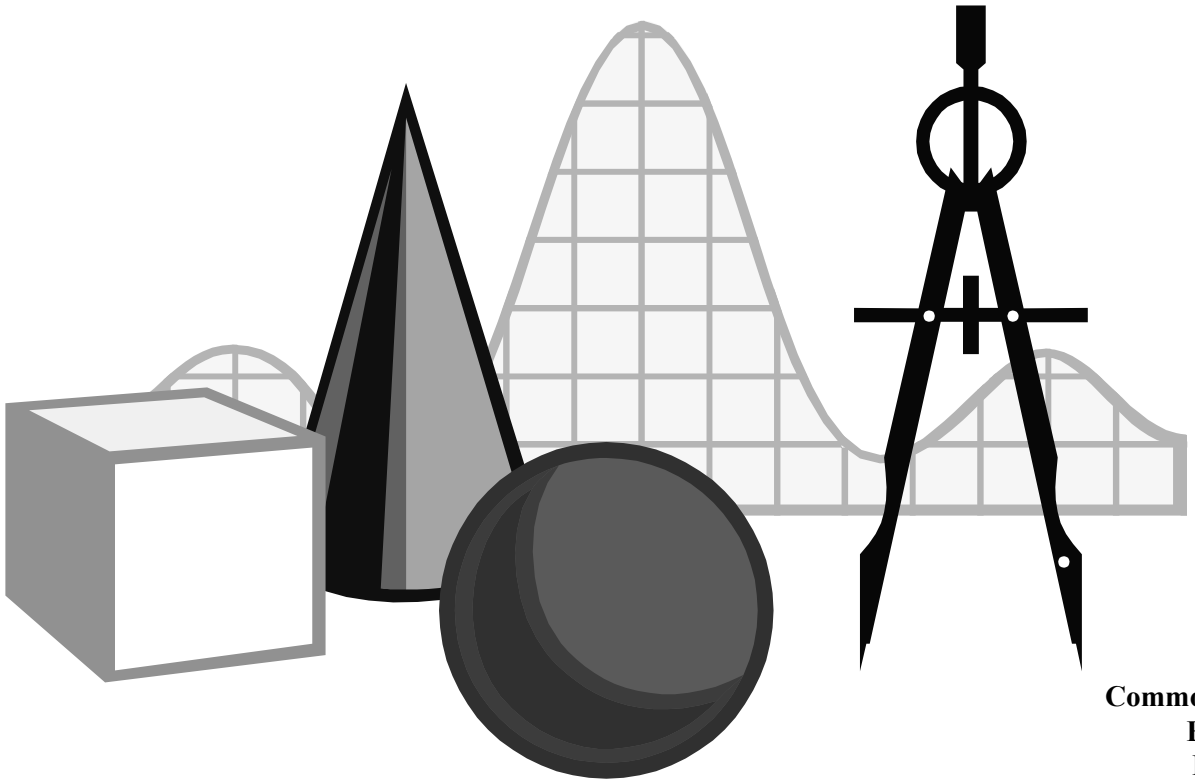
MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 4



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Mathematics instruction in grades 4 and 5 should continue to foster the development of number sense, especially with decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction at grades 4 and 5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names). Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve problems.

4.1 The student will

a) identify (orally and in writing) the place value for each digit in a whole number expressed through millions;

b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the base-10 number system is based upon a simple pattern of tens, in which the value of each place is ten times the value of the place to its right. • Place value refers to the value of each digit and depends upon the position of the digit in the number. For example, in the number 7,864,352, the eight is in the hundred thousands place, and the value of the 8 is eight hundred thousand. • Whole numbers may be written in a variety of formats: – Standard: 1,234,567 – Written: one million, two hundred thirty-four thousand, five hundred sixty-seven – Expanded: (1 × 1,000,000) + (2 × 100,000) + (3 × 10,000) + (4 × 1,000) + (5 × 100) + (6 × 10) + (7 × 1) • Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, …). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students find values of digits in any number as well as read and write numbers. continued	**All students should** • Understand the relationships in the place-value system in which the value of each place is ten times the value of the place to its right. • Use the patterns in the place-value system to read and write numbers. • Understand that reading place value correctly is essential when comparing numbers. • Understand that rounding gives a close number to use when exact numbers are not needed for the situation at hand. • Develop strategies for rounding.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and communicate, both orally and in written form, the place value for each digit in whole numbers expressed through the one millions place. • Read whole numbers through the one millions place that are presented in standard format, and select the matching number in written format. • Write whole numbers through the one millions place in standard format when the numbers are presented orally or in written format. • Identify and use the symbols for *greater than*, *less than*, and *equal to*. • Compare two whole numbers expressed through the one millions, using symbols >, <, or =. • Round whole numbers expressed through the one millions place to the nearest thousand, ten thousand, and hundred-thousand place.

4.1 The student will

a) identify (orally and in writing) the place value for each digit in a whole number expressed through millions;

b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Reading and writing large numbers should be related to numbers that have meanings (e.g., numbers found in the students' environment). Concrete materials such as base-10 blocks and bundles of sticks may be used to represent whole numbers through thousands. Larger numbers may be represented by symbol cards and place-value charts. • Mathematical symbols (>, <) used to compare two unequal numbers are called *inequality symbols*. • A procedure for comparing two numbers by examining place value may include the following: – Line up the numbers by place value by lining up the ones. – Beginning at the left, find the first place value where the digits are different. – Compare the digits in this place value to determine which number is greater (or which is less). – Use the appropriate symbol > or < or words *greater than* or *less than* to compare the numbers in the order in which they are presented. – If both numbers are the same, use the symbol = or words *equal to*. • A strategy for rounding numbers to the nearest thousand, ten thousand, and hundred thousand is as follows: – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero.		

4.2 The student will

a) identify, model, and compare rational numbers (fractions and mixed numbers) , using concrete objects and pictures;

b) represent equivalent fractions; and

c) relate fractions to decimals, using concrete objects.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Rational numbers should be explained as any number that can be written as a fraction (e.g.., $\frac{2}{1}$, $\frac{2}{3}$, $7\frac{1}{4}$). • A variety of fraction models should be used to expand students' understanding of fractions and mixed numbers: – Region/area models: a surface or area is subdivided into smaller equal parts, and each part is compared with the whole (e.g., fraction circles, pattern blocks, geoboards, grid paper, color tiles). – Set models: the whole is understood to be a set of objects, and subsets of the whole make up fractional parts (e.g., counters, chips). – Measurement models: similar to area models but lengths instead of areas are compared (e.g., fraction strips, cuisenaire rods, unifix cubes, number lines, rulers). • Equivalent fractions name the same amount. Students should use a variety of models to identify different names for equivalent fractions. continued	**All students should** • Develop an understanding of fractions as parts of unit wholes, as parts of a collection, and as locations on a number line. • Use models, benchmarks, and equivalent forms to judge the size of fractions. • Recognize and generate equivalent forms of commonly used fractions and decimals.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify, model, and compare fractions and mixed numbers through twelfths, using – region/area models (e.g., fraction circles, pattern blocks, geoboards, color tiles, graph paper); – set models (e.g., two-sided counters, chips); and – measurement models (e.g., cuisenaire rods, unifix cubes, fraction strips, number lines). • Represent fractions for halves, fourths, fifths, and tenths as decimals through thousandths, using concrete objects (e.g., demonstrate the relationship between the fraction $\frac{1}{4}$ and its decimal equivalent 0.25). • Relate fractions to decimals, using concrete objects (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money [coins]). • Identify and represent equivalent fractions through twelfths, using region/area models, set models, and measurement models.

4.2 The student will

a) identify, model, and compare rational numbers (fractions and mixed numbers) , using concrete objects and pictures;

b) represent equivalent fractions; and

c) relate fractions to decimals, using concrete objects.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Students should focus on finding equivalent fractions of familiar fractions such as halves, thirds, fourths, sixths, eighths, tenths, and twelfths. • Decimals and fractions represent the same relationships; however, they are presented in two different formats. Decimal numbers are another way of writing fractions. The base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money).		

4.3 The student will compare the numerical value of fractions (with like and unlike denominators) having denominators of 12 or less, using concrete materials.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction is a way of representing part of a whole (as in a region/area model or a measurement model) or part of a group (as in a set model). A fraction is used to name a part of one thing or a part of a collection of things. • In each fraction model, the parts must be equal (i.e., each pie piece must have the same area; the size of each chip in a set must be equal; the measures such as the red cuisenaire rod or the parts in a fraction strip must be equal). • The denominator (bottom number) tells how many equal parts are in the whole or set. The numerator (top number) tells how many of those parts are being described. • When fractions have the same denominators, they are said to have "common denominators" or "like denominators." Comparing fractions with like denominators involves comparing only the numerators. • Strategies for comparing fractions having unlike denominators may include – comparing fractions to familiar benchmarks (e.g., 0, $\frac{1}{2}$, 1); – finding equivalent fractions, using manipulative models such as fraction strips, number lines, fraction circles, cuisenaire rods, pattern blocks, unifix cubes, base-10 blocks, tangrams, or graph paper; and – finding a common denominator by finding the least common multiple (LCM) of both denominators and then rewriting each fraction as an equivalent fraction, using the LCM as the denominator.	**All students should** • Use models, benchmarks, and equivalent forms to judge the size of fractions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare two fractions having denominators of 12 or less, using manipulative models and drawings, such as – region/area models (e.g., fraction circles, pattern blocks, geoboards, color tiles, graph paper, drawings); – set models (e.g., two-sided counters, chips, drawings); and – measurement models (e.g., cuisenaire rods, unifix cubes, fraction strips, rulers/number lines, drawings). • Compare two fractions with like denominators by comparing numerators (e.g., $\frac{1}{5} < \frac{3}{5}$). • Compare two fractions having unlike denominators of 12 or less by comparing the fractions to common benchmarks (e.g., $\frac{1}{2}$ or 1) to determine their relationship or by finding a common denominator. • Use the symbols >, <, and = to compare the numerical value of two fractions having denominators of 12 or less.

4.4 The student will

a) read, write, represent, and identify decimals expressed through thousandths;

b) round to the nearest whole number, tenth, and hundredth; and

c) compare the value of two decimals, using symbols (<, >, or =), concrete materials, drawings, and calculators.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place-value relationship. • A decimal point separates the whole number places from the places that are less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*. • To read decimals, – read the whole number to the left of the decimal point, if there is one; – read the decimal point as "and"; – read the digits to the right of the decimal point just as you would read a whole number; and – say the name of the place value of the digit in the smallest place. continued	**All students should** • Understand the place-value structure of decimals and use this structure to read, write, and compare decimals. • Understand that decimal numbers can be rounded to an estimate when exact numbers are not needed for the situation at hand. • Understand that decimals are rounded in a way that is similar to the way whole numbers are rounded.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate the ten-to-one place-value relationship for decimals through thousandths, using base-10 manipulatives (e.g., place-value mats/charts, decimal squares, base-10 blocks, money). • Represent and identify decimals expressed through thousandths, using base-10 manipulatives, pictorial representations, calculators, and numerical symbols (e.g., relate the appropriate drawing to 0.005). • Read and write decimals expressed through thousandths, using base-10 manipulatives, drawings, calculators, and numerical symbols. Any decimal less than 1 will include a leading zero (e.g., 0.125). • Round decimals to the nearest whole number, tenth, and hundredth. • Compare the value of two decimals, using the symbols >, <, =.

4.4 The student will

a) read, write, represent, and identify decimals expressed through thousandths;

b) round to the nearest whole number, tenth, and hundredth; and

c) compare the value of two decimals, using symbols (<, >, or =), concrete materials, drawings, and calculators.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Decimals may be written in a variety of forms: – Standard: 26.537 – Written: twenty-six and five hundred thirty-seven thousandths – Expanded: $(2 \times 10) + (6 \times 1) + (5 \times 0.1) + (3 \times 0.01) + (7 \times 0.001)$. • The procedure for rounding decimal numbers is similar to the procedure for rounding whole numbers. • A strategy for rounding decimal numbers to the nearest tenth and hundredth is as follows: – Look one place to the right of the digit you want to round to. – If the digit is 5 or greater, add 1 to the digit in the rounding place, and drop the digits to the right of the rounding place. – If the digit is less than 5, leave the digit in the rounding place as it is, and drop the digits to the right of the rounding place. • Another strategy for rounding decimal numbers utilizes a number line to locate a decimal between two numbers. For example, 18.83 is closer to 18.8 than to 18.9.		

Computation and estimation in grades 4 and 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to working with fractions and decimals. Instruction should focus on computation activities that enable students to model, explain, and develop reasonable proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract with common fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability to identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use (not identify) properties of operations to solve problems [e.g., 7×28 is equivalent to $(7 \times 20) + (7 \times 8)$, or $(7 \times 30) - (7 \times 2)$].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their computations.

Additionally, students should enhance their ability to select an appropriate problem-solving method from among estimation, mental math, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

4.5 The student will estimate whole-number sums and differences and describe the method of estimation. Students will refine estimates, using terms such as *closer to, between*, and *a little more than*.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A sum is the result of adding two or more numbers. • A difference is the amount that remains after one quantity is subtracted from another. • An estimate is a number close to an exact amount. An estimate tells about how much or about how many. • Different strategies for estimating include using compatible numbers to estimate sums and differences and using front-end estimation for sums and differences. – Compatible numbers are numbers that are easy to work with mentally. Number pairs that are easy to add or subtract are compatible. When estimating a sum, replace actual numbers with compatible numbers (e.g., 52 + 74 can be estimated by using the compatible numbers 50 + 75). When estimating a difference, use numbers that are close to the original numbers. Tens and hundreds are easy to subtract (e.g., 83 – 38 is close to 80 – 40). continued	**All students should** • Develop and use strategies to estimate whole-number sums and differences and to judge the reasonableness of such results.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate whole-number sums and differences, using rounding, front-end strategies, and compatible number strategies. Describe the method of estimation used. • Refine estimates by adjusting the final amount, using terms such as *closer to, between*, and *a little more than*.

4.5 The student will estimate whole-number sums and differences and describe the method of estimation. Students will refine estimates, using terms such as *closer to, between*, and *a little more than*.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• continued – The front-end strategy for estimating is computing with the front digits. Front-end estimation for addition can be used even when the addends have a different number of digits. The procedure requires the addition of the values of the digits in the greatest place value of the smallest number. For example: 2367 → 2300 243 → 200 + 1186 → + 1100 3600 • Front-end or leading-digit estimation always gives a sum less than the actual sum; however, the estimate can be adjusted or refined so that it is closer to the actual sum.		

4.6 The student will add and subtract whole numbers written in vertical and horizontal form, choosing appropriately between paper and pencil methods and calculators.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition is the combining of quantities; it uses the following terms: *addend* → 45,623 *addend* → + 37,846 *sum* → 83,469 • Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms: *minuend* → 45,698 *subtrahend* → – 32,741 *difference* → 12,957 • Before adding or subtracting with paper and pencil, addition and subtraction problems in horizontal form should be rewritten in vertical form by lining up the places vertically. • Using base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms. In addition, when the sum in a place is 10 or more, place value is used to regroup the sums so that there is only one digit in each place. In subtraction, when the number (minuend) in a place is not enough from which to subtract, regrouping is required. continued	**All students should** • Understand that addition involves combining quantities and subtraction involves separating quantities. • Develop flexible methods of adding and subtracting whole numbers by taking apart and combining numbers in a variety of ways, most depending on place value. • Understand that addition and subtraction are inverse operations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the sum or difference of two whole numbers, each 999,999 or less, in vertical form with or without regrouping. • Determine the sum or difference of two whole numbers, each 999,999 or less, in horizontal form with or without regrouping. • Find the sum or difference of two whole numbers, each 999,999 or less, using paper and pencil. • Find the sum or difference of two whole numbers, each 999,999 or less, using a calculator.

4.6 The student will add and subtract whole numbers written in vertical and horizontal form, choosing appropriately between paper and pencil methods and calculators.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A certain amount of practice is necessary to develop fluency with computational strategies for multidigit numbers; however, the practice must be meaningful, motivating, and systematic if students are to develop fluency in computation, whether mentally, with manipulative materials, or with paper and pencil. • Calculators are an appropriate tool for computing sums and differences of large numbers, particularly when mastery of the algorithm has been demonstrated.		

4.7 The student will find the product of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer, using estimation and paper and pencil. For larger products (a two-digit numeral times a three-digit numeral), estimation and calculators will be used.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The terms associated with multiplication are *factor* → 376 *factor* → × 23 *product* → 8,648 • One model of multiplication is repeated addition. • Another model of multiplication is the "Partial Product" model. 24 × 3 12 ← Multiply the ones: 3 × 4 = 12 + 60 ← Multiply the tens: 3 × 20 = 60 72 • Students should continue to develop fluency with single-digit multiplication facts and their related division facts. • Calculators should be used to solve problems that require tedious calculations. continued	**All students should** • Understand various meanings of multiplication. • Understand the effects of multiplying whole numbers. • Develop flexible methods of multiplying whole numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate the products of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer. • Find the product of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer, using paper and pencil and calculators.

4.7 The student will find the product of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer, using estimation and paper and pencil. For larger products (a two-digit numeral times a three-digit numeral), estimation and calculators will be used.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Estimation should be used to check the reasonableness of the product. Examples of estimation strategies include the following: – The front-end method: multiply the front digits and then complete the product by recording the number of zeros found in the factors. It is important to develop understanding of this process before using the step-by-step procedure. 523 → 500 × 31 → × 30 15,000 This is 3 × 5 = 15 with 3 zeros. – Compatible numbers: replace factors with compatible numbers, and then multiply. Opportunities for students to discover patterns with 10 and powers of 10 should be provided. 64 → 64 × 11 → × 10		

4.8 The student will estimate and find the quotient of two whole numbers, given a one-digit divisor.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. Both situations may be modeled with base-10 manipulatives. • Multiplication and division are inverse operations. • Terms used in division are *dividend, divisor*, and *quotient.* dividend ÷ divisor = quotient quotient divisor)dividend • Opportunities to invent division algorithms help students make sense of the algorithm. Teachers may teach division by various methods such as repeated multiplication and subtraction before teaching the traditional long-division algorithm.	**All students should** • Understand various meanings of division. • Understand the effects of dividing whole numbers. • Understand various representations of division, i.e., dividend ÷ divisor = quotient quotient divisor)dividend $\frac{\text{dividend}}{\text{divisor}}$ = quotient.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend. • Find the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend.

4.9 The student will

a) add and subtract with fractions having like and unlike denominators of 12 or less, using concrete materials, pictorial representations, and paper and pencil;

b) add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil; and

c) solve problems involving addition and subtraction with fractions having like and unlike denominators of 12 or less and with decimals expressed through thousandths, using various computational methods, including calculators, paper and pencil, mental computation, and estimation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should investigate addition and subtraction of fractions, using a variety of models (e.g. fraction circles, fraction strips, rulers, unifix cubes, pattern blocks). • When adding or subtracting fractions having like denominators, add or subtract the numerators and use the same denominator. • When adding or subtracting fractions having unlike denominators, rewrite them as fractions with a common denominator. The least common multiple (LCM) of the unlike denominators is a common denominator (LCD). • Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money). • For decimal computation, the same ideas developed for whole-number computation may be used, and these ideas may be applied to decimal place values, giving careful attention to the placement of the decimal point in the solution. • Fractions may be related to decimals by using models (e.g., 10-by-10 grids, decimal squares, money).	**All students should** • Develop and use strategies to estimate addition and subtraction involving fractions and decimals. • Use visual models to add and subtract with fractions and decimals.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Add and subtract with fractions having like denominators of 12 or less, using concrete materials, pictorial representations, and paper and pencil. • Add and subtract with fractions having unlike denominators of 12 or less, using concrete materials pictorial representations and paper and pencil. • Solve problems that involve adding and subtracting with fractions having like and unlike denominators of 12 or less. • Add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. • Solve problems that involve adding and subtracting with decimals through thousandths.

Students in grades 4 and 5 should be actively involved in measurement activities that require a dynamic interaction between students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects. The intent is for students to make "ballpark" comparisons and *not* to memorize conversion factors between U.S. Customary and metric units. To fully understand these ballpark comparisons, students must be actively engaged in the process of measurement.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure perimeter, weight/mass, liquid volume/capacity, area, temperature, and time. Students continue to enhance their understanding of measurement by using appropriate tools such as rulers, balances, clocks, and thermometers. The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.

4.10 The student will

a) estimate and measure weight/mass, using actual measuring devices, and describe the results in U.S. Customary/metric units as appropriate, including ounces, pounds, grams, and kilograms;

b) identify equivalent measurements between units within the U.S. Customary system (ounces and pounds) and between units within the metric system (grams and kilograms); and

c) estimate the conversion of ounces and grams and pounds and kilograms, using approximate comparisons (1 ounce is about 28 grams, or 1 gram is about the weight of a paper clip; 1 kilogram is a little more than 2 pounds). *

**The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U. S. Customary and metric units.*

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • Balances are appropriate measuring devices to measure weight in U.S. Customary units (ounces, pounds) and mass in metric units (grams, kilograms). • Practical experience measuring the mass of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate mass. continued	**All students should** • Use benchmarks to estimate and measure weight/mass. • Identify equivalent measures between U.S. Customary and metric measurements.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (e.g., ounce, pound, gram, kilogram) to use when measuring everyday objects in both metric and U.S. Customary units. • Measure objects in both metric and U.S. Customary units (e.g., ounce, pound, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. • Record the mass of an object including the appropriate unit of measure (e.g., 24 grams). • Estimate conversions between U.S. Customary and metric units, using ballpark comparisons, such as – 1 ounce is about 28 grams; – 1 nickel has the mass of about 5 grams; and – 1 kilogram is a little more than 2 pounds.

4.10 The student will

a) estimate and measure weight/mass, using actual measuring devices, and describe the results in U.S. Customary/metric units as appropriate, including ounces, pounds, grams, and kilograms;

b) identify equivalent measurements between units within the U.S. Customary system (ounces and pounds) and between units within the metric system (grams and kilograms); and

c) estimate the conversion of ounces and grams and pounds and kilograms, using approximate comparisons (1 ounce is about 28 grams, or 1 gram is about the weight of a paper clip; 1 kilogram is a little more than 2 pounds). *

**The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U. S. Customary and metric units.*

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Students may use familiar benchmarks to make ballpark comparisons, such as – 1 ounce is about 28 grams (a slice of bread weighs about 1 ounce); – 1 nickel has the mass of about 5 grams; and – 1 kilogram is a little more than 2 pounds (a textbook has a mass of about 1 kilogram). • Students should estimate the mass and weight of everyday objects (e.g., foods, pencils, book bags, shoes), using appropriate metric or U.S. Customary units.		

4.11 The student will

a) estimate and measure length, using actual measuring devices, and describe the results in both metric and U.S. Customary units, including part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, millimeters, centimeters, and meters;

b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters); and

c) estimate the conversion of inches and centimeters, yards and meters, and miles and kilometers, using approximate comparisons (1 inch is about 2.5 centimeters; 1 meter is a little longer than 1 yard; 1 mile is slightly farther than 1.5 kilometers, or 1 kilometer is slightly farther than half a mile). *

**The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U. S. Customary and metric units.*

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Length is the distance along a line or figure from one point to another. • U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter rulers, meter sticks, and tapes. • Practical experience measuring the length of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate length. continued	**All students should** • Use benchmarks to estimate and measure length. • Understand how to convert units of length between the U.S. Customary and metric systems, using ballpark comparisons. • Understand the relationship between U.S. Customary units and the relationship between metric units.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (e.g., inch, foot, yard, millimeter, centimeter, and meter) to use when measuring everyday objects in both metric and U.S. Customary units. • Estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure. • Measure the lengths of objects in both metric and U.S. Customary units, measuring to the nearest inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), foot, yard, millimeter, centimeter, or meter, and record the length including the appropriate unit of measure (e.g., 24 inches). continued

4.11 The student will

a) estimate and measure length, using actual measuring devices, and describe the results in both metric and U.S. Customary units, including part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, millimeters, centimeters, and meters;

b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters); and

c) estimate the conversion of inches and centimeters, yards and meters, and miles and kilometers, using approximate comparisons (1 inch is about 2.5 centimeters; 1 meter is a little longer than 1 yard; 1 mile is slightly farther than 1.5 kilometers, or 1 kilometer is slightly farther than half a mile). *

**The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U. S. Customary and metric units.*

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Students may use familiar benchmarks to make ballpark comparisons, such as – 1 inch is about 2.5 centimeters (the diameter of a quarter is about 1 inch); – 1 meter is a little longer than 1 yard (a doorway is about a yard in width); and – 1 mile is slightly farther than 1.5 kilometers, or 1 kilometer is slightly farther than half a mile. • Students should estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure. • When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), inch, foot, or yard.		continued • Compare estimates of the length of objects with the actual measurement of the length of objects. • Identify equivalent measures of length between U.S. Customary measurements and between metric measurements. • Estimate conversions between the U.S. Customary and metric units, using ballpark comparisons, such as – 1 inch is about 2.5 centimeters; – 1 meter is a little longer than 1 yard; – 1 mile is slightly farther than 1.5 kilometers; and – 1 kilometer is slightly farther than half a mile.

4.12 The student will

a) estimate and measure liquid volume, using actual measuring devices and using metric and U.S. Customary units, including cups, pints, quarts, gallons, milliliters, and liters;

b) identify equivalent measurements between units within the U.S. Customary system (cups, pints, quarts, and gallons) and between units within the metric system (milliliters and liters); and

c) estimate the conversion of quarts and liters, using approximate comparisons (1 quart is a little less than 1 liter, 1 liter is a little more than 1 quart). *

**The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U. S. Customary and metric units.*

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• U.S. Customary units for measurement of liquid volume include cups, pints, quarts, and gallons. Metric units for measurement of liquid volume include milliliters and liters. • Students should measure the liquid volume of everyday objects in both metric and U.S. Customary units, including cups, pints, quarts, gallons, milliliters, and liters, and record the volume including the appropriate unit of measure (e.g., 24 gallons). • Practical experience measuring liquid volume of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate liquid volume. • Students may use familiar benchmarks to make ballpark comparisons, such as 1 quart is a little less than 1 liter, and 1 liter is a little more than 1 quart. • Students should estimate the liquid volume of containers in both metric and U.S. Customary units to the nearest cup, pint, gallon, milliliter, or liter.	**All students should** • Use benchmarks to estimate and measure volume. • Understand how to convert units of volume between the U.S. Customary and metric systems, using ballpark comparisons.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (cups, pints, quarts, gallons, milliliters, or liters) to use when measuring liquid volume in both metric and U.S. Customary units. • Estimate the liquid volume of containers in both metric and U.S. Customary units of measure to the nearest cup, pint, gallon, milliliter, or liter. • Measure the liquid volume of everyday objects in both metric and U.S. Customary units, including cups, pints, quarts, gallons, milliliters, and liters, and record the volume including the appropriate unit of measure (e.g., 24 gallons). • Identify equivalent measures of volume between U.S. Customary and metric measurements. • Estimate conversion between U.S. Customary and metric units, using ballpark comparisons, such as 1 quart is a little less than 1 liter, and 1 liter is a little more than 1 quart.

4.13 The student will

a) identify and describe situations representing the use of perimeter and area; and

b) use measuring devices to find perimeter in both standard and nonstandard units of measure.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Perimeter is the distance around a closed figure. The perimeter of any shape can be found by adding the lengths of the sides. Area is the number of square units needed to cover a surface or figure. • Students need to describe situations from their environment that represent perimeter (e.g., the distance around the edge of a picture frame; the length of fencing needed to enclose a playground). Students should practice associating the word *perimeter* with examples of the concept of perimeter. Students also need to describe situations from their environment that represent area (e.g., laying tile for the floor of a classroom). • Students should be actively involved in measuring perimeter and area in order to fully understand the concepts involved.	**All students should** • Develop strategies to estimate perimeter and area. • Select and apply appropriate tools to determine perimeter.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe situations where the perimeter of an object should be found (e.g., the distance around the edge of walls of the classroom; the length of fencing needed to enclose a playground). • Identify and describe situations in which the area should be found (e.g., laying tile for the floor of the classroom). • Measure the perimeter of an object, using nonstandard units of measure (e.g., unsharpened pencil, board eraser, toothpick, chalk, crayon, paper clip) and record the perimeter including the nonstandard unit of measure used (e.g., 24 paper clips). • Measure the perimeter of concrete objects in both metric and U.S. Customary units of measure to the nearest inch, foot, yard, millimeter, centimeter, or meter. • Determine the perimeter of an object or pictorial representation of an object and label it with the appropriate standard or nonstandard unit of measure.

The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric shapes relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among shapes.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric shapes. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare shapes will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. Discussing ideas, conjecturing, and testing hypotheses precede the development of more formal summary statements. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles. Students also explore coordinate geometry, using the coordinate plane to describe points in the first quadrant.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking Algebra.)

4.14 The student will investigate and describe the relationships between and among points, lines, line segments, and rays.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A point is an exact location in space. It has no length or width. • A line is a collection of points going on and on infinitely in both directions. It has no endpoints. • A line segment is part of a line. It has two endpoints and includes all the points between those endpoints. • A ray is part of a line. It has one endpoint and continues on and on in one direction.	**All students should** • Understand that points, lines, line segments, and rays are fundamental components of noncircular geometric figures.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Differentiate among a point, line, line segment, and ray by using the definitions to compare. • Investigate and describe the relationships between and among points, lines, line segments, and rays.

4.15 The student will

a) identify and draw representations of points, lines, line segments, rays, and angles, using a straightedge or ruler; and

b) describe the path of shortest distance between two points on a flat surface.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A point is an exact location in space. It has no length or width. A point is usually named with a capital letter. • A line is a collection of points going on and on infinitely in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. The line can also be named with a single, lower-case letter. Arrowheads must be drawn to show that the line goes on in both directions infinitely. • A line segment is part of a line. It has two endpoints and includes all the points between those endpoints. To name a line segment, name the endpoints. • A ray is part of a line. It has one endpoint and continues on and on in one direction. To name a ray, say the name of its endpoint first and then say the name of one other point on the ray. • Two rays that have the same endpoint form an angle. This endpoint is called the *vertex*. Angles are found wherever lines and line segments intersect. An angle can be named in three different ways by using – three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle.	**All students should** • Understand that points, lines, line segments, rays and angles are fundamental components of noncircular geometric figures. • Understand that the shortest distance between two points on a flat surface is a line segment.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify points, lines, line segments, rays, and angles, using their definitions. • Draw representations of lines, line segments, rays, and angles, using a straightedge, ruler, or angle ruler.

4.16 The student will identify and draw representations of lines that illustrate intersection, parallelism, and perpendicularity.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Intersecting lines are lines that cross and have one point in common. • Perpendicular lines are special intersecting lines that form right angles (square corners) where they intersect. • Parallel lines are lines that lie on the same flat surface (plane) and never cross. Parallel lines are always the same distance apart and do not share any points. • Students should explore intersection, parallelism, and perpendicularity in both two and three dimensions. For example, students should analyze the edges of a cube. Which line segments are parallel? Which are perpendicular? What plane contains the upper left edge and the lower right edge of the cube? Students can visualize this by using the classroom itself to notice the lines formed by the intersection of the ceiling and walls, of the floor and wall, and of two walls.	**All students should** • Understand that lines in a plane either intersect or are parallel. Perpendicularity is a special case of intersection. • Identify real-world situations that illustrate parallel, intersecting, and perpendicular lines.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify lines that are parallel, intersecting, or perpendicular, using their definitions. • Draw representations of intersecting, parallel, and perpendicular lines.

4.17 The student will

a) analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]);

b) identify congruent and noncongruent shapes; and

c) investigate congruence of plane figures after geometric transformations such as reflection (flip), translation (slide), and rotation (turn), using mirrors, paper folding, and tracing.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. – **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. – **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.) – **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.) continued	**All students should** • Understand that two-dimensional (plane) figures are unique in their defining properties. • Understand that three-dimensional (solid) figures are unique in their defining properties. • Understand the meaning of the term *congruent*. • Understand how to identify congruent shapes.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe the properties of squares, rectangles, triangles, parallelograms, rhombi, and circles. • Identify and describe the properties of spheres, cubes, and rectangular solids (prisms). • Identify congruent and noncongruent figures. • Analyze and compare the properties of – circles and spheres; – squares and cubes; and – rectangles and rectangular solids (prisms). • Recognize the congruence of plane figures resulting from geometric transformations such as translation, reflection, and rotation.

4.17 The student will

a) analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]);

b) identify congruent and noncongruent shapes; and

c) investigate congruence of plane figures after geometric transformations such as reflection (flip), translation (slide), and rotation (turn), using mirrors, paper folding, and tracing.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued – **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusion are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking Algebra.) • A polygon is a two-dimensional geometric figure which has straight sides and can be described as simple and closed. • A triangle is a polygon with three angles and three sides. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a rectangle with four sides of equal length. • A circle is a closed curve with all points in one plane and equidistant from a fixed point (the center). continued		

4.17 The student will

a) analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]);

b) identify congruent and noncongruent shapes; and

c) investigate congruence of plane figures after geometric transformations such as reflection (flip), translation (slide), and rotation (turn), using mirrors, paper folding, and tracing.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Congruent figures are figures having exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models. • A cube is a three-dimensional solid with six congruent square faces and every edge the same length. A cube has 6 faces and 12 edges. • A cylinder is a three-dimensional object formed by two congruent parallel circles joined by a curved surface (usually a solid or hollow object shaped like a can). • A cone is a solid, pointed figure having a flat, round base (usually a circle) that is joined to a vertex by a curved surface. • A rectangular solid (prism) is a three-dimensional figure in which all six faces are rectangles with three pair of parallel, congruent opposite faces. continued		

4.17 The student will

a) analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]);

b) identify congruent and noncongruent shapes; and

c) investigate congruence of plane figures after geometric transformations such as reflection (flip), translation (slide), and rotation (turn), using mirrors, paper folding, and tracing.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A square pyramid is a solid whose base is a square and whose faces are triangles that share a common vertex. • A sphere is a three-dimensional object with all of its points the same distance from the center. • A translation (slide) is a transformation in which an image is formed by moving every point on a figure the same distance in the same direction. • A reflection (flip) is a transformation in which a figure is flipped over a line called the *line of reflection*. All corresponding points in the image and preimage are equidistant from the line of reflection. • A rotation (turn) is a transformation in which an image is formed by turning its preimage about a point.		

4.18 The student will identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A coordinate plane is a way to locate points in a plane. To draw a coordinate plane, draw a horizontal number line and a vertical number line with the two lines intersecting in a right angle at 0 on each line. • The horizontal number line is called the *x-axis* or the *horizontal axis*. • The vertical number line is called the *y-axis* or the *vertical axis*. • Any point on a coordinate plane may be named with two numbers. These two numbers are called *coordinates*. The pair is always named in order — i.e., first x (the location along the x-axis), then y (the location along the y-axis) — and the pair is represented (x, y). • When both numbers in an ordered pair (x, y) are positive, the ordered pair is in the first quadrant. • When plotting a point, start at the origin (0, 0) and let the x-coordinate indicate how far to move horizontally. Then let the y-coordinate indicate how far to move vertically.	**All students should** • Understand how to use two numbers to name a point on a coordinate plane. • Understand that a pair of numbers on a coordinate plane corresponds to one and only one point on the grid.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Identify the ordered pair for a point in the first quadrant of a coordinate plane, given the coordinates (x, y). • Locate points in the first quadrant on a coordinate grid, given the coordinates (x, y).

Students entering grades 4 and 5 have explored the concepts of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students are able to predict whether the game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction at grades 4 and 5 is to deepen their understanding of the concepts of probability by

- developing the continuum of terms to include *impossible*, *unlikely*, *equally likely*, *possible*, and *certain*;
- offering opportunities to set up models simulating real-life events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities imbued with a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data-collection strategies. Students should continue to focus on

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to real-world experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- writing a few sentences to communicate to others their analysis and interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data-analysis methods as powerful means for decision making.

4.19 The student will

a) predict the likelihood of outcomes of a simple event, using the terms *certain, likely, unlikely, impossible*; and

b) determine the probability of a given simple event, using concrete materials.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives. • Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). continued	**All students should** • Understand and apply basic concepts of probability. • Describe events as likely or unlikely and discuss the degree of likelihood, using the terms *certain*, *likely*, *unlikely*, and *impossible*. • Predict the probability of outcomes of simple events and test the prediction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model and determine all possible outcomes of a given simple event where there are no more than 12 possible outcomes, using a variety of manipulatives, such as coins, number cubes, and spinners. • Conduct experiments to determine the probability of an event occurring for a given number of trials (no more than 12 trials), using manipulatives (e.g., the number of times "heads" occurs when flipping a coin 10 times; the chance that when the names of 12 classmates are put in a shoebox, a name that begins with *D* will be drawn). • Write the probability of a given simple event as a fraction, where the total number of possible outcomes is 12 or fewer. • Identify the likelihood of an event occurring and relate it to its fractional representation (e.g., impossible/0; equally likely/$\frac{1}{2}$; certain/1).

4.19 The student will

a) predict the likelihood of outcomes of a simple event, using the terms *certain, likely, unlikely, impossible*; and

b) determine the probability of a given simple event, using concrete materials.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as unlikely*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include real-life examples. • For any event such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up.		

4.20 The student will collect, organize, and display data in line and bar graphs with scale increments of one or greater than one and use the display to interpret the results, draw conclusions, and make predictions.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data analysis helps describe data, recognize patterns or trends, and make predictions. • Investigations involving real-world data should occur frequently, and data can be collected through brief class surveys or through more extended projects taking many days. • Students formulate questions, predict answers to questions under investigation, collect and represent initial data, and consider whether the data answer the questions. • Line graphs are used to show how one variable changes over time. By looking at a single-line graph, it can be determined whether the variable is increasing, decreasing, or staying the same over time. – The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as "continuous data," as it represents data collected over a continuous period of time. continued	**All students should** • Understand the difference between representing categorical data and representing numerical data. • Understand that line graphs show change over time (numerical data). • Understand that bar graphs should be used to compare counts of different categories (categorical data). • Understand how data displayed in bar and line graphs can be interpreted so that informed decisions can be made.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect data, using, for example, observations, measurement, surveys, scientific experiments, polls, or questionnaires. • Organize data into a chart or table. • Construct and display data in bar graphs, labeling one axis with equal whole-number increments of 1 or more (numerical data) (e.g., multiples of 5, 10, or 100) and the other axis with categories related to the title of the graph (categorical data) (e.g., swimming, fishing, boating, and water skiing as the categories of "Favorite Summer Sports"). • Construct and display data in line graphs, labeling the vertical axis with equal whole-number increments of 1 or more and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than four identified points along a continuum for continuous data. For example, growth charts showing age versus height place age on the horizontal axis (e.g., 1 month, 2 months, 3 months, and 4 months). • Title the given graph or identify the title in a given graph and label the axes. continued

4.20 The student will collect, organize, and display data in line and bar graphs with scale increments of one or greater than one and use the display to interpret the results, draw conclusions, and make predictions.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued – The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals, depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data. – Each axis should be labeled, and the graph should be given a title. – A line graph tells whether something has increased, decreased, or stayed the same with the passage of time. Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). • Bar graphs should be used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs. – A bar graph uses parallel horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. continued		continued • Analyze information from simple line and bar graphs by describing the characteristics of the data and the data as a whole (e.g., the category with the greatest/least, categories with the same number of responses, similarities and differences, the total number). Data points will be limited to 20 and categories to 4. • Interpret the data to answer the question posed, and compare the answer to the prediction (e.g., "The summer sport preferred by most is swimming, which is what I predicted before collecting the data."). • Write at least one sentence to describe the analysis and interpretation of the data, identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same. • Select from among four choices a correct analysis of the data presented in a bar or line graph. For example, given a line graph showing the number of soccer players (in millions) in the U.S. over the time period 1980 to 2000 in five-year intervals, select the correct answer response that relates to the graphs, such as, "The greatest increase in number of soccer players occurred between 1985 and 1990."

4.20 The student will collect, organize, and display data in line and bar graphs with scale increments of one or greater than one and use the display to interpret the results, draw conclusions, and make predictions.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued – There is space before, between, and after the bars. – The axis that displays the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Fourth grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, or 10. – Each axis should be labeled, and the graph should be given a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written.		

Students entering grades 4 and 5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships in open number sentences, and they have begun to solve simple equations with one unknown.

The focus of instruction is to help students develop a solid use of patterning as a problem-solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, graphic, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They analyze the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By analyzing mathematical situations and models, students begin to represent these, using symbols and variables to write "rules" for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

4.21 The student will recognize, create, and extend numerical and geometric patterns, using concrete materials, number lines, symbols, tables, and words.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Most patterning activities should involve some form of concrete materials to make up a pattern. • Students will identify and extend a wide variety of patterns, including rhythmic, geometric, graphic, numerical, and algebraic. The patterns will include both growing and repeating patterns. • Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing the relationship symbolically or algebraically. • Tables of values should be analyzed for a pattern to determine what element comes next.	**All students should** • Understand that patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe geometric and numerical patterns, using tables, symbols, or words. • Create geometric and numerical patterns, using concrete materials, number lines, tables, and words. • Extend geometric and numerical patterns, using concrete materials, number lines, tables, and words.

4.22 The student will recognize and demonstrate the meaning of equality, using symbols representing numbers, operations, and relations [e.g., 3 + 5 = 5 + 3 and 15 + (35 + 16) = (15 + 35) + 16].

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. • The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2). • The associative property for addition states that the sum stays the same when the grouping of addends is changed [e.g., 15 + (35 + 16) = (15 + 35) + 16]. The associative property for multiplication states that the product stays the same when the grouping of factors is changed [e.g., 6 × (3 × 5) = (6 × 3) × 5].	**All students should** • Understand that mathematical relationships can be expressed using equations. • Understand that quantities on both sides of an equation must be equal.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize that the equals sign (=) relates equivalent quantities. • Write number sentences to represent equivalent mathematical relationships (e.g., 4 × 3 = 2 × 6). • Identify number sentences that show appropriate use of the equals sign.