MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Geometry
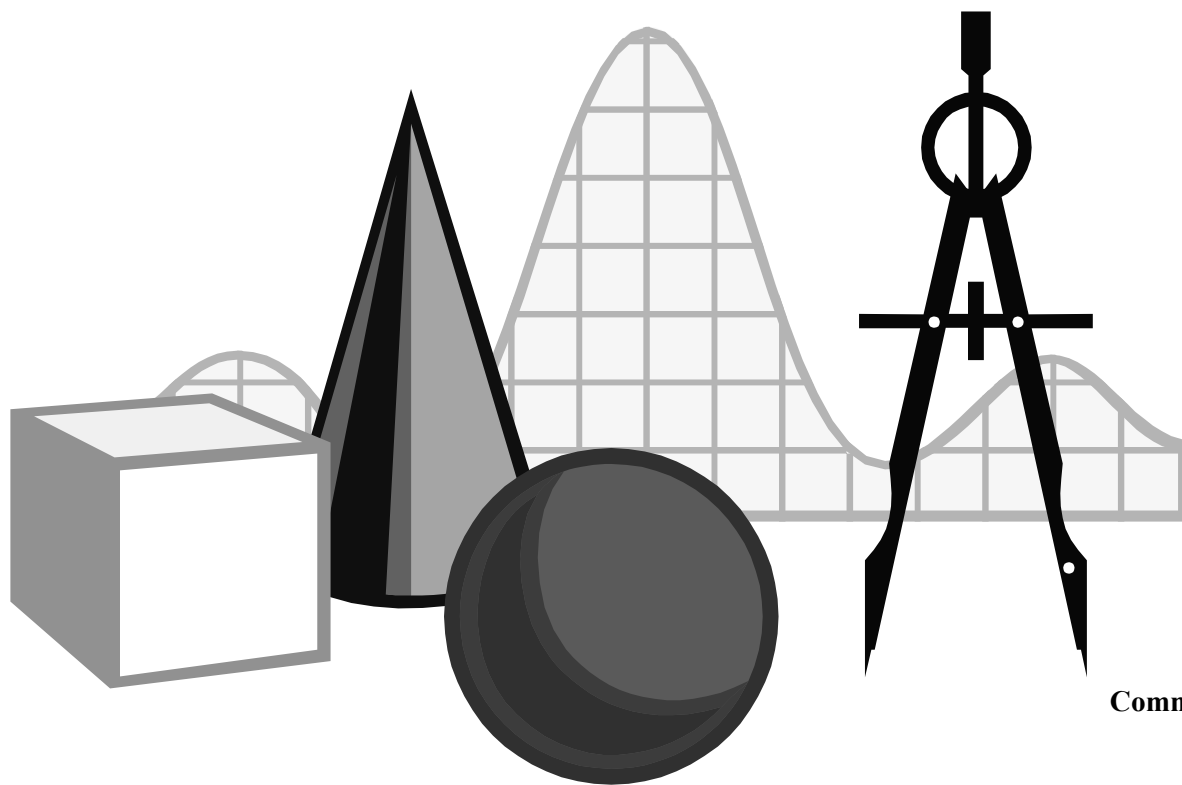
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Introduction

Mathematics content develops sequentially in concert with a set of processes that are common to different bodies of mathematics knowledge. The content of the Mathematics Standards of Learning supports five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. These goals provide a context within which to develop the knowledge and skills identified in the standards.

Geometry offers students a means of describing, analyzing, and understanding aspects of their world. Geometric modeling, visualizing, and spatial reasoning can be used to solve many kinds of problems. Coordinate geometry and other representational systems allow locations to be specified and described. Geometry also focuses on the development of reasoning and proof, using definitions and axioms.

Each topic in the Geometry Curriculum Framework is developed around the Standards of Learning. Each Standard of Learning is expanded in the Essential Knowledge and Skills column. The Essential Understandings column includes concepts, mathematical relationships, and ideas that are important to understanding and teaching the Standard of Learning effectively.

Teachers should help students make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections should be made to other subject areas and fields of endeavor through applications. Using manipulatives, graphing calculators, and computer applications to develop concepts should help students develop and attach meaning to abstract ideas. Throughout the study of mathematics, students should be encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop their competence and their confidence in themselves as mathematics students.

GEOMETRY
STANDARD G.3

The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, angles formed when parallel lines are cut by a transversal, and angles in polygons.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
- Two intersecting lines form angles with specific relationships. - Parallel lines cut by a transversal form angles with specific relationships. - An exterior angle is formed by extending a side of a polygon. - The exterior angle and the corresponding interior angle form a linear pair. - The sum of the measures of the interior angles of a convex polygon may be found by dividing the interior of the polygon into non-overlapping triangles.	- Classify the types of angles formed by two lines and a transversal. - State the relationships between pairs of angles, including a linear pair, vertical angles, corresponding angles, alternate interior angles, same-side (consecutive) interior angles, complementary angles, and supplementary angles. - Solve practical problems involving intersecting and parallel lines in a plane. - Find the sum of the measures of the interior and exterior angles of a convex polygon. - Find the measure of each interior and exterior angle of a regular polygon. - Solve practical problems by using the relationships between pairs of angles such as vertical angles, corresponding angles, alternate interior angles, same-side interior angles, complementary angles, and supplementary angles.

	TOPIC: LINES AND ANGLES
GEOMETRY **STANDARD G.4** The student will use the relationships between angles formed by two lines cut by a transversal to determine if two lines are parallel and verify, using algebraic and coordinate methods as well as deductive proofs.	

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Parallel lines cut by a transversal form angles with specific relationships. • A proof is a chain of logical statements starting with given information and leading to a conclusion. • Some angle relationships can be used to prove that two lines cut by a transversal are parallel. • Parallel lines have the same slope.	• Use properties, postulates, and theorems to determine whether two lines are parallel. • Use algebraic, coordinate, and deductive methods to determine whether two lines are parallel.

GEOMETRY
STANDARD G.11

The student will construct a line segment congruent to a given line segment, the bisector of a line segment, a perpendicular to a given line from a point not on the line, a perpendicular to a given line at a point on the line, the bisector of a given angle, and an angle congruent to a given angle.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS

• Construction techniques are used to solve real-life problems in engineering, architectural design, and building construction.
 	
• Construction techniques may include using a straightedge and compass, paper folding, and dynamic geometry software.
	• Construct
	  – a line segment congruent to a given line segment;
	  – the bisector of a line segment;
	  – a perpendicular to a given line from a point not on the line;
	  – a perpendicular to a given line at a point on the line;
	  – the bisector of a given angle; and
	  – an angle congruent to a given angle.

**GEOMETRY
STANDARD G.1**

The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
a) **identifying the converse, inverse, and contrapositive of a conditional statement;**
b) **translating a short verbal argument into symbolic form;**
c) **using Venn diagrams to represent set relationships; and**
d) **using deductive reasoning, including the law of syllogism.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Reasoning and proof are critical in establishing general claims. • Logical arguments consist of a set of premises or hypotheses and a conclusion. • Proof is a justification that is logically valid and based on initial assumptions, definitions, postulates, and theorems.	• Identify the converse, inverse, and contrapositive of a conditional statement. • Translate short verbal arguments into symbolic form, such as $(p \rightarrow q)$ and $(\sim p \rightarrow \sim q)$. • Use and interpret Venn diagrams. • Determine the validity of a logical argument. • Use valid forms of deductive reasoning, including the law of syllogism. • Select and use various types of reasoning and methods of proof, as appropriate.

GEOMETRY
STANDARD G.5

The student will
a) **investigate and identify congruence and similarity relationships between triangles; and**
b) **prove two triangles are congruent or similar, given information in the form of a figure or statement, using algebraic and coordinate as well as deductive proofs.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Congruence and similarity have real-life applications in a variety of areas, including art, architecture, and the sciences. • Congruence and similarity do not depend on the position of the triangle. • Concepts of logic can demonstrate congruence or similarity. • Congruent figures are also similar, but similar figures are not necessarily congruent.	• Use definitions, postulates, and theorems to determine whether triangles are congruent. • Use definitions, postulates, and theorems to determine whether triangles are similar. • Use algebraic methods, such as properties of proportions, to prove that triangles are similar. • Use coordinate methods, such as the distance formula and the slope formula, to prove two triangles are congruent.

GEOMETRY
STANDARD G.6

The student, given information concerning the lengths of sides and/or measures of angles, will apply the triangle inequality properties to determine whether a triangle exists and to order sides and angles. These concepts will be considered in the context of practical situations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The longest side of a triangle is opposite the largest angle of the triangle. • In a triangle, the measure of an angle and the lengths of the adjacent sides determine the length of the side opposite the angle.	• Given the lengths of three segments, determine whether a triangle could be formed. • Arrange the angles of a triangle in order from smallest to largest when given the lengths of the sides. • Arrange the sides of a triangle in order from smallest to largest when given the measures of the angles. • Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie.

GEOMETRY
STANDARD G.7

The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The Pythagorean Theorem is essential for solving problems involving right triangles. • The relationships between the sides and angles of right triangles are useful in many applied fields. • Some practical problems can be solved by choosing an efficient representation of the problem.	• Given the lengths of two sides of a right triangle, use the Pythagorean Theorem to find the length of the third side. • Determine whether a triangle formed with three given lengths is a right triangle. • Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles. • Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles. • Solve problems involving right triangles, using sine, cosine, and tangent ratios. • Solve practical problems, using right triangle trigonometry and properties of right triangles.

GEOMETRY
STANDARD G.8

The student will

a) **investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;**

b) **prove these properties of quadrilaterals, using algebraic and coordinate methods as well as deductive reasoning; and**

c) **use properties of quadrilaterals to solve practical problems.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Quadrilaterals have a hierarchical nature based on the relationships between their sides, angles, and diagonals. • Properties of quadrilaterals can be used to identify the quadrilateral and to find the measures of sides and angles.	• Solve practical problems, using the properties specific to parallelograms, rectangles, rhombi, squares, and trapezoids. • Prove that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the distance formula, slope, and midpoint formula. • Prove the properties of quadrilaterals, using deductive reasoning.

GEOMETRY
STANDARD G.9

The student will use measures of interior and exterior angles of polygons to solve problems. Tessellations and tiling problems will be used to make connections to art, construction, and nature.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A regular polygon will tessellate the plane if the measure of an interior angle is a factor of 360. • Both regular and nonregular polygons can tessellate the plane.	• Solve problems involving the measures of interior and exterior angles of polygons. • Identify tessellations in art, construction, and nature.

GEOMETRY
STANDARD G.10

The student will investigate and solve practical problems involving circles, using properties of angles, arcs, chords, tangents, and secants. Problems will include finding arc length and area of a sector, and may be drawn from applications of architecture, art, and construction.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Many relationships exist between and among angles, arcs, secants, chords, and tangents of a circle. • All circles are similar. • A chord is part of a secant.	• Given two intersecting chords or two intersecting tangents, find missing lengths. • Calculate the area of a sector of a circle, using proportions. • Given the measure of a central angle in degrees and the radius of the circle, find the related arc length. • Solve practical problems associated with circles, using properties of angles and arcs.

**GEOMETRY
STANDARD G.12**

The student will make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object. Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Three-dimensional figures are a part of everyday life. • Modeling is an overall part of design for products and structures.	• Use properties of three-dimensional objects to make models. • Make a model of a three-dimensional figure from a two-dimensional drawing. • Make a two-dimensional representation of a three-dimensional object. • Solve problems, using scale drawings, perspective drawings, blueprints, or computer drawings as models of three-dimensional objects. • Identify a three-dimensional object from different positions, such as the top view, side view, and front view.

GEOMETRY
STANDARD G.13

The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The surface area of a three-dimensional object is the sum of the areas of all its faces. • The volume of a three-dimensional object is the number of unit cubes that would fill the object.	• Find the total surface area of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas. • Calculate the volume of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas. • Solve practical problems involving total surface area and volume of cylinders, prisms, pyramids, cones, and spheres as well as combinations of three-dimensional figures.

GEOMETRY
STANDARD G.14

The student will
a) **use proportional reasoning to solve practical problems, given similar geometric objects; and**
b) **determine how changes in one dimension of an object affect area and/or volume of the object.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A change in one dimension of an object results in predictable changes in area and/or volume. • A constant ratio exists between corresponding lengths of sides of similar figures.	• Compare perimeters and areas of similar two-dimensional figures, using proportions. • Describe how a change in one measure affects other measures of an object. Measures of an object may include perimeter, area, total surface area, and volume. • Solve practical problems involving similar objects.

	TOPIC: COORDINATE RELATIONS AND TRANSFORMATIONS

GEOMETRY
STANDARD G.2

The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
a) investigating and using formulas for finding distance, midpoint, and slope;
b) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
c) determining whether a figure has been translated, reflected, or rotated.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Transformations and combinations of transformations can be used to describe movement of objects in a plane. • The distance formula is an application of the Pythagorean Theorem. • Geometric figures can be represented in the coordinate plane. • Techniques for investigating symmetry may include paper folding, coordinate methods, and dynamic geometry software.	• Given an image and preimage, identify the transformation that has taken place as a reflection, rotation, or translation. • Apply the distance formula to find the length of a line segment when given the coordinates of the endpoints. • Find the coordinates of the midpoint of a segment, using the midpoint formula. • Find the slope of a line, given the graph or the coordinates of two points on the line. • Determine whether a figure has point symmetry, line symmetry, or neither.