Introduction

In spring 2006, students will use scientific calculators for taking the Virginia Standards of Learning mathematics tests for grades 6, 7, and 8. The best way to prepare students to use a calculator during assessment is to give them ample practice using the tool during instruction.

The Mathematics Standards of Learning for grades 6, 7, and 8 offer many opportunities for using scientific calculators during instruction. The lessons that follow contain a range of instructional practices, from direct instruction on how to use such a calculator to writing-to-learn lessons to hands-on engagement. The lessons show keystrokes for the four VDOE-approved scientific calculators: the TI30Xa-SEVA, the Casio fx-260 School, and the Sharp EL-501V and EL-501WBBK.

Additionally, six lessons that extend the content of the grade 8 mathematics standards are provided for teachers to use in appropriate circumstances. Included also are four grade 8 science lessons that make use of the scientific calculator. These lessons meet or extend the grade 8 Standards of Learning for science.
Acknowledgments

The Virginia Department of Education wishes to express sincere appreciation to the following individuals who have contributed to the writing and editing of the activities in this document.

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Lessons to Implement the Mathematics Standards of Learning
1. **Charitable Cans to Paint**

**Objective**  
Students will find the surface area of a cylinder, using the formula $2\pi r^2 + 2\pi rh$.

**Summary**  
Students gain practical experience in collecting data and calculating the surface area of cylinders.

**Related SOL**  
8.7, 7.8

**Materials**  
Scientific calculators; variety of cylindrical containers (cans); measuring tapes

**Procedure**

1. Set up the context for your lesson by announcing: “We are having a collection for needy preschool students in our area.” (The collection could be for anything — e.g., mittens for the homeless, dog food for the SPCA.) “We have 10 large cans to place around the school to collect school supplies for these children.” You will need rather large cans, such as trash cans or popcorn tins, for collection of items such as mittens, eyeglasses, etc. Number each can for identification purposes.

2. Set up the math connection by explaining: “Our job is to paint each of these cans, including the lids, with our school colors. We need to figure out how much paint to buy.” Use the data that one gallon of paint covers 350 square feet, and one gallon costs $15.00.

3. Measure a sample can with the class, using the formula for surface area ($2\pi r^2 + 2\pi rh$) and the data $r = 2.25$ in. and $h = 4.8$ in.  
Sample keystrokes:

- **TI30Xa-SEVA**
  
  \[
  2 \times \pi \times 2.25 \times 2 + 2 \times \pi \times 2.25 \times 4.8 = 
  \]
  
  Answer: 71.039

- **Casio fx-260 School**
  
  \[
  2 \times \text{Shift} \times \pi \times 2.25 \times 2 + 2 \times \text{Shift} \times \pi \times 2.25 \times 4.8 = 
  \]
  
  Answer: 71.039

- **Sharp EL-501V and EL-501WBBK**
  
  \[
  2 \times \text{2nd F} \times \pi \times 2.25 \times 2 + 2 \times \text{2nd F} \times \pi \times 2.25 \times 4.8 = 
  \]
  
  Answer: 71.039

4. Allow students to work in pairs to calculate the surface area of each numbered can and then to determine the amount of paint they will need to buy and its cost.

5. When each pair has completed the task, compare all the students’ results. Ask: “What would cause some answers to be different?”
2. **Checking Up!**

**Objective** Students will comprehend and apply order of operations on a scientific calculator.

**Summary** Students practice checking their calculated answers to previously solved order-of-operations problems, using a scientific calculator. Much discussion about the function of the keys occurs during the practice that reinforces the order of operations.

**Related SOL** 8.1, 7.2

**Materials** Scientific calculators; previously completed order-of-operations problems with step-by-step solutions

**Procedure**

1. Use an example of the students’ completed problems to demonstrate solving by using keystrokes on the calculator to check order-of-operations work. Demonstrate the keystrokes by displaying them on an overhead acetate or with an overhead calculator. You may want to have the students write the first few problems in keystroke notation to become familiar with it.

Sample problem: \((-6)^3 + 3^2 (2 \times 4 - 9)\)

Keystrokes:

**TI30Xa-SEVA**

\[
( -6 +/– ) ^y \times 3 + 3 x \times ( 2 x 4 – 9 ) =
\]

Answer: \((-225)\)

**Casio fx-260 School**

\[
( -6 +/– ) ^y \times 3 + 3 x \times ( 2 x 4 – 9 ) =
\]

Answer: \((-225)\)

**Sharp EL-501V and EL-501WBBK**

\[
( -6 +/– ) ^y \times 3 + 3 x \times ( 2 x 4 – 9 ) =
\]

Answer: \((-225)\)

Sample problem: \(6^2 (12 – 2^3) – 4(3 + 5 \times 2)\)

Keystrokes:

**TI30Xa-SEVA**

\[
6 ^x x ( 12 - 2 ^{nd} x^3 ) - 4 x ( 3 + 5 x 2 ) =
\]

Answer: 92
2. Work together through various problems until the students have mastered the keystrokes.
3. Once students are conversant with the keystrokes, have them check various problems on their own.
4. **NOTE:** Common errors that students make when using the calculator are
   - forgetting the multiplication symbol before ( )
   - forgetting to press the key at the end of a computation.
3. Collect Them All

Objective  Students will collect data, represent it in a chart, calculate mean, calculate tax, and describe the activity as a simulation.

Summary  Students use the random number generator on the calculator to simulate the purchase of boxes of cereal in order to “collect the whole set” of toys or prizes given by the cereal company. They make a prediction, gather data, record it on a chart, and draw conclusions.

Related SOL  7.4b, 7.14, 7.16, 7.17, 7.18

Materials  Calculators with random number generator; copies of attached worksheet; empty cereal boxes (optional); small prizes (optional)

Procedure
1. Begin the lesson by describing the context as defined in the first paragraph of the worksheet. If your students need visuals or manipulatives to understand the task clearly, use empty cereal boxes and small toys or prizes.
2. Discuss the question: “How many boxes do you think a person will need to buy in this situation to collect all 10 prizes?” After discussion, have students record their predictions on their worksheets.
3. Instruct students in the procedure for generating random numbers. Most calculators are programmed to generate a 3-digit decimal. For this simulation, the numbers will represent a specific prize in the set (i.e., .1xx will represent prize number 1, .2xx will represent prize number 2, etc.)
   Keystrokes:
   Ti30Xa-SEVA
   (No random number generator)

   Casio fx-260 School
   2nd Run#

   Sharp EL-501V and EL-501WBBK
   2nd Random
Keystrokes to calculate an arithmetic mean of 75, 16, 34, 29, 72

**TI30Xa-SEVA**

\[
\begin{array}{cccccc}
75 & \Sigma+ & 16 & \Sigma+ & 34 & \Sigma+ & 29 & \Sigma+ & 72 & \Sigma+ & 2^{\text{nd}} & \bar{x} \\
\end{array}
\]

To clear: \(\Sigma+ \text{ 2}^{\text{nd}} \text{ On/AC}\)

**Casio fx-260 School**

Enter stats mode: \text{Mode} \ \ .

\[
\begin{array}{cccccccc}
\text{Shift} & \text{SAC} & 75 & \text{Data} & 16 & \text{Data} & 34 & \text{Data} & 29 & \text{Data} & 72 & \text{Data} & \text{Shift} & \bar{x} \\
\end{array}
\]

Or \(\text{Shift} \ n\) (for number of data points entered)

Or \(\text{Shift} \ \Sigma x\) (for the sum of the data)

Exit stats mode: \text{Mode} \text{ 0}

**Sharp EL-501V and EL-501WBBK**

Enter stats mode: \(\text{2}^{\text{nd}} \ F \text{ STAT}\)

\[
\begin{array}{ccccccccc}
75 & \text{Data} & 16 & \text{Data} & 34 & \text{Data} & 29 & \text{Data} & 72 & \text{Data} & \bar{x} \\
\end{array}
\]

Exit stats mode: \(\text{2}^{\text{nd}} \text{ On/C} \) or \text{Off}

4. Have students simulate a buyer and generate enough random numbers until they get at least one tally in each column (representing the different toys.) Then, have students total the number of boxes that buyer had to purchase. Have them repeat the process for 10 buyers and determine the mean number of boxes required to get all the toys.

5. Hold a discussion about how the simulation would be different if the manufacturer produced different numbers of toys. Do you think companies produce the same number of toys if they want buyers to collect an entire set of toys?
Collect Them All Worksheet

Name: _______________________________ Date: __________________

The General Lee Cereal Company has started to include toys in its cereal boxes in an effort to increase sales. There are 10 different toys in this set. Assuming the company has distributed an equal number of each toy in the boxes, how many boxes, on average, would a person need to purchase in order to collect all 10 toys? Record your prediction here. _____________________

Use the random number generator on your calculator to produce random numbers. For this simulation, any number that begins with .1 will represent the first toy, .2 will represent the second, etc. The tenth toy is represented by .0.

Simulate 10 different buyers by tallying the number of cereal boxes that each buys, keeping track of how many each had to purchase until he/she collected at least one of each toy.

<table>
<thead>
<tr>
<th>Person</th>
<th>Toy 1</th>
<th>Toy 2</th>
<th>Toy 3</th>
<th>Toy 4</th>
<th>Toy 5</th>
<th>Toy 6</th>
<th>Toy 7</th>
<th>Toy 8</th>
<th>Toy 9</th>
<th>Toy 10</th>
<th>Total Boxes</th>
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<tbody>
<tr>
<td>A</td>
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</tbody>
</table>

1. Determine the mean number of boxes a buyer would have to purchase to get all 10 toys. _____
2. Was your prediction a good estimate? ______ Explain why or why not.

3. What percent represents the tax on food in Virginia? ____________
4. If the cereal costs $2.59 per box, how much money would a person spend, on average, to get all of the toys? (Don’t forget to include the tax.) ______________
5. Suppose the cereal was on sale one week for only $2.29 per box, plus tax. How much money could a person save if she/he bought the cereal during that week? ___________

Extension

6. Suppose that the company did not produce an equal number of each toy. How would the situation be different?
4. **Gifts for the Family**

**Objective**  
Students will compute discounts.

**Summary**  
Students select from catalogs gifts for family members and calculate the cost to determine whether $200 is enough to complete the purchase.

**Related SOL**  
8.3

**Materials**  
Various store catalogs; scientific calculators; copies of attached worksheet

**Procedure**  
1. Demonstrate a sample activity, such as the following:  
   I have $200 to spend on my family. I found a CD player for my dad for $89.00. I have a 15%-off coupon. How much will I spend on this gift?  
   Keystrokes:  
   *TI30Xa-SEVA*  
   \[
   \begin{align*}
   89 &- (89 \times 15 \text{ Shift } %) = \\
   \text{Total amount spent: } &$75.65
   \end{align*}
   
   *Casio fx-260 School*  
   \[
   \begin{align*}
   89 \times 15 \text{ Shift } &\% - \\
   \text{NOTE: The Casio fx-260 School keystrokes for this work are not conducive to mathematical understanding.}
   \end{align*}
   
   *Sharp EL-501V and EL-501WBBK*  
   \[
   \begin{align*}
   89 &- (89 \times 15 \text{ 2nd } F \% ) = \\
   \end{align*}
   
2. Tell students that they have $200 to spend on gifts for their family members. (You may want to use checking accounts.), and have them look through the catalogs, select items for purchase, and record their purchases (item and price) on the worksheet.

3. When students finish shopping, have them draw a secret discount card from a hat, basket, or box. The discounts should range from 5% off to 25% off. Let the students draw one card for each item they recorded as a purchase.

4. Have the students calculate the amounts spent after the discount and record these amounts on the chart. You may wish to have the students work in pairs to check each other’s work.

5. Allow students to find an item for themselves with the total amount of money saved by using the discount cards.

**Extension**  
1. Consider creating tax cards as well, and add a column to the chart for the total cost of the items after the tax is added.
## Gifts for the Family Worksheet

Name: ________________________________  Date: __________________

<table>
<thead>
<tr>
<th>Name of Item</th>
<th>Original Price</th>
<th>Discount Percent</th>
<th>Amount Spent</th>
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</tbody>
</table>

How much would you have spent on all the items if there had been no discounts? _________

How much did you end up spending on all the items by using the discounts? _________

How much money did you save by using the discounts? _________

What item can you purchase for yourself with the money you saved? _________________________
5. *Going Out to Lunch*

**Objective**  
Students will compute tip, sales tax, and total bill for a meal.

**Summary**  
Students calculate final restaurant bills, working with data found in menus from local restaurants.

**Related SOL**  
7.4

**Materials**  
Scientific calculators; area restaurant menus; copies of attached worksheet

**Procedure**
1. Give each student or group of students a menu and worksheet.
2. Work through the sample on the worksheet with the class.
3. Have students choose meals from their menu, working either individually or in groups. Then, have them calculate what their total cost is for the meal, including tax and tip.
Bob gets the $4.89 hamburger platter and a $1.37 slice of apple pie. He had good service, so he decides to leave a 20% tip. The sales tax in his area is 5.5%. How much will he pay for his lunch?

Cost of meal: $4.89 + $1.37 = $6.26
Sales tax is 5.5% of $6.26 = $0.34
Tip is 20% of $6.26 = $1.25

Add cost, tax, and tip for Bob’s total amount: $6.26 + $0.34 + $1.25 = $7.85

Keystrokes:

**TI30Xa-SEVA**

6.26 x 5.5 2nd % =
Answer: round 0.3443 to $0.34

6.26 x 20 2nd % =
Answer: round 1.252 to $1.25

**Casio fx-260 School**

6.26 x 5.5 Shift %
Answer: round 0.3443 to $0.34

6.26 x 20 Shift %
Answer: round 1.252 to $1.25

**Sharp EL-501V and EL-501WBBK**

6.26 x 5.5 2nd F % =
Answer: round 0.3443 to $0.34

6.26 x 20 2nd F % =
Answer: round 1.252 to $1.25

*Remember: Tax and tip are calculated on the original cost.

Now, you need to choose your meal from the menu in front of you. You should include a meal, beverage, and dessert. You may want to get an appetizer as well. Use the guide below to figure out how much money you will spend on this meal.

Food items and their cost: ________________________
__________________________
__________________________
__________________________

Total cost: ________

Area sales tax percent: _________ Amount of tax: ________
Percent for tip: ________ Amount of tip: ________

Total amount spent: ________
6. **Inspecting a Deck**

**Objective**  
Students will solve real-life problems involving right triangles by using the Pythagorean Theorem. They also will verify the Pythagorean Theorem.

**Summary**  
Students play the role of building inspector, checking whether decks built in a neighborhood are “square,” and reporting their findings to the neighborhood association.

**Related SOL**  8.10

**Materials**  
Scientific calculators; copies of attached worksheet

**Procedure**

1. Present students with the problem of a building inspector checking to see if decks have been built “square” in response to the attached letter from a neighborhood association. Explain the term *square* in this case as defining a “quadrilateral with all angles 90 degrees.”

2. Ask the students whether the dimensions shown for the deck below indicate that it is “square.” Work through it together, using the Pythagorean Theorem.

```
3' 6'
```

The diagonal should be 5’, but this deck has a diagonal of 6’. Therefore, this deck is not “square.”

Keystrokes:

* TI30Xa-SEVA *

```
3 x² + 4 x² = √x
```

* Casio fx-260 School *

```
3 x² + 4 x² = Shift √x
```

* Sharp EL-501V and EL-501WBBK *

```
3 x² + 4 x² = √
```

3. Point out the use of the Pythagorean Theorem.

4. Give each student or group of students a copy of the letter from the neighborhood association along with the accompanying diagrams of the decks.

5. Have the students use the Pythagorean Theorem to compute the length of the diagonal (hypotenuse of the two triangles formed) of each rectangular deck shown and determine if each deck is “square.”

6. Have the students put the answers to the decks into a reply letter to the neighborhood association.
Answer key
The answers to the Inspecting a Deck Worksheet are:

1. No
2. Yes
3. Yes
4. No
5. Yes
6. Yes
September 15, 2005

Dear Building Inspector:

The residents in our neighborhood have recently built decks in their backyards. We were told that the decks should be built “square.” It is our understanding that building “square” means that all corners must be right angles. We are not sure whether we did this correctly. Attached are diagrams of our decks with measurements. Please determine which decks were built “square” and inform us in writing which decks are correct and which are not.

Sincerely,

John Q. Smith, President
Short Pump Neighborhood Association
Inspecting a Deck
Worksheet

Name: ________________________________
Date: ________________________________

Use the Pythagorean Theorem to compute the length of the diagonal (hypotenuse of the two triangles formed) of each rectangular deck shown below, and determine if each deck is "square."

Sketches of Homeowners’ Decks

1. 2.
   \begin{array}{c}
   \hspace{0.5cm}
   \begin{array}{c}
   \text{4'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{7'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{5'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{8.6'}
   \end{array}
   \\
   \text{5'}
   \end{array}
   \\

2. 5'
   \begin{array}{c}
   \text{5'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{8.6'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{7'}
   \end{array}

3. 4'
   \begin{array}{c}
   \text{4'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{6'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{7.2'}
   \end{array}

4. 15'
   \begin{array}{c}
   \text{15'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{20.4'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{15'}
   \end{array}

5. 10'
   \begin{array}{c}
   \text{10'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{15.62'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{12'}
   \end{array}

6. 7.5'
   \begin{array}{c}
   \text{7.5'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{14.15'}
   \end{array}
   \hspace{1cm}
   \begin{array}{c}
   \text{12'}
   \end{array}
7. Introducing Exponents with a Calculator

Objective  Students will simplify numbers with positive exponents.

Summary  Students receive direct instruction demonstrating the keystrokes used for exponential form.

Related SOL  6.22

Materials  Scientific calculators

Procedure
1. Review exponential notation with your students.

2. Demonstrate the \( y^x \) key on the calculator.

   Keystrokes for calculating \( 7^4 \):
   
   **TI30Xa-SEVA**
   
   \[
   7 \quad y^x \quad 4 \quad =
   \]

   **Casio fx-260 School**
   
   \[
   7 \quad x^y \quad 4 \quad =
   \]

   **Sharp EL-501V and EL-501WBBK**
   
   \[
   7 \quad y^x \quad 4 \quad =
   \]

3. Have students write each of the following as a product, using their calculators:

   1. \( 4^3 \)  
   2. \( 5^2 \)  
   3. \( 9^1 \)  
   4. \( 10^8 \)  
   5. \( 78^0 \)  
   6. \( 2^6 \)  
   7. \( 4^0 \)  
   8. \( 4^4 \)

Answer key
1. 64  
2. 25  
3. 9  
4. 100,000,000  
5. 1  
6. 64  
7. 1  
8. 256
8. Let the Chips Fall Where They May

Objective Students will gather data and represent it in a chart, find the areas of circles, find the volumes of cylinders, calculate percentages, and calculate the mean.

Summary Students count the number of chocolate chips in representative chocolate chip cookies of various brands to determine which brand contains more chips per cubic unit. They may also calculate the brand with the most chips for the price sold.

Related SOL 6.12a, 6.19, 7.4, 7.8, 7.16, 7.18, 8.3, 8.7

Materials Chocolate chip cookies of 3 to 5 brands, enough so that each student or pair can have 5 cookies; metric rulers; string; 5 cups full of water for each student or pair for soaking the cookies; a graduated cylinder for each student or pair; scientific calculators; copies of attached worksheet

Procedure This is an activity that works best in a science lab so that students have a water source readily available to them; however, this is not a requirement.

1. Give each student or pair of students 5 chocolate chip cookies of a single brand, and have them estimate how many chips they think each cookie contains.
2. Have the students take the appropriate measurements needed to calculate the area of the top of the cookie and the volume of the cookie (see chart on worksheet).
3. Next, have the students “melt off” the cookies from the chips by soaking them in water. Then, have them place the chips in a graduated cylinder to determine their total volume. Have the students answer the various questions on the student worksheet related to volume, percentage volume, etc. (NOTE: You may need to demonstrate how to measure the volume of an object through displacement, making sure to instruct the students in reading the level of the meniscus carefully.) For conversion purposes, give students the conversion fact 1 cubic cm = 1 ml.
4. If the students or student pairs have been given cookies of differing brands, have the whole class make comparisons between brands related to price, chip content, etc.
Keystrokes to calculate an arithmetic mean of 75, 16, 34, 29, 72

**Texas Instruments TI-30Xa-SEVA**

75 \[ \sum + \] 16 \[ \sum + \] 34 \[ \sum + \] 29 \[ \sum + \] 72 \[ \sum + \] 2nd \[ \bar{x} \]

To clear: \[ \sum + \] 2nd On/AC

**Casio fx-260 School**

Enter stats mode: Mode .

Shift SAC 75 Data 16 Data 34 Data 29 Data 72 Data Shift \[ \bar{x} \]

Or Shift n (for number of data points entered)

Or Shift \[ \sum x \] (for the sum of the data)

Exit stats mode: Mode 0

**Sharp EL-501V and EL-501WBBK**

Enter stats mode: 2nd F STAT

75 Data 16 Data 34 Data 29 Data 72 Data \[ \bar{x} \]

Exit stats mode: 2nd On/C or Off
Let the Chips Fall Where They May
Worksheet

Name: ____________________________ Date: __________________

Materials
• 5 cookies of the same brand
• Piece of string
• Metric ruler
• 5 cups, each filled about 2/3 full of water
• Spoon
• Graduated cylinder

1. Estimate how many chips are in each cookie. Record your estimations in column 2 of the table below.
2. Measure the circumference of each cookie by wrapping the string around the edge. Mark off the length of string, and measure it with the ruler, accurate to the nearest millimeter. Put your data in column 3.
3. Using the formula for the circumference of a circle, calculate the diameter of each cookie, and record the results to the nearest tenth of a unit in column 4. Are all the cookies the same? ____
4. Using the diameter that you calculated and the formula for the area of a circle, compute the area of the top surface of each cookie. Record your calculated value to the nearest tenth of a unit in column 5.

<table>
<thead>
<tr>
<th>Cookie</th>
<th>Total Chips (estimated)</th>
<th>Circumference (measured)</th>
<th>Diameter (calculated)</th>
<th>Area of Top (calculated)</th>
<th>No. Chips on Surface (counted)</th>
<th>No. Chips per Square Unit (calculated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
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</tbody>
</table>

5. Count the number of chocolate chips visible on the top surface of each cookie, and record the total number in column 6.
6. Calculate the number of chips per square unit of cookie for each of your cookies. Record the data in column 7.
7. Calculate the mean number of chips per square unit of cookie for your brand of cookies. ______
8. Copy your data for the area of the top of the cookie into column 2 of the table above.
9. Measure to the nearest tenth of a unit the “height” of the cookie. Record your data in column 3.
10. Explain why a chocolate chip cookie is considered a cylinder.

11. Using the formula for the volume of a cylinder, calculate the volume, to the nearest tenth of a unit, of each cookie. Record your data in column 4 in the table above.
12. Since the chocolate chips are not a standard geometric shape, you’ll need to find their volume by displacement. To remove the cookie from around the chips, soak each cookie in its own cup of water until the cookie is soft enough to fall away from the chips. Use a spoon to stir the water to help dissolve the dough and to get the chips out of the water when they are free.
13. Being careful to keep the chips from each cookie separate from the others, count the number of chips in each cookie. How did the actual number compare to your earlier estimate?

14. Fill the graduated cylinder about half full. Carefully drop the chips from cookie 1 into the cylinder to measure their total volume. Record this data in column 5. Once you have finished, pour out the water and the chips. (Don’t let the chips go down the sink: they should go into the trash can.) Repeat the procedure for each of the other four cookies.
15. Using the data from #11, determine the number of chips per cubic unit of cookie. Record the data in column 6.
16. Using the data from #12, determine the volume of chips per cubic unit of cookie. Record the data in column 7.
17. Calculate the percentage of each cookie that consists of chocolate chips, and record the data in the last column.
18. Calculate the mean of the percentages of chip volume for your brand of cookie. _________
Class Data

19. Share all data with your classmates for the different brands of cookies represented. Record the data in the table below.

<table>
<thead>
<tr>
<th>Brand of Cookie</th>
<th>Price per Bag</th>
<th>No. of Cookies per Bag</th>
<th>Price per Cookie</th>
<th>Mean No. Chips per Cookie</th>
<th>Mean % Chip Volume per Cookie</th>
<th>% Chips per Unit Price</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

20. Which brand of cookie provides the most chips per cookie, on average?

21. Which brand of cookie provides the most chips by volume per cookie, on average?

22. Which brand of cookie provides the highest percentage of chips by volume per cookie, on average?

23. Assuming you were a chocolate chip lover, which brand of cookie would you be most likely to buy? ________________ Why?
9. **How to Avoid Making a Mess with pi**

**Objective**  
Students will calculate the circumference of a circle, recognize the difference in answers when using the pi key versus 3.14, and use the “fix” function to set pi at 3.14.

**Summary**  
Students compare and contrast the effect on answers to problems of pi rounded to various numbers of decimal places. This is specifically related to the internal rounding features of scientific calculators.

**Related SOL**  6.12

**Materials**  
Overhead projector; scientific calculators

**Procedure**  
NOTE: This lesson should involve student discussion at each step.

1. Draw a circle, and define the word *circumference* as the distance around a circle.
2. Present the formulas \( C = \pi d \) and \( C = 2\pi r \) for finding circumference to the students, and discuss substituting the values. For example, if the radius of the circle is 3 m, and the value of \( \pi \) is 3.14, an approximation of the circumference of the circle is: \( 2\pi r = (2)(3.14)(3 \text{ m}) = 18.84 \text{ m} \).
3. Repeat the exercise, using the pi key on the calculator instead of entering 3.14 as the value. Keystrokes:  
   - *TI30Xa-SEVA*  
     \[ 2 \times \pi \times 3 = 18.84955592 \]
   - *Casio fx-260 School*  
     \[ 2 \times \text{Shift} \pi \times 3 = 18.84955592 \]
   - *Sharp EL-501V and EL-501WBBK*  
     \[ 2 \times \text{2nd F} \pi \times 3 = 18.84955592 \]
4. Ask the class why the answer using 3.14 differs from that using pi in the calculator. Discuss the fact that pi on the calculator is not rounded to only 2 decimal places, but to 9 decimal places. Ask which answer is more precise. Explain that, in the real world, people most often use pi rounded to two decimal places to calculate circumference. Why would someone use pi to 9 decimal places?
5. Have students compare the two answers, noticing that 18.84 is not 18.84955592 rounded to the nearest tenth. Why?
6. Show students how to “fix” the decimal places of their calculator to get an answer that is already rounded, using the more standard form of pi.

Keystrokes:

**TI30Xa-SEVA**

To fix display to two decimal places:

```
2nd fix 2
```

**Casio fx-260 School**

```
Mode 7 2
```

Your calculator screen should read 0.00

To clear fixed decimal display

```
Mode On
```

**Sharp EL-501V and EL-501WBBK**

To fix display to two decimal places:

```
2nd F TAB 2
```

Your calculator screen should read 0.00.

To return to floating decimal display

```
2nd F TAB .
```

7. Have students key in their problem, using the same keystrokes as before. Make sure they notice that pi comes up as 3.14, but that the answer on their screen is 18.85. Ask why this answer is different from the 18.84 answer they received when they keyed in 3.14 as pi.

8. Allow students to try different problems to see how a rounded pi affects answers. Is there a difference if 2 and 3 are not among their values?
10. Shopping for Groceries

Objective Students will compute a discount and resulting sale price of one discount.

Summary Students calculate the cost of the meal that they plan, using grocery store ads and discount coupons.

Related SOL 7.4, 8.3

Materials Grocery store ads from newspapers; scientific calculators; coupon slips; copies of the two attached worksheets

Procedure
1. Place students into groups of four.
2. Give each student a different discount coupon with each group receiving one for 10% off, one for 15% off, one for 20% off, and one for 25% off.
3. Explain to the students that they are going to plan a meal. Give them grocery store circulars/ad with prices and Worksheet A.
4. Do the sample problem together, using a calculator to illustrate appropriate keystrokes.
5. Have students choose their food items together as a group. They must choose a meat, two side dishes, and a dessert. Have them record their choices on worksheet.
6. Have students find the discounted price of each item, using their individual discount card.
7. As the groups finish, give them Worksheet B. Explain that this task is for the group to plan the most economical meal, using their coupons to maximum advantage.
8. When all groups are done, find out which group saved the most money on their meal.
SAMPLE PROBLEM: Squeaky Clean shampoo regularly costs $2.26. You have a coupon for 18% off one item. What will the cost of the shampoo be? (Do not include tax.)

Find the percentage off, using the following keystrokes:

* TI30Xa-SEVA *

\[ \$2.69 \times 18 \quad 2^{\text{nd}} \% = \]

* Casio fx-260 School *

\[ \$2.69 \times 18 \quad \text{Shift} \% \]

* Sharp EL-501V and EL-501WBBK *

\[ \$2.69 \times 18 \quad 2^{\text{nd}} \text{F} \% = \]

Answer: .4842 rounds to $0.48

Then, subtract: $2.69 – $0.48 = $2.21
(Alternatively, you may find 82% of $2.69 and eliminate the subtraction step.)

My discount card is for _______ % off.

Meat: ________________ Cost: ________
Amount off: ________
Price after discount: ________

Side dish 1: ________________ Cost: ________
Amount off: ________
Price after discount: ________

Side dish 2: ________________ Cost: ________
Amount off: ________
Price after discount: ________

Dessert: ________________ Cost: ________
Amount off: ________
Price after discount: ________
Shopping for Groceries
Worksheet B

Group Name: ___________________________   Date: ______________________

Meat: ________________
Original cost: ________________
Percent of discount: ________________
Amount of discount: ________________
Price after discount: ________________

Side dish 1: ________________
Original cost: ________________
Percent of discount: ________________
Price after discount: ________________

Side dish 2: ________________
Original cost: ________________
Percent of discount: ________________
Amount of discount: ________________
Price after discount: ________________

Dessert: ________________
Original cost: ________________
Percent of discount: ________________
Amount of discount: ________________
Price after discount: ________________

Original cost of meal: ________________
Your cost after discounts: ________________
Total amount of money saved: ________________

*Go back and change who buys which item, and compare the total amount of money you saved.

Describe a strategy for using your coupons to save the most money.
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
11. Who Are You Calling a Nerd?

Objective  Students will gather data and represent it in a chart and in a stem-and-leaf plot, and to find the mean, median, and mode of the data.

Summary  Students count Nerds™ candy and, with the aid of a scientific calculator, determine the values of the measures of central tendency (mean, median and mode) for the data. They also make conjectures based on their data and discuss the concepts of operational definition and outliers in a data set.

Related SOL  6.18, 6.19, 6.20.a, 7.16, 7.17

Materials  One Nerds™ snack-size candy box for each student (these can easily be found in mid-to late-October in bags containing both grape- and strawberry-flavored candies); scientific calculators; copies of attached worksheet

Procedure
1. Provide each student with a snack-size box of Nerds™ candy. Allow them to open the box and spread the candy out on a piece of paper, but do not allow them to eat any yet.
2. Explain that in scientific research, experimenters typically have to formulate an “operational definition” of a concept for the purposes of their study. For example, intelligence may be defined as what an intelligence test measures. Introduce and describe the term operational definition, and have the class agree on what would constitute a good operational definition of Nerd™ for this study. Ask such questions as: Will you count a piece of candy as a Nerd™ only if it measures a specific diameter? Will a piece count as two Nerds™ if it clearly looks like two fused pieces? Does a piece have to be a certain color to be a Nerd™?
3. Once the operational definition of a Nerd™ is established, have each student count the number of Nerds™ in his/her box and report that number to the class to record on the data chart. Have each student record the data from the entire class in the charts provided on the worksheet and then create a stem-and-leaf plot for the strawberry Nerds™ and one for the grape Nerds™. (Consider back-to-back stem-and-leaf plots for this data.)
4. Have students enter the data sets in the calculator to determine the mean of each flavor, and have them determine the median and the mode from the stem-and-leaf plots they created.
5. Have a class discussion of outliers and their effect on the mean as opposed to the median and the mode. (The mean is sensitive to the actual magnitude of the data value as opposed to its location or to the number of times it occurs.)
6. Have students formulate their own questions about Nerds™ that can be answered from the stem-and-leaf plot.
<table>
<thead>
<tr>
<th>Keystrokes to calculate an arithmetic mean of 75, 16, 34, 29, 72</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TI30Xa-SEVA</strong></td>
</tr>
<tr>
<td>75 ( \sum + ) 16 ( \sum + ) 34 ( \sum + ) 29 ( \sum + ) 72 ( \sum + ) 2nd ( \bar{x} )</td>
</tr>
<tr>
<td>To clear: ( \sum + ) 2nd On/AC</td>
</tr>
<tr>
<td><strong>Casio fx-260 School</strong></td>
</tr>
<tr>
<td>Enter stats mode: Mode .</td>
</tr>
<tr>
<td>Shift SAC 75 Data 16 Data 34 Data 29 Data 72 Data Shift ( \bar{x} )</td>
</tr>
<tr>
<td>Or Shift ( n ) (for number of data points entered)</td>
</tr>
<tr>
<td>Or Shift ( \sum x ) (for the sum of the data)</td>
</tr>
<tr>
<td>Exit stats mode: Mode 0</td>
</tr>
<tr>
<td><strong>Sharp EL-501V and EL-501WBBK</strong></td>
</tr>
<tr>
<td>Enter stats mode: 2nd F STAT</td>
</tr>
<tr>
<td>75 Data 16 Data 34 Data 29 Data 72 Data ( \bar{x} )</td>
</tr>
<tr>
<td>Exit stats mode: 2nd On/C or Off</td>
</tr>
</tbody>
</table>
Who Are You Calling a Nerd?
Worksheet

Name: ________________________________ Date: __________________

1. What is an “operational definition?”

2. What is the operational definition of Nerd™ for this class activity?

3. Record the number of Nerds™ of each flavor in your class in the charts below.

<table>
<thead>
<tr>
<th>Strawberry Nerds™</th>
<th>Grape Nerds™</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

4. Create a stem-and-leaf plot for each of the data samples you gathered.

5. Enter the data into the scientific calculator and report the mean number of Nerds™ of each flavor. Round each value to the nearest tenth.

Mean Guitar = _______       Mean Guitar = _______

6. Using the stem-and-leaf plots you created, give the values of the median and the mode for each flavor of candy.

Median Guitar = _______       Median Guitar = _______

Mode Guitar = _______       Mode Guitar = _______

7. An “outlier” is a data value that is very different from the others values in the data set. Does either flavor appear to have any outliers? ______ If so, which values are they?

Suspected outliers Guitar: _______  Suspected outliers Guitar: _______

8. Which of the measures of central tendency were most affected by outliers that you identified in #7? __________________________ Why do you think so?

9. What questions about Nerds™ could you answer from the stem-and-leaf plot?
12. Who Won the Race?

Objective  Students will devise and compare a variety of methods for determining mean, median, and mode.

Summary  Students devise as many ways as possible to determine which team of runners wins a race, which places second, which third, and which fourth. All methods of determining the overall team position must be supported with logical reasoning and appropriate mathematical calculations, when appropriate. As the final project, groups present reports on their conclusions.

Related SOL  8.3, 8.12

Materials  Copies of attached worksheet; scientific calculators

Procedure  
1. Distribute the worksheet outlining the problem description and providing the data.
2. Have students read the introduction and directions. Answer any questions, especially those that relate to interpreting the data table.
3. Have students work individually for 10 minutes to devise as many ways as they can to determine overall team placement. After 10 minutes, have students pair up, compare their proposed methods with their partner, and together attempt to devise more ways to determine the overall winner. Have them work with these partners for about 15 minutes.
4. After the pairs have worked together for 15 minutes, let pairs join one another to make groups of four. Have each group’s members share all of their methods with each other and then pick the one they believe is the most fair to present to the Judging Committee.
5. Instruct each group to prepare a brief presentation for the Judging Committee. The presentation should include the reason(s) for the choice of methods (with logical and mathematical support) and should include a comparison to other methods showing why the chosen method is preferable.

Possible student responses  
1. Look at the first row (the order in which the first person from each team crossed the line): RMS had the fastest runner, WMS the second fastest, JMS the third fastest, and LMS the fourth fastest. Hence, they place: RMS, WMS, JMS, and LMS.
2. Find the average of the top 10 runners from each school. In this case, they finish JMS (19.2), LMS (19.8), RMS (20.9), and WMS (22.2).
3. Find the average for all runners on each team. Will this favor teams with more or fewer runners?
4. Compare the median finisher for each school. Since row 17 is the last row with a finisher from each school, consider row 9 to be the median: places are then JMS, RMS, LMS, and WMS.

Assessment  An open-ended problem is best assessed with the use of a rubric. You will need to determine the weight of each component of the rubric, depending on the emphasis you place on each part of the activity. Suggested items to measure include the following:
• Use of correct mathematics (e.g., mean, median, mode)
• Fluency (lots of methods devised)
• Creativity (methods substantially different from each other)
Who Won the Race?
Worksheet

Name: _____________________________ Date: __________________________

Jefferson Middle School (JMS), Washington Middle School (WMS), Lincoln Middle School (LMS), and Roosevelt Middle School (RMS) competed for the State Championship in Cross Country. The results of the race are displayed in the table at right, which shows how each runner placed compared to his/her own teammates (Within Team Rank), as well as how each runner placed overall. For example, the fastest runner from Jefferson Middle School placed 4th overall; the last runner to finish from Roosevelt Middle School finished 64th overall. Although each team entered 20 runners, 6 runners did not complete the race; therefore, no times are given for them.

When the race results were examined, the Judging Committee disagreed over which team had actually won the race. One judge thought that RMS was the winner, because a runner from that school finished first. Other judges disagreed, citing a variety of reasons.

The Judging Committee has contracted your consulting firm to devise a way to determine the overall winning team. You will need to present a method to them tomorrow that you think is the best and fairest way of determining the overall winner. You must support your method with solid reasoning and mathematics. Your presentation should not only describe the best method, but should also include reference to other methods and a description of why your chosen method is preferred.

Your consulting firm consists of four people. In the past, you have found the following procedure works best:
• Individuals work alone for 10 minutes, brainstorming ideas and making initial calculations and recommendations.
• Pairs form, compare initial ideas, and then brainstorm more ideas.
• The whole firm comes together to discuss results of the work, then makes a final decision, and plans the presentation.

<table>
<thead>
<tr>
<th>Within Team Rank</th>
<th>JMS Overall Finish</th>
<th>LMS Overall Finish</th>
<th>RMS Overall Finish</th>
<th>WMS Overall Finish</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>6</td>
<td>1</td>
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<td>71</td>
<td>–</td>
<td>–</td>
<td>73</td>
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<tr>
<td>20</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>74</td>
</tr>
</tbody>
</table>
Lessons to Extend the Mathematics Standards of Learning
13. Brief Introduction to Trigonometry

Objective
Students will create similar triangles to investigate basic trigonometric relationships, evaluate the three basic trigonometric ratios for acute angles in right triangles, use the calculator to evaluate sine, cosine, and tangent for a given angle, and determine the relationship between the "co-" functions.

Summary
Students discover various trigonometric relationships. The activity introduces them to the sine, cosine, and tangent ratios. The lesson also involves similar triangles and illustrates that the values of the trigonometric ratios are independent of the size of the triangle.

Prerequisite SOL
6.2, 7.11, 8.6, 8.10

Materials
Copies of attached worksheet; graph paper; rulers; protractors; and scientific calculators

Procedure
1. Have students follow the instructions on the worksheet. Answer questions as needed.
2. You may want to follow up this exercise by rotating the triangles in space (so that the triangles in question do not always have a standard angle position) to illustrate that the relationships are true regardless of orientation in space.
3. You may also want to lead a discussion that tests students' number sense by investigating the values that are possible for sine and cosine in a right triangle. (0 < value < 1) Since the hypotenuse has to be the longest side of the right triangle, the denominator has to be larger than the numerator.
4. Continue this discussion by investigating the tangent values. (value > 0)
**Brief Introduction to Trigonometry Worksheet**

Name: ____________________________ Date: ______________________

On graph paper, draw a right triangle in which one leg is 1 unit long and the other leg is 1.5 units long. Label the points as shown at right.

1. Use a protractor to measure $\angle CAB$. __________
2. Use a protractor to measure $\angle ACB$. __________

Extend the leg $\overline{AB}$ by an amount equal to its length and call the new endpoint $D$, as shown at right. Make $DE$ twice as long as $BC$. Draw $CE$.

3. Use a protractor to measure $\angle AED$. _______
4. What is the measure of $\angle EAD$? _______
5. What is the measure of $\angle EDA$? _______
6. What is true about the two triangles that you have drawn?
   _______________________________________________
   _______________________________________________

Extend the leg $\overline{AD}$ by an amount equal to the length of $\overline{AB}$, and call the new endpoint $F$, as shown at right.

7. How long will you have to make $\overline{FG}$? _________
8. Measure $\angle AGF$ with a protractor. _________
9. What is true about the three triangles you have drawn?
   _______________________________________________
   _______________________________________________
   _______________________________________________
The **tangent** of an acute angle in a right triangle is defined as the **ratio of the length of the leg opposite it to the length of the leg adjacent to it**.

10. Find the tangent of $\angle$CAB in $\triangle$CAB. _____________
11. Find the tangent of $\angle$EAD in $\triangle$EAD. _____________
12. Find the tangent of $\angle$GAF in $\triangle$GAF. _____________
13. What does this tell you about the tangent of an angle relative to the size of the triangle to which it belongs?
14. Use the calculator to find the tangent of $\angle$CAB. _____________
15. Use the Pythagorean Theorem to find the length of $\overline{AC}$ _______. Of $\overline{AE}$ _______. Of $\overline{AG}$ _______.

The **sine** of an angle is defined as the **ratio of the length of the leg opposite it to the length of the hypotenuse**.

16. Find the sine of $\angle$CAB in $\triangle$CAB. _____________
17. Find the sine of $\angle$EAD in $\triangle$EAD. _____________
18. Find the sine of $\angle$GAF in $\triangle$GAF. _____________
19. What does this tell you about the sine of an angle relative to the size of the triangle to which it belongs? ______________________________________________________
20. Use the calculator to find the sine of $\angle$CAB. _____________
21. Find the sine of $\angle$ACB in $\triangle$CAB. _____________
22. Find the sine of $\angle$AED in $\triangle$EAD. _____________
23. Find the sine of $\angle$AGF in $\triangle$GAF. _____________
24. What does this tell you about the sine of an angle relative to the size of the triangle to which it belongs?
25. Use the calculator to find the sine of $\angle$ACB. _____________

The **cosine** of an angle is defined as the **ratio of the length of the leg adjacent to it to the length of the hypotenuse**.

26. Find the cosine of $\angle$CAB in $\triangle$CAB. _____________
27. Find the cosine of $\angle$EAD in $\triangle$EAD. _____________
28. Find the cosine of $\angle$GAF in $\triangle$GAF. _____________
29. What does this tell you about the cosine of an angle relative to the size of the triangle to which it belongs? ______________________________________________________
30. Use the calculator to find the cosine of $\angle$CAB. _____________
31. Find the cosine of $\angle$ACB in $\triangle$CAB. _____________
32. Find the cosine of $\angle$AED in $\triangle$EAD. _____________
33. Find the cosine of $\angle$AGF in $\triangle$GAF. _____________
34. What does this tell you about the cosine of an angle relative to the size of the triangle to which it belongs? ______________________________________________________
35. Use the calculator to find the cosine of ∠ACB. _________

36. What is true about \( \sin ∠CAB \) and \( \cos ∠CAB \)? ________________________________
   About \( \sin ∠EAD \) and \( \cos ∠AED \)? ________________________________
   About \( \sin ∠GAF \) and \( \cos ∠AGF \)? ________________________________

37. What is true about \( m∠CAB \) and \( m∠ACB \)? ________________________________

38. The prefix \( \text{co} \) refers to a relationship between the sine and the cosine ratios. Investigate the measurements of the angles, and make a conjecture about what \( \text{co} \) refers to in this context.

39. Test your hypothesis by selecting angles that satisfy the relationship and checking them with your calculator.
14. Building Towers

Objective  Students will expand upon the basic counting principle to find the number of arrangements, using factorials.

Summary  Students use colored cubes to find how many ways they can be arranged. From a chart that the class creates, students work to discover the meaning of the term factorial.

Prerequisite SOL  7.15

Materials  Five different-colored cubes for each student; scientific calculators

Procedure  
1. Give each student a blue cube. Ask them how many different color orders they could have when building a tower with this one cube. (The answer is 1.)
2. Give each student a second color cube (the same for all). Ask how many different color orders they could have when building a tower with these two cubes. (The answer is 2.)
3. Give them a third color cube. Now, it’s more difficult, so ask students to build towers with the three different color cubes and make all of the possible orders of colors. How many different color orders could there be? (The answer is 6.)
4. Give them a fourth color cube. Have the students walk around with their cubes arranged differently so that they can see all of the possible combinations. (If there are fewer than 24 students in the class, you will have to have some students holding two towers.) (The answer here is 24.)
5. Give them a fifth color cube, and have them guess from their seats how many different orders they could have.
6. Stop them in a reasonable amount of time and put up a chart on the board or overhead with the information collected:
   1 cube — 1 order
   2 cube — 2 orders
   3 cube — 6 orders
   4 cube — 24 orders
7. Discuss any patterns they may see. If they do not see any patterns, say: “Look at the combination of four colors:
   How many choices did you have for the first cube? (4)
   How many for the second? (3)
   How many for the third? (2)
   How many for the fourth? (1)
   Use the Basic Counting Principle to determine the possible arrangements:
   $4 \times 3 \times 2 \times 1 = 24$
   Check for the three- and two-block towers:
   $3 \times 2 \times 1 = 6 \quad 2 \times 1 = 2$
8. Define factorial for students, and discuss its uses.
9. Have students practice finding factorials, using a calculator:

Keystrokes for 4!:

**TI30Xa-SEVA**

4 2nd x!

**Casio fx-260 School**

4 Shift x!

**Sharp EL-501V and EL-501WBBK**

4 2nd F n!

10. Have students complete the chart on the board for combinations of up to 10 different colored cubes, using a calculator.
15. Candy? Probably

Objective  Students will determine the difference between combinations and permutations, determine the probability of a future event based on experimental data, list the possible arrangements of various objects in combination and permutation, and use the calculator to determine the values of combinations and permutations.

Summary  Students investigate probabilities and determine the difference between combinations (where order is unimportant) and permutations (where order is significant), using M&M’s™ and Skittles™.

Prerequisite SOL  6.20b, 7.15, 7.18, 8.11

Materials  M&M’s™, Skittles™, copies of attached worksheet

Procedure  
1. Give each student a sample of M&M’s™ and a sample of Skittles™. Have them count the candies and tally the results. Then, have them follow the procedures outlined on the worksheet to investigate the probabilities, combinations, and permutations. (The candies can be used as manipulatives in creating the combinations and permutations for those who have difficulty determining them symbolically.)

Keystrokes: 
**TI30Xa-SEVA**

Permutations for 5 choose 4:  

\[
5 \text{ 2nd } nPr 4 =
\]

Combinations for 5 choose 4:  

\[
5 \text{ 2nd } nCr 4 =
\]

**Casio fx-260 School**

Permutations for 5 choose 4:  

\[
5 \text{ Shift } nPr 4 =
\]

Combinations for 5 choose 4:  

\[
5 \text{ Shift } nCr 4 =
\]

**Sharp EL-501V and EL-501WBBK**

Keys are not available on these models.

2. Follow up with discussion about other examples of combinations and permutations.
Candy? Probably
Worksheet

Name: _______________________________  Date: __________________________

Count the number of candies of each color you have in your sample, and record the data below.

<table>
<thead>
<tr>
<th>M&amp;M’s™:</th>
<th>Brown</th>
<th>Red</th>
<th>Orange</th>
<th>Yellow</th>
<th>Green</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Skittles™:</th>
<th>Red</th>
<th>Orange</th>
<th>Yellow</th>
<th>Green</th>
<th>Purple</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Probabilities
1. If you randomly select a candy, what color are you most likely to get? ______
2. If you randomly select one M&M™, what is the probability that you will get a red one? ______
3. If you randomly select one M&M™, what is the probability that you will get a blue one? ______
4. If you randomly select one M&M™, what is the probability that it will be a green one? ______
5. If you randomly select one Skittle™, what is the probability that it will be an orange one? ______
6. If you randomly select one Skittle™, what is the probability that you will get a purple one? ______
7. If you randomly select one candy, what is the probability that you will get a yellow one? ______
8. If you randomly select one candy, what is the probability that you will get an orange one? ______
9. If you randomly select one candy, what is the probability that you will get a green one? ______
10. If you randomly select a candy and it is blue, what is the probability that it is an M&M™? ______
11. If you randomly select a candy and it is green, what is the probability that it is a Skittle™? ______
12. If you randomly select a candy and it is yellow, what is the probability that it is an M&M™? ______

Combinations
13. How many ways can you get a group of 4 M&M’s™ from among the 6 colors? ______ List the various groups below.

14. How many ways can you get a group of 3 Skittles™ from among the 5 colors? List the groups below.
In the previous two exercises, the order in which you arranged the colors did not matter: Red-Yellow-Green and Green-Yellow-Red are considered the same combination. Your calculator can evaluate combinations, using the $nC_r$ key. Evaluate 5C3 from #14 above. Go back to your list of combinations to see if you forgot any. If you did, include them below.

**Permutations**

Suppose, when you gathered up the candies, the order in which you selected them made a difference so that Red-Yellow-Green is a different group from Yellow-Red-Green. This is called a permutation. (In fact, your locker combination is actually misnamed, since you have to dial the numbers in a particular order; you can’t just dial the three digits in any order. Technically, it should be called a locker “permutation.”)

15. How many permutations are possible using 2 colors from among the 5 Skittles™ colors? _____
   List them below.

16. Your calculator can evaluate permutations, using the $nP_r$ key. Evaluate 5P2: _____
17. Go back and examine your list to see if you forgot any. If you did, find them and include them below.

18. How many permutations are possible using 4 colors from among the 5 Skittles™ colors? _____
19. How many permutations are possible using 5 colors from among the 6 M&M’s™ colors? _____
16. Factorial (!) or Fiction?

Objective
Students will use the factorial key on the calculator to determine the values of 0! through 12!. They also will examine the factorial values to determine the definition of \( n! \) and evaluate various expressions involving factorials, using the definition.

Summary
Students generate a chart of the values of the factorials for 0 through 12 and then discover the formula for \( n! \).

Prerequisite SOL
6.21, 7.19, 8.14

Materials
Scientific calculators capable of calculating factorials through 12; copies of attached worksheet

Procedure
1. Instruct students on how to use the factorial key on their calculators.
   Keystrokes for finding 6!:
   - TI30Xa-SEVA
     \[ 6 \ 2^{nd} \ x! \]
   - Casio fx-260 School
     \[ 6 \ \text{Shift} \ x! \]
   - Sharp EL-501V and EL-501WBBK
     \[ 6 \ 2^{nd} F \ n! \]

2. You may need to lead the students through the beginnings of the patterns to demonstrate how to arrive at the formula \( n! = n(n – 1)! \)

3. Have students follow the instructions on the accompanying worksheet.

Answer key
Answers to #5 on the worksheet: a. 10! b. 6! c. 3! d. 12! e. 9! f. 4! g. 19! h. 7!

<table>
<thead>
<tr>
<th>Number (n)</th>
<th>n!</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1 x 1</td>
</tr>
<tr>
<td>2</td>
<td>2 x 1</td>
</tr>
<tr>
<td>3</td>
<td>6 x 2 x 1</td>
</tr>
<tr>
<td>4</td>
<td>24 x 3 x 2</td>
</tr>
<tr>
<td>5</td>
<td>120 x 4 x 6</td>
</tr>
<tr>
<td>6</td>
<td>720 x 5 x 24</td>
</tr>
<tr>
<td>7</td>
<td>5040 x 6 x 120</td>
</tr>
<tr>
<td>8</td>
<td>40320 x 7 x 720</td>
</tr>
<tr>
<td>9</td>
<td>362880 x 8 x 5040</td>
</tr>
<tr>
<td>10</td>
<td>3628800 x 9 x 40320</td>
</tr>
<tr>
<td>11</td>
<td>39916800 x 10 x 3628800</td>
</tr>
<tr>
<td>12</td>
<td>479001600 x 11 x 36288000</td>
</tr>
</tbody>
</table>

Virginia Department of Education 41
Factorial (!) or Fiction?
Worksheet

Name: __________________________ Date: __________________

1. Use the factorial key on your calculator to complete the chart below.

<table>
<thead>
<tr>
<th>Number (n)</th>
<th>n!</th>
<th>n! as a product of at most three other factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1 x 1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2 x 1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3 x 2 x 1</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. In the third column above, rewrite each n! value as the product of at most three other factors as suggested by the entries provided.

3. In the last column, rewrite each n! value as a product of one integer and another n! value.

4. Given your answers above and the fact that 0! = 1, write the formula for n!

   \[ n! = \] __________________________

5. Evaluate each of the following:
   a. \(10 \times 9!\)  
   b. \(6 \times 5!\)  
   c. \(8 \times 7!\)  
   d. \(14 \times 13!\)  
   e. \(10! \div 10\)  
   f. \(5! \div 5\)  
   g. \(20! \div 20\)  
   h. \(9! \div (9 \times 8)\)
17. **Half Some Candy**

**Objective**
Students will gather data and represent it in a chart and in a graph, use the calculator to evaluate exponential equations, and compare experimental and theoretical data.

**Summary**
Students use the calculator to find the values of the functions $y = 2^x$ and $y = 0.5^x$ and then plot the points to create the graphs of the two functions. They then use M&Ms™ or Skittles™ to simulate a half-life experiment that follows the mathematical model given by $y = 0.5$

**Prerequisite SOL**
7.14, 8.1, 8.14, 8.18

**Materials**
M&M’s™ or Skittles™; small paper drinking cups; scientific calculators, copies of attached worksheet

**Procedure**
1. Begin with a review of equations and their graphs, using the worksheet.
2. Have the students complete #1 and #2 on the worksheet as examples of nonlinear graphs. Have a class discussion of how linear equations differ in form from nonlinear equations.

**Keystrokes:**

*TI30Xa-SEVA*

- $2^x$ where $x = 2$
- $2^x$ where $x = 3$
- $2^x$ where $x = 4$

- $2 \ y^x \ 2 \ \Rightarrow$
- $2 \ y^x \ 3 \ \Rightarrow$
- $2 \ y^x \ 4 \ \Rightarrow$

*Casio fx-260 School*

Because the key on the Casio calculator is labeled $x^y$, you may want to alter the letters used as variables on the worksheet.

- $2^y$ where $y = 2$
- $2^y$ where $y = 3$
- $2^y$ where $y = 4$

- $2 \ x^y \ 2 \ \Rightarrow$
- $2 \ x^y \ 3 \ \Rightarrow$
- $2 \ x^y \ 4 \ \Rightarrow$
Sharp EL-501V and EL-501WBBK

$2^x$ where $x = 2$

$2^x$ where $x = 3$

$2^x$ where $x = 4$

\[
\begin{array}{c}
2 \quad 2 \quad = \\
2 \quad 3 \quad = \\
2 \quad 4 \quad = \\
\end{array}
\]

3. Distribute the candies and cups. Have students read the procedure in step #4 on the worksheet. Answer any questions to help clarify the procedure before letting the students continue with the remainder of the activity.

4. If students have not been exposed to the concept of half-life, one definition is “the period of time it takes for half of a radioactive substance to decay.” (Consider consulting with the eighth-grade science teacher on this lesson for a math/science connection.)

5. End the lesson with a class discussion of question #10 from the worksheet and the difference between experimental and theoretical values.

Extension
If you wish, you could expand this concept to include the topics of domain and range, and independent and dependent variables.
Half Some Candy
Worksheet

Name: ____________________________ Date: __________________

1. Use the calculator to find the values of the function \( y = 2^x \). Complete the table of values, and then draw the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

2. Use the calculator to find the values of the function \( y = 0.5^x \). Complete the table of values, and draw the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 0.5^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
3. Record the number of candies you have in your sample. _______

4. Shake the cup and gently toss the candies out onto a piece of paper. Record the number of candies that landed “face up” (that is, with the letter of the candy showing). _______ Those that landed “face down” will not be used again. (You may just have to eat those! ☺)

5. Complete the table below, and then plot the data values at right.

<table>
<thead>
<tr>
<th>Toss #</th>
<th>Candies Left</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

6. Which of the graphs in steps #1, #2, or #5 does your model most resemble? _______

7. A “perfect” half-life model follows the equation \( y = A_0(1/2)^x \), where \( A_0 \) equals the number in the original sample.

   Write the perfect half-life equation for your sample. _____________________

8. Use your calculator to evaluate the perfect half-life values for your candy sample. Complete the chart, and graph the perfect half-life curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = A_0(1/2)^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
9. How do your actual values compare to the perfect half-life situation?

10. Write a paragraph to explain why you think your experimental values do not match the theoretical, perfect half-life equation values. What about the situation makes it nearly impossible to re-create the perfect equation values? What other sources of error might there be?
18. Way Off Base!

Objective  Students will change numerals from base 10 to other bases.

Summary  Students change numbers from base 10 to other bases, namely 2 (the binary system) and 8 (octal). More advanced students may also convert numbers to base 16 (hexadecimal, a system that uses the digits 0 through 9 and the letters A through F).

Prerequisite SOL  8.1, 8.5

Materials  Scientific calculators, copies of attached worksheet

Procedure  1. Using expanded notation and numbers through 100,000, review with students why our number system is a called a base 10 system.

2. Compare our base 10 system to a base 2 system, using the expanded notation format. Point out to students how a chart, such as the one they are about to complete, can help in converting numbers to different bases. Additional information, such as the way the binary system is the language of computers, is important for students to see relevance in the activity.

3. Once the students have a fundamental understanding of bases, allow them to work individually, in pairs, or in small groups to complete the activity.

Keystrokes for calculating $2^5$:

**TI30Xa-SEVA**

$$2 \yx 5 =$$

**Casio fx-260 School**

$$2 \xy 5 =$$

NOTE: If using the *Casio fx-260 School*, you may want to change the exponent on the worksheet to $y$.

**Sharp EL-501V and EL-501WBBK**

$$2 \yx 5 =$$

4. The values for the initial chart are as follows:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$2^x$</th>
<th>$8^x$</th>
<th>$10^x$</th>
<th>$16^x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>8</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>64</td>
<td>100</td>
<td>256</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>512</td>
<td>1,000</td>
<td>4,096</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>4,096</td>
<td>10,000</td>
<td>65,536</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>32,768</td>
<td>100,000</td>
<td>1,048,576</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
<td>262,144</td>
<td>1,000,000</td>
<td>16,777,216</td>
</tr>
</tbody>
</table>
Answer key
The answers to the questions on the worksheet are as follows:

2. a. 111001  b. 1000111  c. 1000  d. 100111  e. 10001
3. a. 71  b. 107  c. 1102  d. 47  e. 144
4. a. 6  b. 13  c. 22  d. 55  e. 91
5. a. 6  b. 15  c. 26  d. 67  e. 133
6. a. 36  b. 79  c. 12C  d. 1388  e. 15D77
Way Off Base!
Worksheet

Name: ____________________________ Date: ____________________________

1. Using the \( y^x \) key on your calculator, determine the values of each of the following:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2^x )</th>
<th>( 8^x )</th>
<th>( 10^x )</th>
<th>( 16^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When we write our numbers, we use a base 10 system. Each digit represents the number of a specific power of 10 the digit contains. For example, the number 28 means that the value of the number is

\[
(2 \times 10^1) + (8 \times 10^0)
\]

\[
(2 \times 10) + (8 \times 1)
\]

\[
20 + 8
\]

We could write our numbers using a different base system but using the same logic. For example, in a base 2 system, 28 is

\[
(1 \times 16) + (1 \times 8) + (1 \times 4)
\]

\[
(1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)
\]

So in base 2, 28 is 11100.

2. How would you write the following base 10 numbers in base 2? Verify on your calculator.
   a. 57     b. 71     c. 8     d. 39     e. 17

3. How would you write the following base 10 numbers in base 8? Verify on your calculator.
   a. 57     b. 71     c. 578    d. 39     e. 100

4. How would you write the following base 2 numbers in base 10? Verify on your calculator.
   a. 110    b. 1101   c. 10110  d. 110111 e. 1011011

5. How would you write the following base 2 numbers in base 8? Verify on your calculator.
   a. 110    b. 1101   c. 10110  d. 110111 e. 1011011

6. Convert the following base 10 numbers to base 16. Verify on your calculator.
   a. 54     b. 121    c. 300    d. 5,000  e. 89,463

   NOTE: In base 16, once you use the digits 0 through 9, you then use the letters A through F; therefore, 31 is (1 \( \times \) 16\(^0\)) + (15 \( \times \) 16\(^0\)). Since there is no single digit for 15, use the letter F (10 = A, 11 = B, 12 = C, 13 = D, 14 = E, 15 = F); therefore, 31 = 1F.
Lessons to Reinforce/Extend the Grade 8 Science Standards of Learning
19. Demonstration on Inclined Planes

Objective Students will determine the mechanical advantage, work input and output, and efficiency of a simple machine.

Summary Students collect data, using three different types of ramps, calculate work input, output, and percent efficiency for each, and compare the ramps.

Related SOL PS.1, PS.10

Materials A board to represent an inclined plane; sandpaper that can be taped onto the board; wax paper that can be taped onto the board; a wooden block with an eyelet that can be attached to the spring scale and pulled across the board; masking tape; spring scale incremented in Newtons, (one incremented in grams may be used, but then grams must be converted to kilograms, and kilograms to Newtons by multiplying by 9.8); meter stick; copies of attached worksheet; scientific calculators

Prior knowledge needed
• An understanding of the terms work, work input, work output, mechanical advantage, and percent efficiency
• An understanding of the role friction plays in opposing motion
• The ability to use and read a spring scale accurately
• The ability to recognize and exclude outliers
• The ability to round data

Procedure
1. Distribute worksheets, and have the students read the problems.
2. Demonstrate the keystrokes to apply the formula M.A. = length/height.
3. Demonstrate the keystrokes to apply the formula \( W_o = W_i = Fd \), where \( W_o \) stands for work output, \( W_i \) stands for work input, \( F \) stands for force in Newtons, and \( d \) stands for distance in meters.
4. Do the demonstration, as described on the worksheet. You may wish to ask students to volunteer to help with the demonstration.
5. Demonstrate keystrokes to find percent efficiency, using 0.24 for the work output and 0.35 for the work input.

Keystrokes:

Casio fx-260 School

\[
0.24 \div 0.35 \text{ Shift } \% 
\]

Sharp EL-501V and EL-501WBBK

\[
0.24 \div 0.35 \text{ 2nd } F \% 
\]
Demonstration on Inclined Planes
Lab sheet

Name: _____________________________ Date: __________________

Matching
Match the terms with the definitions:
1. _____ Mechanical advantage of an inclined plane a. Work output
2. _____ The formula for percent efficiency b. M.A. = length/height
3. _____ The work done by a simple machine c. Work input
4. _____ The work done on a simple machine d. \( \frac{W_o}{W_i} \times 100 \)

Materials
A board to represent an inclined plane; sandpaper that can be taped onto the board; wax paper that can be taped onto the board; a wooden block with an eyelet that can be attached to the spring scale and pulled across the board; masking tape; spring scale incremented in Newtons, (one incremented in grams may be used, but then grams must be converted to kilograms, and kilograms to Newtons by multiplying by 9.8); meter stick; copies of attached lab sheet; scientific calculator

Procedure
1. Attach the wooden block to the spring scale through the eyelet in the block. Hold up the spring scale so that the block hangs freely, and read the scale. Record this number in the output force box in the chart below. If you do not change the block you are using, the output force will stay the same for all three ramps.
2. Using the meter stick, measure the length of the ramp and the height of the ramp. Record these numbers in meters in the data chart.
3. Place the block at the bottom of the wooden inclined plane. Attach the spring scale to the block. With a consistent force, pull the block up the plane. Read the amount of force it takes to do this; this is the input force. Record this in the data chart as the Trial 1 input force.
4. Repeat step 3 two more times, and record the data.
5. Average your results. Round to the nearest tenth, and record the average input force.
6. Repeat steps 3 through 5, using the sandpaper- and the wax-paper-covered ramps.
<table>
<thead>
<tr>
<th>Ramp Description</th>
<th>Ramp Length</th>
<th>Ramp Height</th>
<th>Input Force</th>
<th>Work Input</th>
<th>Output Force</th>
<th>Work Output</th>
<th>% Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain Wood</td>
<td>T1:</td>
<td></td>
<td>T2:</td>
<td>T3:</td>
<td>Avg:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T1:</td>
<td></td>
<td>T2:</td>
<td>T3:</td>
<td>Avg:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T1:</td>
<td></td>
<td>T2:</td>
<td>T3:</td>
<td>Avg:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sandpaper</td>
<td></td>
<td></td>
<td>T1:</td>
<td>T2:</td>
<td>T3:</td>
<td>Avg:</td>
<td></td>
</tr>
<tr>
<td>Wax Paper</td>
<td></td>
<td></td>
<td>T1:</td>
<td>T2:</td>
<td>T3:</td>
<td>Avg:</td>
<td></td>
</tr>
</tbody>
</table>
Observations

Calculations
1. Mechanical advantage = length ÷ height
2. Work input = input force x ramp length
3. Work output = output force x ramp height
4. Percent efficiency = \((W_o ÷ W_i) \times 100\)

Show the work for your calculations by writing the formula, substituting the appropriate terms (including labels) into the formula, solving with your scientific calculator, and then recording your answer with the correct label.

Questions
1. What does the mechanical advantage of a machine tell you?

2. Why is the work input always greater than the work output?

3. If you always put more work into a simple machine than you get out of it, what is the advantage of using a simple machine?

4. How did the ramp surfaces affect the percent efficiency of the simple machine?

5. Can you think of a way to modify the block that would increase the percent efficiency of the ramp? Explain.
20. Lab on Mass and Momentum

Objective
Students will determine the relationships among mass, momentum, and speed.

Summary
Students roll balls made of different materials down a ramp and collect data on their comparative speeds. They also roll each ball down ramps into cups and collect data.

Related SOL
PS.1, PS.10

Materials
Wooden ramp 1 meter long and with toy car track; 4 textbooks; triple beam balance or electronic balance; 4 balls of equal volume but different densities (brass, aluminum, wood, and steel); tape measure; stopwatch; large foam cup; flat space for the balls to roll at least 15 meters (e.g., hallway or gymnasium); copies of attached lab sheet; scientific calculators

Procedure
1. Begin by reviewing the following concepts with the students:
   - Using the formula \( s = \frac{d}{t} \) to find speed
   - Using the formula \( m_0 = mv \) to find momentum
   - An understanding of the difference between distance and displacement
   - The ability to make line and bar graphs
   - The ability to recognize and exclude outliers
2. Hand out the lab sheet, and read it over with the class.
3. Have students follow the directions on the lab sheet to conduct the experiment and gather their data.
4. After the data has been collected, go through the lab, showing students how to find speed for Part 1 by using \( s = \frac{d}{t} \) and fixing the decimal to one place. Doing this will help students see that there is no difference in the speed of each ball. Be sure to account for the minor differences due to timing and/or other variables.
5. Have students use the data they collected in Part 2 of the lab to find the average distance that the cup moved, fixing the calculator to one decimal place.
6. Have students make the two graphs and answer the questions on the lab sheet.

Keystrokes, using the sample data for the time to roll one meter:
Trial 1: 1.42 sec; Trial 2: 1.39 sec; Trial 3: 1.44 sec

**TI30Xa-SEVA**

\[ \text{2nd Fix} \quad 1 \]

fixes the calculator to one decimal place.

\[ \begin{align*}
\sum+ & \quad 1.42 \\
\sum+ & \quad 1.39 \\
\sum+ & \quad 1.44 \\
\text{2nd} \quad \bar{x} & \quad 1.4
\end{align*} \]

NOTE: If you use the On/AC button to clear the screen, you will lose your fixed places. You will no longer see the word fix on the LCD. You should use the CE/C button to clear the screen and maintain your fixed places.
Casio fx-260 School

\[\text{mode} \ 7 \ 1\] fixes the calculator to one decimal place.

\[1.42 \ \text{Data} \ 1.39 \ \text{Data} \ 1.44 \ \text{Data}\] enters all the data.

\[\text{mode}\] takes you out of fix mode.

Sharp EL-501V and EL-501WBBK

\[2^{\text{nd}} \ F \leftrightarrow E \ 1\] fixes the calculator to one decimal place.

\[1.42 \ \text{Data} \ 1.39 \ \text{Data} \ 1.44 \ \text{Data}\] enters all the data.

\[2^{\text{nd}} \ F \leftrightarrow E \] takes you out of fix mode.
Lab on Mass and Momentum
Lab sheet

Name: __________________________ Date: __________________

Part 1: The effect of the mass of an object on its speed rolling down a ramp

Do you believe that heavier objects roll faster down a ramp? _____ Explain, and make a hypothesis:

Materials
Wooden ramp 1 meter long and with toy car track; 4 textbooks; triple beam balance or electronic balance; 4 balls of equal volume but different densities (brass, aluminum, wood, and steel); tape measure; stopwatch; large foam cup; flat space for the balls to roll at least 15 meters (e.g., hallway or gymnasium); copies of attached lab sheet; scientific calculators

Procedure
1. Make sure your ramp is elevated to a height of 4 textbooks (about 14 cm).
2. Place the first ball on the ramp on the toy car track, and allow it to roll 1 meter down the ramp.
3. Time how long it takes the ball to roll 1 meter, and record the time in the chart below.
4. Repeat steps 2 and 3 twice more with the first ball. Find and record the average time for the three trials with the first ball.
5. Repeat steps 2 through 4, using the remaining balls.
6. Use the balance to find the mass of each ball. Record this data.

<table>
<thead>
<tr>
<th>Ball Type</th>
<th>Mass of Ball (g)</th>
<th>Time to Roll (sec)</th>
<th>Average Time (sec)</th>
<th>Average Speed ( S = \frac{d}{t} ) (show work)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td></td>
<td>T1:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T2:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T3:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td></td>
<td>T1:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T2:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T3:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td></td>
<td>T1:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T2:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T3:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brass</td>
<td></td>
<td>T1:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T2:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T3:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusion
How did an increase in mass affect the speed of a ball?
Part 2: The effect of the mass of the ball on the distance it will push a cup.

Do you believe that the more mass an object has, the farther it will push a cup? _____ Explain and make a hypothesis:

Procedure
1. Cut a large foam cup in half from top to bottom, and place one half (cut side down) on the floor at the bottom of the ramp so that it is propped on the ramp.
2. Allow one of the balls to roll down the ramp and enter the cup.
3. Measure the distance in meters that the cup moves from the ramp. (Be sure to measure the distance the cup travels, not its displacement.)
4. Make two additional trials per ball, and record the data. Find and record the average distance for each ball.

<table>
<thead>
<tr>
<th>Ball</th>
<th>Distance Cup Moved (m)</th>
<th>Average Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>T1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T2:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T3:</td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>T1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T2:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T3:</td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>T1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T2:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T3:</td>
<td></td>
</tr>
<tr>
<td>Brass</td>
<td>T1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T2:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T3:</td>
<td></td>
</tr>
</tbody>
</table>

Conclusion
How did the mass of the ball affect the distance the cup traveled?

Graphing
1. Make a line graph of the mass of each ball vs. the average speed of each ball.
2. Make a line graph of the mass of each ball vs. the average distance each ball moved the cup.

Questions
1. There are two identical trucks, but one has a full load and the other is empty. Which one will roll faster down the same hill? ______________ Explain.

2. How is the experiment you did in Part 2 related to the formula for momentum (mass x velocity = momentum)?
21. Lab on Potential Energy

Objective  Students will determine the loss of potential energy on an object.

Summary  Students conduct an investigation on the transfer of potential to kinetic energy in a bouncing ball. They use a scientific calculator to average data.

Related SOL  PS.1, PS.5, PS.6, PS.10

Materials  Scientific calculators; a ball that bounces; meter stick; graph paper

Procedure
1. Begin by reviewing the following concepts with the students:
   • Introduction to potential and kinetic energy
   • An understanding of the Law of Conservation of Energy and how friction affects motion
   • Knowledge of statistics terms: \( n \) value, summation, and \( x \) bar (The advantage of this in science is the ability to add in new data and the ability to see how many pieces of data have been entered.)
2. Drop the ball from a specified height, and ask the class why the ball never seems to bounce back to the height from which it was dropped. Students should discuss energy conversions like sound or waste due to friction.
3. Hand out lab sheets, and have students read them.
4. Hand out materials, and have students gather data. (You might wish to give different groups different types of balls and later discuss the difference this makes.)
5. Demonstrate calculator keystrokes for finding the average (see below).
6. Have students complete the lab.

Keystrokes (for example):

<table>
<thead>
<tr>
<th>Drop Height</th>
<th>Bounce Height Trial 1</th>
<th>Bounce Height Trial 2</th>
<th>Bounce Height Trial 3</th>
<th>( \sum x )</th>
<th>( \bar{x} )</th>
<th>Bh</th>
<th>Dh</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 m</td>
<td>0.39 m</td>
<td>0.39 m</td>
<td>0.36 m</td>
<td>1.14</td>
<td>0.38</td>
<td>.38/.50</td>
<td>0.76</td>
<td>76%</td>
<td></td>
</tr>
</tbody>
</table>

TI30Xa-SEVA

0.39 \[\sum^+\] 0.39 \[\sum^+\] 0.36 \[\sum^+\] puts the three terms into the calculator.

2nd \[\sum^+\] gives the sum of the data.

2nd \[\bar{x}\] gives the mean.

2nd \[n\] gives the number of terms entered.
Casio fx-260 School
All Statistics functions are in blue above the keys indicated.

Enter stats mode: \( \text{Mode} \) .

0.39 Data 0.39 Data 0.36 Data enters all the data.

\( \text{Shift} \ \sum + \) gives the sum of the data.

\( \text{Shift} \ \bar{x} \) gives the mean.

\( \text{Shift} \ n \) gives the number of terms entered.

Exit stats mode: \( \text{Mode} \ 0 \)

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Enter stats mode: \( \text{2nd} \text{ On/C} \)

0.39 Data 0.39 Data 0.36 Data enters all the data.

\( \text{2nd} \ \sum + \ \bar{x} \) gives the sum of the data.

\( n \) gives the number of terms entered. It is in blue and does not require \( \text{2nd} \) key.

Exit stats mode: \( \text{2nd} \text{ On/C} \)
Lab on Potential Energy
Lab sheet

Name: ________________________________ Date: _______________________

Topic
The effect of drop height on bounce height of a ball

Problem
Does changing the drop height of a ball affect the percent of energy lost on making one bounce?

Purpose
To determine how much potential energy a ball loses on making one bounce

Materials
Scientific calculators; a ball that bounces; meter stick; graph paper

Procedure
1. Drop a ball from the heights shown in meters in column 1 of the table below. After each drop, measure the height the ball reaches after bouncing on the floor, and record the bounce height.

2. For each drop height, do three trials and find the average. Use your scientific calculator to do this, and record your Σx value, your n value, and then your average (x̄).

3. Using the average for each height, find the ratio of the bounce height to the drop height (Bh/Dh).

4. Divide the average bounce height by the drop height to put the ratio into decimal form.

5. Convert each decimal to a percent. This tells you the percent of the original drop height that the bounced ball reaches. Graph these results on a line graph with drop heights on your x-axis and percent of original height on the y-axis.

<table>
<thead>
<tr>
<th>Drop Height</th>
<th>Bounce Height Trial 1</th>
<th>Bounce Height Trial 2</th>
<th>Bounce Height Trial 3</th>
<th>Σx</th>
<th>n</th>
<th>x̄</th>
<th>Bh/Dh</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusion
Does the height of the drop affect the amount of energy lost on a bounce? _____ Explain.
Questions
1. Why should you use percent rather than the actual bounce height?

2. List at least two improvements you could make in this lab.

3. If you dropped a ball from a height of 10 meters, predict how high it would bounce, based on the information you got from your experimentation.
22. Investigating the Pendulum

Objective
Students will determine the effect of string length on the period of a pendulum.

Summary
Students investigate using five different lengths of string for a pendulum and calculate the percent error.

Related SOL
PS.1, PS.6

Materials
One meter stick per group of students; 110 cm common household string; protractor; peg stand with peg; 100 g mass; stopwatch; scissors; graph paper; scientific calculators; copies of attached lab sheet

Procedure
1. Begin by reviewing the following concepts with the students:
   • An understanding of potential and kinetic energy
   • An understanding of the Law of Conservation of Energy
2. Hand out lab sheets, and have the students read them.
3. Explain the formula for calculating theoretical time \( T = 2\pi \sqrt{\frac{L}{g}} \), and then demonstrate the keystrokes, if necessary.
4. Have the students follow the directions on the lab sheet.
5. Demonstrate keystrokes to find percent error, if necessary.

Keystrokes:
For procedure #2, using a value of 0.5 m for \( L \) (length), \( \pi \) for \( \pi \), and 9.8 m/sec\(^2\) for the value of \( g \) (gravity).

*TI30Xa-SEVA*

\[
2 \times \pi \times \left( \frac{0.5}{9.8} \right) \sqrt{x} =
\]

*Casio fx-260 School*

\[
2 \times \text{Shift} \times \pi \times \left( \frac{0.5}{9.8} \right) \text{Shift} \sqrt{x} =
\]

*Sharp EL-501V and EL-501WBBK*

\[
2 \times 2^{nd} F \times \pi \times \left( \frac{0.5}{9.8} \right) \sqrt{} =
\]
For procedure #5, using the theoretical time as 1.42, and the actual time as 1.48:

**TI30Xa-SEVA**

\[1.42 \div 1.48 \quad 2^{nd} \% =\]

**Casio fx-260 School**

\[1.42 \div 1.48 \quad \text{Shift} \quad \%\]

**Sharp EL-501V and EL-501WBBK**

\[1.42 \div 1.48 \quad 2^{nd} \ F \quad \% =\]
Investigating the Pendulum
Lab sheet

Name: ________________________________ Date: __________________

Topic
The effect of the length of the string on the period of the pendulum

Problem

________________________________________________________________________

Hypothesis

________________________________________________________________________

Materials
One meter stick per group of students; 110 cm common household string; protractor; peg stand with peg; 100 g mass; stopwatch; scissors; graph paper; scientific calculators; copies of attached lab sheet

Procedure
1. Set up the peg stand so that a weight tied to a peg will move freely back and forth when released from a pre-determined height.
2. Attach one end of the 110 cm string to the peg and the other end to the weight (100 g mass) so that the length of the string from peg to weight is 100 cm.
3. Lift the weight to a 45-degree angle. Drop the weight while starting the timer. Time how long it takes the weight to make 10 complete periods (back and forth = 1 period). Record this actual time to the nearest thousandth.
4. Repeat step 3 two more times.
5. Find the average time for 10 periods, and record.
6. Repeat steps 2 through 4, using 80, 60, 40, and 20 cm lengths of string from peg to weight. (Cut the 100 cm string shorter each time).
7. Complete the data chart.

<table>
<thead>
<tr>
<th>String Length in cm</th>
<th>Actual Time for 10 Periods (sec)</th>
<th>Average Time for 10 Periods (sec)</th>
<th>Average Time for 1 Period (sec)*</th>
<th>Theoretical Time for 1 Period (sec)</th>
<th>Percent Error (%)</th>
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</thead>
<tbody>
<tr>
<td>100</td>
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<td>20</td>
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</table>

* To find the average time for 1 period, divide your average time for 10 periods by 10.
Observations

Calculations
1. Calculate the theoretical time for one period by using the formula $T = 2\pi \sqrt{\frac{L}{g}}$, where $T$ = the theoretical time in sec, $L$ = length of string in m, and $g$ = the force of gravity in m/sec$^2$. Round to the nearest hundredth. Don’t forget to convert your string length to meters before you do your calculations!
2. Calculate your percent error by using the formula: % error = theoretical time ÷ actual time x 100. Round to the nearest whole number. Alternatively, you may fix the decimal point to 0 places and have the calculator round.

Graphing
Create a line graph of the length of the string vs. the time for 1 period. Graph two best-fit lines, one that represents your actual data, and the other that represents your theoretical data.

Conclusions

Questions
1. Draw a picture of the lab you did. On the picture, label the following points:
   - Point A — the place on the pendulum where the maximum kinetic energy is found
   - Point B — the place on the pendulum where the maximum potential energy is found
   - Point C — the place on the pendulum where the minimum kinetic energy is found
   - Point D — the place on the pendulum where the minimum potential energy is found
2. Explain how the Law of Conservation of Energy was demonstrated by this lab. Include in your explanation why the drop point of the mass is always higher than the rebound point. If energy is conserved, why does this happen? Where does the energy “go”?
3. Take a look at your percent error. Were you ever 100% accurate? Why, or why not? With what lengths of string were you more accurate? Can you offer any explanation for your findings?