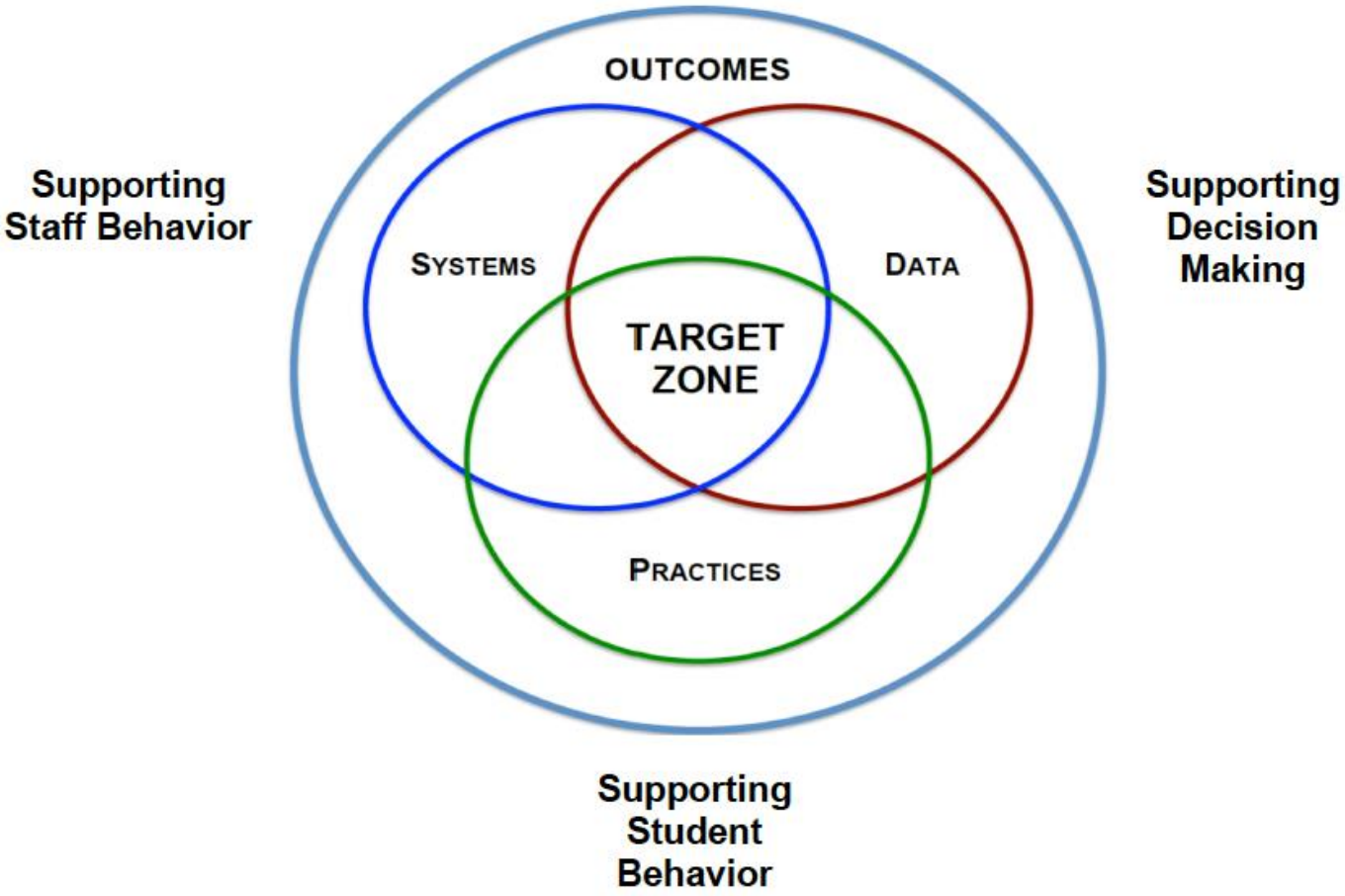
# EVIDENCE BASED PRACTICES:

## FOCUS ON MATHEMATICS IN A COORDINATED SYSTEM OF SUPPORT




## TARGET AREAS: SYSTEMS AND PRACTICES

**"We can, whenever and wherever we choose, successfully teach all children whose schooling is of interest to us. We already know more than we need to do that. Whether or not we do it must finally depend on how we feel about the fact that we haven't so far."**

~Ron Edmonds

**PREPARED FOR THE COLLABORATIVE LEARNING NETWORK OF THE**

## VIRGINIA TIERED SYSTEM OF SUPPORTS

**BY DAN MULLIGAN, FLEXIBLECREATIVITY.COM**

**JANUARY 2015**

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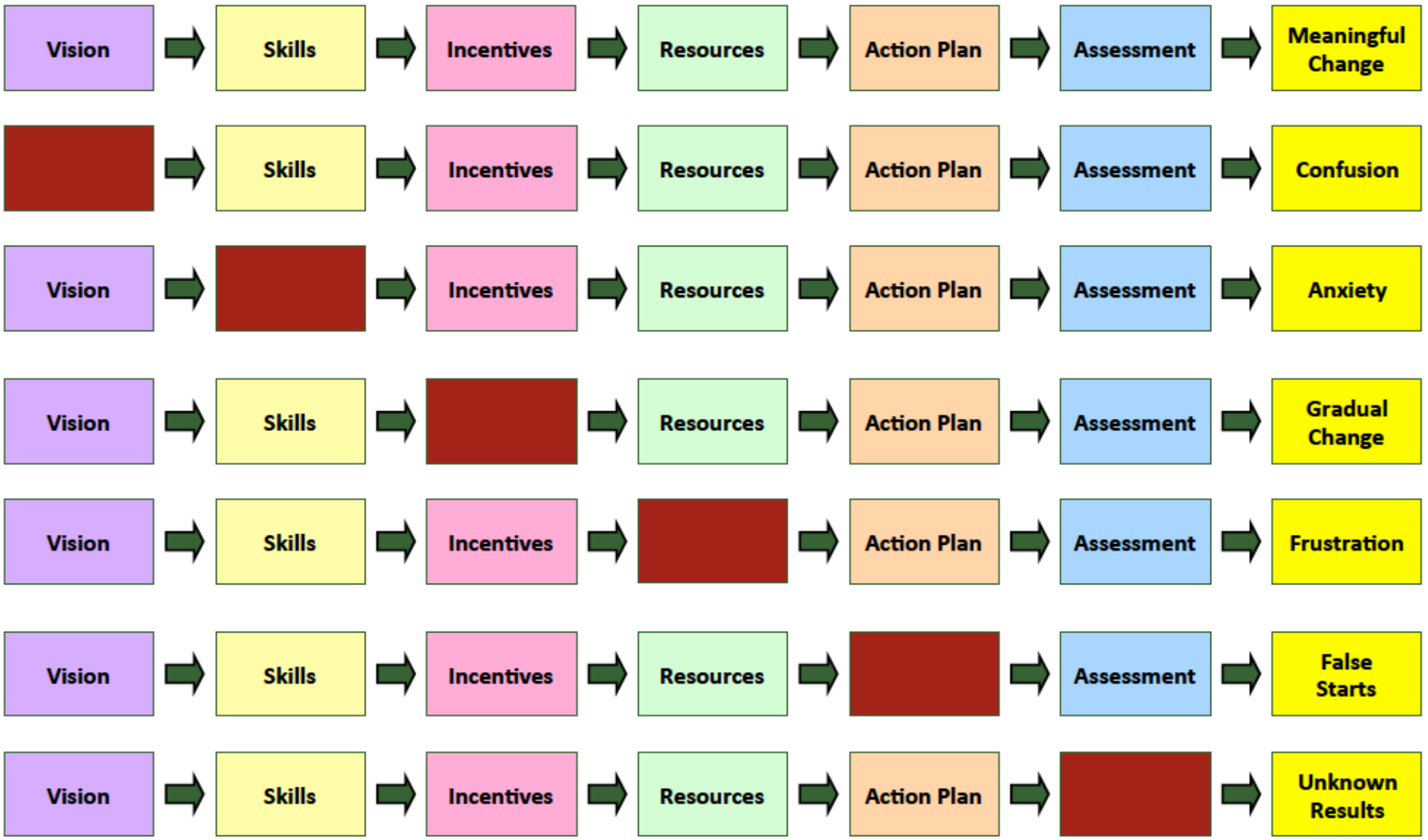
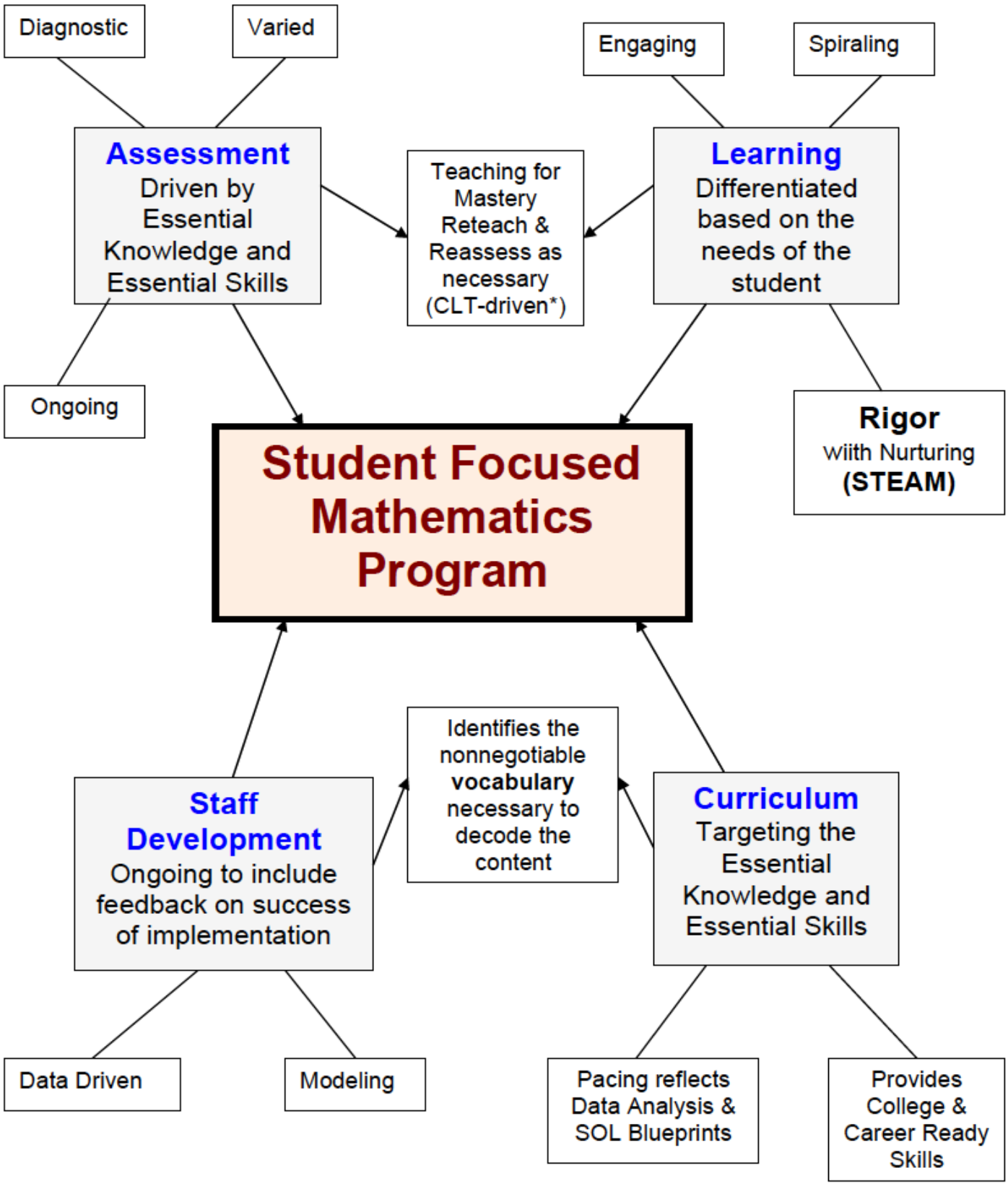
# Managing Complex Change




Adapted from Delores Ambrose, 1987

# Components of a Focused Mathematics Program

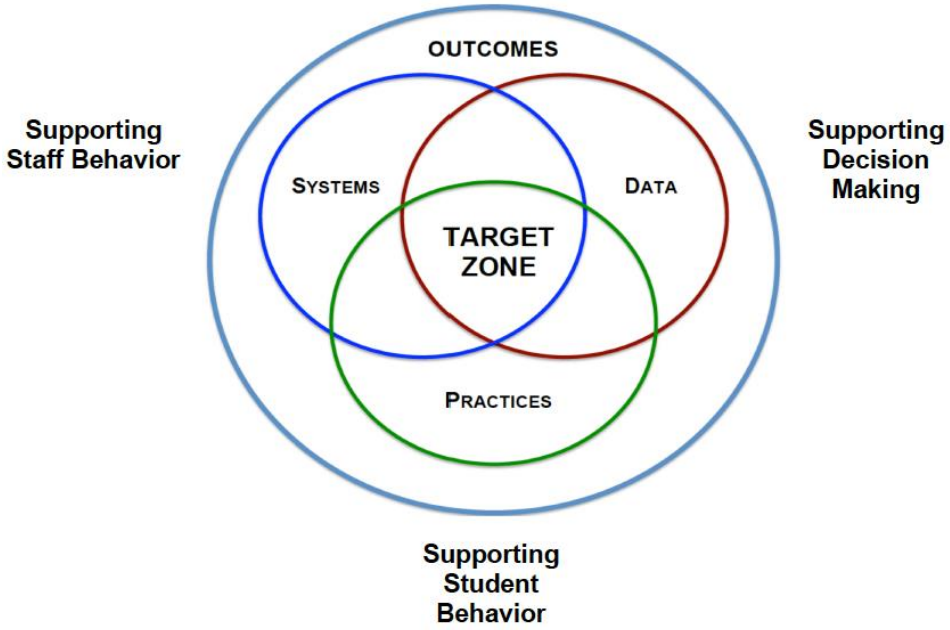



*CLT – Collaborative Learning Team

**Systems:** supports that are needed to enable the accurate and durable implementation of the practices of an inclusive math program.

| | |
|---|---|
| **Know**<br><br>1. The organizational structure (the flow from central office, school, grade level or content area teams, classroom)<br>2. The ways in which we will build communication skills among staff (listen to understand, reciprocity) | **My notes:** |
| **Understand**<br><br>1. Our systems are aligned for the results we get<br>2. Our systems are experienced differently depending on the specific group of stakeholders<br>3. The importance of a guidance document<br>4. Understand communication as critical system component | |
| **Do**<br><br>1. Begin to build a guidance document that integrates data, practices, and systems for Tier 1 | |

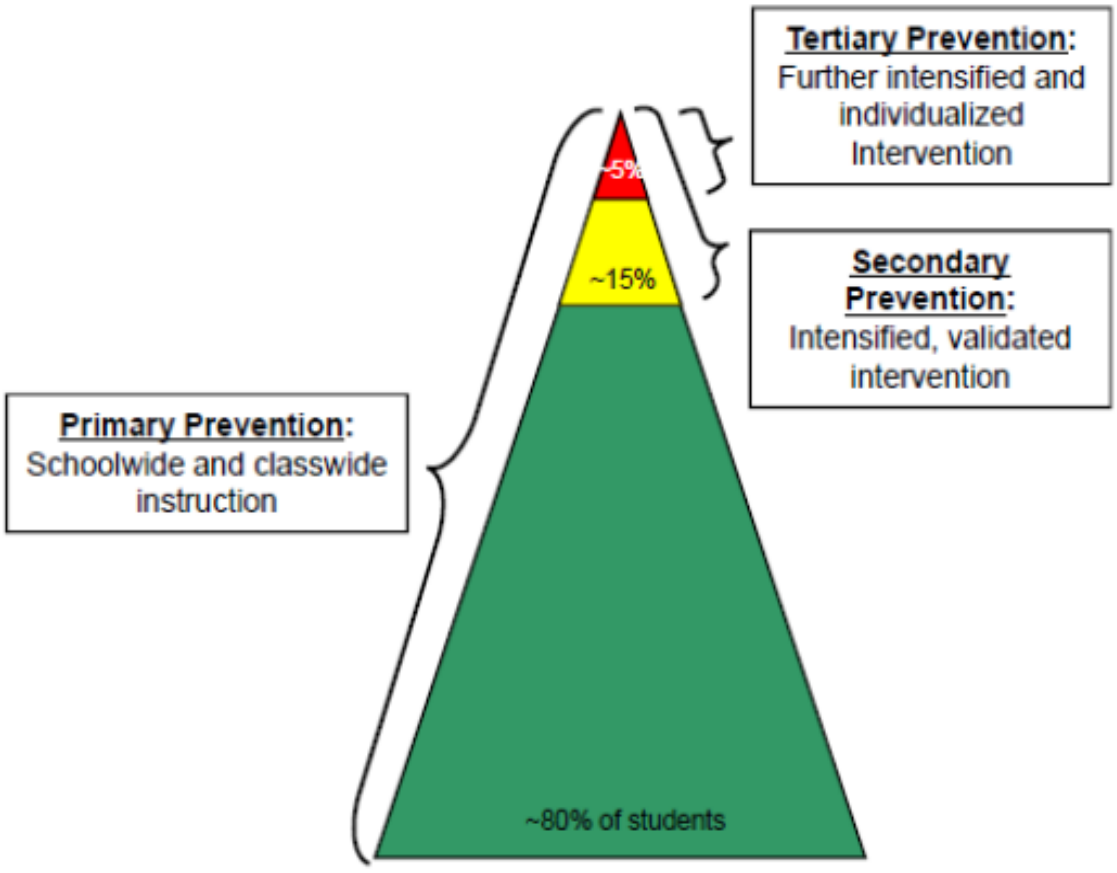
## Evidence Based Practice Organizer



## VIRGINIA TIERED SYSTEM OF SUPPORTS

Eleven considerations serve as the foundations for implementation:

- Implementation is interactive and informing.
- Implementation involves stakeholders at multiple levels.
- Implementation occurs in phases.
- Sustainable implementation requires continuous regeneration.
- Implementation success is based on multiple criteria.
- Implementation selects scalable evidence-based practice.
- Practices must be implemented with integrity.
- Policy and practice inform each other.
- Implementation is systemic.
- Implementation decisions are based on responsiveness to intervention.
- Implementation is team-based, strategic, action planning process.



# SYSTEM A. CURRICULUM

| Guiding Question | Resource Page # | Current Reality | Next Steps |
|---|---|---|---|
| To what extent is the division/school curriculum aligned to the Essential Knowledge of the Curriculum Framework? | | | |
| To what extent is the division/school curriculum aligned to the Essential Skills of the Curriculum Framework and College and Career Readiness skills? | | | |
| To what extent is the division/school curriculum aligned to the Essential Vocabulary of the Curriculum Framework? | | | |
| To what extent are reading comprehension strategies reinforced in the content area? | | | |
| To what extent does the division/school/class/ student pacing aligned to the SOL Blueprints, student achievement data analysis, and considered dynamic? | | | |
| | | | |

**3.11 The student will**

**a) tell time to the nearest minute, using analog and digital clocks; and**

**b) determine elapsed time in one-hour increments over a 12-hour period.**

| UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only) | ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|---|
| • While digital clocks make reading time easy, it is necessary to ensure that students understand that there are sixty minutes in an hour.<br>• Use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise when time is read.<br>• Students need to understand that time has passed or will pass.<br>• Elapsed time is the amount of time that has passed between two given times.<br>• Elapsed time should be modeled and demonstrated using geared analog clocks and timelines.<br>• It is necessary to ensure that students understand that there are sixty minutes in an hour when using analog and digital clocks.<br>• Elapsed time can be found by counting on from the beginning time to the finishing time.<br>– Count the number of whole hours between the beginning time and the finishing time. For example, to find the elapsed time between 7 a.m. and 10 a.m., students can count on to find the difference between the times (7 and 10), so the total elapsed time is 3 hours. | **All students should**<br>• Apply appropriate techniques to determine time to the nearest minute, using analog and digital clocks.<br>• Understand how to determine elapsed time in one-hour increments over a 12-hour period. | **The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**<br>• Tell time to the nearest minute, using analog and digital clocks.<br>• Match the times shown on analog and digital clocks to written times and to each other.<br>• When given the beginning time and ending time, determine the elapsed time in one-hour increments within a 12-hour period (times do not cross between a.m. and p.m.).<br>• Solve practical problems in relation to time that has elapsed. |

| Essential Vocabulary | Driving Question | Minimum Competencies |
|---|---|---|

# TARGET FOR UNDERSTANDING – ESSENTIAL KNOWLEDGE $ SKILLS

**VERSION 1:**
**PROVIDE A SYSTEM FOR PLANNING ASSESSMENT AND LEARNING**

## KINDERGARTEN

## MEASUREMENT

***Essential Knowledge Skills and Processes – At a Glance***

**Target for Understanding:** Instruments and Attributes

| K.7 |
|---|
| a. Describe the properties/characteristics (e.g., color, relative size) of a penny, nickel, dime, and quarter |
| b. Identify a penny, nickel, dime, and quarter |
| c. Identify that a nickel is the same value as five pennies |
| d. Count a randomly placed collection of pennies and/or nickels (or models of pennies and/or nickels) whose value is 10 cents or less, and determine the value of the collection |

| K.8 |
|---|
| a. Identify a ruler as an instrument to measure length |
| b. Identify different types of scales as instruments to measure weight |
| c. Identify different types of clocks (analog and digital) as instruments to measure time |
| d. Identify the components of a calendar, including days, months, and seasons |
| e. Identify different types of thermometers as instruments used to measure temperature |

| K.9 |
|---|
| a. Tell time on an analog clock to the hour |
| b. Tell time on a digital clock to the hour |

| K.10 |
|---|
| a. Compare and describe lengths of two objects (as shorter or longer), using direct comparison or nonstandard units of measure (e.g., foot length, hand span, new pencil, paper clip, block) |
| b. Compare and describe heights of two objects (as taller or shorter), using direct comparison or nonstandard units of measure (e.g., book, hand span, new pencil, paper clip, block) |
| c. Compare and describe weights of two objects (as heavier or lighter), using direct comparison or nonstandard units of measure (e.g., book, cubes, new pencil, paper clip, block) |
| d. Compare and describe temperatures of two objects or environment (as hotter or colder), using direct comparison |

# GRADE 1

## MEASUREMENT

***Essential Knowledge Skills and Processes – At a Glance***

### Target for Understanding: Time and Measurement

| 1.7 |
|---|
| a. Identify the value of a nickel, a dime, and a quarter in terms of pennies |
| b. Recognize the characteristics of pennies, nickels, and dimes (e.g., color, size) |
| c. Count by ones to determine the value of a collection of pennies whose total value is 100 cents or less |
| d. Count by fives to determine the value of a collection of nickels whose total value is 100 cents or less |
| e. Count by tens to determine the value of a collection of dimes whose total value is 100 cents or less |
| f. Count by ones, fives, and tens to determine the value of a collection of pennies and nickels, pennies and dimes, and nickels and dimes whose total value is 100 cents or less |
| g. Count by ones, fives, and tens to determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less |

| 1.8 |
|---|
| a. Tell time shown on an analog clock to the half-hour |
| b. Tell time shown on a digital clock to the half-hour |
| c. **Match a written time to the time shown on a digital and analog clock to the half-hour** |

| 1.9 |
|---|
| a. Measure the length of objects, using various nonstandard units (e.g., connecting cubes, paper clips, erasers) |
| b. **Measure the weight/mass of objects, using a balance scale with various nonstandard units (e.g., paper clips, bean bags, cubes)** |
| c. **Measure the volume of objects, using various nonstandard units (e.g., connecting cubes, blocks, rice, water)** |

| 1.10 |
|---|
| a. Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other, using nonstandard units of measure (e.g., a spoonful or scoopful) |
| b. Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other by pouring the contents of one container into the other |
| c. **Compare the weight/mass of two objects, using the terms *lighter*, *heavier*, or *the same*, using a balance scale. The pan containing less weight/mass will rise and the pan containing more weight/mass will fall. If the objects are of equivalent weight/mass, the two pans will balance** |

| 1.11 |
|---|
| a. **Read a calendar to locate a given day or date** |
| b. **Identify the months of the year** |
| c. **Identify the seven days in a week** |
| d. **Determine the days/dates before and after a given day/date (e.g., yesterday, today, tomorrow)** |
| e. **Determine the date that is a specific number of days or weeks in the past or in the future from a given date, using a calendar** |
| f. **Identify specific dates (e.g., the third Monday in a given month)** |

# GRADE 2

## MEASUREMENT

***Essential Knowledge Skills and Processes – At a Glance***

### Target for Understanding: Time and Measurement

| **2.10** |
|---|
| a. Determine the value of a collection of coins and one-dollar bills whose total value is $2.00 or less |
| b. Compare the values of two sets of coins and one-dollar bills (each set having a total value of $2.00 or less), using the terms *greater than, less than*, or *equal to* |
| c. **Simulate everyday opportunities** to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less |
| d. Use the cent (¢) and dollar ($) symbols and decimal point (.) to write a value of money which is $2.00 or less |

| **2.11** |
|---|
| a. Estimate and **measure** the length of various line segments and objects to the nearest inch and centimeter |
| b. **Estimate and then measure the weight/mass of objects to the nearest pounds/ounces and kilograms/grams, using a scale** |
| c. **Estimate and measure liquid volume in cups, pints, quarts, gallons, and liters** |

| **2.12** |
|---|
| a. Show, tell, and write time to the **nearest five minutes**, using an analog and digital clock |
| b. Match a written time to a time shown on a clock face to the **nearest five minutes** |

| **2.13** |
|---|
| a. Determine the days/dates before and after a given day/date |
| b. Determine the day that is a specific number of days or weeks in the past or in the future from a given date, using a calendar |
| c. Identify specific **days** and dates (e.g., the third Monday in a given month or what day of the week does May 11 fall on) |
| d. Compare and describe temperatures of two objects or environment (as hotter or colder), using direct comparison |

| **2.14** |
|---|
| a. Read temperature to the nearest 10 degrees from real Celsius and Fahrenheit thermometers and from physical models (including pictorial representations) of such thermometers |

# GRADE 3

## MEASUREMENT

***Essential Knowledge Skills and Processes – At a Glance***

**Target for Understanding:** U.S. Customary and Metric Units, Area and Perimeter, Time

| **3.8** |
|---|
| a. Count the value of collections of coins and bills up to $5.00 |
| b. Compare the values of two sets of coins or bills, up to $5.00, using the terms *greater than, less than*, and *equal to* |
| c. Make change from $5.00 or less |

| **3.9** |
|---|
| a. Estimate and use U.S. Customary and metric units to measure lengths of objects to the nearest $\frac{1}{2}$ **of an inch**, inch, foot, yard, centimeter, and meter |
| b. Determine the actual measure of length using U.S. Customary and metric units to measure objects to the nearest $\frac{1}{2}$ of an inch, foot, yard, centimeter, and meter |
| c. Estimate and use U.S. Customary and metric units to measure liquid volume to the nearest cup, pint, quart, gallon, and liter |
| d. Determine the actual measure of liquid volume using U.S. Customary and metric units to measure to the nearest cup, pint, quart, gallon, and liter |
| e. Estimate and use U.S. Customary and metric units to measure the weight/mass of objects to the nearest ounce, pound, gram, and kilogram |
| f. Determine the actual measure of weight/mass using U.S. Customary and metric units to measure the weight/mass of objects to the nearest ounce, pound, gram, and kilogram |
| g. Estimate and use U.S. Customary and metric units to measure area and perimeter |
| h. **Determine the actual measure of area or perimeter using U.S. Customary and metric units** |

| **3.10** |
|---|
| a. **Measure each side of a variety of polygons and add the measures of the sides to determine the perimeter of each polygon** |
| b. **Determine the area of a given surface by estimating and then counting the number of square units needed to cover the surface** |

| **3.11** |
|---|
| a. Tell time to the nearest minute, using analog and digital clocks |
| b. Match the times shown on analog and digital clocks to written times and to each other |
| c. **When given the beginning time and ending time, determine the elapsed time in one-hour increments within a 12-hour period (times do not cross between a.m. and p.m.)** |
| d. **Solve practical problems in relation to time that has elapsed** |

| **3.12** |
|---|
| a. Identify equivalent relationships observed in a calendar, including the number of days in a given month, the number of days in a week, the number of days in a year, and the number of months in a year |
| b. Identify the number of minutes in an hour and the number of hours in a day |

| **3.13** |
|---|
| a. Read temperature to the nearest degree from real Celsius and Fahrenheit thermometers and from physical models (including pictorial representations) of such thermometers |

# GRADE 4

## MEASUREMENT

***Essential Knowledge Skills and Processes – At a Glance***

### Target for Understanding: Equivalence between U.S. Customary and Metric Units

| **4.6** |
|---|
| a. Determine an appropriate unit of measure (e.g., ounce, pound, **ton,** gram, kilogram) to use when measuring everyday objects in both metric and U.S. Customary units. |
| b. Measure objects in both metric and U.S. Customary units (e.g., ounce, pound, **ton**, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. |
| c. Record the mass of an object including the appropriate unit of measure (e.g., 24 grams). |

| **4.7** |
|---|
| a. Determine an appropriate unit of measure (e.g., inch, foot, yard, **mile**, millimeter, centimeter, and meter) to use when measuring everyday objects in both metric and U.S. Customary units. |
| b. Estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure. |
| c. Measure the length of objects in both metric and U.S. Customary units, measuring to the nearest inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), foot, yard, **mile**, millimeter, centimeter, or meter, and record the length including the appropriate unit of measure (e.g., 24 inches). |
| d. Compare estimates of the length of objects with the actual measurement of the length of objects. |
| ~~e.~~ Identify equivalent measures of length between units within the U.S. Customary measurements and between units within the metric measurements. |

| **4.8** |
|---|
| a. Determine an appropriate unit of measure (cups, pints, quarts, gallons) to use when measuring liquid volume in U.S. Customary units. |
| b. Estimate the liquid volume of containers in U.S. Customary units of measure to the nearest cup, pint, quart, and gallon. |
| c. Measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, and gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons). |
| d. Identify equivalent measures of volume between units within the U.S. Customary system. |

| **4.9** |
|---|
| **a. Determine the elapsed time in hours and minutes within a 12-hour period (times can cross between a.m. and p.m.).** |
| **b. Solve practical problems in relation to time that has elapsed.** |

# Target for Understanding – Math Process Standards

## Mathematical Process Skills -"Student Self-Evaluation" Recording Form

Name: _____ Date: _____ Project: _____

| Process Skill | Notes |
|---|---|
| **1- Mathematical Problem Solving** | |
| ☐ I understand the meaning of the problem and look for entry points to its solution<br>☐ I analyze information (givens, constrains, relationships, goals)<br>☐ I make conjectures and plan a solution pathway<br>☐ I monitor and evaluate my progress and change course as necessary<br>☐ I check my answers to problems and ask, "Does this make sense?" | |
| **2- Mathematical Communication** | |
| ☐ I use definitions and previously established causes/effects (results) in constructing arguments<br>☐ I make conjectures and use counterexamples to build a logical progression of statements to explore and support their ideas<br>☐ I communicate and defend my mathematical reasoning using objects, drawings, diagrams, actions<br>☐ I listen to or read the arguments of others<br>☐ I decide if the arguments of others make sense to me and ask probing questions to clarify or improve the arguments | |
| **3- Mathematical Reasoning** | |
| ☐ I make sense of quantities and relationships in problem situations<br>☐ I represent abstract situations symbolically and understand the meaning of quantities<br>☐ I create a coherent representation of the problem at hand<br>☐ I consider the units involved<br>☐ I can flexibly use properties of operations | |
| **4- Mathematical Connections** | |
| ☐ I look for patterns or structure, recognizing that quantities can be represented in different ways<br>☐ I recognize the significance in concepts and models and use the patterns or structure for solving related problems<br>☐ I view complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems<br>☐ I notice repeated calculations and look for general methods and shortcuts<br>☐ I continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on findings | |

# Mathematical Process Skills -"Student Self-Evaluation" Recording Form

Name: _____ Date: _____ Project: _____

| 5- Mathematical Representation | |
|---|---|
| ☐ I apply my prior knowledge to solve real world problems<br>☐ I identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas<br>☐ I make assumptions and approximations to make a problem simpler<br>☐ I check to see if an answer makes sense within the context of a situation and change a model when necessary | |
| **6- Use appropriate tools strategically** | |
| ☐ I make sound decisions about the use of specific tools. Examples might include:<br>☐ Calculator<br>☐ Concrete models<br>☐ Digital Technology<br>☐ Pencil/paper<br>☐ Ruler, compass, protractor<br>☐ I use technological tools to visualize the results of assumptions, explore consequences and compare predications with data<br>☐ I identify relevant external math resources (digital content on a website) and I use them to pose or solve problems<br>☐ I use technological tools to explore and deepen understanding of concepts | |
| **7- Mathematical Precision** | |
| ☐ I communicate precisely using clear definitions<br>☐ I state the meaning of symbols, carefully specifying units of measure, and providing accurate labels<br>☐ I calculate accurately and efficiently, expressing numerical answers with a degree of precision<br>☐ I provide carefully formulated explanations<br>☐ I label accurately when measuring and graphing | |

*Adapted from the Mathematics Process Standards, SOL, Virginia Department of Education, by Dan Mulligan, 2012 – 2013.*

**Additional comments based on observed learning:**

# TARGET FOR UNDERSTANDING: ESSENTIAL VOCABULARY (doe.virginia.gov)

## Algebra I Vocabulary Cards

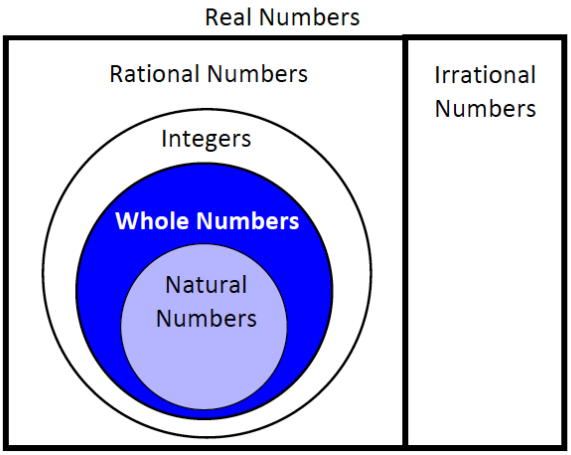
### Table of Contents


**Expressions and Operations**
Natural Numbers
Whole Numbers
Integers
Rational Numbers
Irrational Numbers
Real Numbers
Absolute Value
Order of Operations
Expression
Variable
Coefficient
Term
Scientific Notation
Exponential Form
Negative Exponent
Zero Exponent
Product of Powers Property
Power of a Power Property
Power of a Product Property
Quotient of Powers Property
Power of a Quotient Property
Polynomial
Degree of Polynomial
Leading Coefficient
Add Polynomials (group like terms)
Add Polynomials (align like terms)
Subtract Polynomials (group like terms)
Subtract Polynomials (align like terms)
Multiply Polynomials
Multiply Binomials
Multiply Binomials (model)
Multiply Binomials (graphic organizer)
Multiply Binomials (squaring a binomial)
Multiply Binomials (sum and difference)
Factors of a Monomial
Factoring (greatest common factor)
Factoring (perfect square trinomials)
Factoring (difference of squares)
Difference of Squares (model)
Divide Polynomials (monomial divisor)
Divide Polynomials (binomial divisor)
Prime Polynomial
Square Root
Cube Root
Product Property of Radicals
Quotient Property of Radicals
Zero Product Property
Solutions or Roots
Zeros
x-Intercepts

**Equations and Inequalities**
Coordinate Plane
Linear Equation
Linear Equation (standard form)
Literal Equation
Vertical Line
Horizontal Line
Quadratic Equation
Quadratic Equation (solve by factoring)
Quadratic Equation (solve by graphing)
Quadratic Equation (number of solutions)
Identity Property of Addition
Inverse Property of Addition
Commutative Property of Addition
Associative Property of Addition
Identity Property of Multiplication
Inverse Property of Multiplication
Commutative Property of Multiplication
Associative Property of Multiplication
Distributive Property
Distributive Property (model)
Multiplicative Property of Zero
Substitution Property
Reflexive Property of Equality
Symmetric Property of Equality
Transitive Property of Equality
Inequality
Graph of an Inequality
Transitive Property for Inequality
Addition/Subtraction Property of Inequality
Multiplication Property of Inequality
Division Property of Inequality
Linear Equation (slope intercept form)
Linear Equation (point-slope form)


# Whole Numbers

The set of numbers
0, 1, 2, 3, 4…




**Create a Glossary of Critical Content Terms:**

## Glossary

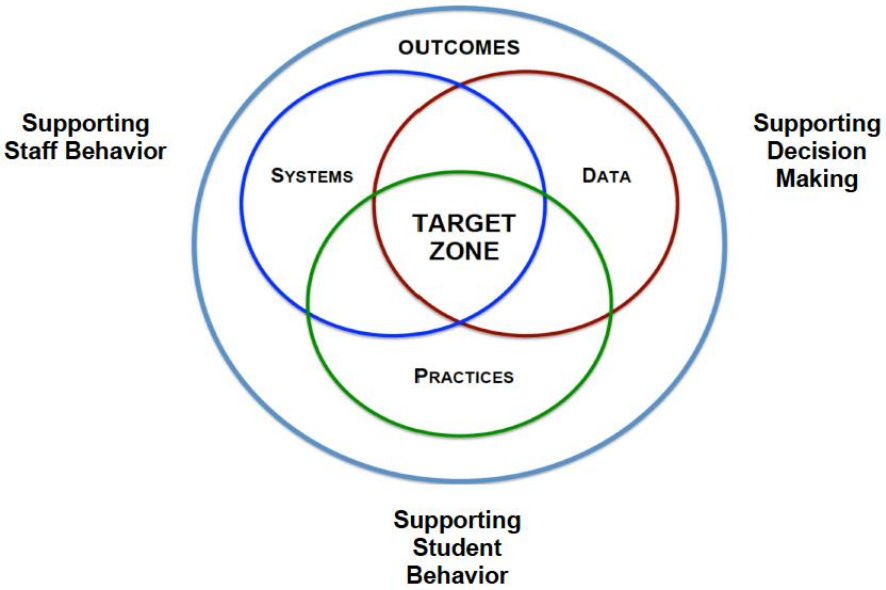
| Term | Page # | Description | Picture/ Example |
|---|---|---|---|
| Alternate interior angles | | | |
| Alternate exterior angles | | | |
| Corresponding angles | | | |
| Midpoint | | | |
| Parallel lines | | | |
| Perpendicular lines | | | |
| Skew lines | | | |
| Transversal | | | |

## WHAT OPPORTUNITIES EXIST IN YOUR SCHOOL/DIVISION TO EXPAND STUDENT VOCABULARY OWNERSHIP?

# **Practices:** interventions and strategies that are evidence based.

| **Know** | **My notes:** |
|---|---|
| 1. Unit design that begins with the end in mind and differentiates the plan for getting there will ensure the success of at least 80% of all students. | |
| **Understand** | |
| 1. Essential knowledge, understandings, and skills are the same for all learners; we use multiple means of representation, engagement, and expression to differentiate learning plans. | |
| **Do** | |
| 1. Map effective design practices onto current structures and processes and identify wishes for professional learning and/or coaching. | |

## Evidence Based Practice Organizer



## B. ASSESSMENT

| Guiding Question | Resource Page # | Current Reality | Next Steps |
|---|---|---|---|
| To what extent is the division/school assessment aligned to the EKSV of the Curriculum Framework? | | | |
| To what extent is the division/school/class assessment diagnostic in nature? *Does assessment encourage 'students' to try?* | | | |
| To what extent is the division/school/class assessment ongoing? *Does assessment inform instructional decisions?* | | | |
| To what extent do students experience a comprehensive system of assessment? *See recommendations of the Standards of Learning Innovation Committee* | | | |
| | | | |

## USING VIRGINIA DOE RESOURCES:

# Spring 2014 Student Performance Analysis

## Geometry

## Standards of Learning

## Suggested Practice for SOL G.12

**Students need additional practice determining the equation of a circle when several steps are necessary.**

**Write the equation of a circle, in standard form, that has a diameter with endpoints at $(-4, 3)$ and $(0, -5)$.**

First, find the midpoint of the diameter which is the center of the circle.

$$\left(\frac{-4+0}{2}, \frac{3+-5}{2}\right) = (-2, -1)$$

Next, find the length of the radius using the center and one of the points on the circle.

$$\sqrt{(-2-0)^2 + \left(-1-(-5)\right)^2} = \sqrt{20}$$

$$(x+2)^2 + (y+1)^2 = 20$$

20

**VERSION 2: ONGOING ASSESSMENT**
**PROVIDE A SYSTEM FOR STUDENT/PARENT/TEACHER/SCHOOL PROGRESS MONITORING**

## KINDERGARTEN

### PATTERNS, FUNCTIONS, AND ALGEBRA

***Essential Knowledge Skills and Processes – At a Glance***

**Target for Understanding:** Attributes and Patterning

| K.15 | PA1 | | PA2 | | PA3 | |
|---|---|---|---|---|---|---|
| | T1 | ML | T2 | M2 | T3 | M3 |
| a. Sort objects into appropriate groups (categories) based on one attribute | | | | | | |
| b. Classify sets of objects into groups (categories) of one attribute | | | | | | |
| c. Label attributes of a set of objects that has been sorted | | | | | | |
| d. Name multiple ways to sort a set of objects | | | | | | |

| K.16 | PA1 | | PA2 | | PA3 | |
|---|---|---|---|---|---|---|
| | T1 | ML | T2 | M2 | T3 | M3 |
| a. Observe and identify the basic repeating pattern (core) found in repeating patterns of common objects, sounds, and movements that occur in practical situations | | | | | | |
| b. Identify the core in a repeating pattern | | | | | | |
| c. Extend a repeating pattern by adding at least two repetitions to the pattern | | | | | | |
| d. Create a repeating pattern | | | | | | |
| e. Compare similarities and differences between patterns | | | | | | |

## GRADE 8

# PATTERNS, FUNCTIONS, AND ALGEBRA

***Essential Knowledge Skills and Processes – At a Glance***

## Target for Understanding: Linear Relationships

| 8.14 | PA1 | | PA2 | | PA3 | |
|---|---|---|---|---|---|---|
| | T1 | ML | T2 | M2 | T3 | M3 |
| a. Graph in a coordinate plane ordered pairs that represent a relation. | | | | | | |
| b. Describe and represent relations and functions, using tables, graphs, words, and rules. Given one representation, students will be able to represent the relation in another form. | | | | | | |
| c. **Relate and compare different representations for the same relation.** | | | | | | |

| 8.15 | PA1 | | PA2 | | PA3 | |
|---|---|---|---|---|---|---|
| | T1 | ML | T2 | M2 | T3 | M3 |
| a. Solve **two- to four-step** linear equations in one variable using concrete materials, pictorial representations, and paper and pencil illustrating the steps performed. | | | | | | |
| b. **Solve two-step inequalities in one variable by showing the steps and using algebraic sentences.** | | | | | | |
| c. **Graph solutions to two-step linear inequalities on a number line.** | | | | | | |
| d. **Identify properties of operations used to solve an equation from among:**<br>**1. the commutative properties of addition and multiplication;**<br>**2. the associative properties of addition and multiplication;**<br>**3. the distributive property;**<br>**4. the identity properties of addition and multiplication;**<br>**5. the zero property of multiplication;**<br>**6. the additive inverse property; and**<br>**d. the multiplicative inverse property.** | | | | | | |

| 8.16 | PA1 | | PA2 | | PA3 | |
|---|---|---|---|---|---|---|
| | T1 | ML | T2 | M2 | T3 | M3 |
| a. Construct a table of ordered pairs by substituting values for *x* in a linear equation to find values for *y*. | | | | | | |
| b. Plot in the coordinate plane ordered pairs (*x*, *y*) from a table. | | | | | | |
| c. Connect the ordered pairs to form a straight line (a continuous function). | | | | | | |
| **d.** Interpret the unit rate of the proportional relationship graphed as the slope of the graph, and compare two different proportional relationships represented in different ways.† | | | | | | |

| 8.17 | PA1 | | PA2 | | PA3 | |
|---|---|---|---|---|---|---|
| | T1 | ML | T2 | M2 | T3 | M3 |
| a. **Apply the following algebraic terms appropriately: *domain, range, independent variable,* and *dependent variable.*** | | | | | | |
| b. **Identify examples of domain, range, independent variable, and dependent variable.** | | | | | | |
| c. **Determine the domain of a function.** | | | | | | |
| d. **Determine the range of a function.** | | | | | | |
| e. **Determine the independent variable of a relationship.** | | | | | | |
| f. **Determine the dependent variable of a relationship** | | | | | | |

**ALGEBRA I** **Name:** _____

# EQUATIONS AND INEQUALITIES

***Essential Knowledge Skills and Processes – At a Glance***

**# of items: 18**

| A.4a-f | PA1 | | PA2 | | PA3 | |
|---|---|---|---|---|---|---|
| | T1 | ML | T2 | M2 | T3 | M3 |
| a. Solve a literal equation (formula) for a specified variable. | | | | | | |
| b. Simplify expressions and solve equations, using the field properties of the real numbers and properties of equality to justify simplification and solution. | | | | | | |
| c. Solve quadratic equations. | | | | | | |
| d. Identify the roots or zeros of a quadratic function over the real number system as the solution(s) to the quadratic equation that is formed by setting the given quadratic expression equal to zero. | | | | | | |
| ~~e.~~ Solve multistep linear equations in one variable. | | | | | | |
| f. Confirm algebraic solutions to linear and quadratic equations, using a graphing calculator. | | | | | | |
| g. Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to find the ordered pair which satisfies both equations. | | | | | | |
| h. Given a system of two linear equations in two variables that has a unique solution, solve the system graphically by identifying the point of intersection. | | | | | | |
| i. Determine whether a system of two linear equations has one solution, no solution, or infinite solutions. | | | | | | |
| j. Write a system of two linear equations that models a real-world situation. | | | | | | |
| k. Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that models a real-world situation. | | | | | | |
| l. Determine if a linear equation in one variable has one, an infinite number, or no solutions.† | | | | | | |

| A.5a-d | PA1 | | PA2 | | PA3 | |
|---|---|---|---|---|---|---|
| | T1 | ML | T2 | M2 | T3 | M3 |
| a. Solve multistep linear inequalities in one variable. | | | | | | |
| b. Justify steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers. | | | | | | |
| c. Solve real-world problems involving inequalities. | | | | | | |
| d. Solve systems of linear inequalities algebraically and graphically. | | | | | | |

| A.6a-b | PA1 | | PA2 | | PA3 | |
|---|---|---|---|---|---|---|
| | T1 | ML | T2 | M2 | T3 | M3 |
| a. Graph linear equations and inequalities in two variables, including those that arise from a variety of real-world situations. | | | | | | |
| b. Use the parent function $y = x$ and describe transformations defined by changes in the slope or $y$-intercept. | | | | | | |
| c. Find the slope of the line, given the equation of a linear function. | | | | | | |
| d. Find the slope of a line, given the coordinates of two points on the line. | | | | | | |
| e. Find the slope of a line, given the graph of a line. | | | | | | |
| f. Recognize and describe a line with a slope that is positive, negative, zero, or undefined. | | | | | | |
| g. Use transformational graphing to investigate effects of changes in equation parameters on the graph of the equation. | | | | | | |
| h. Write equation of a line when given the graph of a line. | | | | | | |
| i. Write an equation of a line when given two points on the line whose coordinates are integers. | | | | | | |
| j. Write an equation of a line when given the slope and a point on the line whose coordinates are integers. | | | | | | |
| k. Write an equation of a vertical line as $x = a$. | | | | | | |
| l. Write the equation of a horizontal line as $y = c$. | | | | | | |

# ASSESSMENT IS VARIED

| Assessment Type | DOK/ Bloom Alignment | Format | Usefulness and Resulting Evidence |
|---|---|---|---|
| **Closed Tasks** | **DOK 1**<br>Recall & Reproduction<br><br>**BLOOM A/ BLOOM B**<br>Remembering, Understanding | • Multiple-choice items<br>• True-False Items<br>• Fill-in-the-Blank Items<br>• Solve (without showing steps)<br>• TEI | • Useful for assessing content-based standards; Not useful for process-based standards<br>• Assess student knowledge of facts, skills, or concepts<br>• Take less time, thus allowing time for open-ended or performance tasks |
| **Open Tasks & Constructed Responses** | **DOK 2**<br>Basic Skills & Concepts<br><br>**BLOOM B, C**<br>Understanding, Applying | • Tasks with different possible answers<br>• Tasks with different possible processes<br>• TEI | Useful for assessing student's ability to<br>• Use processes and strategies<br>• Interpret information<br>• Apply information<br>• Reasoning<br>• Communicate thinking |
| **Performance Assessments** | **DOK 4**<br>Extended Thinking<br><br>**BLOOM F**<br>Create | • Integrative tasks that yield specific products<br>• Authentic assessments<br>• Extended projects | Useful for assessing student:<br>• Ability to organize, synthesize, and apply information and skills<br>• Use of resources |
| **Informal Assessments** | **DOK 2**<br>Basic Skills & Concepts<br><br>**DOK 3**<br>Strategic Thinking & Reasoning | • Teacher observations<br>• Teacher checklists (rubrics)<br>• Conversations or interviews | Depending on what is discussed or observed, these informal assessments may reveal student:<br>• Process or strategy used<br>• Reasoning<br>• Understanding of a topic or concept<br>• Ability to communicate and collaborate |
| **Self-Assessment or Reflection** | **DOK 3**<br>Strategic Thinking & Reasoning<br><br>**BLOOM E**<br>Evaluate | • Student journals or reflection logs<br>• Student checklists<br>• Group (whole class or small group) reflection activities<br>• Daily or weekly self-evaluations<br>• Teacher-student interviews | • Develops student awareness of strengths and areas for improvement; conscious use of thinking skills (metacognitive skills)<br>• Can show student process, thinking, and reasoning skills<br>• Reveals student disposition toward topic or learning<br>• Helps teachers and students identify student's personal goals |

# CLOSED TASKS

**SOL 8.1 The student will**

a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and

b) compare and order decimals, fractions, percents, and numbers written in scientific notation.

**Essential Knowledge, Skills and Processes**

a. Simplify numerical expressions containing: 1) exponents (where the base is a rational number and the exponent is a positive whole number); 2) fractions, decimals, integers and square roots of perfect squares; and 3) grouping symbols (no more than 2 embedded grouping symbols). Order of operations and properties of operations with real numbers should be used.

b. Compare and order no more than five fractions, decimals, percents, and numbers written in scientific notation using positive and negative exponents. Ordering may be in ascending or descending order.

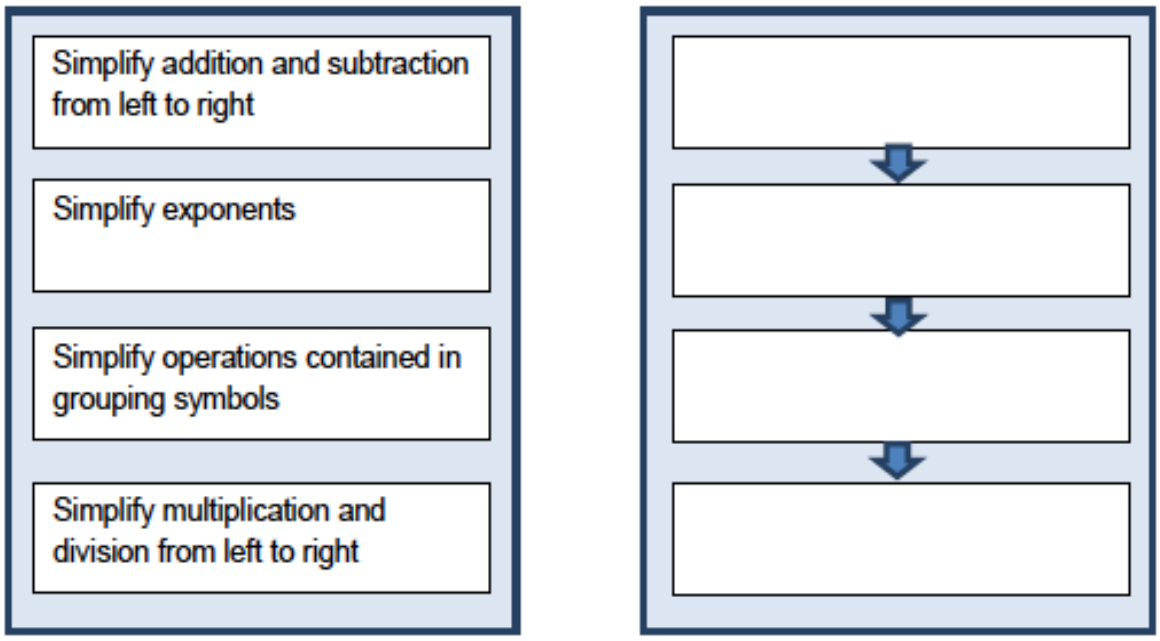
## Checkpoint Items (Do *not* use a calculator to solve these problems)

1. Directions: After showing your thinking, write each answer in the box below each item. Your answers must be in simplest form.

**Simplify each of the following expressions.**

a. $4^3$ b. $\left(\frac{2}{5}\right)^2$ c. $(\sqrt{25})^4$ d. $(-3)^3$

2. Write each phrase in the correct box on the right based on the sequence of steps in the order of operations, provided on the left.




3. Directions: After showing your thinking, write your answer in the box below each item. Your answer must be in simplest form.

**Simplify each of the following expressions.**

a. $6^2 - (4 - 1)^3 + 1$ b. $2\ |3 - 2^2| + 5$ c. $(\sqrt{16})^3 - (5 - 4)^3$

4. Place a circle around each box you want to select. You must select all correct answers.

**Identify each expression that has a value equal to the absolute value of 4.**

| | |
|---|---|
| $(4-2)^3-\sqrt{16}$ | $(1+1)^2-(4-2^2)$ |
| $\lvert(7-2^3)\rvert-3$ | $(\frac{4}{2})^2$ |

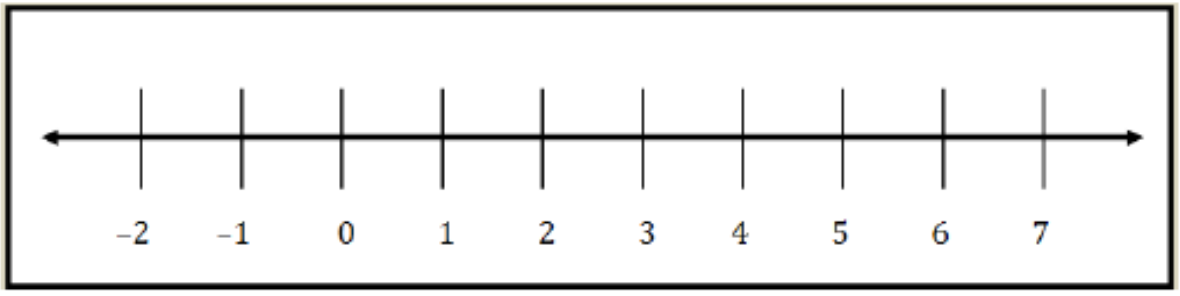
5. Directions: After showing your thinking, write your answers in the series of boxes below.

**Compare and order the given numbers in *descending* order.**

a. $\frac{2}{3}$, 2.89, 55%, $1.2 \times 10^2$, $2.3 \times 10^{-2}$

5. **Directions: Place the letter above the number line that corresponds to the location of its assigned value.**

a. $(\sqrt{4})^2$ b. 4% c. $|6 - 2^2|$




## OPEN TASKS AND CONSTRUCTED RESPONSE

7. After showing your thinking, write your answer in the box provided below.

**Create a mathematical expression of four terms that includes exponents, grouping symbols, and a square root. Your challenge is to ensure that the value of the expression is 5.**
**Include an explanation of the process used to create your expression.**

8. After showing your thinking, write your answer in the boxes provided. Add only one entry into each box following the direction below.

**Create a list of four numbers in ascending order. Ensure that your list includes a number written in scientific notation with a positive exponent, a decimal, a percent, and a number in scientific notation with a negative exponent. Show your thinking!**

# Math 8.1 Checkpoint Solutions

**SOL 8.1 The student will**
a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and
b) compare and order decimals, fractions, percents, and numbers written in scientific notation.

**Essential Knowledge, Skills and Processes**

a. Simplify numerical expressions containing: 1) exponents (where the base is a rational number and the exponent is a positive whole number); 2) fractions, decimals, integers and square roots of perfect squares; and 3) grouping symbols (no more than 2 embedded grouping symbols). Order of operations and properties of operations with real numbers should be used.
b. Compare and order no more than five fractions, decimals, percents, and numbers written in scientific notation using positive and negative exponents. Ordering may be in ascending or descending order.

| Item | Answer | EKS Code | EKS |
|---|---|---|---|
| 1a | 64 | 8.1aa1 | Simplify numerical expressions containing: 1) exponents (where the base is a rational number and the exponent is a positive whole number) |
| 1b | $\frac{4}{25}$ | 8.1aa2 | Simplify numerical expressions containing: 2) fractions, decimals, integers and square roots of perfect squares |
| 1c | 625 | 8.1aa2 | Simplify numerical expressions containing: 2) fractions, decimals, integers and square roots of perfect squares |
| 1d | -27 | 8.1aa1 | Simplify numerical expressions containing: 1) exponents (where the base is a rational number and the exponent is a positive whole number) |
| 2 | 3, 2, 4, 1 | 8.1aa | Simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers |
| 3a | 10 | 8.1aa | Simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers |
| 3b | 7 | 8.1aa | Simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers |
| 3c | 63 | 8.1aa | Simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers |
| 4 | $(4-2)^3-\sqrt{16}$<br>$(1+1)^2-(4-2^2)$ | 8.1aa | Simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers |

## Task 2: The Quilt Quandary

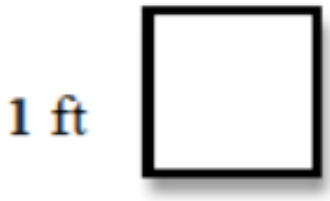
Mrs. Smith is making a quilt in the shape of a square for her younger sister. The border of the quilt is made of 1-foot by 1-foot patches.

She asks one of her students, Maria, to use the picture of the quilt to the right to write an expression to illustrate the number of patches needed to border the square quilt with side length s.


The square quilt has side of length s.
(Not drawn to scale)

Maria writes:

$(s + s + s + s) + (1 + 1 + 1 + 1)$




Patch (Not drawn to scale)

**Part A:**
Is Maria's expression correct? Explain your reasoning.

**Part B:**
Mrs. Smith asks four students (LaToya, Jerry, Nadia, and Joseph) to generate expressions that are equivalent to Maria's expression.

5. LaToya's expression: $4s + 4$
6. Jerry's expression: $4(s + 1)$
7. Nadia's expression: $4s + 1$
8. Joseph's expression: $2s + 2(s + 2)$

How many of the students wrote a correct or an incorrect expression? Justify your answer with a detailed explanation.

## The Quilt Quandary Rubric:

| Score | Description |
|---|---|
| 3 | I have answered completely and correctly all aspects of the question My responses effectively communicate my mathematical understanding. My strategies and ability to carryout my strategies meet the content demands of all parts of the task. |
| 2 | I have answered some aspects of the question completely and correctly. My responses demonstrate adequate evidence and understanding necessary to complete the task with minor errors in execution. I demonstrate some mathematical understanding and will be able to revise my work with discussion and/or feedback from my peers or teacher.<br>My errors may include:<br>• Incomplete justifications for written responses<br>• Incorrectly identifying 1 to 2 responses from Part B<br>• Minor numerical error on Part A or Part B |
| 1 | Many of my answers do not provide complete and correct responses. I demonstrated effort to complete the task; however, there are many conceptual errors throughout the task. My work will require significant revisions.<br>My errors may include:<br>• Incomplete justifications for written responses<br>• Incorrect solutions with incomplete work<br>• Many calculation errors or mathematical misconceptions<br>• Only Part A or Part B shows limited understanding |
| 0 | I did not attempt the challenge. |

| Student Comments: | Teacher Comments: |
|---|---|
| | |

# PERFORMANCE TASKS

## GRADE 6

### COMPUTATION AND ESTIMATION

***Essential Knowledge Skills and Processes – At a Glance***

**Target for Understanding:** Applications of Operations with Rational Numbers

| **6.6** |
|---|
| **a. Multiply and divide with fractions and mixed numbers. Answers are expressed in simplest form.** |
| b. Solve single-step and multistep practical problems that involve addition and subtraction with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form. |
| **c. Solve single-step and multistep practical problems that involve multiplication and division with fractions and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.** |

Jerome used the following steps to simplify each problem. Review the steps he used and explain whether you agree with him.

$$\frac{1}{4} \times 1200$$

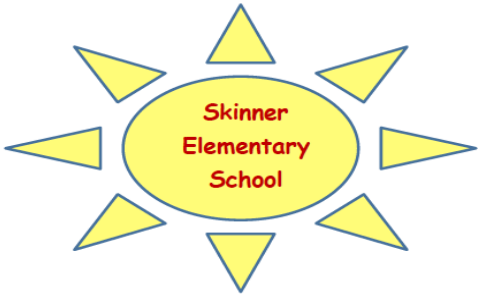
1. Jerome made 1200 into a fraction by putting 1 over 1200. $\frac{1}{1200}$
2. He then multiplied 1 times 1 and 4 times 1200 $\frac{1}{4} \times \frac{1}{1200} = \frac{1 \times 1}{4 \times 1200}$.
3. Jerome wrote his answer as $\frac{1}{4800}$

| Provide Jerome detailed feedback on his steps. |
|---|
| |
| |
| |
| |

Sonia was absent from class today. Explain to her all that you learned about multiplying fractions. Include an example to illustrate your words.

**What is the best design to the new elementary school in Hickman Mills?**

Superintendent Carpenter has requested you and your team to serve as the design committee for the new Skinner Elementary School. Using your patterns blocks design the layout for the new school. Transfer your design to paper and label each section of the school (e.g., library, gym, classroom pods, etc.). The cost of materials of each hexagon section used in construction is $120,000.




Dr. Carpenter has tasked your committee to submit to him:

a. A sketch of the building (with each section of the building labeled)
b. The fractional part of the entire building that is represented by each section.
c. The cost of construction each section.
d. The total cost of materials for Skinner Elementary School.
e. Write a persuasive letter or a presentation to justifying your design of the new school to Dr. Carpenter.

Fraction Friction

Name: _____

Simplify each mathematical expression: (Remember to show your work)

$\frac{1}{24} \times 1200$

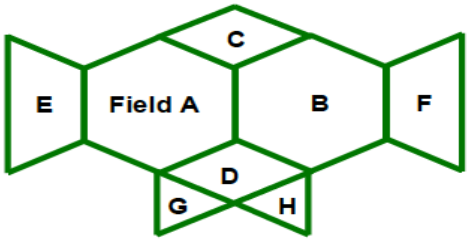
$\frac{1}{8} \times 64$

$\frac{1}{6} \times 120$

$\frac{1}{5} \times 400$

**Farmer Fred**

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.




Note: The diagram above is not necessarily drawn to scale. It would be wise to build Farmer Fred's fields with your pattern blocks.

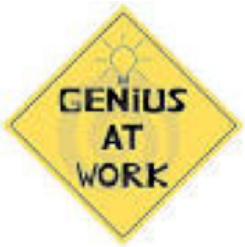
# SAMPLE INFORMAL ASSESSMENT (MATHEMATICS)

## *Rigor-Infused* Mathematics Process Skills Prompts

| Process Skill | What does the 'MATH TALK' sound like? |
|---|---|
| **A. Mathematical Problem Solving** | 1. How would you describe the problem in your own words?<br>2. How would you describe what you are trying to find?<br>3. What do you notice about...?<br>4. What information is given in the problem?<br>5. Describe the relationship between the quantities.<br>6. Describe what you have already tried. What might you change?<br>7. Talk me through the steps you've used to this point.<br>8. What steps in the process are you most confident about?<br>9. What are some other strategies you might try?<br>10. What are some other problems that are similar to this one?<br>11. How might you use one of your previous problems to help you begin?<br>12. How else might you organize...represent... show...? |
| **B. Mathematical Communication** | 1. What mathematical evidence would support your solution?<br>2. How can we be sure that...? / How could you prove that...?<br>3. Will it still work if...?<br>4. What were you considering when...?<br>5. How did you decide to try that strategy?<br>6. How did you test whether your approach worked?<br>7. How did you decide what the problem was asking you to find? (What was unknown?)<br>8. Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not?<br>9. What is the same and what is different about...?<br>10. How could you demonstrate a counter-example? |
| **C. Mathematical Reasoning** | 1. Will the same strategy work in other situations?<br>2. Is this always true, sometimes true or never true?<br>3. How would we prove that...?<br>4. What do you notice about...?<br>5. What is happening in this situation?<br>6. What would happen if...?<br>7. Is there a mathematical rule for...?<br>8. What predictions or generalizations can this pattern support?<br>9. What mathematical consistencies do you notice ?<br>10. What do the numbers used in the problem represent?<br>11. What is the relationship of the quantities?<br>12. How is _____ related to _____?<br>13. What is the relationship between _____and _____?<br>14. What does_____mean to you? (e.g. symbol, quantity, diagram)<br>15. What properties might we use to find a solution?<br>16. How did you decide in this task that you needed to use...?<br>17. Could we have used another operation or property to solve this task? Why or why not? |

| | |
|---|---|
| **D. Mathematical Connections** | 1. What observations do you make about...?<br>2. What do you notice when...?<br>3. What parts of the problem might you eliminate..., simplify...?<br>4. What patterns do you find in...?<br>5. How do you know if something is a pattern?<br>6. What ideas that we have learned before were useful in solving this problem?<br>7. What are some other problems that are similar to this one?<br>8. How does this relate to...?<br>9. In what ways does this problem connect to other mathematical concepts? |
| **E. Mathematical Representations** | 1. What number model could you construct to represent the problem?<br>2. What are some ways to represent the quantities?<br>3. What's an equation or expression that matches the diagram..., number line.., chart..., table..?<br>4. Where did you see one of the quantities in the task in your equation or expression?<br>5. Would it help to create a diagram, graph, table...?<br>6. What are some ways to visually represent...?<br>7. What formula might apply in this situation? |
| **F. Use Appropriate Tools Strategically** | 1. What mathematical tools could we use to visualize and represent the situation?<br>2. What information do you have?<br>3. What do you know that is not stated in the problem?<br>4. What approach are you considering trying first?<br>5. What estimate did you make for the solution?<br>6. In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative?<br>7. Why was it helpful to use...?<br>8. What can using a _____ show us that _____may not?<br>9. In what situations might it be more informative or helpful to use...? |
| **G. Mathematical Precision** | 1. What mathematical terms apply in this situation?<br>2. How did you know your solution was reasonable?<br>3. Explain how you might show that your solution answers the problem.<br>4. Is there a more efficient strategy?<br>5. How are you showing the meaning of the quantities?<br>6. What symbols or mathematical notations are important in this problem?<br>7. What mathematical language...,definitions..., properties can you use to explain...?<br>8. How could you test your solution to see if it answers the problem? |

# It's All About the Second Question




| | |
|---|---|
| What if…? | Is _____ the reason for ___? |
| I wonder why _____? | Can…? |
| If…? | Would you rather…? |
| What is it that…? | What would it take to…? |
| When is it…? | Why is it that…? |
| Who could…? | Would _____ be possible if…? |
| How is _____ like _____? | Is it possible to…? |
| When is…? | Could…? |
| What could happen if…? | How can…? |
| If it were possible…? | What is your opinion about…? |
| Are there…? | Is it right to…? |
| Why is…? | I wonder when…? |
| How…? | I'm wondering if…? |
| Where did…? | How could it…? |
| Do you…? | Why are…? |
| Does it matter if…? | If it ___, could _____? |
| When is it…? | What can…? |

The impact of a first question can be enriched by following with:

"How do you know?"

"What makes you say that?"

## C. STAFF DEVELOPMENT – MODELING AND DATA DRIVEN (DIFFERENTIATED)

| Guiding Question | Resource Page # | Current Reality | Next Steps |
|---|---|---|---|
| To what extent is the division/school/teacher staff development aligned to EKSV of the Curriculum Framework? | | | |
| To what extent is the division/school/teacher staff development grounded in research-based strategies? | | | |
| To what extent is the division/school/teacher staff development based on analysis of student achievement data? | | | |
| To what extent is the division/school/class differentiated based on individual staff needs? | | | |
| To what extent is the division/school/teacher staff development ongoing in CLTs (including constructive feedback)? | | | |
| To what extent do administrators (as instructional leaders) participate in staff development with teachers? | | | |
| | | | |

# DIFFERENTIATED STAFF DEVELOPMENT

**This K-12 Math BINGO board links to popular resources shared during the October 2014 VASS conference. Use it to challenge yourself and your staff to access and use the materials!**

**Good luck!**

| B | I | N | G | O |
|---|---|---|---|---|
| AMO Chart and Math SOL Blueprints | Frayer Model | Vertical Articulation by Strand K-5 or Gr 3-8 or EOC | What I Know About | Process Goals |
| Developing Mathematical Thinking for Questioning | Lesson Planning Guide | Essential Vocabulary | Where Do I Belong | Sample Math Observation Form Geometry |
| Differentiation | Web Resources (File to Links) |   VASS Presentation VASS Handout | Working on the Right Work | Target for Understanding K-EOC Math |
| Process Thinking Rubric | Sample Performance Tasks | SOL & Bloom's / Depth of Knowledge – Rigor and Assessment | Virginia Teacher Standards | Fan and Pick Board Sample Cards |
| What's My Rule | Sample Checkpoint Test Math 7.15 | Learning Contract | Problem Solving | Sample SOL Scrimmages: Gr3-5 |

# STAFF DEVELOPMENT ALIGNED WITH THE EKSV OF THE CURRICULUM FRAMEWORK

MATHEMATICS

## INSTRUCTIONAL VIDEOS FOR TEACHERS

These instructional videos are provided as support for the implementation of the 2009 Mathematics Standards of Learning (SOL).

ON THIS PAGE: ▪ Strategies Across the Strands ▪ Number & Number Sense ▪ Computation & Estimation ▪ Measurement ▪ Geometry ▪ Probability & Statistics ▪ Patterns, Functions & Algebra ▪ Additional Resources

The videos on this page link to Flash files which require the Adobe Flash Player plug-in. Download the free player.

### Strategies Across the Strands

- Play Video Working with Vocabulary / Concept Development (grades 4-8)
  Dr. Lois Williams, VDOE mathematics specialist, on the Frayer Model for vocabulary/concept development.
  *Handout available:* Working with Vocabulary / Concept Development (Word)
- Play Video Vocabulary (grades 6-8)
  Dr. Lois Williams shares a technique to assist students who confuse common mathematics terms such as radius and diameter.
- Play Video Notebooks for Organization (grades 6-8)
  Dr. Lois Williams explains an organizational system for middle school mathematics classes.
- Play Video Multi-Step Problem Solving (grades 4-8)
  Cheryl Gray, Spotsylvania County Schools middle school mathematics specialist, on how to approach multi-step problems with data presented in a table. *Handout available:* Multi-Step Problem Solving (PPT)
- Play Video Scientific Calculator Use (grades 6-8)
  Dr. Lois Williams explains the use of scientific calculators in Virginia middle schools.
  *Handout available:* Scientific Calculator Manual (PDF)

### Number & Number Sense

- Play Video Developing Early Number Sense (grades K-2)
  Laura Domalik, Hanover County provides instructional strategies for counting and vocabulary. The strategies include counting on, counting back, one more than (+1), one less than (-1), basic fact concepts of +1 and -1, and missing addends.
- Play Video Using a Beaded Number Line (grades K-2)
  Laura Scearce, Hanover County, provides instructional strategies for using beaded number lines to develop the students' skills in counting, counting backwards, rounding, adding, and subtracting.
- Play Video Modeling Equality (grades K-2)
  Debi Godfrey, Henrico County, provides activities with pan and number balances to represent and build the concept of equality in kindergarten.
- Play Video Understanding Fractions (grades K-2)
  Laura Scearce, Hanover County, provides instructional strategies for developing fraction sense amongst second graders by comparing unit fractions.
- Play Video Ordering Fractions (grades K-2)
  Laura Scearce, Hanover County, models the use of an instructional card game to reinforce the concept of comparing fractions.
- Play Video Models for Teaching Fractions (grades 3-8)
  Alfreda Jernigan, Norfolk Public Schools, highlights three key concepts that should be addressed when providing instruction on fractions.
- Play Video Rules for Order of Operations (grades 5-8)
  Dr. Lois Williams shares an activity to get students to use writing in mathematics to remember the rules for the order of operations.
  *Handout available:* Rules for Order of Operations (Word)

## D. LEARNING – DIFFERENTIATED BASED ON THE NEED OF THE STUDENT

| Guiding Question | Resource Page # | Current Reality | Next Steps |
|---|---|---|---|
| To what extent is the division/school/teacher teaching/learning practices grounded in valid evidence-based practices? | | | |
| To what extent does the division/school provide constructive feedback on effectiveness of the learning process? | | | |
| To what extent does the division/school provide coaching to assist teachers in refining their craft? | | | |
| To what extent is the division/school/class teaching/learning differentiated based on individual staff needs? | | | |
| To what extent are division/school/teacher teaching/learning strategies discussed in CLTs? | | | |
| To what extent do administrators (as instructional leaders) monitor the fidelity of instruction, intervention, and assessment? | | | |
| | | | |

# Framework for Instructional Planning

## 1. Create an Environment for Learning

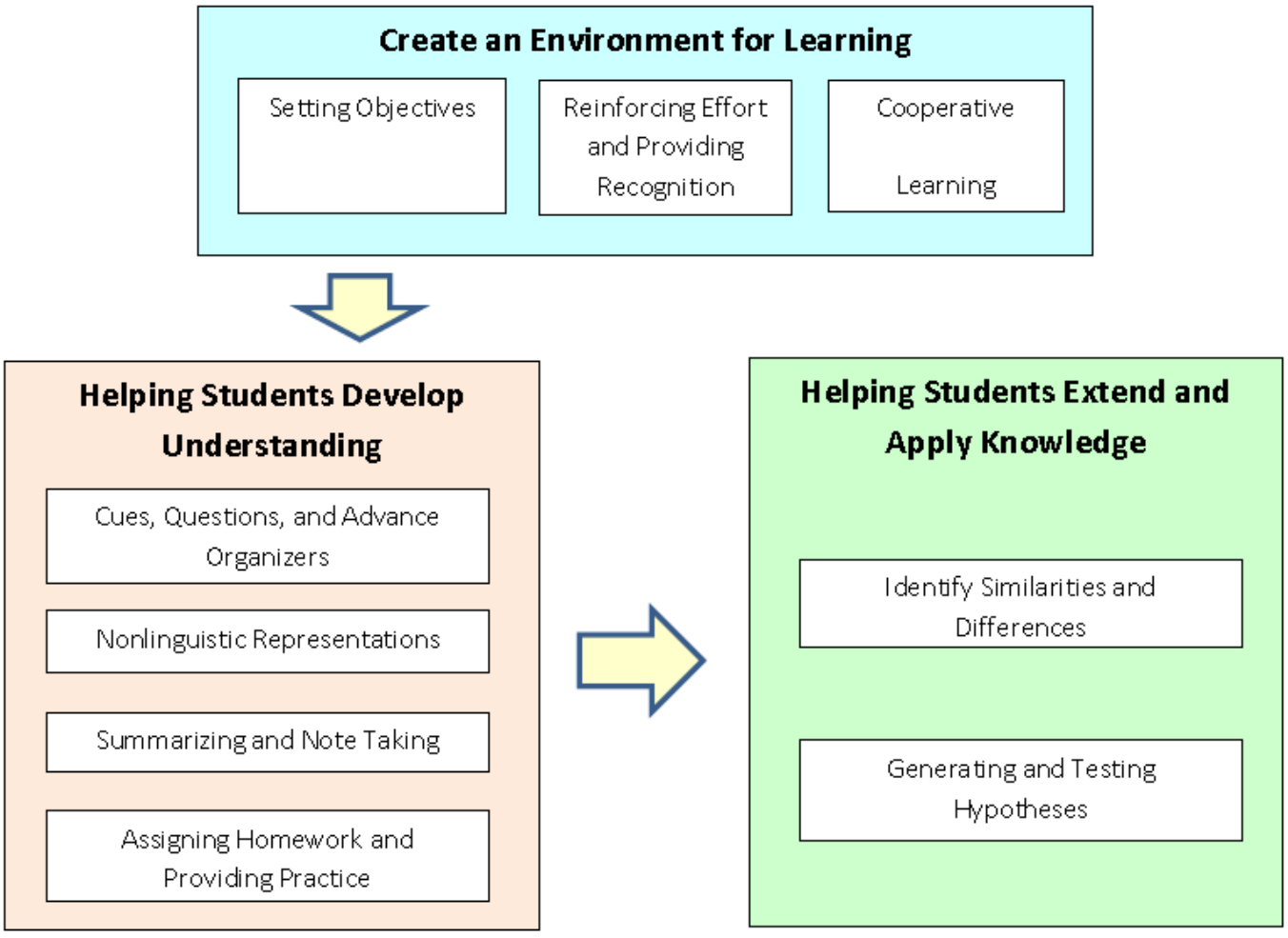
- Helping students know what is expected of them, providing students with opportunities for regular feedback on progress, assuring students they are capable of learning content and skills

## 2. Helping Students Develop Understanding*

- Integrating prior knowledge with new knowledge
- Procedural knowledge: constructing a model of the steps required of the process and practicing its variations; using the process or skill fluently or without any conscious thought

## 3. Helping Students Extend and Apply Knowledge

- Moving beyond 'right answer' learning to an expanded understanding and use of concepts and skills in real-world contexts.



# Mathematics Standards-Based Lesson Plan Blueprint

Infusing 21st century mathematics process skills for students and learning strategies for educators

**Subject:** _____

**Duration:** _____

**Standard(s) (including Essential Knowledge and Skills):** (underline the student thinking)

**Essential Vocabulary (including Prior Knowledge Vocabulary):**

**Materials needed:**

**Student Process Skills being addressed: (circle the skills that apply)**

| | | |
|---|---|---|
| **Problem Solving** | **Communication** | **Reasoning** |
| **Connections** | **Representations** | **Strategic use of Tools** |

**Assessment/Expected Outcomes:** (As a result of this lesson what will students be able to do/demonstrate..? What will be accepted as evidence? Assessments should match the level of thinking of the objective (the verbs)).

- ☐ Student self-assessment
- ☐ Performance tasks
- ☐ Oral reports
- ☐ RAFT
- ☐ Technology enhanced
- ☐ Essays
- ☐ Forced choice
- ☐ Short written response
- ☐ Teacher observation
- ☐ Demonstration
- ☐ Authentic Assessment
- ☐ Rubric Provided
- ☐ Free Response
- ☐ Other _____

Dan Mulligan, 2015

**Procedures/Lesson Overview** (Specify the steps of the lesson presentation)

**NOTE: Include *at least one* kinesthetic activity in your lesson plan.**

**Creating an Environment for Learning:** (Activate prior knowledge. Provide background information. How will students be hooked?

- ☐ Set objectives
- ☐ Provide feedback
- ☐ Reinforcing Effort
- ☐ Cooperative Learning
- ☐ Identify similarities and differences
- ☐ Providing Recognition

**Helping Students Develop Understanding:** (What strategies and activities will be used to support the learning objectives? How will students receive feedback on their understanding and progress?)

- ☐ Nonlinguistic representations
- ☐ Summarizing and note taking
- ☐ Questions, cues, advance organizers
- ☐ Cooperative learning
- ☐ Identify similarities and differences
- ☐ Assigning Homework and Providing Practice

**Helping Students Extend and Apply Knowledge:** (Tie new knowledge to existing knowledge and future knowledge. ***Reflect. Evaluate. Think. Create.***)

- ☐ Identifying Similarities and Differences
- ☐ Generating and testing Hypotheses
- ☐ Evaluate
- ☐ Self-assessment
- ☐ Summarizing and Note Taking

**Additional Notes:**

Dan Mulligan, 2015

# Mathematical Practice Look Fors

| Student Behaviors | |
|---|---|
| **Actively solving problems**<br>*Students are:*<br>• Working and reading rich problems carefully<br>• Drawing, pictures, diagrams, tables, or using objects to make sense of problems<br>• Discussing the meaning of problems with classmates<br>• Making choices about which solution path to take<br>• Trying out potential solution paths and making changes as needed<br>• Checking answers and making sure solutions are reasonable and make sense<br>• Persisting in efforts to solve challenging problems, even after reaching a point of frustration | |
| **Consistently reason mathematically**<br>*Students are:*<br>• Using mathematical symbols to represent situations<br>• Taking quantities out of context to work with them<br>• Putting quantities back in context to see if they make sense<br>• Considering units when determining if the answer makes sense in terms of the situation | |
| **Collaboratively justify own reasoning and the reasoning of others**<br>*Students are:*<br>• Making and testing conjectures<br>• Explaining and justifying their thinking using words, objects, and drawings<br>• Listening to the ideas of others and determining if they make sense<br>• Asking useful questions<br>• Identifying flaws in logic when responding to the approaches of teammates<br>• Elaborating with a second sentence to explain their thinking and connect it to the first sentence<br>• Talking about and asking questions about each other's thinking<br>• Revising their work based upon the justification and explanations of others | |
| **Model their mathematical thinking**<br>*Students are:*<br>• Using mathematical models (e.g., formulas, equations, symbols) to solve problems in the world<br>• Using appropriate tools such as objects, drawings, and tables to create mathematical models<br>• Making connections between different mathematical representations (concrete, verbal, algebraic, numerical, graphical, pictorial, etc.)<br>• Checking if an answer makes sense within the context of a situation and changing the model as needed | |
| **Use appropriate tools strategically**<br>*Students are:*<br>• Using technological tools to explore and deepen understanding of concepts<br>• Deciding which tool will best help solve the problem<br>  ○ Calculator<br>  ○ Concrete models<br>  ○ Digital technology<br>  ○ Pencil/paper<br>  ○ Ruler, compass, protractor<br>• Estimating solutions before using a tool<br>• Comparing estimates to solutions to see if the tool was effective | |
| **Regularly demonstrate mathematical precision**<br>*Students are:*<br>• Communicating using clear language and accurate mathematical vocabulary<br>• Deciding when to estimate or give an accurate answer<br>• Calculating accurately and efficiently, expressing answers with an appropriate degree of precision<br>• Using appropriate units; appropriately labeling diagrams and graphs | |
| **Make Mathematical Connections**<br>*Students are:*<br>• Finding and explaining patterns in numbers<br>• Finding and explaining patterns in diagrams and graphs<br>• Using patterns to make rules about math<br>• Using math rules to help them solve problems | |

# Mathematical Practice Look Fors

| Teacher Behaviors | |
|---|---|
| **Actively solving Problems**<br>*Teachers are:*<br>• Providing rich problems aligned to the Essential Knowledge and Skills of the standards<br>• Providing appropriate time for students to engage in the productive struggle of problem solving<br>• Asking open-ended questions: *What do you know? What do you need to find out? What can we do?*<br>• Asking rigorous questions: *How is ___'s way of solving the problem like/different from yours? Why?*<br>• Asking guiding questions: *What tools/manipulatives might help? How can we get past this?* | |
| **Consistently reason mathematically**<br>*Teachers are:*<br>• Providing a variety of problems in different contexts that allow students to arrive at a solution in different ways<br>• Using think aloud strategies (math talk) as they model the 'thinking' involved in problem solving<br>• Attentively listening for strategies students are using to solve problems<br>• Asking meaningful questions: *What does the number ____ (or variable) represent in the problem? How can you represent the problem with symbols and numbers? Can you make a chart, table, or graph?* | |
| **Collaboratively justify own reasoning and the reasoning of others**<br>*Teachers are:*<br>• Posing tasks that require students to explain, justify, argue, or critique<br>• Providing many opportunities for student discourse in pairs, groups, and during whole group instruction<br>• Create discourse by asking: *What examples could prove or disprove your reasoning? What questions would you ask about___?* | |
| **Model their mathematical thinking**<br>*Teachers are;*<br>• Providing opportunities for students to solve problems in real world contexts<br>• Identifying problem solving connects connected to student interests<br>• Asking questions: *Can you write a number sentence to describe this situation? What do you already know about solving this problem? What connections do you see? Why do the results make sense? Is this working or do you need to change your model?* | |
| **Use appropriate tools strategically**<br>*Teachers are:*<br>• Using Making a variety of tools readily accessible to students and allowing them to select appropriate tools for themselves<br>• Helping students understand the benefits and limitations of a variety of math tools<br>• Asking questions: *Which tool/manipulative would be best for this problem? Why? What other resources would help you solve tis problem?* | |
| **Regularly demonstrate mathematical precision**<br>*Teachers are:*<br>• Explicitly teaching mathematics vocabulary<br>• Insisting on accurate use of academic language from students<br>• Modeling precise communication<br>• Requiring students to answer problems with complete sentences, including units<br>• Providing students to check the accuracy of their work<br>• Asking questions: *Illustrate or give an example of the word in context. How do know your answer is accurate? Explain.* | |
| **Make Mathematical Connections**<br>*Teachers are:*<br>• Providing sense making experiences for all students<br>• Allowing students to do the work of using structure to find patterns for themselves rather than doing the work for students<br>• Asking questions: *Why does this happen? How is ___ related to ___? Why is this important to the problem? What do you know about ___ that you can apply to this situation? How can you use what you know to explain why it works? What patterns do you see?* | |