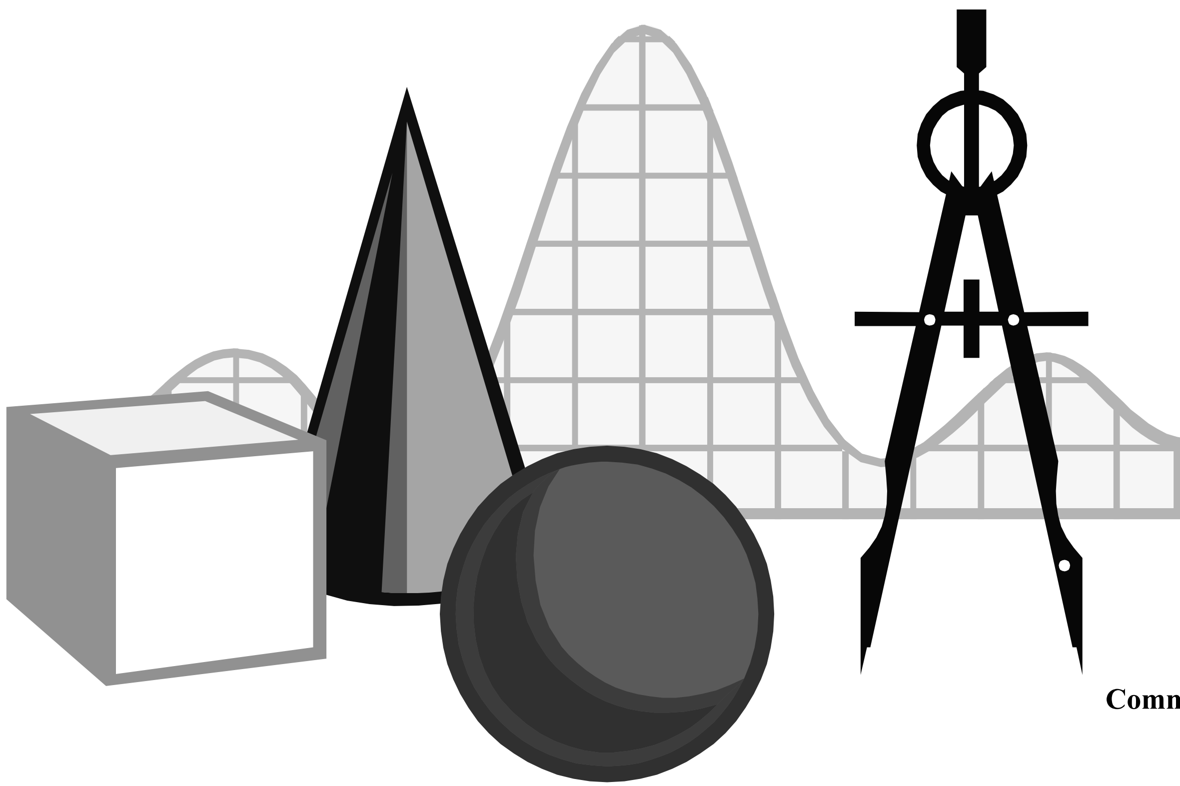
# MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

## *Discrete Mathematics*



**Commonwealth of Virginia**
**Board of Education**
**Richmond, Virginia**

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The 2002 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's website at http://www.pen.k12.va.us.

## Introduction

Mathematics content develops sequentially in concert with a set of processes that are common to different bodies of mathematics knowledge. The content of the Mathematics Standards of Learning supports five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. These goals provide a context within which to develop the knowledge and skills identified in the standards.

Discrete Mathematics involves applications using discrete variables rather than continuous variables. Modeling and understanding finite systems is central to the development of the economy, the natural and physical sciences, and mathematics itself. Discrete Mathematics introduces the topics of social choice as a mathematical application, matrices and their uses, graph theory and its applications, and counting and finite probability, as well as the processes of optimization, existence, and algorithm construction.

Each topic in the Discrete Mathematics Curriculum Framework is developed around the Standards of Learning. Each Standard of Learning is expanded in the Essential Knowledge and Skills column. The Essential Understandings column includes concepts, mathematical relationships, and ideas that are important to understanding and teaching the Standard of Learning effectively.

Teachers should help students make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections should be made to other subject areas and fields of endeavor through applications. Using manipulatives, graphing calculators, and computer applications to develop concepts should help students develop and attach meaning to abstract ideas. Throughout the study of mathematics, students should be encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop their competence and their confidence in themselves as mathematics students.

**TOPIC: GRAPHS**

## DISCRETE MATHEMATICS
## STANDARD DM.1

**The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • A tournament is a digraph that results from giving directions to the edges of a complete graph.<br>• Adjacent vertices are connected by an edge.<br>• In a connected graph, every pair of vertices is adjacent. | • Find the valence of each vertex in a graph.<br>• Use graphs to model situations in which the vertices represent objects, and edges (drawn between vertices) represent a particular relationship between objects.<br>• Represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems.<br>• Investigate and describe valence and connectedness.<br>• Determine whether a graph is planar or nonplanar.<br>• Use directed graphs (digraphs) to represent situations with restrictions in traversal possibilities. |

**TOPIC: GRAPHS**

# DISCRETE MATHEMATICS
# STANDARD DM.2

**The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • Euler's Theorem states: If *G* is a connected graph and all its valences are even, then *G* has an Euler Circuit.<br>• Pairs of routes (circuits) correspond to the same Hamilton Circuit because one route can be obtained from the other by traversing the vertices in reverse order.<br>There are $\frac{(n-1)!}{2}$ Hamilton Circuits.<br>• A multigraph is connected if there is a path between every pair of vertices. | • Determine if a graph has an Euler Circuit or Path, and find it.<br>• Determine if a graph has a Hamilton Circuit or Path, and find it.<br>• Count the number of Hamilton Circuits for a complete graph with *n* vertices.<br>• Use the Euler Circuit algorithm to solve optimization problems. |

TOPIC: GRAPHS

## DISCRETE MATHEMATICS
## STANDARD DM.3

**The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
| --- | --- |
| • Every planar graph has a chromatic number that is less than or equal to four (the four-color-map theorem).<br>• A graph can be colored with two colors if and only if it contains no cycle of odd length.<br>• The chromatic number of a graph cannot exceed one more than the maximum number of degrees of the vertices of the graph. | • Model projects consisting of several subtasks, using a graph.<br>• Use graphs to resolve conflicts that arise in scheduling. |

**TOPIC: GRAPHS**

## DISCRETE MATHEMATICS
## STANDARD DM.4

**The student will apply algorithms, such as Kruskal's, Prim's, or Dijkstra's, relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • A spanning tree of a connected graph *G* is a tree that is a subgraph of *G* and contains every vertex of *G*. | • Use Kruskal's Algorithm to find the shortest spanning tree of a connected graph.<br>• Use Prim's Algorithm to find the shortest spanning tree of a connected graph.<br>• Use Dijkstra's Algorithm to find the shortest spanning tree of a connected graph. |

## TOPIC: ELECTION THEORY AND FAIR DIVISION

## DISCRETE MATHEMATICS
## STANDARD DM.7

**The student will analyze and describe the issue of fair division (e.g., cake cutting, estate division). Algorithms for continuous and discrete cases will be applied.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • Group decision making combines the wishes of many to yield a single fair result.<br>• A fair division problem may be discrete or continuous.<br>• The success of the estate division algorithm requires that each heir be capable of placing a value on each object in the estate.<br>• A fair division problem consists of $n$ individuals (players) who must partition some set of goods, $s$, into $n$ disjoint sets. | • Investigate and describe situations involving discrete division (e.g., estate division).<br>• Use an algorithm for fair division for a group of indivisible objects.<br>• Investigate and describe situations involving continuous division of an infinitely divisible set (e.g., cake cutting).<br>• Use an algorithm for fair division of an infinitely divisible set. |

**TOPIC: ELECTION THEORY AND FAIR DIVISION**

**DISCRETE MATHEMATICS**
**STANDARD DM.8**

**The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • Historically, popular voting methods have often led to counterintuitive results.<br><br>• A candidate who wins over every other candidate in a one-on-one ballot is a Condorcet winner.<br><br>• A Borda count assigns points in descending order to each voter's subsequent ranking and then adds these points to arrive at a group's final ranking.<br><br>• To select a voting system is to compromise between the shortcomings inherent in each system. | • Determine in how many different ways a voter can rank choices.<br><br>• Investigate and describe the following voting procedures:<br>– weighted voting;<br>– plurality;<br>– majority;<br>– sequential (winners run off);<br>– sequential (losers are eliminated);<br>– Borda count; and<br>– Condorcet winner.<br><br>• Compare and contrast different voting procedures.<br><br>• Describe the possible effects of approval voting, insincere and sincere voting, a preference schedule, and strategic voting on the election outcome. |

**TOPIC: ELECTION THEORY AND FAIR DIVISION**

**DISCRETE MATHEMATICS**
**STANDARD DM.9**

**The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • The apportionment of Congressional representatives is based on the latest census. | • Compare and contrast the Hamilton and Jefferson methods of political apportionment with the Hill-Huntington method (currently in use in the U.S. House of Representatives) and the Webster-Willcox method.<br>• Solve allocation problems, using apportionment methods.<br>• Investigate and describe how salary caps affect apportionment. |

**TOPIC: COMPUTER MATHEMATICS**

# DISCRETE MATHEMATICS
# STANDARD DM.11

**The student will describe and apply sorting algorithms and coding algorithms used in storing, processing, and communicating information. These will include**
**a) bubble sort, merge sort, and network sort; and**
**b) ISBN, UPC, Zip, and banking codes.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • A bubble sort orders elements of an array by comparing adjacent elements.<br><br>• A merge sort combines two sorted lists into a single sorted list.<br><br>• Coding algorithms must account for the number of possible codes within the constraints of the coding system. | • Select and apply a sorting algorithm, such as a<br>– bubble sort;<br>– merge sort; and<br>– network sort.<br><br>• Describe and apply a coding algorithm, such as<br>– ISBN numbers;<br>– UPC codes;<br>– Zip codes; and<br>– banking codes. |

**TOPIC: COMPUTER MATHEMATICS**

# DISCRETE MATHEMATICS
# STANDARD DM.12

**The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • Two-valued (Boolean) algebra serves as a workable method for interpreting the logical truth and falsity of compound statements.<br><br>• Venn diagrams provide pictures of topics in set theory, such as intersection and union, mutually exclusive sets, and the empty set. | • Generate truth tables that encode the truth and falsity of two or more statements.<br><br>• Use Venn diagrams to codify and solve logic problems.<br><br>• Use matrices as arrays of data to solve logic problems. |

**TOPIC: RECURSION AND OPTIMIZATION**

# DISCRETE MATHEMATICS
# STANDARD DM.5

**The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • Critical path scheduling sometimes yields optimal solutions. | • Specify in a digraph the order in which tests are to be performed.<br>• Identify the critical path to determine the earliest completion time (minimum project time).<br>• Use the list-processing algorithm to determine an optimal schedule.<br>• Create and test scheduling algorithms. |

**TOPIC: RECURSION AND OPTIMIZATION**

# DISCRETE MATHEMATICS
# STANDARD DM.6

**The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • Linear programming models an optimization process.<br>• A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized.<br>• Any maximum or minimum value for a system of inequalities will occur at a corner point of a feasible region. | • Model practical problems with systems of linear inequalities.<br>• Identify the feasibility region of a system of linear inequalities with no more than four constraints.<br>• Identify the coordinates of the corner points of a feasibility region.<br>• Find the maximum or minimum value of the system.<br>• Describe the meaning of the maximum or minimum value in terms of the original problem. |

**TOPIC: RECURSION AND OPTIMIZATION**

## DISCRETE MATHEMATICS
## STANDARD DM.10

**The student will use the recursive process and difference equations with the aid of appropriate technology to generate**
**a) compound interest;**
**b) sequences and series;**
**c) fractals;**
**d) population growth models; and**
**e) the Fibonacci sequence.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • Recursion is a process that creates new objects from existing objects that were created by the same process.<br>• A fractal is a figure whose dimension is not a whole number.<br>• Fractals are self-similar. | • Use finite differences and recursion to model compound interest and population-growth situations.<br>• Model arithmetic and geometric sequences and series recursively.<br>• Compare and contrast the recursive process, and create fractals.<br>• Compare and contrast the recursive process and the Fibonacci sequence.<br>• Find a recursive relationship that generates the Fibonacci sequence. |

**TOPIC: RECURSION AND OPTIMIZATION**

## DISCRETE MATHEMATICS
## STANDARD DM.13

**The student will apply the formulas of combinatorics in the areas of**
**a) the Fundamental (Basic) Counting Principle;**
**b) knapsack and bin-packing problems;**
**c) permutations and combinations; and**
**d) the pigeonhole principle.**

| ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
|---|---|
| • The branch of mathematics that addresses the number of ways objects can be arranged or combined is combinatorics.<br>• If $n$ and $r$ are positive integers and $n \geq r$, $nPr = \frac{n!}{(n-r)!}$ and $nCr = \frac{n!}{r!\,(n-r)!}$.<br>• A bin-packing problem determines the minimum number of containers of fixed volume (bins) required to hold a set of objects.<br>• A knapsack problem determines the most valuable set of objects that fit into a container (knapsack) of fixed volume.<br>• Bin packing and knapsack packing are optimization techniques. | • Use the Fundamental (Basic) Counting Principal to determine the number of possible outcomes of an event.<br>• Find the number of permutations possible when $r$ objects selected from $n$ objects are ordered.<br>• Find the number of combinations possible when subsets of $r$ elements are selected from a set of $n$ elements without regard to order.<br>• Use the pigeonhole principle to solve packing problems to facilitate proofs.<br>• Use the bin-packing and knapsack-packing algorithms to solve practical problems. |