triangle 2.
4. Demonstrate how to express that the corresponding sides are proportional: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$. Point out that typically the sides of one figure are placed in the numerators and the corresponding sides of the second figure are placed in the denominators.
5. If students are struggling with these concepts, walk through another example with them, using figures 5 and 6. Ask students to write the statement for corresponding sides being proportional: $\frac{AB}{EF} = \frac{AD}{EH} = \frac{BC}{FG} = \frac{CD}{GH}$. Ask students to look closely at this proportion together with the statement about the angles being congruent. AB in the numerator connects $\angle A$ and $\angle B$. $\angle E$ and $\angle F$ are equal in measure to $\angle A$ and $\angle B$, so the corresponding side is EF. Have students examine each of the other combinations. Students should be moving toward being able to set up a proportion based on the angles with equal measurements.
6. Have students trace, cut out, and do the same process to find the proportion for figures 3 and 4 and for figures 7 and 8. Review these with the students when they are finished with the work.
7. Assign the "Similar Figures and Proportions, 2" worksheet. If students need to trace and cut out the smaller figure in each pair of figures to do the problems, encourage them to do so.

Reflection

Have students discuss the procedure for setting up a proportion when given the fact that two figures are similar figures. Allow them to use diagrams to make their explanation thorough.

Sample Assessment

Released 2002 grade 8 mathematics SOL test question



If ΔABC is similar to ΔDEF, which of the following must be true?

A $\frac{AB}{AC} = \frac{DE}{EF}$

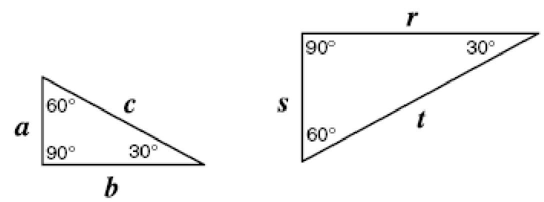
B $\frac{AB}{DF} = \frac{AC}{EF}$

C $\frac{AB}{BC} = \frac{DE}{DF}$

*D $\frac{AB}{DE} = \frac{AC}{DF}$

Released 2004 grade 8 mathematics SOL test question

This is a pair of similar triangles.



Which of the following proportions is true for these triangles?

*F $\frac{a}{s} = \frac{c}{t}$

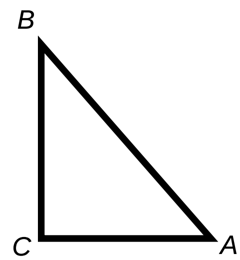
G $\frac{a}{s} = \frac{b}{t}$

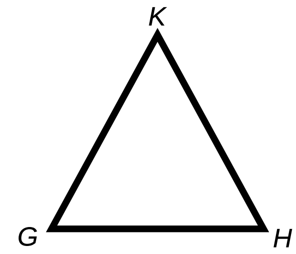
H $\frac{a}{s} = \frac{c}{r}$

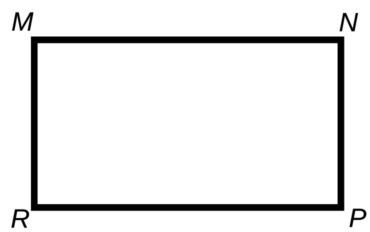
J $\frac{a}{s} = \frac{s}{b}$

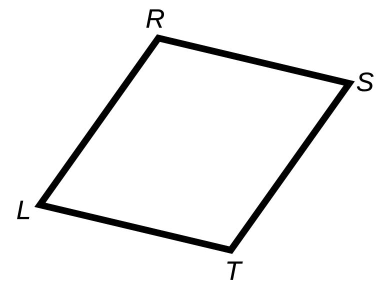
Name: ________________________

Warm-up









1. In triangle *ABC*, m$\angle A$ is 35° and m$\angle C$ is 90°. What is the measure of $\angle B$? ________

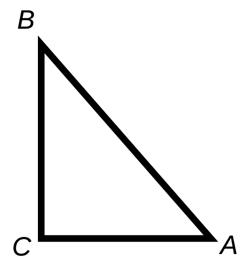
2. In triangle *GHK*, m$\angle G$ is 70° and m$\angle H$ is 65°. What is the measure of $\angle K$? ________

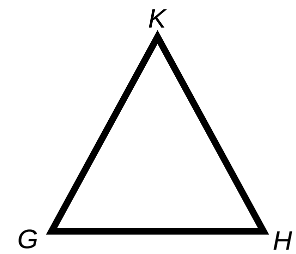
3. If quadrilateral *MNPR* is a rectangle, what is the measure of each angle? ________

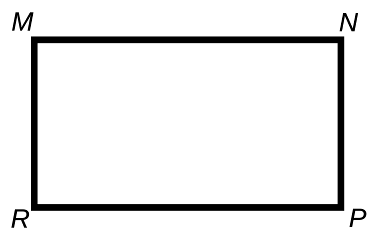
4. In quadrilateral *LRST*, m$\angle L$ is 85°, m$\angle R$ is 95°, and m$\angle S$ is 85°. What is the measure of $\angle T$? ________

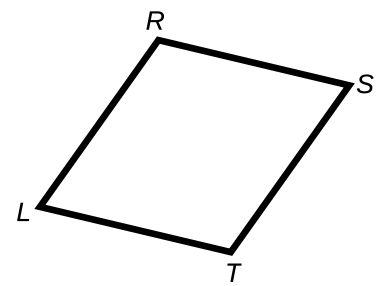
Name: ANSWER KEY

Warm-up









1. In triangle *ABC*, m∠*A* is 35° and m∠*C* is 90°. What is the measure of ∠*B*? ___55°___

2. In triangle *GHK*, m∠*G* is 70° and m∠*H* is 65°. What is the measure of ∠*K*? ___45°___

3. If quadrilateral *MNPR* is a rectangle, what is the measure of each angle? ___90°___

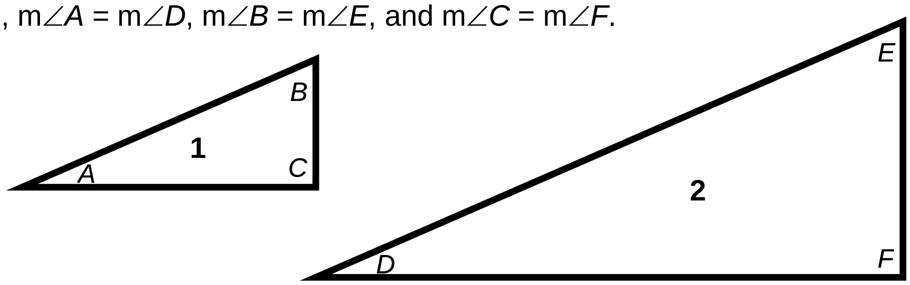
4. In quadrilateral *LRST*, m∠*L* is 85°, m∠*R* is 95° and m∠*S* is 85°. What is the measure of ∠*T*? ___95°___

Name: ______________________

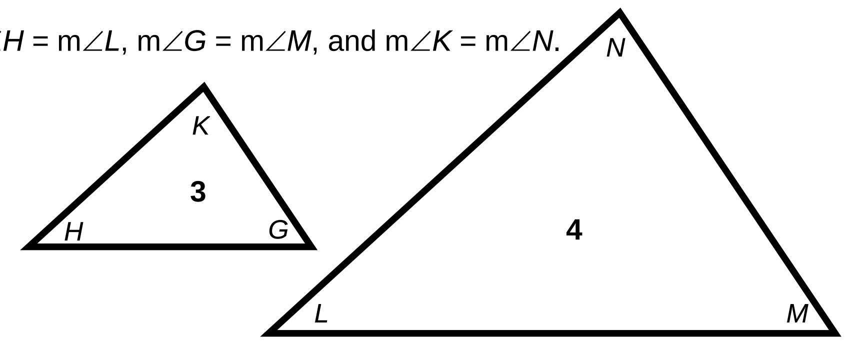
Similar Figures and Proportions, 1

The pairs of figures below are *similar figures.* By definition of similar figures, the lengths of their corresponding sides are *proportional* or *in proportion*. Determine the corresponding sides, and write the proportion for corresponding sides.

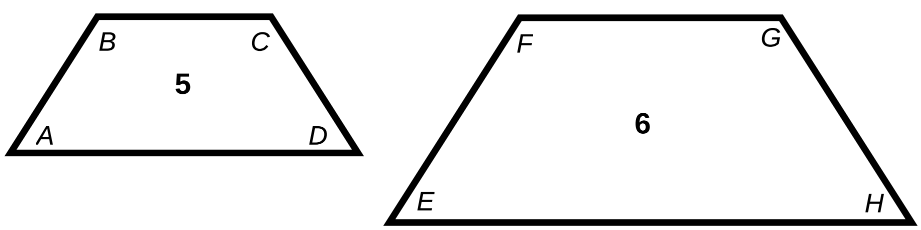
1. For triangles 1 and 2, m$\angle A$ = m$\angle D$, m$\angle B$ = m$\angle E$, and m$\angle C$ = m$\angle F$.



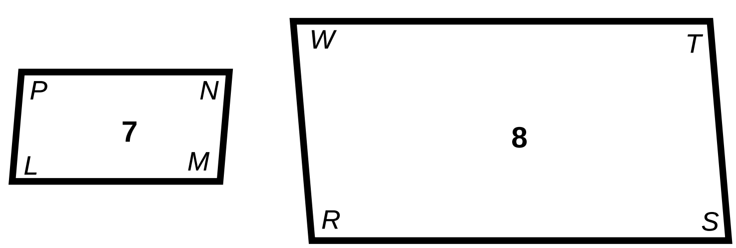
2. For triangles 3 and 4, m$\angle H$ = m$\angle L$, m$\angle G$ = m$\angle M$, and m$\angle K$ = m$\angle N$.



3. For quadrilaterals 5 and 6, m$\angle A$ = m$\angle E$, m$\angle B$ = m$\angle F$, m$\angle C$ = m$\angle G$, and m$\angle D$ = m$\angle H$.



4. For quadrilaterals 7 and 8, m$\angle L$ = m$\angle S$, m$\angle M$ = m$\angle R$, m$\angle P$ = m$\angle T$, and m$\angle N$ = m$\angle W$.

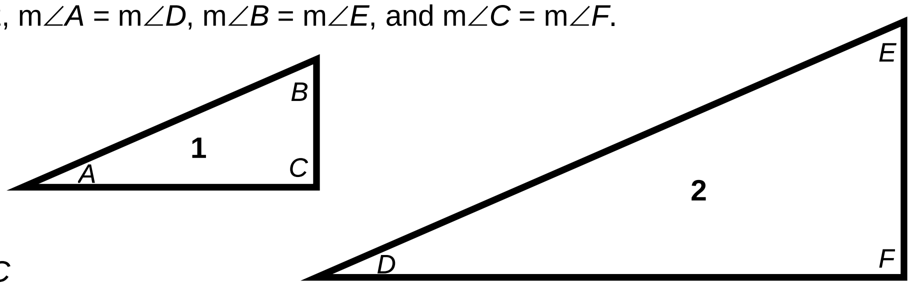


Name: ANSWER KEY

Similar Figures and Proportions, 1

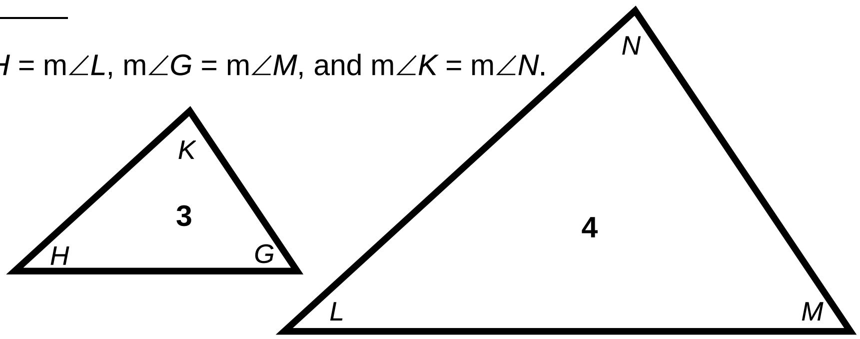
The pairs of figures below are *similar figures.* By definition of similar figures, the lengths of their corresponding sides are *proportional* or *in proportion*. Determine the corresponding sides, and write the proportion for corresponding sides.

1. For triangles 1 and 2, $m\angle A = m\angle D$, $m\angle B = m\angle E$, and $m\angle C = m\angle F$.



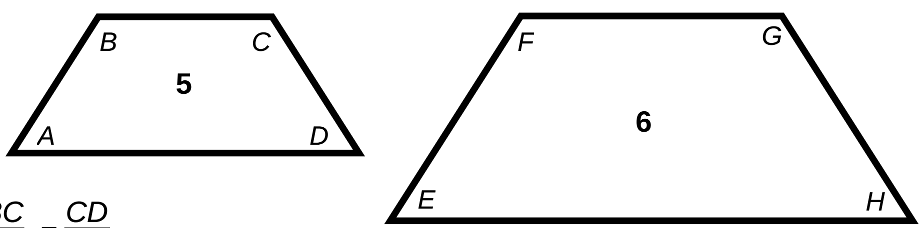
$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

2. For triangles 3 and 4, $m\angle H = m\angle L$, $m\angle G = m\angle M$, and $m\angle K = m\angle N$.



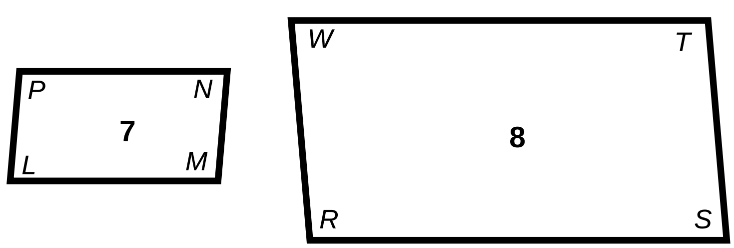
$\frac{HK}{LN} = \frac{HG}{LM} = \frac{GK}{MN}$

3. For quadrilaterals 5 and 6, $m\angle A = m\angle E$, $m\angle B = m\angle F$, $m\angle C = m\angle G$, and $m\angle D = m\angle H$.



$\frac{AB}{EF} = \frac{AD}{EH} = \frac{BC}{FG} = \frac{CD}{GH}$

4. For quadrilaterals 7 and 8, $m\angle L = m\angle S$, $m\angle M = m\angle R$, $m\angle P = m\angle T$, and $m\angle N = m\angle W$.



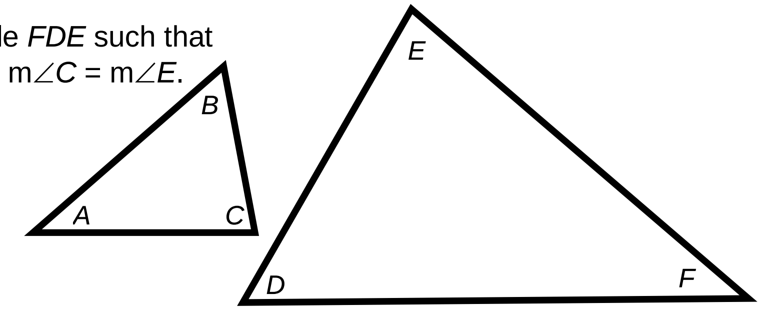
$\frac{LP}{ST} = \frac{LM}{SR} = \frac{PN}{TW} = \frac{MN}{RW}$

Name: ______________________________

Similar Figures and Proportions, 2

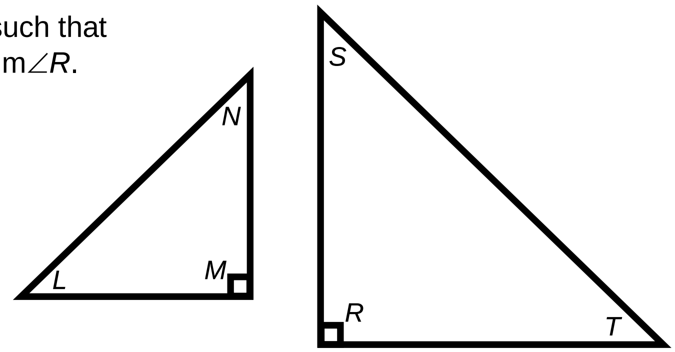
The pairs of figures below are *similar figures.* By definition of similar figures, the lengths of their corresponding sides are *proportional* or *in proportion*. Determine the corresponding sides, and write the proportion for corresponding sides.

1. Triangle ABC is similar to triangle *FDE* such that m∠*A* = m∠*F*, m∠*B* = m∠*D*, and m∠*C* = m∠*E*.



____

2. Triangle LMN is similar to triangle *TRS* such that m∠*L* = m∠*T*, m∠*N* = m∠*S*, and m∠*M* = m∠*R*.



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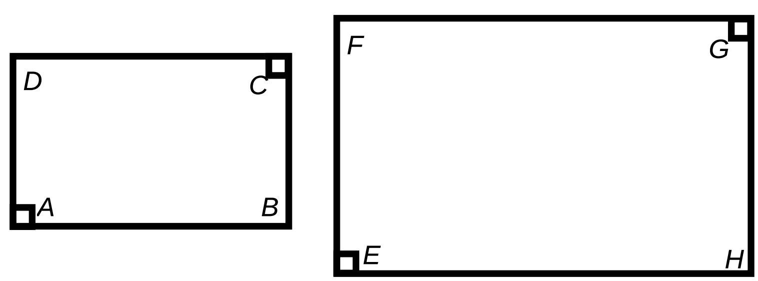
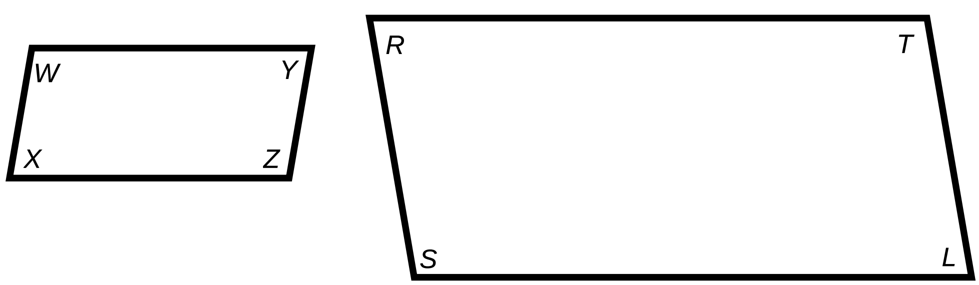
3. Rectangle *ABCD* is similar to rectangle *EHGF*.



____

4. Quadrilateral WXZY is similar to quadrilateral *RSLT* such that m∠*Z* = m∠*S*, m∠*Y* = m∠*R*, m∠*X* = m∠*L*, and m∠*W* = m∠*T*.



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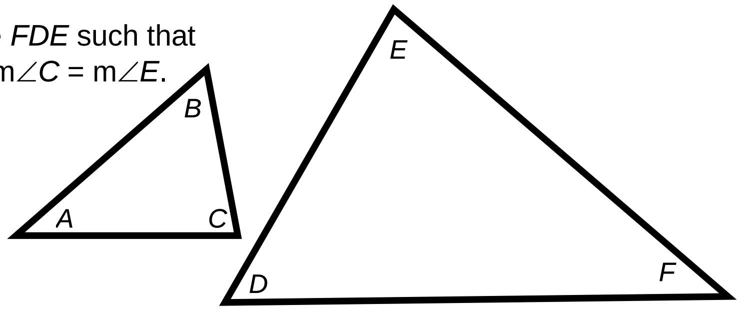
Name: <u>ANSWER KEY</u>

Similar Figures and Proportions, 2

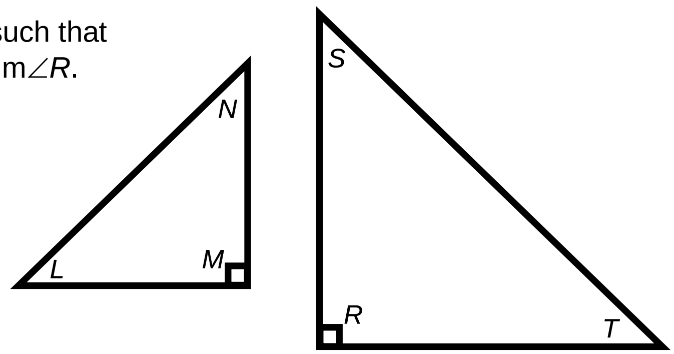
The pairs of figures below are *similar figures.* By definition of similar figures, the lengths of their corresponding sides are *proportional* or *in proportion*. Determine the corresponding sides, and write the proportion for corresponding sides.

1. Triangle ABC is similar to triangle *FDE* such that m$\angle A$ = m$\angle F$, m$\angle B$ = m$\angle D$, and m$\angle C$ = m$\angle E$.



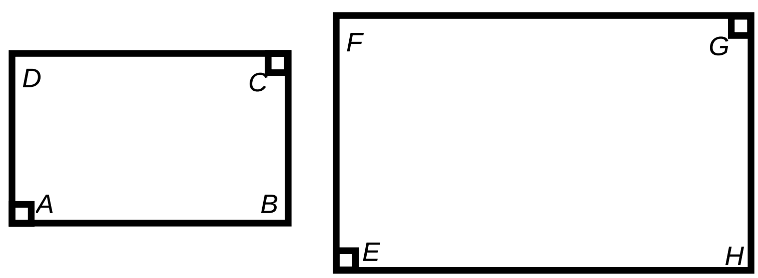
$\frac{AB}{FD} = \frac{AC}{FE} = \frac{BC}{DE}$

2. Triangle *LMN* is similar to triangle *TRS* such that m$\angle L$ = m$\angle T$, m$\angle N$ = m$\angle S$, and m$\angle M$ = m$\angle R$.



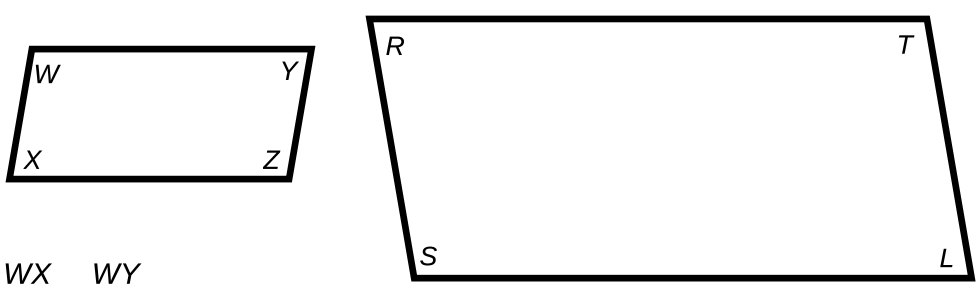
$\frac{LN}{TS} = \frac{LM}{TR} = \frac{NM}{SR}$

3. Rectangle *ABCD* is similar to rectangle *EHGF*.



$\frac{AB}{EH} = \frac{BC}{HG} = \frac{CD}{GF} = \frac{AD}{EF}$

4. Quadrilateral WXZY is similar to quadrilateral *RSLT* such that m$\angle Z$ = m$\angle S$, m$\angle Y$ = m$\angle R$, m$\angle X$ = m$\angle L$, and m$\angle W$ = m$\angle T$.



$\frac{YZ}{RS} = \frac{XZ}{LS} = \frac{WX}{TL} = \frac{WY}{TR}$