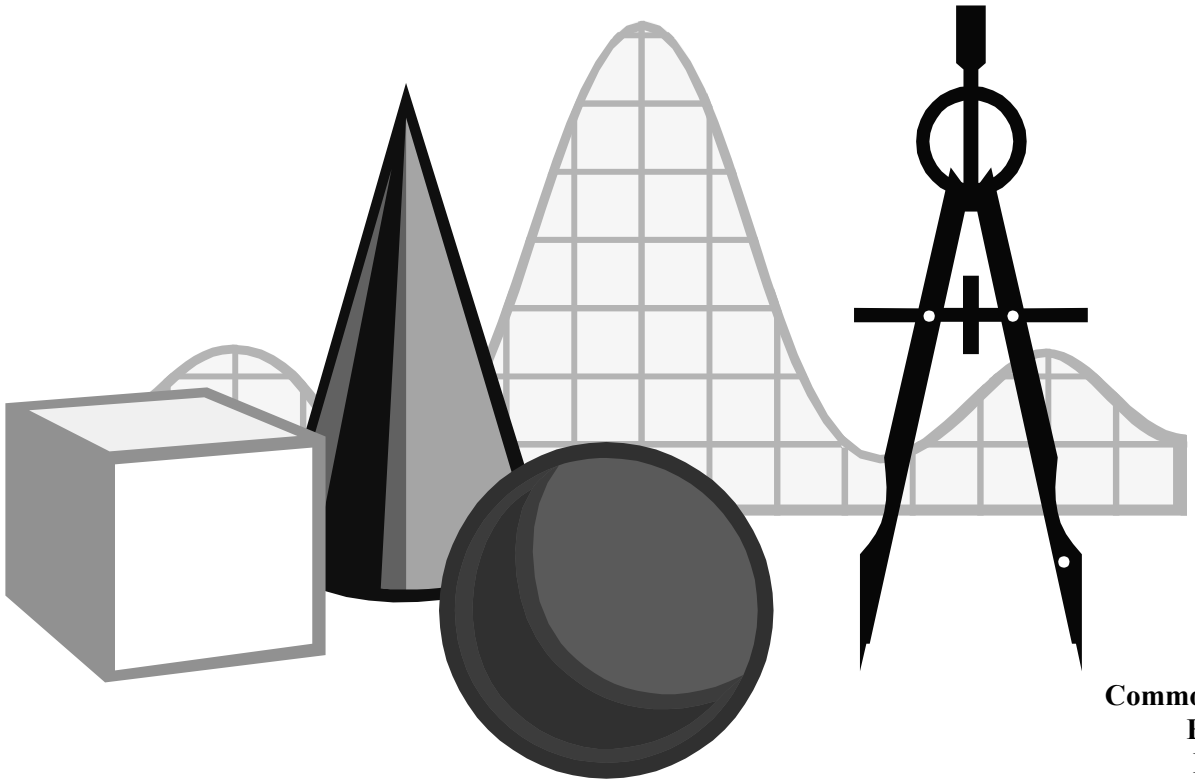
MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 6



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In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving real-life problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.
- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational-number representations is encouraged when students solve problems.
- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.
- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

6.1 The student will identify representations of a given percent and describe orally and in writing the equivalence relationships among fractions, decimals, and percents.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• *Percent* means "per 100" or how many "out of 100"; *percent* is another name for *hundredths*. • A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $30\% = \frac{30}{100} = \frac{3}{10} = 0.3$). • Percents can be expressed as fractions with a denominator of 100 (e.g., $75\% = \frac{75}{100} = \frac{3}{4}$). • Percents can be expressed as decimals (e.g., $38\% = \frac{38}{100} = 0.38$). • A fraction can be rewritten as an equivalent fraction with a denominator of 100, and, thus, as a decimal or percent (e.g., $\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$). • Decimals, fractions, and percents can be represented using concrete materials (e.g., base-10 blocks, decimal squares, or grid paper). • Percents should be represented by drawing a shaded region on a 10-by-10 grid to represent a given percent. • Percents are used in real life for taxes, sales, data description, and data comparison.	**All students should** • Understand that percent is a way of representing fractions and decimals. • Understand that a number can be written as a fraction, decimal, or percent. • Understand that percent is a method of standardization that is efficient because each number is always based on 100ths. • Understand that percents are used in real-life applications to compare or describe data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize that *percent* means "out of 100" or *hundredths*, using the percent symbol (%). • Identify the decimal and percent equivalents for halves, thirds, fourths, fifths, and tenths. • Describe orally and in writing the equivalent relationship among decimals, percents, and fractions that have denominators that are factors of 100. • Draw a shaded region on a 10-by-10 grid to represent a given percent. • Represent in decimal, fraction, and/or percent form a given shaded region of a 10-by-10 grid.

6.2 The student will describe and compare two sets of data, using ratios, and will use appropriate notations such as *a*/*b*, *a* to *b*, and *a*:*b*.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A ratio is a comparison of any two quantities and conveys an idea that cannot be expressed as a single number. A ratio is used to represent a variety of relationships within a set and between two sets. • A ratio can compare part of a set to the entire set (part-whole comparison). • A ratio can compare part of a set to another part of the same set (part-part comparison). • A ratio can compare part of a set to a corresponding part of another set (part-part comparison). • A ratio can compare all of a set to all of another set (whole-whole comparison). • A ratio can be written using a fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3). • A ratio is a multiplicative comparison of two numbers, measures, or quantities. • All fractions are ratios. • Ratios can compare two parts of the same whole. • Rates can be expressed as ratios.	**All students should** • Understand that a ratio is a comparison of two quantities. • Understand that ratios can be represented in more than one way.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe a relationship within a set by comparing part of the set to the entire set. • Describe a relationship between two sets by comparing part of one set to a corresponding part of the other set. • Describe a relationship between two sets by comparing all of one set to all of the other set. • Describe a relationship within a set by comparing one part of the set to another part of the same set. • Represent the relationship that makes a comparison by using the notations $\frac{a}{b}$, *a*:*b*, and *a* to *b*.

6.3 The student will

a) find common multiples and factors, including least common multiple and greatest common factor;

b) identify and describe prime and composite numbers; and

c) identify and describe the characteristics of even and odd integers.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Number theory is a part of mathematics in which characteristics of numbers and relationships between numbers are examined. • The set of natural numbers is the set of counting numbers (1, 2, 3, …). • The set of whole numbers is the set of natural numbers and zero (0, 1, 2, 3, …). • A factor of a number is an integer that divides evenly into that number. • A factor of a number is a divisor of the number. • A multiple of a number is the product of the number and any natural number. • A common factor of two or more numbers is a divisor that all of the numbers share. • The least common multiple of two or more numbers is the smallest common multiple of the given numbers. • The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share. • A prime number is a natural number that has exactly two different factors, one and the number itself. continued	**All students should** • Understand and use the unique characteristics of certain sets of numbers, including factors; multiples; and prime, composite, even, and odd numbers. • Understand that an integer and its opposite are the same distance from zero on a number line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify common multiples and the least common multiple for up to three numbers less than or equal to 50. • Identify common factors and the greatest common factor for up to three numbers less than or equal to 50. • Identify which numbers are prime for numbers less than or equal to 50. • Identify which numbers are composite for numbers less than or equal to 50. • Explain orally and in writing why a number is prime or composite. • Explain orally and in writing why a given integer is even (divisible by two) or odd (not divisible by two).

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c) identify and describe the characteristics of even and odd integers.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A composite number is a natural number that has more than two different factors. • The number 1 is neither prime nor composite because it has only one factor, itself. • The prime factorization of a number is a representation of the number as the product of its prime factors. For example, the prime factorization of 18 is $2 \times 3 \times 3$. • Zero is not a natural number. Although it has an infinite number of factors, it is neither prime nor composite. • Prime or composite numbers can be represented by rectangular models or rectangular arrays on grid paper. A prime number can be represented by only one rectangular array (e.g., 7 can only be represented by a 7×1). A composite number can always be represented by more than one rectangular array (e.g., 9 can be represented by a 9×1 or a 3×3). • Divisibility rules are useful tools in identifying prime and composite numbers. • The set of integers is the set of whole numbers and their opposites.		

6.4 The student will compare and order whole numbers, fractions, and decimals, using concrete materials, drawings or pictures, and mathematical symbols.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The decimal point is a symbol that indicates the location of the ones place and all other subsequent place values in the decimal system. • The decimal point separates a whole number amount from a number that is less than one. • Decimals can be represented and compared, using decimal manipulatives, drawings, pictures, or symbols. • Fractions can be represented and compared by using fraction manipulatives, drawings, pictures, or symbols.	**All students should** • Understand how the magnitude of a number represented by a whole number, fraction, or decimal compares to another number represented by a whole number, fraction, or decimal. • Understand how to represent the same whole number, fraction, or decimal in multiple ways, using concrete materials, a drawing, a symbol, or a statement.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare two whole numbers by representing the numbers with concrete objects or picture representations or by using the symbols <, ≤, >, ≥, or =. • Compare two fractions with denominators of 12 or less by representing the fractions with fraction manipulatives or picture representations or by using the symbols <, ≤, >, ≥, or =. • Compare two decimals through thousandths by representing the decimals with decimal manipulatives or picture representations or by using place-value charts or the symbols <, ≤, >, ≥, or =.

6.5 The student will identify, represent, order, and compare integers.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Integers are the set of whole numbers and their opposites. • Positive integers are greater than zero. • Negative integers are less than zero. • Zero is neither positive nor negative. • A negative integer is always less than a positive integer. • When comparing two negative numbers, the negative number that is closer to zero is greater. • An integer and its opposite are the same distance from zero on a number line. • On a conventional number line, a smaller number is always located to the left of a larger number (e.g., –7 lies to the left of –3; thus –7 < –3). • Comparison between integers can be made by using the mathematical symbols: < (less than), > (greater than), or = (equal to).	**All students should** • Understand how to identify, represent, order, and compare integers. • Understand that an integer and its opposite are the same distance from zero on a number line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify an integer represented by a point on a number line. • Represent an integer on a number line. • Compare and order integers, using a number line. • Compare integers, using the mathematical symbols <, >, and =.

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem-solving and application activities.

6.6 The student will

a) solve problems that involve addition, subtraction, multiplication, and/or division with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less, and express their answers in simplest form; and

b) find the quotient, given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Simplifying fractions to their simplest form assists with uniformity of answers and concepts. • Equivalent forms are needed to perform the operations of addition and subtraction with fractions. • Multiplication and division are inverse operations. • Rewriting an improper fraction as a mixed numeral assists with uniformity of answers and concepts. • There is implied addition of the whole number part and the fractional part in mixed numerals. • Estimation is an essential skill used to make sense of the placement of the decimal point in performing operations on decimals and for checking the decimal point's correct placement. • Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. For example: What is an approximate answer for $2.19 \div 0.8$? The answer is around 2 because $2 \div 1 = 2$. continued	**All students should** • Understand that fraction computation uses the same ideas as whole-number computation, applying those concepts to fractional parts. • Understand that decimal division uses the same ideas as whole-number division with an added focus on the placement of the decimal point. • Understand that using estimation helps determine the reasonableness of answers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Convert fractions to equivalent forms to perform the operations of addition and subtraction. • Simplify fractional answers to simplest form. • Solve problems that involve addition and/or subtraction with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less, and express answers in simplest form. • Solve problems that involve multiplication and/or division with fractions and mixed numbers that include denominators of 12 or less, and express answers in simplest form. • Given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit, find the quotient. • Given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with more than one non-zero digit, find the quotient by using a calculator.

6.6 The student will

a) solve problems that involve addition, subtraction, multiplication, and/or division with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less, and express their answers in simplest form; and

b) find the quotient, given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Understanding the placement of the decimal point is very important when finding quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, by 0.06, and by 0.006. • Using an area model assists with students' developing understanding of multiplication and division of fractions. • Using manipulatives to build conceptual understanding and using pictures and sketches to link concrete examples to the symbolic enhance students' understanding of operations with fractions and help students connect the meaning of whole-number computation to fraction computation. • It is helpful to use estimation to develop computational strategies. For example, $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3. • It is helpful for students to simplify before they multiply fractions, using the commutative property of multiplication to reduce fractions to simplest form before multiplying.		

6.7 The student will use estimation strategies to solve multistep practical problems involving whole numbers, decimals, and fractions (rational numbers).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimation and checking the reasonableness of a result enhance computational proficiency. • Various estimation strategies, such as front-end, compatible numbers, or rounding, are effective for various operations and situations. • Solving multistep problems in the context of real-life situations enhances interconnectedness and proficiency with estimation strategies. • Examples of real-life situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, deciding what time to leave for school or the movies, and sharing a pizza or the prize money from a contest. • Rational numbers is the set of numbers that can be written as a ratio or fraction.	**All students should** • Be able to produce an approximate answer for a given problem. • Understand that an estimated answer helps validate the reasonableness of a computed answer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve multistep practical problems involving whole numbers, decimals, and fractions by using estimation strategies and checking for the reasonableness of results.

6.8 The student will solve multistep consumer-application problems involving fractions and decimals and present data and conclusions in paragraphs, tables, or graphs. Planning a budget will be included.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A consumer application problem is defined as the type of problem that is normally encountered in daily living, such as problems related to money, travel, work, recreation, and home life. • A budget may be kept for short or long periods of time. Students may keep a short-term budget to enable the purchase of an expensive item or a long-term budget to facilitate a long-term spending plan.	**All students should** • Understand how mathematics relates to problems in daily life. • Understand how to represent problems within various contexts. • Understand the importance of planning and maintaining a budget.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine essential information necessary to solve consumer application problems. • Choose the operation or operations required to solve the problem. • Solve multistep consumer application problems involving fractions with denominators not greater than 12 and decimals not greater than hundredths, where solutions require at least a two-step process. • Represent the solution as a data table or graph. • Present and justify the solution orally or in writing. • Plan and maintain a budget.

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

6.9 The student will compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system and estimate conversions between units in each system:

a) length — part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;
b) weight/mass — ounces, pounds, tons, grams, and kilograms;
c) liquid volume — cups, pints, quarts, gallons, milliliters, and liters; and
d) area — square units. *

** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.*

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Making sense of various units of measure is an essential life skill, requiring reasonable estimates of what measurements mean, particularly in relation to other units of measure. – 1 inch is about 2.5 centimeters. – 1 foot is about 30 centimeters. – 1 meter is a little longer than a yard, or about 40 inches. – 1 mile is slightly farther than 1.5 kilometers. – 1 kilometer is slightly farther than half a mile. – 1 ounce is about 28 grams. – 1 nickel has the mass of about 5 grams. – 1 kilogram is a little more than 2 pounds. – 1 quart is a little less than 1 liter. – 1 liter is a little more than 1 quart. – Water freezes at 0°C and 32°F. – Water boils at 100°C and 212°F. – Normal body temperature is about 37°C and 98°F. – Room temperature is about 20°C and 70°F. continued	**All students should** • Understand that there is a structured relationship between and among units of measure for length, area, weight/mass, and volume in the metric and U.S. Customary systems. • Understand that weight and mass are different.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system. • Estimate the conversion of units of length, area, weight/mass, and volume between the U.S. Customary system and the metric system by using ballpark comparisons. • Determine the most appropriate unit of measure for a given situation.

6.9 The student will compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system and estimate conversions between units in each system:

a) length — part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;
b) weight/mass — ounces, pounds, tons, grams, and kilograms;
c) liquid volume — cups, pints, quarts, gallons, milliliters, and liters; and
d) area — square units. *

** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.*

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Multiple experiences with using nonstandard and standard units of measure to measure physical objects help students develop an intuitive understanding of size. • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight*, as shown by the questions: "How much does it weigh?" versus "What is its mass?"		

6.10 The student will estimate and then determine length, weight/mass, area, and liquid volume/capacity, using standard and nonstandard units of measure.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Chunking or benchmarks are strategies used to make measurement estimates. • Chunks of length, such as a window's length, can be used to estimate the length of a classroom wall. • Benchmarks, such as the two-meter height of a standard doorway, can be used to estimate height. • The degree of accuracy of measurement required is determined by the situation. • Whether to use an underestimate or an overestimate is determined by the situation. • Physically measuring objects along with using visual and symbolic representations improves student understanding of both the concepts and processes of measurement.	**All students should** • Understand that measures are determined by quantitative comparison to a standard unit. • Understand that units of measure are determined by the attributes of the object being measured. • Understand that measures of length are expressed in linear units, measures of area are expressed in square units, and measures of volume are expressed in cubic units.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate measurements by comparing the object to be measured against a benchmark. • Solve measurement problems by estimating and determining length, using standard and nonstandard units of measure. • Solve measurement problems by estimating and determining weight/mass, using standard and nonstandard units of measure. • Solve measurement problems by estimating and determining area, using standard and nonstandard units of measure. • Solve measurement problems by estimating and determining liquid volume/capacity, using standard and nonstandard units of measure.

6.11 The student will determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area and apply the appropriate formula.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The perimeter of a polygon is the measure of the distance around the polygon. • The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve. • The perimeter of a square whose side measures s is 4 times s ($P = 4s$), and its area is side times side ($A = s^2$). • The perimeter of a rectangle is the sum of twice the length and twice the width [$P = 2l + 2w$, or $P = 2(l + w)$], and its area is the product of the length and the width ($A = l \times w$). • The area of a triangle is one half of the measure of the base times the height: $A = \frac{1}{2}bh$, or $A = bh \div 2$. • Experiences in using a variety of measuring devices and making real measurements promote an understanding of measurements and the formulas associated with measurements. • Experiences in deriving the formulas for area and perimeter, using manipulatives such as tiles, one-inch cubes, adding machine tape, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and facility in their use.	**All students should** • Understand the attributes of polygons and the use of measures to determine area and perimeter. • Understand the derivation of formulas related to area and perimeter of polygons and how to determine which is used in problem situations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area. • Apply formulas to solve problems involving area and perimeter of triangles and rectangles.

6.12 The student will

a) solve problems involving the circumference and/or area of a circle when given the diameter or radius; and

b) derive approximations for pi (π) from measurements for circumference and diameter, using concrete materials or computer models.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of pi (π) is the ratio of the circumference of a circle to its diameter. • The ratio of the circumference to the diameter of a circle is a constant value, pi (π), which can be approximated by measuring various sizes of circles. • The fractional approximation of pi generally used is $\frac{22}{7}$. • The decimal approximation of pi generally used is 3.14. • The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle. • The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle.	**All students should** • Select the appropriate approximation for pi (π) when solving problems. • Understand the derivation of pi and formulas for finding circumference and area of a circle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. • Find the circumference of a circle by substituting a value for the diameter or the radius into the formula $C = \pi d$ or $C = 2\pi r$. • Find the area of a circle by using the formula $A = \pi r^2$. • Determine the circumference and/or area of a circle, using various tools. • Create and solve problems that involve finding the circumference and/or area of a circle when given the diameter or radius.

6.13 The student will

a) estimate angle measures, using 45°, 90°, and 180° as referents, and use the appropriate tools to measure the given angles; and

b) measure and draw right, acute, and obtuse angles and triangles.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A referent for a 45° angle is the angle created by a diagonal of a square and one of the sides of the square that share a common vertex. • A referent for a 90° angle (right angle) is the angle created by the corner of a square. • A referent for a 180° angle (straight angle) is the angle created by a straight line. • Protractors and angle rulers are tools commonly used to measure angles. • An acute angle has a measure between 0° and 90°. • A right angle (square angle) has a measure of 90°. • An obtuse angle has a measure between 90° and 180°. • An acute triangle has three acute angles. • A right triangle has one right angle and two acute angles. • An obtuse triangle has one obtuse angle and two acute angles.	**All students should** • Understand that an angle is two rays diverging from a common point. • Understand names for angles and triangles by defining referents and characteristics.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate visually the angle measure of a given angle by using 45°, 90°, and 180° as referents, and use appropriate tools to check the reasonableness of the estimate. • Draw and measure acute, right, and obtuse angles, using appropriate tools. • Draw and measure acute, right, and obtuse triangles, using appropriate tools.

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.
- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.
- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking Algebra.)
- **Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking Geometry.)

6.14 The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The sum of the measures of the angles of a triangle is 180°. • A triangle can be classified by angles: an acute triangle has three acute angles; an obtuse triangle has one obtuse angle and two acute angles; a right triangle has one right angle and two acute angles. • Acute angles are greater than 0° and less than 90°, obtuse angles are greater than 90° and less than 180°, and right angles measure exactly 90°. • Triangles can be classified by the measures of their sides: all three sides of a scalene triangle have different measures; at least two sides of an isosceles triangle have the same measure; all three sides of an equilateral triangle have the same measure. • Quadrilaterals can be classified by the number of parallel sides: a parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has only one pair of parallel sides; other quadrilaterals have no parallel sides. • Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a trapezoid may have none, one, or two 90° angles. continued	**All students should** • Understand that plane figures are identified and described by their similarities, differences, and defining properties. • Understand that plane figures are classified by their defining properties.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Classify triangles, quadrilaterals, pentagons, and hexagons. • Classify a triangle based on the size of its angles and/or its sides. • Identify the sum of the measures of the angles of any triangle or quadrilateral. • Determine that the sum of the measures of the angles of a triangle is 180°. • Classify a triangle by its angles. • Classify and describe the similarities and differences in sets of triangles by sorting. • Classify quadrilaterals by pairs of parallel sides by sorting. • Identify and describe the similarities and differences in sets of quadrilaterals by sorting.

6.14 The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Quadrilaterals can be classified by the number of congruent sides: a rhombus has four congruent sides; a square, which is a rhombus with four right angles, also has four congruent sides; a parallelogram and a rectangle each have two pairs of congruent sides. • A parallelogram has two pairs of opposite sides congruent. A rectangle, which is a parallelogram with four right angles, also has two pairs of opposite sides congruent. A square, which is a parallelogram with four right angles and four congruent sides, also has two pair of opposite sides congruent. • A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals. • The sum of the measures of the angles of a quadrilateral is 360°. • A trapezoid with congruent, non-parallel sides is called an *isosceles trapezoid*.		

6.15 The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes. Examples of noncongruent and congruent figures will be included.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Congruent figures have exactly the same size and the same shape. • Noncongruent figures may have the same shape but not the same size. • The matching or corresponding angles of congruent polygons have the same measure, and the matching or corresponding sides of congruent polygons have the same measure. • The direct comparison of congruent or noncongruent figures can be accomplished by placing one figure on top of the other or by measuring all sides and angles. • Construction of congruent line segments, angles, and polygons helps students understand congruency.	**All students should** • Understand the meaning of congruence.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Characterize polygons as congruent and noncongruent according to the measures of their sides and angles. • Determine the congruence of segments, angles, and polygons by direct comparison, given their attributes.

6.16 The student will construct the perpendicular bisector of a line segment and an angle bisector.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A perpendicular bisector of a line segment is a line, line segment, or ray that forms a right angle with the line segment and divides the line segment into two equal parts. • The bisector of an angle separates the interior of the angle into two congruent angles. • To bisect means to divide into two equal parts. • A line of symmetry for an angle or line segment bisects the angle or line segment.	**All students should** • Understand the attributes of perpendicular lines. • Understand the attributes of a bisector.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct the perpendicular bisector of a line segment by using a variety of tools. • Construct the bisector of an angle by using a variety of tools.

6.17 The student will sketch, construct models of, and classify solid figures (rectangular prism, cone, cylinder, and pyramid).

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Solid figures are three-dimensional and can be represented by drawings on a flat surface. • A prism is a three-dimensional figure having two congruent parallel polygonal bases. The faces connecting the corresponding edges of the bases are rectangles. A prism is classified by the shape of its bases. • A rectangular prism is a three-dimensional solid figure that has six rectangular faces. • A cylinder is a three-dimensional solid figure that has two parallel circular bases and a curved surface that connects the two bases. • A cone is a three-dimensional solid figure that has a circular base, all the points on which are connected to a vertex not on the base. • A pyramid is a three-dimensional solid figure that has a polygon as its base, all the points on which are connected to a vertex not on the base. The sides of a pyramid are triangles. The base of the pyramid determines its name (e.g., a "square pyramid" has a square base).	**All students should** • Understand how to interpret a picture of a solid figure from a two-dimensional diagram and vice versa. • Understand the decomposition of a solid figure into a discrete set of surfaces.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Sketch rectangular prisms, cones, cylinders, and pyramids from two-dimensional representations and three-dimensional models. • Construct models for rectangular prisms, cones, cylinders, and pyramids. • Classify rectangular prisms, cones, cylinders, and pyramids by their two-dimensional representations. • Identify a three-dimensional model of a prism, cone, cylinder, or pyramid from its two-dimensional representation.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving real-life problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of central tendency and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including

a) line, bar, and circle graphs;
b) stem-and-leaf plots; and
c) box-and-whisker plots.

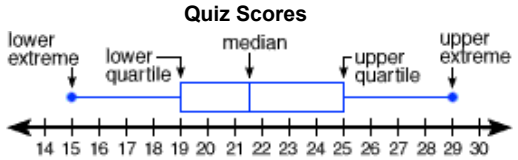
Circle graphs will be limited to halves, fourths, and eighths.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to answer the problem. • Data can be discrete or continuous. • Different types of graphs are used to display different types of data. – Bar graphs use categorical (discrete) data (e.g., months or eye color). – Line graphs use continuous data (e.g., temperature and time). – Circle graphs show a relationship of the parts to a whole. • All graphs include a title, and data categories should have labels. • A scale should be chosen that is appropriate for the data. • A key is essential to explain how to read the graph. • A title is essential to explain what the graph represents. continued	**All students should** • Understand that data can be displayed in a variety of graphical representations. • Select and use appropriate statistical methods to analyze data. • Understand that different types of representations can tell different things about the same data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect data sets of no more than 20 items by using tally sheets, surveys, observations, questionnaires, interviews, and polls. • Organize data by using lists, charts, and tables. • Organize and display data in bar and line graphs, displaying the information as clearly as possible by using increments of whole numbers, fractions, and decimals rounded to the nearest tenth. • Organize and display data in circle graphs by depicting information as fractional parts that are limited to halves, fourths, and eighths. • Organize and display data sets of no more than 20 numbers in stem-and-leaf plots where the stem is listed in ascending order and the leaves are in ascending order, with or without commas between leaves. continued

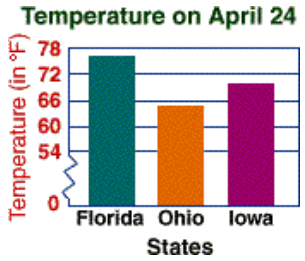
6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including
a) line, bar, and circle graphs;
b) stem-and-leaf plots; and
c) box-and-whisker plots.
Circle graphs will be limited to halves, fourths, and eighths.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • An understanding of place value is used to organize a set of data into a stem-and-leaf plot. Stem-and-leaf plots organize data consecutively. A stem-and-leaf plot shows the frequency of data and can be used to find the range, the median, and the mode. **Stem-and-Leaf Plot** **Number of Sit-Ups** Stem | Leaves 3 | 4 6 8 8 4 | 0 3 6 7 7 5 | 0 0 1 2 Each tens digit is called the *stem*. The ones digits are called the *leaves*. • A box-and-whisker plot shows how a set of data clusters around the middle (median). It is a graph that uses a rectangle to represent the middle 50% of a set of data and "whiskers" at both ends to represent the remainder of the data. The five critical points in a box-and-whisker plot are lower extreme, lower quartile, median, upper quartile, and upper extreme. Each of these points represents the bounds for the four quartiles. The range is the difference between the upper extreme and the lower extreme. The interquartile range is the difference between the upper quartile and the lower quartile. continued		continued • Organize and display data sets of no more than 20 numbers in box-and-whisker plots, identifying the lower extreme (minimum), lower quartile, median, upper quartile, and upper extreme (maximum). Use the critical points in a box-and-whisker plot to determine the range and the interquartile range. • Decide which type of graph is appropriate for a given situation. – Bar graphs are used to display categorical (discrete) data. – Line graphs are used to display continuous data. – Circle graphs are used to show a relationship of the parts to a whole. • Interpret data from line, bar, and circle graphs and from stem-and-leaf and box-and-whisker plots.

6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including
a) line, bar, and circle graphs;
b) stem-and-leaf plots; and
c) box-and-whisker plots.
Circle graphs will be limited to halves, fourths, and eighths.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued **Box-and-Whisker Plot**  • Data are analyzed by describing the various features and elements of a graph. • Inferences and convincing arguments are based on data analysis. • Bar graphs are used to compare counts of different categories (categorical or discrete data). – A bar graph uses either horizontal or vertical parallel bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis displaying the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. The values should represent equal increments. continued		

6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including
a) line, bar, and circle graphs;
b) stem-and-leaf plots; and
c) box-and-whisker plots.
Circle graphs will be limited to halves, fourths, and eighths.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued – Each axis should be labeled, and the graph should have a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, mode, least and greatest) should be included. **Bar Graph**  • Line graphs should be utilized to show how one variable changes over time. By looking at a single-line graph, it can be determined whether the variable is increasing, decreasing, or staying the same with the passage of time. continued		

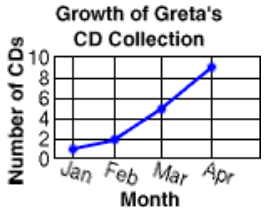
6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including
a) line, bar, and circle graphs;
b) stem-and-leaf plots; and
c) box-and-whisker plots.
Circle graphs will be limited to halves, fourths, and eighths.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued – The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as continuous data because it represents data collected over a continuous period of time. – The values along the vertical axis represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals, depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data. – Each axis should be labeled, and the graph should have a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., trends of increase and/or decrease, least and greatest) should be included. continued		

6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including

a) line, bar, and circle graphs;
b) stem-and-leaf plots; and
c) box-and-whisker plots.

Circle graphs will be limited to halves, fourths, and eighths.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued **Line Graph**  		

6.19 The student will describe the mean, median, and mode as measures of central tendency, describe the range, and determine their meaning for a set of data.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Measures of central tendency are types of averages for a data set. They represent numbers that best describe a data set. Mean, median, and mode are measures of central tendency that are useful for describing the average for different situations. – Mean works well for sets of data with no very high or low numbers. – Median is a good choice when data sets have a couple of values much higher or lower than most of the others. – Mode is a good descriptor to use when the set of data has some identical values. • The mean is the numerical average of the data set and is found by adding the numbers in the data set together and dividing the sum by the number of data pieces in the set. • The median is the middle value of a data set in ranked order. If there is an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values. continued	**All students should** • Understand that measures of central tendency are types of averages for a data set. • Understand that mean, median, and mode are measures of central tendency that are useful for describing data in different situations. • Understand that the range describes the spread of a set of data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the mean for a set of data. • Find the median for a set of data. • Find the mode for a set of data. • Find the range for a set of data. • Describe the three measures of central tendency and a situation in which each would best represent a set of data.

6.19 The student will describe the mean, median, and mode as measures of central tendency, describe the range, and determine their meaning for a set of data.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal. – For 2, 3, 4, 5, 5, 6, 7, 8, 8, 8, 9, 11, the mode is 8. – For 2, 3, 4, 5, 5, 5, 7, 8, 8, 8, 9, 11, the modes are 5 and 8 (bimodal). – For 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 17, there is no mode. • The range is the difference between the greatest and least values in a set of data and shows the spread in a set of data.		

6.20 The student will

a) make a sample space for selected experiments and represent it in the form of a list, chart, picture, or tree diagram; and

b) determine and interpret the probability of an event occurring from a given sample space and represent the probability as a ratio, decimal, or percent, as appropriate for the given situation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A sample space is the set of all possible outcomes of an experiment. • A sample space can be organized by using a list, chart, picture, or tree diagram. The sample space for tossing two coins is (H,H), (H,T), (T,H), and (T,T). • The probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes (sample space). • The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent. • The probability of an event occurring is a ratio between 0 and 1. – A probability of 0 means the event will never occur. – A probability of 1 means the event will always occur. • A simple event is one event (e.g., pulling one sock out of a drawer and examining the probability of getting one color).	**All students should** • Understand how to use and interpret information given in a sample space. • Understand that a probability can be expressed as a ratio, decimal, or percent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Plan and carry out experiments that use concrete materials to find a sample space. • Determine the sample space for selected experiments and represent the sample space for up to 20 possibilities as a list, chart, picture, and/or tree diagram. • Given a sample space, determine the probability of a simple event. Represent the probability as a ratio, fraction, decimal, or percent where the fraction's denominator does not exceed 20, decimals are rounded to tenths, and percent is rounded to $\frac{1}{10}$ of a percent.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving real-life problems.

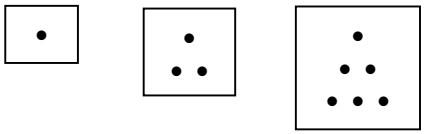
Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

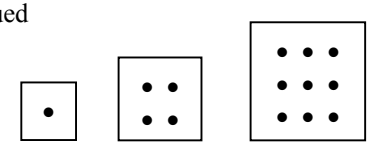
6.21 The student will investigate, describe, and extend numerical and geometric patterns, including triangular numbers, patterns formed by powers of 10, and arithmetic sequences.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In the numerical pattern of an arithmetic sequence, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Sample numerical patterns are 6, 9, 12, 15, 18, …; and 5, 7, 9, 11, 13, …. • Strategies to recognize and describe the differences between terms in numerical patterns include, but are not limited to, examining the change between consecutive terms, looking for prime numbers, and finding common factors. An example is the pattern 1, 2, 4, 7, 11, 16, …. • Numerical patterns may include linear and exponential growth, perfect squares, triangular and other polygonal numbers, or Fibonacci numbers. • Strategies to recognize and describe geometric patterns include, but are not limited to, examining flips, slides, turns, growth, and symmetry. Rotation (turn) is the result of turning a figure around a point or a vertex. Translation (slide) is the result of sliding a figure in any direction within a plane. Dilatation (scale increase or decrease) is the result of enlarging or shrinking a figure by a scale amount. Reflection (flip) is the result of flipping a figure over a line. continued	**All students should** • Understand that mathematical patterns can be represented in various forms, geometrically or numerically. • Understand that patterns regularly occur in everyday life. • Understand that patterns can be recognized, extended, or generalized. • Understand that numerical patterns may involve adding or multiplying by the same number. • Understand that geometric patterns may involve shape, size, angles, transformations of shapes, and growth.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and apply strategies to recognize and describe the change between terms in numerical patterns. • Investigate and apply strategies to recognize and describe geometric patterns. • Describe verbally and in writing the relationships between consecutive terms in a numerical or geometric pattern. • Extend and apply numerical and geometric patterns to similar situations. • Create numerical and geometric patterns by using a given rule or mathematical relationship. • Describe numerical and geometric patterns, including triangular numbers.

6.21 The student will investigate, describe, and extend numerical and geometric patterns, including triangular numbers, patterns formed by powers of 10, and arithmetic sequences.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • When multiplying tens together, the product is called a "power of 10." The numbers 1, 10, 100, 1000, and 10,000 are powers of 10. A power of 10 can be expressed with an exponent. The exponent gives the number of times that 10 is a factor, i.e., $10 \times 10 \times 10 = 10^3$. • The first five triangular numbers are 1, 3, 6, 10, and 15. A triangular number can be represented geometrically as a certain number of dots arranged in a triangle, with one dot in the first (top) row and each succeeding lower row having one more dot that the row above it. To find the next triangular number, a new row is added to an existing triangle, and total number of dots counted. Students should make the connection between the number of new dots in the triangle and the corresponding triangular number. Triangular numbers can be represented as a growing pattern of triangles.  • A square number can be represented geometrically as the number of dots in a square array. Square numbers are perfect squares and are the numbers that result from multiplying any whole number by itself (e.g., $36 = 6 \times 6$). Square numbers (1, 4, 9, 16, …) can be represented as a growing pattern of squares. For example: continued		

6.21 The student will investigate, describe, and extend numerical and geometric patterns, including triangular numbers, patterns formed by powers of 10, and arithmetic sequences.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued  • The possible number of patterns is infinite. • The simplest types of patterns are repeating patterns. In such patterns, students need to identify the basic unit of the pattern and repeat it. • Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns both in numerical and geometric formats. • In geometric number patterns, students must determine what each number is multiplied by to obtain the next number in the geometric sequence. This multiplier is called the *common ratio*. Sample geometric number patterns include 2, 4, 8, 16, 32, …; 1, 5, 25, 125, 625, …; and 80, 20, 5, 1.25, ….		

6.22 The student will investigate and describe concepts of positive exponents, perfect squares, square roots, and, for numbers greater than 10, scientific notation. Calculators will be used to develop exponential patterns.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A power of a number represents repeated multiplication of the number by itself (e.g., $8^3 = 8 \times 8 \times 8$ and is read "8 to the third power"). • In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. • Any real number other than zero raised to the zero power is 1. • Perfect squares are the numbers that result from multiplying any whole number by itself (e.g., $36 = 6 \times 6$). • Perfect squares can be represented geometrically as the areas of squares the length of whose sides are whole numbers (e.g., 1×1, 2×2, or 3×3). • A square root of a number is a number which, when multiplied by itself, produces the given number (e.g., the square root of 49 is 7 since $7 \times 7 = 49$). • Patterns in place-value charts provide visual meaning of exponents: $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$. • Scientific notation for a number is expressed by writing the number as a number greater than or equal to 1 but less than 10 times a power of 10 (e.g., 3.2×10^3 is scientific notation for 3,200).	**All students should** • Understand that a power of a number is repeated multiplication of that number by itself. • Understand that squaring a number and taking a square root of a number are inverse operations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Recognize and describe patterns with exponents by using a calculator. • Recognize and describe patterns of perfect squares. • Recognize and describe patterns with square roots and squares by using squares, grid paper, and calculators. • Recognize powers of ten by examining patterns in a place-value chart: $10^4 = 10{,}000$, $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$. • Write scientific notation for a number greater than 10.

6.23 The student will

a) model and solve algebraic equations, using concrete materials;

b) solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions; and

c) use the following algebraic terms appropriately*: variable, coefficient, term,* and *equation*.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A one-step linear equation is an equation that requires one operation to solve. • A coefficient is the numerical factor in a term. For example, in the term $3xy^2$, 3 is the coefficient; in the term x, 1 is the coefficient. • Positive rational solutions are limited to whole numbers and positive fractions and decimals. • An equation is a mathematical sentence stating that two expressions are equal. • A variable is a symbol (placeholder) used to represent an unspecified member of a set. • A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3.	**All students should** • Understand that physical objects can be used to represent and solve algebraic equations. • Understand that in an equation, the equal sign indicates that the value on the left side of the sign is the same as the value on the right side. • Understand that to maintain equality, an operation performed on one side of an equation must be performed on the other side.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent a one-step equation, using a variety of concrete materials such as colored chips on an equation mat, algebra tiles, or weights on a balance scale. • Solve a one-step equation by demonstrating the steps algebraically. • Use the following algebraic terms appropriately: *equation*, v*ariable, term,* and *coefficient*. • Identify examples of equations, variables, terms, and coefficients.