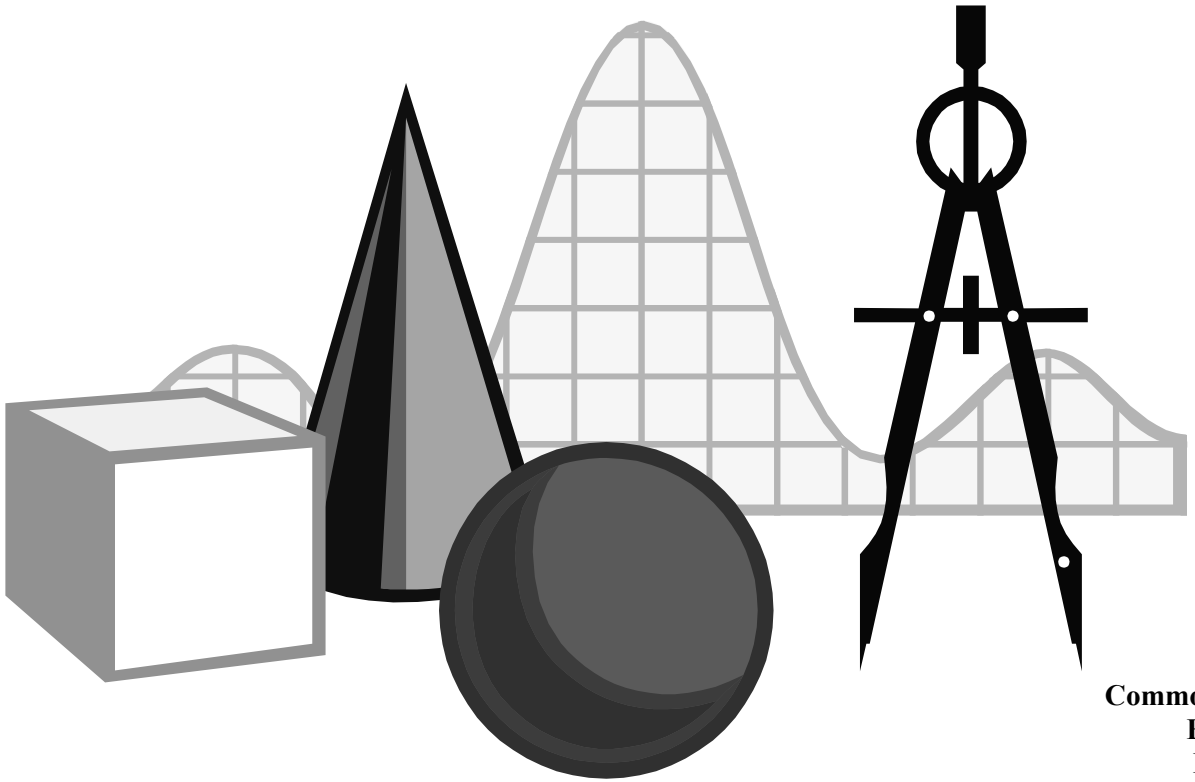
MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 3



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Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, simple fractions, number relationships ("more than," "less than," and "as many as"), and the effects of simple operations on numbers (fact families). These learning experiences should allow students to engage actively in a variety of problem-solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

3.1 The student will read and write six-digit numerals and identify the place value for each digit.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place-value relationship. • The structure of the base-10 blocks is based on the ten-to-one place-value relationship (e.g., 10 cubes make a long, 10 longs make a flat, 10 flats make a big cube). • Place value refers to the value of each digit and depends upon the position of the digit in the number. In the number 7,864, the eight is in the hundreds place, and the value of the 8 is eight hundred. • Flexibility in thinking about numbers — or "decomposition" of numbers (e.g., 12,345 is 123 hundreds, 4 tens, and 5 ones) — is critical and supports understandings essential to multiplication and division. • Whole numbers may be written in a variety of formats: – Standard: 123,456; – Written: one hundred twenty-three thousand, four hundred fifty-six; and – Expanded: $(1 \times 100{,}000) + (2 \times 10{,}000) + (3 \times 1{,}000) + (4 \times 100) + (5 \times 10) + (6 \times 1)$. continued	**All students should** • Understand the relationships in the place-value system, where each place is ten times the value of the place to its right. • Use the patterns in the place-value system to read and write numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and identify the place value for each digit in a six-digit numeral, using base-10 manipulatives (e.g., base-10 blocks). • Read six-digit numerals orally. • Write six-digit numerals that are stated verbally or written in words.

3.1 The student will read and write six-digit numerals and identify the place value for each digit.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, and so on). Places within the periods repeat (hundreds, tens, ones). The base-10 blocks of shapes repeat every period (e.g., cube – long – flat, big cube – big long – big flat). Commas are used to separate the periods. Knowing the place value and period of a number helps students find values of digits in any number as well as read and write numbers. • To read a whole number through the hundred thousands place, – read the number to the first comma; – say the name of the period (e.g., "thousands"); and – read the last three digits, but do not say the name of the ones period. • Reading and writing large numbers should be related to numbers that have meanings (e.g., numbers found in the students' environment). Concrete materials, such as base-10 blocks and bundles of sticks, may be used to represent whole numbers through thousands. Larger numbers may be represented by symbol cards on place-value charts.		

3.2 The student will round a whole number, 9,999 or less, to the nearest ten, hundred, and thousand.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Rounding is one of the estimation strategies and is often used to assess the reasonableness of a solution or to give a rough idea of an amount. • Students should explore reasons for estimation, using real-life experiences, and use rounding to solve real-life situations. • Opportunities to explore the concept of rounding on number lines reinforce an understanding of rounding to the nearest ten or nearest hundred. Use the strategy of identifying the number on the number line and locating the multiple of ten or one hundred to which it is closest. • A strategy for rounding numbers to the nearest ten, hundred, or thousand is as follows: – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place, and change the digits to the right of the rounding place to zero.	**All students should** • Understand that rounding gives an estimate to use when exact numbers are not needed for the situation at hand.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Round a given whole number, 9,999 or less, to the nearest ten, hundred, and thousand. • Solve problems, using rounding of numbers, 9,999 or less, to the nearest ten, hundred, and thousand.

3.3 The student will compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than*, *less than*, or *equal to*).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Mathematical symbols (>, <) used to compare two unequal numbers are called *inequality symbols*. • A procedure for comparing two numbers by examining place value may include the following: – Line up the numbers by place value by lining up the ones. – Beginning at the left, find the first place value where the digits are different. – Compare the digits in this place value to determine which number is greater (or which is less). – Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented. – If both numbers are the same, use the symbol = or the words *equal to*.	**All students should** • Understand that knowledge of place value is essential when comparing numbers. • Understand the relative magnitude of numbers by comparing numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the meaning of the terms *greater than, less than*, and *equal to*. • Determine which of two whole numbers between 0 and 9,999 is greater. • Determine which of two whole numbers between 0 and 9,999 is less. • Compare two whole numbers between 0 and 9,999, using the symbols >, <, or =.

3.4 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. Students will use these relationships to solve problems such as 5 + 3 = 8 and 8 – 3 = __.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition and subtraction are inverse operations, as are multiplication and division; that is, one undoes the other. • In building thinking strategies for subtraction, an emphasis is placed on connecting the subtraction fact to the related addition fact. The same is true for division, where the division fact is tied to the related multiplication fact. Building fact sentences helps strengthen this relationship.	**All students should** • Understand how addition and subtraction are related. • Understand how multiplication and division are related.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the inverse relationships between addition/subtraction and multiplication/division to solve related basic fact sentences. For example, 5 + 3 = 8 and 8 – 3 = __; 4 × 3 = 12 and 12 ÷ 4 = __. • Write three related basic fact sentences when given one basic fact sentence for addition/subtraction and for multiplication/division. For example, given 3 × 2 = 6, write __ × 3 = 6, 6 ÷ 3 = __, and 6 ÷ __ = 3.

3.5 The student will

a) divide regions and sets to represent a fraction; and

b) name and write the fractions represented by a given model (area/region, length/measurement, and set). Fractions (including mixed numbers) will include halves, thirds, fourths, eighths, and tenths.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction is a way of representing part of a whole (as in a region/area model or a measurement model) or part of a group (as in a set model). Fractions are used to name a part of one thing or a part of a collection of things. • In each fraction model, the parts must be equal (i.e., each pie piece must have the same area; the size of each chip in a set must be equal; the measures such as the red cuisenaire rod or the connecting cube must be equal). Wholes are broken into equal-sized parts and reassembled into wholes. • The denominator (bottom number) tells how many equal parts are in the whole or set. The numerator (top number) tells how many of those parts are being described. • Students should have opportunities to make connections among fraction representations by connecting concrete or pictorial representations with oral language and symbolic representations. • Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., *thirds* means "three equal parts of a whole," $\frac{1}{3}$ represents one of three equal-size parts when a pizza is shared among three students, or *three-fourths* means "three of four equal parts of a whole") furthers this development.	**All students should** • Understand that the denominator tells the number of equal parts in a whole and the numerator tells how many equal size parts are being considered.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Name and write fractions and mixed numbers represented by drawings or concrete materials for halves, thirds, fourths, eighths, and tenths. • Represent a given fraction or mixed number, using concrete materials, pictures, and symbols for halves, thirds, fourths, eighths, and tenths. For example, write the symbol for one-fourth, and represent it with concrete materials and pictures.

3.6 The student will compare the numerical value of two fractions having like and unlike denominators, using concrete or pictorial models involving areas/regions, lengths/measurements, and sets.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Comparing unit fractions (a fraction in which the numerator is one) builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (e.g., $\frac{1}{5}$ of a bar is smaller than $\frac{1}{4}$ of a bar). • The denominator tells the number of equal parts into which a whole is divided. The numerator tells how many equal parts are described by the fraction. • Comparing fractions to a benchmark (e.g., close to 0, less than $\frac{1}{2}$, exactly $\frac{1}{2}$, greater than $\frac{1}{2}$, or close to 1) facilitates the comparison of fractions when using concrete materials or pictorial models. • Appropriate concrete materials for this standard include fraction circles, fraction bars, and pattern blocks.	**All students should** • Understand that the value of a fraction is dependent on both the number of parts in a whole (denominator) and the number of those parts being considered (numerator).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare the values of two fractions having like denominators where the denominators are 2, 3, 4, 8, or 10, using concrete or pictorial models. Use the terms *greater than, less than*, or *equal to* or symbols >, <, or = to compare their values. • Compare the values of two unit fractions (a fraction in which the numerator is one), having unlike denominators, where the denominators are 2, 3, 4, 8, or 10, using concrete or pictorial models. Use the terms *greater than, less than*, or *equal to* or symbols >, <, or = to compare their values. • Compare the values of two fractions having unlike denominators where the denominators are 2, 3, 4, 8, and 10, using concrete or pictorial models. Use the terms *greater than, less than*, or *equal to* or symbols >, <, or = to compare their values.

3.7 The student will read and write decimals expressed as tenths and hundredths, using concrete materials and models.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Models and real-life examples of decimals should be used to represent decimals as a part of a whole. • Models for decimals include ten-frames (a 2-by-5 array that can be used to represent tenths), grid paper, and base-10 blocks (which can be used with the flat representing 1 whole, the long representing 1 tenth, and the small unit cube representing 1 hundredth). • Decimals may be written in a variety of forms: – Standard: 26.53 – Written: twenty-six and fifty-three hundredths • Reading decimals from standard format is not required in this standard. However, reading a decimal presented through concrete materials follows a similar procedure: – Read the representation for the whole number (if there is a whole number presented). – Say the word *and* before stating the representation for the tenths and/or hundredths. – Read the representation for the tenths and/or hundredths just as you would read a whole number. – Say the name of the place value of the representation of the digit in the smallest place.	**All students should** • Understand that decimals are another way of representing fractions. • Understand that decimals are written as an extension of the place-value system and that each place to the right of the decimal gets ten times smaller than the previous place.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate the ten-to-one relationship of the decimal places, using base-10 place-value models. • Read and write decimals expressed as tenths, which are represented with base-10 blocks, grid paper, circular fraction pieces, and/or ten-frames. • Read and write decimals expressed as hundredths, which are represented with base-10 blocks and/or grid paper.

A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling and discussing a variety of problem situations to help students move from the concrete to the abstract and develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, and multiplication facts and related fact families;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions and decimals.

3.8 The student will solve problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping, using various computational methods, including calculators, paper and pencil, mental computation, and estimation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition is the combining of quantities; it uses the following terms: *addend* → 423 *addend* → + 246 *sum* → 669 • Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms: *minuend* → 7,698 *subtrahend* → – 5,341 *difference* → 2,357 • An example of an approach to solving problems is Polya's four-step plan: – Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new. – Plan: Decide what operation(s) and sequence of steps to use to solve the problem. – Solve: Follow the plan and work accurately. If the first attempt does not work, try another plan. – Look back: Does the answer make sense? continued	**All students should** • Understand that estimation skills are valuable, time-saving tools particularly in real-life situations when exact answers are not required or needed. • Develop flexible methods of adding whole numbers by combining numbers in a variety of ways, most depending on place values. • Develop and use strategies to estimate whole-number sums and differences and to judge the reasonableness of such estimations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether to add or subtract in problem situations. • Determine whether an estimate is an appropriate solution for addition and subtraction problems. • Add or subtract two whole numbers, each 9,999 or less. • Estimate and find the sum of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation. • Estimate and find the difference of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation. • Solve problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping.

3.8 The student will solve problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping, using various computational methods, including calculators, paper and pencil, mental computation, and estimation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Knowing whether to find an exact answer or to make an estimate is learned through practical experiences in recognizing which is appropriate. When an exact answer is required, opportunities to practice recognizing whether the answer can be determined mentally or must involve paper and pencil or calculators help students select the correct approach. • Determining whether an estimate is appropriate and using a variety of strategies to estimate require experiences with problem situations involving estimation. • There are a variety of mental math strategies for each basic operation, and opportunities to practice these strategies give students the tools to use them at appropriate times. For example, with addition, mental math strategies include – Adding 9: add 10 and subtract 1; and – Making 10: for column addition, look for numbers that group together to make 10. continued		

3.8 The student will solve problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping, using various computational methods, including calculators, paper and pencil, mental computation, and estimation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Using base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms. When adding, if the sum in a place is 10 or more, place value is used to regroup the sums so that there is just one digit in each place. When subtracting, if the number in a place is not enough to subtract from, regrouping is required. • A certain amount of practice is necessary to develop fluency with computational strategies for multidigit numbers; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil.		

3.9 The student will recall the multiplication and division facts through the nines table.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The development of computational fluency relies on quick access to basic number facts. • Strategies to learn the multiplication facts through the nines table include an understanding of multiples/skip counting, properties of zero and one as factors, square numbers, pattern of nines, commutative property, and fact families (the two related multiplication and two division problems). • In order to develop and use strategies to learn the multiplication facts through the nines table, students should use concrete materials, hundred chart, and mental math. • Multiplication is a shortcut for adding same-size groups. To extend the understanding of multiplication, three models may be used: – The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces repeated addition or skip counting. – The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array) helps build the commutative property. – The length model (e.g., a number line) also reinforces repeated addition or skip counting. • A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil.	**All students should** • Develop fluency with basic number combinations for multiplication and division. • Understand that multiplication is repeated addition. • Understand that division is the inverse of multiplication. • Understand that patterns and relationships exist in the basic facts. • Understand that number relationships can be used to learn and retain the basic facts.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recall and state the multiplication and division facts through the nines table. • Recall and write the multiplication and division facts through the nines table.

3.10 The student will represent multiplication and division, using area and set models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The multiplication and division facts through the nines tables should be modeled. • Multiplication is a shortcut for repeated addition. The terms associated with multiplication are listed below: *factor* → 54 *factor* → × 3 *product* → 162 • Creating real-life problems and solving them facilitates the connection between mathematics and everyday experiences (e.g., area problems). • The use of base-10 blocks and repeated addition can serve as a model. For example, 17 × 4 is four sets consisting of one rod and seven units. The sum is renamed as six rods and eight units or 68. This can be thought of as (10 + 7) × 4 = (10 × 4) + (7 × 4) 40 + 28 = 40 + (20 + 8) (40 + 20) + 8 = 68. (SET) • The use of base-10 blocks and the array model can be used to solve the same problem. A rectangle array that is one rod and seven units long by four units wide is formed. The area of this array is represented by 4 longs and 28 units and can be thought of as (10 + 7) × 4 = (10 × 4) + (7 × 4) 40 + 28 = 40 + (20 + 8) (40 + 20) + 8 = 68. (AREA) • An algorithm is a step-by-step method for computing.	**All students should** • Understand various meanings of multiplication and division. • Understand the effects of multiplying and dividing whole numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model multiplication, using area and set models. • Model division, using area and set models. • Solve multiplication problems, using the standard multiplication algorithm, where one factor is 99 or less and the second factor is 5 or less. • Create and solve word problems involving multiplication, where one factor is 99 or less and the second factor is 5 or less.

3.11 The student will add and subtract with proper fractions having like denominators of 10 or less, using concrete materials and pictorial models representing areas/regions, lengths/measurements, and sets.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A proper fraction is a fraction whose numerator is smaller than its denominator. A proper fraction is another name for a fraction between zero and one. • The concepts of addition and subtraction applied to fractions are the same as these concepts applied to whole numbers. • Reasonable answers to problems involving addition and subtraction of fractions can be established by using benchmarks such as 0, $\frac{1}{2}$, and 1. For example, $\frac{3}{5}$ and $\frac{4}{5}$ are each greater than $\frac{1}{2}$, so their sum is greater than 1. • Concrete materials and pictorial models representing area/regions (circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters) can be used to add and subtract fractions having like denominators of 10 or less.	**All students should** • Understand that computation with fractions uses the same concepts as whole number computation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate a fractional part (halves, thirds, fourths, eighths, and tenths) of a whole, using – region/area models (e.g., pie pieces, pattern blocks, geoboards, drawings); – set models (e.g., chips, counters, cubes, drawings); and – measurement models (e.g., nonstandard units such as cuisenaire rods, connecting cubes, and drawings). • Name and write fractions and mixed numbers represented by drawings or concrete materials for halves, thirds, fourths, eighths, and tenths. • Represent a given fraction or mixed number, using concrete materials, pictures, and symbols, for halves, thirds, fourths, eighths, and tenths. For example, write the symbol for one-fourth and represent it with concrete materials and/or pictures. • Add and subtract with proper fractions having denominators of 10 or less, using concrete materials and pictorial models representing area/regions (circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters).

3.12 The student will add and subtract with decimals expressed as tenths, using concrete materials, pictorial representations, and paper and pencil.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Several models that can be used for addition and subtraction of decimals include base-10 blocks, grid paper strips, rectangles and circles divided into tenths, coins, and decimal squares. • When using base-10 pieces to add and subtract tenths, identify the piece that represents the one, and then determine the pieces (each of which should be one-tenth of the one piece) that represent the tenths. For example, with base-10 blocks, if the long represents the one, the small cubes represent the tenths. • In order to make the connection between the base-10 pieces and the paper and pencil method, students should record the numeric representation as they work with the base-10 pieces.	**All students should** • Understand that decimal numbers are another way of representing fractions. • Understand that decimal computation uses the same concepts as whole number-computation and is based on place-value concepts.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Add and subtract with decimals expressed as tenths, using concrete materials (e.g., grid paper, base-10 materials, and circular regions divided into tenths). • Add and subtract with decimal numbers expressed as tenths, using paper and pencil.

Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

3.13 The student will determine by counting the value of a collection of bills and coins whose total value is $5.00 or less, compare the value of the coins or bills, and make change.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills. • A variety of skills can be used to determine the change after a purchase, including – counting on, i.e., starting with the amount to be paid (purchase price), counting forward to the next dollar, and then counting forward by dollar bills to reach the amount from which to make change; – mentally calculating the difference; and – using coins and bills.	**All students should** • Understand that a collection of coins has a value that can be counted. • Understand how to make change from $5.00 or less.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count the value of collections of coins and bills up to $5.00. • Compare the values of two sets of coins or bills, up to $5.00, using the terms *greater than, less than*, and *equal to*. • Make change from $5.00 or less.

3.14 The student will estimate and then use actual measuring devices with metric and U.S. Customary units to measure

a) length — inches, feet, yards, centimeters, and meters;

b) liquid volume — cups, pints, quarts, gallons, and liters; and

c) weight/mass — ounces, pounds, grams, and kilograms.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • The concept of a standard measurement unit is one of the major ideas in understanding measurement. Familiarity with standard units is developed through hands-on experiences of comparing, estimating, measuring, and constructing. • Benchmarks of common objects need to be established for each of the specified units of measure (e.g., the mass of a math book is about one kilogram). Practical experience measuring the mass of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate measures. • One unit of measure may be more appropriate than another to measure an object, depending on the size of the object and the degree of accuracy desired. • Correct use of measurement tools is essential to understanding the concepts of measurement.	**All students should** • Understand how to estimate measures of length, liquid volume, and weight/mass. • Understand how to determine the actual measure of length, liquid volume, and weight/mass.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and use the following units of length: centimeters, meters, inches, feet, and yards. • Identify and use the following units of liquid volume: cups, pints, quarts, gallons, and liters. • Identify and use the following units of weight/mass: ounces, pounds, grams, and kilograms. • Estimate and then measure lengths of objects to the nearest centimeter and meter and the nearest inch, foot, and yard. • Estimate and then measure the weight/mass of objects to the nearest ounce and pound and the nearest gram and kilogram. • Estimate and then measure liquid volume to the nearest cup, pint, quart, gallon, and liter.

3.15 The student will tell time to the nearest five-minute interval and to the nearest minute, using analog and digital clocks.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Telling time is applying reading skills to a clock face. • While digital clocks make reading time easy, it is necessary to ensure that students understand that there are sixty minutes in an hour. • Use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise when time is read.	**All students should** • Apply appropriate techniques to determine time to the nearest five-minute interval, using analog and digital clocks.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Tell time to the hour, half-hour, quarter-hour, nearest five-minute interval, and nearest minute, using analog and digital clocks. • Match the times shown on analog and digital clocks to written times.

3.16 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The knowledge that a year has $365 + \frac{1}{4}$ days will help students understand the necessity of adding a full day every fourth year, called a *leap year*. • The use of a calendar facilitates the understanding of time relationships between days and months, days and weeks, days and years, and months and years. • The use of an analog clock facilitates the understanding of time relationships between minutes and hours and hours and days.	**All students should** • Understand the relationship that exists among periods of time, using calendars and clocks.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify equivalent relationships observed in a calendar, including the number of days in a given month, the number of days in a week, the number of days in a year, and the number of months in a year. • Identify the number of minutes in an hour and the number of hours in a day.

3.17 The student will read temperature to the nearest degree from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used.

UNDERSTANDING THE STANDARD (TEACHER NOTES)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimating and measuring temperatures in the environment in Fahrenheit and Celsius require the use of real thermometers. • A physical model can be used to represent the temperature determined by a real thermometer. • The symbols for degrees in Celsius (°C) and degrees in Fahrenheit (°F) should be used to write temperatures. • Celsius and Fahrenheit temperatures should be related to everyday occurrences by measuring the temperature of the classroom, the outside, liquids, body temperature, and other things found in the environment.	**All students should** • Understand how to measure temperature in Celsius and Fahrenheit with a thermometer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Read temperature to the nearest degree from real Celsius and Fahrenheit thermometers and from physical models (including pictorial representations) of such thermometers.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of shapes and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent two- and three-dimensional shapes through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, comparing, and investigating three-dimensional objects and their two-dimensional faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, sorting, and classifying shapes; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these shapes but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

3.18 The student will analyze two-dimensional (plane) and three-dimensional (solid) geometric figures (circle, square, rectangle, triangle, cube, rectangular solid [prism], square pyramid, sphere, cone, and cylinder) and identify relevant properties, including the number of corners, square corners, edges, and the number and shape of faces, using concrete models.

UNDERSTANDING THE STANDARD (TEACHER NOTES)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. – **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. – **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know that the door is a rectangle."). – **Level 2: Analysis**. Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it's closed, it has four sides and four right angles, and opposite sides are parallel."). • The study of geometric figures must be active, using visual images and concrete materials. continued	**All students should** • Understand more precise ways to describe shapes by focusing on properties and the specialized vocabulary associated with these shapes and properties.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify by name, models and pictures of plane geometric figures (circle, square, rectangle, and triangle) and solid geometric figures (cube, rectangular solid, square pyramid, sphere, cone, and cylinder). • Identify plane geometric figures by counting the number of sides, corners, and square corners. • Identify geometric solids by counting the number of corners, square corners, and edges, and by the shapes of the faces. • Classify, compare, and contrast plane and solid geometric figures (e.g., circle/sphere, square/cube, triangle/pyramid, and rectangle/rectangular solid), using corners, square corners, faces, and edges.

3.18 The student will analyze two-dimensional (plane) and three-dimensional (solid) geometric figures (circle, square, rectangle, triangle, cube, rectangular solid [prism], square pyramid, sphere, cone, and cylinder) and identify relevant properties, including the number of corners, square corners, edges, and the number and shape of faces, using concrete models.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Access to a variety of concrete tools such as graph paper, pattern blocks, geoboards, and geometric solids is greatly enhanced by computer software tools that support exploration. • Opportunity must be provided for building and using geometric vocabulary to describe two-dimensional (plane) and three-dimensional (solid) shapes. • It is appropriate at this level to refer to "square corner." However, teachers should begin to introduce the term *right angle* for square corner as well. • A cube is a three-dimensional figure with six congruent square faces and with every edge the same length. A cube has 8 corners and 12 edges. • A cylinder is a three-dimensional figure formed by two congruent parallel circles joined by a curved surface. • A cone is a three-dimensional, pointed figure that has a flat, round base (usually a circle) that is joined to a vertex by a curved surface. continued		

3.18 The student will analyze two-dimensional (plane) and three-dimensional (solid) geometric figures (circle, square, rectangle, triangle, cube, rectangular solid [prism], square pyramid, sphere, cone, and cylinder) and identify relevant properties, including the number of corners, square corners, edges, and the number and shape of faces, using concrete models.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A rectangular solid is a three-dimensional figure in which all six faces are rectangles with three pair of parallel congruent opposite faces. • A sphere is a three-dimensional figure with all of its points the same distance from its center. • A square pyramid is a three-dimensional figure whose base is a square and whose other faces are triangles that share a common vertex. • A face is a polygon that serves as one side of a three-dimensional figure (e.g., a square is a face of a cube). • An edge is the line segment where two faces of a three-dimensional figure meet. • A corner is the point at which three or more edges meet. • Students should be reminded that a concrete three-dimensional object, even when called a "solid," may actually be hollow rather than solid.		

3.19 The student will identify and draw representations of line segments and angles, using a ruler or straightedge.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A point is an exact location in space. It has no length or width. Usually, a point is named with a capital letter. • A line is a collection of points going on and on in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. The line can also be named with a single, lower-case letter. Arrowheads must be drawn to show that the line goes on in both directions infinitely. • A line segment is part of a line. It has two endpoints and includes all the points between those endpoints. The endpoints are used to name a line segment. • An angle is formed by two rays having a common endpoint. This endpoint is called the *vertex*. Angles are found wherever lines and line segments intersect. An angle can be named in three different ways by using – three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle. • Angle rulers may be particularly useful in developing the concept of an angle.	**All students should** • Understand that line segments and angles are fundamental components of noncircular, plane polygons. • Understand that a line segment is a part of a line having two end points and that it contains all the points between those two endpoints.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and locate examples of a point, line segment, and angle. • Draw line segments and angles, using a ruler or straightedge.

3.20 The student, given appropriate drawings or models, will identify and describe congruent and symmetrical, two-dimensional (plane) figures, using tracing procedures.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Congruent figures are figures having exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models. • A figure or shape is symmetrical when one-half of the figure is the mirror image of the other half. Opportunities for exploring symmetry in figures can best be accomplished by using physical models. • A line of symmetry divides a symmetrical figure, object, or arrangement of objects into two parts that are congruent if one part is reflected (flipped) over the line of symmetry. • While investigating symmetry, students move shapes, such as pattern blocks, intuitively, thereby exploring transformations of those shapes. A transformation is the movement of a figure — either a translation, rotation, or reflection. A translation (slide) is the result of sliding a figure in any direction; a rotation (turn) is the result of turning a figure around a point or a vertex; and a reflection (flip) is the result of flipping a figure over a line.	**All students should** • Understand that congruent shapes match exactly. • Understand that symmetrical figures can be divided into two halves that are the mirror image of each other.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Locate examples of symmetrical figures, and verify their symmetry by using tracing procedures. • Determine if given figures have a line or lines of symmetry (vertical, horizontal, diagonal), using tracing procedures. • Locate examples of congruent figures and verify their congruency by laying one on top of the other. • Determine if given figures are congruent, using tracing procedures.

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

3.21 The student, given grid paper, will

a) collect and organize data on a given topic of his/her choice, using observations, measurements, surveys, or experiments; and

b) construct a line plot, a picture graph, or a bar graph to represent the results. Each graph will include an appropriate title and key.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Investigations involving data should occur frequently and relate to students' experiences, interests, and environment. • Formulating questions for investigations is student-generated at this level. For example: What is the favorite cafeteria lunch preferred by students in the class when four lunch menus are offered? • The purpose of a graph is to represent data gathered to answer a question. • Bar graphs are used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs. – A bar graph uses parallel horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis displaying the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Third grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, or 10. continued	**All students should** • Understand how gathered data can be collected and organized. • Understand that data can be displayed in different types of graphs depending on the questions to be answered.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Formulate questions to investigate. • Design data investigations to answer formulated questions, limiting the number of categories for data collection to four. • Collect data, using surveys, polls, questionnaires, scientific experiments, and observations. • Organize data and construct a bar graph on grid paper representing 16 or fewer data points for no more than four categories. • Label bar graphs with a title, a description of each axis, and a key where appropriate. Limit increments on the numerical axis to whole numbers representing multiples of 1, 2, 5, or 10.

3.21 The student, given grid paper, will

a) collect and organize data on a given topic of his/her choice, using observations, measurements, surveys, or experiments; and

b) construct a line plot, a picture graph, or a bar graph to represent the results. Each graph will include an appropriate title and key.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued – Each axis should be labeled, and the graph should be given a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written. • Line plots show the spread of data. To construct a line plot, – draw a number line on grid paper (The scale of numbers should include the greatest value and the least value in a set of numbers.); – for each piece of data, draw an *x* above the corresponding number; and – title the plot.		

3.22 The student will read and interpret data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The purpose of a graph is to represent data gathered to answer a question. • When data are displayed in an organized manner, the results of the investigations can be described and the posed question answered. • Recognition of appropriate and inappropriate statements begins at this level with graph interpretations.	**All students should** • Understand that data sets can be analyzed to provide different kinds of information.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Read the information presented on a simple bar or picture graph (e.g., the title, the categories, the description of the two axes, the key). • Read information presented in line plots. • Analyze and interpret information from simple picture and bar graphs, with data points limited to 16 and categories to 4, by writing at least one statement. • Analyze and interpret information from line plots, with data points limited to 16, by writing at least one statement. • Describe the categories of data and the data as a whole (e.g., data were collected on four types of eggs — scrambled, fried, hard boiled, and egg salad — eaten by students). • Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs). • Select a correct interpretation of a graph from a set of interpretations of the graph, where one is correct and the remaining three are incorrect. For example, a bar graph containing data on four types of eggs — scrambled, fried, hard boiled, and egg salad — eaten by students shows that more students prefer scrambled eggs. A correct answer response, if given, would be that more students prefer scrambled eggs than any other type of eggs.

3.23 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives. • Investigation of experimental probability is continued at this level through informal activities using two-colored counters, spinners, and random number generators (number cubes). • Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event $= \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). continued	**All students should** • Understand and apply basic concepts of probability. • Understand that probability is the chance of an event happening.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define probability as the chance that an event will happen. • List all possible outcomes for a given situation (e.g., heads and tails are the two possible outcomes of flipping a coin). • Identify the possible outcomes for a common event, using terms such as *impossible*, *unlikely*, *equally likely*, *likely*, and *certain*.

3.23 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include real-life examples. • For any event, such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. • A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, table, or chart.		

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns in the real world. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

3.24 The student will recognize and describe a variety of patterns formed using concrete objects, numbers, tables, and pictures, and extend the pattern, using the same or different forms (concrete objects, numbers, tables, and pictures).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Exploring patterns requires active physical and mental involvement. • The use of materials to extend patterns permits experimentation or trial-and-error approaches that are almost impossible without them. • Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing numbers symbolically or algebraically. • The simplest types of patterns are repeating patterns. In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. • Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats. • Sample numeric patterns include – 6, 9, 12, 15, 18, …; – 1, 2, 4, 7, 11, 16, …; – 2, 4, 8, 16, 32, …; and – 1, 5, 25, 125, 625, …. continued	**All students should** • Understand that numeric and geometric patterns can be expressed in words or symbols. • Understand the structure of a pattern and how it grows or changes. • Understand that mathematical relationships exist in patterns. • Understand that patterns can be translated from one representation to another.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize repeating and growing numeric and geometric patterns (e.g., skip counting, addition tables, and multiplication tables). • Describe repeating and growing numeric and geometric patterns formed using concrete objects, numbers, tables, and/or pictures, using the same or different forms. • Extend repeating and growing numeric and geometric patterns formed using concrete objects, numbers, tables, and/or pictures, using the same or different forms.

3.24 The student will recognize and describe a variety of patterns formed using concrete objects, numbers, tables, and pictures, and extend the pattern, using the same or different forms (concrete objects, numbers, tables, and pictures).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation (turn) is the result of turning a figure around a point or a vertex, and reflection (flip) is the result of flipping a figure over a line. • A table of values can be analyzed to determine the pattern that has been used, and that pattern can then be used to find the next value.		

3.25 The student will

a) investigate and create patterns involving numbers, operations (addition and multiplication), and relations that model the identity and commutative properties for addition and multiplication; and

b) demonstrate an understanding of equality by recognizing that the equal sign (=) links equivalent quantities such as $4 \cdot 3 = 2 \cdot 6$.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. • The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., $4 + 3 = 3 + 4$). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., $2 \times 3 = 3 \times 2$). • The identity property for addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. • A number sentence is an equation with numbers (e.g., $6 + 3 = 9$; or $6 + 3 = 4 + 5$).	**All students should** • Understand that mathematical relationships can be expressed using number sentences. • Understand that quantities on both sides of an equals sign must be equal.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize that the equals sign relates equivalent quantities. • Write number sentences to represent equivalent mathematical relationships (e.g., $4 \cdot 3 = 2 \cdot 6$). • Identify number sentences that show appropriate use of the equals sign.