Mathematics and RtI

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What Are the Major Changes in the 2009 Mathematics K-8 Standards of Learning

2009 *Mathematics Standards of Learning* Vertical Articulation by Reporting Category

K-8 Number and Number Sense

K: count to 100 (1’s, 5’s, 10’s)/back from 10/unit fractions - half, fourth

1: write numerals 0-100/count to 100 (2’s)/back from 30/unit fractions (2, 3, 4)

2: 3-digit place value/ordinality/advanced counting

3: multistep problem solving/mixed numbers

4: 7-digit place value/equivalent fractions/compare numbers (magnitude/decimals)

5: thousandths place value/fractions—decimals/prime, composite, even, odd numbers

6: ratios/fractions—decimals—percent/integers/positive exponents

7: scientific notation (negative exponents)/absolute value/represent sequences with variable expressions

8: comparing and ordering fractions, decimals, percent, scientific notation

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K-8 Computation and Estimation

K: ---

1: + facts to 18 and –/ orders of magnitude in place value

2: + facts to 20 and –/ +, – 2 digit numbers

3: x facts to 12's and ÷/ +, - multi-step problems

4: +, -, x, ÷ whole numbers/ multiples/ factors/ +, - fractions and decimals

5: +, -, x, ÷ decimals/ +, - fraction problems/ order of operations

6: x, ÷ fractions/ order of operations

7: +, -, x, ÷ integers/ proportional reasoning

8: proportional reasoning/ percent increase decrease/ squares and square roots

K-8 Measurement

K: attributes/ money/ time
1: money/ time/calendar/ nonstandard units
2: money/ time/ calendar/ temperature/ standard units
3: money/ elapsed time/ standard units/ perimeter and area
4: elapsed time/ standard units/ equivalent measurements
5: elapsed time/ standard units/ equivalent measurements/perimeter and area/ circles-circumference and area/ angles
6: US Customary and metric-convert between systems/ π/ area, perimeter, and volume
7: volume and surface area – prisms and cylinders
8: applications with volume and surface area/ changing attributes′ affect on volume and surface area

K-8 Geometry

K: attributes – plane figures
1: attributes – plane figures
2: attributes – plane and solid figures
3: characteristics of plane and solid figures/ congruence
4: plane figures/ congruence/ transformations/ polygons
5: angles/ triangles/ define plane figures
6: coordinate plane/ congruence/ quadrilaterals
7: quadrilaterals in detail/ transformations in the coordinate plane
8: 3-D models given 2-D representations/ Pythagorean Theorem/ applications with plane figures

K-8 Probability and Statistics

K: data collection and display

1: data collection and display

2: data collection and display/ elementary analysis of data

3: data collection and display (more complex displays)/ simple probability

4: data collection and display/ simple analysis of data/ simple probability

5: sample space and probability/ measures of center/ range/ mean as fair share/ data collection and display - stem and leaf

6: more complex data displays/ mean as balance point/ dependent and independent events – probability

7: probability-experimental and theoretical/ Fundamental Counting Principle/ complex data displays

8: dependent and independent events/ scatterplots

K-8 Patterns, Functions, and Algebra

K: Attributes and patterns
1: attributes and patterns (growing and repeating)/ =
2: patterns/ simple equations; =, ≠
3: patterns/ properties – identity and commutative
4: numerical and geometric patterns/ = in equation/ associative property
5: variable/ distributive property
6: geometric and arithmetic sequences (simple)/ 1-step linear equations/ identity, * property of 0/ multiplicative inverse/ graphing inequalities
7: multiple representations/ 1 and 2-step equations and inequalities/ apply properties
8: representations of relations and functions/ multi-step equations/ graphing linear equations

Arithmetic and Mathematics

What are they?

Arithmetic

Arithmetic is traditionally defined as the oldest and most elementary branch of mathematics. Used by almost everyone in tasks ranging from simple counting to advanced science and business calculations.

Arithmetic involves the study of quantity, especially as the result of combining numbers. It refers to the simpler properties when using the traditional operations of addition, subtraction, multiplication and division with smaller values of numbers.

Arithmetic Operations

The basic arithmetic operations are addition, subtraction, multiplication and division.
This subject also includes more advanced operations, such as manipulations of percentages, square roots, exponentiation, and logarithmic functions.
Arithmetic is performed according to an order of operations. Any set of objects upon which all four operations of arithmetic can be performed (except division by zero), and wherein these four operations obey the usual laws, is called a field.

Mathematics

Mathematics is traditionally defined as the study of quantity, structure, space, and change.

Mathematicians seek out patterns, formulate new conjectures (theorem), and establish truth by rigorous deduction (reasoning) from appropriately chosen axioms (hypotheses) and definitions.

Mathematics

Mathematics is very broadly divided into foundations, algebra, analysis, geometry, and applied mathematics, which includes theoretical computer science.

Arithmetic, Mathematics, and Algebra

The fundamentals of algebra are rooted in the algebraic structure of arithmetic. This is not arithmetic in the traditional sense.

This concept is reflected by bringing to light the algebraic character inherent in arithmetic.

How Do We Bring to Light the Algebraic Character Inherent in Arithmetic?

It takes much understanding of mathematics to recognize that arithmetic is far more than adding, subtracting, multiplying, and dividing numbers in a traditional sense.

Traditional arithmetic relies on procedural knowledge. A more nontraditional approach considers the need for conceptual understanding.

To enhance the algebraic character of arithmetic, we need to understand that all whole-number computations are nothing but a sequence of single-digit computations artfully put together.

This is the kind of thinking students will need to succeed in algebra and advanced mathematics.

How Do We Bring to Light the Algebraic Character Inherent in Arithmetic?

Adding whole numbers:

$$\begin{array}{r} 45 \\ \underline{+31} \\ 6 \end{array}$$

Add the 5 and 1 to get 6. Record the 6, then

$$\begin{array}{r} 45 \\ \underline{+31} \\ 76 \end{array}$$

Add the 4 and 3 to get 7. Record the 7.
This gives the answer 76. Right?

Not exactly.
This is a procedural, traditional approach to teaching addition of two-digit numerals.

How can we teach this skill conceptually and reflect the algebraic character that is inherent in this procedural, traditional model?

Maybe Like This:

Adding whole numbers:

45	40 + 5	40 + 5	45
+31	30 + 1	30 + 1	+31
76	? ?	70 + 6	76

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The essence of the addition algorithm lies in the abstract understanding that the arithmetic computations with whole numbers, no matter how large, can all be reduced to computations with single-digit numbers.

45	45723
+31	+31251
76	76974

Whether the 4 stands for 40 or 40,000 and the 3 stands for 30 or 30,000 is completely irrelevant.

If students have been given the proper foundation in grade 2, then in grade 5, they should be able to do the following:

$$45 + 31 = (4 \times 10) + 5 + (3 \times 10) + 1$$
$$= (4 \times 10) + (3 \times 10) + 5 + 1$$
$$= (4 + 3) \times 10 + (5 + 1)$$

This is the Distributive Property

This method focuses on the conceptual aspect of the addition and multiplication skill with emphasis on place value.

The procedural, traditional model uses counts of "one" to derive an answer without any understanding of place value. Without a model representation, there is no connection between the arithmetic, place value concepts, and algebra.

Rules of Arithmetic

The Rules of Arithmetic refer to properties. There are nine:

Commutative + x
Associative + x
Distributive a, b, and c, a(b+c) = ab + ac
Identity + x
Inverse + x

These "rules" are found in the Patterns, Functions, and Algebra strand beginning in grade 3 and continuing through grade 7.

Researchers have found that young students have implicit knowledge of properties in mathematics. Many students recognize that numbers in a simple addition problem are interchangeable,
e.g., that 5 + 2 is equivalent to 2 + 5. It is simply a property that has not been taught explicitly, therefore, students do not make the connection to other contexts such as algebra.
They may not understand that the commutative property applies to more "complicated mathematics" like larger numbers and fractions. Elementary students can learn to think about arithmetic in ways that both enhance their early learning of arithmetic and provide a foundation for learning algebra.

The Commutative Rule for Addition: The value of a sum does not depend on the order of the addends. In other words, for any two nonnegative integers, a and b,

$$a + b = b + a.$$

The Associative Rule for Addition: When adding three numbers, the value of the sum does not depend on the way the numbers are combined into pairwise sums. More precisely, for any three nonnegative integers a, b, and c,

$$(a + b) + c = a + (b + c).$$

The Distributive Rule: The product of one nonnegative integer times a sum of two others is obtained by adding the first number times the second plus the first number times the third. More precisely, for any nonnegative integers a, b, and c,

$$a \times (b + c) = a \times b + a \times c.$$

The Commutative Rule for Multiplication: For any two nonnegative integers n and m, the products $m \times n$ and $n \times m$ are the same:

$$m \times n = n \times n.$$

The Associative Rule for Multiplication: For any nonnegative integers ℓ, m, and n,

$$\ell \times (m \times n) = (\ell \times m) \times n.$$

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4. Base 10 Components and Algebra

The Role of the Rules of Arithmetic in Algebra: There are several ways in which having students make explicit use of single-place numbers in arithmetic can help prepare them for algebra. One is that, as they develop facility in applying the Rules of Arithmetic to solve problems with them, they are, in effect, practicing the same computational techniques they will use in algebra. The reason is that the Rules of Arithmetic are also the rules for manipulating algebraic expressions. The Rules provide a remarkably compact summary for much of what constitutes legitimate algebraic manipulation. Moreover, they themselves are most succinctly expressed as algebraic identities. So, for students who have developed an increased awareness of them, writing them symbolically can become part of a natural transition to the use of algebraic notation for expressing general properties.

Base 10 Form and Polynomials in 10: Working with base 10 components can also prepare students for algebra through the close relationship between decimal arithmetic and the algebra of polynomials. In a certain sense base 10 notation exploits algebra in the service of arithmetic because decimal numbers can be usefully thought of as "polynomials in 10." Emphasizing this relationship can both shed light on arithmetic and make algebra more familiar and learnable.

For example, consider the numbers 21 and 13 and the expressions $2x + 1$ and $x + 3$, and observe the similarity between the computations for $21 + 13$ and $(2x + 1) + (x + 3)$, with the symbol x playing the role of the number 10 and using juxtaposition of symbols to represent multiplication in algebraic expressions.

Line 1	$21 + 13$	
Line 2	$= (20 + 1) + (10 + 3)$	$(2x + 1) + (x + 3)$
Line 3	$= (2 \times 10 + 1) + (10 + 3)$	$= (2x + 1) + (x + 3)$
Line 4	$= (2 \times 10 + 10) + (1 + 3)$	$= (2x + x) + (1 + 3)$
Line 5	$= (2 \times 10 + 1 \times 10) + 4$	$= (2x + 1x) + 4$
Line 6	$= (2 + 1) \times 10 + 4$	$= (2 + 1)x + 4$
Line 7	$= 3 \times 10 + 4$	$= 3x + 4$
Line 8	$= 30 + 4$	$= 3x + 4$
Line 9	$= 34$	

Part of the genius of the place-value system for representing numbers is that the same strategy used to express and compute with nonnegative integers extends with only minor modifications to a much broader class of numbers that involve fractional quantities.

Place Value

When the language of arithmetic, specifically place value, is omitted, students develop misconceptions about the basic arithmetic operations.

These misconceptions can and do create barriers for learning and applying computational skills to problems, specifically those of an algebraic nature.

Six Essential Components for Developing Readiness for Algebra

1. **Targeted Concepts**
2. **Algebraic Structure of Arithmetic**
3. **Prior Knowledge and Misconceptions**
4. **Mathematics as a Language**
5. **How and What of Learning Mathematics**
6. **Instructional Support**

How is This Done in the Classroom?

By teaching mathematics as

1. **Problem Solving**
2. **Communication**
3. **Reasoning**
4. **Connections**
5. **Representations**

How is This Done in the Classroom?

1. **Targeted Concepts**
   a. focused problem solving (not just to solve)
   b. identified misconceptions such as equals means an answer, letters represent specific numbers, shapes with bigger areas have bigger perimeters, the more digits a number has the larger it is, small numbers cannot be divided by larger numbers, etc.)

How is This Done in the Classroom?

2. Algebraic Structure of Arithmetic

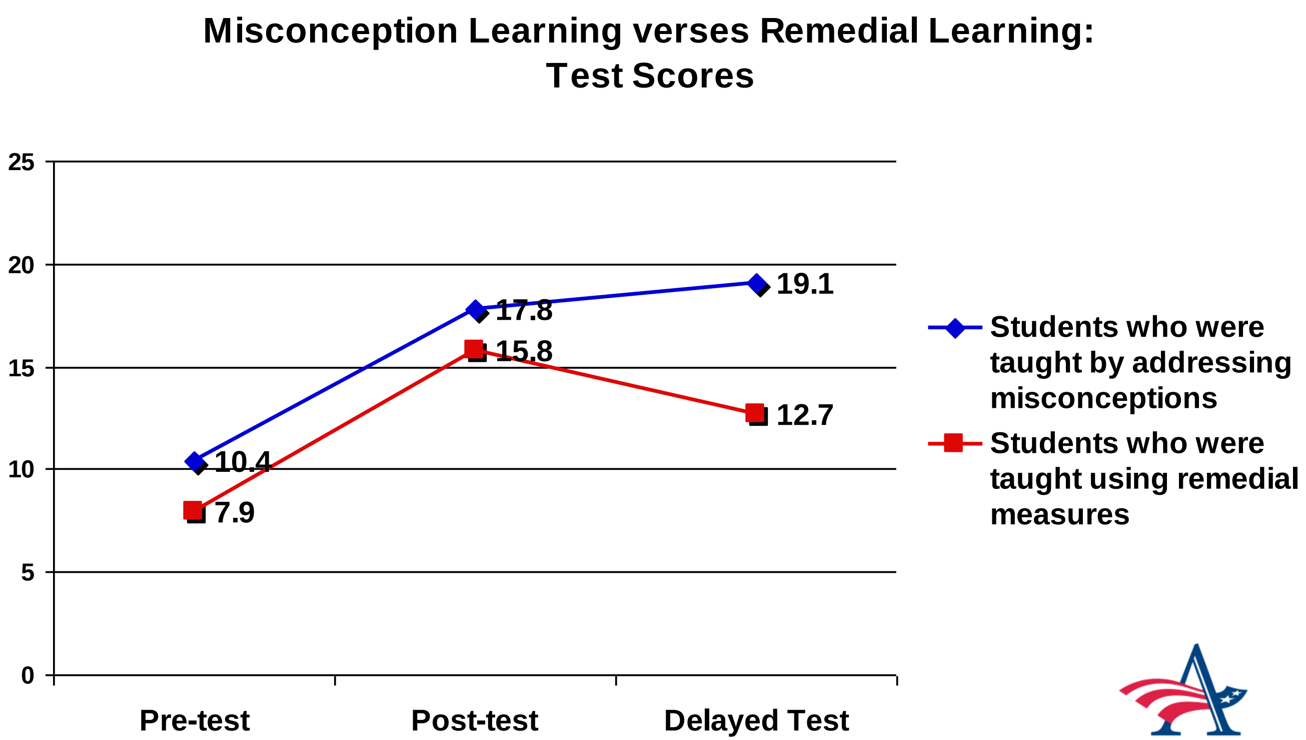
a. use of Algebraic symbols or notation
b. number lines
c. function tables (T-tables)
d. graphs

Equals does not always mean an answer
Function in the context of addition [f(x) = x + 3]
Problem solving not just as quantities but as sets of possible values

How is This Done in the Classroom?

3. **Prior Knowledge and Misconceptions**
   a. deeper understanding and long-term learning
   b. errors reveal student reasoning and focus is on the mathematics rather than the answer (create new understanding)
   c. focus on explaining and using mathematical ideas for transfer to other topics
   d. identified misconceptions such as equals means an answer, letters represent specific numbers, shapes with bigger areas have bigger perimeters, the more digits a number has the larger it is, small numbers cannot be divided by larger numbers, etc.)

Remediation vs. Revising Misconceptions



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How is This Done in the Classroom?

4. **Language**
   a. tool for organizing thinking
   b. words used in the context of mathematics
   c. assists in concept development
   d. builds connections among mathematical ideas, skills

How is This Done in the Classroom?

5. The How and What of Learning Mathematics
   a. knowledge gained through experiences – tasks
   b. asking questions
   c. intelligence is the process by which one acquires knowledge and cognitive ability
   d. collaboration promotes deep conceptual insights and shifts in perspective that lead to increases in student understanding and retention of concepts

How is This Done in the Classroom?

6. **Instructional Support**
   a. focused professional development
   b. aligned curriculum
   c. assessments that mirror and inform instruction by measuring a specific outcome, competency

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The current grade 5 student SOL data from grade 4 indicated:

49% passed the grade 4 SOL leaving 51% as failing.

These failing students fell into three AYP subgroups:

- disadvantaged,
- disabled, and
- minority

Is behavior an issue? Yes!

Why?

The traditional approach to mathematics did not work. The traditional approach is seatwork with paper and pencil without teacher support.

Why?

The students did not have the prior knowledge to do the work independently.

Engagement?

Students were disengaged and this set them up for continued failure.

- Incomplete class work became homework. The cycle of not learning anything continued.

How do we break this cycle? Jean said instruction needed to change and time is of the essence. It is now January.

How is this done?

Suggestions:
-put homework aside
-use formative assessment to determine instructional needs (division of whole numbers)
What next?

Assessment errors:

Jean determined that the students were unable to divide a 4 digit by one digit (456 divided by 8)

The teacher had modeled 4 strategies for division, all of which only served to confuse the students. They were confused because of division misconceptions.

The misconceptions were:

1. No connection to multiplication (division was taught as a discrete skill)
2. Fragmented place value concepts
3. General misconception-division was an action that we do with numbers without understanding (procedural/conceptual)-lack of number sense

Knowing this, what is the intervention step that needs taking?

The classroom teacher ranked students based on their assessment Score-Range: 15%-100% (65 out of 114 grade 5 students scored below the 70% SOL benchmark)

Jean selected students across the quartile bands (0%-25% and 26%-50%) to provide appropriate interventions.

What were the interventions?

Remember, this is only one strand of the 6 strands.

-no pencil/paper

-whiteboards/markers/eraser as a tool for improving engagement

-teacher modeled and provided appropriate mathematics vocabulary

-discourse (why did Jean do what she did-they were getting it!)

Estimation

- played a key role in developing conceptual understanding of division, and
- connecting division to its inverse operation.

What are some of the more common mathematical errors students make?

How do we recognize them, diagnose them, and then provide appropriate interventions?