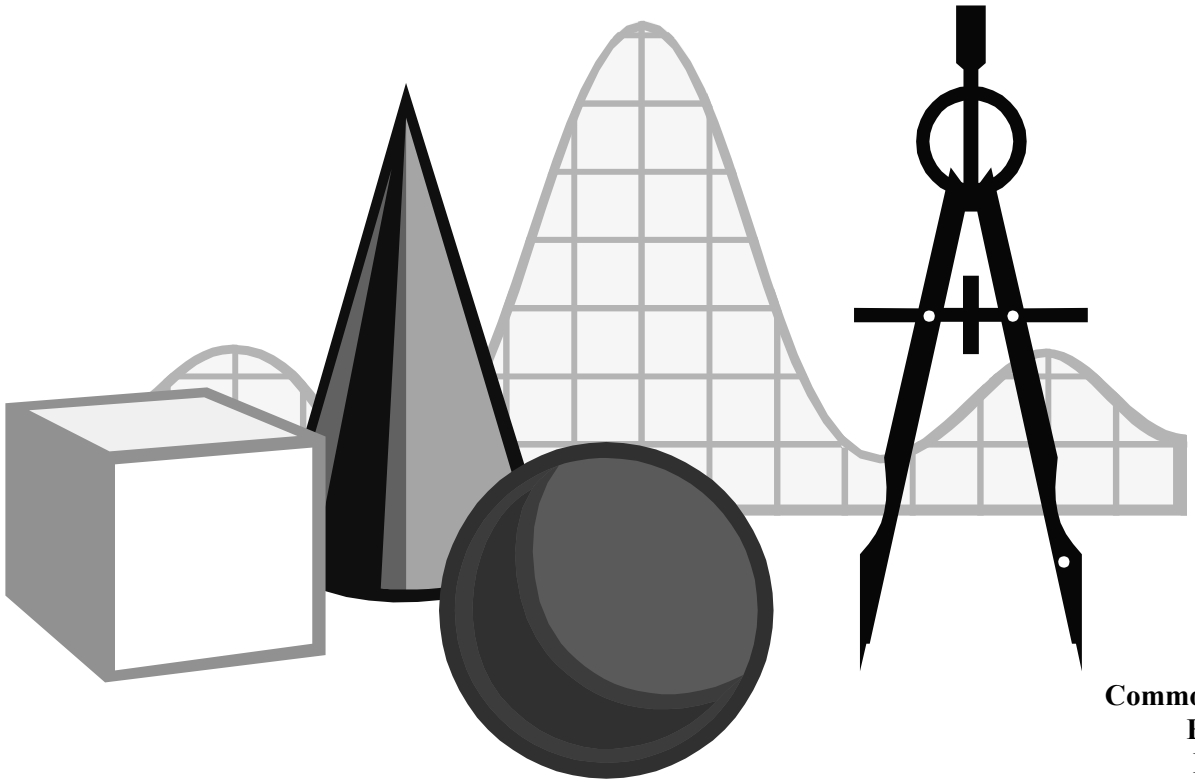
MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 7



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The 2002 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's website at http://www.pen.k12.va.us.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving real-life problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.
- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational-number representations is encouraged when students solve problems.
- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.
- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

7.1 The student will compare, order, and determine equivalent relationships between fractions, decimals, and percents, including use of scientific notation for numbers greater than 10.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Fractions, decimals, and percents are three different representations of the same number. Some numbers also can be represented in scientific notation. • A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $30\% = \frac{30}{100} = \frac{3}{10} = 0.3$). • Scientific notation is used to represent very large or very small numbers. • A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$). • Decimals, fractions, and percents can be compared using concrete materials (e.g., base ten blocks, decimal squares, and grid paper). • Equivalent relationships among fractions, decimals, and percents can be determined by using manipulatives (e.g., fraction bars, base-ten blocks, fraction circles, graph paper, and calculators).	**All students should** • Understand that a number can be represented as a decimal, fraction, percent, and/or in scientific notation. • Develop strategies to compare, order, and determine equivalency among fractions, decimals, and percents. • Understand real-life uses of scientific notation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent a number in fraction, decimal, and percent forms. Fractions will have denominators of 12 or less. • Compare, order, and determine equivalent relationships among fractions, decimals, and percents. Decimals are limited to the thousandths place, and percents are limited to the tenths place. • Write a number greater than 10 in scientific notation. • Compare, order, and determine equivalent relationships between numbers larger than 10 written in scientific notation. • Compare very large numbers, using scientific notation. • Order no more than five numbers written as fractions, decimals, percents, and numbers larger than 10 written in scientific notation in ascending (least to greatest) or descending (greatest to least) order.

7.2 The student will simplify expressions that contain rational numbers (whole numbers, fractions, and decimals) and positive exponents, using order of operations, mental mathematics, and appropriate tools.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A rational number can be expressed as a ratio of two integers. • An expression, like a phrase, has no equal sign. • Expressions are simplified by using the order of operations. • The order of operations defines the computation order to follow in simplifying an expression. • The order of operations is as follows: – First, complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right. • The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent. • Any number, except 0, raised to the zero power is 1.	**All students should** • Understand that the order of operations describes the order to use to simplify expressions containing more than one operation. • Select appropriate strategies and tools to simplify expressions. • Understand that whole numbers, fractions, and decimals are rational numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify expressions by using the order of operations in a demonstrated step-by-step approach. • Find the value of numerical expressions, using order of operations, mental mathematics, and appropriate tools. Exponents are limited to positive values. Fractions are limited to having denominators of 12 or less. Decimals are limited to the thousandth place.

7.3 The student will identify and apply the following properties of operations with real numbers:

a) the commutative and associative properties for addition and multiplication;
b) the distributive property;
c) the additive and multiplicative identity properties;
d) the additive and multiplicative inverse properties; and
e) the multiplicative property of zero.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The commutative property for addition states that changing the order of the addends does not change the sum (e.g., 5 + 4 = 4 + 5). • The commutative property for multiplication states that changing the order of the factors does not change the product (e.g., 5 · 4 = 4 · 5). • The associative property of addition states that regrouping the addends does not change the sum [e.g., 5 + (4 + 3) = (5 + 4) + 3]. • The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5 · (4 · 3) = (5 · 4) · 3]. • Subtraction and division are neither commutative nor associative. • The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number [e.g., 5 · (3 + 7) = (5 · 3) + (5 · 7), or 5 · (3 – 7) = (5 · 3) – (5 · 7)]. continued	**All students should** • Understand that using the properties of operations with real numbers helps with understanding mathematical relationships. • Understand how to use these properties when computing.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the real number equation that represents each property of operations with real numbers, when given several real number equations. • Explore the properties of real numbers, using diagrams and manipulatives. • Test the validity of properties by using examples of the properties of operations on real numbers. • Identify the property of operations with real numbers that is illustrated by a real number equation.

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c) the additive and multiplicative identity properties;
d) the additive and multiplicative inverse properties; and
e) the multiplicative property of zero.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division. • The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$). • The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$). • Inverses are numbers that combine with other numbers and result in identity elements [e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$]. • The additive inverse property states that the sum of a number and its additive inverse always equals zero [e.g., $5 + (-5) = 0$]. continued		

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b) the distributive property;
c) the additive and multiplicative identity properties;
d) the additive and multiplicative inverse properties; and
e) the multiplicative property of zero.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$). • Zero has no multiplicative inverse. • The multiplicative property of zero states that the product of any real number and zero is zero. • Division by zero is not a possible arithmetic operation.		

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem-solving and application activities.

7.4 The student will

a) solve practical problems using rational numbers (whole numbers, fractions, decimals) and percents; and

b) solve consumer-application problems involving tips, discounts, sales tax, and simple interest.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Tips are computed on the cost of the service generally without including the tax on the service. • The sale price is determined by subtracting the discount from the original price of the merchandise. • Simple interest for a number of years is determined by multiplying the principal by the rate of interest by the number of years of the loan or investment. ($I = prt$) • The total value of an investment is equal to the sum of the original investment and the interest earned. The total cost of a loan is equal to the sum of the original cost and the interest paid.	**All students should** • Select appropriate methods for computing with rational numbers according to the context of the problem. • Understand how mathematics relates to problems in daily life. • Understand that tips, discounts, sales tax, and simple interest are computed using the same procedure — i.e., multiplying the base (cost of service, merchandise, or principal) by a rate expressed as a percent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine solutions to everyday problems involving whole numbers, decimals, fractions, and percents. Fractions are limited to having denominators no greater than 12. • Compute the tip and total bill for a meal. • Compute a discount and the resulting (sale) price for one discount. • Compute the sales tax for purchased items. • Compute the simple interest earned on an investment or a loan for a specified number of years and/or months.

7.5 The student will formulate rules for and solve practical problems involving basic operations (addition, subtraction, multiplication, and division) with integers.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The set of integers is the set of whole numbers and their opposites (e.g., … –3, –2, –1, 0, 1, 2, 3, …). • Integers are used in real-life situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), and changes in altitude (above/below sea level) • Concrete experiences in formulating rules for adding and subtracting integers should be explored by examining patterns along a number line and using manipulatives, such as two-color counters, or by using algeblocks. • Concrete experiences in formulating rules for multiplying and dividing integers should be explored by examining patterns along a number line and using manipulatives, such as two-color counters, or by using algeblocks.	**All students should** • Develop and apply strategies involving mathematical operations with integers. • Understand how problems in daily life can be represented and solved by using integers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Formulate rules for adding integers. • Formulate rules for subtracting integers. • Formulate rules for multiplying integers. • Formulate rules for dividing integers. • Solve practical problems involving addition, subtraction, multiplication, and division with integers.

7.6 The student will use proportions to solve practical problems, which may include scale drawings, that contain rational numbers (whole numbers, fractions, and decimals) and percents.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A proportion is a statement of equality between two ratios. • A common property relates the numerators of the two ratios, and another common property relates the denominators of the two ratios. For example, both numerators relate to one property, such as length, while both denominators relate to another property, such as width. Alternatively, both numerators could relate to scale lengths, while both denominators relate to actual lengths. • The dimensions of a scale model are proportional to the corresponding dimensions of the object (e.g., a blueprint of a house floor plan is proportional to the actual dimensions of the floor). • A proportion can be written as $\frac{a}{b} = \frac{c}{d}$, $a:b = c:d$, or a is to b as c is to d. • A proportion can be solved by finding the product of the means and the product of the extremes. For example, in the proportion $a:b = c:d$, a and d are the extremes and b and c are the means. If values are substituted for a, b, c, and d such as 5:12 = 10:24, then the product of extremes (5×24) is equal to the product of the means (12×10). continued	**All students should** • Understand that a proportion is an equation showing that two ratios are equal. • Understand how to set up a proportion, given the relationship between two items. • Understand that when two quantities are proportional, a change in one quantity corresponds to a predictable change in the other. • Understand that proportions are useful in solving many types of problems.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write proportions that represent equivalent relationships between two sets. • Solve a proportion to find a missing term. • Apply proportions to solve problems that involve percents. • Apply proportions to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and/or decimals no less than tenths**.**

7.6 The student will use proportions to solve practical problems, which may include scale drawings, that contain rational numbers (whole numbers, fractions, and decimals) and percents.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • In a proportional situation, both quantities increase or decrease together. • In a proportional situation, two quantities increase multiplicatively. Both are multiplied by the same factor. • A proportion can be solved by finding equivalent fractions. • There is a distinction between a proportion and the idea of equivalent fractions. Equivalent fractions are symbols for the same quantity or amount and they represent the same rational number in different forms. • A rate is a special ratio that always has a denominator of 1. Examples of rates include miles/hour and revolutions/minute. A rate compares measures of different types. • A percent is a special ratio in which the denominator is 100. • Proportions are used in every-day contexts, such as speed, recipe conversions, scale drawings, map reading, reducing and enlarging, comparison shopping, and monetary conversions.		

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- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

7.7 The student, given appropriate dimensions, will
a) estimate and find the area of polygons by subdividing them into rectangles and right triangles; and
b) apply perimeter and area formulas in practical situations.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a simple, closed plane figure with sides that are line segments. • The perimeter of a polygon is the distance around the figure. • The area of a rectangle is computed by multiplying the lengths of two adjacent sides. • The area of a triangle is computed by multiplying the measure of its base by the measure of its height and dividing that product by 2. • The area of any polygon is based upon knowing how to find the area of a rectangle. • The area of a parallelogram is computed by multiplying the measure of its base by the measure of its height. • The area of a trapezoid is computed by taking the average of the measures of the two bases and multiplying this average by the height. • An estimate of the area of a polygon can be made by subdividing the polygon into rectangles and right triangles, estimating their areas, and adding the areas together.	**All students should** • Understand the derivation of formulas related to area and perimeter of polygons. • Understand how to apply area or perimeter in real-life situations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Subdivide a polygon into rectangles and right triangles, estimate the area of the rectangles and/or right triangles to estimate the area of the polygon, and find the area of the rectangles and/or right triangles to determine the area of the polygon. • Apply perimeter formulas to solve real-life problems. • Apply area formulas to solve real-life problems.

7.8 The student will investigate and solve problems involving the volume and surface area of rectangular prisms and cylinders, using concrete materials and practical situations to develop formulas.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The area of a rectangle is computed by multiplying the lengths of two adjacent sides. • The area of a circle is computed by squaring the radius and multiplying that product by π ($A = \pi r^2$, where π = 3.14 or $\frac{22}{7}$). • A rectangular prism can be represented on a flat surface as a net that contains six rectangles — two that have measures of the length and width of the base, two others that have measures of the length and height, and two others that have measures of the width and height. The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$). • A cylinder can be represented on a flat surface as a net that contains two circles (bases for the cylinder) and one rectangular region whose length is the circumference of the circular base and whose width is the height of the cylinder. The surface area of the cylinder is the area of the two circles and the rectangle ($SA = 2\pi r^2 + 2\pi rh$). • The volume of a rectangular prism is computed by multiplying the area of the base, B, (length times width) by the height of the prism ($V = lwh = Bh$). • The volume of a cylinder is computed by multiplying the area of the base, B, (πr^2) by the height of the cylinder ($V = \pi r^2 h = Bh$).	**All students should** • Understand how to apply volume and surface area in real-life situations. • Understand the derivation of formulas related to volume and surface area of polygons.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Develop a procedure and formula for finding the surface area of a rectangular prism. • Solve practical problems that require finding the surface area of a rectangular prism. • Develop a procedure and formula for finding the surface area of a cylinder. • Solve practical problems that require finding the surface area of a cylinder. • Develop a procedure and formula for finding the volume of a rectangular prism. • Solve practical problems that require finding the volume of a rectangular prism. • Develop a procedure and formula for finding the volume of a cylinder. • Solve practical problems that require finding the volume of a cylinder.

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.
- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.
- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking Algebra.)
- **Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking Geometry.)

7.9 The student will compare and contrast the following quadrilaterals: a parallelogram, rectangle, square, rhombus, and trapezoid. Deductive reasoning and inference will be used to classify quadrilaterals.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A quadrilateral is a closed planar (two-dimensional) figure with four sides that are line segments. • A parallelogram is a quadrilateral whose opposite sides are parallel and congruent. • A rectangle is a parallelogram with four right angles. • A square is a rectangle with four congruent sides or a rhombus with four right angles. • A rhombus is a parallelogram with four congruent sides. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. • A trapezoid with congruent, non-parallel sides is called an *isosceles trapezoid*. • Quadrilaterals can be sorted according to common attributes, using a variety of materials. • A chart or graphic organizer can be made to organize quadrilaterals according to attributes such as sides and/or angles. • For all parallelograms, both pairs of opposite sides and both pairs of opposite angles are congruent. • Parallelograms have special characteristics (such as both pairs of opposite sides are parallel and congruent) that are true for any parallelogram. • Rectangles have special characteristics (such as diagonals are perpendicular bisectors) that are true for any rectangle.	**All students should** • Understand that quadrilaterals can be classified according to the attributes of their sides and/or angles. • Understand that a quadrilateral can belong to one or more subsets of the set of quadrilaterals. • Understand that every quadrilateral in a subset has all of the defining attributes of the subset. (If a quadrilateral is a rhombus, it has all the attributes of a rhombus.)	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the classification(s) to which a quadrilateral belongs. • Compare and contrast attributes of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. • Classify quadrilaterals, using deductive reasoning and inference.

7.10 The student will identify and draw the following polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a simple closed curve in a plane that has line segments as sides. • A pentagon is a polygon with five sides. • A hexagon is a polygon with six sides. • A heptagon is a polygon with seven sides. • An octagon is a polygon with eight sides. • A nonagon is a polygon with nine sides. • A decagon is a polygon with ten sides. • Prefixes in the names of polygons tell the number of sides: penta = 5, hexa = 6, hepta = 7, octa = 8, nona = 9, and deca = 10.	**All students should** • Understand the meaning of prefixes associated with the number of sides of a polygon.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify by the number of sides or number of angles the following polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon. • Draw a pentagon, hexagon, heptagon, octagon, nonagon, and decagon, using a variety of tools.

7.11 The student will determine if geometric figures — quadrilaterals and triangles — are similar and write proportions to express the relationships between corresponding parts of similar figures.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Two polygons are similar if corresponding (matching) angles are congruent and the lengths of corresponding sides are proportional. • Congruent polygons are similar polygons for which the ratio of the corresponding sides is 1:1.	**All students should** • Understand that similar geometric figures have the same shape but may have different sizes. • Understand how ratios and proportions can be used to determine the length of something that cannot be measured directly.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify corresponding sides and corresponding angles of similar figures. • Write proportions to express the relationships between the lengths of corresponding sides of similar figures. • Examine congruence of corresponding angles and proportionality of corresponding sides to determine if quadrilaterals or triangles are similar.

7.12 The student will identify and graph ordered pairs in the four quadrants of a coordinate plane.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In a coordinate plane, the horizontal axis is typically called the *x-axis,* and the vertical axis is typically called the *y-axis*. The coordinates of a point are typically represented by the ordered pair (x, y), where x is the first coordinate and y is the second coordinate. However, any letters may be used to label the axes and the corresponding ordered pairs. • In a plane, a point can be located by its distances from two intersecting perpendicular number lines. The distance from one line is measured along a line parallel to the other line. • The first coordinate of a point is its distance from the vertical number line along a horizontal line. • The second coordinate of a point is its distance from the horizontal number line along a vertical line. • The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular number lines. Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II, (–,+); for quadrant III, (–, –); and for quadrant IV, (+,–).	**All students should** • Understand that the coordinates of a point define its location in a coordinate plane.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and label the axes of a coordinate plane. • Identify and label the quadrants of a coordinate plane. • Identify the quadrant in which an ordered pair is positioned by examining the ordered pair. • Graph ordered pairs in the four quadrants of a coordinate plane. • Identify ordered pairs represented by points in the four quadrants of the coordinate plane.

7.13 The student, given a polygon in the coordinate plane, will represent transformations — rotation and translation — by graphing the coordinates of the vertices of the transformed polygon and sketching the resulting figure.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A rotation of a geometric figure is a turn of the figure around a fixed point. The point may or may not be on the figure. The fixed point is called the *center of rotation*. • A translation of a geometric figure is a slide of the figure in which all the points on the figure move the same distance in the same direction.	**All students should** • Understand that the size or shape of a figure does not change by a translation or rotation. • Understand that translations and rotations usually change the position of the figure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the coordinates of the image of a polygon that has been translated either vertically or horizontally. • Identify the coordinates of the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin. • Sketch the image of a polygon translated vertically or horizontally. • Sketch the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin.

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of central tendency and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

7.14 The student will investigate and describe the difference between the probability of an event found through simulation versus the theoretical probability of that same event.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Theoretical probability of an event is the expected probability and can be found with a formula. • Theoretical probability of an event = $\frac{\text{number of possible favorable outcomes}}{\text{total number of possible outcomes}}$ • The experimental probability of an event is determined by carrying out a simulation or an experiment. • The experimental probability = $\frac{\text{number of times desired outcomes occur}}{\text{number of trials in the experiment}}$ • In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability (Law of Large Numbers).	**All students should** • Understand the meaning of theoretical probability. • Understand that experimental probability of an event is determined by carrying out a simulation or an experiment. • Understand the difference between theoretical and experimental probability. • Understand that in experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the theoretical probability of an event. • Describe changes in the experimental probability as the number of trials increases. • Investigate and describe the difference between the probability of an event found through simulation versus the theoretical probability of that same event.

7.15 The student will identify and describe the number of possible arrangements of several objects, using a tree diagram or the Fundamental (Basic) Counting Principle.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Tree diagrams are used to illustrate possible arrangements of objects.  • The Fundamental Counting Principle is a computational procedure to determine the number of possible arrangements of several objects. It is the product of the number of ways each object can be chosen individually (e.g., the possible arrangements of four shirts, two pants, and three shoes is $4 \cdot 2 \cdot 3$ or 24).	**All students should** • Understand that tree diagrams are used to illustrate the possible combinations (outcomes) in a sample space. • Understand that the Fundamental (Basic) Counting Principle is a computational procedure used to determine the number of possible arrangements (combinations or outcomes) of several objects.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the number of possible arrangements of no more than three types of objects, using a tree diagram. • Compute the number of possible arrangements of no more than three types of objects by using the Fundamental (Basic) Counting Principle.

7.16 The student will create and solve problems involving the measures of central tendency (mean, median, mode) and the range of a set of data.

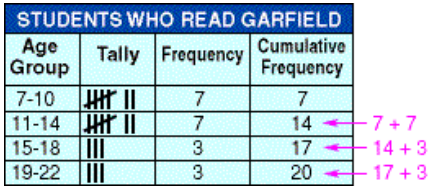
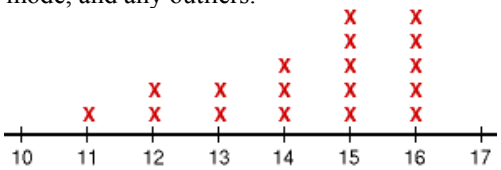
UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Measures of central tendency are types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of central tendency that are useful for describing the average for different situations. • Mean works well for sets of data with no very high or low numbers. • Median is a good choice when data sets have a couple of values much higher or lower than most of the others. • Mode is a good descriptor to use when the set of data has some identical values. • The mean of a set of numbers is the sum of the set of numbers divided by the number of numbers in the set. • The median is the middle number of a set of data when the numbers are arranged from least to greatest or the mean of the two middle numbers when the set has two middle numbers. • The mode is the number that appears most frequently in a set of data. There may be one, more than one, or no mode. • The range is the difference between the greatest number and the least number in a set of data. continued	**All students should** • Understand and appropriately use measures of central tendency for a data set. • Understand that range indicates how data is spread out or dispersed.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Examine the range to understand spread or dispersion of the data. • Describe the three measures of central tendency (mean, median, or mode) and situations in which each would best represent the data for data sets with no more than 20 data points. • Solve problems by finding the mean of a set of no more than 20 numbers. • Solve problems by finding the median of a set of data of no more than 20 numbers when the numbers are arranged from least to greatest, including data sets that have one middle number and data sets that have two middle numbers. • Solve problems by finding the mode of a set of data of no more than 20 numbers. • Identify the mode in a set of data, given that there may be one, more than one, or no mode. • Solve problems by finding the range of a set of data of no more than 20 numbers. • Solve problems with multiple identical data points for which the mode is the best descriptor. Two modes are sufficient for data sets with no more than 20 data points. continued

7.16 The student will create and solve problems involving the measures of central tendency (mean, median, mode) and the range of a set of data.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • Range indicates how data is spread out or dispersed. • For any given problem situation involving a set of data, the analysis is likely to include examination of measures of central tendency and dispersion of this data.		continued • Solve problems in which some data points are outliers and thus, for which the median is the best descriptor. The data sets will contain no more than 20 data points. • Solve problems in which the data points are similar and thus, for which the mean is the best descriptor. The data sets will contain no more than 20 data points.

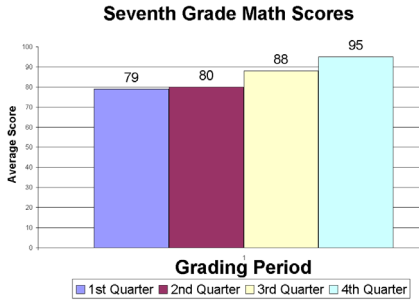
7.17 The student, given a problem situation, will collect, analyze, display, and interpret data, using a variety of graphical methods, including

a) frequency distributions;
b) line plots;
c) histograms;
d) stem-and-leaf plots;
e) box-and-whisker plots; and
f) scattergrams.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• All graphs tell a story and include a title and labels that describe the data. • A frequency distribution shows how often an item, a number, or a range of numbers occurs.  • A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range, mode, and any outliers.  continued	**All students should** • Understand that graphs tell a story. • Understand that data can be displayed in a variety of graphical representations. • Select and use appropriate statistical methods to analyze data. • Understand that different types of graphs can be used to represent the same data in a variety of ways.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect, analyze, display, and interpret a data set of no more than 20 items, using frequency distributions. • Collect, analyze, display, and interpret a data set of no more than 20 items, using line plots. • Collect, analyze, display, and interpret a data set of no more than 20 items, using histograms. • Collect, analyze, display, and interpret a data set of no more than 20 items, using stem-and-leaf plots where the stem is listed in ascending order and the leaves are in ascending order with or without commas between leaves. • Collect, analyze, display, and interpret a data set of no more than 20 items, using box-and-whisker plots that identify the minimum, maximum, median, the upper- and lower-extremes, range, and interquartile range. • Collect, analyze, display, and interpret a data set of no more than 20 items, using scattergrams. • Interpret data points in scattergrams as having positive, negative, or no relationship.

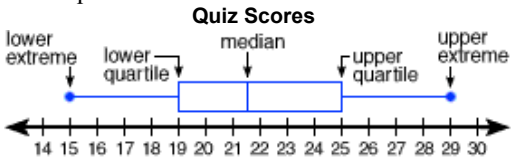
7.17 The student, given a problem situation, will collect, analyze, display, and interpret data, using a variety of graphical methods, including

a) frequency distributions;
b) line plots;
c) histograms;
d) stem-and-leaf plots;
e) box-and-whisker plots; and
f) scattergrams.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A histogram is a form of bar graph in which the categories are consecutive and equal intervals. The length or height of each bar is determined by the number of data elements falling into a particular interval.  • A stem-and-leaf plot displays data from least to greatest using the digits of the greatest place value to group data. **Number of Sit-Ups** Stem | Leaves 3 | 4 6 8 8 4 | 0 3 6 7 7 5 | 0 0 1 2 Each tens digit is called the *stem.* The ones digits are called the *leaves.* continued		

7.17 The student, given a problem situation, will collect, analyze, display, and interpret data, using a variety of graphical methods, including

a) frequency distributions;
b) line plots;
c) histograms;
d) stem-and-leaf plots;
e) box-and-whisker plots; and
f) scattergrams.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A box-and-whisker plot uses a rectangle to represent the middle 50% of a set of data and "whiskers" at both ends to represent the remainder of the data. The five critical points in a box-and-whisker plot are lower extreme, lower quartile, median, upper quartile, and upper extreme. Each of these points represents the bounds for the four quartiles. The range is the difference between the upper extreme and the lower extreme. The interquartile range is the difference between the upper quartile and the lower quartile.  • A scattergram illustrates the relationship between two sets of data. A scattergram consists of points. The coordinates of a point represent the measures of two attributes of the point.		

7.18 The student will make inferences, conjectures, and predictions based on analysis of a set of data.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Inferences, conjectures, and predictions are based on careful data analysis. • Methods of collecting and analyzing data are factors in determining the validity of any inferences or arguments based on the data.	**All students should** • Understand that data analysis supports inferences, conjectures, and predictions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Organize data not exceeding 20 items into tables and/or graphs that provide a clear representation of dispersion or convergence of the data. • Determine patterns and relationships within data sets (e.g., trends). • Make inferences, conjectures, and predictions based on analysis of a set of data not exceeding 20 items.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving real-life problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

7.19 The student will represent, analyze, and generalize a variety of patterns, including arithmetic sequences and geometric sequences, with tables, graphs, rules, and words in order to investigate and describe functional relationships.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A function is a rule that pairs exactly one element of a set with one and only one element of another set. An example of a function is that which relates the sum, s, of the measures of the interior angles of a polygon to the number, n, of sides: $s = (n - 2) \cdot 180$. • In the numeric pattern of an arithmetic sequence, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. • In geometric sequences, students must determine what each number is multiplied by in order to obtain the next number in the geometric sequence. This multiplier is called the *common ratio*. Sample geometric sequences include – 2, 4, 8, 16, 32, …; – 1, 5, 25, 125, 625, …; and – 80, 20, 5, 1.25, ….	**All students should** • Understand that patterns in mathematics are often represented by using a rule that relates elements in one set to elements in another set. • Understand that rules that relate elements in two sets can be represented by word sentences, equations, tables of values, or graphs.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Analyze situations to discover a variety of patterns. • Analyze numeric and geometric sequences to discover a variety of patterns. • Represent a variety of patterns, using tables, graphs, rules, and words, in order to investigate and describe functional relationships. • Generalize a variety of patterns.

7.20 The student will write verbal expressions as algebraic expressions and sentences as equations.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A verbal expression is a word phrase (e.g., "the sum of two consecutive integers"). • A verbal sentence is a complete word statement (e.g., "The sum of two consecutive integers is five."). • An algebraic expression is a variable expression that contains at least one variable (e.g., $2x - 5$). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x + 1 = 5$). • Key words in translating verbal expressions/ sentences to algebraic expressions/equations may include words and their translations such as: *is* to =, *of* to *multiplication*, *more than* to +, *less than* to –, *increased by* to +, and *decreased by* to –.	**All students should** • Understand that word phrases and sentences can be represented as algebraic expressions and equations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write verbal expressions as algebraic expressions. • Write verbal sentences as algebraic equations.

7.21 The student will use the following algebraic terms appropriately: *equation, inequality,* and *expression*.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An expression is a name for a number. • An expression that contains a variable is a variable expression. • An expression that contains only numbers is a numerical expression. • An equation is a mathematical sentence that states that two expressions are equal. • An inequality is a mathematical sentence that states that one quantity is less than (or greater than) another quantity.	**All students should** • Understand that different words have specific and different meanings to describe algebraic relationships.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Apply the following algebraic terms appropriately: *equation, inequality,* and *expression*. • Identify examples of equations, inequalities, and expressions.

7.22 The student will

a) solve one-step linear equations and inequalities in one variable with strategies involving inverse operations and integers, using concrete materials, pictorial representations, and paper and pencil; and

b) solve practical problems requiring the solution of a one-step linear equation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A one-step equation is defined as an equation that requires the use of one operation to solve (e.g., $x + 3 = -4$). • A one-step inequality is defined as an inequality that requires the use one operation to solve (e.g., $x - 4 > 9$). • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • When both expressions of an inequality are multiplied or divided by a negative number, the inequality symbol reverses (e.g., $-3x < 15$ is equivalent to $x > -5$).	**All students should** • Understand that an operation that is performed on one side of an equation must be performed on the other side to maintain equality. • Understand procedures for solving inequalities. • Understand that when both expressions are multiplied or divided by a negative number, the inequality symbol reverses.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent and demonstrate steps in solving equations in one variable, using concrete materials, pictorial representations, and algebraic sentences. • Represent and demonstrate steps in solving inequalities in one variable, using concrete materials, pictorial representations, and algebraic sentences. • Translate one-step word problems and practical problems into algebraic equations and solve them.