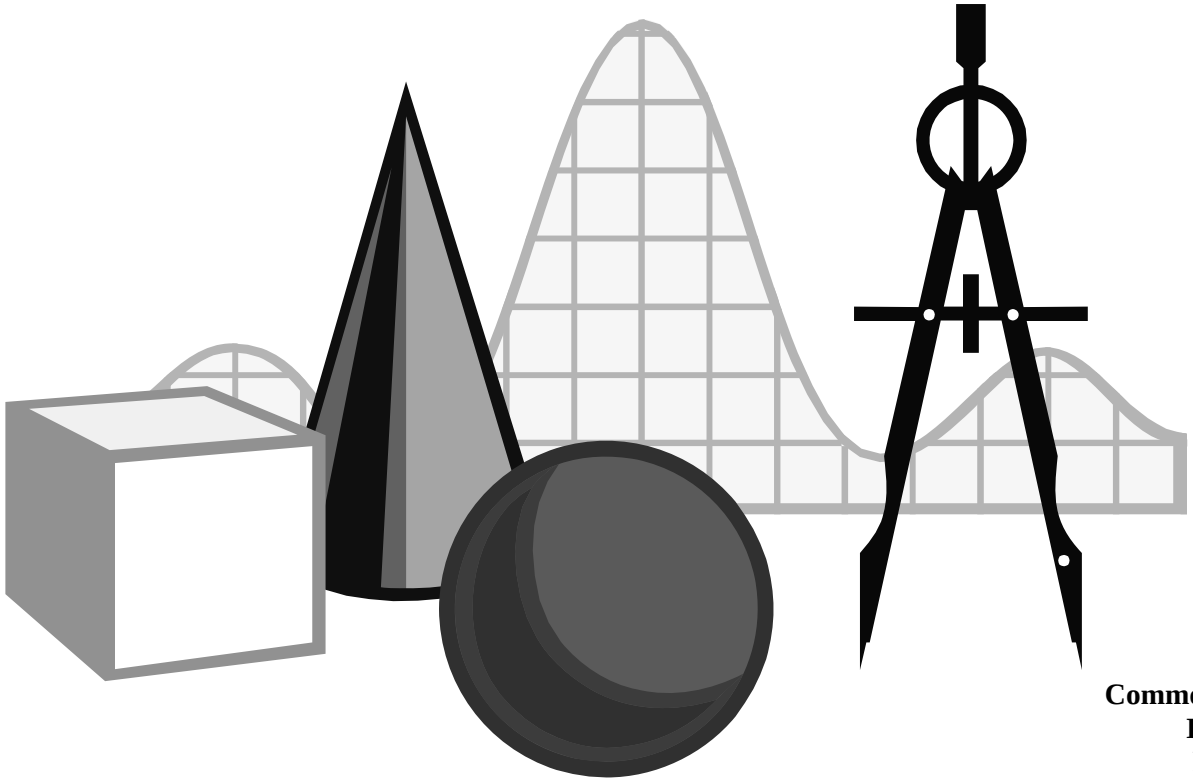
MATHEMATICS STANDARDS OF LEARNING CURRICULUM FRAMEWORK

Kindergarten



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The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each ~~S~~s<u>s</u>tandard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ <u>Standards of Learning</u> assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. <u>Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.</u>

Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, ~~simple~~ fractions, number relationships ("more than," "less than," and "~~as many as~~ equal to"), and the effects of ~~simple~~ single-step and multistep computations ~~operations on numbers (fact families)~~. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

K.1 The student, given two sets, each containing 10 or fewer concrete objects, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of one-to-one correspondence.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A set is a collection of distinct elements or items. • A one-to-one correspondence exists when two sets have an equal number of items. • Strategies for developing the concept of one-to-one matching involve set comparisons without counting. Hands-on experiences in matching items between two sets by moving, touching, and aligning objects, using one-to-one correspondence, enable visual as well as kinesthetic comparisons of the number of items in the two sets. • Students can also ~~use the strategy of~~ count~~ing~~ to make comparisons between two sets without matching the sets, using one-to-one correspondence. • Students are generally familiar with the concept of *more*, but have had little experience with the term *less*. It is important to use the terms together to build an understanding of their relationship. For example, when asking which group has more, follow with which group has less and vice versa.	**All students should** • Understand how quantities relate to each other, which leads to an understanding of how numbers are related to each other.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Match each member of one set with each member of another set, using the concept of one-to-one correspondence to compare the number of members between sets, where each set contains 10 or fewer objects. • Compare and describe two sets of 10 or fewer objects, using the terms *more*, *fewer*, and *the same*. • Given a set of objects, construct a second set which has more, fewer or the same number of objects.

K.2 The student, given a set containing 15 or fewer concrete objects, will

a) tell how many are in the set by counting the number of objects orally;

b) write the numeral to tell how many are in the set; and

c) select the corresponding numeral from a given set of numerals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Counting involves two separate skills: verbalizing the list of standard number words in order ("one, two, three, …") and connecting this sequence with the objects in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken. Objects may be presented in random order or arranged for easy counting. • Kinesthetic involvement (e.g., tracing the numbers, using tactile materials, such as sand, sandpaper, carpeting, or finger paint) facilitates the writing of numerals. • Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals. • Zero (0) is both a number and a digit. As a number, it plays a central role in mathematics as the additive identity of the integers, real numbers, and many other algebraic structures. As a digit, zero is used as a placeholder in systems. • *Conservation of number* and *cardinality principle* are two important milestones in development to attaching meaning to counting. • The cardinality principle refers to the concept that the last counted number describes the total amount of the counted set. It is an extension of one-to-one correspondence.	**All students should** • Read and write numerals from 0 through ~~10~~ 15. • Understand that the total number of objects can be found by counting. • Understand that the last counted number describes the total amount in the set. • Understand that if the set is empty, it has 0 elements. • Understand that changing the spatial arrangement of a set of objects does not change the total amount of the set.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count orally the number of objects in a set containing ~~10~~ 15 or fewer concrete objects, using one-to-one correspondence, and identify the corresponding numeral. • Identify written numerals from 0 through ~~10~~ 15 represented in random order. • Select the numeral from a given set of numerals that corresponds to a set of ~~10~~ 15 or fewer concrete objects. • Write the numerals from 0 through ~~10~~ 15. • Write a numeral that corresponds to a set of ~~10~~ 15 or fewer concrete objects. • Construct a set of objects that corresponds to a given numeral, including an empty set.

K.2 The student, given a set containing 15 or fewer concrete objects, will

a) tell how many are in the set by counting the number of objects orally;

b) write the numeral to tell how many are in the set; and

c) select the corresponding numeral from a given set of numerals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Conservation of number is the understanding that the number of objects remains the same when they are rearranged spatially.		

K.3 The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Understanding the cardinal and ordinal meanings of numbers is necessary to quantify, measure, and identify the order of objects. • An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). *Ordered position*, *ordinal position*, and *ordinality* are terms that refer to the place or position of an object in a sequence or set. • The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects. • The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., the student's position in line when students are lined up alphabetically by first name).	**All students should** • Use ordinal numbers to describe the ~~order~~ position of objects in a sequence.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the ordinal positions first~~, second, and third~~ through tenth using ordered sets of ~~three~~ ten concrete objects and/or pictures of such sets presented from – left-to-right; – right-to-left; – top-to-bottom; and/or – bottom-to-top.

K.4 The student will

a) count forward to 100 and backward from 10;

b) identify one more than a number and one less than a number; and

c) count by fives and tens to 100.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Counting skills are essential components of the development of number ideas; however, they are only one of the indicators of the understanding of numbers. • Counting forward by rote advances the child's development of sequencing. The natural numbers are 1, 2, 3, 4…. The whole numbers are 0, 1, 2, 3, 4…. Students should count the ~~natural~~ whole numbers 0, 1, 2, 3, 4,…. ~~These are not to be confused with the whole numbers that begin with the integer zero.~~ • Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with 10, 9, 8,… through …3, 2, 1, 0. • Counting forward and backward leads to the development of counting on and counting back. • The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include, but are not limited to, counters, hundred chart, and calculators. • Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five. • Skip counting by tens is a precursor for use of place value, addition, counting money, and	**All students should** • Use the correct oral counting sequence in both forward and backward counting situations. • Understand that skip counting can be used to count a collection of objects. • Describe patterns in skip counting and use those patterns to predict the next number or numbers in the skip counting sequence. • Understand that numeric relationships include one more than, one less than, two more than, two less than, etc. • Understand benchmarks of five and ten.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count forward from ~~1~~ 0 to ~~30~~ 100. • Count backward from 10 to ~~1~~ 0. • Recognize the relationship of one more than and one less than a number using objects (i.e., five and one more is six; and one less than ten is nine). • Group ~~30~~ 100 or fewer objects together into sets of fives or tens and then count them by fives or by tens. • ~~Investigate and recognize the pattern of counting by fives and tens, using 30 or fewer concrete objects.~~ • Investigate and recognize the pattern of counting by fives ~~and tens~~ to ~~30~~ 100, using a ~~calculator~~ variety of tools. • Investigate and recognize the pattern of counting by ~~fives and~~ tens to ~~30~~ 100, using a ~~calculator~~ variety of tools.

K.4 The student will

a) count forward to 100 and backward from 10;

b) identify one more than a number and one less than a number; and

c) count by fives and tens to 100.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
multiplying by multiples of 10. • Calculators can be used to display the numeric patterns that result from skip counting. ~~Use the constant feature of the four-function calculator to display the numbers in the sequence when skip counting by that constant. For example, when skip counting by fives, press 5 + 5 = = =… to produce 5, 10, 15,….~~		

K.5 The student will identify the parts of a set and/or region that represent fractions for halves and fourths.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction is a way of representing part of a whole (as in a region/area model) or part of a group (as in a set model). • In each fraction model, the parts must ~~be equal (i.e., each pie piece must~~ have the same area~~)~~. ~~In problems with fractions, a whole is broken into equal-size parts and reassembled into one whole.~~ • The fractional parts of a set model are subsets of an equal number. For example, in a set of ten cubes, each half would be a subset of five cubes. • Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions furthers this development (e.g., ~~*thirds*~~ *fourths* means "~~three~~ four equal parts of a whole" or $\frac{1}{4}$ represents one of ~~three~~ four ~~equal-size~~ parts of equal size when a pizza is shared among ~~three~~ four students).	**All students should** • Understand that fractional parts are equal shares of a whole region or a whole set. • Understand that the fraction name (*half, fourth*) tells the number of equal parts in the whole. • Understand that the fraction name (*half, fourth*) of the set model is a subset of the whole set with equal numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize fractions as representing parts of equal size of a whole. • Given a region, identify half and/or a fourth of the region. • Given a set, identify half and/or a fourth of the set.

A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling ~~and~~, discussing, and recording a variety of problem situations. This approach ~~to~~ helps students ~~move~~ transition from the concrete to the representation to the symbolic ~~the abstract~~ ~~and~~ in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, multiplication, division ~~facts~~ and related facts ~~families~~;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions ~~and decimals~~.

K.6 The student will model adding and subtracting whole numbers, using up to 10 concrete objects.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Whole numbers are 0, 1, 2, 3, 4, 5, 6, and so on. • Addition is the process of combining or joining sets. • Subtraction can be viewed as a ~~"~~taking away~~"~~ or ~~"~~separating~~"~~ process or as compare to find the difference between two sets. • Counting on from the larger set to determine the sum of the combined sets is a strategy for finding a sum. • Counting backward from the larger set to determine the difference between two sets is a strategy for subtraction. • Number relationships, including the following, help students develop strategies for adding and subtracting. – Instant recognition of the amount in a set of objects that are arranged in a familiar pattern such as the dots on number cubes – One more than, one less than, two more than, two less than	**All students should** • Understand that addition j~~oins~~ means putting things ~~objects~~ together and that subtraction is the inverse of addition and means ~~separates~~ to separate ~~objects~~ things out.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Combine two sets with known quantities in each set, and count the combined set using up to 10 concrete objects, to determine the sum, where the sum is not greater than 10 ~~concrete objects~~. • Given a set of 10 or fewer concrete objects, ~~Remove~~ remove, ~~"~~take away,~~"~~ or separate part of ~~a~~ the set ~~from a given set to~~ and determine the result ~~of subtraction~~.

Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

K.7 The student will recognize a penny, nickel, dime, and quarter and will determine the value of a collection of pennies and/or nickels whose total value is 10 cents or less.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Involvement in varied activities such as physically manipulating coins and making comparisons about their sizes, colors, and values is prerequisite to the skills of coin recognition and valuation. • Counting money helps students gain an awareness of consumer skills and the use of money in everyday life. • A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to ensure competence with using money. • ~~Establishing a one-to-one correspondence between the number names and the objects in a set of coins (pennies and/or nickels) is essential for an accurate count.~~ • Students need experiences to develop the concept that a nickel has a value of five cents even though it is one object.	**All students should** • Develop common referents for identifying pennies, nickels, dimes, and quarters. • Understand the value of a collection of coins whose value is 10 cents or less.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the properties/characteristics (e.g., color, relative size) of a penny, nickel, dime, and quarter. • Identify a penny, nickel, dime, and quarter. • Identify that a nickel is the same value as five pennies. • Count a randomly placed collection of pennies and/or nickels (or models of pennies and/or nickels) whose value is 10 cents or less, and determine the value of the collection.

K.8 The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences in measuring physical objects, using nonstandard and standard units of measure, help to develop an intuitive understanding of measurement and will help students connect a tool with its purpose in measuring. • Selecting from among various measuring instruments and determining which can be used to solve various real-life problems are introduced at this level. • A precursor to connecting tools to a type of measurement is an introduction to the concepts of length, weight, time, and temperature.	**All students should** • Identify an appropriate measuring tool for a given unit of measure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a ruler as an instrument to measure length. • Identify different types of scales as instruments to measure weight. • Identify different types of clocks (analog and digital) as instruments to measure time. • Identify the components of a calendar, including days, months, and seasons. • Identify different types of thermometers as instruments used to measure temperature.

K.9 The student will tell time to the hour, using analog and digital clocks.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences in relating time on the hour to daily routines and school schedules (e.g., catching the bus, lunch time, recess time, and resource time) help students develop personal referents for time. • Making sense of telling time to the nearest hour is reinforced when students recognize the positions of the hands on an analog clock and identify the corresponding time to the hour.	**All students should** • Apply an appropriate technique, depending on the type of clock, to determine time to the nearest hour.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Tell time on an analog clock to the hour. • Tell time on a digital clock to the hour.

K.10 The student will compare two objects or events, using direct comparisons or nonstandard units of measure, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder). Examples of nonstandard units include foot length, hand span, new pencil, paper clip, and block.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Length is the distance ~~along a line or figure from one point to another~~ between two points. • Height is the vertical length of a perpendicular to its base. • Weight is a measure of the heaviness of an object. • Temperature is the degree of hotness or coldness of an object (e.g., a body) or environment. • Students need to identify the attribute that they are measuring (e.g., length, height, weight, temperature) before they begin to measure. • ~~Extensive opportunities~~ Multiple hands-on experiences are needed to gain the ability to compare the attributes of objects.	**All students should** • Compare and order objects according to their attributes. • Develop an understanding of measuring with nonstandard and standard units of measure. • Recognize attributes (length, height, weight, temperature) that can be measured.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and describe lengths of two objects (as shorter or longer), using direct comparison or nonstandard units of measure (e.g., foot length, hand span, new pencil, paper clip, block). • Compare and describe heights of two objects (as taller or shorter), using direct comparison or nonstandard units of measure (e.g., book, hand span, new pencil, paper clip, block). • Compare and describe weights of two objects (as heavier or lighter), using direct comparison or nonstandard units of measure (e.g., book, cubes, new pencil, paper clip, block). • Compare and describe temperatures of two objects or environment (as hotter or colder), using direct comparison.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of ~~shapes~~ figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent ~~two- and three-dimensional~~ plane and solid ~~shapes~~ figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting and investigating ~~three-dimensional~~ solid objects and their ~~two-dimensional~~ faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying ~~shapes~~ figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to ~~these shapes~~ figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.

K.11 The student will

a) identify, describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and

b) compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An important part of the geometry strand in grades K through 2 is the naming and describing of ~~shapes~~ figures. Children move from their own vocabulary and begin to incorporate conventional terminology as the teacher uses geometric terms. • A plane geometric figure is any ~~two-dimensional~~ plane, closed ~~shape~~ figure. Circles and polygons are examples of plane geometric figures. • Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed. • The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. – **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. – **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know	**All students should** • Use their knowledge of ~~two-dimensional~~ plane figures to help them systematically represent and describe their world. • ~~Develop an understanding of the shapes of geometric figures by using various methods.~~ • Identify the characteristics of plane geometric figures (circle, triangle, square, and rectangle). • Compare the size and shape of plane geometric figures by using strategies to sort and/or group and begin to refine the vocabulary used to explain their strategies.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a circle, triangle, square, and rectangle. • Describe the ~~properties~~ characteristics of triangles, squares, and rectangles, including number of sides and number of ~~corners~~ angles. • Describe a circle using terms such as round and curved. • ~~Draw~~ Trace a circle, triangle, square, and rectangle. • Compare and group plane geometric figures (circle, triangle, square, and rectangle) according to their relative sizes (larger, smaller). • Compare and group plane geometric figures (circle, triangle, square, and rectangle) according to their shapes. • Distinguish between examples and nonexamples of identified geometric figures (circle, triangle, square, and rectangle).

K.11 The student will

a) identify, describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and

b) compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
that a door is a rectangle.") – **Level 2: Analysis.** Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it is closed; it has four sides and four right angles."). • A polygon is a geometric figure that – has sides that are ~~straight~~ line segments; – is simple (its sides do not cross); – is closed; and – is two-dimensional (~~it~~ lies in a plane). • A triangle is a polygon with three angles and three sides. Children should be shown different types of triangles such as equilateral, isosceles, scalene, right, acute, and obtuse; however, they are not expected to name the various types. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a rectangle with all four sides of equal length. • A circle is a closed curve with all points in one plane and the same distance from a fixed point (the center). • Early experiences with comparing and sorting ~~shapes~~ figures assist students in analyzing the characteristics ~~and properties of two-dimensional geometric shapes~~ of plane geometric figures. • Attribute blocks, relational attribute blocks, ~~power blocks~~, and tangrams are among the		

K.11 The student will

a) identify, describe, and trace plane geometric figures (circle, triangle, square, and rectangle); and

b) compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
manipulatives that are particularly appropriate for sorting and comparing size. – Clay, straws, and paper and scissors are several manipulatives that are appropriate for constructing geometric figures.		

K.12 The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of ~~shape~~ figures those things represent. • Children are often confused when a ~~shape~~ figure such as a square is rotated: they frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — i.e., a square is a square regardless of its location in space; there is no such geometric ~~shape~~ figure as a diamond. ~~• Geometric manipulatives that can be used to combine plane geometric figures to create familiar shapes are~~ ~~– tangrams;~~ ~~– attribute blocks;~~ ~~– pattern blocks;~~ ~~– power blocks;~~ ~~– relational attribute blocks; and~~ ~~– transformations (slides, flips, turns) on shapes, which can be applied to change the orientation of the shape.~~	**All students should** • ~~Use a variety of skills that relate to direction, distance, and position in space in order to enhance their navigation skills.~~ • Understand that objects can have different orientations in space.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify pictorial representations of a circle, triangle, square, and rectangle, regardless of their position and orientation in space. • Describe the location of one object relative to another, using the terms *above*, *below*, and *next to*.

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

K.13 The student will gather data by counting and tallying.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are pieces of information collected about people or things. The primary purpose of collecting data is to answer questions. • Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence. • When data are presented in an organized manner, students can describe the results of their investigation (i.e., identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same number of responses). • In the process of gathering data, students make decisions about what is relevant to their investigation (e.g., when collecting data on their classmates' favorite pets, deciding to limit the categories to common pets). • When students begin to collect data, they recognize the need to categorize, which helps develop the understanding of "things that go together." Categorical data are used when constructing ~~pictorial~~ picture graphs and bar graphs.	**All students should** • Pose questions and gather data. • Understand how data are collected and presented in an organized manner by counting and tallying.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • ~~Gather~~ Use counting and tallying to gather data on ~~given~~ categories identified by ~~counting and tallying~~ the teacher and/or student (e.g., favorites, number of days of various types of weather during a given month, types of pets, types of shoes).

K.14 The student will display gathered data in object graphs, picture graphs, and tables, and will answer questions related to the data.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student). • ~~Pictorial~~ Picture graphs are graphs that use pictures to show and compare information. ~~A key is often used to indicate what each picture represents (e.g., one picture of a dog represents five dogs).~~ • Tables are an orderly arrangement of data in which the data are arranged in columns and rows in an essentially rectangular format. Tables may be used to display some type of numerical relationship or organized lists (e.g., input/output functions, tables showing one candy costs five cents and two candies cost 10 cents). • Students represent data to convey results of their investigations at a glance, using concrete objects, pictures, and numbers to give a "picture" of the organized data. • When data are displayed in an organized manner, children can describe the results of their investigations. • Graphs can be used to make connections between mathematics and social studies and/or science (e.g., job areas and the different people that work in these areas: health — doctors and nurses; education — teachers and principals). • Statements representing an analysis and interpretation of the characteristics of the data in	**All students should** • Understand that data can be represented using concrete objects, pictures, and graphs. • Understand that different types of representations emphasize different things about the same data. • Understand that picture graphs use pictures to show and compare information; object graphs use concrete materials to represent categorical data; and tables can be used to show an orderly arrangement of data in columns and rows. • Answer questions related to the gathered data from object graphs, picture graphs, and tables. • Relate their ideas about the data to concepts such as part-part-whole and number relationships.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Display data by arranging concrete objects into organized groups to form a simple object graph. • Display gathered data, using ~~pictorial representations of the data~~ pictures to form a simple ~~pictorial~~ picture graph (e.g., a picture graph of the types of shoes worn by students on a given day). • Display ~~information~~ gathered data in tables, either in rows or columns ~~(e.g., a table showing the number of bunnies in one column and the number of ears the bunnies have in another or a table showing the time schedule for classroom activities)~~. • Answer questions related to the gathered data displayed in object graphs, picture graphs, and tables by: – Describing the categories of data and the data as a whole (e.g., the total number of responses) and its parts. – Identifying parts of the data that represent numerical relationships, including categories with the greatest, the least, or the same.

K.14 The student will display gathered data in object graphs, picture graphs, and tables, and will answer questions related to the data.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be asked.		

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns ~~in the real world~~. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to ~~A~~algebra.

K.15 The student will sort and classify objects according to attributes.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic ~~of a thing~~). An object has many attributes such as color, size. shape, thickness, etc. • General similarities and differences among objects are easily observed by children entering kindergarten, who are able to focus on any one attribute. The teacher's task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves with different colors and different ~~shapes~~ figures).	**All students should** • Understand that the same set of objects can be sorted and classified in different ways.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Sort objects into appropriate groups (categories) based on one attribute~~, such as size, shape, or color~~. • Classify sets of objects into ~~three~~ groups (categories) of one attribute ~~(e.g., for size — small, medium, and large)~~. • Label attributes of a set of objects that has been sorted. • Name multiple ways to sort a set of objects.

K.16 The student will identify, describe, and extend repeating patterns.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Pattern recognition is a fundamental cornerstone of mathematics, particularly algebra. • In a repeating pattern, the core of the pattern is the string of elements that repeats. By identifying the core, students demonstrate their understanding of the pattern. • Students should recognize that the sound pattern 'snap, clap, snap, clap' is the same in form as the color pattern 'red, blue, red, blue'. • Pattern recognition and the extension of the pattern allows students to make predictions. • The simplest types of patterns are repeating patterns. The patterns can be oral, such as the refrain in "Old MacDonald's Farm" ("e-i-e-i-o"), or physical with clapping and snapping patterns, or combinations of both, such as is found in songs like the "Hokey Pokey." In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. • Sample repeating patterns (repeating the ~~basic~~ core ~~unit)~~ are – ABABABAB; – ABCABC; – AABBAABBAABB; – AABAAB; – AABCAABC; and – ABACABAC.	**All students should** • Understand that patterns are a way to recognize order and organize their world and to predict what comes next in an arrangement. • Understand that the sound pattern 'snap, clap, snap, clap' is the same in form as the color pattern 'red, blue, red, blue'.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Observe and identify the basic repeating pattern (core) found in repeating patterns of common objects, sounds, and movements that occur in ~~real-life~~ practical situations~~, where there are four or fewer elements in the basic repeating pattern~~. • ~~Describe~~ Identify the ~~basic repeating pattern~~ core ~~found~~ in a repeating pattern~~, where there are four or fewer elements in the basic repeating pattern~~. • Extend a repeating pattern by adding at least two repetitions to the pattern. • Create a repeating pattern. • Compare similarities and differences between patterns.