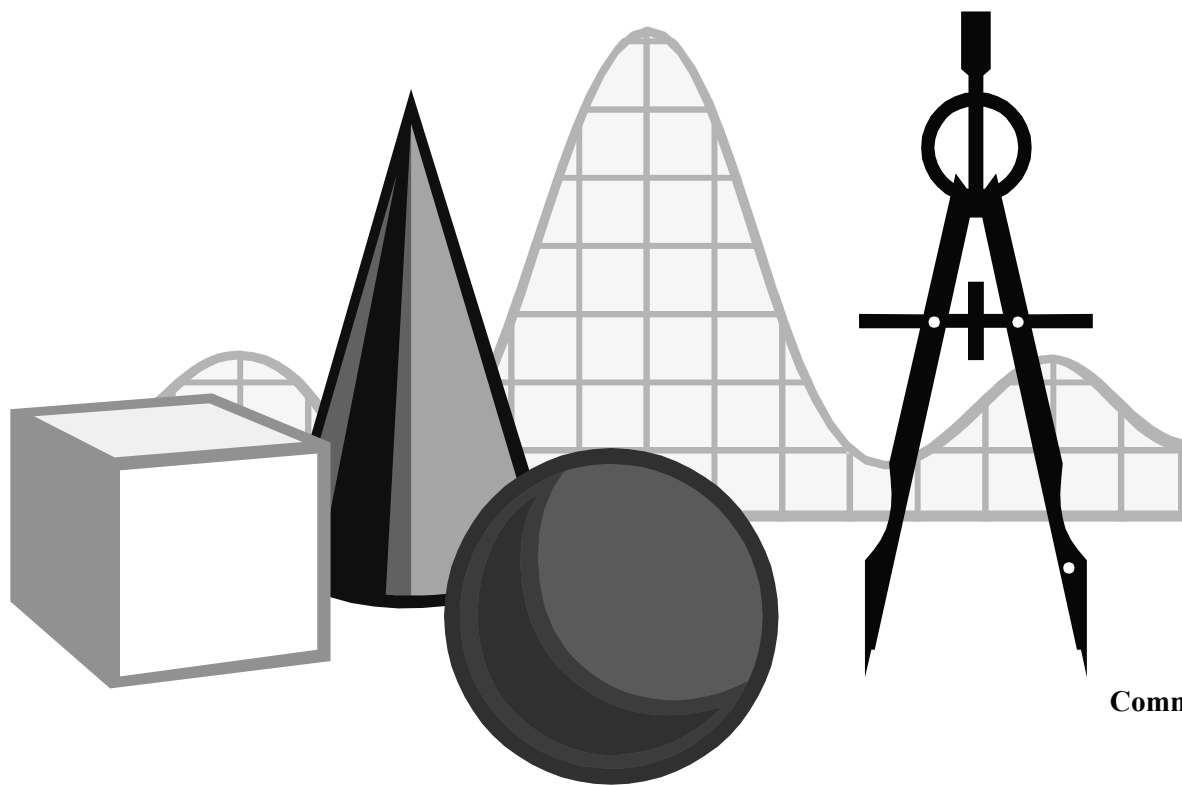


MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Mathematical Analysis



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The 2002 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's website at <http://www.pen.k12.va.us>.

Introduction

Mathematics content develops sequentially in concert with a set of processes that are common to different bodies of mathematics knowledge. The content of the Mathematics Standards of Learning supports five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. These goals provide a context within which to develop the knowledge and skills identified in the standards.

Mathematical Analysis serves as a preparatory course for a study of calculus. Mathematical Analysis extends the study of families of functions and also includes sequences and series, polar equations, vectors, and parametric equations. An intuitive introduction to the concept of the limit of an algebraic function may be enhanced with algebraic methods or numerical substitution.

Each topic in the Mathematical Analysis Curriculum Framework is developed around the Standards of Learning. Each Standard of Learning is expanded in the Essential Knowledge and Skills column. The Essential Understandings column includes concepts, mathematical relationships, and ideas that are important to understanding and teaching the Standard of Learning effectively.

Teachers should help students make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections should be made to other subject areas and fields of endeavor through applications. Using manipulatives, graphing calculators, and computer applications to develop concepts should help students develop and attach meaning to abstract ideas. Throughout the study of mathematics, students should be encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop their competence and their confidence in themselves as mathematics students.

**MATHEMATICAL ANALYSIS
STANDARD MA.1**

The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, y -intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.

ESSENTIAL UNDERSTANDINGS

- The graphs of polynomial and rational functions can be determined by exploring characteristics and components of the functions.

ESSENTIAL KNOWLEDGE AND SKILLS

- Identify a polynomial function, given an equation or graph.
- Identify rational functions, given an equation or graph.
- Identify zeros, upper and lower bounds, y -intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, points of discontinuity, end behavior, and maximum and minimum points, given a graph of a function.
- Sketch the graph of a polynomial function.
- Sketch the graph of a rational function.
- Investigate and verify characteristics of a polynomial or rational function, using a graphing calculator.

**MATHEMATICAL ANALYSIS
STANDARD MA.2**

The student will find compositions of functions and inverses of functions. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions.

STANDARD MA.3

The student will investigate and describe the continuity of functions, using graphs. The functions will include absolute value, piecewise, and step functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> • Continuous and discontinuous functions can be identified by their equations or graphs. • In composition of functions, a function serves as input for another function. • A graph of a function and its inverse are symmetric about the line $y = x$. 	<ul style="list-style-type: none"> • Find the composition of functions. • Find the inverse of a function algebraically and graphically. • Determine the domain and range of the composite functions. • Determine the domain and range of the inverse of a function. • Identify absolute value, step, and piece-wise-defined functions. • Describe continuity of a function. • Investigate the continuity of absolute value, step, rational, and piece-wise-defined functions. • Use transformations to sketch absolute value, step, and rational functions. • Verify the accuracy of sketches of functions, using a graphing utility.

**MATHEMATICAL ANALYSIS
STANDARD MA.7**

The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.

ESSENTIAL UNDERSTANDINGS

ESSENTIAL KNOWLEDGE AND SKILLS

- The limit of a function is the value approached by $f(x)$ as x approaches a given value or infinity.

- Verify intuitive reasoning about the limit of a function, using a graphing utility.
- Find the limit of a function algebraically, and verify with a graphing utility.
- Find the limit of a function numerically, and verify with a graphing utility.

**MATHEMATICAL ANALYSIS
STANDARD MA.9**

The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems. This will include the role of e , natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. Graphing utilities will be used to investigate and verify the graphs and solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
<ul style="list-style-type: none"> Exponential and logarithmic functions are inverse functions. 	<ul style="list-style-type: none"> Identify exponential functions from an equation or a graph. Identify logarithmic functions from an equation or a graph. Define e, and know its approximate value. Write logarithmic equations in exponential form and vice versa. Identify common and natural logarithms. Use laws of exponents and logarithms to solve equations and simplify expressions. Model practical problems, using exponential and logarithmic functions. Graph exponential and logarithmic functions, using a graphing utility, and identify asymptotes, intercepts, domain, and range.

**MATHEMATICAL ANALYSIS
STANDARD MA.4**

The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal’s Triangle.

ESSENTIAL UNDERSTANDINGS

- The Binomial Theorem provides a formula for calculating the product $(a + b)^n$ for any positive integer n .
- Pascal’s Triangle is a triangular array of binomial coefficients.

ESSENTIAL KNOWLEDGE AND SKILLS

- Expand binomials having positive integral exponents.
- Use the Binomial Theorem, the formula for combinations, and Pascal’s Triangle to expand binomials.

**MATHEMATICAL ANALYSIS
STANDARD MA.5**

The student will solve practical problems involving arithmetic and geometric sequences and series. This will include finding the sum (sigma notation included) of finite and infinite convergent series that will lead to an intuitive approach to a limit.

ESSENTIAL UNDERSTANDINGS

- Examination of infinite sequences and series may lead to a limiting process.
- Arithmetic sequences have a common difference between any two consecutive terms.
- Geometric sequences have a common factor between any two consecutive terms.

ESSENTIAL KNOWLEDGE AND SKILLS

- Use and interpret the notation: Σ , n , n th, and a_n .
- Given the formula, find the n th term, a_n , for an arithmetic or geometric sequence.
- Given the formula, find the sum, S_n , if it exists, of an arithmetic or geometric series.
- Model and solve problems, using sequence and series information.
- Distinguish between a convergent and divergent series.
- Discuss convergent series in relation to the concept of a limit.

**MATHEMATICAL ANALYSIS
STANDARD MA.6**

The student will use mathematical induction to prove formulas/statements.

ESSENTIAL UNDERSTANDINGS

- Mathematical induction is a method of proof that depends on a recursive process.
- Mathematical induction allows reasoning from specific true values of the variable to general values of the variable.

ESSENTIAL KNOWLEDGE AND SKILLS

- Compare inductive and deductive reasoning.
- Prove formulas/statements, using mathematical induction.

TOPIC: POLAR AND PARAMETRIC EQUATIONS

**MATHEMATICAL ANALYSIS
STANDARD MA.10**

The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

ESSENTIAL UNDERSTANDINGS

- The real number system is represented geometrically on the number line, and the complex number system is represented geometrically on the plane where $a + bi$ corresponds to the point (a, b) in the plane.

ESSENTIAL KNOWLEDGE AND SKILLS

- Recognize polar equations (rose, cardioid, limacon, lemniscate, spiral, and circle), given the graph or the equation.
- Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility.
- Convert complex numbers from rectangular form to polar form and vice versa.
- Find the intersection of the graphs of two polar equations, using a graphing utility.

TOPIC: POLAR AND PARAMETRIC EQUATIONS

**MATHEMATICAL ANALYSIS
STANDARD MA.12**

The student will use parametric equations to model and solve application problems. Graphing utilities will be used to develop an understanding of the graph of parametric equations.

ESSENTIAL UNDERSTANDINGS

- Parametric equations are used to express two dependent variables, x and y , in terms of an independent variable (parameter), t .
- Some curves cannot be represented as a function, $f(x)$. Parametric graphing enables the representation of these curves in terms of functions.

ESSENTIAL KNOWLEDGE AND SKILLS

- Graph parametric equations, using a graphing utility.
- Use parametric equations to model motion over time.
- Determine solutions to parametric equations, using a graphing utility.
- Compare and contrast traditional solution methods with parametric methods.

**MATHEMATICAL ANALYSIS
STANDARD MA.8**

The student will investigate and identify the characteristics of conic section equations in (h, k) and standard forms. The techniques of translation and rotation of axes in the coordinate plane will be used to graph conic sections.

ESSENTIAL UNDERSTANDINGS

- Matrices can be used to represent transformations of figures in the plane.

ESSENTIAL KNOWLEDGE AND SKILLS

- Given a translation or rotation matrix, find an equation for the transformed function or conic section.
- Investigate and verify graphs of transformed conic sections, using a graphing utility.

**MATHEMATICAL ANALYSIS
STANDARD MA.11**

The student will perform operations with vectors in the coordinate plane and solve practical problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

ESSENTIAL UNDERSTANDINGS

- Every vector has an equal vector that has its initial point at the origin.
- The magnitude and direction of a vector with the origin as the initial point are completely determined by the coordinates of its terminal point.

ESSENTIAL KNOWLEDGE AND SKILLS

- Use vector notation.
- Perform the operations of addition, subtraction, scalar multiplication, and inner (dot) product on vectors.
- Graph vectors and resultant vectors.
- Express complex numbers in vector notation.
- Define *unit vector*, and find the unit vector in the same direction as a given vector.
- Identify properties of vector addition, scalar multiplication, and dot product.
- Find vector components.
- Find the norm (magnitude) of a vector.
- Use vectors in simple geometric proofs.

**MATHEMATICAL ANALYSIS
STANDARD MA.13**

The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

ESSENTIAL UNDERSTANDINGS

- Real-life problems can be modeled using trigonometry and vectors.

ESSENTIAL KNOWLEDGE AND SKILLS

- Solve and create problems, using trigonometric functions.
- Solve and create problems, using the Pythagorean Theorem.
- Solve and create problems, using the Law of Sines and the Law of Cosines.