**Institute Objective:**
To improve mathematics instruction by providing district-level trainers with professional development resources focused on facilitating students' mathematical understanding through mathematical problem solving, communication, reasoning and representation.

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<tr>
<th>Time</th>
<th>Facilitators Notes</th>
<th>Materials Needed Additional Notes</th>
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<tr>
<td>10 minutes</td>
<td><strong>Welcome and Introductions:</strong>&lt;br&gt;• Facilitators introduce themselves.&lt;br&gt;• Ask participants to introduce themselves to other people at their table.&lt;br&gt;• Show slide with VDOE Institute Objective (To improve mathematics instruction by providing district-level trainers with professional development resources focused on facilitating students' mathematical understanding through mathematical problem solving, communication, reasoning and representation.)</td>
<td>Computer&lt;br&gt;Projector&lt;br&gt;Screen&lt;br&gt;PowerPoint slides 1, 2</td>
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| 30 minutes | **Problem Solving Using Rich Task: Comparing and Ordering Fractions**<br>Part I (15 minutes) Individual and Partner sort:<br>• The following fractions are on a PowerPoint slide 3 in random order: (1/15, 1/20, 10/11, 8/9, 5/8, 5/7, 3/5, 4/9, ¼, 6/12, 13/11)<br>• Instructions on PowerPoint slides 3, and 4<br>  - Take 5 minutes to put the fractions in order from least to greatest without getting a common denominator or converting to decimals.<br>  - After 5 minutes working alone direct the group to work with a shoulder partner to finish the process and to justify that the fractions are in the correct order.<br>• Facilitators walk around the room and observe the ways that participants are attacking the problem. Ask questions such as: How do you know that 3/5 is greater than 4/9? How do you know that 10/11 is more than 8/9? Facilitators want to listen to see if participants are beginning to use the benchmarks 0, ½, and 1 as well as for other ways of thinking.<br>Part II (10 minutes)<br>• Table groups: PowerPoint slide 5. At your table discuss the big ideas about fractions brought forward by this task or activity. Which fractions were the easiest to
### Facilitators Notes

- Facilitators will listen for ideas such as the denominator determines the size of the parts that make up the whole and the numerator determines how many parts of the whole are being examined; some discussion that provides evidence that participants are thinking, for instance, of 8/9 as eight 1/9s; comparing the distance or area fractions are from the benchmarks of 0, ½, and 1; and why some fractions are more challenging than others.
- Whole Group Closure: What are the big ideas about fractions brought forward by this task or activity?
- If time does not permit transition to next part by saying: “Now we want to look at how students can be actively engaged by tasks such as this one.”

### Mathematical Process Skills—“Student Look-fors”

This activity will introduce the “Student Look-fors” for the Mathematical Process Skills – and relate the Ordering Fractions Task to these Mathematical Process Skills.

**Part I (10 minutes)** To prepare to use the Mathematical Process Skills – “Student Look-fors” the group will first work through becoming familiar with the information on the Mathematics Standards of Learning for Virginia Public Schools, 2009 Handout which shares information about **Virginia’s Mathematics Process Goals** found in the introduction to the Virginia Standards of Learning on pages iv-v.

- Ask participants to count off by 6’s at each table to take responsibility for one of the following. The following are on PowerPoint slide 6
  1. Goals Introductory Paragraph
  2. Mathematical Problem Solving Goal
  3. Mathematical Communication Goal
  4. Mathematical Reasoning Goal
  5. Mathematical Connections Goal
6. Mathematical Representations Goal

- Ask the participants to quickly read their assigned part of the Mathematics Standards of Learning for Virginia Public Schools, 2009 Handout and underline key ideas and to prepare to share the key ideas in their section. (2 minutes to read)
- After 2-3 minutes ask each person at the table to share the key ideas for the part they read beginning with number 1. Each person has about a minute to share. (8 minutes)

Closure for Part I to transition to Part II: (5 minutes)

- Facilitator will now share with the group the Mathematical Process Skills – “Student Look-fors” Handout (example is on slide 7) and tell the group that this is a tool for observing classrooms and for discussing student learning around the Mathematical Process Goals.
- Facilitators will make explicit that the “Student Look-fors” sheet includes the 5 Mathematical Process goals and also captures the information in the opening paragraph about tools and mathematical precision from the Mathematics Standards of Learning for Virginia Public Schools, 2009 Handout.
- Directions on slide 8. Ask the participants to work individually to read over the “Student Look-fors” and then work with a shoulder partner to find the key ideas that came out in your previous table discussion of the Mathematics Process Goals and other information in the Mathematics Standards of Learning for Virginia Public Schools, 2009 Handout.

Part II (15 minutes) Relating the Mathematical Process Skills for Students to the Ordering Fractions Task:

- Facilitator will say: “If a visitor walked into the ordering fractions math activity using the “Student Look-fors” sheet which indicators might they observe?”
- Ask participants to turn to their shoulder partner and discuss what indicators the visitor might have observed and what about the activity or engagement in the
### Problem Solving and the Fraction Tracks Activity:

Note for Facilitator. To prepare participants to think about the Mathematical Process Skills and how to use the “Student Look-fors” tool, participants will engage in the Fraction Tracks Activity that will be used to observe the classroom lesson shown in the video in the next section. There are many versions of the Fraction Tracks Activity and the version being used in this workshop has been adapted from NCTM Illuminations.

Background reading to prepare the Facilitator to complete this activity with teachers.

- Fraction Tracks is an activity that uses a linear model for fractions and consists of seven parallel number lines on a mat and that show the relationships between fractions with the denominators 2, 3, 4, 5, 6, 8, and 10. This set of fraction tracks is like a set of number lines all lined up, one below the other. The top track shows halves, the next track shows thirds, and so on.

- After playing the game the participants will look at a video of a fifth grade classroom engaged in this same activity.

- There are many versions of this game. We will be looking at just one of the ways to play the game during this activity. One different version of the game uses a mat with tracks numbered 0 to 2 and fraction cards with proper and improper fractions.

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<td>10 minutes</td>
<td>activity gave evidence of the particular Mathematical Process Skill? Ask certain tables to begin with certain “Student Look-fors.” (Facilitators listen to conversations and help clarify if there is confusion over directions or focus.) (5 minutes)</td>
<td>Fraction Track Game Board (Will be posted on the website)</td>
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<td>• Whole Group Closure: Facilitator will go through each skill and ask what table can site evidence for an indicator of that skill. Collect comments from 1 or 2 tables based on time remaining to be able to get through all seven skills. There is not discussion here but just a statement of evidence. (10 minutes)</td>
<td>Fraction Cards (Will be posted on the website)</td>
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<td>20 minutes</td>
<td><strong>Problem Solving and the Fraction Tracks Activity:</strong></td>
<td>Beans</td>
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<td>Note for Facilitator. To prepare participants to think about the Mathematical Process Skills and how to use the “Student Look-fors” tool, participants will engage in the Fraction Tracks Activity that will be used to observe the classroom lesson shown in the video in the next section. There are many versions of the Fraction Tracks Activity and the version being used in this workshop has been adapted from NCTM Illuminations.</td>
<td>PowerPoint slides 9 and 10</td>
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<td>Background reading to prepare the Facilitator to complete this activity with teachers.</td>
<td>Video: <em>Fraction Tracks</em> <a href="http://www.learner.org/resources/series33.html?pop=yes&amp;pid=916">http://www.learner.org/resources/series33.html?pop=yes&amp;pid=916</a></td>
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<td></td>
<td>• Fraction Tracks is an activity that uses a linear model for fractions and consists of seven parallel number lines on a mat and that show the relationships between fractions with the denominators 2, 3, 4, 5, 6, 8, and 10. This set of fraction tracks is like a set of number lines all lined up, one below the other. The top track shows halves, the next track shows thirds, and so on.</td>
<td><em>Fraction Tracks</em>, from Teaching Math: A Video Library (5-8/9-12) used with permission by Annenberg Learner. <a href="http://www.learner.org">www.learner.org</a></td>
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### Facilitator's Notes

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<td>The goal is to move beans so that they land exactly on 2.</td>
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<td>- Facilitators should watch the video BEFORE facilitating the workshop.</td>
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<td></td>
<td><strong>Introducing and engaging in the Fraction Tracks Activity with workshop participants: (15 minutes)</strong></td>
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Participants will work in pairs to play the game.

- Facilitators will say to participants: “We are going to engage in an activity that uses the linear model for representing fractions to have students explore the relationships between fractions using denominators 2, 3, 4, 5, 6, 8, and 10. As you engage in the activity think about the mathematics ideas that students will be using during the activity. After this activity we will watch a video of a fifth grade classroom where students are engaging in this same activity.”

Use PowerPoint slide 9 with the following directions to prepare participants to play the game.

- The game is played by pairs. Each pair has a Game Board and 20 beans. Each pair of participants will have one set of fraction cards. This introductory game uses only the fraction cards equal to or less than one whole (The mat for the game and the fraction card sheets will be posted on the website.) The goal of the game is to move beans from 0 so they land exactly on 1. When you land on one you win the bean. You then place another bean at 0 on this track.
- Place seven beans on the game board on the 0’s, one on each track. Mix the cards and place the deck facedown.
- Participants take turns drawing the top card and moving a bean or beans so that the move(s) are equivalent to the fraction on the card. Once the card has been drawn place the card in a discard pile.
- (Optional part of the directions or you can let players discover this strategy.) You can use one fraction card to move on one track or on several.
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|             | • If a player is not able to move the total amount of the fraction card, that person loses a turn.  
• Facilitators will circulate as the participants are playing the game and make note of the mathematical ideas that come up in the conversations and the strategies that participants are using.                                                                                           |                  |                  |
|             | **Closing Reflection Questions:** Use PowerPoint slide 10  
• Have individual participants take 1-2 minutes to write down the strategies they used and what mathematical concepts are connected to the strategies.  
• If time permits have some individuals share out in the whole group.                                                                                                                                                                                                 |                  |                  |
|             | **Transition to Next Part of the Workshop:**  
• Tell participants that they are going to watch Hilory Paster’s fifth-grade class engaged in the Fraction Tracks activity and they will be using Student “Look-fors” Note Recording Handout observation tool.                                                                                                                                 |                  |                  |
| 40 minutes  | **Observing a classroom of students to identify Mathematical Process Skills in place as they engage in the Fraction Tracks activity.**  
Part I: Watching the Video (25 minutes)  
• Directions on PowerPoint slide 11  
• Participants need a copy of the Mathematical Process Skills – “Student Look-fors” Recording Handout.  
• Facilitator will assign a skill area to each table in a way that ensures 7 groups and ask the table to star their assigned skill area. Each person will use the recording handout to collect specific evidence for their table’s skill area as they watch the video.  
• Tell the group they will be watching a group of fifth graders engaged a classroom lesson that uses the fraction tracks activity.  
• Have participants take 2 minutes to look at the skill area they will be observing to |                  |                  |
|             | **Mathematical Process Skills - “Student Look-fors” Recording Form Handout**  
Video: *Fraction Tracks*  
*Fraction Tracks*, from Teaching Math: A Video Library (5-8/9-12) used with permission by Annenberg Learner.  
[www.learner.org](http://www.learner.org)  
Chart paper |                  |                  |
### Time | Facilitators Notes | Materials Needed | Additional Notes
---|---|---|---
50 minutes | become familiar with the indicators.  
- Watch the 17 minute video and have table groups’ record specific evidence to support the indicators observed in their table’s skill area.  
Part II: Table Group Debriefing  
- Directions on PowerPoint slide 12  
- After watching the video each table will use a sheet of chart paper to display their observations.  
  - At the top of the chart paper write the Skill Area your table observed.  
  - As a group identify the one or two indicators in your assigned skill area that seemed to be most visible on this video.  
  - On the chart paper record specific evidence for the one or two indicators identified in the skill area.  
  - Post your chart on the wall.  
- When a group has posted their chart they can go back to their tables and discuss the other skill areas in relationship to the video until everyone is finished.  
- If time permits, do a gallery walk to observe the charts from all the groups. If there is not sufficient time before lunch, ask the groups to take time to look at all the posters during their lunch break.  
- Transition to lunch. Ask participants to think about the impact of being intentional in planning to incorporate the Mathematical Skills classroom instruction. | Markers  
PowerPoint slide 11 and 12 | ---

20 minutes | **Understanding the Standards and How the Mathematical Process Skills Inform Instructional Planning**  
In this part of the workshop participants will identify the instructional moves for SOL 4.2 a & b using SOL 4.2 a & b using the Understanding the Standard Handout which shows the Mathematical Process Skills | SOL 4.2 a & b using the Understanding the Standard Handout.  
Mathematical Process Skills | ---
### Facilitators Notes

unpacking of this standard. The goal is to help participants understand how to make decisions about instruction by using the Curriculum Framework and including the Mathematical Process Goals for Students. Point out the Mathematical Process Skills “Student Look-fors” handout. The handout and discussion prior to lunch can assist them in their planning.

- Use PowerPoint slide 13 which shows page 5 of the Curriculum Framework SOL 4.2 a&b to illustrate how to unpack the information in the Curriculum Framework.
- Give participants the Understanding the Standards Handout for 4.2 a & b and use PowerPoint slide 14 to show the unpacked information from the Framework and the Classroom Instruction column blank.
- Share with participants that the increased rigor in the 2009 SOL has brought to light the need to carefully unpack the Standards to make more purposeful and strategic decisions when choosing instructional activities.
- Say to participants: “What instruction will enable students to develop a deeper understanding of the concepts in 4.2 a & b?” Do not engage in discussion, this is a leading question to prepare for the activity.
- Participants will work with a shoulder partner (10 minutes) to analyze the unpacked standard and identify the Mathematical Process Skills and where possible specific indicators that are implicit and/or explicit in the Standard. Participants will record their ideas in the Classroom Instruction column. Directions on PowerPoint slide 15.

### Closure:

- Whip around the room and collect one idea from each table. Facilitators will chart the ideas. As you move to different table groups ask for one that has not already been given. If a group does not have a new idea they can say pass. Time will determine how many times the facilitator can go around the room.
- The facilitator will summarize and bring closure by carefully considering the ideas.

### Materials Needed

“Student Look-fors” Handout

PowerPoint slides 13, 14 and 15
shared and make a statement such as, “The more we interact with the Curriculum Frameworks the more prepared we will be to help our teachers and ultimately our students better understand the SOL. And through the attention given to the Mathematical Process Skills in planning instruction we can achieve the rigor expected with the Standards.”

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| 10 minutes | **Introduce Instructional Talk Moves** Background information for the facilitator, The Instructional Talk moves that encourage mathematical communication are provided on PowerPoint slide 16.  
1- Teacher revoices a student’s reasoning for the purpose of clarification and advancing student thinking. This will allow more think time for all students. Why: To get some vocabulary out that has not been used  
2- Student revoices another student’s reasoning to make sense themselves and advance the other students’ reasoning and understanding.  
3- Asking students to justify or prove someone else’s reasoning. Using justification or proof to allow for respectful discussion of ideas.  
4- Asking students to build on the group’s reasoning by connecting and extending another student’s idea.  
5- Wait time (means to make the other things happen)  
- The facilitator will use the Encouraging Communication in Mathematics Class Handout to review with the participants each of the talk moves and give one or two examples in the observed column.  
- Let folks know they will be watching a video of classroom teaching to see each of these ideas in play. | Encouraging Communication in Mathematics Class Handout | PowerPoint slides 16 |
| 25 minutes | **Observe The Fraction Tracks Classroom to Identify Talk Moves**  
- The facilitator will assign each table one of the first four talk moves to collect | Encouraging Communication in Mathematics Class Note Taking Handout |
Facilitators' Notes

- Give everyone the Encouraging Communication in Mathematics Class Recording Handouts which has a place to record their notes.
- Before watching the video, ask each table to take a few minutes (3) minutes to further examine the Talk Move they are assigned and discuss the information on the handout.
- Tell the participants they are going to watch the first 10 minutes of the Fraction Tracks video again. This time they will watch the classroom on the video to record specific evidence from the classroom that supports the use of their table’s assigned Talk Move, everyone can make notes about wait time. Also pay attention to what the teacher is doing and what the children are doing both in large group and small groups.
- Give the table 5 minutes to share and review what was recorded by individuals at their table.
- As a whole group each table shares 2 things they saw on the video that was evidence for their assigned talk move. Facilitators should capture these on chart paper.

Materials Needed

- Chart paper, markers
- Handout of 2 tasks on related topic

Facilitators will bring closure to this part of the workshop by making explicit that the Mathematical Process Goal of Communication or classroom discourse happens between students and the teacher and also between students. Teachers will need to help their students learn how to engage in mathematical communication. In planning instruction teachers need to provide something mathematically significant for students to talk about. In the next part of the workshop we will delve more into ways teachers can plan instruction to encourage communication in the mathematics class.
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<tr>
<td></td>
<td>Participants will work in groups of 4 or 5 at their tables to do the tasks. Large tables may have 2 working groups.</td>
<td>Handout of math tasks with 3 tasks per page with work space.</td>
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<td>Partner Work (10 minutes)</td>
<td>Table group recording sheet for collecting table group’s high and low designations handout</td>
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<td></td>
<td>• Directions on PowerPoint slide 17 for participants.</td>
<td>Whole Group Chart paper for recording high and low (modeled on the table group recording handout)</td>
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<td>• In your table group, divide up the tasks so that at least 2 people will complete each task in the set. Then write on the task whether you would consider the task low or high cognitively demanding.</td>
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<td>• Each person should work independently before discussing with your partner whether you thought it was a low or high cognitively demanding task.</td>
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<td>• After discussing the two tasks that you and your partner solved go back read the remainder of the tasks and without working these tasks indicate if you think the task is a low or high cognitively demanding tasks.</td>
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<td>Table Group Sort (15 minutes)</td>
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<td>• Have table groups share and discuss individual results for each task and come to an agreement on which are high and which are low cognitively demanding tasks. Discuss the reasoning for decisions about the level of the task. (Facilitators should listen for disagreements and make note of resolutions that may need to be brought before the whole group.)</td>
<td>PowerPoint slides 16, 17, 18, 19, 20, 21, 22</td>
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<td>• Ask groups to make notes about the characteristics of the task that help them to make decisions about high and low cognitively demanding tasks.</td>
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<td>Whole Group Summary (15 minutes)</td>
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<td>• The facilitator will prepare one big chart for the whole group collection of data using the table group handout as a model. Ask for a show of one hand per table for each task. Facilitator counts and records the high and low decisions on chart paper.</td>
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<td>• Discuss any disagreements with participants referring back to characteristics they generated in their table groups.</td>
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<td></td>
<td>Whole Group (15 minutes)</td>
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### Facilitators Notes

- If time permits the facilitator can collect from the whole group and record on a chart characteristics that differentiate low level from high level cognitively demanding tasks.
- If time does not permit move directly to PowerPoint slide 18 introduce the term Rich Mathematical Tasks and to examine researched based descriptors for Rich Tasks.
- Recall the Ordering Fractions Activity and the Fraction Tracks Activity and think about these activities in light of the characteristics the class has identified for high cognitively demanding tasks and the research based descriptors for Rich Tasks.
- Use PowerPoint slides 19, 20, and 21 to examine the characteristics of high and low cognitively demanding tasks presented in the *Implementing Standards Based Instruction* book your district received today. You will note that the framework in the book has 4 levels but it is helpful to begin with two levels.
- Facilitator should tie the characteristics generated during the sharing back to the Mathematical Process Skills to make explicit that paying attention to these skills will increase cognitive demand for tasks, thus increasing the rigor.
- Pose the directions and question: “Take 2 minutes and think for yourself what factors impede a high cognitively demanding task?”
- Show PowerPoint slides 22 on factors impeding high cognitive demand.
- If time permits discuss how to make the Low level tasks be more cognitively demanding. Consider the characteristics and the Mathematical Process Skills.

### Exit Card: Consolidating Today’s Work

Today we have looked at several important factors for teachers to consider in planning and facilitating learning experiences for students. See PowerPoint slide 23.

- Using rich task to engage students
- Using the Cognitive Demand Framework to analyze mathematical tasks.
- Identifying the Mathematical Process Goals called for in the standards in the

### Materials Needed

- PowerPoint slides 23 and 24
- 3 x 5 index card
### Time | Facilitators Notes | Materials Needed | Additional Notes
--- | --- | --- | ---

| 5 minutes | Virginia SOL Curriculum Framework  
- Using the Process Goals and “Student Look-fors” in the classroom setting  
- Encouraging mathematical conversation using Talk Moves in the classroom setting.  

Question Prompt is on PowerPoint slide 24.  
As you reflect on our work together today respond to the following question: When a classroom teacher pays attention to each of these how is student engagement and ultimately student learning impacted? |  |  |

| 5 minutes | VDOE Evaluation |  |  |
