2014 Mathematics Institutes
The purpose of the 2014 Mathematics Institutes is to provide teachers with professional development focused on the instruction that supports process goals for students in mathematics. The emphasis of this professional development will be on fostering students' abilities to make mathematical connections and use effective and appropriate representations in mathematics.

Professional Development Instructions
A product of the 2014 Mathematics Institutes is a set of online professional development modules designed to be used by a group of teachers of a specific grade level or course, facilitated by a member of the team. Modifications could be made to adapt the professional development for more than one grade level/course or for large groups. Facilitators should review the activities and handouts in this guide prior to facilitating this professional development.

Approximate Time | Facilitator Instructions | Materials
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45 minutes total | **Module 1: Participant personal reflection on representations and connections**  
**Essential Question:** What does it mean for students to represent and make connections in the mathematics classroom?  

1) To frame the Essential Question, given above, within a context, have participants solve the Staircase problem. Encourage participants to solve the problem geometrically as well as algebraically. Have participants think geometrically about the nth arrangement. (Think about using area to arrive at the formula $\frac{n(n+1)}{2}$.)  
2) Think (individually)/Pair (share with a partner)/Share (with the whole group):  
   - **Pair:** When you hear the term representations, what does it mean to you? Take turns sharing thoughts.  
   - **Whole group:** What does it mean for students to use effective and appropriate representations in the mathematics classroom?  
   - **Table group:** Why is it important for students to make connections in mathematics? Share an example of a connection students should make?  
   - **Whole Group:** What does it mean for students to make connections in the mathematics classroom?  
3) Whole group discussion: Compare and contrast the groups’ discussion points with Process Goals for Students: Representations and Connections document.  

**Virginia’s Process Goals for Students**
SOL Vertical Articulation Technical Assistance Documents  
- [Grades K-3](#)  
- [Grades 3-6](#)  
- [Grades 5-8](#)  
- [Algebra](#)  
- [Geometry](#)  

**Materials**  
- [Staircase Problem](#)  
- Linking Cubes  
- [Process Goals for Students: Representations and Connections](#)
### Facilitator Instructions

<table>
<thead>
<tr>
<th>Approximate Time</th>
<th>Facilitator Instructions</th>
<th>Materials</th>
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<tbody>
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<td>4) Whole group discussion: What questions should be considered regarding representations and connections when planning for instruction? Record a list that will be referenced in Module 5.</td>
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| 45 minutes total | **Module 2: Mathematical task** | • Geometry task  
• Rulers, Compasses, Patty paper  
• Five Types of Mathematical Representations  
• Learning Progression for Geometric Constructions |
| 1) Each person should solve the geometry task in at least two different ways.  
2) Partner discussion: Identify representations and connections within the task and strategies for solving the task.  
   - Table group: Look at the Five Types of Mathematical Representations document. What representations were used to solve the task? Explain your representation and strategy to others at your table? What connections are evident in your solution?  
   - Whole group: Have selected participants explain their solution to the group and share why a particular representation and strategy was used. Other participants ask questions and look for connections among the representations.  
3) Whole group discussion: Identify the mathematical content addressed in the task. Where is this content found in the Learning Progression for Geometric Constructions document? | |
| 45 minutes total | **Module 3: Looking at student work**  
**Essential Question:** How are students using representations and making connections in their work? | • Student work  
• Analyzing Student Work chart |
| 1) Complete the Analyzing Student Work chart for each Student Work Sample A through I.  
2) Partner or small group discussion:  
   - What representations are evident in each student’s work? What are the similarities and differences?  
   - What does the student work tell us about their understanding? How do the representations give clues to student understanding?  
   - What misconceptions are evident in the student work?  
3) Partner or small group discussion:  
   - Is there evidence that students are making connections within their own work?  
   - How could the student work be used to help all students make connections  
   - among representations, | |
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</tr>
</thead>
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| 40 minutes total | Module 4 Part 1: Planning for the use of representations and connections in instruction regarding constructions | • Rulers, Compasses, Patty paper  
• Construction Exercise  
• Learning Progression for Geometric Constructions  
• Mathematics Instructional Connections for Physical and Visual Representations  
• Chart paper, Markers  
• VDOE End-of-Course Tools Practice  
• Using Dynamic Geometry Software, begin at 6:42, The Teaching Channel  
• Process Goals Video Chart  
• Construction Connections |

Essential Questions: What should be considered regarding representations and connections when planning for instruction on constructions?

1) Complete the Construction Exercise. While completing the constructions, think carefully about why these constructions work.
2) Small group or whole group discussion: Considering Construction Exercise and the mathematical task from Module 2.
   - How can these constructions be justified?
   - When might these constructions be taught?
   - When might the construction exercises and bus stop task from Module 2 be used?
   - Refer to the Learning Progression for Geometric Constructions document.
     - What is the related content (prior knowledge) from previous and future grade(s)/course(s)?
     - How is the objective connected to the related content and to the real world?
   - Refer to the Mathematics Instructional Connections for Physical and Visual Representations document.
     - What representations or strategies may have been used to develop prior knowledge?
     - What representations could be used to develop and reinforce understanding of the content?
     - Which representations or strategies will model the mathematics and deepen and extend students’ mathematical understanding?
     - What are the strengths and limitations of the representation or strategy?

4) Whole group discussion:
   - How do the various student representations communicate
     - student understanding of the mathematics,
     - student generalizations of the mathematics, and
     - mathematical relationships?
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<th>Materials</th>
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| 40 minutes total | **Module 4 Part 2: Planning for the use of representations and connections in instruction regarding constructions**  
**Essential Questions: How can constructions be best integrated into the Geometry curriculum?**  
1) Using the VDOE Geometry Construction Cards, have participants work in partners or small groups to create a poster for each of the constructions.  
   - Assign each group one of the constructions. Have participants do the actual construction on chart paper.  
   - List the mathematical connections inherent in the construction.  
   - Justify why the constructions work.  
   - Explain when or where in the Geometry curriculum the construction might be taught.  
2) Whole group: Share posters with one another and explain your work.  
3) Compare your posters to the Construction Connections document.                                                                                                                                                                                                                                                                                                                  | • Rulers, Compasses, Patty paper  
• Chart paper, Markers  
• VDOE Geometry Construction Cards  
• Learning Progression for Geometric Constructions  
• Construction Connections |
| 45 minutes total | **Module 5: Closure**  
1) Small group discussion: Review and revise the list of planning questions brainstormed in Module 1: “What questions should be considered regarding representations and connections when planning for instruction?”  
2) Whole group discussion: In what ways does the discussion in Part 1 support the following quote: “Representation should be an important element of lesson planning. Teachers must ask themselves, ‘What models or materials (representations) will help convey the mathematical focus of today’s lesson?’” – Skip Fennell                                                                                                                                                                                                 | • Skip Fennell’s NCTM President’s Message  
• Planning Mathematics Instruction: Essential Questions  
• Current Ideal Next Steps |
3) Small group discussion: Compare the list of questions to be considered regarding representations and connections when planning for instruction with the Planning Mathematics Instruction: Essential Questions document.
   – Highlight the questions you already think about when planning.
   – Which questions are new for you to think about?
   – Are the questions on the list you created earlier reflected on this document?
4) Small group discussion: What practical implications does our work with constructions have for your classroom, school, and/or division? How will it inform instruction and pacing?
5) Individually: Complete the Current Ideal Next Steps form.