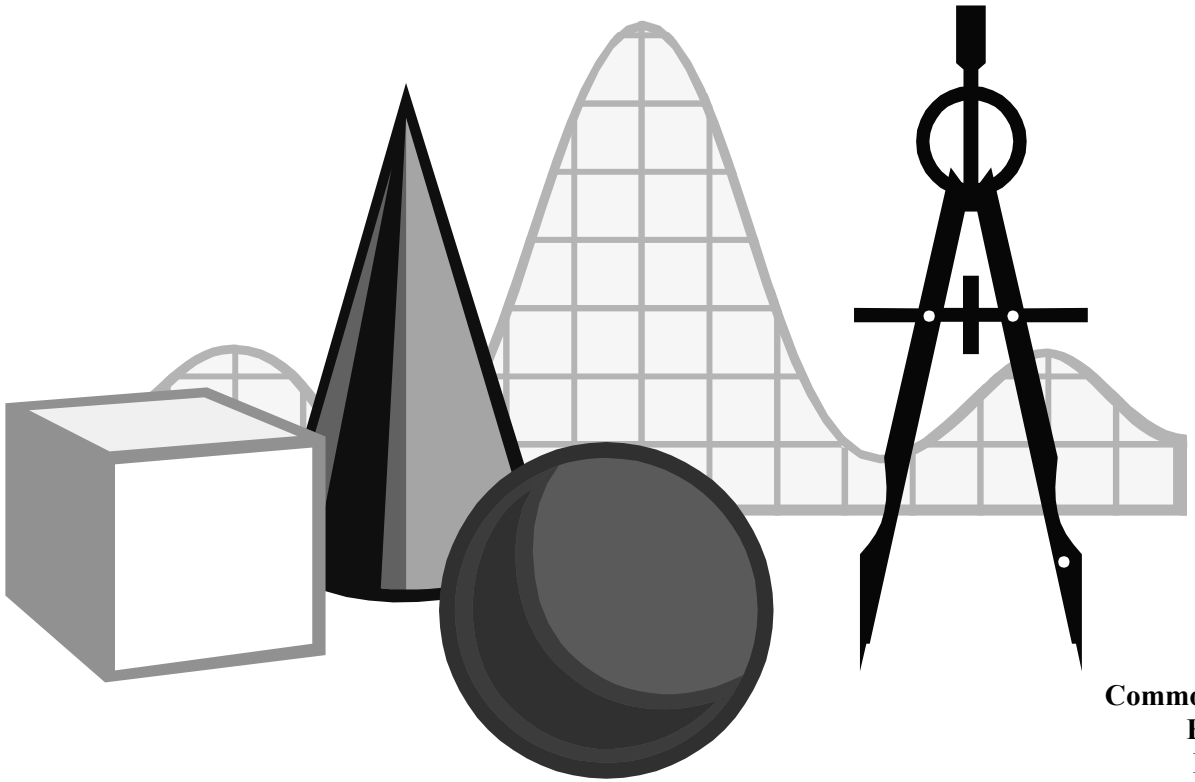
MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 5



Commonwealth of Virginia
Board of Education
Richmond, Virginia

Virginia Department of Education
P.O. Box 2120
Richmond, Virginia 23218-2120
http://www.pen.k12.va.us

Superintendent of Public Instruction
Jo Lynne DeMary

Deputy Superintendent
M. Kenneth Magill

Assistant Superintendent for Instruction
Patricia I. Wright

Office of Elementary Instructional Services
James S. Heywood, Director
Karen Grass, Mathematics Specialist

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Mathematics instruction in grades 4 and 5 should continue to foster the development of number sense, especially with decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction at grades 4 and 5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names). Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve problems.

5.1 The student will

a) read, write, and identify the place values of decimals through thousandths;

b) round decimal numbers to the nearest tenth or hundredth; and

c) compare the values of two decimals through thousandths, using the symbols >, <, or =.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the base-10 number system is based upon a simple pattern of tens in which each place is ten times the value of the place to its right. This is known as a ten-to-one place-value relationship. • A decimal point separates the whole number places from the places less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*. • To read decimals, – read the whole number to the left of the decimal point, if there is one; – read the decimal point as "and"; – read the digits to the right of the decimal point just as you would read a whole number; and – say the name of the place value of the digit in the smallest place. • Decimals may be written in a variety of forms: – Standard: 23.456 – Written: Twenty-three and four hundred fifty-six thousandths – Expanded: $(2 \times 10) + (3 \times 1) + (4 \times 0.1) + (5 \times 0.01) + (6 \times 0.001)$ continued	**All students should** • Understand the place-value structure of decimals and use this structure to read, write, and compare decimals. • Understand that decimals are rounded in a way that is similar to the way whole numbers are rounded. • Understand that decimal numbers can be rounded to an estimate when exact numbers are not needed for the situation at hand.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the place values for each digit in decimals through thousandths. • Read decimal numbers through thousandths from written words or place-value format. • Write decimal numbers through thousandths from written words or from decimal numbers presented orally. • Round decimal numbers to the nearest tenth or hundredth. • Identify the symbols for the terms *greater than*, *less than*, and *equal to*. • Compare the value of two decimal numbers through thousandths, using the symbols >, <, or =.

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a) read, write, and identify the place values of decimals through thousandths;

b) round decimal numbers to the nearest tenth or hundredth; and

c) compare the values of two decimals through thousandths, using the symbols >, <, or =.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • To help students identify the ten-to-one place-value relationship for decimals through thousandths, use base-10 manipulatives, such as place-value mats/charts, decimal squares, base-10 blocks, and money. • To help students compare the value of two decimals through thousandths, use manipulatives, such as place-value mats/charts, 10-by-10 grids, decimal squares, base-10 blocks, and money. • Decimals can be rounded to the nearest tenth or hundredth in situations when exact numbers are not needed. • A strategy for rounding decimal numbers to the nearest tenth and hundredth is as follows: – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero. continued		

5.1 The student will

a) read, write, and identify the place values of decimals through thousandths;

b) round decimal numbers to the nearest tenth or hundredth; and

c) compare the values of two decimals through thousandths, using the symbols >, <, or =.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A procedure for comparing two decimals by examining place value may include the following: – Line up the decimal numbers at their decimal points. – Beginning at the left, find the first place value where the digits are different. – Compare the digits in this place value to determine which number is greater (or which is less). – Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented. – If both numbers are the same, use the symbol = or words *equal to*. Two numbers can be compared by examining place value and/or using a number line.		

5.2 The student will

a) recognize and name commonly used fractions (halves, fourths, fifths, eighths, and tenths) in their equivalent decimal form and vice versa; and

b) order a given set of fractions and decimals from least to greatest. Fractions will include like and unlike denominators limited to 12 or less, and mixed numbers.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should focus on finding equivalent decimals of familiar fractions such as halves, fourths, fifths, eighths, and tenths. • Decimals and fractions represent the same relationships; however, they are presented in two different formats. Decimal numbers are another way of writing fractions. Base-10 models (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money) concretely relate fractions to decimals.	**All students should** • Understand the relationship between commonly used fractions and their decimal form. • Understand that fractions and decimals can be ordered from least to greatest.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent fractions (halves, fourths, fifths, eighths, and tenths) in their equivalent decimal form. • Represent decimals in their equivalent fraction form (halves, fourths, fifths, eighths, and tenths). • Determine equivalent relationships between decimals and fractions with denominators up to 12. • Order from least to greatest a given set of no more than five numbers written as decimals and as fractions and mixed numbers with denominators of 12 or less.

Computation and estimation in grades 4 and 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to working with fractions and decimals. Instruction should focus on computation activities that enable students to model, explain, and develop reasonable proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract with common fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability not only to identify but to use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use (not identify) properties of operations to solve problems [e.g., 7×28 is equivalent to $(7 \times 20) + (7 \times 8)$, or $(7 \times 30) - (7 \times 2)$].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their computations. Additionally, students should enhance their ability to select an appropriate problem-solving method from among estimation, mental math, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

5.3 The student will create and solve problems involving addition, subtraction, multiplication, and division of whole numbers, using paper and pencil, estimation, mental computation, and calculators.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An example of an approach to solving problems is Polya's four-step plan: – Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new. – Plan: Decide what operation(s) to use and what sequence of steps to use to solve the problem. – Solve: Follow the plan and work accurately. If the first attempt doesn't work, try another plan. – Look back: Does the answer make sense? • Estimation gives a rough idea of an amount. Strategies such as front-end, rounding, and mental computation may be used to estimate addition, subtraction, multiplication, and division of whole numbers. • Examples of problems to be solved by using estimation strategies are encountered in shopping for groceries, buying school supplies, budgeting allowance, and sharing the cost of a pizza or the prize money from a contest. • Estimation can be used to check the reasonableness of the results.	**All students should** • Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with whole numbers. • Understand the meaning of mathematical operations and how these operations relate to one another when creating and solving word problems.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Create problems involving the operations of addition, subtraction, multiplication, and/or division of whole numbers, using real-life situations. • Estimate the sum, difference, product, and quotient of whole-number computations. • Solve problems involving addition, subtraction, multiplication, and division of whole numbers, using paper and pencil, mental computation, and calculators, in which – sums, differences, and products will not exceed five digits; – multipliers will not exceed two digits; – divisors will not exceed two digits; or – dividends will not exceed four digits.

5.4 The student will find the sum, difference, and product of two numbers expressed as decimals through thousandths, using an appropriate method of calculation, including paper and pencil, estimation, mental computation, and calculators.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition and subtraction with decimals may be investigated using a variety of models (e.g., 10-by-10 grids, number lines, money). • Decimal computation uses similar procedures as those developed for whole-number computation and applies them to decimal place values, giving careful attention to the placement of the decimal point in the solution. • Multiplication of decimals follows the same procedure as multiplication of whole numbers. The only difference is that a decimal point must be correctly placed in the product. • The product of decimals is dependent upon the two factors being multiplied. Factors = Products tenths × tenths = hundredths tenths × hundredths = thousandths hundredths × hundredths = ten-thousandths tenths × thousandths = ten-thousandths • In cases where an exact product is not required, the product of decimals can be estimated using strategies for multiplying whole numbers, such as front-end and compatible numbers, or rounding. In each case, the student needs to determine where to place the decimal point to ensure that the product is reasonable.	**All students should** • Use similar procedures as those developed for whole-number computation and apply them to decimal place values, giving careful attention to the placement of the decimal point in the solution. • Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with decimal numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate method of calculation to find the sum, difference, and product of two numbers expressed as decimals through thousandths, selecting from among paper and pencil, estimation, mental computation, and calculators. • Estimate the sum, difference, and product of two numbers expressed as decimals through thousandths. • Find the sum, difference, and product of two numbers expressed as decimals through thousandths, using paper and pencil. • Find the sum, difference, and product of two numbers expressed as decimals through thousandths, using mental computation. • Find the sum, difference, and product of two numbers expressed as decimals through thousandths, using calculators. • Use estimation to check the reasonableness of a sum, difference, and product.

5.5 The student, given a dividend of four digits or fewer and a divisor of two digits or fewer, will find the quotient and remainder.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Division is the operation of making equal groups or shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. Both situations may be modeled with base-10 manipulatives. • Division is the inverse of multiplication; therefore, multiplication and division are inverse operations. • Terms used in division are *dividend, divisor*, and *quotient.* dividend ÷ divisor = quotient quotient divisor)dividend • There are a variety of algorithms for division such as repeated multiplication and subtraction. Experience with these algorithms may enhance understanding of the traditional long-division algorithm.	**All students should** • Understand the various meanings of *division* and its effect on whole numbers. • Understand various representations of division, i.e., dividend ÷ divisor = quotient quotient divisor)dividend $\frac{\text{dividend}}{\text{divisor}}$ = quotient.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate the quotient of two whole numbers when given a dividend of four digits or fewer and a divisor of two digits or fewer. • Determine the quotient with no remainder of two whole numbers when given a dividend of four digits or fewer and a divisor of two digits or fewer. • Determine the quotient and remainder of two whole numbers when given a dividend of four digits or fewer and a divisor of two digits or fewer. • Use estimation to check the reasonableness of a quotient.

5.6 The student, given a dividend expressed as a decimal through thousandths and a single-digit divisor, will find the quotient.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Division of decimals is performed the same way as division of whole numbers. The only difference is the placement of the decimal point in the quotient. • Division is the operation of making equal groups or shares. • The fair-share concept of decimal division can be modeled, using manipulatives (e.g., base-10 blocks). • The quotient can be estimated, given a dividend expressed as a decimal through thousandths (and no annexing of zeros during the division process) and a single-digit divisor. • Estimation can be used to check the reasonableness of a quotient.	**All students should** • Use the procedures developed for whole-number division and apply these procedures to decimal place values, giving careful attention to the placement of the decimal point in the solution.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the quotient, given a dividend expressed as a decimal through thousandths (and no annexing of zeros during the division process) and a single-digit divisor. All dividends should be evenly divisible by the divisor.

5.7 The student will add and subtract with fractions and mixed numbers, with and without regrouping, and express answers in simplest form. Problems will include like and unlike denominators limited to 12 or less.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. • A fraction is in simplest form when its numerator and denominator have no common factors other than 1. • Fractions having like denominators means the same as fractions having common denominators. • Equivalent fractions name the same amount. To find equivalent fractions, multiply or divide the numerator and denominator by the same non-zero number. • Addition and subtraction with fractions and mixed numbers can be modeled using a variety of concrete materials and pictorial representations as well as paper and pencil. • To add, subtract, and compare fractions and mixed numbers, it often helps to find the least common denominator. The least common denominator (LCD) of two or more fractions is the least common multiple (LCM) of the denominators. • To add or subtract with fractions having the same or like denominators, add or subtract the numerators and write in simplest form. continued	**All students should** • Develop and use strategies to estimate and compute addition and subtraction of fractions. • Understand the concept of least common multiple and least common denominator.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Add and subtract fractions having like and unlike denominators. Denominators should be limited to 12 or less, and answers should be expressed in simplest form. • Add and subtract with mixed numbers having like and unlike denominators, with and without regrouping. Denominators should be limited to 12 or less, and answers should be expressed in simplest form. • Use estimation to check the reasonableness of a sum or difference.

5.7 The student will add and subtract with fractions and mixed numbers, with and without regrouping, and express answers in simplest form. Problems will include like and unlike denominators limited to 12 or less.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • To add or subtract with fractions that do not have the same denominator, first find equivalent fractions with the least common denominator. Then add or subtract and write the answer in simplest form. • A mixed number has two parts: a whole number and a fraction. • To add or subtract with mixed numbers, students may use a number line, draw a picture, rewrite fractions with like denominators, or rewrite mixed numbers as fractions.		

Students in grades 4 and 5 should be actively involved in measurement activities that require a dynamic interaction among students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects. The intent is for students to make "ballpark" comparisons and *not* to memorize conversion factors between U.S. Customary and metric units. To fully understand these ballpark comparisons, students must be actively engaged in the process of measurement.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure perimeter, weight/mass, liquid volume/capacity, area, temperature, and time. Students continue to enhance their understanding of measurement by using appropriate tools such as rulers, balances, clocks, and thermometers. The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.

5.8 The student will describe and determine the perimeter of a polygon and the area of a square, rectangle, and right triangle, given the appropriate measures.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Perimeter is the distance around a figure. • To find the perimeter of any polygon, add the lengths of the sides. • Area is the number of square units needed to cover a surface or figure. • Students should investigate the area of a square, rectangle, and triangle by using manipulatives (e.g., tiles, geoboards, graph paper). • Students should investigate, using manipulatives, to discover the formulas for the area of a square, rectangle, and triangle: – Area of a rectangle = Length × Width – Area of a square = Side × Side – Area of a triangle = $\frac{1}{2}$ Base × Height • Students should label the perimeter or area with the appropriate unit of linear or square measure.	**All students should** • Understand the concepts of perimeter and area. • Understand and use appropriate units of measure for perimeter and area.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the perimeter of a polygon, with or without diagrams, when – the lengths of all sides of a polygon that is not a rectangle or a square are given; – the length and width of a rectangle are given; or – the length of a side of a square is given. • Determine the area of a square, with or without diagrams, when the length of a side is given. • Determine the area of a rectangle, with or without diagrams, when the length and width are given. • Determine the area of a right triangle, with or without diagrams, when the base and the height are given. • Determine the perimeter of a polygon and area of a square, rectangle, and triangle, following the parameters listed above, using only whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 24 square inches).

5.9 The student will identify and describe the diameter, radius, chord, and circumference of a circle.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A circle is a set of points on a flat surface (plane) with every point equidistant from a given point called the *center*. • A chord is a line segment connecting any two points on a circle. • A diameter is a chord that goes through the center of a circle. • A radius is a segment from the center of a circle to any point on the circle. Two radii end-to-end form a diameter of a circle. • Circumference is the distance around or perimeter of a circle.	**All students should** • Identify the parts of a circle. • Understand that the circumference is the distance around the circle. • Understand the relationship between the measures of diameter and radius and the relationship between the measures of radius and circumference.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the diameter, radius, chord, and circumference of a circle. • Describe the relationship between – diameter and radius; and – radius and circumference. • Identify the diameter, radius, chord, and circumference of a given circle.

5.10 The student will differentiate between perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Perimeter is the distance around an object. It is a measure of length. Area is the number of square units needed to cover a surface. Volume is a measure of capacity.	**All students should** • Understand the difference between using perimeter, area, and volume in a given situation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Differentiate between the concepts of area, perimeter, and volume. • Describe real-life situations where area, perimeter, and volume are appropriate measures to use, and justify their choices orally or in writing. • Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.

5.11 The student will choose an appropriate measuring device and unit of measure to solve problems involving measurement of

a) length — part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;
b) weight/mass — ounces, pounds, tons, grams, and kilograms;
c) liquid volume — cups, pints, quarts, gallons, milliliters, and liters;
d) area — square units; and
e) temperature — Celsius and Fahrenheit units.

Problems also will include estimating the conversion of Celsius and Fahrenheit units relative to familiar situations (water freezes at 0°C and 32°F, water boils at 100°C and 212°F, normal body temperature is about 37°C and 98.6°F).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Length is the distance along a line or figure from one point to another. • U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter rulers, meter sticks, and tapes. • When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$), foot, yard, or mile. • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight that an object changes is dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). continued	**All students should** • Understand how to select a measuring device and unit of measure to solve problems involving measurement.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve problems involving measurement by selecting an appropriate measuring device and a U.S. Customary or metric unit of measure for the following: – length: part of an inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers; – weight: ounces, pounds, and tons; – mass: grams and kilograms; – liquid volume: cups, pints, quarts, gallons, milliliters, and liters; – area: square units; and – temperature: Celsius and Fahrenheit units. • Estimate the conversion of Celsius and Fahrenheit units relative to familiar situations: – Water freezes at 0°C and 32°F. – Water boils at 100°C and 212°F. – Normal body temperature is about 37°C and 98.6°F.

5.11 The student will choose an appropriate measuring device and unit of measure to solve problems involving measurement of

a) length — part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;

b) weight/mass — ounces, pounds, tons, grams, and kilograms;

c) liquid volume — cups, pints, quarts, gallons, milliliters, and liters;

d) area — square units; and

e) temperature — Celsius and Fahrenheit units.

Problems also will include estimating the conversion of Celsius and Fahrenheit units relative to familiar situations (water freezes at 0°C and 32°F, water boils at 100°C and 212°F, normal body temperature is about 37°C and 98.6°F).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Appropriate measuring devices to measure mass in U.S. Customary units (ounces, pounds) and metric units (grams, kilograms) are balances. • U.S. Customary units to measure liquid volume (capacity) include cups, pints, quarts, and gallons. Metric units to measure liquid volume (capacity) include milliliters and liters. • Area is the number of square units needed to cover a surface or figure. • Temperature is measured using a thermometer. The U.S. Customary unit of measure is degrees Fahrenheit; the metric unit of measure is degrees Celsius. • Practical experience measuring familiar objects helps students establish benchmarks and facilitates students' ability to use the units of measure to make estimates.		

5.12 The student will determine an amount of elapsed time in hours and minutes within a 24-hour period.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Elapsed time is the amount of time that has passed between two given times. • Elapsed time can be found by counting on from the beginning time to the finishing time. – Count the number of whole hours between the beginning time and the finishing time. – Count the remaining minutes. – Add the hours and minutes. For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count on as follows: – from 10:15a.m. to 1:15 p.m., count 3 hours; – from 1:15 p.m. to 1:25 p.m., count 10 minutes; and then – add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.	**All students should** • Understand the "counting on" strategy for determining elapsed time in hours and minutes.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine elapsed time in hours and minutes within a 24-hour period.

5.13 The student will measure and draw right, acute, and obtuse angles and triangles, using appropriate tools.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Angles are measured in degrees. A degree is $\frac{1}{360}$ of a complete rotation. • To measure the number of degrees in an angle, use a protractor or an angle ruler. • A right angle measures exactly 90°. • An acute angle measures less than 90°. • An obtuse angle measures greater than 90° but less than 180°. • Before measuring an angle, students should first compare it to a right angle to determine whether the measure of the angle is less than or greater than 90°. • Students should understand how to work with a protractor or angle ruler as well as available computer software to measure and draw angles and triangles. • A right triangle has one right angle. • An obtuse triangle has one obtuse angle and two acute angles. • An acute triangle has three acute angles.	**All students should** • Understand how to measure and draw acute, right, and obtuse angles. • Understand how to identify a triangle as either acute, right, or obtuse.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the appropriate tools (e.g., protractor and straightedge or angle ruler as well as available software) used to measure and draw angles and triangles. • Draw right, acute, and obtuse angles, using appropriate tools. • Measure right, acute, and obtuse angles, using appropriate tools, and identify their measures in degrees. • Measure the angles of right, acute, and obtuse triangles, using appropriate tools, and identify their measures in degrees.

The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric shapes relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among shapes.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric shapes. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare shapes will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. Discussing ideas, conjecturing, and testing hypotheses precede the development of more formal summary statements. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles. Students also explore coordinate geometry, using the coordinate plane to describe points in the first quadrant.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking Algebra.)

5.14 The student will classify angles and triangles as right, acute, or obtuse.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A right angle is an angle that forms a square corner. A right angle measures exactly 90°. • An acute angle forms an angle less than a right angle. An acute angle measures greater than 0° but less than 90°. • An obtuse angle forms an angle greater than a right angle. An obtuse angle measures greater than 90° but less than 180°. • A right triangle has one right angle. • An obtuse triangle has one obtuse angle. • An acute triangle has three acute angles (or no angle measuring 90° or greater). • To facilitate the exploration of relationships, ask students whether a right triangle can have an obtuse angle. Why or why not? Can an obtuse triangle have more than one obtuse angle? Why or why not? What type of angles are the two angles other than the right angle in a right triangle? What type of angles are the two angles other than the obtuse angle in an obtuse triangle?	**All students should** • Understand that triangles can be classified by the measures of their angles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Classify angles as right, acute, and obtuse. • Classify triangles as right, acute, and obtuse.

5.15 The student, using two-dimensional (plane) figures (square, rectangle, triangle, parallelogram, rhombus, kite, and trapezoid) will

a) recognize, identify, describe, and analyze their properties in order to develop definitions of these figures;
b) identify and explore congruent, non-congruent, and similar figures;
c) investigate and describe the results of combining and subdividing shapes;
d) identify and describe a line of symmetry; and
e) recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A triangle is a polygon with three sides. Triangles may be classified according to the measure of their angles, i.e., right, acute, or obtuse. Triangles may also be classified according to the measure of their sides, i.e., scalene (no sides congruent), isosceles (at least two sides congruent) and equilateral (all sides congruent). • A quadrilateral is a polygon with four sides. • A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. Properties of a parallelogram include the following: – A diagonal (a segment that connects two vertices of a polygon but is not a side) divides the parallelogram into two congruent triangles. – The opposite sides of a parallelogram are congruent. – The opposite angles of a parallelogram are congruent. – The diagonals of a parallelogram bisect each other. continued	**All students should** • Understand that the defining properties and symmetry of various plane figures are unique. • Understand that simple plane figures can be combined to make more complicated figures and that complicated figures can be subdivided into simple plane figures.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Recognize and identify the properties of squares, rectangles, triangles, parallelograms, rhombi, kites and trapezoids. • Describe the properties of squares, rectangles, triangles, parallelograms, rhombi, kites and trapezoids. • Analyze the properties of squares, rectangles, triangles, parallelograms, rhombi, kites and trapezoids. • Identify congruent, non-congruent, and similar figures. • Describe the results of combining and subdividing shapes. • Identify and describe a line of symmetry. • Recognize the images of figures resulting from geometric transformations such as translation, reflection, or rotation.

5.15 The student, using two-dimensional (plane) figures (square, rectangle, triangle, parallelogram, rhombus, kite, and trapezoid) will

a) recognize, identify, describe, and analyze their properties in order to develop definitions of these figures;
b) identify and explore congruent, non-congruent, and similar figures;
c) investigate and describe the results of combining and subdividing shapes;
d) identify and describe a line of symmetry; and
e) recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A rectangle is a parallelogram with four right angles. Since a rectangle is a parallelogram, a rectangle has the same properties as those of a parallelogram. • A square is a rectangle with four congruent sides. Since a square is a rectangle, a square has all the properties of a rectangle and of a parallelogram. • A rhombus is a parallelogram with four congruent sides. Opposite angles of a rhombus are congruent. Since a rhombus is a parallelogram, the rhombus has all the properties of a parallelogram. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases,* and the non-parallel sides are called *legs*. If the legs have the same length, then the trapezoid is an isosceles trapezoid. • A kite is a quadrilateral with two distinct pairs of adjacent congruent sides. continued		

5.15 The student, using two-dimensional (plane) figures (square, rectangle, triangle, parallelogram, rhombus, kite, and trapezoid) will

a) recognize, identify, describe, and analyze their properties in order to develop definitions of these figures;
b) identify and explore congruent, non-congruent, and similar figures;
c) investigate and describe the results of combining and subdividing shapes;
d) identify and describe a line of symmetry; and
e) recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A line of symmetry is a line that divides a figure into congruent halves, each of which is the reflected image of the other. • A translation (slide) is a transformation in which an image is formed by moving every point on a figure the same distance in the same direction. • A reflection (flip) is a transformation in which a figure is flipped over a line, called the *line of reflection*. All corresponding points in the image and preimage are equidistant from the line of reflection. • A rotation (turn) is a transformation in which an image is formed by turning its preimage about a point. • Two figures are said to be congruent if they have exactly the same size and shape. • Two figures are said to be similar if they have exactly the same shape but not necessarily the same size. • Two figures can be combined to form a new shape. Students should be able to identify the figures that have been combined. • A polygon may be subdivided into two or more figures. Students should understand how to divide a polygon into familiar figures.		

5.16 The student will identify, compare, and analyze properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Three-dimensional figures are called *solid figures* or simply *solids*. Solids enclose a region of space. Solids are classified by the types of surfaces they have. These surfaces may be flat, curved, or both. • A cylinder is a solid bounded by two congruent, parallel circular bases joined by a curved surface whose cross-section perpendicular to the axis is always a circle congruent to the bases. • A cone is a solid bounded by a circular base and a curved surface with one vertex. • A cube is a solid with six congruent square faces and every edge the same length. A cube has 6 faces and 12 edges. • A square pyramid is a solid whose base is a square and whose other faces are triangles that share a common vertex. • A rectangular prism is a solid in which all six faces are rectangles with three pairs of parallel, congruent, opposite faces.	**All students should** • Understand that solid figures are unique in their defining properties.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism). • Analyze and compare properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism).

Students entering grades 4 and 5 have explored the concepts of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students are able to predict whether the game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction at grades 4 and 5 is to deepen their understanding of the concepts of probability by

- developing the continuum of terms to include *impossible*, *unlikely*, *equally likely, possible*, and *certain*;
- offering opportunities to set up models simulating real-life events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities imbued with a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data-collection strategies. Students should continue to focus on

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to real-world experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions ("How many?" "How much?") from the data displays;
- identifying and justifying comparisons ("Which is the most? Which is the least?" "Which is the same? Which is different?") about the information;
- comparing their initial predictions to the actual results; and
- writing a few sentences to communicate to others their analysis and interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data-analysis methods as powerful means for decision making.

5.17 The student will

a) solve problems involving the probability of a single event by using tree diagrams or by constructing a sample space representing all possible results;

b) predict the probability of outcomes of simple experiments, representing it with fractions or decimals from 0 to 1, and test the prediction; and

c) create a problem statement involving probability based on information from a given problem situation. Students will not be required to solve the created problem statement.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g. the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as unlikely*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include real-life examples. continued	**All students should** • Understand and apply basic concepts of probability to make predictions of outcomes of simple experiments. • Understand that a sample space represents all possible outcomes of an experiment. • Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a sample space, using a tree diagram to identify all possible outcomes of a single event. • Construct a sample space, using a list or chart to represent all possible outcomes of a single event. • Determine the probability of a single event when the total number of possible outcomes is 12 or less. • Determine the outcome of an event that is least likely to occur (0) or most likely to occur (1) when the number of possible outcomes is 12 or less. • Create a problem statement involving probability based on information from a given problem situation. Students will not be expected to solve the problem.

5.17 The student will

a) solve problems involving the probability of a single event by using tree diagrams or by constructing a sample space representing all possible results;

b) predict the probability of outcomes of simple experiments, representing it with fractions or decimals from 0 to 1, and test the prediction; and

c) create a problem statement involving probability based on information from a given problem situation. Students will not be required to solve the created problem statement.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • For any event such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. • A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, chart, or tree diagram. • Tree diagrams are drawn to show all of the possible combinations (outcomes) in a sample space. The counting principle tells how to find the number of outcomes when there is more than one way to put things together. For example, how many different outfit combinations can you make from 2 shirts (red and blue) and 3 pants (black, white, khaki)? The sample space displayed in a tree diagram would show that there are $2 \times 3 = 6$ (counting principle) outfit combinations: red-black; red-white; red-khaki; blue-black; blue-white; blue-khaki). • A spinner with eight equal-size sections is equally likely to land on any one of the sections, three of which are red, three green, and two yellow. Have students write a problem statement involving probability, such as, "What is the probability that the spinner will land on green?"		

5.18 The student will, given a problem situation, collect, organize, and display a set of numerical data in a variety of forms, using bar graphs, stem-and-leaf plots, and line graphs, to draw conclusions and make predictions.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The emphasis in all work with statistics should be on the analysis and the communication of the analysis, rather than on a single correct answer. Data analysis should include opportunities to describe the data, recognize patterns or trends, and make predictions. • Statistical investigations should be active, with students formulating questions about something in their environment and finding quantitative ways to answer the questions. • Investigations can be brief class surveys or more extended projects taking many days. • Through experiences displaying data in a variety of graphical representations, students learn to select an appropriate representation. • Bar graphs are used to compare counts of different categories (categorical data). – A bar graph uses either horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. continued	**All students should** • Understand that bar graphs compare categorical data, stem-and-leaf plots list data in a meaningful array, and line graphs show changes over time. • Understand how to propose and justify conclusions and predictions that are based on displays of data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect data, using observations (e.g., weather), measurement (e.g., shoe sizes), surveys (e.g., favorite television shows), or experiments (e.g., plant growth). • Organize the data into a chart or table. • Construct bar graphs, labeling one axis with equal whole-number or decimal increments and the other axis with attributes of the topic (categorical data) (e.g., skiing, basketball, ice hockey, skating, and sledding as the categories of "Favorite Winter Sports"). Bar graphs will have no more than six categories. • Display data in line graphs, bar graphs, and stem-and-leaf plots. • Construct line graphs, labeling the vertical axis with equal whole-number, decimal, or fractional increments and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than six identified points along a continuum for continuous data (e.g., the decades: 1950s, 1960s, 1970s, 1980s, 1990s, and 2000s). • Construct a stem-and-leaf plot to organize and display data, where the stem is listed in ascending order and the leaves are in ascending order, with or without commas between leaves. continued

5.18 The student will, given a problem situation, collect, organize, and display a set of numerical data in a variety of forms, using bar graphs, stem-and-leaf plots, and line graphs, to draw conclusions and make predictions.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued – The axis displaying the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Fifth-grade students should collect data that is recorded in increments of multiples of whole numbers, decimals, and fractions. – Each axis should be labeled, and the graph should have a title. – Statements representing an analysis and interpretation of the characteristics of the data (e.g., similarities and differences, mode, and least and greatest) in the graph should be included. • Line graphs are used to show how one variable changes over time. By looking at a single-line graph, it can be determined whether the variable is increasing, decreasing, or staying the same over time. – The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as "continuous data" because it represents data collected over a continuous period of time. continued		continued • Title the given graph, or identify the title. • Interpret the data to compare the answer to the prediction. • Write a few sentences to describe the interpretation of the data.

5.18 The student will, given a problem situation, collect, organize, and display a set of numerical data in a variety of forms, using bar graphs, stem-and-leaf plots, and line graphs, to draw conclusions and make predictions.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued – The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data. – Each axis should be labeled and the graph should have a title. – A line graph tells whether something has increased, decreased, or stayed the same with the passage of time. Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). • Stem-and-leaf plots allow the exact values of data to be listed in a meaningful array. Data covering a range of 25 numbers are best displayed in a stem-and-leaf plot and are utilized to organize numerical data from least to greatest, using the digits of the greatest place value to group data. – The data is organized from least to greatest. – Each value should be separated into a stem and a leaf [e.g., two-digit numbers are separated into stems (tens) and leaves (ones)]. continued		

5.18 The student will, given a problem situation, collect, organize, and display a set of numerical data in a variety of forms, using bar graphs, stem-and-leaf plots, and line graphs, to draw conclusions and make predictions.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued – The stems are listed vertically from least to greatest with a line to their right. The leaves are listed horizontally, also from least to greatest, and can be separated by spaces or commas. Every value is recorded, regardless of the number of repeats. – A key is often included to explain how to read the plot.		

5.19 The student will find the mean, median, mode, and range of a set of data.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The mean, median, and mode are three of the various ways that data can be analyzed. • The mean is the numerical average of the data set and is found by adding all the values in the set and dividing the sum by the number of values. • The median is the piece of data that lies in the middle of the set of data arranged in order. • The mode is the piece of data that occurs most frequently in the data set. There may be one, more than one, or no mode in a data set. • The range is the spread of a set of data. It is determined by subtracting the smallest number in the data from the largest number in the data. • Students need to learn more than how to identify the mean, median, mode, and range of a set of data. They need to build an understanding of what the number tells them about the data, and they need to see those values in the context of other characteristics of the data.	**All students should** • Understand how to determine the mean, median, mode, and range of a set of data. • Understand that the mean is the numerical average of a data set; the median is the number in the middle of a set of data; the mode is the piece of data that occurs most often; and the range is the spread of a set of data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Calculate the mean of a group of numbers representing data from a given context. • Determine the median of a group of numbers representing data from a given context. • Determine the mode of a group of numbers representing data from a given context. • Determine the range of a group of numbers representing data from a given context.

Students entering grades 4 and 5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships in open number sentences, and they have begun to solve simple equations with one unknown.

The focus of instruction is to help students develop a solid use of patterning as a problem-solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, graphic, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They analyze the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By analyzing mathematical situations and models, students begin to represent these, using symbols and variables to write "rules" for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

5.20 The student will analyze the structure of numerical and geometric patterns (how they change or grow) and express the relationship, using words, tables, graphs or a mathematical sentence. Concrete materials and calculators will be used.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• There are an infinite number of patterns. • The simplest types of patterns are repeating patterns. In such patterns, students need to identify the basic unit of the pattern and repeat it. • Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both numerical and geometric formats. • Sample numerical patterns are 6, 9, 12, 15, 18, …; 5, 7, 9, 11, 13, …; 1, 2, 4, 7, 11, 16, …; 2, 4, 8, 16, 32, …; and 1, 5, 25, 125, 625, …. • In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation (turn) is the result of turning a figure around a point or a vertex, and reflection (flip) is the result of flipping a figure over a line.	**All students should** • Understand that patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols. • Understand the structure of a pattern and how it grows or changes. • Understand that mathematical relationships exist in patterns.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe numerical and geometric patterns formed by using concrete materials and calculators. • Express the relationship found in numerical and geometric patterns, using words, tables, graphs, or a mathematical sentence.

5.21 The student will

a) investigate and describe the concept of variable;

b) use a variable expression to represent a given verbal quantitative expression involving one operation; and

c) write an open sentence to represent a given mathematical relationship, using a variable.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A variable is a symbol that can stand for any one of a set of numbers or other objects. • A variable is a quantity that can have different values. Any letter can be used as a variable. • A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an equals sign (=). • A verbal quantitative expression involving one operation can be represented by a variable expression that describes what is going on. Numbers are used when they are known; variables are used when the numbers are unknown. For example, "a full box of cookies and four extra" can be represented by $b + 4$; "three full boxes of cookies" by $3b$; "a full box of cookies shared among four" by $\frac{b}{4}$. • An open sentence is a mathematical sentence containing a variable and an equals sign (=). For example, the sentence, "A full box of cookies and four extra equal 24 cookies." can be written as $b + 4 = 24$, where b stands for the number of cookies in one full box. "Three full boxes of cookies equal 60 cookies." can be written as $3b = 60$. • At this level, discuss how the symbol × used to represent multiplication can often be confused with the variable x. Students can minimize this confusion by using parentheses [e.g., $4(x) = 20$] or a small dot raised off the line to represent multiplication.	**All students should** • Understand that a variable is a symbol that can stand for any member of a set of numbers. • Understand that a variable expression is a variable or combination of variables, numbers, and symbols that represents a mathematical relationship. • Understand that verbal quantitative expressions can be translated to variable expressions. • Understand that an open sentence is a mathematical sentence with a variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the concept of a variable (presented as boxes, letters, or other symbols) as a representation of an unknown quantity. • Use a variable expression to represent a given verbal expression involving one operation (e.g., "5 more than a number" can be represented by $x + 5$). • Write an open sentence with addition, subtraction, multiplication, or division, using a variable to represent a missing number.

5.22 The student will create a problem situation based on a given open sentence using a single variable.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An open sentence is a mathematical sentence containing a variable. It also contains an equals sign (=). For example, $b + 3 = 23$ represents the answer to the word problem, "How many cookies are in a box if the box plus three more equals 23 cookies, where b stands for the number of cookies in the box? • By using story problems and numerical sentences, students begin to explore forming equations and representing quantities, using variables.	**All students should** • Understand that an open sentence is a mathematical sentence with a variable. • Understand that problem situations can be expressed as open sentences.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Create and write a word problem to match a given open sentence with a single variable and one operation.