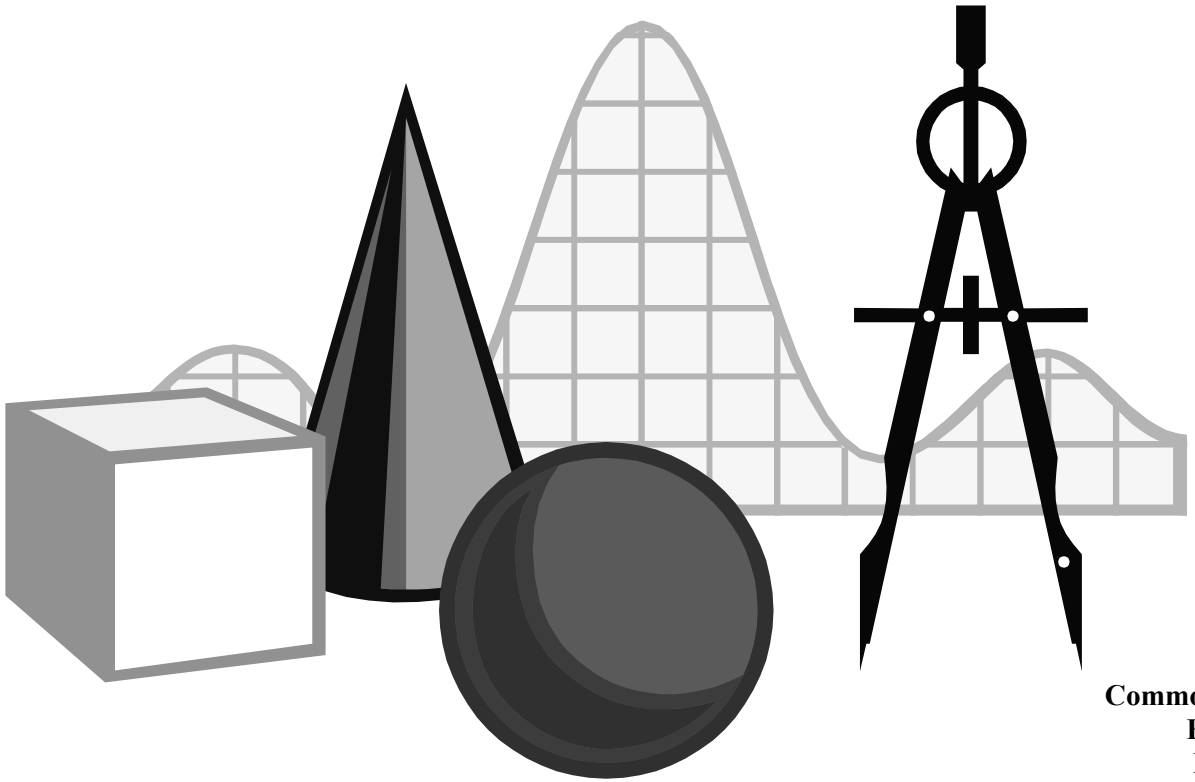
MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Algebra II



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The 2002 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's website at http://www.pen.k12.va.us.

Introduction

Mathematics content develops sequentially in concert with a set of processes that are common to different bodies of mathematics knowledge. The content of the Mathematics Standards of Learning supports five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. These goals provide a context within which to develop the knowledge and skills identified in the standards.

Algebra provides a systematic way to represent mathematical relationships and analyze change. Students need to understand the concepts and symbols of algebra, the structures that govern the manipulation of the symbols, and ways that the symbols can be used to record ideas and events. Students should explore patterns that are exponential and logarithmic and continue to develop the notion of families of functions.

Each topic in the Algebra II Curriculum Framework is developed around the Standards of Learning. Each Standard of Learning is expanded in the Essential Knowledge and Skills column. The Essential Understandings column includes concepts, mathematical relationships, and ideas that are important to understanding and teaching the Standard of Learning effectively.

Teachers should help students make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections should be made to other subject areas and fields of endeavor through applications. Using manipulatives, graphing calculators, and computer applications to develop concepts should help students develop and attach meaning to abstract ideas. Throughout the study of mathematics, students should be encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop their competence and their confidence in themselves as mathematics students.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.1

The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Complex numbers are organized into a hierarchy of subsets with properties applicable to each subset. • Complex numbers are a superset of real numbers and, as a system, contain solutions for equations that are not solvable over the set of real numbers.	• Identify examples of field properties: commutative, associative, identity, inverse, and distributive. • Identify examples of axioms of equality: reflexive, symmetric, transitive, substitution, addition, and multiplication. • Identify examples of axioms of inequality and order: trichotomy, transitive, addition, and multiplication. • Place the following sets of numbers in a hierarchy of subsets: complex, pure imaginary, real, rational, irrational, integers, whole, and natural. • Add and multiply matrices, and determine which field properties hold.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.2

The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Computational skills applicable to numerical fractions also apply to rational expressions involving variables.	• Add, subtract, multiply, and divide rational expressions whose denominators are monomials or polynomial expressions in completely factored form. • Simplify a rational expression with common monomial or binomial factors. • Recognize a complex fraction, and simplify it as a quotient or product of simple fractions.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.3

The student will

a) add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and

b) write radical expressions as expressions containing rational exponents and vice versa.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Radical expressions can be written and simplified using rational exponents. • Only radicals with a common radicand and index can be added or subtracted.	• Simplify radical expressions containing positive rational numbers and variables. • Convert from radical notation to exponential notation, and vice versa. • Add and subtract radical expressions with like radicands. • Multiply and divide radical expressions not requiring rationalizing the denominators.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.5

The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The complete factorization of polynomials has occurred when each factor is a prime polynomial. • Pattern recognition can be used to determine complete factorization of a polynomial.	• Determine the greatest monomial factor as a first step in complete factorization. • Recognize squares and cubes of positive integers. • Recognize examples of general patterns: difference of squares, sum and difference of cubes, and perfect square trinomials. • Factor polynomials by applying general patterns.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA II
STANDARD AII.17

The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of *i*.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Complex numbers are a superset of real numbers.	• Recognize that the square root of –1 is represented as i. • Define and identify a complex number. • Apply the definition of i to simplify square roots of negative numbers. • Simplify powers of i. • Add, subtract, and multiply complex numbers.

TOPIC: RELATIONS AND FUNCTIONS

ALGEBRA II
STANDARD AII.8

The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The graphs/equations for a family of functions can be determined using a transformational approach.	• Recognize graphs of parent functions for linear, quadratic, absolute value, step, and exponential functions. • Given an equation of a function, identify the function as linear, quadratic, absolute value, step, or exponential. • Write the equation of a linear (slope-intercept form), quadratic ([h, k] form), absolute value, step, or exponential function, given the graph of the parent function or an integral translation of a parent function. • Given an equation, graph a linear, quadratic, absolute value, step, or exponential function with the aid of a graphing calculator.

TOPIC: RELATIONS AND FUNCTIONS

ALGEBRA II
STANDARD AII.9

The student will find the domain, range, zeros, and inverse of a function; the value of a function for a given element in its domain; and the composition of multiple functions. Functions will include exponential, logarithmic, and those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Functions describe the relationship between two variables. • Graphs of functions that are inverses of each other are reflections across the line $y = x$. • The composition of a function and its inverse is the identity function. • Functions arise from practical situations. • If (a, b) is an element of a function, then (b, a) is an element of the inverse of the function.	• Identify the domain, range, zeros, and inverse of a function presented algebraically or graphically. • Distinguish between relations and functions that are expressed algebraically and graphically. • Recognize restricted/discontinuous domains and ranges. • Use interchange of variables to find the inverse of a function. • Given the graphs, recognize that exponential and logarithmic functions are inverses of each other. • Find the composition of two functions. • Find the value of a function for a given element from the domain. • Investigate exponential and logarithmic functions, using the graphing calculator.

TOPIC: RELATIONS AND FUNCTIONS

ALGEBRA II
STANDARD AII.15

The student will recognize the general shape of polynomial, exponential, and logarithmic functions. The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Shapes and behavior of graphs of polynomials can be determined by analyzing transformations of parent functions. • Using graphing calculators is a strategy for investigating the shape and behavior of polynomial functions. • The Fundamental Theorem of Algebra (Carl Fredrich Gauss) states that in the complex number system, an n^{th} degree polynomial equation has n zeros. • Exponential and logarithmic functions are either strictly increasing or strictly decreasing.	• Investigate the shape and behavior of linear, quadratic, and cubic functions. Behaviors will include intercepts, number of turning points, and end behavior. • Investigate the shape and behavior of exponential ($a^x = y$) and logarithmic ($\log_b x = y$) functions, including intercepts and end behavior. • Using the general shape of the graph of a function, identify the family of graphs to which a particular graph belongs. Characteristics of a graph may include the x- and y-intercepts, number and location of turning points, and end behaviors.

TOPIC: RELATIONS AND FUNCTIONS

ALGEBRA II
STANDARD AII.16

The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first *n* terms, finding the n^{th} term, and evaluating summation formulas. Notation will include $\sum$ and a_n.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Sequences and series arise from practical situations. • The study of sequences and series is an application of investigation of patterns.	• Distinguish between a sequence and a series. • Recognize patterns in a sequence. • Distinguish between arithmetic and geometric sequences. • Use and interpret the notations $\sum$, n, n^{th} term, and a_n. • Write the first n terms in an arithmetic or geometric sequence. • Given the formula, find a_n (the n^{th} term) for an arithmetic or a geometric sequence. • Given formulas, find the sum, S_n, of the first n terms of an arithmetic or geometric series, including infinite series.

TOPIC: RELATIONS AND FUNCTIONS

ALGEBRA II
STANDARD AII.19

The student will collect and analyze data to make predictions and solve practical problems. Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit. Models will include linear, quadratic, exponential, and logarithmic functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data and scatterplots may indicate patterns that can be modeled with an algebraic equation. • Graphing calculators can be used to collect, organize, picture, and create an algebraic model of the data. • Data that fit linear, quadratic, exponential, and logarithmic models arise from practical situations.	• Collect and analyze data. • Investigate scatterplots to determine if patterns exist, and then identify the patterns. • Find an equation for the curve of best fit for data, using a graphing calculator. Models will include linear, quadratic, exponential, and logarithmic functions. • Make predictions, using data, scatterplots, or curve of best fit. • Given a set of data, determine the model that would best describe the data.

TOPIC: RELATIONS AND FUNCTIONS

ALGEBRA II
STANDARD AII.20

The student will identify, create, and solve practical problems involving inverse variation and a combination of direct and inverse variations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Practical problems can be modeled and solved by using direct and/or inverse variations. • Joint variation is a combination of direct variations.	• Translate "y is directly proportional to x" as $y = kx$. • Translate "y is inversely proportional to x" as $y = \frac{k}{x}$. • Translate "y varies jointly as x and z" as $y = kxz$. • Determine the value of the constant of proportionality, k, given initial conditions for x and y. • Set up and solve practical problems, using combinations of direct and inverse variation.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.4

The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used as a primary method of solution and to verify algebraic solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Absolute value equations and inequalities can be used to model practical problems.	• Solve absolute value equations in one variable algebraically and graphically, using a graphing calculator. • Solve absolute value inequalities in one variable algebraically and graphically. • Express the solutions to absolute value equations and inequalities in one variable graphically and as an algebraic inequality. • Graph absolute value equations in two variables. • Verify solutions to absolute value equations and inequalities in two variables, using a graphing calculator.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.6

The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and for confirming the algebraic solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A quadratic equation whose graph does not intersect the *x*-axis has only complex solutions. • Complex solutions occur in pairs (conjugates). • The quadratic formula can be used to solve any quadratic equation.	• Recognize a quadratic equation. • Select an appropriate strategy for solving a quadratic equation (factoring, using the quadratic formula, or graphing). • Solve a quadratic equation over the set of complex numbers. • Identify from a graph the real solutions to a quadratic equation. • Find the real roots of a quadratic equation, using a graphing calculator.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.7

The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used for solving and for confirming the algebraic solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A solution of an equation makes the equation true. • Equations can be solved in a variety of ways. • The solution of an equation in one variable can be found by graphing each side of the equation separately and finding the *x*-coordinate of the point of intersection. • Practical problems can be interpreted, represented, and solved using equations.	• Solve equations containing rational expressions with monomial denominators algebraically and graphically. • Solve equations containing a radical expression algebraically and graphically. The equation will contain a linear expression under the radical, and all terms outside the radical will be constants. • Identify from a graph the solutions to an equation containing rational or radical expressions. • Solve an equation containing rational or radical expressions, using a graphing calculator. • Check possible solutions to an equation containing rational or radical expressions, using a graphing calculator.

TOPIC: ANALYTICAL GEOMETRY

ALGEBRA II
STANDARD AII.10

The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, *x*-intercept of a graph, and factors of a polynomial expression.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The Fundamental Theorem of Algebra states that, including complex and repeated solutions, an n^{th} degree polynomial equation has exactly n roots (solutions). • The following statements are equivalent: – k is a zero of the polynomial function f; – $(x - k)$ is a factor of $f(x)$; – k is a solution of the polynomial equation $f(x) = 0$; and – k is an x-intercept for the graph of the polynomial.	• Identify the x-intercept(s) of a graph. • Identify the zero(s) of a function, given a graph. • Determine the linear factors of a polynomial expression when the zeros of the corresponding polynomial function are displayed on a graph.

TOPIC: ANALYTICAL GEOMETRY

ALGEBRA II
STANDARD AII.18

The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in (*h*, *k*) form, the student will sketch graphs of conic sections, using transformations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Changing parameters (*h*, *k*) affects the graph of a conic section in a predictable pattern. • Conic sections can be distinguished by their equations.	• Identify types of conic sections, given (*h*, *k*) form of an equation. • Identify types of conic sections from a graph. • Sketch the graph of a conic section in (*h*, *k*) form, using knowledge of transformations.

TOPIC: SYSTEMS OF EQUATIONS/INEQUALITIES

ALGEBRA II
STANDARD AII.11

The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Matrices can be used to model and solve practical problems.	• Recognize matrices that can be multiplied. • Perform matrix multiplication with a graphing calculator or a computer program with matrix capabilities. • Use matrix multiplication to solve practical problems.

TOPIC: SYSTEMS OF EQUATIONS/INEQUALITIES

ALGEBRA II
STANDARD AII.12

The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Matrices are a convenient shorthand for solving systems of equations. • Matrices can model a variety of linear systems. • Solutions of a linear system are values that satisfy every equation in the system.	• Model problems with a system of no more than three linear equations. • Represent a system of no more than three linear equations in matrix form. • Solve a matrix equation using a graphing calculator or computer program with matrix capability. • Find the inverse of a matrix with a graphing calculator. • Express a system of linear equations as a matrix equation.

TOPIC: SYSTEMS OF EQUATIONS/INEQUALITIES

ALGEBRA II
STANDARD AII.13

The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Linear programming models an optimization process. • A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized. • Any maximum or minimum value will occur at a corner point of a feasible region.	• Model practical problems with systems of linear inequalities. • Solve systems of linear inequalities. • Identify the feasibility region of a system of linear inequalities with no more than five constraints. • Identify the coordinates of the corner points of a feasibility region. • Find the maximum or minimum value for the function defined over the feasibility region. • Describe the meaning of the maximum or minimum value.

TOPIC: SYSTEMS OF EQUATIONS AND INEQUALITIES

ALGEBRA II
STANDARD AII.14

The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Solutions of a nonlinear system of equations are numerical values that satisfy every equation in the system. • The coordinates of points of intersection in nonlinear systems are solutions to the system.	• Identify nonlinear systems of equations as linear-quadratic or quadratic-quadratic. • Visualize a nonlinear system of two equations, and predict the number of solutions, using the graphing calculator. • Solve a linear-quadratic system of two equations algebraically and graphically. • Solve a quadratic-quadratic system of two equations algebraically and graphically.