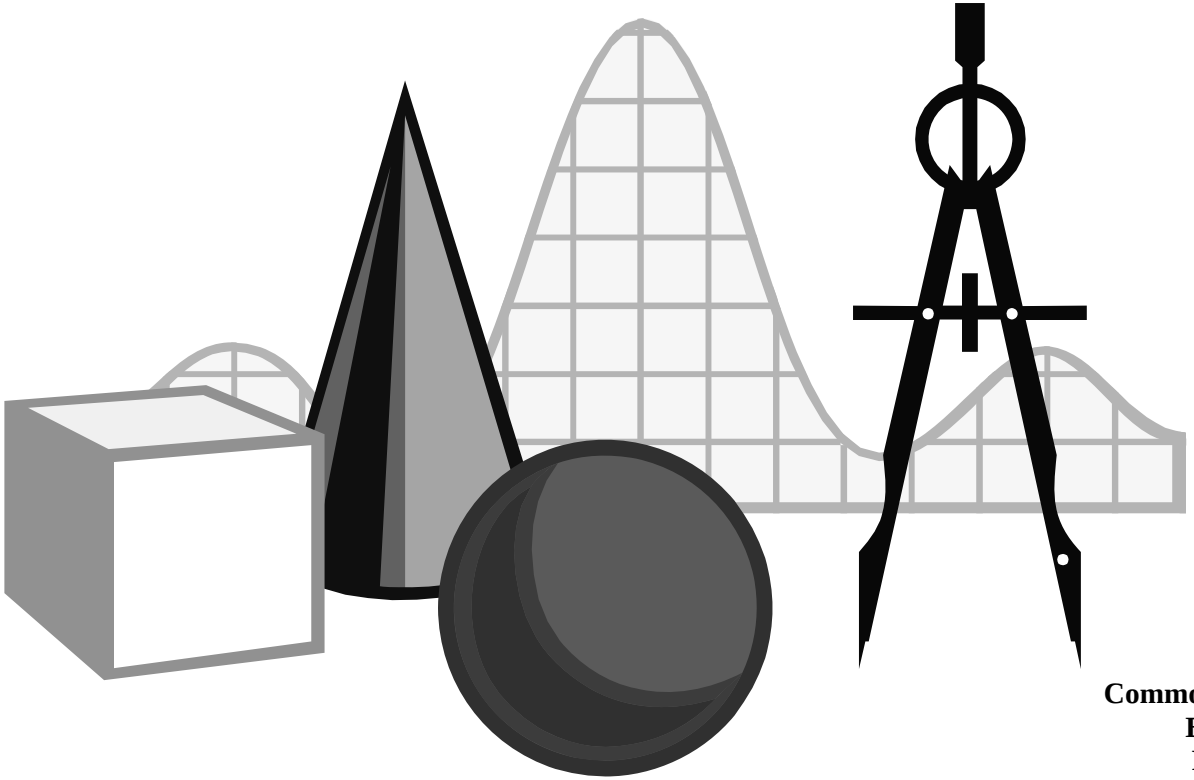
MATHEMATICS STANDARDS OF LEARNING PROPOSED CURRICULUM FRAMEWORK

Probability and Statistics



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The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia 2009 *Mathematics Standards of Learning* Curriculum Framework
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills~~ and Essential Understandings~~. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills
Each ~~S~~standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for ~~SOL~~ Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.1

The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Measures of central tendency describe how the data cluster or group. • Measures of dispersion describe how the data spread (disperse) around the center of the data. • Graphical displays of data may be analyzed informally. • Data analysis must take place within the context of the problem.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Create and interpret graphical displays of data, including dotplots, stem-and-leaf plots, and histograms. • Examine graphs of data for clusters and gaps, and relate those phenomena to the data in context. • Examine graphs of data for outliers, and explain the outlier(s) within the context of the data. • Examine graphs of data and identify the central tendency of the data as well as the spread. Explain the central tendency and the spread of the data within the context of the data.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.2

The student will analyze numerical characteristics of univariate data sets to describe patterns and departure from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning within a context. • Analysis of the descriptive statistical information generated by a univariate data set should include the interplay between central tendency and dispersion as well as among specific measures. • Data points identified algorithmically as outliers should not be excluded from the data unless sufficient evidence exists to show them to be in error.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem's context. • Identify possible outliers, using an algorithm. • Explain the influence of outliers on a univariate data set. • Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.3

The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Statistical tendency refers to typical cases but not necessarily to individual cases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast two or more univariate data sets by analyzing measures of center and spread within a contextual framework. • Describe any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data. • Analyze in context kurtosis and skewness in conjunction with other descriptive measures.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.4

The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A scatterplot serves two purposes: – to determine if there is a useful relationship between two variables, and – to determine the family of equations that describes the relationship. • Data are collected for a purpose and have meaning in a context. • Association between two variables considers both the direction and strength of the association. • The strength of an association between two variables reflects how accurately the value of one variable can be predicted based on the value of the other variable. • Outliers are observations with large residuals and do not follow the pattern apparent in the other data points.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Examine scatterplots of data, and describe skewness, kurtosis, and correlation within the context of the data. • Describe and explain any unusual features of the data, such as clusters, gaps, or outliers, within the context of the data. • Identify influential data points (observations that have great effect on a line of best fit because of extreme *x*-values) and describe the effect of the influential points.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.5

The student will find and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Least squares regression generates the equation of the line that minimizes the sum of the squared distances from the data points to the line. • Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured). • Residual = Actual – Fitted • A correlation coefficient measures the degree of association between two variables that are related linearly. • Two variables may be strongly associated without a cause-and-effect relationship existing between them.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Calculate a correlation coefficient. • Explain how the correlation coefficient, *r*, measures association by looking at its formula. • Use regression lines to make predictions, and identify the limitations of the predictions. • Use residual plots to determine if a linear model is satisfactory for describing the relationship between two variables. • Describe the errors inherent in extrapolation beyond the range of the data. • Use least squares regression to find the equation of the line of best fit for a set of data. • Explain how least squares regression generates the equation of the line of best fit by examining the formulas used in computation.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.6

The student will make logarithmic and power transformations to achieve linearity.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A logarithmic transformation reduces positive skewness because it compresses the upper tail of the distribution while stretching the lower tail. • Nonlinear transformations do not preserve relative spacing between data points.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Apply a logarithmic transformation to data. • Explain how a logarithmic transformation works to achieve a linear relationship between variables. • Apply a power transformation to data. • Explain how a power transformation works to achieve a linear relationship between variables.

TOPIC: DESCRIPTIVE STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.7

The student, using two-way tables, will analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Simpson's paradox refers to the fact that aggregate proportions can reverse the direction of the relationship seen in the individual parts. • Two categorical variables are independent if the conditional frequencies of one variable are the same for every category of the other variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Produce a two-way table as a summary of the information obtained from two categorical variables. • Calculate marginal, relative, and conditional frequencies in a two-way table. • Use marginal, relative, and conditional frequencies to analyze data in two-way tables within the context of the data.

TOPIC: DATA COLLECTION

PROBABILITY AND STATISTICS
STANDARD PS.8

The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of a sample statistic varies from sample to sample if the simple random samples are taken repeatedly from the population of interest. • Poor data collection can lead to misleading and meaningless conclusions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast controlled experiments and observational studies and the conclusions one can draw from each. • Compare and contrast population and sample and parameter and statistic. • Identify biased sampling methods. • Describe simple random sampling. • Select a data collection method appropriate for a given context.

TOPIC: DATA COLLECTION

PROBABILITY AND STATISTICS
STANDARD PS.9

The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random and stratified) and methods to reduce bias.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The purpose of sampling is to provide sufficient information so that population characteristics may be inferred. • Inherent bias diminishes as sample size increases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. • Determine which sampling technique is best, given a particular context. • Plan a survey to answer a question or address an issue. • Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. • Design a survey instrument. • Conduct a survey.

TOPIC: DATA COLLECTION

PROBABILITY AND STATISTICS
STANDARD PS.10

The student will plan and conduct an experiment. The plan will address control, randomization, and measurement of experimental error.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Experiments must be carefully designed in order to detect a cause-and-effect relationship between variables. • Principles of experimental design include comparison with a control group, randomization, and blindness.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Plan and conduct an experiment. The experimental design should address control, randomization, and minimization of experimental error.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.11

The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The complement of event A consists of all outcomes in which event A does not occur. • Two events, A and B, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If A and B are not independent, then they are said to be dependent. • Events A and B are mutually exclusive if they cannot occur simultaneously.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. • Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.12

The student will find probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data are collected for a purpose and have meaning in a context. • Venn diagrams may be used to find conditional probabilities. • The "Law of Large Numbers" states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Calculate relative frequency and expected frequency. • Find conditional probabilities for dependent, independent, and mutually exclusive events.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.13

The student will develop, interpret, and apply the binomial probability distribution for discrete random variables, including computing the mean and standard deviation for the binomial variable.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A probability distribution is a complete listing of all possible outcomes of an experiment together with their probabilities. The procedure has a fixed number of independent trials. • A random variable assumes different values depending on the event outcome. • A probability distribution combines descriptive statistical techniques and probabilities to form a theoretical model of behavior.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Develop the binomial probability distribution within a real-world context. • Calculate the mean and standard deviation for the binomial variable. • Use the binomial distribution to calculate probabilities associated with experiments for which there are only two possible outcomes.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.14

The student will simulate probability distributions, including binomial and geometric.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A probability distribution combines descriptive methods and probabilities to form a theoretical model of behavior. • A probability distribution gives the probability for each value of the random variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Design and conduct an experiment that simulates a binomial distribution. • Design and conduct an experiment that simulates a geometric distribution.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.15

The student will identify random variables as independent or dependent and find the mean and standard deviations for sums and differences of independent random variables.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A random variable is a variable that has a single numerical value, determined by chance, for each outcome of a procedure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast independent and dependent random variables. • Find the standard deviation for sums and differences of independent random variables.

TOPIC: PROBABILITY

PROBABILITY AND STATISTICS
STANDARD PS.16

The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation. • Areas under the curve represent probabilities associated with continuous distributions. • The normal curve is a probability distribution and the total area under the curve is 1.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a normal probability distribution. • Describe how the standard deviation and the mean affect the graph of the normal distribution. • Determine the probability of a given event, using the normal distribution.

TOPIC: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.17

The student, given data from a large sample, will find and interpret point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired).

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A primary goal of sampling is to estimate the value of a parameter based on a statistic. • Confidence intervals use the sample statistic to construct an interval of values that one can be reasonably certain contains the true (unknown) parameter. • Confidence intervals and tests of significance are complementary procedures. • Paired comparisons experimental design allows control for possible effects of extraneous variables.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct confidence intervals to estimate a population parameter, such as a proportion or the difference between two proportions; or a mean or the difference between two means. • Select a value for alpha (Type I error) for a confidence interval. • Interpret confidence intervals in the context of the data. • Explain the importance of random sampling for confidence intervals. • Calculate point estimates for parameters and discuss the limitations of point estimates.

TOPIC: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.18

The student will apply and interpret the logic of a hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, and difference between two means (independent and paired) and Chi-squared tests for goodness of fit, homogeneity of proportions, and independence.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Confidence intervals and tests of significance are complementary procedures. • Paired comparisons experimental design allows control for possible effects of extraneous variables. • Tests of significance assess the extent to which sample data support a hypothesis about a population parameter. • The purpose of a goodness of fit test is to decide if the sample results are consistent with results that would have been obtained if a random sample had been selected from a population with a known distribution. • Practical significance and statistical significance are not necessarily congruent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the Chi-squared test for goodness of fit to decide if the population being analyzed fits a particular distribution pattern. • Use hypothesis-testing procedures to determine whether or not to reject the null hypothesis. The null hypothesis may address proportion, mean, difference between two proportions or two means, goodness of fit, homogeneity of proportions, and independence. • Compare and contrast Type I and Type II errors. • Explain how and why the hypothesis-testing procedure allows one to reach a statistical decision.

TOPIC: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.19

The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The Central Limit Theorem states: – The mean of the sampling distribution of means is equal to the population mean. – If the sample size is sufficiently large, the sampling distribution approximates the normal probability distribution. – If the population is normally distributed, the sampling distribution is normal regardless of sample size. • Sampling distributions have less variability with larger sample sizes.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the use of the Central Limit Theorem for drawing inferences about a population parameter based on a sample statistic. • Describe the effect of sample size on the sampling distribution and on related probabilities. • Use the normal approximation to calculate probabilities of sample statistics falling within a given interval. • Identify and describe the characteristics of a sampling distribution of a sample proportion, mean, difference between two sample proportions, or difference between two sample means.

TOPIC: INFERENTIAL STATISTICS

PROBABILITY AND STATISTICS
STANDARD PS.20

The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures, using tables or graphing calculators.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Paired comparisons experimental design allows control for possible effects of extraneous variables. • The sampling distribution of means with a small sample size follows a t-distribution.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a t-distribution. • Compare and contrast a t-distribution and a normal distribution. • Use a t-test for single-sample and two-sample data.