Supporting the Mathematics Process Goals through Research-based Teaching Practices: Part II

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Essential Understandings

• The mathematics process goals (problem solving, communication, reasoning, connections, and representations) play an instrumental role in the teaching and learning of mathematics with understanding.

• Teaching Practice: **Use and Connect Mathematical Representations**
  Students demonstrate a deeper mathematical understanding and enhanced problem solving abilities when they learn to represent, discuss and make connections among mathematical ideas in multiple forms.
Mathematics Process Goals

- Mathematical Understanding
- Problem Solving
- Connections
- Communication
- Representations
- Reasoning

Mathematical Understanding
Five goals...for students to

become mathematical problem solvers that

• communicate mathematically;  
• reason mathematically;  
• make mathematical connections; and  
• use mathematical representations to model and interpret practical situations

Process Goals
Process Goals: Connections and Representations

Fostering students’ ability to make mathematical connections and use effective and appropriate representations

- What does it mean?
- What does it look like in the classroom?
  - Role of teacher
  - Role of student
- How do you support teachers as they strive to help students make connections and use effective and appropriate representations in the classroom?

VDOE Mathematics Updates and Resources
Defining Connections and Representations

• What does it mean to make mathematical connections and use effective and appropriate representations?

• What words come to mind when you think about ...

  making connections?

  using effective and appropriate representations?

• Turn-n-Talk
Mathematical Connections

Students will **relate concepts and procedures** from different topics in mathematics to one another and see mathematics as an integrated field of study. Through the application of content and process skills, students will **make connections between different areas of mathematics and between mathematics and other disciplines, especially science**. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.

*From the Introduction to the 2009 Mathematics Standards of Learning*
Quality instruction allows all students to:

• recognize and use connections among mathematical ideas;
• understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
• recognize and apply mathematics in contexts outside of mathematics.

_NCTM Principles and Standards, 2000_
Teaching Elementary Students the Magic of Math

Think of a time when you saw (or taught) a lesson where students were making GREAT connections! Share with a partner or small group.
Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships with a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should move easily among different representations — graphical, numerical, algebraic, verbal, and physical — and recognize that representation is both a process and a product.

*From the Introduction to the 2009 Mathematics Standards of Learning*
Five Types of Mathematical Representations

Representations are useful in all areas of mathematics because they help us develop, share, and preserve our mathematical thoughts. They help to portray, clarify, or extend a mathematical idea by focusing on its essential features."

*NCTM Principles and Standards, 2000, p. 206*
Taking a Look Inside an Elementary Classroom

- First/Second Grade Combination Class
- Teacher – Tim Sears (14 year veteran)
- Probability and Animal Habitats

**Video Reflection Questions**

- What type of connections are being made in this lesson?
- What types of representations are students using?
- What does this tell you about their understanding of the concepts being developed in this lesson?
Use and Connect Mathematical Representations

WHAT ARE TEACHERS DOING:

- Selecting tasks that allow students to decide which representations to use in making sense of the problems.
- Allocating substantial instructional time for students to use, discuss, and make connections among representations.
- Introducing forms of representations that can be useful to students.
- Asking students to make math drawings or use other visual supports to explain and justify their reasoning.
- Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.
- Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems.

WHAT ARE STUDENTS DOING:

- Using multiple forms of representations to make sense of and understand mathematics.
- Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.
- Making choices about which forms of representations to use as tools for solving problems.
- Sketching diagrams to make sense of problem situations.
- Contextualizing mathematical ideas by connecting them to real-world situations.
- Considering the advantages or suitability of using various representations when solving problems.

The Value of Making Connections in the Mathematics Classroom

An emphasis on mathematical connections helps students build a disposition to use connections in solving mathematical problems, rather than see mathematics as a set of disconnected, isolated concepts and skills.

*NCTM Principles and Standards, 2000*
Experiencing Mathematics in Context

• Students should connect mathematical concepts to their daily lives.
• When students connect mathematical ideas, their understanding is deeper and more lasting.
• Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics.

*NCTM Principles and Standards, 2000*
Role of the Student

- *Create* and *use* representations to organize, record, and communicate mathematical ideas
- *Select, apply,* and *translate* among mathematical representations to solve problems
- *Use* representations to model and interpret physical, social, and mathematical phenomena

The Role of the Teacher

- **Create** a learning environment that encourages and supports the use of multiple representations
- **Model** the use of a variety of representations
- **Orchestrate** discussions where students share their representations and thinking
- **Support** students in making connections among multiple representations, to other math content and to real world contexts

Beliefs about Teaching and Learning Mathematics

STAND UP, PAIR UP, SPEAK UP

• *Introduce yourself to your new partner*

• *Review the Beliefs about Teaching and Learning Mathematics with your partner (p. 1 of Principles to Actions handout).*

• What productive beliefs are evident in the Mr. Sear’s classroom?

• How do those beliefs support students in making connections among different representations of the problem?
Students must be actively engaged in developing, interpreting, and critiquing a variety of representations. This type of work will lead to better understanding and effective, appropriate use of representation as a mathematical tool.

_NCTM Principles and Standards, 2000, p. 206_
Teachers must ask themselves,

“What models or materials (representations) will help convey the mathematical focus of today’s lesson?”

- Skip Fennell, Past-President NCTM
Mathematics Instructional Connections for Physical and Visual Representations

This document serves as a resource to assist teachers in connecting physical and visual representations to mathematical content. It is not meant to be an exhaustive list, nor does it mean that these representations may only be used with the identified content. Challenges or limitations may arise when using some representations to model content.

**Algebra Tiles**

- Operations with integers and polynomial expressions
- Model and factor linear and quadratic expressions

**Balance Scale**

- Properties of equality
- Equality
- Model one-variable equations
- Solve one-variable equations

**Base Ten Blocks**

- One-to-one correspondence
- Count and skip count
- Place value
- Represent whole numbers and decimals
- Compare and order whole numbers and decimals
- Operations with whole numbers and decimals
- Powers of 10
Role of the Mathematics Teacher Leader

How do you support teachers as they strive to help students make connections and use effective and appropriate representations in the classroom?
Mathematics Classroom LOOK FORS

- Are multiple representations and connections encouraged within the classroom? this lesson?
- What types of representation and/or connections are present in this activity?
- How are students creating and using representations to make sense of the mathematics?
- Are students describing and justifying their reasoning with drawings, diagrams, and other representations?
- How is the classroom discussion enhanced by students’ representations and/or connections?
- In what ways does the teacher assess students’ abilities to use representations meaningfully to solve problems?
Fostering Connections and Representations in Instruction

• Use data to determine areas of weakness (students and teachers)
• Be knowledgeable about and encourage the use of available resources
• Structure PLCs to explore resources provided
  – Process goals for students
  – Vertical articulation of content
  – Instructional strategies
  – Development of quality assessments
  – Effective use of data (formative and summative)
Five goals...for students to

become mathematical **problem solvers** that

- **communicate** mathematically;
- **reason** mathematically;
- make mathematical **connections**; and
- use mathematical **representations** to model and interpret practical situations

**Process Goals**
VDOE Updates and Resources
Mathematics SOL Revision Process

Anticipated Timeline

– February 2015 – timeline goes to the BOE
– March 2015 – public comment on 2009 standards
– Spring/Summer/Fall 2015 – review process
VDOE – Resources

• Standards of Learning
• Curriculum Framework
• Testing Blueprints
• SOL Practice Items and Tools Practice
• 2013 Released Test Items
• 2013 Student Performance Analysis
• Technical assistance document
• Plain English Mathematics Test Information
• ESS Sample Lesson Plans
• Instructional Videos
• Vocabulary Resources – K-8, Geometry, and Algebra Cards
<table>
<thead>
<tr>
<th>Mathematics Teaching Practices</th>
<th>VDOE Mathematics Professional Development Resources (available online)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Establish mathematics goals to focus learning. Effective teaching</td>
<td>Making Mathematical Connections and Using Representations (2014)</td>
</tr>
<tr>
<td>of mathematics establishes clear goals for the mathematics that</td>
<td>provides teachers with professional development focused on fostering</td>
</tr>
<tr>
<td>students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</td>
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<tr>
<td>Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</td>
<td></td>
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<tr>
<td>Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</td>
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</tr>
<tr>
<td>Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</td>
<td></td>
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<tr>
<td>Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.</td>
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<tr>
<td>Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</td>
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</tr>
<tr>
<td>Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</td>
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<tr>
<td>Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</td>
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</tbody>
</table>

Professional Development

VDOE provides targeted professional development through Mathematics Institutes. Since 2009, the institutes, framed by the five goals for students of becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations, have:

- outlined the content standard changes from the 2001 Mathematics SOL to the 2009 Mathematics Standards of Learning (SOL) – 2009;
- provided training in the vertical progression of content and pedagogy – 2010, 2014;
- provided instructional guidance in content areas of greatest challenge – 2010, 2014;
- provided professional development resources focused on facilitating students’ mathematical understanding through problem solving, communication, and reasoning – 2011;
- provided professional development resources focused on the use of formative assessment resources to drive instructional decisions – 2012;
- provided professional development on the analysis and modification of existing resources to match student learning expectations and on promoting problem solving – 2013; and
- provided professional development on fostering students’ abilities to make mathematical connections and use effective and appropriate representations in mathematics – 2014.

Mathematics SOL Institutes

- 2014 – Making Mathematical Connections and Using Representations
- 2013 – Modifying Resources to Promote Problem Solving and Critical Thinking
- 2012 – Using Formative Assessment Resources to Drive Instructional Decisions

STANDARDS OF LEARNING

Mathematics K-12
Includes:
- Curriculum Frameworks
- Enhanced Scope & Sequence Guides
- Test Blueprints
- Released Tests and
- Practice Items
“The content of the mathematics standards is intended to support the five goals for students”

- 2009 Mathematics Standards of Learning
2009 – Changes in Mathematics Standards

Changes to Standards of Learning (SOL) Content

- Increase rigor through an emphasis on multistep problems and application
- Move content between grade levels/subjects to improve the vertical progression
- Remove content from a grade levels/subjects
- Remove content repeated among grade levels
- Add new content
- Reduce language that limits content
### Mathematical Problem Solving

- Mathematical Communication
- Mathematical Reasoning
- Mathematical Connections
- Mathematical Representations

#### Vertical Articulation Technical Assistance Document - Grade 3 Through Grade 6

<table>
<thead>
<tr>
<th>Counting/ Cardinality/ Place Value</th>
<th>Grade 3</th>
<th>Grade 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 a) read/write 6-digit numerals, ID place value/value of each digit; b) round whole numbers 9,999 or less to nearest 10/100/1000; c) compare two whole numbers 0 - 9,999 w/ symbols/words</td>
<td>4.1 a) ID orally/in writing place value for each digit in a whole number through millions; b) compare two whole numbers through millions w/ symbols; c) round whole numbers through millions to nearest 10/100/1,000/10,000/100,000/1,000,000</td>
<td></td>
</tr>
<tr>
<td>3.2 a) model fractions, equivalence</td>
<td>4.2 a) compare and order fractions/mixed numbers; b) represent equivalent fractions; c) ID division statement that represents a fraction</td>
<td></td>
</tr>
<tr>
<td>3.3 a) name/write fractions rep by model; model/write fraction’s names; c) compare fractions w/like/unlike denominators</td>
<td>4.3 a) read/write/represent/ID decimals through thousandths; b) round to whole, tenth, hundredth; c) compare/order; d) write decimal and fraction equiv from a model</td>
<td></td>
</tr>
<tr>
<td>3.4 a) represent multi/div using area/set/number line models, create/solve problems involving multi of two whole numbers 99 or less and 5 or less</td>
<td>5.1 a) determine common multiples/factors</td>
<td></td>
</tr>
<tr>
<td>3.5 a) compare/order/make decimals</td>
<td>5.2 a) equivalent decimals</td>
<td></td>
</tr>
<tr>
<td>3.6 represent multi/div using area/set/number line models, create/solve problems involving multi of two whole numbers 99 or less and 5 or less</td>
<td>5.3 a) compare/order/make decimals</td>
<td></td>
</tr>
</tbody>
</table>

#### Mathematics INSTRUCTIONAL VIDEOS FOR TEACHERS

These instructional videos are provided as support for the implementation of the 2009 Mathematics Standards.

- **ON THIS PAGE**
  - Strategies Across the Strands
  - Number & Number Sense
  - Computation & Estimation
  - Probability & Statistics
  - Patterns, Functions & Algebra
  - Additional Resources

- **Mathematical Problem Solving**
  - Play Video: "Working with Vocabulary / Concept Development (grades 4-6)"
    - Dr. Lois Williams, VDOE mathematics specialist, on the Frayer Model for vocabulary/concept development
    - Handout available: "Working with Vocabulary / Concept Development (Word)"
  - Play Video: "Vocabulary (grades K-2)"
    - Dr. Lois Williams shares a technique to assist students who confuse common mathematics terms such as "difference," "sum," and "total.
  - Play Video: "Notebooks for Organization (grades K-2)"
    - Dr. Lois Williams explains an organizational system for middle school mathematics classes.
  - Play Video: "Multi-Step Problem Solving (grades 4-8)"
    - Cheryl Gray, Sophisylvania County Schools middle school mathematics specialist, on how to approach a word problem
    - Handout available: "Multi-Step Problem Solving (PPT)"
  - Play Video: "Scientific Calculator Use (grades K-2)"
    - Dr. Lois Williams explains the use of scientific calculators in Virginia middle schools
    - Handout available: "Scientific Calculator Manual (PDF)"

#### Number & Number Sense

- **NEW** Play Video: "Developing Early Number Sense (grades K-2)"
  - Laura Domanik, Hanover County, provides instructional strategies for counting and vocabulary. The strategies focus on more than (+1), one less than (-1), basic fact concepts of +1 and -1, and missing addends.
- **NEW** Play Video: "Using a Beaded Number Line (grades K-2)"
  - Laura Scardino, Hanover County, provides instructional strategies for using beaded number lines to determine backwards, rounding, adding, and subtracting.
- **NEW** Play Video: "Modeling Equality (grades K-2)"
  - Delia Gerino, Hanover County, provides activities with one and number balances to represent and understand equal.
Mathematical Problem Solving
Mathematical Communication
Mathematical Reasoning
Mathematical Connections
Mathematical Representations
2012 – Using Formative Assessment Tools

Mathematical Problem Solving
Mathematical Communication
Mathematical Reasoning
Mathematical Connections
Mathematical Representations
2013 – Modifying Instructional Resources to Promote Problem Solving and Critical Thinking

Virginia Department of Education
Fall 2013 Mathematics Standards of Learning Institutes

Facilitator’s Guide
Grade Band 3-5

2013 Mathematics SOL Institutes
The purpose of the 2013 Mathematics SOL Institutes is to provide teachers with professional development focused on the relationship between curriculum, instruction, and assessment, by targeting the processes of analysis and modification of existing resources to match student learning expectations and promote problem solving.

Introduction and Instructions
This grade-band professional development will be comprised of two components:

- Module 1 Parts 1-4: Analyzing and modifying assessments – Participants will compare expectations of SOL and Curriculum Framework to an assessment and modify it to meet intended expectations.
- Module 2 Parts 1-3: Modifying mathematical tasks to promote problem solving – Participants will modify existing mathematical tasks to emphasize the use of process skills and problem solving.

The product of the 2013 Mathematics SOL Institutes is a set of online professional development modules designed to be used by a group of teachers of a specific grade level or course. Modifications could be made to adapt the professional development for more than one grade level/course or for large groups. Each group of teachers should select a facilitator for which this Facilitator’s Guide was written. Facilitators should review the activities and handouts prior to facilitating this professional development.

<table>
<thead>
<tr>
<th>Approximate Time</th>
<th>Facilitator Instructions</th>
<th>Materials</th>
</tr>
</thead>
</table>
| 30 minutes total | Module 1 Part 1: Analysis of Assessments | - Mathematics Assessment (select the assessment for your course)  
  - Grade 3  
  - Grade 4  
  - Grade 5  
  - Assessment Analysis Guiding Questions |
| 15 minutes        | 1. Select and distribute the appropriate assessment for your whole group.  
  2. Ask participants to complete the assessment individually. While working, participants should be thinking about whether or not it is a “good” assessment and why.  
  3. Have participants discuss their conclusions with a partner or small group.  
  4. Have small groups share and record their conclusions and justifications with the whole group.  
  5. View the assessment analysis guiding questions document and discuss similarities and differences. |
| 15 minutes        | Module 1 Part 2: Analysis of Assessments | - Mathematics Communication  
  - Mathematical Reasoning  
  - Mathematical Connections  
  - Mathematical Representations |

Assessment Analysis Guiding Questions

SOL ALIGNMENT

- Does the assessment assess the standard and targeted components of the Essential Knowledge and Skills in the Curriculum Framework?
- Does the assessment reflect the requirements of the verbs found in the Curriculum Framework?
- Did instruction go beyond the standard?
  - If so, does the assessment reflect your instruction?
  - If not, does the assessment adhere to the parameters of the SOL?

LEVEL OF COGNITIVE DEMAND

- Does the assessment have an appropriate variety of questions requiring various levels of cognitive demand?
- Does the assessment require students to explain and justify?

FORMAT

- Does the assessment provide various ways for students to demonstrate understanding (open response, multiple...
# Levels of Classroom Discourse

<table>
<thead>
<tr>
<th>Teacher role</th>
<th>Questioning</th>
<th>Explaining mathematical thinking</th>
<th>Mathematical representations</th>
<th>Building student responsibility within the community</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>Teacher is at the front of the room and dominates conversation. Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.</td>
<td>Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.</td>
<td>Representations are missing, or teacher shows them to students.</td>
<td>Culture supports students keeping ideas to themselves or just providing answers when asked.</td>
</tr>
<tr>
<td>Level 1</td>
<td>Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only. Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.</td>
<td>Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.</td>
<td>Students learn to create math drawings to depict their mathematical thinking.</td>
<td>Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.</td>
</tr>
<tr>
<td>Level 2</td>
<td>Teacher facilitates conversation between students, and encourages students to ask questions of one another. Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.</td>
<td>Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.</td>
<td>Students label their math drawings so that others are able to follow their mathematical thinking.</td>
<td>Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.</td>
</tr>
<tr>
<td>Level 3</td>
<td>Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others. Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask “why” and call for justification. Teacher questions may still guide discourse.</td>
<td>Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.</td>
<td>Students follow and help shape the descriptions of others’ math thinking through math drawings and may suggest edits in others’ math drawings.</td>
<td>Students believe that they are math leaders and can help shape the thinking of others. They help shape others’ math thinking in supportive, collegial ways and accept the same support from others.</td>
</tr>
</tbody>
</table>

Fig. 11. Levels of classroom discourse. From Hufford-Ackles, Fuson, and Sherin (2014), table 1.
QUESTIONS?

debra.delozier@doe.virginia.gov
Resources

*Edutopia. Teaching Children the Magic of Math, YouTube. June 23, 2014, [https://www.youtube.com/watch?v=E91gAjunqOM](https://www.youtube.com/watch?v=E91gAjunqOM)*


*National Council of Teachers of Mathematics. 2000 Principals and Standards for School Mathematics. Reston, VA.*

*Virginia Standards of Learning, 2009.*

*Virginia Department of Education Website*