Supporting the Mathematics Process Goals through Research-Based Teaching Practices

Middle/High School
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“The content of the mathematics standards is intended to support the five goals for students”

- 2009 *Mathematics Standards of Learning*
Essential Understandings

• The mathematics process goals (problem solving, communication, reasoning, connections, and representations) play an instrumental role in the teaching and learning of mathematics with understanding.

• There are specific research-based teaching practices that support the implementation of the mathematics process goals.
Mathematics Process Goals

- Mathematical Understanding
- Problem Solving
- Reasoning
- Connections
- Representations
- Communication
Mathematical Problem Solving

• Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities.

• Students also will recognize and create problems from real-life data and situations within and outside mathematics and then apply appropriate strategies to find acceptable solutions.

To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students become competent mathematical problem solvers.
Mathematical Connections

- Students will relate concepts and procedures from different topics in mathematics to one another and see mathematics as an integrated field of study.

Through the application of content and process skills, students will make connections between different areas of mathematics and between mathematics and other disciplines.

Mathematical Communication

• Students will use the language of mathematics, including specialized vocabulary and symbols, to express mathematical ideas precisely.

Representing, discussing, reading, writing, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied.

Mathematical Representations

• Students will represent and describe mathematical ideas, generalizations, and relationships with a variety of methods.
• Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics.

Students should move easily among different representations — graphical, numerical, algebraic, verbal, and physical — and recognize that representation is both a process and a product.
Mathematical Reasoning

• Students will recognize reasoning and proof as fundamental aspects of mathematics.
• Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures.
• Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.
• In addition, students will learn to apply proportional and spatial reasoning and to reason from a variety of representations such as graphs, tables, and charts.
Five goals...for students to

- become mathematical problem solvers that
- communicate mathematically;
- reason mathematically;
- make mathematical connections; and
- use mathematical representations to model and interpret practical situations.

TABLE GROUP DISCUSSION
What teaching practices should be employed to ensure students are engaged in these processes in ways that deepen their mathematical understanding?
MATHEMATICS THROUGH PROBLEM SOLVING

VERSUS

MATHEMATICS FOR PROBLEM SOLVING
Mathematics through problem solving

- Interactions among students and the teacher
- Communication of mathematical ideas by students
- Students analyzing, interpreting, and developing processes for solving “rich” mathematical tasks
- Teachers facilitating learning by asking strategic questions and building on ideas that students bring to mathematical tasks
- Teachers facilitating students’ sharing of ideas, processes, and conclusions
Principles to Actions: Ensuring Mathematical Success for All

- Describes the **supportive conditions, structures, and policies** required to give all students the power of mathematics
- Focuses on **teaching and learning**
- Engages students in **mathematical thinking**
- How to ensure that mathematics achievement is maximized **for every student**
- Not specific to any standards; **it’s universal**

**Mathematics Teaching Practices**

- **Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

- **Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

- **Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

- **Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

- **Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.

- **Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

- **Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

- **Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.
Take a few minutes to look at the process goal indicators and compare it with the Mathematics Teaching Practices…

Where do you see the process goals represented within the research-based teaching practices?

Share at least two connections you found with a partner ….
Eight High-Leverage Instructional Practices

• Establish mathematics goals to focus learning
• Implement tasks that promote reasoning and problem solving
• **Use and connect mathematical representations**
• Facilitate meaningful mathematical discourse
• Pose purposeful questions
• Build procedural fluency from conceptual understanding
• Support productive struggle in learning mathematics
• Elicit and use evidence of student thinking
Mathematics Process Goals

- Connections
- Communication
- Representations
- Problem Solving
- Mathematical Understanding
- Reasoning
Defining Connections and Representations

- What does it mean to **make mathematical connections** and use effective and appropriate representations?
- What words come to mind when you think about ...  
  *making connections?*
  *using effective and appropriate representations?*
- **Turn-n-Talk**
Students must be actively engaged in developing, interpreting, and critiquing a variety of representations.
This type of work will lead to better understanding and effective, appropriate use of representation as a mathematical tool.

_NCTM Principles and Standards, 2000, p. 206_
Five Types of Mathematical Representations

Representations are useful in all areas of mathematics because they help us **develop, share, and preserve** our mathematical thoughts. They help to **portray, clarify, or extend a mathematical idea** by focusing on its essential features."

_NCTM Principles and Standards, 2000, p. 206_
Experiencing Mathematics in Context

• Students should connect mathematical concepts to their daily lives, as well as to situations from science, the social sciences, medicine, and commerce. For example,
  – high school students working with a drug store chain to determine where it should locate a new pharmacy in their neighborhood on the basis of analyses of demographic and economic data.

• When students connect mathematical ideas, their understanding is deeper and more lasting.

• Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics.

NCTM Principles and Standards, 2000
Quality instruction allows all students to:

• recognize and use connections among mathematical ideas;
• understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
• recognize and apply mathematics in contexts outside of mathematics.

_NCTM Principles and Standards, 2000_
An emphasis on mathematical connections helps students build a disposition to use connections in solving mathematical problems, rather than see mathematics as a set of disconnected, isolated concepts and skills.

_NCTM Principles and Standards, 2000_
The Role of the Teacher

- **Create** a learning environment that encourages and supports the use of multiple representations
- **Model** the use of a variety of representations
- **Orchestrate** discussions where students share their representations and thinking
- **Support** students in making connections among multiple representations, to other math content and to real world contexts
Role of the Student

- **Create** and **use** representations to organize, record, and communicate mathematical ideas
- **Select, apply**, and **translate** among mathematical representations to solve problems
- **Use** representations to model and interpret physical, social, and mathematical phenomena
Taking a Peek inside a Middle School Mathematics Classroom

- Use the Mathematical Process Goals for Students sheet to review the indicators under ‘connections’ and ‘representations’ while viewing the video.

- In the Note’s section, record specific evidence from the video that the indicators for the process goals are evident in this classroom.

http://www.learner.org/ Building Viewpoints from Annenberg Learner Teaching Math: A Video Library, 5-8
Taking a Peek inside a 9th Grade Mathematics Classroom

- Use the Mathematical Process Goals for Students sheet to review the indicators under ‘connections’ and ‘representations’ while viewing the video.

- In the Note’s section, record specific evidence from the video that the indicators for the process goals are evident in this classroom.

http://galileo.org/cbe/top-speed/  Top Speed from The Galileo Educational Network
At your table talk about...

- What evidence did you find that the process goals (representations or connections) were present in this lesson?

- Discuss how the purposeful use of representations and connections in the mathematics classroom can create opportunities to elicit evidence of student thinking.
<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selecting tasks that allow students to decide which representations to use in making</td>
<td>Using multiple forms of representations to make sense of and understand mathematics.</td>
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<tr>
<td>sense of the problems.</td>
<td>Describing and justifying their mathematical understanding and reasoning with drawings,</td>
</tr>
<tr>
<td>Allocating substantial instructional time for students to use, discuss, and make</td>
<td>diagrams, and other representations.</td>
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<tr>
<td>connections among representations.</td>
<td>Making choices about which forms of representations to use as tools for solving problems.</td>
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<tr>
<td>Introducing forms of representations that can be useful to students.</td>
<td>Sketching diagrams to make sense of problem situations.</td>
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<td>Asking students to make math drawings or use other visual supports to explain and</td>
<td>Contextualizing mathematical ideas by connecting them to real-world situations.</td>
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<tr>
<td>justify their reasoning.</td>
<td>Considering the advantages or suitability of using various representations when solving</td>
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<tr>
<td>Focusing students’ attention on the structure or essential features of mathematical</td>
<td>problems.</td>
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<tr>
<td>ideas that appear, regardless of the representation.</td>
<td></td>
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<tr>
<td>Designing ways to elicit and assess students’ abilities to use representations</td>
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<tr>
<td>meaningfully to solve problems.</td>
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</tbody>
</table>
Teachers must ask themselves,

“What models or materials (representations) will help convey the mathematical focus of today’s lesson?”

- Skip Fennell, Past-President NCTM

http://www.nctm.org/uploadedFiles/About_NCTM/President/2006_09pres.pdf
Planning Mathematics Instruction: Essential Questions

Determining Mathematical Objective(s) for Students

- What should students know, understand, and demonstrate?
- Which bullets from the Essential Knowledge and Skills will be addressed?
- What criteria will be used to determine student mastery?

Connecting to Prior Knowledge and Future Content

- What is the related content (prior knowledge) from previous grade(s)/course(s)?
- What is the related content in future grade(s)/course(s)?
- What representations/strategies were used to develop prior knowledge?
- How is the objective connected to the related content and to the real world?

Developing and Reinforcing Content

- What are students’ common misconceptions?
- What vocabulary is essential?
- Which representations/strategies will model the mathematics and deepen and extend students’ mathematical understanding?
  - What are the strengths and limitations of the representation/strategy?
  - How will the selected representation assist in student understanding?
  - How might the representation/strategy highlight common misconceptions?
- In what order will the content and different representations be introduced?
- What mathematical tasks will be used to develop content?
  - How is the task connected to the objective and representation/strategy?
  - How does the task require an increased level of cognitive demand?
  - How does the task address the learning needs of students with diverse language learners, and gifted students?
- In what ways could the task be extended to connect to other materials?

Mathematics Instructional Connections for Physical and Visual Representations

- Base 10 Blocks
- Number Lines
- Bar Model
- Five and Ten Frames, Rekenreks
- Arrays

Content Connections

- One-to-one correspondence
- Counting and skip counting
- Place value
- Representing fractions (length model, decimals, and integers)
- Ordering and comparing whole numbers, fractions, decimals, and integers
- Operations with whole numbers, fractions, decimals, and integers
- Measurement
- Representing data (line plots, balance point/mean)
- Probability
- Representing absolute value

Challenges could include:

- Modeling multiplication versus division (motion required to build understanding)
- Modeling division by negative integers
- Operations with whole numbers and fractions
- Related facts (addition and subtraction)
- Related facts
- Area and perimeter
- Distributive Property
- Counting and skip counting
VDOE provides targeted professional development through Mathematics Institutes. Since 2009, the institutes, framed by the five goals for students of becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations, have:

- outlined the content standard changes from the 2001 Mathematics SOL to the 2009 Mathematics Standards of Learning (SOL) – 2009;
- provided training in the vertical progression of content and pedagogy – 2010, 2014;
- provided instructional guidance in content areas of greatest challenge – 2010, 2014;
- provided professional development resources focused on facilitating students’ mathematical understanding through problem solving, communication, and reasoning – 2011;
- provided professional development resources focused on the use of formative assessment resources to drive instructional decisions – 2012;
- provided professional development on the analysis and modification of existing resources to match student learning expectations and on promoting problem solving – 2013; and
- provided professional development on fostering students’ abilities to make mathematical connections and use effective and appropriate representations in mathematics – 2014.

Mathematics SOL Institutes


- 2014 – Making Mathematical Connections and Using Representations
- 2012 – Using Formative Assessment Resources to Drive Instructional Decisions
Role of the Mathematics Leader

How do you support teachers as they strive to help students make connections and use effective and appropriate representations in the classroom?
Beliefs About Teaching and Learning Mathematics

<table>
<thead>
<tr>
<th>Unproductive beliefs</th>
<th>Productive beliefs</th>
</tr>
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<tbody>
<tr>
<td>Mathematics learning should focus on practicing procedures and memorizing basic number combinations.</td>
<td>Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.</td>
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<td>Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.</td>
<td>All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.</td>
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<tr>
<td>Students can learn to apply mathematics only after they have mastered the basic skills.</td>
<td>Students can learn mathematics through exploring and solving contextual and mathematical problems.</td>
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<td>The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.</td>
<td>The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.</td>
</tr>
<tr>
<td>The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.</td>
<td>The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.</td>
</tr>
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<td>An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.</td>
<td>An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.</td>
</tr>
</tbody>
</table>

NCTM,. (p 11, 2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA. NCTM
Mathematics Classroom LOOK FORS

• Are multiple representations and connections encouraged within the classroom? this lesson?
• What types of representation and/or connections are present in this activity?
• How are students creating and using representations to communicate their mathematical ideas?
• How is the classroom discussion enhanced by students’ representations and/or connections?
• Who is doing the thinking? students or teachers?
Using and connecting mathematical representations

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<td>• Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.</td>
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<td>• Designing ways to elicit and address students’ abilities to use representations meaningfully to solve problems.</td>
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<td>• Using multiple forms of representations to make sense of and understand mathematics.</td>
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<td>• Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.</td>
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<td>• Seeking to understand the approaches used by peers by asking clarifying questions, trying out others’ strategies, and describing the approaches used by others.</td>
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<td>• Identifying how different approaches to solving a task are the same and how they are different.</td>
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Fostering Connections and Representations in Instruction

- Use data to determine areas of weakness (students and teachers)
- Be knowledgeable about and encourage the use of available resources
- Structure PLCs to explore resources provided
  - Process goals for students
  - Vertical articulation of content
  - Instructional strategies
  - Development of quality assessments
  - Effective use of data
Five goals...for students to become mathematical problem solvers that

• communicate mathematically;
• reason mathematically;
• make mathematical connections; and
• use mathematical representations to model and interpret practical situations

Process Goals
VDOE – Resources

- Standards of Learning
- Curriculum Framework
- Testing Blueprints
- SOL Practice Items and Tools Practice
- 2013 Released Test Items
- 2013 Student Performance Analysis
- Technical assistance document
- Plain English Mathematics Test Information (May 2014)
- ESS Sample Lesson Plans
- Instructional Videos
- Vocabulary Resources – K-8, Geometry, and Algebra Cards
Mathematics SOL Revision Process

Anticipated Timeline

– February 2015 – timeline goes to the BOE
– March 2015 – public comment on 2009 standards
– Spring/Summer/Fall 2015 – review process
Thank You

Questions???

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Eliciting and Using Evidence of Student Thinking

Effective mathematics teaching elicits evidence of students’ current mathematical understanding and uses it as the basis for making instructional decisions. This attention to both eliciting and using evidence is an essential component of formative assessment (Wiliam 2007a).

A focus on evidence includes:

- identifying indicators of what is important to notice in students’ mathematical thinking,
- planning for ways to elicit that information,
- interpreting what the evidence means with respect to students’ learning, and then
- deciding how to respond on the basis of students’ understanding (Jacobs, Lamb, and Philipp 2010; Sleep and Boerst 2010; van Es 2010).
Evidence of student thinking should:

• Provide a window into students’ thinking;
• Help the teacher determine the extent to which students are reaching the math learning goals; and
• Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.
Students need opportunities to...

• organize and consolidate their mathematical thinking though communication;
• communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
• analyze and evaluate the mathematical thinking and strategies of others; and
• use the language of mathematics to express mathematical ideas precisely.