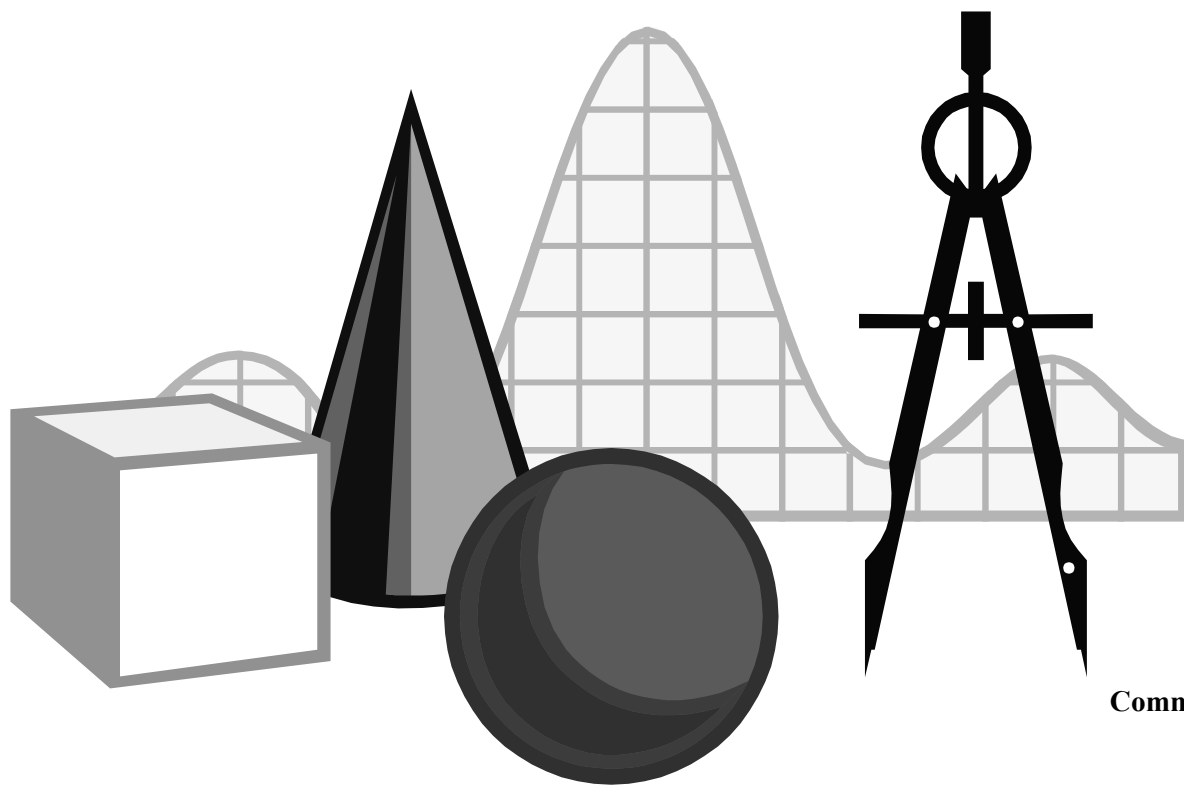


# MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

*Grade 1*



Commonwealth of Virginia  
Board of Education  
Richmond, Virginia  
© 2002

Copyright © 2002

by the

Virginia Department of Education

P.O. Box 2120

Richmond, Virginia 23218-2120

<http://www.pen.k12.va.us>

All rights reserved. Reproduction of materials contained herein for instructional purposes in Virginia classrooms is permitted.

**Superintendent of Public Instruction**

Jo Lynne DeMary

**Deputy Superintendent**

M. Kenneth Magill

**Assistant Superintendent for Instruction**

Patricia I. Wright

**Office of Elementary Instructional Services**

James S. Heywood, Director

Karen Grass, Mathematics Specialist

**NOTICE TO THE READER**

The Virginia Department of Education does not unlawfully discriminate on the basis of sex, race, color, religion, handicapping conditions, or national origin in employment or in its educational programs and activities.

The 2002 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's website at <http://www.pen.k12.va.us>.

Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, simple fractions, number relationships (“more than,” “less than,” and “as many as”), and the effects of simple operations on numbers (fact families). These learning experiences should allow students to engage actively in a variety of problem-solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

## 1.1 The student will count objects in a given set containing between 1 and 100 objects and write the corresponding numeral.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• There are three developmental levels of counting:               <ul style="list-style-type: none"> <li>– rote sequence;</li> <li>– one-to-one correspondence; and</li> <li>– the cardinality of numbers.</li> </ul> </li> <li>• Counting involves two separate skills: verbalizing the list of standard number words in order (“one, two, three,...”) and connecting this sequence with the items in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken.</li> <li>• The last number stated represents the number of objects in the set. This is known as the cardinality of the set.</li> <li>• Rote counting is a prerequisite skill for the understanding of addition, subtraction, and the ten-to-one concept of place value.</li> <li>• Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Use the correct oral counting sequence to tell how many objects are in a set.</li> <li>• Write numerals correctly.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Count by rote from 1 to 100.</li> <li>• Write numerals for the numbers 1 to 100.</li> <li>• Count a randomly placed collection of objects containing between 1 and 100 items and write the corresponding numeral.</li> </ul>

**1.2 The student will group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.**

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• The number system is based on a simple pattern of tens where each place has ten times the value of the place to its right. This is known as the ten-to-one concept of place value.</li> <li>• Opportunities to experience the relationship between tens and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place-value concept of our number system. Ten-to-one trading activities with manipulatives on place-value mats provide excellent experiences for developing the understanding of the places in the base-10 system.</li> <li>• Models that clearly illustrate the relationship between tens and ones are physically proportional (e.g., the tens piece is ten times larger than the ones piece).</li> <li>• Recording the numeral when using physical and pictorial models leads to an understanding that the position of each digit in a numeral determines the size of the group it represents.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand that groups of tens and ones can be used to tell how many.</li> <li>• Understand that numerals are written to show how many tens and how many ones.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Group a collection of objects into sets of tens and ones.</li> <li>• Write the numeral that corresponds to the total number of objects in a given collection of objects that have been grouped into sets of tens and ones.</li> </ul>

## 1.3 The student will count forward by ones, fives, and tens to 100, by twos to 20, and backward by ones from 20.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include counters, hundred chart, and calculators.</li> <li>• Skip counting by twos supports the development of the concept of even numbers.</li> <li>• Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five.</li> <li>• Skip counting by tens is a precursor for use of place value, addition, counting money, and multiplying by multiples of 10.</li> <li>• Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with 20, 19, 18, ... through ...3, 2, 1.</li> <li>• Calculators can be used to reinforce skip counting. Use the constant feature of the four-function calculator to display the numbers in the sequence when skip counting by that constant. For example, when skip counting by twos, press <math>2 + 2 = = \dots</math> to produce 2, 4, 6, 8, 10, ...; or when skip counting by fives, press <math>5 + 5 = = \dots</math> to produce 5, 10, 15, ...</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand that collections of objects can be grouped and skip counting can be used to count the collection.</li> <li>• Describe patterns in counting by ones (both forward and backward) and skip counting and use those patterns to predict the next number in the counting sequence.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Count by ones, fives, and tens to 100, using concrete objects, such as counters, connecting cubes, pennies, nickels, and dimes.</li> <li>• Skip count orally by fives and tens to 100.</li> <li>• Count by twos to 20, using concrete objects, such as counters, connecting cubes, and pennies.</li> <li>• Skip count orally by twos to 20.</li> <li>• Count backward by ones from 20.</li> </ul>

## 1.4 The student will recognize and write numerals 0 through 100.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• Correct formation of the numerals 0 through 100 can be developed through kinesthetic activities, using tactile materials, such as sand, sandpaper, carpeting, or finger paint.</li> <li>• Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Associate oral number names with the correct numeral.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Say the correct name for each numeral 0 through 100.</li> <li>• Write each numeral 0 through 100, using correct numeral formation.</li> </ul>

## 1.5 The student will identify the ordinal positions first through tenth, using an ordered set of objects.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• Understanding the ordinal meaning of numbers is necessary to quantify, measure, and identify the order of objects.</li> <li>• An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). <i>Ordered position, ordinal position, and ordinality</i> are terms that refer to the place or position of an object in a sequence or set.</li> <li>• The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects. Ordinal position can also be emphasized through sequencing events (e.g., months in a year, sequencing in a story).</li> <li>• The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., a student's position in line when students are lined up alphabetically by first name).</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Use ordinal numbers to describe the position of an object in a sequence or set.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Count an ordered set of objects, using the ordinal number words <i>first</i> through <i>tenth</i>.</li> <li>• Identify the ordinal positions, first through tenth, using an ordered set of objects.</li> <li>• Identify the ordinal positions, first through tenth, using an ordered set of objects presented in lines or rows from <ul style="list-style-type: none"> <li>– left-to-right;</li> <li>– right-to-left;</li> <li>– top-to-bottom; and</li> <li>– bottom-to-top.</li> </ul> </li> </ul>



## 1.6 The student will identify and represent the concepts of one-half and one-fourth, using appropriate materials or a drawing.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• A fraction is a way of representing part of a whole.</li> <li>• In each fraction model, the parts must be equal (i.e., each pie piece must have the same area). In problems with fractions, a whole is broken into equal-size parts and reassembled into one whole.</li> <li>• The words <i>denominator</i> and <i>numerator</i> are not required at this grade, but the concepts of part and whole are required for understanding of a fraction.</li> <li>• At this level, students should not be expected to recognize or use symbolic representations for fractions (e.g., <math>\frac{1}{2}</math> or <math>\frac{1}{4}</math>), until they understand the part-whole relationship.</li> <li>• Students should have opportunities to make connections among fraction representations by connecting concrete or pictorial representations with spoken representations (e.g., “one-half,” but not <math>\frac{1}{2}</math>).</li> <li>• Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., <i>thirds</i> means “three equal parts of a whole” or <math>\frac{1}{3}</math> represents one of three equal-size parts when a pizza is shared among three students) furthers this development.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand that a fraction represents a part of a whole.</li> <li>• Understand that fractional parts are equal shares of a whole.</li> <li>• Understand that the fraction name (<i>half</i>, <i>fourth</i>) tells the number of equal parts in the whole.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Represent a whole to show it having two equal parts.</li> <li>• Represent a whole to show it having four equal parts.</li> <li>• Identify and model one-half and one-fourth of a whole, using <ul style="list-style-type: none"> <li>– region/area models (e.g., pie pieces, pattern blocks, geoboards, drawings); and</li> <li>– measurement models (e.g., cuisenaire rods, connecting cubes, fraction strips, drawings).</li> </ul> </li> </ul>

A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as “How many books do we have altogether?” or “How many cookies are left if I eat two?” or “I have three more candies than you do.” Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling and discussing a variety of problem situations to help students move from the concrete to the abstract and develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, and multiplication facts and related fact families;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions and decimals.

- 1.7 The student, given a familiar problem situation involving magnitude, will
- select a reasonable magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, and 500); and
  - explain the reasonableness of his/her choice.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>Magnitude refers to the size of a set.</li> <li>Exploring ways to estimate the number of objects in a set, based on appearance, (e.g., clustering, grouping, comparing) enhances the development of number sense.</li> <li>To estimate means to find a number that is close to the exact amount. When asking for an estimate, teachers might ask: “<i>about</i> how much?” or “<i>about</i> how many?”</li> <li>Students should be provided opportunities to estimate a quantity, given a benchmark of 10 and/or 100 objects.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>Develop an understanding of the magnitude (size) of whole numbers and use this knowledge to estimate quantities.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>Select a reasonable magnitude for a given set from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, and 500 jelly beans in jars) in a familiar problem situation.</li> <li>Given a familiar problem situation involving magnitude, explain why a particular estimate was chosen as the most reasonable from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral .</li> </ul>

## 1.8 The student will recall basic addition facts — i.e., sums to 10 or less — and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• Associate the terms <i>addition</i>, <i>adding</i>, and <i>sum</i> with the concept of joining or combining.</li> <li>• Associate the terms <i>subtraction</i>, <i>subtracting</i>, <i>minus</i>, and <i>difference</i> with the process of “taking away” or separating (i.e., removing a set of objects from the given set of objects, finding the difference between two numbers, or comparing two numbers).</li> <li>• Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include               <ul style="list-style-type: none"> <li>– counting back;</li> <li>– “one-more-than,” “two-more-than facts”;</li> <li>– “one-less-than,” “two-less-than facts”;</li> <li>– “doubles” to recall addition facts (e.g., <math>2 + 2 = \underline{\quad}</math>; <math>3 + 3 = \underline{\quad}</math>);</li> <li>– “near doubles” [e.g., <math>3 + 4 = (3 + 3) + 1 = \underline{\quad}</math>];</li> <li>– “make-ten facts” (e.g., at least one addend of 8 or 9);</li> <li>– “think addition for subtraction” (e.g., for <math>9 - 5 = \underline{\quad}</math>, think “5 and what number makes 9?”);</li> <li>– use of the commutative property, without naming the property (e.g., <math>4 + 3</math> is the same as <math>3 + 4</math>);</li> <li>– use of fact families (e.g., <math>4 + 3 = 7</math>, <math>3 + 4 = 7</math>, <math>7 - 4 = 3</math>, and <math>7 - 3 = 4</math>);</li> <li>– use of the zero property (e.g., <math>4 + 0 = 4</math>), without naming the property but saying, “When you add zero to a number, you always get the original number.”</li> </ul> </li> </ul> <p>continued</p>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Develop fluency with basic number combinations for addition and subtraction.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Identify + as a symbol for addition and – as a symbol for subtraction.</li> <li>• Recall and state orally the basic addition facts for sums to 10 or less and the corresponding subtraction facts.</li> <li>• Recall and write the basic addition facts for sums to 10 or less and the corresponding subtraction facts, when addition or subtraction problems are presented in either horizontal or vertical written format.</li> </ul>

1.8 The student will recall basic addition facts — i.e., sums to 10 or less — and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<p>continued</p> <ul style="list-style-type: none"><li>Manipulatives should be used initially to develop an understanding of addition and subtraction facts and to engage students in meaningful memorization. Rote recall of the facts is often achieved through constant practice and may come in a variety of formats, including presentation through flash cards, practice sheets, and/or games.</li></ul>		

- 1.9 The student will create and solve story and picture problems involving one-step solutions, using basic addition and subtraction facts.**

<b>UNDERSTANDING THE STANDARD (Teacher Notes)</b>	<b>ESSENTIAL UNDERSTANDINGS</b>	<b>ESSENTIAL KNOWLEDGE AND SKILLS</b>
<ul style="list-style-type: none"> <li>• The problem-solving process is enhanced when students               <ul style="list-style-type: none"> <li>– create their own story problems; and</li> <li>– model word problems, using manipulatives or drawings.</li> </ul> </li> <li>• Students should be exposed to a variety of problem types related to addition and subtraction, including               <ul style="list-style-type: none"> <li>– join and separate problems (action involved);</li> <li>– part-part-whole problems (no action involved); and</li> <li>– comparison problems.</li> </ul> </li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand various meanings of addition and subtraction in a variety of situations.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Interpret and solve oral or written story and picture problems involving one-step solutions, using basic addition and subtraction facts (sums to 10 or less and the corresponding subtraction facts).</li> <li>• Identify a correct number sentence to solve an oral or written story or picture problem, selecting from among basic addition and subtraction facts.</li> </ul>

Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

## 1.10 The student will

- a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and
- b) determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• Many experiences with coins help students develop an understanding of money, such as               <ul style="list-style-type: none"> <li>– drawing pennies to show the value of a given coin (e.g., a nickel, a dime, or a quarter);</li> <li>– playing store and purchasing classroom object, using play money (pennies);</li> <li>– representing the value of a nickel, a dime, and a quarter, using pennies; and</li> <li>– trading the equivalent value of pennies for a nickel, a dime, and a quarter, using play money.</li> </ul> </li> <li>• Counting money helps students gain an awareness of consumer skills and the use of money in everyday life.</li> <li>• A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to ensure competence with counting money.</li> <li>• Establishing a one-to-one correspondence between the number names and the items in a set of coins (pennies, nickels, or dimes) is essential for an accurate count.</li> <li>• The last number stated represents the value of a collection of coins begin counted. This is known as the cardinality of the set.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Develop an understanding of exchanging the appropriate number of pennies for a nickel, a dime, or a quarter.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Identify the value of a nickel, a dime, and a quarter in terms of pennies.</li> <li>• Recognize the characteristics of pennies, nickels, and dimes (e.g., color, size).</li> <li>• Identify the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.</li> <li>• Count by ones to determine the total value of a collection of pennies whose total value is 100 cents or less.</li> <li>• Count by fives to determine the total value of a collection of nickels whose total value is 100 cents or less.</li> <li>• Count by tens to determine the total value of a collection of dimes whose total value is 100 cents or less.</li> </ul>



## 1.11 The student will tell time to the half hour, using an analog or digital clock.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• Many experiences using clocks help students develop an understanding of the telling of time to the half hour, including               <ul style="list-style-type: none"> <li>– identifying the parts of an analog clock (minute and hour hands);</li> <li>– demonstrating a given time to the half hour, using a model clock;</li> <li>– writing correct digital time to the half hour; and</li> <li>– relating time on the half hour to daily routines and school schedules (e.g., the times of TV programs, bedtime, resource time, lunch time, recess time).</li> </ul> </li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand how to tell time to the half hour, using an analog or digital clock.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Tell time shown on an analog clock to the half hour.</li> <li>• Tell time shown on a digital clock to the half hour.</li> <li>• Match a written time to the time shown on a digital or analog clock to the half hour.</li> </ul>

## 1.12 The student will use nonstandard units to measure length and weight.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• The process of measurement involves selecting a unit of measure, comparing the object to be measured to the unit, counting the number of times the object can be measured by the unit, and arriving at an approximate total number of units.</li> <li>• Premature use of instruments or formulas leaves children without the understanding necessary for solving measurement problems.</li> <li>• When children’s initial explorations of length and weight involve the use of nonstandard units, they develop some understanding about the need for standard measurement units for length and weight, especially when they communicate about these measures.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand how to measure length and weight, using nonstandard units of measure.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Measure the length of objects, using nonstandard units (e.g., connecting cubes, paper clips, erasers).</li> <li>• Measure the weight of objects, using nonstandard units (e.g., paper clips, bean bags, cubes).</li> </ul>

**1.13 The student will compare the volumes of two given containers by using concrete materials (e.g., jelly beans, sand, water, rice).**

<b>UNDERSTANDING THE STANDARD</b> (Teacher Notes)	<b>ESSENTIAL UNDERSTANDINGS</b>	<b>ESSENTIAL KNOWLEDGE AND SKILLS</b>
<ul style="list-style-type: none"> <li>• Estimation is a commonly used strategy to compare the volumes of two containers.</li> <li>• Determining the volume of a container by counting the number of nonstandard units (e.g., a spoonful, scoopful, or teacupful of concrete material, such as jelly beans, sand, water, or rice) that can be held by the container is a precursor to comparing volumes.</li> <li>• A variety of activities that focus on directly comparing the volume of objects leads to an understanding of volume.</li> <li>• The level of difficulty in measuring volume can be increased by varying and mixing the sizes of the containers.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand how to fill containers with objects to determine their volume and compare the volumes two containers.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Compare the volumes of two containers to determine if the volume of one is greater than, less than, or the same as the other, using nonstandard units of measure (e.g., a spoonful, scoopful, or teacupful).</li> <li>• Compare the volumes of two containers to determine if the volume of one is greater than, less than, or the same as the other by pouring the contents of one container into the other.</li> </ul>

## 1.14 The student will compare the weights of two objects, using a balance scale.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• Balance scales are instruments used for weighing. A balance scale usually has a beam that is supported in the center. On each side of the beam are two identical trays. When the trays hold equal weights, the beam is level, and the scale is “balanced.”</li> <li>• Physically measuring the weights of objects, using a balance scale, helps students develop an intuitive idea of what it means to say something is “lighter,” “heavier,” or “the same.”</li> <li>• Experience estimating the weights of two objects, using the terms <i>lighter</i>, <i>heavier</i>, or <i>the same</i>, prior to weighing the objects promotes an understanding of the concept of balance.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand that a balance beam can be used to compare the weights of two objects.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <p>Compare the weights of two objects, using the terms <i>lighter</i>, <i>heavier</i>, or <i>the same</i>, using a balance scale.</p>

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of shapes and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent two- and three-dimensional shapes through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, comparing, and investigating three-dimensional objects and their two-dimensional faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, sorting, and classifying shapes; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these shapes but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

**1.15** The student will describe the proximity of objects in space (*near, far, close by, below, above, up, down, beside, and next to*).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• Students should have opportunities to demonstrate the proximity of objects in space, including               <ul style="list-style-type: none"> <li>– acting out stories that address spatial orientations and spatial relationships (e.g., “The rain was coming down, and the giant lifted up the town and set it down close by on a mountaintop.”);</li> <li>– drawing objects to show the spatial relationships of one object to another (e.g., a balloon next to a student holding it; a tree next to a house); and</li> <li>– using concrete objects to model the spatial relationship of one object to another.</li> </ul> </li> <li>• Building geometric and spatial capabilities fosters enthusiasm for mathematics while providing a context to develop spatial sense.</li> <li>• Translations (slides, flips, and turns) can be used to change the location of objects.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Accurately describe the proximity of objects in space in order to gain essential navigation skills.</li> <li>• Use familiarity with shape, structure, and location to develop spatial reasoning.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Identify the spatial relationships of objects, using the terms <i>near, far, close by, below, above, up, down, beside, and next to</i>.</li> </ul>

**1.16 The student will draw, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, corners, and square corners.**

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• A plane geometric figure is any two-dimensional closed shape. Circles and polygons are examples of plane geometric figures.</li> <li>• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.               <ul style="list-style-type: none"> <li>– <b>Level 0: Pre-recognition.</b> Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.</li> <li>– <b>Level 1: Visualization.</b> Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., “I know it’s a rectangle because it looks like a door, and I know that a door is a rectangle.”)</li> <li>– <b>Level 2: Analysis.</b> Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., “I know it’s a rectangle because it is closed; it has four sides and four right angles, and opposite sides are parallel.”).</li> </ul> </li> </ul> <p>continued</p>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Develop strategies to sort and/or group plane geometric figures and refine the vocabulary used to explain their strategies.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Draw triangles, squares, rectangles, and circles.</li> <li>• Describe triangles, squares, and rectangles by the number of sides, corners, and square corners.</li> <li>• Describe circles.</li> <li>• Identify the name of the shape when given information about the number of sides, corners, and/or square corners.</li> </ul>

**1.16 The student will draw, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, corners, and square corners.**

<b>UNDERSTANDING THE STANDARD (Teacher Notes)</b>	<b>ESSENTIAL UNDERSTANDINGS</b>	<b>ESSENTIAL KNOWLEDGE AND SKILLS</b>
<p>continued</p> <ul style="list-style-type: none"> <li>• A polygon is a geometric figure which               <ul style="list-style-type: none"> <li>– has sides that are straight;</li> <li>– is simple (its sides do not cross);</li> <li>– is closed; and</li> <li>– is two-dimensional (it lies in a plane).</li> </ul> </li> <li>• A triangle is a polygon with three angles and three sides.</li> <li>• A quadrilateral is a polygon with four sides.</li> <li>• A rectangle is a quadrilateral with four right angles.</li> <li>• A square is a rectangle with four sides of equal length.</li> <li>• A circle is a closed curve with all points in one plane and the same distance from a fixed point (the center).</li> <li>• Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed.</li> </ul>		



- 1.17 The student will identify and describe objects in his/her environment that depict plane geometric figures (triangle, rectangle, square, and circle).**

<b>UNDERSTANDING THE STANDARD (Teacher Notes)</b>	<b>ESSENTIAL UNDERSTANDINGS</b>	<b>ESSENTIAL KNOWLEDGE AND SKILLS</b>
<ul style="list-style-type: none"> <li>• Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of shape those things represent.</li> <li>• Recognizing shapes once transformations (slides, flips, turns) have been applied is a skill that students should begin to develop during the primary grades.</li> <li>• Children are often confused when a shape such as a square is rotated: they frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — i.e., a square is a square regardless of its location in space; there is no such geometric shape as a diamond.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand that geometric figures are integral parts of his/her environment.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Identify representations of circles, squares, rectangles, and triangles in the environment at school and home and tell why they represent those shapes.</li> <li>• Describe representations of circles, squares, rectangles, and triangles in the environment (e.g., "I know it's a rectangle because it looks like a door, and I know that a door is a rectangle.&gt;").</li> </ul>

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible*, *unlikely*, *equally likely*, *more likely*, and *certain*.

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

**1.18 The student will investigate, identify, and describe various forms of data collection in his/her world (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream), using tables, picture graphs, and object graphs.**

<b>UNDERSTANDING THE STANDARD (Teacher Notes)</b>	<b>ESSENTIAL UNDERSTANDINGS</b>	<b>ESSENTIAL KNOWLEDGE AND SKILLS</b>
<ul style="list-style-type: none"> <li>• Students' questions about the physical world can often be answered by collecting data and observing the results.</li> <li>• Data is information collected about people or things. The primary purpose of collecting data is to answer questions.</li> <li>• After generating questions, students decide what information is needed and how it can be collected.</li> <li>• The collection of the data often leads to new questions to be investigated.</li> <li>• The entire process broadens children's views of mathematics and its usefulness.</li> <li>• Data collection could involve voting, informal surveys, tallying, and charts.</li> <li>• Surveys, which are data-collecting tools that list choices, should have a limited number of questions at the primary grades.</li> <li>• Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand how data can be collected and presented in an organized manner.</li> <li>• Understand that data gathered and analyzed from observations and surveys can have an impact on our everyday lives.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Investigate various forms of data collection, including counting and tallying, informal surveys, observations, and voting.</li> <li>• Identify and describe various forms of data collection in his or her world (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream.)</li> </ul>

- 1.19** The student will interpret information displayed in a picture or object graph, using the vocabulary *more, less, fewer, greater than, less than, and equal to*.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• Statistics are used to describe and interpret with numbers the world around us; it is a tool for problem solving.</li> <li>• Students' questions about everyday life can often be answered by collecting and interpreting data.</li> <li>• Organized data provides a clearer picture for interpretation.</li> <li>• Picture graphs are graphs that use pictures to show and compare information. A key is often used to indicate what each picture represents (e.g., one picture of a dog represents five dogs).</li> <li>• Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student).</li> <li>• Interpretation of the data could lead to additional questions to be investigated.</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand that picture graphs use pictures to represent and compare data while object graphs use concrete objects to represent and compare data.</li> <li>• Understand that data can be analyzed and interpreted, using the terms <i>more, less, fewer, greater than, less than, and equal to</i>.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Demonstrate the meaning of the terms <i>more, less, fewer, greater than, less than, and equal to</i>, using concrete materials.</li> <li>• Compare one category to another in a graph, indicating which has more or which has less.</li> <li>• Interpret information displayed in object graphs and picture graphs, using the words <i>more, less, fewer, greater than, less than, and equal to</i>.</li> <li>• Find answers to questions, using graphs (e.g., "Which category has more?" "Which category has less").</li> </ul>

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children’s mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns in the real world. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

**1.20 The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.**

<b>UNDERSTANDING THE STANDARD</b> (Teacher Notes)	<b>ESSENTIAL UNDERSTANDINGS</b>	<b>ESSENTIAL KNOWLEDGE AND SKILLS</b>
<ul style="list-style-type: none"> <li>• Sorting, classifying, and ordering objects facilitate work with patterns, geometric shapes, and data.</li> <li>• To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic of a thing).</li> <li>• General similarities and differences among items are easily observed by primary students, who can begin to focus on more than one attribute at a time. During the primary grades, the teacher's task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves with different colors and different shapes).</li> </ul>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand the same set of objects can be sorted and classified in different ways.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Sort and classify objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, or thickness.</li> </ul>

- 1.21 The student will recognize, describe, extend, and create a wide variety of patterns, including rhythmic, color, shape, and numerical. Patterns will include both growing and repeating patterns. Concrete materials and calculators will be used by students.**

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> <li>• Patterns allow children to recognize order, to generalize, and to predict.</li> <li>• Patterning should include               <ul style="list-style-type: none"> <li>– reproducing a given pattern, using manipulatives;</li> <li>– recording a pattern with pictures or symbols;</li> <li>– transferring a pattern into a different form or different representation (e.g., blue–blue–red to an AAB repeating pattern); and</li> <li>– analyzing patterns in the real world (e.g., calendar, seasons, days of the week).</li> </ul> </li> <li>• The simplest types of patterns are repeating patterns. The patterns can be oral, such as the refrain in “Old MacDonald’s Farm” (“e-i-e-i-o”), or physical with clapping and snapping patterns, or combinations of both, such as is found in songs like the “Hokey Pokey.” In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience.</li> <li>• Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats.</li> </ul> <p>continued</p>	<p><b>All students should</b></p> <ul style="list-style-type: none"> <li>• Understand that patterns are a way to recognize order, to organize their world, and to predict what comes next in an arrangement.</li> <li>• Analyze how both repeating and growing patterns are generated.</li> </ul>	<p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Recognize the pattern in a given rhythmic, color, shape, or numerical sequence.</li> <li>• Describe the pattern in a given rhythmic, color, shape, or numerical sequence.</li> <li>• Extend a pattern, using manipulatives, geometric figures, numbers, or calculators.</li> <li>• Create a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators (e.g., the growing patterns 2, 3, 2, 4, 2, 5, 2, 6, 2, ...).</li> <li>• Create an arithmetic number pattern, using a calculator (e.g., when skip counting by fives, use the constant feature on the calculator by pressing <math>5 + 5 = = = \dots</math> to produce the pattern 5, 10, 15, 20, ...).</li> </ul>

**1.21 The student will recognize, describe, extend, and create a wide variety of patterns, including rhythmic, color, shape, and numeric. Patterns will include both growing and repeating patterns. Concrete materials and calculators will be used by students.**

<b>UNDERSTANDING THE STANDARD (Teacher Notes)</b>	<b>ESSENTIAL UNDERSTANDINGS</b>	<b>ESSENTIAL KNOWLEDGE AND SKILLS</b>
<p>continued</p> <ul style="list-style-type: none"> <li>• Sample numeric patterns are                6, 9, 12, 15, 18, ...;                5, 7, 9, 11, 13, ... (repeating numeric pattern);                and                1, 2, 4, 7, 11, 16, ... (growing numeric pattern).</li> <li>• In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation (turn) is the result of turning a figure around a point or a vertex, and reflection (flip) is the result of flipping a figure over a line.</li> </ul>		