2014 Mathematics Institutes
The purpose of the 2014 Mathematics Institutes is to provide teachers with professional development focused on the instruction that supports process goals for students in mathematics. The emphasis of this professional development will be on fostering students’ abilities to make mathematical connections and use effective and appropriate representations in mathematics.

Professional Development Instructions
A product of the 2014 Mathematics Institutes is a set of online professional development modules designed to be used by a group of teachers of a specific grade level or course, facilitated by a member of the team. Modifications could be made to adapt the professional development for more than one grade level/course or for large groups. Facilitators should review the activities and handouts in this guide prior to facilitating this professional development.

Approximate Time | Facilitator Instructions | Materials
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45 minutes total | **Module 1: Participant personal reflection on representations and connections**
**Essential Question:** What does it mean for students to represent and make connections in the mathematics classroom?
1) Think (individually)/Pair (share with a partner)/Share (with the whole group):
   - What does it mean for students to use effective and appropriate representations in the mathematics classroom?
   - What does it mean for students to make connections in the mathematics classroom?
2) Whole group discussion: Compare and contrast the groups’ discussion points with Process Goals for Students: Representations and Connections document.
3) Whole group discussion: What questions should be considered regarding representations and connections when planning for instruction? This list should be recorded for use in Module 6.

45 minutes total | **Module 2: Mathematical task**
1) Each individual should solve the mathematical task in at least two different ways, and consider potential student misconceptions or common errors.
2) Partner discussion: Identify representations and connections within the task and strategies for solving the task.
3) Whole group discussion: Identify the mathematical content addressed in the task.

*Virginia’s Process Goals for Students*

SOL Vertical Articulation Technical Assistance Documents
- Grades K-3
- Grades 3-6
- Grades 5-8
- Algebra
- Geometry
### Approximate Time | Facilitator Instructions | Materials
--- | --- | ---
45 minutes total | **Module 3: Looking at student work**  
**Essential Question:** How are students using representations and making connections in their work?  
1) Review the student work on the mathematical task solved in Module 2.  
2) Partner or small group discussion:  
   - What representations are evident in each student’s work?  
   - How are the representations alike?  
   - What does the student work tell us about their understanding?  
3) Partner or small group discussion:  
   - What evidence is there that students are making connections within their own work?  
   - How could the student work be used to help all students make connections  
     - among representations,  
     - among strategies, or  
     - among mathematical ideas?  
4) Whole group discussion:  
   How do the various student representations communicate  
   - student understanding of the mathematics,  
   - student generalizations of the mathematics, and  
   - mathematical relationships? | • Student work

40 minutes total | **Module 4: Planning for the use of representations and connections in instruction**  
**Essential Question:** What should be considered regarding representations and connections when planning for instruction?  
1) Small group or whole group discussion: Consider the mathematical task from Module 2.  
   - Refer to the Grades 4-8 Fraction and Decimal Learning Progression document.  
     - What is the related content (prior knowledge) from previous and future grade(s)/course(s)?  
     - How is the objective connected to the related content and to the real world?  
   - Refer to the Mathematics Instructional Connections for Physical and Visual Representations document.  
     - What representations/strategies may have been used to develop prior knowledge?  
     - What representations could be used to develop and reinforce understanding of the content? | • Grades 4-8 Fraction and Decimal Learning Progression  
• Mathematics Instructional Connections for Physical and Visual Representations
### Module 5 Part 1: Exploring fraction division with equation strings

**Essential Questions:** How can different fraction models help students develop their understanding of fraction concepts?

1. **Small group exploration:** Examine and discuss the relationship between each successive equation in the string in the Equation String and Word Problems document. The Equation Strings Recording Chart may be helpful in organizing your thoughts.
   - How is each equation related to the previous equation?
   - How could you apply your reasoning used in solving one to help you solve the next?
   - Consider each equation string in context. Read each word problem included in the document and solve the examples on page 3, 6, and 7 using the suggested model to illustrate your thinking.
   - Discuss your successes and challenges you encountered using the suggested model.
   - Use a different model to explain examples 3, 6, and 7.

2. **Small group discussion:**
   - How can we help students work flexibly among and with each of these representations?
   - Is it important that students use more than one model/representation to solve problems? Why or why not?

### Module 5 Part 2: Exploring fraction division with equation strings

**Essential Questions:** How can different fraction models help students develop their understanding of fraction concepts?

1. **Partner/Small group exploration:** Create a “real life” fraction division word problem and use your choice of models. Share and justify the choice with the small group.

2. **Small group discussion:**
   - What are the advantages of teaching fraction division with contextual problems while emphasizing the use of models? Disadvantages?
   - Were there limitations of specific models?
   - How could equation strings be used with decimal and integer operations?

### Module 6: Closure

- Skip Fennell’s NCTM
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| 20 minutes       | 1) Small group discussion: Review and revise the list of questions brainstormed in Module 1: “What questions should be considered regarding representations and connections when planning for instruction?”  
2) Whole group discussion: In what ways did your small group discussion support the following quote?  
“Representation should be an important element of lesson planning. Teachers must ask themselves, ‘What models or materials (representations) will help convey the mathematical focus of today’s lesson?’” – Skip Fennell | President’s Message  
• Planning Mathematics Instruction: Essential Questions |
| 25 minutes       | 1) Small group discussion: Compare the list of questions to be considered regarding representations and connections when planning for instruction with the Planning Mathematics Instruction: Essential Questions document. |          |