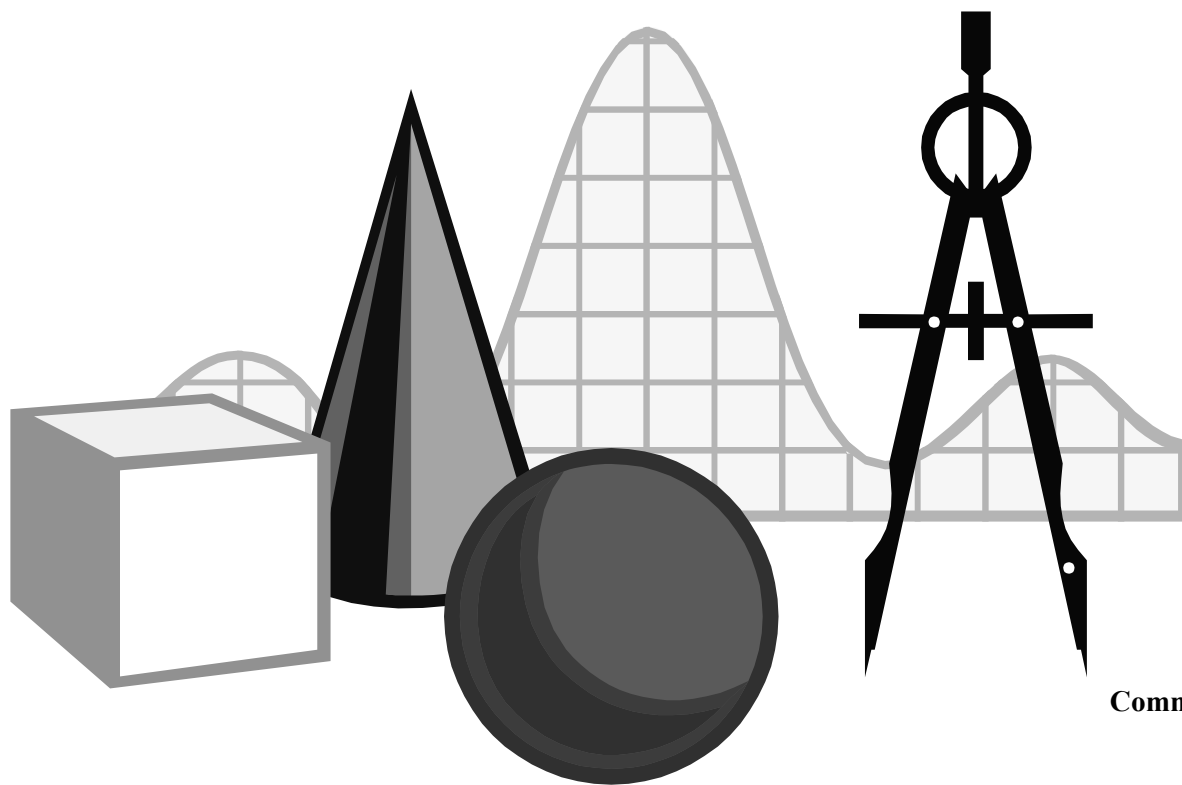


MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Trigonometry



Commonwealth of Virginia
Board of Education
Richmond, Virginia
© 2002

Copyright © 2002

by the

Virginia Department of Education

P.O. Box 2120

Richmond, Virginia 23218-2120

<http://www.pen.k12.va.us>

All rights reserved. Reproduction of materials contained herein for instructional purposes in Virginia classrooms is permitted.

Superintendent of Public Instruction

Jo Lynne DeMary

Deputy Superintendent

M. Kenneth Magill

Assistant Superintendent for Instruction

Patricia I. Wright

Office of Secondary Instructional Services

Linda M. Wallinger, Director

Deborah Kiger Lyman, Mathematics Specialist

NOTICE TO THE READER

The Virginia Department of Education does not unlawfully discriminate on the basis of sex, race, color, religion, handicapping conditions, or national origin in employment or in its educational programs and activities.

The 2002 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's website at <http://www.pen.k12.va.us>.

Introduction

Mathematics content develops sequentially in concert with a set of processes that are common to different bodies of mathematics knowledge. The content of the Mathematics Standards of Learning supports five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. These goals provide a context within which to develop the knowledge and skills identified in the standards.

Trigonometry functions model periodic phenomena, such as those associated with alternating electrical current and the Dow Jones industrial average. The study of trigonometry includes trigonometric definitions, and graphing and solving trigonometric equations and inequalities within the context of applications. Students should be encouraged to make connections among right triangle ratios, circular functions, and trigonometric functions. Communication using the language of mathematics, logic of problem solving and mathematical procedures, and interpretation of results in context are essential skills for students.

Each topic in the Trigonometry Curriculum Framework is developed around the Standards of Learning. Each Standard of Learning is expanded in the Essential Knowledge and Skills column. The Essential Understandings column includes concepts, mathematical relationships, and ideas that are important to understanding and teaching the Standard of Learning effectively.

Teachers should help students make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections should be made to other subject areas and fields of endeavor through applications. Using manipulatives, graphing calculators, and computer applications to develop concepts should help students develop and attach meaning to abstract ideas. Throughout the study of mathematics, students should be encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop their competence and their confidence in themselves as mathematics students.

TOPIC: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY
STANDARD T.1**

The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. Circular function definitions will be connected with trigonometric function definitions.

ESSENTIAL UNDERSTANDINGS

- Triangular trigonometric function definitions are related to circular trigonometric function definitions.
- Both degrees and radians are units for measuring angles.
- Drawing an angle in standard position will force the terminal side to lie in a specific quadrant.
- A point on the terminal side of an angle determines a reference triangle from which the values of the six trigonometric functions may be derived.

ESSENTIAL KNOWLEDGE AND SKILLS

- Define the six triangular trigonometric functions of an angle in a right triangle.
- Define the six circular trigonometric functions of an angle in standard position.
- Make the connection between the triangular and circular trigonometric functions.
- Recognize and draw an angle in standard position.
- Show how a point on the terminal side of an angle determines a reference triangle.

TOPIC: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY
STANDARD T.2**

The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.

ESSENTIAL UNDERSTANDINGS

- If one trigonometric function value is known, then a triangle can be formed to use in finding the other five trigonometric function values.
- Knowledge of the unit circle is a useful tool for finding all six trigonometric values for special angles.

ESSENTIAL KNOWLEDGE AND SKILLS

- Given one trigonometric function value, find the other five trigonometric function values.
- Develop the unit circle, using both degrees and radians.
- Solve problems, using the circular function definitions and the properties of the unit circle.
- Recognize the connections between the coordinates of points on a unit circle and
 - coordinate geometry;
 - cosine and sine values; and
 - lengths of sides of special right triangles (30° - 60° - 90° and 45° - 45° - 90°).

TOPIC: TRIANGULAR AND CIRCULAR TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY
STANDARD T.3**

The student will find without the aid of a calculating utility the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting radians to degrees and vice versa.

ESSENTIAL UNDERSTANDINGS

- Special angles are widely used in mathematics.
- Unit circle properties will allow special-angle and related-angle trigonometric values to be found without the aid of a calculator.
- Degrees and radians are units of angle measure.
- A radian is the measure of the central angle that is determined by an arc whose length is the same as the radius of the circle.

ESSENTIAL KNOWLEDGE AND SKILLS

- Find trigonometric function values of special angles and their related angles in both degrees and radians.
- Apply the properties of the unit circle without using a calculator.
- Use a conversion factor to convert from radians to degrees and vice versa without using a calculator.

TOPIC: INVERSE TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY
STANDARD T.4**

The student will find with the aid of a calculator the value of any trigonometric function and inverse trigonometric function.

ESSENTIAL UNDERSTANDINGS

- The trigonometric function values of any angle can be found by using a calculator.
- The inverse trigonometric functions can be used to find angle measures whose trigonometric function values are known.
- Calculations of inverse trigonometric function values can be related to the triangular definitions of the trigonometric functions.

ESSENTIAL KNOWLEDGE AND SKILLS

- Use a calculator to find the trigonometric function values of any angle in either degrees or radians.
- Define inverse trigonometric functions.
- Find angle measures by using the inverse trigonometric functions when the trigonometric function values are given.

TOPIC: INVERSE TRIGONOMETRIC FUNCTIONS

**TRIGONOMETRY
STANDARD T.7**

The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

ESSENTIAL UNDERSTANDINGS

- Restrictions on the domains of some inverse trigonometric functions exist.

ESSENTIAL KNOWLEDGE AND SKILLS

- Find the domain and range of the inverse trigonometric functions.
- Use the restrictions on the domains of the inverse trigonometric functions in finding the values of the inverse trigonometric functions.
- Identify the graphs of the inverse trigonometric functions.

TOPIC: TRIGONOMETRIC IDENTITIES

**TRIGONOMETRY
STANDARD T.5**

The student will verify basic trigonometric identities and make substitutions, using the basic identities.

ESSENTIAL UNDERSTANDINGS

- Trigonometric identities can be used to simplify trigonometric expressions, equations, or identities.
- Trigonometric identity substitutions can help solve trigonometric equations, verify another identity, or simplify trigonometric expressions.

ESSENTIAL KNOWLEDGE AND SKILLS

- Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include
 - reciprocal identities;
 - Pythagorean identities;
 - sum and difference identities;
 - double-angle identities; and
 - half-angle identities.

TOPIC: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

TRIGONOMETRY STANDARD T.6

The student, given one of the six trigonometric functions in standard form [e.g., $y = A \sin (Bx + C) + D$, where A , B , C , and D are real numbers], will

- a) state the domain and the range of the function;
- b) determine the amplitude, period, phase shift, and vertical shift; and
- c) sketch the graph of the function by using transformations for at least a one-period interval.

The graphing calculator will be used to investigate the effect of changing A , B , C , and D on the graph of a trigonometric function.

ESSENTIAL UNDERSTANDINGS

- The domain and range of a trigonometric function determine the scales of the axes for the graph of the trigonometric function.
- The amplitude, period, phase shift, and vertical shift are important characteristics of the graph of a trigonometric function, and each has a specific purpose in applications using trigonometric equations.
- The graph of a trigonometric function can be used to display information about the periodic behavior of a real-world situation, such as wave motion or the motion of a Ferris wheel.

ESSENTIAL KNOWLEDGE AND SKILLS

- Determine the amplitude, period, phase shift, and vertical shift of a trigonometric function from the equation of the function and from the graph of the function.
- Describe the effect of changing A , B , C , or D in the standard form of a trigonometric equation {e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D$ }.
- State the domain and the range of a function written in standard form {e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D$ }.
- Sketch the graph of a function written in standard form {e.g., $y = A \sin (Bx + C) + D$ or $y = A \cos [B(x + C)] + D$ } by using transformations for at least one period or one cycle.

TOPIC: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

**TRIGONOMETRY
STANDARD T.8**

The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, check for reasonableness of results, and verify algebraic solutions.

ESSENTIAL UNDERSTANDINGS

- Solutions for trigonometric equations will depend on the domains.
- A calculator can be used to find the solution of a trigonometric equation as the points of intersection of the graphs when one side of the equation is entered in the calculator as Y_1 and the other side is entered as Y_2 .

ESSENTIAL KNOWLEDGE AND SKILLS

- Solve trigonometric equations with restricted domains algebraically and by using a graphing utility.
- Solve trigonometric equations with infinite solutions algebraically and by using a graphing utility.
- Check for reasonableness of results, and verify algebraic solutions, using a graphing utility.

TOPIC: TRIGONOMETRIC EQUATIONS, GRAPHS, AND PRACTICAL PROBLEMS

**TRIGONOMETRY
STANDARD T.9**

The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

ESSENTIAL UNDERSTANDINGS

- A practical problem may be solved by using one of a variety of techniques associated with triangles.

ESSENTIAL KNOWLEDGE AND SKILLS

- Write a practical problem involving triangles.
- Solve practical problems involving triangles.
- Use the trigonometric functions, Pythagorean Theorem, Law of Sines, and Law of Cosines to solve practical problems.
- Identify a solution technique that could be used with a given problem.