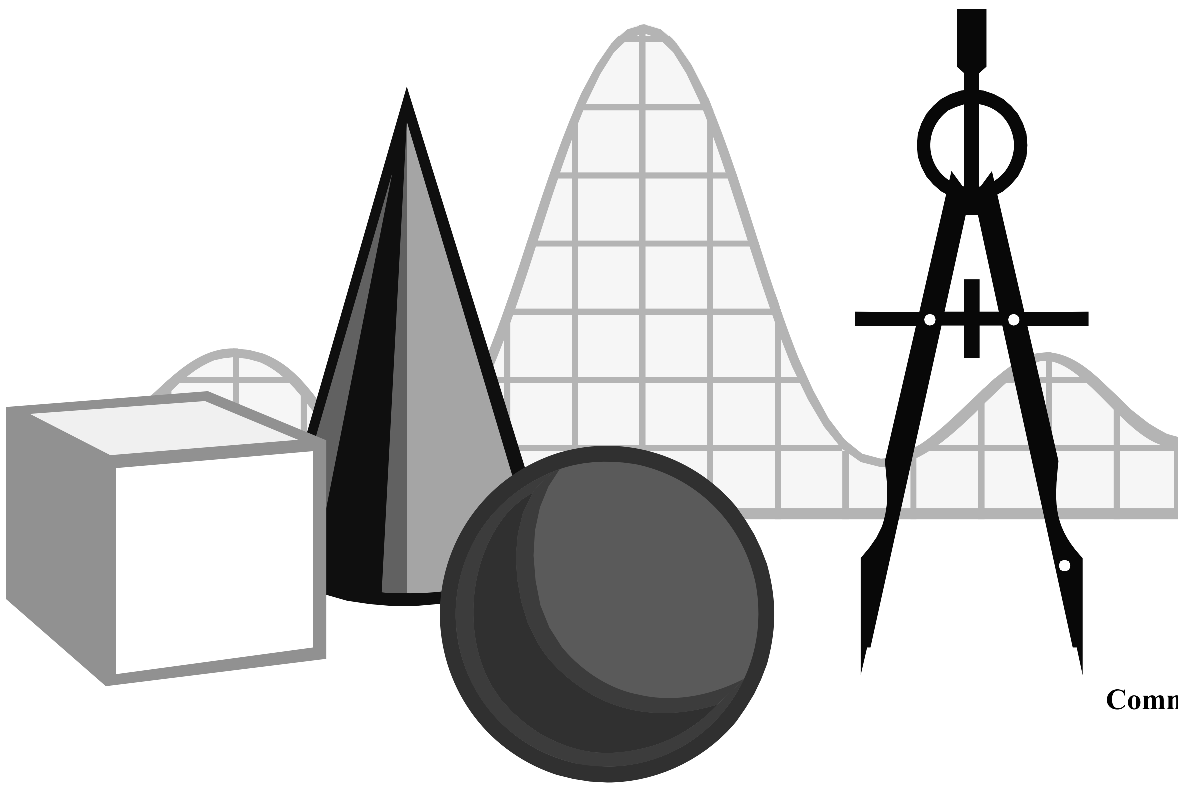
MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Grade 6



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NOTICE TO THE READER

The Virginia Department of Education does not discriminate on the basis of race, color, national origin, sex, disability, or age in its programs and activities.

The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.
- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.
- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.
- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

6.1 The student will describe and compare data, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A ratio is a comparison of any two quantities ~~and conveys an idea that cannot be expressed as a single number~~. A ratio is used to represent ~~a variety of~~ relationships within ~~a set~~ and between ~~two~~ sets. • A ratio can compare part of a set to the entire set (part-whole comparison). • A ratio can compare part of a set to another part of the same set (part-part comparison). • A ratio can compare part of a set to a corresponding part of another set (part-part comparison). • A ratio can compare all of a set to all of another set (whole-whole comparison). • The order of the quantities in a ratio is directly related to the order of the quantities expressed in the relationship. For example, if asked for the ratio of the number of cats to dogs in a park, the ratio must be expressed as the number of cats to the number of dogs, in that order. • A ratio is a multiplicative comparison of two numbers, measures, or quantities. • All fractions are ratios and vice versa. • Ratios may or may not be written in simplest form. • Ratios can compare two parts of a whole. • Rates can be expressed as ratios.	~~**All students should**~~ • ~~Understand that a ratio is a comparison of two quantities.~~ • ~~Understand that ratios can be represented in more than one way.~~ • What is a ratio? A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a set and between two sets. A ratio can be written using ~~a~~ fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe a relationship within a set by comparing part of the set to the entire set. • Describe a relationship between two sets by comparing part of one set to a corresponding part of the other set. • Describe a relationship between two sets by comparing all of one set to all of the other set. • Describe a relationship within a set by comparing one part of the set to another part of the same set. • Represent ~~the~~ a ~~verbal~~ relationship in words that makes a comparison by using the notations $\frac{a}{b}$, *a*:*b*, and *a to b*. • Create a relationship in words for a given ratio expressed symbolically.

6.1 The student will describe and compare data, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/		

6.2 The student will

a) investigate and describe fractions, decimals and percents as ratios;
b) identify a given fraction, decimal or percent from a representation;
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
d) compare and order fractions, decimals, and percents.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• *Percent* means “per 100” or how many “out of 100”; *percent* is another name for *hundredths*. • A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., 30% = $\frac{30}{100}$ = $\frac{3}{10}$ = 0.3). • Percents can be expressed as fractions with a denominator of 100 (e.g., 75% = $\frac{75}{100}$ = $\frac{3}{4}$). • Percents can be expressed as decimal (e.g., 38% = $\frac{38}{100}$ = 0.38). • ~~A~~ Some fractions can be rewritten as ~~an~~ equivalent fractions with ~~a~~ denominators of ~~100~~ powers of 10, and~~, thus,~~ ~~as a~~ can be represented as decimals or percents (e.g., $\frac{3}{5}$ = $\frac{6}{10}$ = $\frac{60}{100}$ = 0.60 = 60%). • Decimals, fractions, and percents can be represented using concrete materials (e.g., ~~b~~Base-10 blocks, number lines, decimal squares, or grid paper). • Percents ~~should~~ can be represented by drawing ~~a~~ shaded regions on ~~a 10-by-10~~ grids or by finding a location on number lines ~~to represent a given percents~~. • Percents are used in real life for taxes, sales, data description, and data comparison.	~~**All students should**~~ • ~~Understand that percent is a way of representing fractions and decimals.~~ • ~~Understand that a number can be written as a fraction, decimal, or percent.~~ • ~~Understand that percent is a method of standardization that is efficient because each number is always based on 100ths~~ hundredths. • ~~Understand that percents are used in real-life applications to compare or describe data.~~ • What is the relationship among fractions, decimals and percents? Fractions, decimals, and percents are three different ways to express the same number. A ratio can be written using ~~a~~ fraction form ($\frac{2}{3}$), a colon (2:3), or the word *to* (2 to 3). Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Recognize that *percent* means “out of 100” or *hundredths*, using the percent symbol (%).~~ • Identify the decimal and percent equivalents for ~~halves, thirds, fourths, fifths, and tenths~~ numbers written in fraction form including repeating decimals. • Represent fractions, decimals, and percents on a number line. • Describe orally and in writing the equivalent relationships among decimals, percents, and fractions that have denominators that are factors of 100. • ~~Draw a shaded region on a 10-by-10 grid to represent a given fraction, decimal, and percent~~. • Represent, by shading a grid, a fraction, decimal, and percent. • Represent in ~~decimal,~~ fraction, decimal, and~~/or~~ percent form a given shaded region of a ~~10-by-10~~ grid. • Compare two decimals through thousandths ~~by representing the decimals with decimal~~ using manipulatives ~~or,~~ ~~picture~~ pictorial representations, ~~or by using place-value charts~~ number lines, ~~or~~ and

6.2 The student will

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b) identify a given fraction, decimal or percent from a representation;
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
d) compare and order fractions, decimals, and percents.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Fractions, decimals and percents are equivalent forms representing a given number. • The decimal point is a symbol that ~~indicates the location of the ones place and all other subsequent in the decimal system~~separates the whole number part from the fractional part of a number. • The decimal point separates the whole number amount from the part of a number that is less than one. • The symbol • can be used in Grade 6 in place of X to indicate multiplication. • ~~Decimals can be represented and compared, using decimal manipulatives, drawings, pictures, or symbols.~~ • ~~Fractions can be represented and compared by using fraction manipulatives, drawings, pictures, or symbols.~~ • Strategies using 0, $\frac{1}{2}$ and 1 as benchmarks can be used to compare fractions. • When comparing two fractions, use $\frac{1}{2}$ as a benchmark. Example: Which is greater, $\frac{4}{7}$ or $\frac{3}{9}$?		~~the~~ symbols (<, ≤, ≥, .>, ~~or~~ =). • ~~Compare two whole numbers by representing the numbers with concrete objects or picture representations or by using the symbols <, ≤, >, ≥, or =.~~ • Compare two fractions with denominators of 12 or less ~~by representing the fractions with fraction~~ using manipulatives ~~or picture~~ , pictorial representations, number lines, ~~or~~ ~~by using the~~ and symbols (<, ≤, ≥, .>, ~~or~~ =). • Compare two percents using pictorial representations and symbols (<, ≤, ≥, .>, =). • Order no more than 3 fractions, decimals, and~~/or~~ percents (decimals through thousandths, fractions with denominators of 12 or less), in ascending or descending order.

6.2 The student will

a) investigate and describe fractions, decimals and percents as ratios;
b) identify a given fraction, decimal or percent from a representation;
c) demonstrate equivalent relationships among fractions, decimals, and percents; and
d) compare and order fractions, decimals, and percents.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
$\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. The denominator tells the number of parts that make the whole. $\frac{3}{9}$ is less than $\frac{1}{2}$ because 3, the numerator, is less than half of 9, the denominator, which tells the number of parts that make the whole. Therefore, $\frac{4}{7} > \frac{3}{9}$. • When comparing two fractions close to 1, use distance from 1 as your benchmark. Example: Which is greater, $\frac{6}{7}$ *or* $\frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{9}$ *is* $\frac{1}{9}$ away from 1 whole. Since $\frac{1}{7} > \frac{1}{9}$, then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$ so $\frac{8}{9} > \frac{6}{7}$. • Students should have experience with fractions such as $\frac{1}{8}$, whose decimal representation is a terminating		

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UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
decimal (e. g., $\frac{1}{8} = 0.125$) and with fractions such as $\frac{2}{9}$, whose decimal representation does not end but continues to repeat (e. g., $\frac{2}{9} = 0.222\ldots$). The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/ under computation and estimation.		

6.3 The student will
a) identify and represent integers;
b) order and compare integers; and
c) identify and describe absolute value of integers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Integers are the set of whole numbers, ~~and~~ their opposites, and zero. • Positive integers are greater than zero. • Negative integers are less than zero. • Zero is an integer that is neither positive nor negative. • A negative integer is always less than a positive integer. • When comparing two negative ~~numbers~~ integers, the negative ~~number~~ integer that is closer to zero is greater. • An integer and its opposite are the same distance from zero on a number line. For example, the opposite of 3 is -3. • The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented as $\lvert -6 \rvert = 6$. • On a conventional number line, a smaller number is always located to the left of a larger number (e.g., –7 lies to the left of –3, thus –7 < –3; 5 lies to the left of 8 thus 5 is less than 8).	~~**All students should**~~ • ~~Understand how to identify, represent, order, and compare integers.~~ • What role do negative integers play in practical situations? Some examples of the use of negative integers are found in temperature (below 0), finance (owing money), below sea level. There are many other examples. • How does the absolute value of an integer compare to the absolute value of its opposite? They are the same because an integer and its opposite are the same distance from zero on a number line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify an integer represented by a point on a number line. • Represent integers on a number line. • ~~Compare and order~~ Order and compare integers using a number line. • Compare integers, using ~~the~~ mathematical symbols (<, >, ~~and~~ =).

6.4 The student will demonstrate multiple representations of multiplication and division of fractions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Using manipulatives to build conceptual understanding and using pictures and sketches to link concrete examples to the symbolic enhance students' understanding of operations with fractions and help students connect the meaning of whole number computation to fraction computation. • Multiplication and division of fractions can be represented with arrays, paper folding, repeated addition, repeated subtraction, fraction strips, pattern blocks and area models. • When multiplying a whole by a fraction such as 3 x $\frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. • When multiplying a fraction by a fraction such as $\frac{2}{3}\cdot\frac{3}{4}$, we are asking for part of a part. • When multiplying a fraction by a whole number such as $\frac{1}{2}$ x 6, we are trying to find a part of the whole. • ~~Using an area model assists with students' developing understanding of multiplication and division of fractions.~~ • For measurement division, the divisor is the number of groups. You want to know how many are in each of those groups . Division of fractions can be explained as how many of a given divisor are	• When multiplying fractions, what is the meaning of the operation? When multiplying a whole by a fraction such as 3 x $\frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. When multiplying a fraction by a fraction such as $\frac{2}{3}\bullet\frac{3}{4}$, we are asking for part of a part. When multiplying a fraction by a whole number such as $\frac{1}{2}$ x 6, we are trying to find a part of the whole. • What does it mean to divide with fractions? For measurement division, the divisor is the number of groups and the quotient will be the number of groups in the dividend. Division of fractions can be explained as how many of a given divisor are needed to equal the given dividend. In other words, for $\frac{1}{4}\div\frac{2}{3}$ the question is, "How many $\frac{2}{3}$ make $\frac{1}{4}$?" For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?"	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Demonstrate the concepts of multiplication and division of fractions.~~ • Demonstrate multiplication and division of fractions using multiple representations~~ including, but not limited to, arrays, paper folding, fractions strips, pattern blocks and area models~~. • ~~Discover and then m~~Model algorithms for multiplying and dividing with fractions using appropriate representations.

6.4 The student will demonstrate multiple representations of multiplication and division of fractions.

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needed to equal the given dividend. In other words, for $\frac{1}{4} \div \frac{2}{3}$, the question is, "How many $\frac{2}{3}$ make $\frac{1}{4}$?" • For partition division the divisor is the size of the group, so the quotient answers the question, "How much is the whole?" or "How much for one?" Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#nns		

6.5 The student will investigate and describe concepts of positive exponents and perfect squares.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In 8^3, 8 is the base and 3 is the exponent. • A power of a number represents repeated multiplication of the number by itself (e.g., $8^3 = 8 \times 8 \times 8$ and is read "8 to the third power"). • Any real number other than zero raised to the zero power is 1. Zero to the zero power (0) is undefined. • ~~Patterns in place-value charts provide visual meaning of exponents: $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$.~~ • Perfect squares are the numbers that result from multiplying any whole number by itself (e.g., $36 = 6 \times 6 = 6^2$). • Perfect squares can be represented geometrically as the areas of squares the length of whose sides are whole numbers (e.g., 1×1, 2×2, or 3×3). This can be modeled with grid paper, tiles, geoboards and virtual manipulatives.	~~**All students should**~~ • ~~Understand that a power of a number is repeated multiplication of that number by itself.~~ • ~~Understand that squaring a number and taking a square root of a number are inverse operations.~~ • What does exponential form represent? Exponential form is a short way to write repeated multiplication of a common factor such as $5 \times 5 \times 5 \times 5 = 5^4$. • What is the relationship between perfect squares and a geometric square? A perfect square is the area of a geometric square ~~where~~ whose side length is a whole number.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and describe patterns with exponents that are natural numbers, by using a calculator. • Recognize and describe patterns of perfect squares not to exceed 20^2, by using grid paper, square tiles, tables, and calculators. • ~~Recognize and describe patterns with square roots and squares by using squares, grid paper, and calculators.~~ • Recognize powers of ten by examining patterns in a place value chart: $10^4 = 10{,}000$, $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$, $10^0 = 1$.

In the middle grades, the focus of mathematics learning is to
- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.

6.6 The student will

a) multiply and divide fractions and mixed numbers; and

b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Simplifying fractions to ~~their~~ simplest form assists with uniformity of answers ~~and concepts~~. • ~~Equivalent forms are needed to perform the operations of addition and subtraction with fractions.~~ • Addition and subtraction are inverse operations as are ~~M~~ multiplication and division. ~~are inverse operations.~~ • ~~Rewriting an improper fraction as a mixed numeral assists with uniformity of answers and concepts.~~ • ~~There is implied addition of the whole number part and the fractional part in mixed numerals.~~ • ~~Using manipulatives to build conceptual understanding and using pictures and sketches to link concrete examples to the symbolic enhance students' understanding of operations with fractions and help students connect the meaning of whole number computation to fraction computation.~~ • It is helpful to use estimation to develop computational strategies. For example, $2\frac{7}{8}\cdot\frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3. • When multiplying a whole by a fraction such as	**~~All students should~~** • How are multiplication and division of fractions and multiplication and division of whole numbers alike? ~~Understand that f~~ Fraction computation ~~uses the same ideas~~ can be approached in the same way as whole number computation, applying those concepts to fractional parts. • What is the role of estimation in solving problems? ~~Use e~~ Estimation ~~to~~ helps determine the reasonableness of answers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Simplify fractional answers to simplest form.~~ • Multiply and divide with fractions and mixed numbers. Answers ~~may be~~ are expressed in simplest form. • Solve single and multistep practical problems that involve addition and~~/or~~ subtraction with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less ~~and express answers in simplest form~~. Answers ~~may be~~ are expressed in simplest form. • Solve single and multistep practical problems that involve multiplication and~~/or~~ division with fractions and mixed numbers that include denominators of 12 or less ~~and express answers in simplest form~~. Answers ~~may be~~ are expressed in simplest form.

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$3\cdot\frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. • When multiplying a fraction by a fraction such as $\frac{2}{3}\cdot\frac{3}{4}$, we are asking for part of a part. • When multiplying a fraction by a whole number such as $\frac{1}{2}\cdot 6$, we are trying to find a part of the whole. • ~~It is helpful for students to simplify before they multiply fractions, using the commutative property of multiplication to reduce fractions to simplest form before multiplying.~~		

6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• ~~Estimation is an essential skill used to make sense of the placement of the decimal point in performing operations on decimals and for checking the decimal point's correct placement.~~ • Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. For example: What is an approximate answer for 2.19 ÷ 0.8? The answer is around 2 because 2 ÷ 1 = 2. • Understanding the placement of the decimal point is very important when finding quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, by 0.06, and by 0.006. • Solving multistep problems in the context of real-life situations enhances interconnectedness and proficiency with estimation strategies. • Examples of ~~real-life~~ practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, deciding what time to leave for school or the movies, and sharing a pizza or the prize money from a contest. • ~~A consumer application problem is defined as the type of problem that is normally encountered in daily living, such as problems related to money, travel, work, recreation, and home life.~~ • ~~A budget may be kept for short or long periods of~~	~~**All students should**~~ • ~~Be able to produce an approximate answer for a given problem.~~ • ~~Understand that an estimated answer helps validate the reasonableness of a computed answer.~~ • What is the role of estimation in solving problems? Estimation gives a reasonable solution to a problem when an exact answer is not required. If an exact answer is required, estimation allows you to know if the calculated answer is reasonable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Solve multistep practical problems involving decimals by using estimation strategies and checking for the reasonableness of results.~~ • ~~Given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit, find the quotient.~~ • ~~Given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with more than one non-zero digit, find the quotient by using a calculator.~~ • Solve single-step and multistep practical problems involving addition, subtraction, multiplication and division ~~of~~ with decimals expressed to thousandths with no more than two operations.

6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~time. Students may keep a short-term budget to enable the purchase of an expensive item or a long-term budget to facilitate a long-term spending plan.~~		

6.8 The student will evaluate whole number numerical expressions, using the order of operations.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The order of operations is a convention that defines the computation order to follow in simplifying an expression. • The order of operations is as follows: – First, complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right. * ~~Grouping symbols include p~~Parentheses ( ), brackets [ ], braces {}, and the division bar – as in $\frac{3+4}{5+6}$ should be treated as grouping symbols. • The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent. • Any number, except 0, raised to the zero power is 1. Zero to the zero power is undefined.	**~~All students should~~** • What is the significance of the order of operations? ~~Understand that the~~ The order of operations ~~describes~~ prescribes the order to use to simplify expressions containing more than one operation. It ensures that there is only one correct answer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify expressions by using the order of operations in a demonstrated step-by-step approach. The expressions should be limited to positive values and not include ~~braces~~ braces { } or absolute value ||. • Find the value of numerical expressions, using order of operations, mental mathematics, and appropriate tools. Exponents are limited to positive values.

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- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

6.9 The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Making sense of various units of measure is an essential life skill, requiring reasonable estimates of what measurements mean, particularly in relation to other units of measure. – 1 inch is about 2.5 centimeters. – 1 foot is about 30 centimeters. – 1 meter is a little longer than a yard, or about 40 inches. – 1 mile is slightly farther than 1.5 kilometers. – 1 kilometer is slightly farther than half a mile. – 1 ounce is about 28 grams. – 1 nickel has the mass of about 5 grams. – 1 kilogram is a little more than 2 pounds. – 1 quart is a little less than 1 liter. – 1 liter is a little more than 1 quart. – Water freezes at 0°C and 32°F. – Water boils at 100°C and 212°F. – Normal body temperature is about 37°C and 98°F. – Room temperature is about 20°C and 70°F. • ~~Multiple experiences with using nonstandard and standard units of measure to measure physical objects help students develop an intuitive understanding of size.~~ • Mass is the amount of matter in an object. Weight is the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight*, as shown by the questions: "How much	~~**All students should**~~ • ~~Understand that there is a structured relationship between and among units of measure for length, area, weight/mass, and volume in the metric and U.S. Customary systems.~~ • What is the difference between weight and mass? ~~Understand that w~~ Weight and mass are different. Mass is the amount of matter in an object. Weight is the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. • ~~Understand that measures are determined by quantitative comparison to a standard unit.~~ • How do you determine which units to use at different times? Units of measure are determined by the attributes of the object being measured. ~~Understand that m~~ Measures of length are expressed in linear units, measures of area are expressed in square units, and measures of volume are expressed in cubic units. • Why are there two different measurement systems? Measurement systems are conventions invented by different cultures to meet their needs. The U.S. Customary System is the preferred method in the United States. The metric system is the preferred system worldwide.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system.~~ • Estimate the conversion of units of length, ~~area,~~ weight/mass, ~~and~~ volume, and temperature between the U.S. Customary system and the metric system by using ballpark comparisons. Ex: 1 L $\approx$ 1qt. Ex: 4L $\approx$ 4 qts. • ~~Determine the most appropriate unit of measure for a given situation.~~ • Estimate measurements by comparing the object to be measured against a benchmark. • ~~Solve measurement problems by estimating and determining length, using standard and nonstandard units of measure.~~ • ~~Solve measurement problems by estimating and determining weight/mass, using standard and nonstandard units of measure~~. • ~~Solve measurement problems by estimating and determining area, using standard and nonstandard units of measure~~. • ~~Solve measurement problems by estimating and determining liquid volume/capacity, using standard and nonstandard units of measure.~~

6.9 The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
does it weigh?" versus "What is its mass?" • ~~Chunking or benchmarks are strategies used to make measurement estimates.~~ • ~~Chunks of length, such as a window's length, can be used to estimate the length of a classroom wall~~. • ~~Benchmarks, such as the two-meter height of a standard doorway, can be used to estimate height.~~ • The degree of accuracy of measurement required is determined by the situation. • Whether to use an underestimate or an overestimate is determined by the situation. • Physically measuring objects along with using visual and symbolic representations improves student understanding of both the concepts and processes of measurement. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#measurement		

6.10 The student will
a) define pi (π) as the ratio of the circumference of a circle to its diameter;
b) solve practical problems involving circumference and area of a circle, given the diameter or radius;
c) solve practical problems involving area and perimeter; and
d) describe and determine the volume and surface area of a rectangular prism.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Experiences in deriving the formulas for area and perimeter, using manipulatives such as tiles, one-inch cubes, adding machine tape, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and facility in their use. • The perimeter of a polygon is the measure of the distance around the polygon. • Circumference is the distance around or perimeter of a circle. • The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve. • The perimeter of a square whose side measures s is 4 times s ($P = 4s$), and its area is side times side ($A = s^2$). • The perimeter of a rectangle is the sum of twice the length and twice the width [$P = 2l + 2w$, or $P = 2(l + w)$], and its area is the product of the length and the width ($A = l \cancel{\times} w$). • ~~Experiences in using a variety of measuring devices and making real measurements promote an understanding of measurements and the formulas associated with measurements.~~ • The value of pi (π) is the ratio of the circumference	~~**All students should**~~ • ~~Understand the attributes of polygons and the use of measures to determine area and perimeter.~~ • ~~Understand the derivation of formulas related to area and perimeter of polygons and how to determine which is used in problem situations.~~ • ~~Select the appropriate approximation for pi (π) when solving problems.~~ • ~~Understand the derivation of pi and formulas for finding circumference and area of a circle.~~ • What is the relationship between the circumference and diameter of a circle? The circumference of a circle is about 3 times the measure of the diameter. • What is the difference between area and perimeter? Perimeter is the distance around the outside of a figure while area is the measure of the amount of space enclosed by the perimeter. • What is the relationship between area and surface area? Surface area is calculated for a three-dimensional figure. It is the sum of the areas of the two-dimensional surfaces that make up the three-dimensional figure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. • Find the circumference of a circle by substituting a value for the diameter or the radius into the formula $C = \pi d$ or $C = 2\pi r$. • Find the area of a circle by using the formula $A = \pi r^2$. • Apply formulas to solve practical problems involving area and perimeter of triangles and rectangles. • Create and solve problems that involve finding the circumference and~~/or~~ area of a circle when given the diameter or radius. • Solve problems that require finding the surface area of a rectangular prism, given a diagram of the prism with the necessary dimensions labeled. • Solve problems that require finding the volume of a rectangular prism given a diagram of the prism with the necessary dimensions labeled.

6.10 The student will
a) define pi (π) as the ratio of the circumference of a circle to its diameter;
b) solve practical problems involving circumference and area of a circle, given the diameter or radius;
c) solve practical problems involving area and perimeter; and
d) describe and determine the volume and surface area of a rectangular prism.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
of a circle to its diameter. • The ratio of the circumference to the diameter of a circle is a constant value, pi (π), which can be approximated by measuring various sizes of circles. • The fractional approximation of pi generally used is $\frac{22}{7}$. • The decimal approximation of pi generally used is 3.14. • The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle. • The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle. • The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$). • The volume of a rectangular prism is computed by multiplying the area of the base, B, (length x width) by the height of the prism ($V = lwh = Bh$ ~~or B=Bh~~). Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#measurement		• ~~Determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area.~~ • ~~Determine the circumference and/or area of a circle, using various tools.~~

In the middle grades, the focus of mathematics learning is to
- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.
- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.
- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking ~~A~~algebra.)
- **Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking ~~G~~geometry.)

6.11 The student will
a) identify the coordinates of a point in a coordinate plane; and
b) graph ordered pairs in a coordinate plane.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (x, y), where x is the first coordinate and y is the second coordinate. However, any letters may be used to label the axes and the corresponding ordered pairs. • ~~The first coordinate of a point is its distance from the vertical number line along a horizontal line.~~ • ~~The second coordinate of a point is its distance from the horizontal number line along a vertical line.~~ • ~~In a plane, a point can be located by its distances from two intersecting perpendicular number lines. The distance from one line is measured along a line parallel to the other line.~~ • The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular number lines. Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II, (–,+); for quadrant III, (–, –); and for quadrant IV, (+,–). • In a coordinate plane, the origin is the point at the intersection of the x-axis and y-axis; the coordinates of this point are (0,0). • For all points on the x-axis, the y-coordinate is 0. For all points on the y-axis, the x-coordinate is 0. ~~When a point lies on the y-axis, the x-coordinate is always 0.~~ • The coordinates may be used to name the point. (e.g., the point (2,7)). It is not necessary to say "the point	**~~All students should~~** • Can any given point be represented by more than one ordered pair? ~~Understand that t~~ The coordinates of a point define its unique location in a coordinate plane. Any given point is defined by only one ordered pair. • In naming a point in the plane, does the order of the two coordinates matter? Yes. The first coordinate tells the location of the point to the left or right of the y-axis and the second point tells the location of the point above or below the x-axis. Point (0, 0) is at the origin.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and label the axes of a coordinate plane. • Identify and label the quadrants of a coordinate plane. • Identify the quadrant ~~or axis~~ ~~in which~~ or the axis on which a ~~an ordered pair~~ point is positioned by examining the ~~ordered pair~~ coordinates (ordered pair) of the point. • Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. • Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane.

6.11 The student will

a) identify the coordinates of a point in a coordinate plane; and

b) graph ordered pairs in a coordinate plane.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
whose coordinates are (2,7)”.		

6.12 The student will determine congruence of segments, angles, and polygons.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Congruent figures have exactly the same size and the same shape. • Noncongruent figures may have the same shape but not the same size. • The symbol for congruency is $\cong$. • The ~~matching or~~ corresponding angles of congruent polygons have the same measure, and the ~~matching or~~ corresponding sides of congruent polygons have the same measure. • The ~~direct comparison~~ determination of ~~congruent or noncongruent~~ the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of ~~measuring~~ all sides and angles. • Construction of congruent line segments, angles, and polygons helps students understand congruency.	~~**All students should**~~ • ~~Understand the meaning of congruence.~~ • Given two congruent figures, what inferences can be drawn about how the figures are related? The congruent figures will have exactly the same size and shape. • Given two congruent polygons, what inferences can be drawn about how the polygons are related? Corresponding angles of congruent polygons will have the same measure. ~~and~~ ~~c~~Corresponding sides of congruent polygons will have the same ~~length~~ measure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Characterize polygons as congruent and noncongruent according to the measures of their sides and angles. • Determine the congruence of segments, angles, and polygons ~~by direct comparison,~~ given their attributes.

6.13 The student will describe and identify properties of quadrilaterals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A quadrilateral is a closed planar (two-dimensional) figure with four sides that are line segments. • A parallelogram is a quadrilateral whose opposite sides are parallel and <u>opposite angles are</u> congruent. • ~~For all parallelograms, both pairs of opposite sides and both pairs of opposite angles are congruent.~~ • ~~Parallelograms have special characteristics (such as both pairs of opposite sides are parallel and congruent) that are true for any parallelogram.~~ • A rectangle is a parallelogram with four right angles. • Rectangles have special characteristics (such as diagonals are perpendicular bisectors) that are true for any rectangle. • <u>To bisect means to divide into two equal parts.</u> • A square is a rectangle with four congruent sides or a rhombus with four right angles. • A rhombus is a parallelogram with four congruent sides. • ~~A trapezoid is a quadrilateral with exactly one pair of parallel sides.~~ • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases*, and the nonparallel sides are called *legs*. If the legs have the same length, then the trapezoid is an isosceles trapezoid. • ~~A trapezoid with congruent, nonparallel sides is called an isosceles trapezoid.~~	~~**All students should**~~ • <u>Can a figure belong to more than one subset of quadrilaterals?</u> <u>Any figure that has the attributes of more than one subset of quadrilaterals can belong to more than one subset. For example, rectangles have opposite sides of equal length. Squares have all 4 sides of equal length thereby meeting the attributes of both subsets.</u>	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • <u>Sort and classify polygons as quadrilaterals, parallelograms, rectangles, trapezoids, kites, rhombi, and squares based on their properties. Properties include number of parallel sides, angle measures and number of congruent sides.</u> • <u>Identify the sum of the measures of the angles of a quadrilateral as 360°.</u>

6.13 The student will describe and identify properties of quadrilaterals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A kite is a quadrilateral with two pairs of adjacent congruent sides. One pair of opposite angles is congruent. • Quadrilaterals can be sorted according to common attributes, using a variety of materials. • Quadrilaterals can be classified by the number of parallel sides: a parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has only one pair of parallel sides; other quadrilaterals have no parallel sides. • Quadrilaterals can be classified by the measures of their angles: a rectangle has four 90° angles; a trapezoid may have none, one, or two 90° angles. • Quadrilaterals can be classified by the number of congruent sides: a rhombus has four congruent sides; a square, which is a rhombus with four right angles, also has four congruent sides; a parallelogram and a rectangle each have two pairs of congruent sides. • A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals. • The sum of the measures of the angles of a quadrilateral is 360°. • A chart ~~or~~ , graphic organizer or Venn Diagram can be made to organize quadrilaterals according to attributes such as sides and/or angles. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#geometry		

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of ~~central tendency~~ center and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

6.14 The student, given a problem situation, will
a) construct circle graphs;
b) draw conclusions and make predictions, using circle graphs; and
c) compare and contrast graphs that present information from the same data set.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to answer the problem. • ~~Data can be discrete or continuous.~~ • Different types of graphs are used to display different types of data. – Bar graphs use categorical (discrete) data (e.g., months or eye color). – Line graphs use continuous data (e.g., temperature and time). – Circle graphs show a relationship of the parts to a whole. • All graphs include a title, and data categories should have labels. • A scale should be chosen that is appropriate for the data. • A key is essential to explain how to read the graph. • A title is essential to explain what the graph represents. • Data are analyzed by describing the various features and elements of a graph. Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#statistics	~~**All students should**~~ • ~~Understand that data can be displayed in a variety of graphical representations.~~ • ~~Select and use appropriate statistical methods to analyze data.~~ • ~~Understand that different types of representations can tell different things about the same data.~~ • What type of data are best presented in a circle graph? Circle graphs are best used for data showing a relationship of the parts to the whole.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Collect data sets of no more than 20 items by using tally sheets, surveys, observations, questionnaires, interviews, and polls.~~ • ~~Organize data by using lists, charts, and tables.~~ • ~~Organize and display data in bar and line graphs, displaying the information as clearly as possible by using increments of whole numbers, fractions, and decimals rounded to the nearest tenth.~~ • Collect, ~~O~~organize and display data in circle graphs by depicting information as fractional. ~~parts that are limited to halves, fourths, and eighths.~~ • Draw conclusions and make predictions about data presented in a circle graph. • Compare and contrast data presented in a circle graph with the same data represented in other graphical forms ~~studied in the previous grade level~~. • ~~Decide which type of graph is appropriate for a given situation.~~ ~~– Bar graphs are used to display categorical (discrete) data.~~ ~~– Line graphs are used to display continuous data.~~

6.15 The student will
a) describe mean as balance point; and
b) decide which measure of center is appropriate for a given purpose.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Measures of ~~central tendency~~ center are types of averages for a data set. They represent numbers that ~~best~~ describe a data set. Mean, median, and mode are measures of ~~central tendency~~ center that are useful for describing the average for different situations. – Mean works well for sets of data with no very high or low numbers. – Median is a good choice when data sets have a couple of values much higher or lower than most of the others. – Mode is a good descriptor to use when the set of data has some identical values or when data are not conducive to computation of other measures of central tendency, as when working with data in a yes or no survey. • The mean is the numerical average of the data set and is found by adding the numbers in the data set together and dividing the sum by the number of data pieces in the set. • In grade 5 mathematics, mean is defined as fair-share. • Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point. • Defining mean as balance point is a prerequisite for understanding standard deviation.	**~~All students should~~** • ~~Understand that measures of central tendency are types of averages for a data set.~~ • ~~Understand that mean, median, and mode are measures of central tendency that are useful for describing data in different situations.~~ • ~~Understand that the range describes the spread of a set of data.~~ • What does the phrase "measure of center" mean? This is a collective term for the 3 types of averages for a set of data – mean, median and mode. • What is meant by mean as balance point? Mean can be defined as the point on a number line where the data distribution is balanced. This means that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances of all the data points below the mean. This is the concept of mean as the balance point.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the mean for a set of data. • Describe the three measures of ~~central tendency~~ center and a situation in which each would best represent a set of data. • Identify and draw a number line that demonstrates the concept of mean as balance point for a set of data.

6.15 The student will
a) describe mean as balance point; and
b) decide which measure of center is appropriate for a given purpose.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values. • The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal. ~~– For 2, 3, 4, 5, 5, 6, 7, 8, 8, 8, 9, 11, the mode is 8.~~ ~~– For 2, 3, 4, 5, 5, 5, 7, 8, 8, 8, 9, 11, the modes are 5 and 8 (bimodal).~~ ~~– For 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 17, there is no mode.~~ • ~~The range is the difference between the greatest and least values in a set of data and shows the spread in a set of data.~~		

6.16 The student will
a) compare and contrast dependent and independent events; and
b) determine probabilities for dependent and independent events.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes (sample space). • The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent. • The probability of an event occurring is a ratio between 0 and 1. – A probability of 0 means the event will never occur. – A probability of 1 means the event will always occur. • A simple event is one event (e.g., pulling one sock out of a drawer and examining the probability of getting one color). • Events are independent when the outcome of one has no effect on the outcome of the other. For example, rolling a number cube and flipping a coin are independent events. • The probability of two independent events is found by using the following formula: $P(A \, and \, B) = P(A) \cdot P(B)$ Ex: When rolling two number cubes simultaneously, what is the probability of rolling a 3 on one cube and a 4 on the other? $P(3 \, and \, 4) = P(3) \cdot P(4) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ • Events are dependent when the outcome of one	~~**All students should**~~ • ~~Understand that a probability can be expressed as a ratio, decimal, or percent~~. • How can you determine if a situation involves dependent or independent events? Events are independent when the outcome of one has no effect on the outcome of the other. Events are dependent when the outcome of one event is influenced by the outcome of the other.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether two events are dependent or independent. • Compare and contrast dependent and independent events. • Determine the probability of two dependent events. • Determine the probability of two independent events.

6.16 The student will

a) compare and contrast dependent and independent events; and

b) determine probabilities for dependent and independent events.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
event is influenced by the outcome of the other. For example, when drawing two marbles from a bag, *not* replacing the first after it is drawn affects the outcome of the second draw. • The probability of two dependent events is found by using the following formula: $P(A \text{ } and \text{ } B) = P(A) \cdot P(B \text{ } after \text{ } A)$ Ex: You have a bag holding a blue ball, a red ball, and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick and then *without* replacing the blue ball in the bag, picking a red ball on the second pick? $P(\text{blue } and \text{ red}) = P(\text{blue}) \cdot P(\text{red } after \text{ blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$		

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- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving ~~real-life~~ practical problems.

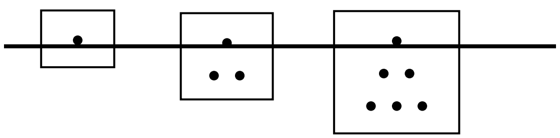
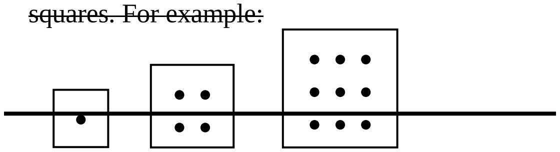
Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through life experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

6.17 The student will identify and extend geometric and arithmetic sequences.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Numerical patterns may include linear and exponential growth, perfect squares, triangular and other polygonal numbers, or Fibonacci numbers. • Arithmetic and geometric sequences are types of numerical patterns. • In the numerical pattern of an arithmetic sequence, students must determine the difference, called the *common difference,* between each succeeding number in order to determine what is added to each previous number to obtain the next number. Sample numerical patterns are 6, 9, 12, 15, 18, …; and 5, 7, 9, 11, 13, …. • In geometric number patterns, students must determine what each number is multiplied by to obtain the next number in the geometric sequence. This multiplier is called the *common ratio*. Sample geometric number patterns include 2, 4, 8, 16, 32, …; 1, 5, 25, 125, 625, …; and 80, 20, 5, 1.25, … • Strategies to recognize and describe the differences between terms in numerical patterns include, but are not limited to, examining the change between consecutive terms~~, looking for prime numbers,~~ and finding common factors. An example is the pattern 1, 2, 4, 7, 11, 16,… • ~~Strategies to recognize and describe geometric patterns include, but are not limited to, examining flips, slides, turns, growth, and symmetry. Rotation (turn) is the result of turning a figure around a point or a vertex. Translation (slide) is the result of sliding a figure in any direction within a plane. Dilatation (scale increase or decrease) is the result of enlarging or shrinking a figure by a scale amount. Reflection~~	**~~All students should~~** • ~~Understand that mathematical patterns can be represented in various forms, geometrically or numerically.~~ • ~~Understand that patterns regularly occur in everyday life.~~ • ~~Understand that patterns can be recognized, extended, or generalized~~. • ~~Understand that numerical patterns may involve adding or multiplying by the same number.~~ • ~~Understand that geometric patterns may involve shape, size, angles, transformations of shapes, and growth.~~ • What is the difference between an arithmetic and a geometric sequence? While both are numerical patterns, arithmetic sequences are additive and geometric sequences are multiplicative.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and apply strategies to recognize and describe the change between terms in ~~numerical~~ arithmetic patterns. • Investigate and apply strategies to recognize and describe geometric patterns. • Describe verbally and in writing the relationships between consecutive terms in an ~~numerical~~ arithmetic or geometric ~~pattern~~ sequence. • Extend and apply ~~numerical~~ arithmetic and geometric ~~patterns~~ sequences to similar situations. • ~~Create~~ ~~Using a table as an organizing tool, e~~Extend ~~numerical~~ arithmetic and geometric ~~patterns~~ sequences in a table by using a given rule or mathematical relationship. • ~~Describe numerical and geometric patterns, including triangular numbers.~~ • Compare and contrast arithmetic and geometric sequences. • Identify the common difference for a given arithmetic sequence. • Identify the common ratio for a given geometric sequence.

6.17 The student will identify and extend geometric and arithmetic sequences.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
~~(flip) is the result of flipping a figure over a line.~~ • ~~The first five triangular numbers are 1, 3, 6, 10, and 15. A triangular number can be represented geometrically as a certain number of dots arranged in a triangle, with one dot in the first (top) row and each succeeding lower row having one more dot that the row above it. To find the next triangular number, a new row is added to an existing triangle, and total number of dots counted. Students should make the connection between the number of new dots in the triangle and the corresponding triangular number. Triangular numbers can be represented as a growing pattern of triangles.~~  • ~~A square number can be represented geometrically as the number of dots in a square array. Square numbers are perfect squares and are the numbers that result from multiplying any whole number by itself (e.g., $36 = 6 \times 6$). Square numbers (1, 4, 9, 16, …) can be represented as a growing pattern of squares. For example:~~  • ~~The possible number of patterns is infinite.~~ • ~~The simplest types of patterns are repeating patterns. In such patterns, students need to identify the basic unit of the pattern and repeat it.~~ • ~~Growing patterns are more difficult for students to~~		

6.17 The student will identify and extend geometric and arithmetic sequences.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
~~understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns both in numerical and geometric formats.~~		

6.18 The student will solve one-step linear equations in one variable involving whole number coefficients and positive rational solutions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A one-step linear equation is an equation that requires one operation to solve. • A mathematical expression contains a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship. An expression cannot be solved. • A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3. • A coefficient is the numerical factor in a term. For example, in the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient. • Positive rational solutions are limited to whole numbers and positive fractions and decimals. • An equation is a mathematical sentence stating that two expressions are equal. • A variable is a symbol (placeholder) used to represent an unspecified member of a set.	~~**All students should**~~ • ~~Understand that physical objects can be used to represent and solve algebraic equations.~~ • ~~Understand that in an equation, the equal sign indicates that the value on the left side of the sign is the same as the value on the right side.~~ • When solving an equation, why is it necessary to perform the same operation on both sides of an equal sign? ~~Understand that to~~ To maintain equality, an operation performed on one side of an equation must be performed on the other side.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Represent and solve a one-step equation, using a variety of concrete materials such as colored chips ~~on an equation mat~~, ~~algebra tiles~~, algeblocks or weights on a balance scale. • Solve a one-step equation by demonstrating the steps algebraically. • Identify and ~~U~~use the following algebraic terms appropriately: *equation*, *variable*, *expression*, *term*, and *coefficient*. • ~~Identify examples of equations, variables, terms, and coefficients.~~

6.19 The student will investigate and recognize
a) the identity properties for addition and multiplication;
b) the multiplicative property of zero; and
c) the inverse property for multiplication.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division.
- The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., $5 + 0 = 5$).
- The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$).
- Inverses are numbers that combine with other numbers and result in identity elements.
- The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$).
- Zero has no multiplicative inverse.
- The multiplicative property of zero states that the product of any real number and zero is zero.
- Division by zero is not a possible arithmetic operation. Division by zero is undefined.

Additional resources on this topic can be found at http://www.doe.virginia.gov/VDOE/middle-math-strategies/#nns

ESSENTIAL UNDERSTANDINGS

~~All students should~~

- ~~Understand that using the properties of operations with real numbers helps with understanding mathematical relationships.~~
- ~~Understand how to use these properties when computing.~~
- How are the identity properties for multiplication and addition the same? Different?
  For each operation the identity elements are numbers that combine with other numbers without changing the value of the other numbers. The additive identity is zero (0). The multiplicative identity is one (1).
- What is the result of multiplying any real number by zero?
  The product is always zero.
- Do all real numbers have a multiplicative inverse?
  No. Zero has no multiplicative inverse because there is no real number that can be multiplied by zero resulting in a product of one.

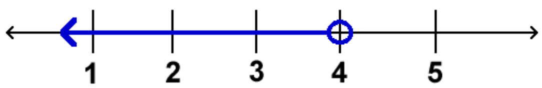
ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify the real number equation that represents each property of operations with real numbers, when given several real number equations.
- ~~Explore the properties of real numbers, using diagrams and manipulatives~~.
- Test the validity of properties by using examples of the properties of operations on real numbers.
- Identify the property of operations with real numbers that is illustrated by a real number equation.

NOTE: The commutative, associative and distributive properties are taught in previous grades.

6.20 The student will graph inequalities on a number line.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Inequalities using the < or > symbols are represented on a number line with an open circle on the number and a shaded line over the solution set. Ex: x < 4  • When graphing x ≤ 4 fill in the circle above the 4 to indicate that the 4 is included. • Inequalities using the ≤ *or* ≥ symbols are represented on a number line with a closed circle on the number and shaded line in the direction of the solution set. • The solution set to an inequality is the set of all numbers that make the inequality true. • It is important for students to see inequalities written with the variable before the inequality symbol and after. For example x > -6 and 7 > y.	~~**All students should**~~ • ~~Does the order of the elements~~ ~~i~~In an inequality, does the order of the elements matter? Yes, the order does matter. For example, x > 5 is not the same relationship as 5 > x. However, x > 5 is the same relationship as 5 < x.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Given a simple inequality with integers, graph the relationship on a number line. • Given the graph of a simple inequality with integers, represent the inequality two different ways using the symbols <, >, ≤ *and* ≥.