Supporting the Mathematics Process Goals through Research-based Teaching Practices: Part II

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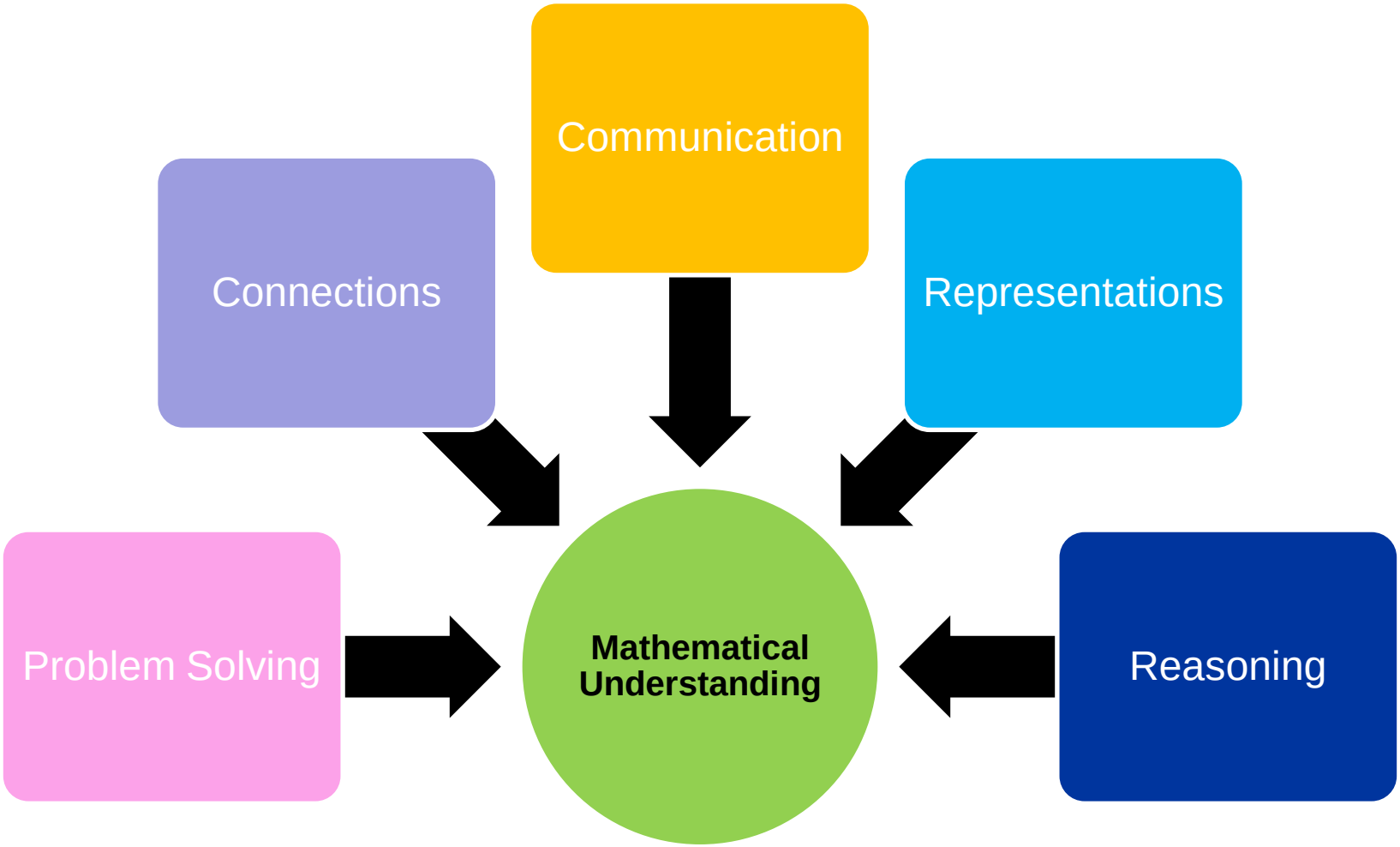
Essential Understandings

- The mathematics process goals (problem solving, communication, reasoning, connections, and representations) play an instrumental role in the teaching and learning of mathematics with understanding.

- Teaching Practice: Use and Connect Mathematical Representations
  Students demonstrate a deeper mathematical understanding and enhanced problem solving abilities when they learn to represent, discuss and make connections among mathematical ideas in multiple forms.

Mathematics Process Goals



Five goals…for students to

become mathematical problem solvers that

- communicate mathematically;
- reason mathematically;
- make mathematical connections; and
- use mathematical representations to model and interpret practical situations

Process Goals

❖ Process Goals: Connections and Representations

Fostering students' ability to make mathematical connections and use effective and appropriate representations

- What does it mean?
- What does it look like in the classroom?
  - Role of teacher
  - Role of student
- How do you support teachers as they strive to help students make connections and use effective and appropriate representations in the classroom?

❖ VDOE Mathematics Updates and Resources

Defining Connections and Representations

- What does it mean to **make mathematical connections** and **use effective and appropriate representations**?
- What words come to mind when you think about …

  making connections?

  using effective and appropriate representations?
- *Turn-n-Talk*

Mathematical Connections

Students will relate concepts and procedures from different topics in mathematics to one another and see mathematics as an integrated field of study. Through the application of content and process skills, students will make connections between different areas of mathematics and between mathematics and other disciplines, especially science. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.

From the Introduction to the 2009 Mathematics Standards of Learning

Quality instruction allows all students to:

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

NCTM Principles and Standards, 2000

Making Connections in Elementary School

Teaching Elementary Students the

Magic of Math

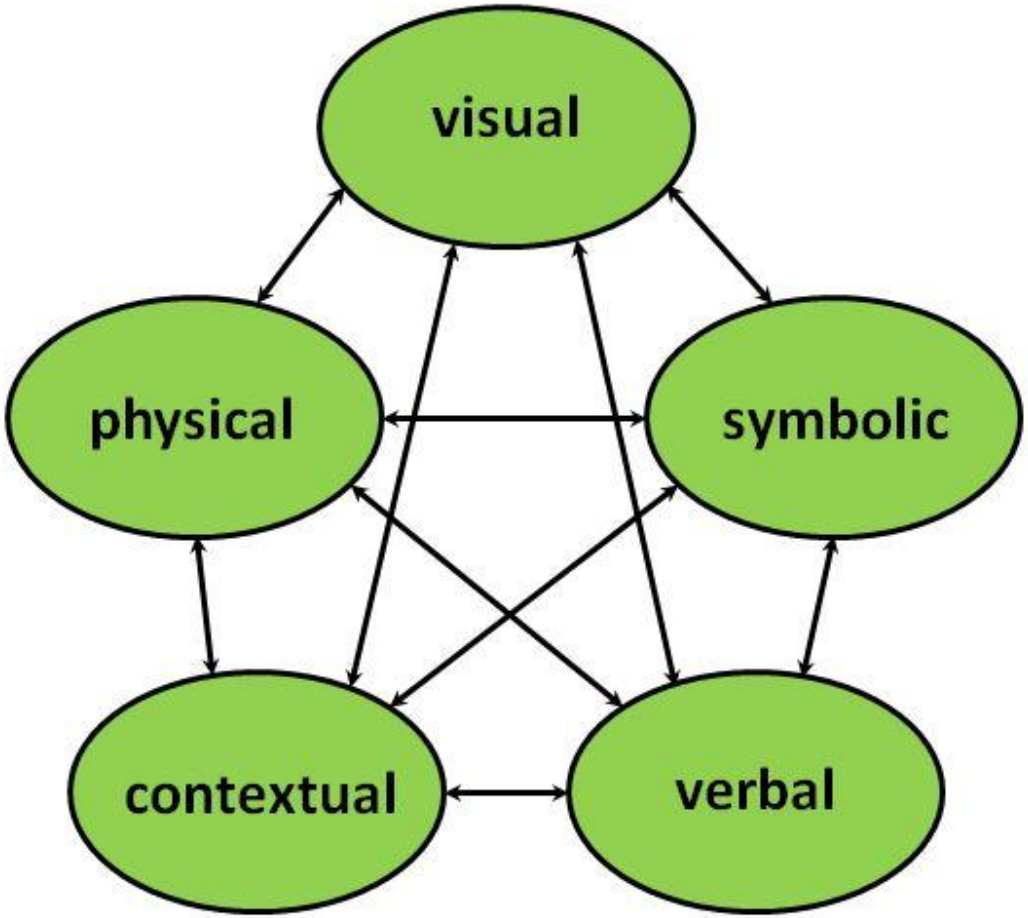
Think of a time when you saw (or taught) a lesson where students were making GREAT connections!
Share with a partner or small group.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships with a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should move easily among different representations – graphical, numerical, algebraic, verbal, and physical – and recognize that representation is both a *process and a product*.

From the Introduction to the 2009 Mathematics Standards of Learning

Five Types of Mathematical Representations



Source: The National Council of Teachers of Mathematics. (2014). *Principles to Actions Ensuring Mathematical Success for All*, Reston, VA: NCTM.

Representations are useful in all areas of mathematics because they help us **develop, share**, and **preserve** our mathematical thoughts.
They help to **portray**, **clarify**, or **extend a mathematical idea** by focusing on its essential features."

NCTM Principles and Standards, 2000, p. 206

Taking a Look Inside an Elementary Classroom

- First/Second Grade Combination Class
- Teacher – Tim Sears (14 year veteran)
- Probability and Animal Habitats

Video Reflection Questions

- What type of connections are being made in this lesson?
- What types of representations are students using?
- What does this tell you about their understanding of the concepts being developed in this lesson?

Use and Connect Mathematical Representations

WHAT ARE TEACHERS DOING:

- Selecting tasks that allow students to decide which representations to use in making sense of the problems.
- Allocating substantial instructional time for students to use, discuss, and make connections among representations.
- Introducing forms of representations that can be useful to students.
- Asking students to make math drawings or use other visual supports to explain and justify their reasoning.
- Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.
- Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.

WHAT ARE STUDENTS DOING:

- Using multiple forms of representations to make sense of and understand mathematics.
- Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.
- Making choices about which forms of representations to use as tools for solving problems.
- Sketching diagrams to make sense of problem situations.
- Contextualizing mathematical ideas by connecting them to real-world situations.
- Considering the advantages or suitability of using various representations when solving problems.

NCTM,. (p 29, 2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA. NCTM

The Value of Making Connections in the Mathematics Classroom

An emphasis on mathematical connections helps students build a disposition to use connections in solving mathematical problems, rather than see mathematics as a set of disconnected, isolated concepts and skills.

NCTM Principles and Standards, 2000

Experiencing Mathematics in Context

- Students should connect mathematical concepts to their daily lives.
- When students connect mathematical ideas, their understanding is deeper and more lasting.
- Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics.

NCTM Principles and Standards, 2000

Role of the Student

- *Create* and *use* representations to organize, record, and communicate mathematical ideas
- *Select*, *apply*, and *translate* among mathematical representations to solve problems
- *Use* representations to model and interpret physical, social, and mathematical phenomena

Adapted from: National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA.

The Role of the Teacher

- *Create* a learning environment that encourages and supports the use of multiple representations
- *Model* the use of a variety of representations
- *Orchestrate* discussions where students share their representations and thinking
- *Support* students in making connections among multiple representations, to other math content and to real world contexts

Adapted from: National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA.

Beliefs about Teaching and Learning Mathematics

STAND UP, PAIR UP, SPEAK UP

- *Introduce yourself to your new partner*
- *Review the Beliefs about Teaching and Learning Mathematics with your partner (p. 1 of Principles to Actions handout).*
- **What productive beliefs are evident in the Mr. Sear's classroom?**
- **How do those beliefs support students in making connections among different representations of the problem?**

Students must be actively engaged in developing, interpreting, and critiquing a variety of representations.
This type of work will lead to better understanding and effective, appropriate use of representation as a mathematical tool.

NCTM Principles and Standards, 2000, p. 206

Planning for the Use of Representations

Teachers must ask themselves,

"What models or materials (representations) will help convey the mathematical focus of today's lesson?"

- Skip Fennell, Past-President NCTM

http://www.nctm.org/uploadedFiles/About_NCTM/President/2006_09pres.pdf

2014 – Making Connections and Using Representations

Planning Mathematics Instruction: Essential Questions

Determining Mathematical Objective(s) for Students

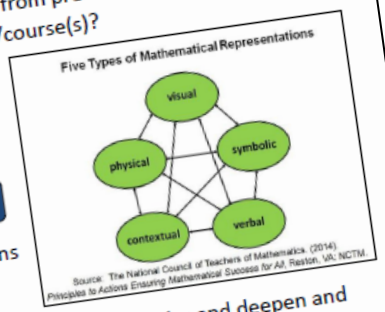
- What should students know, understand, and demonstrate?
- Which bullets from the Essential Knowledge and Skills will be addressed?
- What criteria will be used to determine student understanding?

Connecting to Prior Knowledge and Future Content

- What is the related content (prior knowledge) from previous grade(s)/course(s)?
- What is the related content in future grade(s)/course(s)?
- What representations/strategies were used to develop prior knowledge?
- How is the objective connected to the related content and to the real world?

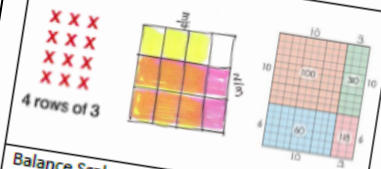
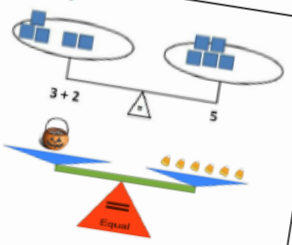
Developing and Reinforcing Content

- What are students' common misconceptions and how will they be addressed?
- What vocabulary is essential?
- Which representations/strategies will model the mathematics and deepen and extend students' mathematical understanding?
  - What are the strengths and limitations of the representation/strategy?
  - How will the selected representation assist in student understanding?
- In what order will the content and different representations be introduced?
- What mathematical tasks will be used to develop content?
  - How is the task connected to the objective and representation/strategy?
  - Does the task require an appropriate level of cognitive demand?
  - In what ways could the task be extended to connect to other mathematical concepts or other disciplines?
- How will students communicate their understanding?
- What differentiation will address the learning needs of students with disabiliti language learners, and gifted students?
- How will student understanding be monitored throughout instruction (forma assessment)?



Mathematics Instructional Connections for Physical and Visual Representations

This document serves as a resource to assist teachers in connecting physical and visual representations to mathematical content. It is not meant to be an exhaustive list, nor does it mean that these representations may only be used with the identified content. Challenges or limitations may arise when using some representations to model content.

Representation	Content Connections
Algebra Tiles 	• Operations with integers and polynomial expressions • Model and factor linear and quadratic expressions *Challenges or limitations could include:* • Modeling expressions other than linear and quadratic • Factoring non-factorable quadratic expressions
Arrays 	• Multiplication and division of whole numbers • Operations with fractions • Area and perimeter • Distributive Property • Commutative Property
Balance Scale 	• Properties of equality • Equality • Model one-variable equations • Solve one-variable equations
Base Ten Blocks 	• One-to-one correspondence • Count and skip count • Place value • Represent whole numbers and decimals • Compare and order compare whole numbers and decimals • Operations with whole numbers and decimals • Powers of 10

Role of the Mathematics Teacher Leader

How do you support teachers as they strive to help students make connections and use effective and appropriate representations in the classroom?

Mathematics Classroom LOOK FORS

- Are multiple representations and connections encouraged within the classroom? this lesson?
- What types of representation and/or connections are present in this activity?
- How are students creating and using representations to make sense of the mathematics?
- Are students describing and justifying their reasoning with drawings, diagrams, and other representations?
- How is the classroom discussion enhanced by students' representations and/or connections?
- In what ways does the teacher assess students' abilities to use representations meaningfully to solve problems?

Fostering Connections and Representations in Instruction

- Use data to determine areas of weakness (students and teachers)
- Be knowledgeable about and encourage the use of available resources
- Structure PLCs to explore resources provided
  - Process goals for students
  - Vertical articulation of content
  - Instructional strategies
  - Development of quality assessments
  - Effective use of data (formative and summative)

Five goals…for students to

become mathematical problem solvers that

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Process Goals

VDOE Updates and Resources

Mathematics SOL Revision Process

Anticipated Timeline

- February 2015 – timeline goes to the BOE
- March 2015 – public comment on 2009 standards
- Spring/Summer/Fall 2015 – review process
- Implement (2009/2016 SOL – 2017-18)
- Implement (2016 SOL – 2018-19)

VDOE – Resources

- Standards of Learning
- Curriculum Framework
- Testing Blueprints
- SOL Practice Items and Tools Practice
- 2013 Released Test Items
- 2013 Student Performance Analysis
- Technical assistance document
- Plain English Mathematics Test Information
- ESS Sample Lesson Plans
- Instructional Videos
- Vocabulary Resources – K-8, Geometry, and Algebra Cards
- Mathematics Institutes (2009, 2010, 2011, 2012, 2013, 2014)

NEW

Mathematics Teaching Practices	VDOE Mathematics Professional Development Resources (available online)
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	**Making Mathematical Connections and Using Representations (2014)** provides teachers with professional development focused on fostering students' abilities to make mathematical connections and use effective and appropriate representations in mathematics. **Modifying Resources to Promote Problem Solving and Critical Thinking (2013)** provides teachers with professional development focused on analysis and modification of existing resources to match student learning expectations and promote problem solving. **Using Formative Assessment Resources to Drive Instructional Decisions (2012)** provides teachers with professional development on the use of formative assessment resources to drive instructional decisions. **Facilitating Student's Mathematical Understanding Through a Focus on the Process Goals for Students (2011)** provides professional development resources focused on facilitating students' mathematical understanding through problem solving, communication and reasoning. **Implementing New Content and Increased Rigor of the 2009 Mathematics Standards of Learning (2010)** provides teachers with professional development to support the implementation of the 2009 Mathematics Standards of Learning. **Overview of Changes from the 2001 to 2009 Mathematics Standards of Learning (2009)** provides teachers with professional development containing an overview of the changes to the mathematics standards. All available online at: http://www.doe.virginia.gov/instruction/mathematics/professional_development/index.shtml
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.	
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.	
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.	
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.	
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.	
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.	

Adapted from: The National Council of Teachers of Mathematics. (p. 9-10, 2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA. NCTM

Professional Development

Board of Education
News
For Public Education Administrators
For Students & Parents
Education Directories
Standards of Learning (SOL) & Testing
Instruction
Special Education
Student & School Support
Teaching in Virginia
Federal Programs
Statistics & Reports
Information Management
School Finance

VDOE provides targeted professional development through Mathematics Institutes. Since 2009, the institutes, framed by the five goals for students of becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations, have:

- outlined the content standard changes from the 2001 Mathematics SOL to the 2009 Mathematics Standards of Learning (SOL) – 2009;
- supported district leaders and teachers in the implementation of the 2009 Mathematics SOL – 2009, 2010, 2011, 2012, 2013, 2014;
- provided training in the vertical progression of content and pedagogy – 2010, 2014;
- provided instructional guidance in content areas of greatest challenge – 2010, 2014;
- provided professional development resources focused on facilitating students' mathematical understanding through problem solving, communication, and reasoning – 2011;
- provided professional development resources focused on the use of formative assessment resources to drive instructional decisions – 2012;
- provided professional development on the analysis and modification of existing resources to match student learning expectations and on promoting problem solving – 2013; and
- provided professional development on fostering students' abilities to make mathematical connections and use effective and appropriate representations in mathematics – 2014.

STANDARDS OF LEARNING

Mathematics K-12

Includes:

- Curriculum Frameworks
- Enhanced Scope & Sequence Guides
- Test Blueprints
- Released Tests and
- Practice Items

Mathematics SOL Institutes

Expand All | Collapse All

2014 – Making Mathematical Connections and Using Representations

2013 – Modifying Resources to Promote Problem Solving and Critical Thinking

2012 – Using Formative Assessment Resources to Drive Instructional Decisions

"The content of the mathematics standards is intended to support the five goals for students"

- 2009 *Mathematics Standards of Learning*

2009 – Changes in Mathematics Standards

Changes to Standards of Learning (SOL) Content

- Increase rigor through an emphasis on multistep problems and application
- Move content between grade levels/subjects to improve the vertical progression
- Remove content from a grade levels/subjects
- Remove content repeated among grade levels
- Add new content
- Reduce language that limits content

2010 – Vertical Articulations and Pedagogy

Vertical Articulation Technical Assistance Document - Grade 3 Through Grade 6

	Grade 3	Grade 4	
Counting/ Cardinality/Place Value	3.1 a) read/write 6-digit numerals, ID place value/value of each digit; b) round whole numbers 9,999 or less to nearest 10/100/1000; c) compare two whole numbers 0 - 9,999 w/ symbols/words	4.1 a) ID orally/in writing place value for each digit in a whole number through millions; b) compare two whole numbers through millions w/ symbols; c) round whole numbers through millions to nearest 10/100/1,000/10,000/100,000	5.1 ro neare
Modeling/Comparing/Ordering	3.3 a) name/write fractions rep by model; b) model/write fraction's names; c) compare fractions w/like/unlike denominators	4.2 a) compare and order fractions/mixed numbers; b) represent equivalent fractions; c) ID division statement that represents a fraction	5.2 a) equiv comp
	3.6 represent mult/div using area/set/number line models, create/solve problems involving mult of two whole numbers 99 or less and 5 or less	4.3 a) read/write/represent/ID decimals through thousandths; b) round to whole, tenth, hundredth; c) compare/order; d) write decimal and fraction equiv from a model	5.18 c using
		4.5 a) determine common multiples/ factors	5.3 a) comp chara

Mathematical Problem Solving

Mathematical Communication

Mathematical Reasoning

Mathematical Connections

Mathematical Representations

MATHEMATICS

INSTRUCTIONAL VIDEOS FOR TEACHERS

These instructional videos are provided as support for the implementation of the 2009 Mathematics Standar

ON THIS PAGE: Strategies Across the Strands · Number & Number Sense · Computation & E · Probability & Statistics · Patterns, Functions & Algebra · Additional Resources

The videos on this page link to Flash files which require the Adobe Flash Player plug-in. Download the free

Strategies Across the Strands

- Play Video Working with Vocabulary / Concept Development (grades 4-8)
  Dr. Lois Williams, VDOE mathematics specialist, on the Frayer Model for vocabulary/concept develop
  Handout available: Working with Vocabulary / Concept Development (Word)
- Play Video Vocabulary (grades 6-8)
  Dr. Lois Williams shares a technique to assist students who confuse common mathematics terms su
- Play Video Notebooks for Organization (grades 6-8)
  Dr. Lois Williams explains an organizational system for middle school mathematics classes.
- Play Video Multi-Step Problem Solving (grades 4-8)
  Cheryl Gray, Spotsylvania County Schools middle school mathematics specialist, on how to approach table. *Handout available:* Multi-Step Problem Solving (PPT)
- Play Video Scientific Calculator Use (grades 6-8)
  Dr. Lois Williams explains the use of scientific calculators in Virginia middle schools.
  Handout available: Scientific Calculator Manual (PDF)

Number & Number Sense

- NEW Play Video Developing Early Number Sense (grades K-2)
  Laura Domalik, Hanover County provides instructional strategies for counting and vocabulary. The str more than (+1), one less than (-1), basic fact concepts of +1 and -1, and missing addends.
- NEW Play Video Using a Beaded Number Line (grades K-2)
  Laura Scearce, Hanover County, provides instructional strategies for using beaded number lines to de backwards, rounding, adding, and subtracting.
- NEW Play Video Modeling Equality (grades K-2)
  Debi Godfrey, Henrico County, provides activities with pan and number balances to represent and buil

2011 – Cognitive Demand and Process Goals

Mathematical Problem Solving

Mathematical Communication

Mathematical Reasoning

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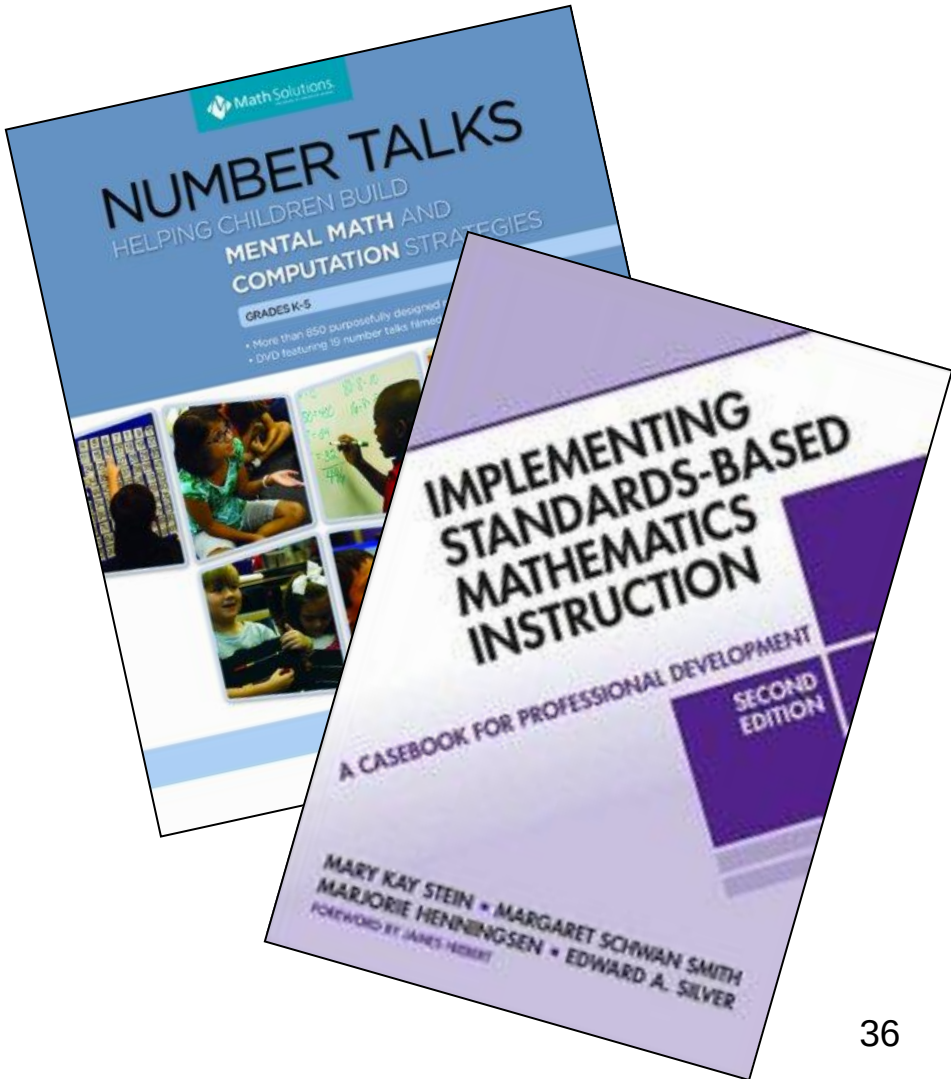


2012 – Using Formative Assessment Tools

Virginia Department of Education
Fall 2012 Mathematics Standards of Learning Institutes

Facilitator's Guide
Grade Band 6-8

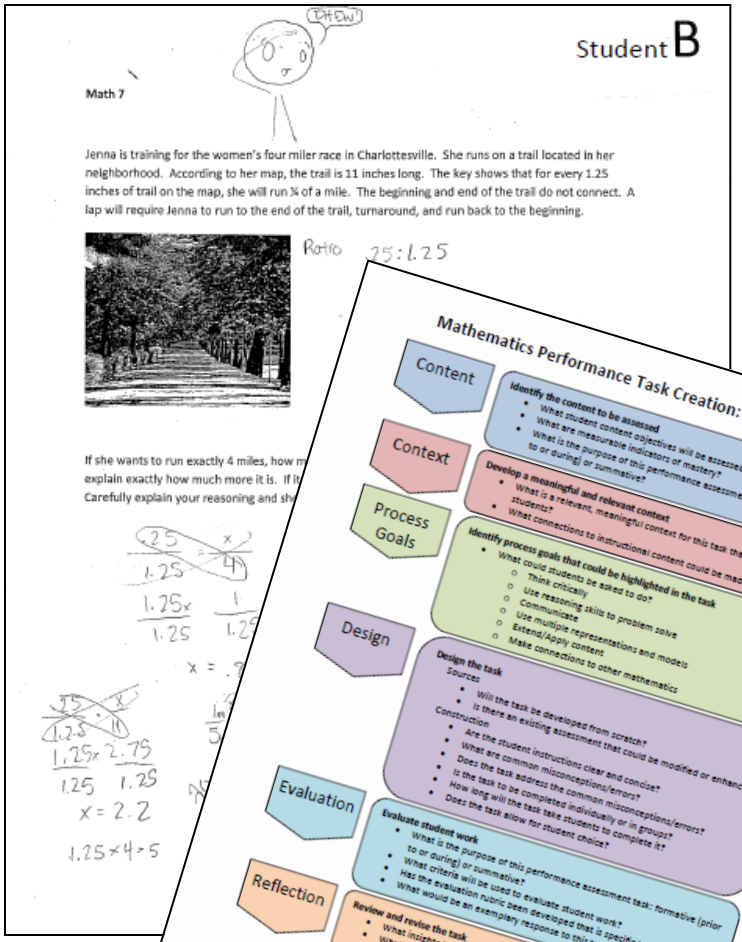
2012 Mathematics SOL Institutes
The purpose of the 2012 Mathematics Standards of Learning (SOL) Institutes was to provide teachers with professional development on using formative assessment resources to drive instructional decisions.

Introduction and Instructions
The product of the 2012 Mathematics SOL Institutes is a set of six online professional development modules designed to be used by a group of teachers of a specific grade level or course. Modifications could be made to adapt the professional development for more than one grade level or for large groups. Each group of teachers should select a facilitator for which this Facilitator's Guide was written. Facilitators should review the activities and handouts prior to facilitating this professional development. When scheduling for this professional development, please allow adequate time to complete all six of the modules while minimizing the amount of time between module sessions.

A **Mathematics Performance Task** is an assessment that requires students to synthesize mathematical content in a problem-solving setting that requires communication, reasoning, use of multiple representations, and making mathematical connections. Read more about Virginia's Process Goals for students.

Time	Facilitator Instructions	Materials
60 minutes total	**Module 1: Performance Tasks** Purpose: To anticipate common student misconceptions and errors associated with a mathematics performance task.	Handouts Needed: - Mathematics Performance Task (the Grade 7 performance task may be used by teachers of grades 6-8) – Grade 7 - Common Misconceptions and Errors Recording Sheet (a summary recording sheet will be used in Module 3) Other: - Grade 7 student video
15 minutes	1) Distribute the performance task. 2) Ask participants to work on the performance task individually. While working on the task, participants should anticipate and record common student misconceptions and errors on the Common Misconceptions and Errors Recording Sheet.	
45 minutes	1) Have participants discuss their solution strategy with a partner or small group and then the facilitator should select varied solutions to be shared with the whole group. 2) Give participants guiding questions to answer as they view the video. Show student video (6:30 min). (Facilitator may suggest looking for: independent student work; small group discussion; facilitator enhancing task through guided learning; group share of strategies) Have participants view the video keeping the following statements in mind: - Give examples of the teacher acting as a facilitator of learning (i.e., wait	

Virginia Department of Education – 1



Mathematical Problem Solving

Mathematical Communication

Mathematical Reasoning

Mathematical Connections

Mathematical Representations

2013 –Modifying Instructional Resources to Promote Problem Solving and Critical Thinking

Virginia Department of Education
Fall 2013 Mathematics Standards of Learning Institutes

Facilitator's Guide
Grade Band 3-5

2013 Mathematics SOL Institutes

The purpose of the 2013 Mathematics SOL Institutes is to provide teachers with professional development focused on the relationship between curriculum, instruction, and assessment, by targeting the processes of analysis and modification of existing resources to match student learning expectations and promote problem solving.

Introduction and Instructions

This grade-band professional development will be comprised of two components:

- Module 1 Parts 1-4: Analyzing and modifying assessments – Participants will compare expectations of SOL and Curriculum Framework to an assessment and modify it to meet intended expectations.
- Module 2 Parts 1-3: Modifying mathematical tasks to promote problem solving – Participants will modify existing mathematical tasks to emphasize the use of process skills and problem solving.

Virginia's Process Goals for Students

SOL Vertical Articulation Technical Assistance Documents

- Grades K-3
- Grades 3-6
- Grades 5-8
- Algebra
- Geometry

The product of the 2013 Mathematics SOL Institutes is a set of online professional development modules designed to be used by a group of teachers of a specific grade level or course. Modifications could be made to adapt the professional development for more than one grade level/course or for large groups. Each group of teachers should select a facilitator for which this Facilitator's Guide was written. Facilitators should review the activities and handouts prior to facilitating this professional development.

Approximate Time	Facilitator Instructions	Materials
30 minutes total	**Module 1 Part 1: Analysis of Assessments**	
15 minutes	1) Select and distribute the appropriate assessment for your whole group. 2) Ask participants to complete the assessment individually. While working, participants should be thinking about whether or not it is a "good" assessment and why.	• Mathematics Assessment (select the assessment for your course) – Grade 3 – Grade 4 – Grade 5 • Assessment Analysis Guiding Questions
15 minutes	3) Have participants discuss their conclusions with a partner or small group. 4) Have small groups share and record their conclusions and justifications with the whole group. 5) View the Assessment Analysis Guiding Questions document and discuss similarities	

Mathematical Problem Solving

Mathematical Communication

Mathematical Reasoning

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Mathematical Representations

Assessment Analysis Guiding Questions

SOL ALIGNMENT

- Does the assessment assess the standard and targeted components of the Essential Knowledge and Skills in the Curriculum Framework?
- Does the assessment reflect the requirements of the verbs found in the Curriculum Framework?
- Did instruction go beyond the standard?
  - If so, does the assessment reflect your instruction?
  - If not, does the assessment adhere to the parameters of the SOL?

LEVEL OF COGNITIVE DEMAND

- Does the assessment have an appropriate variety of questions requiring various levels of cognitive demand?
- Does the assessment require students to explain and justify?

FORMAT

- Does the assessment provide various ways for students to demonstrate understanding (open response, multiple

Levels of Classroom Discourse

	Teacher role	Questioning	Explaining mathematical thinking	Mathematical representations	Building student responsibility within the community
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so that others are able to follow their mathematical thinking.	Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

Fig. 11. Levels of classroom discourse. From Hufford-Ackles, Fuson, and Sherin (2014), table 1.

The National Council of Teachers of Mathematics. (p. 32, 2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA. NCTM

QUESTIONS?

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Resources

Edutopia. Teaching Children the Magic of Math, YouTube. June 23, 2014, https://www.youtube.com/watch?v=E91gAjunqOM

Annenburg Learner: Applying Representations (K-2) http://www.learner.org/courses/teachingmath/gradesk_2/session_05/section_04_d.html?pop=yes&pid=3068#

National Council of Teachers of Mathematics. 2000 *Principals and Standards for School Mathematics*. Reston, VA.

Virginia Standards of Learning, 2009.

Virginia Department of Education Website