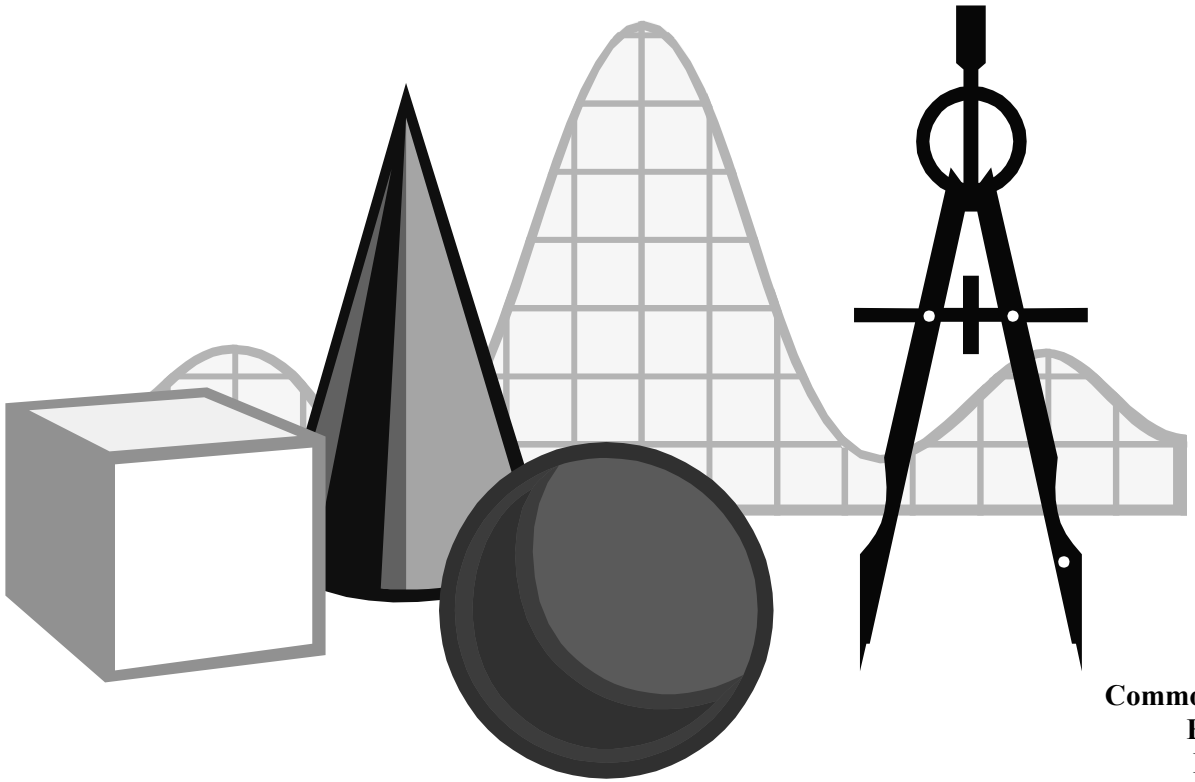
MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 8



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The 2002 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's website at http://www.pen.k12.va.us.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving real-life problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.
- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational-number representations is encouraged when students solve problems.
- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.
- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

8.1 The student will

a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers;

b) recognize, represent, compare, and order rational numbers expressed in scientific notation; and

c) compare and order decimals, fractions, percents, and numbers written in scientific notation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• *Expression* is a word used to designate any symbolic mathematical phrase that may contain numbers and/or variables. Expressions do not contain an equal sign. • A numerical expression contains only numbers and the operations on those numbers. • Expressions are simplified using the order of operations and the properties for operations with real numbers, i.e., associative, commutative, and distributive properties. • The order of operations defines the order in which operations are performed to simplify an expression. • The order of operations is as follows: – Complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols, do the innermost operation first. – Evaluate all exponential expressions. – Multiply and/or divide in order from left to right. – Add and/or subtract in order from left to right. continued	**All students should** • Understand that any real number can be shown on a number line. • Understand that integers, positive and negative fractions, and positive and negative decimals are rational numbers. • Understand that an expression is like a phrase in that it has no equal sign. • Understand that the order of operations describes the order to use to compute with rational numbers. • Understand that numbers can be represented as decimals, fractions, percents, and in scientific notation. • Understand and use strategies to simplify expressions and to compare and order numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify numerical expressions containing exponents where the base is a rational number and the exponent is a positive whole number, using the order of operations and properties of operations with real numbers. • Recognize, represent, compare, and order rational numbers expressed in scientific notation, using both positive and negative exponents. • Compare and order fractions, decimals, percents, and numbers written in scientific notation.

8.1 The student will

a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers;

b) recognize, represent, compare, and order rational numbers expressed in scientific notation; and

c) compare and order decimals, fractions, percents, and numbers written in scientific notation.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
continued • A power of a number represents repeated multiplication of the number. For example, $(-5)^3$ means $(-5) \cdot (-5) \cdot (-5)$. The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In this example, (–5) is the base, and 3 is the exponent. • Scientific notation is used to represent very large or very small numbers. • A number written in scientific notation is the product of two factors: a decimal greater than or equal to one but less than 10 multiplied by a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $3.1 \times 10^{-5} = 0.000031$). • Any real number raised to the zero power is 1. The only exception to this rule is zero itself. • Scientific calculators, graphing calculators, and some four-function calculators follow the rules of order of operations.		

8.2 The student will describe orally and in writing the relationship between the subsets of the real number system.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The set of natural numbers is the set of counting numbers (1, 2, 3, 4, ...). • The set of whole numbers is the set of all the natural numbers and zero. • The set of integers is the set of whole numbers and their opposites. • The set of rational numbers is the set of all numbers that can expressed as fractions in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero. • The set of irrational numbers is the set of all non-repeating, nonterminating decimals. • The set of real numbers is the set of all rational and irrational numbers. An irrational number cannot be expressed as an integer or the quotient of integers.	**All students should** • Understand the relationship between the subsets of the real number system.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe orally and in writing the relationships among the sets of Natural or Counting Numbers, Whole Numbers, Integers, Rational Numbers, Irrational Numbers, and Real Numbers. • Illustrate the relationships among the subsets of the real number system by using graphic organizers such as Venn diagrams. Subsets include real numbers, rational numbers, irrational numbers, integers, whole numbers, and natural numbers. • Identify the subsets of the real number system to which a given number belongs. • Determine whether a given number is a member of a particular subset of the real number system, and explain why. • Describe each subset of the set of real numbers.

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem-solving and application activities.

8.3 The student will solve practical problems involving rational numbers, percents, ratios, and proportions. Problems will be of varying complexities and will involve real-life data, such as finding a discount and discount prices and balancing a checkbook.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Practical problems may include, but not be limited to, those related to economics, sports, science, social sciences, transportation, and health. Some examples include problems involving the amount of a pay check per month, the discount price on a product, temperature, simple interest, sales tax, and installment buying. • A percent is a special ratio in which the denominator is 100. • A discount is a percentage of the original price. The discount price is the original price minus the discount.	**All students should** • Understand how mathematics relates to problems in daily life. • Select an appropriate method or methods for computing with rational numbers and percents according to the context of the problem. • Understand how to set up a proportion given the relationship between two items.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve practical problems by using computation procedures for whole numbers, integers, rational numbers, percents, ratios, and proportions. • Maintain a checkbook and check registry for five or fewer transactions. • Compute a discount and the resulting (sale) price for one discount.

8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables. Problems will be limited to positive exponents.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Algebraic expressions use operations with algebraic symbols (variables). • Algebraic expressions are evaluated by replacing the variables with numbers and applying the order of operations to simplify the resulting expression. • The replacement values are the numbers that replace the variables in an algebraic expression. • The order of operations is as follows: – Complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols, do the innermost operation first. – Evaluate all exponential expressions. – Multiply and/or divide in order from left to right. – Add and/or subtract in order from left to right.	**All students should** • Evaluate an algebraic expression by substituting a number for each variable and then simplifying the result. • Understand how to apply the order of operations after substituting given values for variables in algebraic expressions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Substitute numbers for variables in an algebraic expression and simplify the expression by using the order of operations. Exponents used are whole numbers less than 4. • Apply the order of operations to evaluate formulas.

8.5 The student, given a whole number from 0 to 100, will identify it as a perfect square or find the two consecutive whole numbers between which the square root lies.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A perfect square is a whole number whose square root is a whole number (e.g., $\sqrt{25}$ is 5; thus, 25 is a perfect square). • The square root of a number is that number which when multiplied by itself equals the number. • Any whole number other than a perfect square has a square root that lies between two consecutive whole numbers. • The square root of a whole number that is not a perfect square is an irrational number (e.g., $\sqrt{2}$ is an irrational number). An irrational number cannot be expressed exactly as a ratio.	**All students should** • Understand that a perfect square is the product of a number multiplied by itself. • Develop strategies for finding the square root of a number.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the perfect squares from 0 to 100. • Identify the two consecutive whole numbers between which the square root of a given whole number from 0 to 100 lies (e.g., $\sqrt{57}$ lies between 7 and 8 since $7^2 = 49$ and $8^2 = 64$).

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- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

8.6 The student will verify by measuring and describe the relationships among vertical angles, supplementary angles, and complementary angles and will measure and draw angles of less than 360°.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Vertical angles are the opposite angles formed by two intersecting lines. Vertical angles are congruent. • Complementary angles are any two angles such that the sum of their measures is 90°. • Supplementary angles are any two angles such that the sum of their measures is 180°.	**All students should** • Understand the meaning of the term *angle*. • Understand how to use angle-measuring tools. • Understand that pairs of angles are named by their defining attributes.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Measure angles of less than 360° to the nearest degree, using appropriate tools. • Identify and describe the relationships among the angles formed by two intersecting lines. • Identify and describe pairs of angles that are vertical. • Identify and describe pairs of angles that are supplementary. • Identify and describe pairs of angles that are complementary.

8.7 The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polyhedron is a solid figure whose faces are all polygons. • A pyramid is a polyhedron with a base that is a polygon and other faces that are triangles with a common vertex. – The lateral area of a pyramid is the sum of the areas of the triangular faces. – The area of the base of a pyramid is the area of the polygon which is the base. – The total surface area of a pyramid is the sum of the lateral area and the area of the base. – The volume of a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height. • A circular cone is a geometric solid whose base is a circle and whose side is a surface composed of line segments connecting points on the base to a fixed point (the vertex) not on the base. – The lateral area of a circular cone is the area of the surface connecting the base with the vertex and is equal to πrl, where l is the slant height. – The area of the base of a circular cone is πr^2. – The total surface area of a circular cone is $\pi r^2 + \pi rl$. – The volume of a cone is $\frac{1}{3}\pi r^2 h$, where h is the height and πr^2 is the area of the base.	**All students should** • Understand the derivation of formulas for volume and surface area of prisms, cylinders, cones, and pyramids. • Understand the differences between volume and surface area.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compute the surface area of a pyramid by finding the sum of the areas of the triangular faces and the base. • Compute the surface area of a cone by calculating the sum of the areas of the side and the base, using formulas. • Compute the volume and surface area of rectangular solids (prisms), cylinders, cones, and square pyramids, using formulas. • Investigate and solve problems involving volume and surface area of rectangular solids (prisms), cylinders, cones and pyramids.

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.
- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.
- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking Algebra.)
- **Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking Geometry.)

8.8 The student will apply transformations (rotate or turn, reflect or flip, translate or slide, and dilate or scale) to geometric figures represented on graph paper. The student will identify applications of transformations, such as tiling, fabric design, art, and scaling.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A rotation of a geometric figure is a turn of the figure around a fixed point. The point may or may not be on the figure. The fixed point is called the *center of rotation.* • The rotation of a figure can be made clockwise or counterclockwise around a fixed point. • A reflection of a geometric figure is a flip of the figure across a line. Each point on the reflected figure is the same distance from the line as the corresponding point in the original figure. • A translation of a geometric figure is a slide of the figure in which all the points on the figure move the same distance in the same direction. • A dilation of a geometric figure is a transformation that changes the size of a figure by a scale factor to create a similar figure. • Real-life applications may include the following: – A rotation of the hour hand of a clock from 2:00 to 3:00 shows a turn of 30° clockwise. – A reflection of a boat in water shows an image of the boat flipped upside down with the water line being the line of reflection. – A translation of a shape on a wallpaper pattern shows the same shape slid the same distance in the same direction. – A dilation of a model airplane is the production model of the airplane.	**All students should** • Understand the relationship between transformations in a coordinate plane and their application in real-life.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the geometric transformations (rotation, reflection, translation, and dilation) by using a variety of real-life examples • Demonstrate the reflection of a figure over a vertical or horizontal line on a coordinate grid. • Demonstrate 90° 180°, 270°, and 360° rotations of a figure on a coordinate grid. • Demonstrate the translation of a figure on a coordinate grid. • Demonstrate the dilation of a figure from a fixed point on a coordinate grid.

8.9 The student will construct a three-dimensional model, given the top, side, and/or bottom views.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Three-dimensional models of geometric solids can be used to understand perspective and provide tactile experiences in determining two-dimensional perspectives. • Three-dimensional models of geometric solids can be represented on isometric paper.	**All students should** • Understand that a three-dimensional object can be represented as a two-dimensional model that represents views of the object from different perspectives.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct three-dimensional models, given top, side, and bottom views.

8.10 The student will

a) verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement; and

b) apply the Pythagorean Theorem to find the missing length of a side of a right triangle when given the lengths of the other two sides.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the legs (altitude and base). This relationship is known as the Pythagorean Theorem: $a^2 + b^2 = c^2$. • The Pythagorean Theorem is used to find the measure of any one of the three sides of a right triangle if the measures of the other two sides are known. • Whole number triples that are the measures of the sides of right triangles, such as (3,4,5), (6,8,10), (9,12,15), and (5,12,13), are commonly known as Pythagorean triples.	**All students should** • Understand that, for a right triangle, the square of the measure of the hypotenuse equals the sum of the squares of the measures of the base and altitude. • Understand that the Pythagorean Theorem is a tool to find the measure of any side of a right triangle, given the measures of the other two sides.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the parts of a right triangle (the hypotenuse and the legs). • Verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement. • Find the measure of a side of a right triangle, given the measures of the other two sides. The measures of the sides of the triangle may be whole numbers no larger than 15 or decimals in tenths. • Solve real-life problems involving right triangles by using the Pythagorean Theorem.

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of central tendency and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

8.11 The student will analyze problem situations, including games of chance, board games, or grading scales, and make predictions, using knowledge of probability.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The probability of an event occurring is the ratio of the desired outcomes to the total number of possible outcomes. • Choices that involve chance are based on an understanding of the reasonableness of obtaining a specific outcome. • The probability that an event is likely to occur is close to one. • The probability that an event is not likely to occur is close to zero. • The probability that an event is as likely to occur as it is not to occur is close to one-half. • Knowledge of probability can be used to determine the likelihood of winning such events as a prize through a raffle or lottery ticket, earning a high score in a dart game, or winning a card game.	**All students should** • Understand how to make predictions based on knowledge of probability. • Understand that choices that involve chance are based on an understanding of the reasonableness of obtaining a specific outcome. • Understand that knowledge of probability can be used to determine the likelihood of winning such events as contests.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Analyze a problem situation, and determine the likelihood of an event occurring, using knowledge of probability. • Predict the outcome of an event by analyzing its probability. • Explain the consequences of making different choices, using knowledge of probability. • Make predictions about the outcomes of games of chance, board games, and grading scales by using knowledge of probability.

8.12 The student will make comparisons, predictions, and inferences, using information displayed in frequency distributions; box-and-whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • The information displayed in different graphs may be examined to determine how data are or are not related, ascertaining differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inferences). • Scattergrams can be used to predict trends and estimate a line of best fit.	**All students should** • Understand that comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations in order to draw conclusions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Make comparisons, predictions, and inferences, given data sets of no more than 20 items that are displayed in frequency distributions; box-and-whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms.

8.13 The student will use a matrix to organize and describe data.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Matrices are used to sort, list, and organize data, such as sorting by gender the number of soccer, football, basketball, and softball players in a set. • A matrix is a rectangular array of numbers in rows and columns that are enclosed with brackets. A matrix organizes a data set visually. • A matrix is identified by its dimensions, rows, and columns (e.g., a 2-by-3 matrix has two rows and three columns).	**All students should** • Understand that a matrix is a rectangular array of numbers in rows and columns that organizes a data set visually.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the characteristics of a matrix, including designating labels for rows and columns. • Use a matrix of no more than 12 entries to organize and describe a data set. • Identify the position of an element by row and column. • Transfer data from a chart to a matrix.

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- Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

8.14 The student will

a) describe and represent relations and functions, using tables, graphs, and rules; and

b) relate and compare tables, graphs, and rules as different forms of representation for relationships.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A relation is any set of ordered pairs. For each first member (domain), there may be many second members (range). • A function is a relation in which there is one and only one second member (range) for each first member (domain). • As a graph, a function is any curve (including straight lines) such that any vertical line would pass through the curve only once. • As a table of values, a function has a unique value assigned to the second variable (range) for each value of the first variable (domain). • Some relations are functions; all functions are relations.	**All students should** • Understand the difference between functions and relations. • Understand that a function is a one-to-one relationship between the domain and range.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Graph in a coordinate plane ordered pairs that represent a relation. • Write a rule that represents a relation from a table of values. • Write a table of values from the rule that represents a relation. • Write a table of values from the graph of ordered pairs of a relation. • Describe and represent relations and functions, using tables, graphs, and rules. • Relate and compare different representations of the same relation.

8.15 The student will solve two-step equations and inequalities in one variable, using concrete materials, pictorial representations, and paper and pencil.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A two-step equation is defined as an equation that requires the use of two different operations to solve (e.g., $2x + 3 = -4$). • A two-step inequality is defined as an inequality that requires the use of two different operations to solve (e.g., $3x - 4 > 9$). • In an equation, the equal sign indicates that the value on the left is the same as the value on the right. • To maintain equality, an operation that is performed on one side of an equation must be performed on the other side. • The same procedures that work for equations work for inequalities. When both expressions of an inequality are multiplied or divided by a negative number, the inequality sign reverses.	**All students should** • Understand that an operation that is performed on one side of an equation must be performed on the other side to maintain equality. • Understand the procedures for solving inequalities. • Understand that when both expressions are multiplied or divided by a negative number, the inequality symbol reverses.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve two-step linear equations by showing the steps and using algebraic sentences. • Solve two-step inequalities by showing the steps and using algebraic sentences.

8.16 The student will graph a linear equation in two variables, in the coordinate plane, using a table of ordered pairs.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A linear equation is an equation in two variables whose graph is a line. • Graphing a linear equation requires determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. • The axes of a coordinate plane are generally labeled *x* and *y*; however, any letters may be used that are appropriate for the function.	**All students should** • Understand that the graph of a linear equation in two variables is the set of all ordered pairs that satisfy the equation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a table of ordered pairs by substituting values for *x* in a linear equation to find values for *y*. • Plot in the coordinate plane ordered pairs (*x*, *y*) from a table. • Connect the ordered pairs to form a straight line.

8.17 The student will create and solve problems, using proportions, formulas, and functions.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A proportion is a statement of equality between two ratios. • A formula is a statement based on logical mathematical conclusions or observation and experimental evidence (e.g., the volume of a circular cylinder, $V = \pi r^2 h$). • A function is a relation (any set of ordered pairs) in which there is one and only one second member (range) for each first member (domain). • Proportional situations are based on multiplicative relationships. Equal ratios result from multiplication or division, not from addition or subtraction.	**All students should** • Understand that proportions, formulas, and functions are ways to express relationships mathematically and are tools for solving problems.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write problems that require establishing a relationship between ratios. • Solve problems by using proportions. • Create problems that require the use of a formula. • Substitute known values for variables in a formula. • Solve a formula by using algebraic procedures. • Create problems that involve a functional relationship. • Solve problems that involve functions.

8.18 The student will use the following algebraic terms appropriately: *domain, range, independent variable,* and *dependent variable.*

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The domain is the possible set of all the input values for the independent variable in a given situation. • The range is the possible set of all the output values for the dependent variable in a given situation. • The independent variable is the input value. • The dependent variable depends on the independent variable and is the output value. • Below is a table of values for finding the circumference of circles, $C = \pi d$, where the value of π is approximated as 3.14. • The independent variable, or input, is the diameter of the circle. The values for the diameter make up the domain. • The dependent variable, or output, is the circumference of the circle. The set of values for the circumference make up the range.	**All students should** • Understand that the domain represents all the values for the independent variables and the range represents all the values for the dependent variables. • Understand that the independent variable is the value that causes the change in the dependent variable and the dependent variable is affected by the independent variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Apply the following algebraic terms appropriately: *domain, range, independent variable,* and *dependent variable.* • Identify examples of domain, range, independent variable, and dependent variable. • Determine the domain of a function. • Determine the range of a function. • Determine the independent variable of a relationship. • Determine the dependent variable of a relationship.

Diameter	Circumference
1 in.	3.14 in.
2 in.	6.28 in.
3 in.	9.42 in.
4 in.	12.56 in.