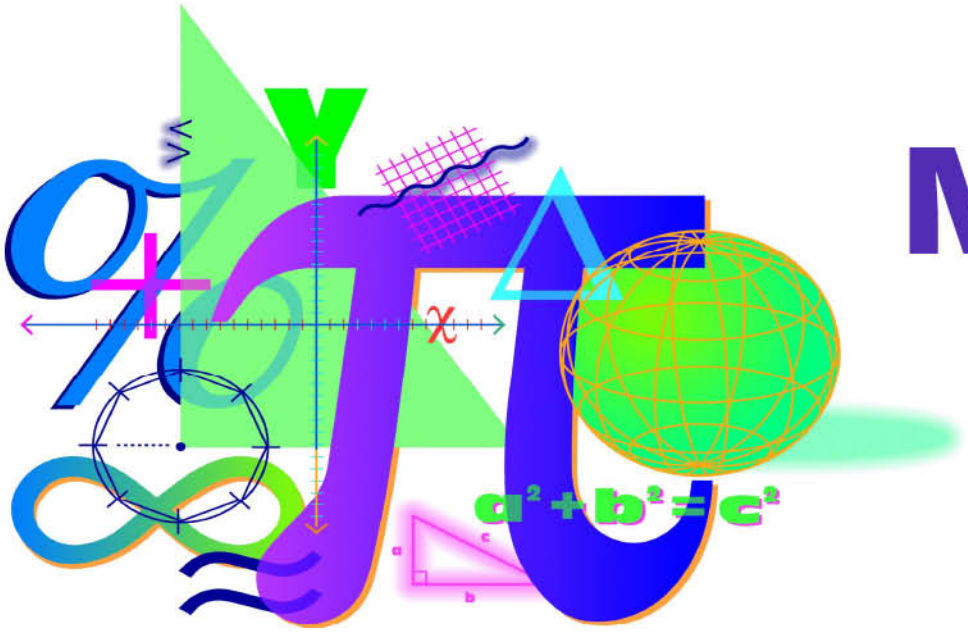

Mathematics
Standards of Learning

Curriculum Framework 2009

Grade 4

Board of Education
Commonwealth of Virginia

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Virginia *Mathematics Standards of Learning* Curriculum Framework 2009
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

Mathematics instruction in grades 4 and 5 should continue to foster the development of number sense, especially with decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction at grades 4 and 5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names). Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve problems.

4.1 The student will

a) identify orally and in writing the place value for each digit in a whole number expressed through millions;

b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the Base-10 number system is based upon a simple pattern of tens, in which the value of each place is ten times the value of the place to its right. • Place value refers to the value of each digit and depends upon the position of the digit in the number. For example, in the number 7,864,352, the eight is in the hundred thousands place, and the value of the 8 is eight hundred thousand or 800,000. • Whole numbers may be written in a variety of formats: – Standard: 1,234,567 – Written: one million, two hundred thirty-four thousand, five hundred sixty-seven – Expanded: $(1 \times 1{,}000{,}000) + (2 \times 100{,}000) + (3 \times 10{,}000) + (4 \times 1{,}000) + (5 \times 100) + (6 \times 10) + (7 \times 1)$ • Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, …). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students find values of digits in any number as well as read and write numbers.	**All students should** • Understand the relationships in the place value system in which the value of each place is ten times the value of the place to its right. • Use the patterns in the place value system to read and write numbers. • Understand that reading place value correctly is essential when comparing numbers. • Understand that rounding gives a close number to use when exact numbers are not needed for the situation at hand. • Develop strategies for rounding.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and communicate, both orally and in written form, the placed value for each digit in whole numbers expressed through the one millions place. • Read whole numbers through the one millions place that are presented in standard format, and select the matching number in written format. • Write whole numbers through the one millions place in standard format when the numbers are presented orally or in written format. • Identify and use the symbols for *greater than*, *less than*, and *equal to*. • Compare two whole numbers expressed through the one millions, using symbols >, <, or =. • Round whole numbers expressed through the one millions place to the nearest thousand, ten thousand, and hundred-thousand place.

4.1 The student will

a) identify orally and in writing the place value for each digit in a whole number expressed through millions;

b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Reading and writing large numbers should be meaningful for students. Experiences can be provided that relate practical situations (e.g., numbers found in the students' environment including population, number of school lunches sold statewide in a day, etc.). Concrete materials such as Base-10 blocks and bundles of sticks may be used to represent whole numbers through thousands. Larger numbers may be represented by digit cards and place value charts. • Mathematical symbols (>, <) used to compare two unequal numbers are called *inequality symbols*. • A procedure for comparing two numbers by examining place value may include the following: – Compare the digits in the numbers to determine which number is greater (or which is less). – Use a number line to identify the appropriate placement of the numbers based on the place value of the digits. – Use the appropriate symbol > or < or words *greater than* or *less than* to compare the numbers in the order in which they are presented. – If both numbers have the same value, use the symbol = or words *equal to*.		

4.1 The student will

a) identify orally and in writing the place value for each digit in a whole number expressed through millions;

b) compare two whole numbers expressed through millions, using symbols (>, <, or =); and

c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A strategy for rounding numbers to the nearest thousand, ten thousand, and hundred thousand is as follows: – Use a number line to determine the rounded number (e.g., when rounding 4,367,925 to the nearest thousand, identify the 'thousands' the number would fall between on the number line, then determine the thousand that the number is closest to): 4,367**,000** ? 4,368**,000** ◄——————————► – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero.		

4.2 The student will

a) compare and order fractions and mixed numbers;

b) represent equivalent fractions; and

c) identify the division statement that represents a fraction.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction is a way of representing part of a whole (as in a region/area model or a measurement model) or part of a group (as in a set model). A fraction is used to name a part of one thing or a part of a collection of things. • In the area/region and length/measurement fraction models, the parts must be equal. In the set model, the elements of the set do not have to be equal (i.e., "What fraction of the class is wearing the color red?"). • The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being counted or described. • When fractions have the same denominator, they are said to have "common denominators" or "like denominators." Comparing fractions with like denominators involves comparing only the numerators. • Strategies for comparing fractions having unlike denominators may include – comparing fractions to familiar benchmarks (e.g., 0, $\frac{1}{2}$, 1); – finding equivalent fractions, using manipulative models such as fraction strips, number lines, fraction circles, rods, pattern blocks, cubes, Base-10 blocks,	**All students should** • Develop an understanding of fractions as parts of unit wholes, as parts of a collection, and as locations on a number line. • Understand that a mixed number is a fraction that has two parts: a whole number and a proper fraction. The mixed number is the sum of these two parts. • Use models, benchmarks, and equivalent forms to judge the size of fractions. • Recognize that a whole divided into nine equal parts has smaller parts than if the whole had been divided into five equal parts. • Recognize and generate equivalent forms of commonly used fractions and decimals. • Understand the division statement that represents a fraction. • Understand that the more parts the whole is divided into, the smaller the parts (e.g., $\frac{1}{5} < \frac{1}{3}$).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and order fractions having denominators of 12 or less, using manipulative models and drawings, such as region/area models. • Compare and order fractions with like denominators by comparing number of parts (numerators) (e.g., $\frac{1}{5} < \frac{3}{5}$). • Compare and order fractions with like numerators and unlike denominators by comparing the size of the parts (e.g., $\frac{3}{9} < \frac{3}{5}$). • Compare and order fractions having unlike denominators of 12 or less by comparing the fractions to benchmarks (e.g., 0, $\frac{1}{2}$ or 1) to determine their relationships to the benchmarks or by finding a common denominator. • Compare and order mixed numbers having denominators of 12 or less. • Use the symbols >, <, and = to compare the numerical value of fractions and mixed numbers having denominators of 12 or less. • Represent equivalent fractions through twelfths, using region/area models, set models, and measurement models.

4.2 The student will

a) compare and order fractions and mixed numbers;
b) represent equivalent fractions; and
c) identify the division statement that represents a fraction.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
tangrams, graph paper, or a multiplication chart and patterns; and – finding a common denominator by finding the least common multiple (LCM) of both denominators and then rewriting each fraction as an equivalent fraction, using the LCM as the denominator. • A variety of fraction models should be used to expand students' understanding of fractions and mixed numbers: – Region/area models: a surface or area is subdivided into smaller equal parts, and each part is compared with the whole (e.g., fraction circles, pattern blocks, geoboards, grid paper, color tiles). – Set models: the whole is understood to be a set of objects, and subsets of the whole make up fractional parts (e.g., counters, chips). – Measurement models: similar to area models but lengths instead of areas are compared (e.g., fraction strips, rods, cubes, number lines, rulers). • A mixed number has two parts: a whole number and a fraction. • Equivalent fractions name the same amount. Students should use a variety of models to identify different names for equivalent fractions.		• Identify the division statement that represents a fraction (e.g., $\frac{3}{5}$ means the same as 3 divided by 5).

4.2 The student will

a) compare and order fractions and mixed numbers;

b) represent equivalent fractions; and

c) identify the division statement that represents a fraction.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Students should focus on finding equivalent fractions of familiar fractions such as halves, thirds, fourths, sixths, eighths, tenths, and twelfths. • Decimals and fractions represent the same relationships; however, they are presented in two different formats. The decimal 0.25 is written as $\frac{1}{4}$. When presented with the fraction $\frac{3}{5}$, the division expression representing a fraction is written as 3 divided by 5.		

4.3 The student will

a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number, tenth, and hundredth;
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the Base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. • Understanding the system of tens means that ten tenths represents one whole, ten hundredths represents one tenth, ten thousandths represents one hundredth. • A decimal point separates the whole number places from the places that are less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*. • To read decimals, – read the whole number to the left of the decimal point, if there is one; – read the decimal point as "and"; – read the digits to the right of the decimal point just as you would read a whole number; and – say the name of the place value of the digit in the smallest place. • Any decimal less than 1 will include a leading zero (e.g., 0.125). • Decimals may be written in a variety of forms: – Standard: 26.537 – Written: twenty-six and five hundred thirty-seven thousandths	**All students should** • Understand the place value structure of decimals and use this structure to read, write, and compare decimals. • Understand that decimal numbers can be rounded to an estimate when exact numbers are not needed for the situation at hand. • Understand that decimals are rounded in a way that is similar to the way whole numbers are rounded. • Understand that decimals and fractions represent the same relationship; however, they are presented in two different formats. • Understand that models are used to show decimal and fraction equivalents.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate the ten-to-one place value relationship for decimals through thousandths, using Base-10 manipulatives (e.g., place value mats/charts, decimal squares, Base-10 blocks, money). • Represent and identify decimals expressed through thousandths, using Base-10 manipulatives, pictorial representations, and numerical symbols (e.g., relate the appropriate drawing to 0.05). • Identify and communicate, both orally and in written form, the position and value of a decimal through thousandths. For example, in 0.385, the 8 is in the hundredths place and has a value of 0.08. • Read and write decimals expressed through thousandths, using Base-10 manipulatives, drawings, and numerical symbols. • Round decimals to the nearest whole number, tenth, and hundredth. • Compare decimals, using the symbols >, <, =. • Order a set of decimals from least to greatest or greatest to least.

4.3 The student will

a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number, tenth, and hundredth;
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
– Expanded: $(2 \times 10) + (6 \times 1) + (5 \times 0.1) + (3 \times 0.01) + (7 \times 0.001)$. • Decimals and fractions represent the same relationships; however, they are presented in two different formats. The decimal 0.25 is written as $\frac{1}{4}$. Decimal numbers are another way of writing fractions. When presented with the fraction $\frac{3}{5}$, the division expression representing a fraction is written as 3 divided by 5. The Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money). • The procedure for rounding decimal numbers is similar to the procedure for rounding whole numbers. • A strategy for rounding decimal numbers to the nearest tenth and hundredth is as follows: – Look one place to the right of the digit you want to round to. – If the digit is 5 or greater, add 1 to the digit in the rounding place, and drop the digits to the right of the rounding place. – If the digit is less than 5, leave the digit in the rounding place as it is, and drop the digits to the right of the rounding place. • Different strategies for rounding decimals include: – Use a number line to locate a decimal between two numbers. For example, 18.83 is closer to 18.8 than to 18.9.		• Represent fractions for halves, fourths, fifths, and tenths as decimals through hundredths, using concrete objects (e.g., demonstrate the relationship between the fraction $\frac{1}{4}$ and its decimal equivalent 0.25). • Relate fractions to decimals, using concrete objects (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money [coins]). • Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4} = 0.25$ or $0.25 = \frac{1}{4}$).

4.3 The student will

a) read, write, represent, and identify decimals expressed through thousandths;

b) round decimals to the nearest whole number, tenth, and hundredth;

c) compare and order decimals; and

d) given a model, write the decimal and fraction equivalents.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
– Compare the digits in the numbers to determine which number is greater (or which is less). – Compare the value of decimals, using the symbols >, <, = (e.g., 0.83 > 0.8 or 0.19 < 0.2). – Order the value of decimals, from least to greatest and greatest to least (e.g., 0.83, 0.821, 0.8). • Decimal numbers are another way of writing fractions (halves, fourths, fifths, and tenths). The Base-10 models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles money). • Provide a fraction model (halves, fourths, fifths, and tenths) and ask students for its decimal equivalent. • Provide a decimal model and ask students for its fraction equivalent (halves, fourths, fifths, and tenths).		

Computation and estimation in grades 4 and 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract with common fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability to identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use (not identify) properties of operations to solve problems [e.g., 7×28 is equivalent to $(7 \times 20) + (7 \times 8)$].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their computations. Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

4.4 The student will

a) estimate sums, differences, products, and quotients of whole numbers;
b) add, subtract, and multiply whole numbers;
c) divide whole numbers, finding quotients with and without remainders; and
d) solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A sum is the result of adding two or more numbers. • A difference is the amount that remains after one quantity is subtracted from another. • An estimate is a number close to an exact solution. An estimate tells about how much or about how many. • Different strategies for estimating include using compatible numbers to estimate sums and differences and using front-end estimation for sums and differences. – Compatible numbers are numbers that are easy to work with mentally. Number pairs that are easy to add or subtract are compatible. When estimating a sum, replace actual numbers with compatible numbers (e.g., 52 + 74 can be estimated by using the compatible numbers 50 + 75). When estimating a difference, use numbers that are close to the original numbers. Tens and hundreds are easy to subtract (e.g., 83 – 38 is close to 80 – 40). – The front-end strategy for estimating is computing with the front digits. Front-end estimation for addition can be used even when the addends have a different number	**All students should** • Develop and use strategies to estimate whole number sums and differences and to judge the reasonableness of such results. • Understand that addition and subtraction are inverse operations. • Understand that division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. • Understand that multiplication and division are inverse operations. • Understand various representations of division and the terms used in division are *dividend*, *divisor*, and *quotient.* $dividend \div divisor = quotient$ $divisor \overline{)\,dividend}$ *quotient* • Understand how to solve single-step and multistep problems using whole number operations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate whole number sums, differences, products, and quotients. • Refine estimates by adjusting the final amount, using terms such as *closer to*, *between*, and *a little more than*. • Determine the sum or difference of two whole numbers, each 999,999 or less, in vertical and horizontal form with or without regrouping, using paper and pencil, and using a calculator. • Estimate and find the products of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer, using paper and pencil and calculators. • Estimate and find the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend. • Solve single-step and multistep problems using whole number operations. • Verify the reasonableness of sums, differences, products, and quotients of whole numbers using estimation.

4.4 The student will

a) estimate sums, differences, products, and quotients of whole numbers;
b) add, subtract, and multiply whole numbers;
c) divide whole numbers, finding quotients with and without remainders; and
d) solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
of digits. The procedure requires the addition of the values of the digits in the greatest of the smallest number. For example: 2367 → 2300 243 → 200 + 1186 → + 1100 3600 • Front-end or leading-digit estimation always gives a sum less than the actual sum; however, the estimate can be adjusted or refined so that it is closer to the actual sum. • Addition is the combining of quantities; it uses the following terms: *addend* → 45,623 *addend* → + 37,846 *sum* → 83,469 • Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms: *minuend* → 45,698 *subtrahend* → – 32,741 *difference* → 12,957		

4.4 The student will

a) estimate sums, differences, products, and quotients of whole numbers;
b) add, subtract, and multiply whole numbers;
c) divide whole numbers, finding quotients with and without remainders; and
d) solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Before adding or subtracting with paper and pencil, addition and subtraction problems in horizontal form should be rewritten in vertical form by lining up the places vertically. • Using Base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the pictorial, to the abstract. Regrouping is used in addition and subtraction algorithms. In addition, when the sum in a place is 10 or more, is used to regroup the sums so that there is only one digit in each place. In subtraction, when the number (minuend) in a place is not enough from which to subtract, regrouping is required. • A certain amount of practice is necessary to develop fluency with computational strategies for multidigit numbers; however, the practice must be meaningful, motivating, and systematic if students are to develop fluency in computation, whether mentally, with manipulative materials, or with paper and pencil. • Calculators are an appropriate tool for computing sums and differences of large numbers, particularly when mastery of the algorithm has been demonstrated.		

4.4 The student will

a) estimate sums, differences, products, and quotients of whole numbers;
b) add, subtract, and multiply whole numbers;
c) divide whole numbers, finding quotients with and without remainders; and
d) solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The terms associated with multiplication are *factor* → 376 *factor* → × 23 *product* → 8,648 • One model of multiplication is repeated addition. • Another model of multiplication is the "Partial Product" model. 24 × 3 12 ← Multiply the ones: 3 × 4 = 12 + 60 ← Multiply the tens: 3 × 20 = 60 72 • Another model of multiplication is the "Area Model" (which also represents partial products) and should be modeled first with Base-10 blocks. (e.g., 23 x 68) • Students should continue to develop fluency with single-digit multiplication facts and their related division facts. • Calculators should be used to solve problems that require tedious calculations.		

4.4 The student will

a) estimate sums, differences, products, and quotients of whole numbers;
b) add, subtract, and multiply whole numbers;
c) divide whole numbers, finding quotients with and without remainders; and
d) solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimation should be used to check the reasonableness of the product. Examples of estimation strategies include the following: – The front-end method: multiply the front digits and then complete the product by recording the number of zeros found in the factors. It is important to develop understanding of this process before using the step-by-step procedure. 523 → 500 × 31 → × 30 15,000 – This is $3 \times 5 = 15$ with 3 zeros. – Compatible numbers: replace factors with compatible numbers, and then multiply. Opportunities for students to discover patterns with 10 and powers of 10 should be provided. 64 → 64 × 11 → × 10 • Division is the operation of making equal groups or equal shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. Both situations may be modeled with Base-10 manipulatives.		

4.4 The student will

a) estimate sums, differences, products, and quotients of whole numbers;
b) add, subtract, and multiply whole numbers;
c) divide whole numbers, finding quotients with and without remainders; and
d) solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Multiplication and division are inverse operations. • Terms used in division are *dividend*, *divisor*, and *quotient.* dividend ÷ divisor = quotient quotient divisor)dividend • Opportunities to invent division algorithms help students make sense of the algorithm. Teachers should teach division by various methods such as repeated multiplication and subtraction (partial quotients) before teaching the traditional long division algorithm.		

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;

c) add and subtract with decimals; and

d) solve single-step and multistep practical problems involving addition and subtraction with fractions and with decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A factor of a number is an integer that divides evenly into that number with a remainder of zero. • A factor of a number is a divisor of the number. • A multiple of a number is the product of the number and any natural number. • A common factor of two or more numbers is a divisor that all of the numbers share. • The least common multiple of two or more numbers is the smallest common multiple of the given numbers. • The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share. • Students should investigate addition and subtraction with fractions, using a variety of models (e.g., fraction circles, fraction strips, rulers, linking cubes, pattern blocks). • When adding or subtracting with fractions having like denominators, add or subtract the numerators and use the same denominator. Write the answer in simplest form using common multiples and factors.	**All students should** • Understand and use common multiples and common factors for simplifying fractions. • Develop and use strategies to estimate addition and subtraction involving fractions and decimals. • Use visual models to add and subtract with fractions and decimals.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find common multiples and common factors of numbers. • Determine the least common multiple and greatest common factor of numbers. • Use least common multiple and/or greatest common factor to find a common denominator for fractions. • Add and subtract with fractions having like denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. • Add and subtract with fractions having unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors. • Solve problems that involve adding and subtracting with fractions having like and unlike denominators whose denominators are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction using common multiples and factors.

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;

c) add and subtract with decimals; and

d) solve single-step and multistep practical problems involving addition and subtraction with fractions and with decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• When adding or subtracting with fractions having unlike denominators, rewrite them as fractions with a common denominator. The least common multiple (LCM) of the unlike denominators is a common denominator (LCD). Write the answer in simplest form using common multiples and factors. • Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money). • For decimal computation, the same ideas developed for whole number computation may be used, and these ideas may be applied to decimals, giving careful attention to the placement of the decimal point in the solution. Lining up tenths to tenths, hundredths to hundredths, etc. helps to establish the correct placement of the decimal. • Fractions may be related to decimals by using models (e.g., 10-by-10 grids, decimal squares, money).		• Add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. • Solve single-step and multistep problems that involve adding and subtracting with fractions and decimals through thousandths.

Students in grades 4 and 5 should be actively involved in measurement activities that require a dynamic interaction between students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure perimeter, weight/mass, liquid volume/capacity, area, temperature, and time. Students continue to enhance their understanding of measurement by using appropriate tools such as rulers, balances, clocks, and thermometers. The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.

4.6 The student will

a) estimate and measure weight/mass and describe the results in U.S. Customary and metric units as appropriate; and

b) identify equivalent measurements between units within the U.S. Customary system (ounces, pounds, and tons) and between units within the metric system (grams and kilograms).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • Balances are appropriate measuring devices to measure weight in U.S. Customary units (ounces, pounds) and mass in metric units (grams, kilograms). • Practical experience measuring the mass of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate weight/mass. • Students should estimate the mass/weight of everyday objects (e.g., foods, pencils, book bags, shoes), using appropriate metric or U.S. Customary units.	**All students should** • Use benchmarks to estimate and measure weight/mass. • Identify equivalent measures between units within the U.S. Customary and between units within the metric measurements.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (e.g., ounce, pound, ton, gram, kilogram) to use when measuring everyday objects in both metric and U.S. Customary units. • Measure objects in both metric and U.S. Customary units (e.g., ounce, pound, ton, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. • Record the mass of an object including the appropriate unit of measure (e.g., 24 grams).

4.7 The student will

a) estimate and measure length, and describe the result in both metric and U.S. Customary units; and

b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Length is the distance along a line or figure from one point to another. • U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure. • Practical experience measuring the length of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate length. • Students should estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure. • When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$), inch, foot, or yard.	**All students should** • Use benchmarks to estimate and measure length. • Understand how to convert units of length between the U.S. Customary and metric systems, using ballpark comparisons. • Understand the relationship between U.S. Customary units and the relationship between metric units.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (e.g., inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring everyday objects in both metric and U.S. Customary units. • Estimate the length of everyday objects (e.g., books, windows, tables) in both metric and U.S. Customary units of measure. • Measure the length of objects in both metric and U.S. Customary units, measuring to the nearest inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$), foot, yard, mile, millimeter, centimeter, or meter, and record the length including the appropriate unit of measure (e.g., 24 inches). • Compare estimates of the length of objects with the actual measurement of the length of objects. • Identify equivalent measures of length between units within the U.S. Customary measurements and between units within the metric measurements.

4.8 The student will

a) estimate and measure liquid volume and describe the results in U.S. Customary units; and

b) identify equivalent measurements between units within the U.S. Customary system (cups, pints, quarts, and gallons).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• U.S. Customary units for measurement of liquid volume include cups, pints, quarts, and gallons. • The measurement of the object must include the unit of measure along with the number of iterations. • Students should measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons). • Practical experience measuring liquid volume of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate liquid volume. • Students should estimate the liquid volume of containers in U.S. Customary units to the nearest cup, pint, quart, and gallon.	**All students should** • Use benchmarks to estimate and measure volume. • Identify equivalent measurements between units within the U.S. Customary system.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (cups, pints, quarts, gallons) to use when measuring liquid volume in U.S. Customary units. • Estimate the liquid volume of containers in U.S. Customary units of measure to the nearest cup, pint, quart, and gallon. • Measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, and gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons). • Identify equivalent measures of volume between units within the U.S. Customary system.

4.9 The student will determine elapsed time in hours and minutes within a 12-hour period.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Elapsed time is the amount of time that has passed between two given times. • Elapsed time should be modeled and demonstrated using analog clocks and timelines. • Elapsed time can be found by counting on from the beginning time to the finishing time. – Count the number of whole hours between the beginning time and the finishing time. – Count the remaining minutes. – Add the hours and minutes. For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count 10 minutes; and then, add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.	**All students should** • Understanding the "counting on" strategy for determining elapsed time in hour and minute increments over a 12-hour period from a.m. to a.m. or p.m. to p.m.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the elapsed time in hours and minutes within a 12-hour period (times can cross between a.m. and p.m.). • Solve practical problems in relation to time that has elapsed.

The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

4.10 The student will

a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and

b) identify representations of lines that illustrate intersection, parallelism, and perpendicularity.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A point is a location in space. It has no length, width, or height. A point is usually named with a capital letter. • A line is a collection of points going on and on infinitely in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. Arrows must be drawn to show that the line goes on in both directions infinitely (e.g., $\overleftrightarrow{AB}$, read as "the line AB"). • A line segment is part of a line. It has two endpoints and includes all the points between those endpoints. To name a line segment, name the endpoints (e.g., $\overline{AB}$, read as "the line segment AB"). • A ray is part of a line. It has one endpoint and continues infinitely in one direction. To name a ray, say the name of its endpoint first and then say the name of one other point on the ray (e.g., $\overrightarrow{AB}$, read as "the ray AB"). • Two rays that have the same endpoint form an angle. This endpoint is called the *vertex*. Angles are found wherever lines and line segments intersect. An angle can be named in three different ways by using – three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or	**All students should** • Understand that points, lines, line segments, rays, and angles, including endpoints and vertices are fundamental components of noncircular geometric figures. • Understand that the shortest distance between two points on a flat surface is a line segment. • Understand that lines in a plane either intersect or are parallel. Perpendicularity is a special case of intersection. • Identify practical situations that illustrate parallel, intersecting, and perpendicular lines.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices. • Understand that lines in a plane can intersect or are parallel. Perpendicularity is a special case of intersection. • Identify practical situations that illustrate parallel, intersecting, and perpendicular lines.

4.10 The student will

a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices; and

b) identify representations of lines that illustrate intersection, parallelism, and perpendicularity.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
– a number written inside the rays of the angle. • Intersecting lines have one point in common. • Perpendicular lines are special intersecting lines that form right angles where they intersect. • Parallel lines are lines that lie in the same place and do not intersect. Parallel lines are always the same distance apart and do not share any points. • Students should explore intersection, parallelism, and perpendicularity in both two and three dimensions. For example, students should analyze the relationships between the edges of a cube. Which edges are parallel? Which are perpendicular? What plane contains the upper left edge and the lower right edge of the cube? Students can visualize this by using the classroom itself to notice the lines formed by the intersection of the ceiling and walls, of the floor and wall, and of two walls.		

4.11 The student will

a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and

b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. – **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. – **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.) – **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.) – **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusion are understood, but the role and significance of deduction is not understood. (Students should transition to	**All students should** • Understand the meaning of the term *congruent*. • Understand how to identify congruent figures. • Understand that the orientation of figures does not affect congruency or noncongruency.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize the congruence of plane figures resulting from geometric transformations such as translation, reflection, and rotation, using mirrors, paper folding and tracing.

4.11 The student will

a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and

b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
this level during grades 5 and 6 and fully attain it before taking algebra.) • Congruent figures are figures having exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models. • A translation is a transformation in which an image is formed by moving every point on a figure the same distance in the same direction. • A reflection is a transformation in which a figure is flipped over a line called the *line of reflection*. All corresponding points in the image and preimage are equidistant from the line of reflection. • A rotation is a transformation in which an image is formed by turning its preimage about a point. • The resulting figure of a translation, reflection, or rotation is congruent to the original figure.		

4.12 The student will

a) define polygon; and

b) identify polygons with 10 or fewer sides.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a closed plane geometric figure composed of at least three line segments that do not cross. None of the sides are curved. • A triangle is a polygon with three angles and three sides. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a rectangle with four sides of equal length. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. • A parallelogram is a quadrilateral with both pairs of opposite sides parallel. • A rhombus is a quadrilateral with 4 congruent sides. • A pentagon is a 5-sided polygon. • A hexagon is a 6-sided polygon. • A heptagon is a 7-sided polygon. • An octagon is an 8-sided polygon. • A nonagon is a 9-sided polygon. • A decagon is a 10-sided polygon.	**All students should** • Identify polygons with 10 or fewer sides in everyday situations. • Identify polygons with 10 or fewer sides in multiple orientations (rotations, reflections, and translations of the polygons).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Define and identify properties of polygons with 10 or fewer sides. • Identify polygons by name with 10 or fewer sides in multiple orientations (rotations, reflections, and translations of the polygons).

Students entering grades 4 and 5 have explored the concepts of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students are able to predict whether the game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction at grades 4 and 5 is to deepen their understanding of the concepts of probability by

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same?” “Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- writing a few sentences to communicate to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

4.13 The student will

a) predict the likelihood of an outcome of a simple event; and

b) represent probability as a number between 0 and 1, inclusive.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives. • Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Conduct experiments to determine the probability of an event occurring for a given number of trials (no more than 25 trials), using manipulatives (e.g., the number of times "heads" occurs when flipping a coin 10 times; the chance that when the	**All students should** • Understand and apply basic concepts of probability. • Describe events as likely or unlikely and discuss the degree of likelihood, using the terms *certain*, *likely*, *equally likely*, *unlikely*, and *impossible*. • Predict the likelihood of an outcome of a simple event and test the prediction. • Understand that the measure of the probability of an event can be represented by a number between 0 and 1, inclusive.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model and determine all possible outcomes of a given simple event where there are no more than 24 possible outcomes, using a variety of manipulatives, such as coins, number cubes, and spinners. • Write the probability of a given simple event as a fraction, where the total number of possible outcomes is 24 or fewer. • Identify the likelihood of an event occurring and relate it to its fractional representation (e.g., impossible/0; equally likely/$\frac{1}{2}$; certain/1). • Determine the outcome of an event that is least likely to occur (less than half) or most likely to occur (greater than half) when the number of possible outcomes is 24 or less. • Represent probability as a point between 0 and 1, inclusively, on a number line.

4.13 The student will

a) predict the likelihood of an outcome of a simple event; and

b) represent probability as a number between 0 and 1, inclusive.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
names of 12 classmates are put in a shoebox, a name that begins with *D* will be drawn). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as unlikely*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. • Activities should include practical examples. • For an event such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. • For another event such as spinning a spinner that is one-third red and two-thirds blue, the two outcomes, red and blue, are not equally likely. This is an unfair spinner (since it is not divided equally), therefore, the outcomes are not equally likely.		

4.14 The student will collect, organize, display, and interpret data from a variety of graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Data analysis helps describe data, recognize patterns or trends, and make predictions. • Investigations involving practical data should occur frequently, and data can be collected through brief class surveys or through more extended projects taking many days. • Students formulate questions, predict answers to questions under investigation, collect and represent initial data, and consider whether the data answer the questions. • Line graphs are used to show how two continuous variables are related. Line graphs may be used to show how one variable changes over time. If this one variable is not continuous, then a broken line is used. By looking at a line graph, it can be determined whether the variable is increasing, decreasing, or staying the same over time. – The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as "continuous data," as it represents data collected over a continuous period of time. – The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals, depending upon the data being collected. The scale should extend one increment above the greatest recorded piece	**All students should** • Understand the difference between representing categorical data and representing numerical data. • Understand that line graphs show change over time (numerical data). • Understand that bar graphs should be used to compare counts of different categories (categorical data). • Understand how data displayed in bar and line graphs can be interpreted so that informed decisions can be made. • Understand that the title and labels of the graph provide the foundation for interpreting the data.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect data, using, for example, observations, measurement, surveys, scientific experiments, polls, or questionnaires. • Organize data into a chart or table. • Construct and display data in bar graphs, labeling one axis with equal whole number increments of 1 or more (numerical data) (e.g., 2, 5, 10, or 100) and the other axis with categories related to the title of the graph (categorical data) (e.g., swimming, fishing, boating, and water skiing as the categories of "Favorite Summer Sports"). • Construct and display data in line graphs, labeling the vertical axis with equal whole number increments of 1 or more and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than 10 identified points along a continuum for continuous data. For example, growth charts showing age versus height place age on the horizontal axis (e.g., 1 month, 2 months, 3 months, and 4 months). • Title or identify the title in a given graph and label or identify the axes. • Interpret data from simple line and bar graphs by describing the characteristics of the data and the data as a whole (e.g., the category with the greatest/least, categories with the same number of responses, similarities and differences, the total number). Data points will be limited to 30 and categories to 8.

4.14 The student will collect, organize, display, and interpret data from a variety of graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
of data. – Each axis should be labeled, and the graph should be given a title. – A line graph tells whether something has increased, decreased, or stayed the same with the passage of time. Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). • Bar graphs should be used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs. – A bar graph uses parallel, horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis that displays the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Fourth-grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, 10, or 100. – Each axis should be labeled, and the graph should be given a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written.		• Interpret the data to answer the question posed, and compare the answer to the prediction (e.g., "The summer sport preferred by most is swimming, which is what I predicted before collecting the data."). • Write at least one sentence to describe the analysis and interpretation of the data, identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same.

Students entering grades 4 and 5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships in open number sentences.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

4.15 The student will recognize, create, and extend numerical and geometric patterns.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Most patterning activities should involve some form of concrete materials to make up a pattern. – Students will identify and extend a wide variety of patterns, including rhythmic, geometric, graphic, numerical, and algebraic. The patterns will include both growing and repeating patterns. • Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing the relationship symbolically or algebraically. • Tables of values should be analyzed for a pattern to determine the next value.	**All students should** • Understand that patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe geometric and numerical patterns, using tables, symbols, or words. • Create geometric and numerical patterns, using concrete materials, number lines, tables, and words. • Extend geometric and numerical patterns, using concrete materials, number lines, tables, and words.

4.16 The student will

a) recognize and demonstrate the meaning of equality in an equation; and

b) investigate and describe the associative property for addition and multiplication.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. • The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., $4 + 3 = 3 + 4$). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., $2 \times 3 = 3 \times 2$). • The associative property for addition states that the sum stays the same when the grouping of addends is changed [e.g., $15 + (35 + 16) = (15 + 35) + 16$]. The associative property for multiplication states that the product stays the same when the grouping of factors is changed [e.g., $6 \times (3 \times 5) = (6 \times 3) \times 5$].	**All students should** • Understand that mathematical relationships can be expressed using equations. • Understand that quantities on both sides of an equation must be equal. • Understand that the associative property for addition means you can change the groupings of three or more addends without changing the sum. • Understand that the associative property for multiplication means you can change the groupings of three or more factors without changing the product**.**	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and demonstrate that the equals sign (=) relates equivalent quantities in an equation. • Write an equation to represent equivalent mathematical relationships (e.g., $4 \times 3 = 2 \times 6$). • Recognize and demonstrate appropriate use of the equals sign in an equation. • Investigate and describe the associative property for addition as $(6 + 2) + 3 = 6 + (2 + 3)$. • Investigate and describe the associative property for multiplication as $(3 \times 2) \times 4 = 3 \times (2 \times 4)$.