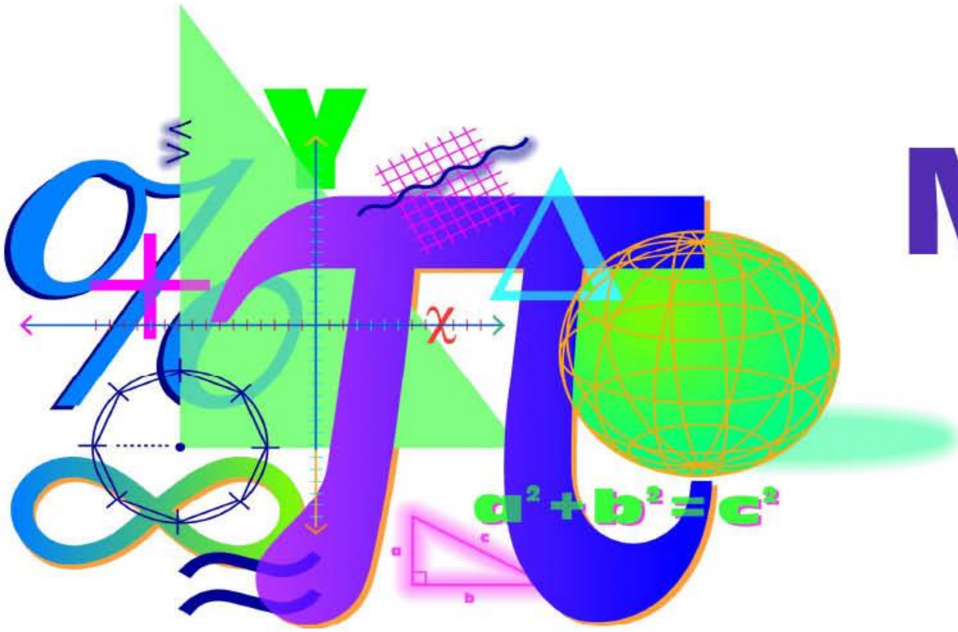

Mathematics
Standards of Learning

Curriculum Framework 2009

Grade 5

Board of Education
Commonwealth of Virginia

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The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia *Mathematics Standards of Learning* Curriculum Framework 2009
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

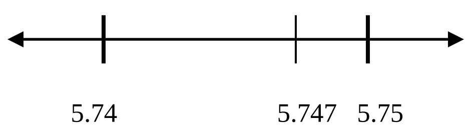
Mathematics instruction in grades 4 and 5 should continue to foster the development of number sense, especially with decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction at grades 4 and 5 allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names). Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve problems.

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the Base-10 number system is based upon a simple pattern of tens in which each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. • A decimal point separates the whole number places from the places less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*. • To read decimals, – read the whole number to the left of the decimal point, if there is one; – read the decimal point as "and"; – read the digits to the right of the decimal point just as you would read a whole number; and – say the name of the place value of the digit in the smallest place. • Decimals may be written in a variety of forms: – Standard: 23.456 – Written: Twenty-three and four hundred fifty-six thousandths – Expanded: $(2 \times 10) + (3 \times 1) + (4 \times 0.1) + (5 \times 0.01) + (6 \times 0.001)$ • To help students identify the ten-to-one place value relationship for decimals through thousandths, use Base-10 manipulatives, such as place value mats/charts, decimal squares, Base-10 blocks, and money.	**All students should** • Understand that decimals are rounded in a way that is similar to the way whole numbers are rounded. • Understand that decimal numbers can be rounded to estimate when exact numbers are not needed for the situation at hand.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Round decimal numbers to the nearest whole number, tenth, or hundredth.

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Decimals can be rounded to the nearest whole number, tenth or hundredth in situations when exact numbers are not needed. • Strategies for rounding decimal numbers to the nearest whole number, tenth and hundredth are as follows: – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place and change the digits to the right of the rounding place to zero. • Create a number line that shows the decimal that is to be rounded. • The position of the decimal will help children conceptualize the number's placement relative for rounding. An example is to round 5.747 to the nearest hundredth: 		

5.2 The student will

a) recognize and name fractions in their equivalent decimal form and vice versa; and

b) compare and order fractions and decimals in a given set from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should recognize, name, and focus on finding equivalent decimals of familiar fractions such as halves, fourths, fifths, eighths, and tenths. • Students should be able to determine equivalent relationships between decimals and fractions with denominators up to 12. • Students should have experience with fractions such as $\frac{1}{8}$, whose decimal representation is a terminating decimal (e. g., $\frac{1}{8} = 0.125$) and with fractions such as $\frac{2}{9}$, whose decimal representation does not end but continues to repeat (e. g., $\frac{2}{9} = 0.222\ldots$). The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$. • To help students compare the value of two decimals through thousandths, use manipulatives, such as place value mats/charts, 10-by-10 grids, decimal squares, Base-10 blocks, meter sticks, number lines, and money.	**All students should** • Understand the relationship between fractions and their decimal form and vice versa. • Understand that fractions and decimals can be compared and ordered from least to greatest and greatest to least.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent fractions (halves, fourths, fifths, eighths, tenths, and twelfths) in their equivalent decimal form and vice versa. • Recognize and name equivalent relationships between decimals and fractions with denominators up to 12. • Compare and order from least to greatest and greatest to least a given set of no more than five numbers written as decimals, fractions, and mixed numbers with denominators of 12 or less.

5.2 The student will
a) recognize and name fractions in their equivalent decimal form and vice versa; and
b) compare and order fractions and decimals in a given set from least to greatest and greatest to least.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A procedure for comparing two decimals by examining may include the following: – Line up the decimal numbers at their decimal points. – Beginning at the left, find the first place value where the digits are different. – Compare the digits in this place value to determine which number is greater (or which is less). – Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented. – If both numbers are the same, use the symbol = or words *equal to*. Two numbers can be compared by examining place value and/or using a number line. • Decimals and fractions represent the same relationships; however, they are presented in two different formats. Decimal numbers are another way of writing fractions. Base-10 models (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money) concretely relate fractions to decimals and vice versa.		

5.3 The student will

a) identify and describe the characteristics of prime and composite numbers; and

b) identify and describe the characteristics of even and odd numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A prime number is a natural number that has exactly two different factors, one and the number itself. • A composite number is a natural number that has more than two different factors. • The number 1 is neither prime nor composite because it has only one factor, itself. • The prime factorization of a number is a representation of the number as the product of its prime factors. For example, the prime factorization of 18 is $2 \times 3 \times 3$. • Prime factorization concepts can be developed by using factor trees. • Prime or composite numbers can be represented by rectangular models or rectangular arrays on grid paper. A prime number can be represented by only one rectangular array (e.g., 7 can be represented by a 7×1 and a 1 x 7). A composite number can always be represented by more than two rectangular arrays (e.g., 9 can be represented by a 9×1, a 1 x 9, or a 3×3). • Divisibility rules are useful tools in identifying prime and composite numbers. • Students should use manipulatives (e.g., Base-10 blocks, cubes, tiles, hundreds board, etc.) to explore and categorize numbers into groups of odd or even.	**All students should** • Understand and use the unique characteristics of certain sets of numbers, including prime, composite, even, and odd numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify prime numbers less than or equal to 100. • Identify composite numbers less than or equal to 100. • Explain orally and in writing why a number is prime or composite. • Identify which numbers are even or odd. • Explain and demonstrate with manipulatives, pictorial representations, oral language, or written language why a number is even or odd.

5.3 The student will
a) identify and describe the characteristics of prime and composite numbers; and
b) identify and describe the characteristics of even and odd numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should use rules to categorize numbers into groups of odd or even. Rules can include: – An odd number does *not* have 2 as a factor or is *not* divisible by 2. – The sum of two even numbers is even. – The sum of two odd numbers is even. – The sum of an even and an odd is odd. – Even numbers have an even number or zero in the ones place. – Odd numbers have an odd number in the ones place. – An even number has 2 as a factor or is divisible by 2.		

Computation and estimation in grades 4 and 5 should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract with common fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability not only to identify but to use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use (not identify) properties of operations to solve problems [e.g., 7×28 is equivalent to $(7 \times 20) + (7 \times 8)$].

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their computations. Additionally, students should enhance their ability to select an appropriate problem solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

<table>
<tr><th>UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)</th><th>ESSENTIAL UNDERSTANDINGS</th><th>ESSENTIAL KNOWLEDGE AND SKILLS</th></tr>
<tr><td>• An example of an approach to solving problems is Polya’s four-step plan:
– Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new.
– Plan: Decide what operation(s) to use and what sequence of steps to use to solve the problem.
– Solve: Follow the plan and work accurately. If the first attempt doesn’t work, try another plan.
– Look back: Does the answer make sense?
• Estimation gives a rough idea of an amount. Strategies such as front-end, rounding, and mental computation may be used to estimate addition, subtraction, multiplication, and division of whole numbers.
• Examples of problems to be solved by using estimation strategies are encountered in shopping for groceries, buying school supplies, budgeting allowance, and sharing the cost of a pizza or the prize money from a contest.
• Estimation can be used to check the reasonableness of the results.</td><td>**All students should**
• Understand the meaning of mathematical operations and how these operations relate to one another when creating and solving single-step and multistep word problems.</td><td>**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**
• Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with whole numbers.
• Create single-step and multistep problems involving the operations of addition, subtraction, multiplication, and division with and without remainders of whole numbers, using practical situations.
• Estimate the sum, difference, product, and quotient of whole number computations.
• Solve single-step and multistep problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers, using paper and pencil, mental computation, and calculators in which
– sums, differences, and products will not exceed five digits;
– multipliers will not exceed two digits;
– divisors will not exceed two digits; or
– dividends will not exceed four digits.
• Use two or more operational steps to solve a multistep problem. Operations can be the same or different.</td></tr>
</table>

5.5 The student will

a) find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths (divisors with only one nonzero digit); and

b) create and solve single-step and multistep practical problems involving decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition and subtraction of decimals may be investigated using a variety of models (e.g., 10-by-10 grids, number lines, money). • Decimal computation uses similar procedures as those developed for whole number computation and applies them to decimal place values, giving careful attention to the placement of the decimal point in the solution. • Multiplication of decimals follows the same procedure as multiplication of whole numbers. The only difference is that a decimal point must be correctly placed in the product giving careful attention to the placement of the decimal point in the solution. • The product of decimals is dependent upon the two factors being multiplied. • In cases where an exact product is not required, the product of decimals can be estimated using strategies for multiplying whole numbers, such as front-end and compatible numbers, or rounding. In each case, the student needs to determine where to place the decimal point to ensure that the product is reasonable. • Division is the operation of making equal groups or shares. When the original amount and the number of shares are known, divide to find the size of each share. When the original amount and the size of each share are known, divide to find the number of shares. Both situations may be modeled with Base-10 manipulatives.	**All students should** • Use similar procedures as those developed for whole number computation and apply them to decimal place values, giving careful attention to the placement of the decimal point in the solution. • Select appropriate methods and tools from among paper and pencil, estimation, mental computation, and calculators according to the context and nature of the computation in order to compute with decimal numbers. • Understand the various meanings of *division* and its effect on whole numbers. • Understand various representations of division, i.e., $\textit{dividend} \div \textit{divisor} = \textit{quotient}$ $\textit{divisor}\,\overset{\textit{quotient}}{\overline{)\,\textit{dividend}}}$ $\dfrac{\textit{dividend}}{\textit{divisor}} = \textit{quotient}.$	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate method of calculation to find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths, selecting from among paper and pencil, estimation, mental computation, and calculators. • Estimate to find the number that is closest to the sum, difference, and product of two numbers expressed as decimals through thousandths. • Find the sum, difference, and product of two numbers expressed as decimals through thousandths, using paper and pencil, estimation, mental computation, and calculators. • Determine the quotient, given a dividend expressed as a decimal through thousandths and a single-digit divisor. For example, 5.4 divided by 2 and 2.4 divided by 5. • Use estimation to check the reasonableness of a sum, difference, product, and quotient. • Create and solve single-step and multistep problems. • A multistep problem needs to incorporate two or more operational steps (operations can be the same or different).

5.5 The student will

a) find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths (divisors with only one nonzero digit); and

b) create and solve single-step and multistep practical problems involving decimals.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The fair-share concept of decimal division can be modeled, using manipulatives (e.g., Base-10 blocks). • Division with decimals is performed the same way as division of whole numbers. The only difference is the placement of the decimal point in the quotient. • The quotient can be estimated, given a dividend expressed as a decimal through thousandths (and no adding of zeros to the dividend during the division process) and a single-digit divisor. • Estimation can be used to check the reasonableness of a quotient. • Division is the inverse of multiplication; therefore, multiplication and division are inverse operations. • Terms used in division are *dividend*, *divisor*, and *quotient.* *dividend* ÷ *divisor* = *quotient* $\text{divisor}\,\overline{)\text{dividend}}$ with *quotient* above • There are a variety of algorithms for division such as repeated multiplication and subtraction. Experience with these algorithms may enhance understanding of the traditional long division algorithm. • A multistep problem needs to incorporate no more than two operational steps (operations can be the same or different).		

5.6 The student will solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers and express answers in simplest form.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. • When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form. • Fractions having like denominators means the same as fractions having common denominators. • Equivalent fractions name the same amount. To find equivalent fractions, multiply or divide the numerator and denominator by the same nonzero number. • Addition and subtraction with fractions and mixed numbers can be modeled using a variety of concrete materials and pictorial representations as well as paper and pencil. • To add, subtract, and compare fractions and mixed numbers, it often helps to find the least common denominator. The least common denominator (LCD) of two or more fractions is the least common multiple (LCM) of the denominators. • To add or subtract with fractions having the same or like denominators, add or subtract the numerators and write in simplest form.	**All students should** • Develop and use strategies to estimate and compute addition and subtraction of fractions. • Understand the concept of least common multiple and least common denominator as they are important when adding and subtracting fractions. • Understand that a fraction is in simplest form when its numerator and denominator have no common factors other than 1. The numerator can be greater than the denominator.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve single-step and multistep practical problems involving addition and subtraction with fractions having like and unlike denominators. Denominators in the problems should be limited to 12 or less (e.g., $\frac{1}{5} + \frac{1}{4}$) and answers should be expressed in simplest form. • Solve single-step and multistep practical problems involving addition and subtraction with mixed numbers having like and unlike denominators, with and without regrouping. Denominators in the problems should be limited to 12 or less, and answers should be expressed in simplest form. • Use estimation to check the reasonableness of a sum or difference.

5.6 The student will solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers and express answers in simplest form.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• To add or subtract with fractions that do not have the same denominator, first find equivalent fractions with the least common denominator. Then add or subtract and write the answer in simplest form. • A mixed number has two parts: a whole number and a fraction. The value of a mixed number is the sum of its two parts. • To add or subtract with mixed numbers, students may use a number line, draw a picture, rewrite fractions with like denominators, or rewrite mixed numbers as fractions.		

5.7 The student will evaluate whole number numerical expressions, using the order of operations limited to parentheses, addition, subtraction, multiplication, and division.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An expression, like a phrase, has no equal sign. • Expressions are simplified by using the order of operations. • The order of operations defines the computation order to follow in simplifying an expression. • The order of operations is as follows: – First, complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right.	**All students should** • Understand that the order of operations describes the order to use to simplify expressions containing more than one operation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Simplify expressions by using the order of operations in a demonstrated step-by-step approach. • Find the value of numerical expressions, using the order of operations. • Given an expression involving more than one operation, describe which operation is completed first, which is second, etc.

Students in grades 4 and 5 should be actively involved in measurement activities that require a dynamic interaction among students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure perimeter, weight/mass, liquid volume/capacity, area, temperature, and time. Students continue to enhance their understanding of measurement by using appropriate tools such as rulers, balances, clocks, and thermometers. The process of measuring is identical for any attribute (i.e., length, weight/mass, liquid volume/capacity, area): choose a unit, compare that unit to the object, and report the number of units.

5.8 The student will

a) find perimeter, area, and volume in standard units of measure;

b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;

c) identify equivalent measurements within the metric system;

d) estimate and then measure to solve problems, using U.S. Customary and metric units; and

e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Perimeter is the distance around an object. It is a measure of length. Area is the number of square units needed to cover a surface. Volume is a measure of capacity and is measured in cubic units. • To find the perimeter of any polygon, add the lengths of the sides. • Students should label the perimeter, area, and volume with the appropriate unit of linear, square, or cubic measure. • Area is the number of square units needed to cover a surface or figure. • Students should investigate, using manipulatives, to discover the formulas for the area of a square, rectangle, and right triangle; and volume of a rectangular solid. – Area of a rectangle = Length × Width – Area of a square = Side × Side – Area of a right triangle = $\frac{1}{2}$ Base × Height – Volume of a rectangular solid = Length x Width x Height • Length is the distance along a line or figure from one point to another.	**All students should** • Understand the concepts of perimeter, area, and volume. • Understand and use appropriate units of measure for perimeter, area, and volume. • Understand the difference between using perimeter, area, and volume in a given situation. • Understand how to select a measuring device and unit of measure to solve problems involving measurement.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the perimeter of a polygon, with or without diagrams, when – the lengths of all sides of a polygon that is not a rectangle or a square are given; – the length and width of a rectangle are given; or – the length of a side of a square is given. • Estimate and determine the perimeter of a polygon, and area of a square, rectangle, and right triangle following the parameters listed above, using only whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 24 square inches). • Estimate and determine the area of a square, with or without diagrams, when the length of a side is given. • Estimate and determine the area of a rectangle, with or without diagrams, when the length and width are given. • Estimate and determine the area of a right triangle, with or without diagrams, when the base and the height are given. • Differentiate among the concepts of area, perimeter, and volume.

5.8 The student will

a) find perimeter, area, and volume in standard units of measure;
b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;
c) identify equivalent measurements within the metric system;
d) estimate and then measure to solve problems, using U.S. Customary and metric units; and
e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure. • When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), foot, or yard. • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight that an object changes is dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • Appropriate measuring devices to measure mass in U.S. Customary units (ounces, pounds) and metric units (grams, kilograms) are balances.		• Develop a procedure for finding volume using manipulatives (e.g., cubes). • Determine volume in standard units. • Describe practical situations where area, perimeter, and volume are appropriate measures to use, and justify their choices orally or in writing. • Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. • Identify equivalent measurements within the metric system for the following: – length: millimeters, centimeters, meters, and kilometers; – mass: grams and kilograms; – liquid volume: milliliters, and liters.

5.8 The student will

a) find perimeter, area, and volume in standard units of measure;
b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;
c) identify equivalent measurements within the metric system;
d) estimate and then measure to solve problems, using U.S. Customary and metric units; and
e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• U.S. Customary units to measure liquid volume (capacity) include cups, pints, quarts, and gallons. Metric units to measure liquid volume (capacity) include milliliters and liters. • Temperature is measured using a thermometer. The U.S. Customary unit of measure is degrees Fahrenheit; the metric unit of measure is degrees Celsius. • Practical experience measuring familiar objects helps students establish benchmarks and facilitates students' ability to use the units of measure to make estimates.		• Solve problems involving measurement by selecting an appropriate measuring device and a U.S. Customary or metric unit of measure for the following: – length: part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$), inches, feet, yards, millimeters, centimeters, meters, and kilometers; – weight: ounces, pounds, and tons; – mass: grams and kilograms; – liquid volume: cups, pints, quarts, gallons, milliliters, and liters; – area: square units; and – temperature: Celsius and Fahrenheit units. – Water freezes at 0°C and 32°F. – Water boils at 100°C and 212°F. – Normal body temperature is about 37°C and 98.6°F.

5.9 The student will identify and describe the diameter, radius, chord, and circumference of a circle.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A circle is a set of points on a flat surface (plane) with every point equidistant from a given point called the *center*. • A chord is a line segment connecting any two points on a circle. Students will benefit from understanding that a chord goes from one side of the circle to the other, but does not need to pass through the center. • A diameter is a chord that goes through the center of a circle. The diameter is two times the radius. A radius is a segment from the center of a circle to any point on the circle. Two radii end-to-end form a diameter of a circle. • Circumference is the distance around or perimeter of a circle. The circumference is about 3 times larger than the diameter of a circle.	**All students should** • Understand that a chord is a line segment that extends between any two unique points of a circle. • Understand that a diameter is also a special chord that goes through the center of a circle. • Understand the relationship between the measures of diameter and radius and the relationship between the measures of radius and circumference. • Understand that a radius is a line segment that extends between the center and the circumference of the circle. • Understand that the circumference is the distance around the circle. Perimeter is the measure of the circumference.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe the diameter, radius, chord, and circumference of a circle. • Describe the relationship between – diameter and radius; – diameter and chord; – radius and circumference; and – diameter and circumference. • The length of the diameter of a circle is twice the length of the radius.

5.10 The student will determine an amount of elapsed time in hours and minutes within a 24-hour period.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Elapsed time is the amount of time that has passed between two given times. • Elapsed time can be found by counting on from the beginning time to the finishing time. – Count the number of whole hours between the beginning time and the finishing time. – Count the remaining minutes. – Add the hours and minutes. For example, to find the elapsed time between 10:15 a.m. and 1:25 p.m., count on as follows: from 10:15 a.m. to 1:15 p.m., count 3 hours; from 1:15 p.m. to 1:25 p.m., count 10 minutes; and then add 3 hours to 10 minutes to find the total elapsed time of 3 hours and 10 minutes.	**All students should** • Understand that elapsed time can be found by counting on from the beginning time to the finishing time.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine elapsed time in hours and minutes within a 24-hour period.

5.11 The student will measure right, acute, obtuse, and straight angles.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Angles are measured in degrees. There are up to 360 degrees in an angle. A degree is $\frac{1}{360}$ of a complete rotation of a full circle. There are 360 degrees in a circle. • To measure the number of degrees in an angle, use a protractor or an angle ruler. • A right angle measures exactly 90°. • An acute angle measures less than 90°. • An obtuse angle measures greater than 90° but less than 180°. • A straight angle measures exactly 180°. • Before measuring an angle, students should first compare it to a right angle to determine whether the measure of the angle is less than or greater than 90°. • Students should understand how to work with a protractor or angle ruler as well as available computer software to measure and draw angles and triangles.	**All students should** • Understand how to measure acute, right, obtuse, and straight angles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the appropriate tools (e.g., protractor and straightedge or angle ruler as well as available software) used to measure and draw angles and triangles. • Measure right, acute, straight, and obtuse angles, using appropriate tools, and identify their measures in degrees. • Recognize angle measure as additive. When an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.† • Solve addition and subtraction problems to find unknown angle measures on a diagram in practical and mathematical problems, (e.g., by using an equation with a symbol for the unknown angle measure).† †Revised March 2011

The study of geometry helps students represent and make sense of the world. At the fourth- and fifth-grade levels, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, and draw representations and describe the relationships between and among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence, other lines such as parallel and perpendicular ones, and classifications of triangles.

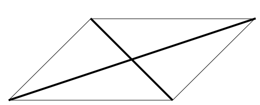
The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)

5.12 The student will classify
a) angles as right, acute, obtuse, or straight; and
b) triangles as right, acute, obtuse, equilateral, scalene, or isosceles.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A right angle measures exactly 90°. • An acute angle measures greater than 0° but less than 90°. • An obtuse angle measures greater than 90° but less than 180°. • A straight angle forms an angle that measures exactly 180°. • A right triangle has one right angle. • An obtuse triangle has one obtuse angle. • An acute triangle has three acute angles (or no angle measuring 90° or greater). • A scalene triangle has no congruent sides. • An isosceles triangle has two congruent sides. • To facilitate the exploration of relationships, ask students whether a right triangle can have an obtuse angle. Why or why not? Can an obtuse triangle have more than one obtuse angle? Why or why not? What type of angles are the two angles other than the right angle in a right triangle? What type of angles are the two angles other than the obtuse angle in an obtuse triangle?	**All students should** • Understand that angles can be classified as right, acute, obtuse, or straight according to their measures. • Understand that a triangle can be classified as either right, acute, or obtuse according to the measure of its largest angle. • Understand that a triangle can be classified as equilateral, scalene, or isosceles according to the number of sides with equal length.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Classify angles as right, acute, straight, or obtuse. • Classify triangles as right, acute, or obtuse. • Classify triangles as equilateral, scalene, or isosceles.

5.13 The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
a) develop definitions of these plane figures; and
b) investigate and describe the results of combining and subdividing plane figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A triangle is a polygon with three sides. Triangles may be classified according to the measure of their angles, i.e., right, acute, or obtuse. Triangles may also be classified according to the measure of their sides, i.e., scalene (no sides congruent), isosceles (at least two sides congruent) and equilateral (all sides congruent). • A quadrilateral is a polygon with four sides. • A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. Properties of a parallelogram include the following: – A diagonal (a segment that connects two vertices of a polygon but is not a side) divides the parallelogram into two congruent triangles. – The opposite sides of a parallelogram are congruent. – The opposite angles of a parallelogram are congruent. – The diagonals of a parallelogram bisect each other. To bisect means to cut a geometric figure into two congruent halves. A bisector is a line segment, line, or plane that divides a geometric figure into two congruent halves. A sample of a bisected parallelogram is below. 	**All students should** • Understand that simple plane figures can be combined to make more complicated figures and that complicated figures can be subdivided into simple plane figures.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Develop definitions for squares, rectangles, triangles, parallelograms, rhombi, and trapezoids. • Investigate and describe the results of combining and subdividing plane figures.

5.13 The student, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid), will
a) develop definitions of these plane figures; and
b) investigate and describe the results of combining and subdividing plane figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A rectangle is a parallelogram with four right angles. Since a rectangle is a parallelogram, a rectangle has the same properties as those of a parallelogram. • A square is a rectangle with four congruent sides. Since a square is a rectangle, a square has all the properties of a rectangle and of a parallelogram. • A rhombus is a parallelogram with four congruent sides. Opposite angles of a rhombus are congruent. Since a rhombus is a parallelogram, the rhombus has all the properties of a parallelogram. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases*, and the nonparallel sides are called *legs*. If the legs have the same length, then the trapezoid is an isosceles trapezoid. • Two or more figures can be combined to form a new figure. Students should be able to identify the figures that have been combined. • The region of a polygon may be subdivided into two or more regions that represent figures. Students should understand how to divide the region of a polygon into familiar figures.		

Students entering grades 4 and 5 have explored the concepts of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students are able to predict whether the game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction at grades 4 and 5 is to deepen their understanding of the concepts of probability by

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- writing a few sentences to communicate to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

5.14 The student will make predictions and determine the probability of an outcome by constructing a sample space.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is “impossible” if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is “certain” if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as unlikely*, *as likely as, equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include practical examples. • For any event such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. • A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, chart, or tree diagram.	**All students should** • Understand that the basic concepts of probability can be applied to make predictions of outcomes of simple experiments. • Understand that a sample space represents all possible outcomes of an experiment.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a sample space, using a tree diagram to identify all possible outcomes of a single event. • Construct a sample space, using a list or chart to represent all possible outcomes of a single event. • Predict and determine the probability of an outcome by constructing a sample space. The sample space will have a total of 24 or less possible outcomes.

5.14 The student will make predictions and determine the probability of an outcome by constructing a sample space.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Tree diagrams show all possible outcomes in a sample space. The Fundamental Counting Principle describes how to find the number of outcomes when there are multiple choices. For example, how many different outfit combinations can you make from 2 shirts (red and blue) and 3 pants (black, white, khaki)? The sample space displayed in a tree diagram would show that there are $2 \times 3 = 6$ (Fundamental Counting Principle) outfit combinations: red-black; red-white; red-khaki; blue-black; blue-white; blue-khaki. • A spinner with eight equal-sized sections is equally likely to land on any one of the sections, three of which are red, three green, and two yellow. Have students write a problem statement involving probability, such as, "What is the probability that the spinner will land on green?"		

5.15 The student, given a problem situation, will collect, organize, and interpret data in a variety of forms, using stem-and-leaf plots and line graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The emphasis in all work with statistics should be on the analysis and the communication of the analysis, rather than on a single correct answer. Data analysis should include opportunities to describe the data, recognize patterns or trends, and make predictions. • Statistical investigations should be active, with students formulating questions about something in their environment and finding quantitative ways to answer the questions. • Investigations can be brief class surveys or more extended projects taking many days. • Through experiences displaying data in a variety of graphical representations, students learn to select an appropriate representation. • Line graphs are used to show how two continuous variables are related. Line graphs may be used to show how one variable changes over time. If one variable is not continuous, then a broken line is used. By looking at a line graph, it can be determined whether the variable is increasing, decreasing, or staying the same over time. – The values along the horizontal axis represent continuous data on a given variable, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as “continuous data” because it represents data collected over a continuous period of time.	**All students should** • Understand how to interpret collected and organized data. • Understand that stem-and-leaf plots list data in a meaningful array. It helps in finding median, modes, minimum and maximum values, and ranges. • Understand that line graphs show changes over time.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Formulate the question that will guide the data collection. • Collect data, using observations (e.g., weather), measurement (e.g., shoe sizes), surveys (e.g., hours watching television), or experiments (e.g., plant growth). • Organize the data into a chart, table, stem-and-leaf plots, and line graphs. • Display data in line graphs and stem-and-leaf plots. • Construct line graphs, labeling the vertical axis with equal whole number, decimal, or fractional increments and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years, and age). Line graphs will have no more than six identified points along a continuum for continuous data (e.g., the decades: 1950s, 1960s, 1970s, 1980s, 1990s, and 2000s). • Construct a stem-and-leaf plot to organize and display data, where the stem is listed in ascending order and the leaves are in ascending order, with or without commas between leaves. • Title the given graph or identify the title. • Interpret the data in a variety of forms (e.g., orally or in written form).

5.15 The student, given a problem situation, will collect, organize, and interpret data in a variety of forms, using stem-and-leaf plots and line graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
– The values along the vertical axis are the scale and represent the frequency with which those values occur in the data set. The values should represent equal increments of multiples of whole numbers, fractions, or decimals depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data. – Each axis should be labeled and the graph should have a title. – A line graph tells whether something has increased, decreased, or stayed the same with the passage of time. Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). A broken line is used if the data collected is not continuous data (such as test scores); a solid line is used if the data is continuous (such as height of a plant). • Stem-and-leaf plots allow the exact values of data to be listed in a meaningful array. Data covering a range of 25 numbers are best displayed in a stem-and-leaf plot and are utilized to organize numerical data from least to greatest, using the digits of the greatest to group data. – The data is organized from least to greatest. – Each value should be separated into a stem and a leaf [e.g., two-digit numbers are separated into stems (tens) and leaves (ones)].		

5.15 The student, given a problem situation, will collect, organize, and interpret data in a variety of forms, using stem-and-leaf plots and line graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
– The stems are listed vertically from least to greatest with a line to their right. The leaves are listed horizontally, also from least to greatest, and can be separated by spaces or commas. Every value is recorded regardless of the number of repeats. – A key is often included to explain how to read the plot.		

5.16 The student will
a) describe mean, median, and mode as measures of center;
b) describe mean as fair share;
c) find the mean, median, mode, and range of a set of data; and
d) describe the range of a set of data as a measure of variation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Statistics is the science of conducting studies to collect, organize, summarize, analyze, and draw conclusions from data. • A measure of center is a value at the center or middle of a data set. Mean, median, and mode are measures of center. • The mean, median, and mode are three of the various ways that data can be analyzed. • Mean represents a fair share concept of the data. Dividing the data constitutes a fair share. This is done by equally dividing the data points. This should be demonstrated visually and with manipulatives. The arithmetic way is to add all of the data points then divide by the number of data points to determine the average or mean. • The median is the piece of data that lies in the middle of the set of data arranged in order. • The mode is the piece of data that occurs most frequently in the data set. There may be one, more than one, or no mode in a data set. Students should order the data from least to greatest so they can better find the mode. • The range is the spread of a set of data. The range of a set of data is the difference between the greatest and least values in the data set. It is determined by subtracting the least number in the data set from the greatest number in the data set.	**All students should** • Understand that mean, median, and mode are described as measures of center. • Understand that mean, median, and mode are three of the various ways that data can be described or summarized. • Understand that mean as fair share is described as equally dividing the data set or the data set has already been divided equally. • Understand how to find the mean, median, and mode of a set of data as measures of center. • Understand values in the context of other characteristics of the data in order to best describe the results.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe and find the mean of a group of numbers representing data from a given context as a measure of center. • Describe and find the median of a group of numbers representing data from a given context as a measure of center. • Describe and find the mode of a group of numbers representing data from a given context as a measure of center. • Describe mean as fair share. • Describe and find the range of a group of numbers representing data from a given context as a measure of variation. • Describe the impact on measures of center when a single value of a data set is added, removed, or changed.† †Revised March 2011

5.16 The student will
a) describe mean, median, and mode as measures of center;
b) describe mean as fair share;
c) find the mean, median, mode, and range of a set of data; and
d) describe the range of a set of data as a measure of variation.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
An example is ordering test scores from least to greatest: 73, 77, 84, 87, 89, 91, 94. The greatest score in the data set is 94 and the least score is 73, so the least score is subtracted from the greatest score or 94 - 73 = 21. The range of these test scores is 21. • Students need to learn more than how to identify the mean, median, mode, and range of a set of data. They need to build an understanding of what the number tells them about the data, and they need to see those values in the context of other characteristics of the data in order to best describe the results.		

Students entering grades 4 and 5 have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships in open number sentences.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

5.17 The student will describe the relationship found in a number pattern and express the relationship.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- There are an infinite number of patterns.
- The simplest types of patterns are repeating patterns. In such patterns, students need to identify the basic unit of the pattern and repeat it.
- Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns.
- Sample numerical patterns are
  6, 9, 12, 15, 18, …;
  5, 7, 9, 11, 13, …;
  1, 2, 4, 7, 11, 16, …;
  2, 4, 8, 16, 32, …;
  32, 30, 28, 26, 24…; and
  1, 5, 25, 125, 625,….
- An expression, like a phrase, has no equal sign.
- When the pattern data are expressed in a T-table, an expression can represent that data. An example is:

X	Y
6	9
7	10
11	14
15	18

  This example defines the relationship as $x + 3$.
- Expressions are simplified by using the order of operations.

ESSENTIAL UNDERSTANDINGS

All students should

- Understand that patterns and functions can be represented in many ways and described using words, tables, and symbols.
- Understand the structure of a pattern and how it grows or changes using concrete materials and calculators.
- Understand that mathematical relationships exist in patterns.
- Understand that an expression uses symbols to define a relationship and shows how each number in the list, after the first number, is related to the preceding number.
- Understand that expressions can be numerical or variable or a combination of numbers and variables.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Describe numerical and geometric patterns formed by using concrete materials and calculators.
- Describe the relationship found in patterns, using words, tables, and symbols to express the relationship.

5.17 The student will describe the relationship found in a number pattern and express the relationship.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A verbal quantitative expression involving one operation can be represented by a variable expression that describes what is going on. Numbers are used when they are known; variables are used when the numbers are unknown. For example, "a full box of cookies and four extra" can be represented by $b + 4$; "three full boxes of cookies" by $3b$; "a full box of cookies shared among four" by $\frac{b}{4}$. • A mathematical expression contains a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship. An expression cannot be solved.		

5.18 The student will

a) investigate and describe the concept of variable;

b) write an open sentence to represent a given mathematical relationship, using a variable;

c) model one-step linear equations in one variable, using addition and subtraction; and

d) create a problem situation based on a given open sentence, using a single variable.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A variable is a symbol that can stand for an unknown number or object. • A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an equals sign (=). • A verbal expression involving one operation can be represented by a variable expression that describes what is going on. Numbers are used when they are known; variables are used when the numbers are unknown. For example, "a full box of cookies and four extra" can be represented by $b + 4$; "three full boxes of cookies" by $3b$; "a full box of cookies shared among four" by $\frac{b}{4}$. • An open sentence contains a variable and an equals sign (=). For example, the sentence, "A full box of cookies and four extra equal 24 cookies." can be written as $b + 4 = 24$, where b stands for the number of cookies in one full box. "Three full boxes of cookies equal 60 cookies." can be written as $3b = 60$. • Another example of an open sentence is $b + 3 = 23$ and represents the answer to the word problem, "How many cookies are in a box if the box plus three more equals 23 cookies, where b stands for the number of cookies in the box?	**All students should** • Understand that a variable is a symbol that can stand for an unknown number or object. • Understand that a variable expression is a variable or combination of variables, numbers, and symbols that represents a mathematical relationship. • Understand that verbal expressions can be translated to variable expressions. • Understand that an open sentence has a variable and an equal sign (=). • Understand that problem situations can be expressed as open sentences.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the concept of a variable (presented as boxes, letters, or other symbols) as a representation of an unknown quantity. • Write an open sentence with addition, subtraction, multiplication, or division, using a variable to represent a missing number. • Model one-step linear equations using a variety of concrete materials such as colored chips on an equation mat or weights on a balance scale. • Create and write a word problem to match a given open sentence with a single variable and one operation.

5.18 The student will

a) investigate and describe the concept of variable;
b) write an open sentence to represent a given mathematical relationship, using a variable;
c) model one-step linear equations in one variable, using addition and subtraction; and
d) create a problem situation based on a given open sentence, using a single variable.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• At this level, discuss how the symbol × used to represent multiplication can often be confused with the variable *x*. Students can minimize this confusion by using parentheses [e.g., 4(*x*) = 20 or 4x = 20] or a small dot raised off the line to represent multiplication [4 • x = 20]. • By using story problems and numerical sentences, students begin to explore forming equations and representing quantities using variables. • An open sentence containing a variable is neither true nor false until the variable is replaced with a number.		

5.19 The student will investigate and recognize the distributive property of multiplication over addition.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products (e.g., 3(4 + 5) = 3 x 4 + 3 x 5, 5 x (3 + 7) = (5 x 3) + (5 x 7); or (2 x 3) + (2 x 5) = 2 x (3 + 5). • The distributive property can be used to simplify expressions (e.g., 9 x 23 = 9(20+3) =180+ 27 = 207; or 5 x 19 = 5(10 + 9) = 50 + 45 = 95).	**All students should** • Understand that the distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. • Understand that using the distributive property with whole numbers helps with understanding mathematical relationships. • Understand when and why the distributive property is used.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and recognize the distributive property of whole numbers, limited to multiplication over addition using diagrams and manipulatives. • Investigate and recognize an equation that represents the distributive property, when given several whole number equations, limited to multiplication over addition.