Spring 2013 Student Performance Analysis

Algebra I Standards of Learning

Presentation may be paused and resumed using the arrow keys or the mouse.
Representing and Evaluating Expressions

SOL A.1
The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.
Suggested Practice for SOL A.1

Students need additional practice translating expressions.

Select each phrase that verbally translates this algebraic expression:

\[ \frac{1}{4} \cdot \sqrt[3]{x} - 5 \]

- One fourth times the cube root of \( x \) less five.
- One fourth times the cube root of \( x \) less than five.
- Five subtract one fourth times the cube root of \( x \).
- Five less than one fourth times the cube root of \( x \).
Suggested Practice for SOL A.1

Students need additional practice evaluating expressions with cube roots, square roots, and the square of a number, particularly when the replacement variable has a negative value.

Evaluate the following expressions:

a. \(-3\sqrt{a} + b^2\) when \(a = 27\) and \(b = -6\) \(33\)

b. \(-2(a + 8) - a^2\) when \(a = -3\) \(-19\)

c. \(3a - \sqrt{a^2 + b^2}\) when \(a = -6\) and \(b = 8\) \(-28\)
Suggested Practice for SOL A.1

Students need additional practice evaluating expressions that contain an absolute value.

Evaluate the following expressions:

a. \(-2|x + 3| \text{ when } x = -8\) \quad \(-10\)

b. \(3|2x - 9| - x \text{ when } x = 2\) \quad \(13\)
Performing Operations on Polynomials

SOL A.2
The student will perform operations on polynomials, including

a) applying the laws of exponents to perform operations on expressions;

b) adding, subtracting, multiplying, and dividing polynomials; and

c) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.
Suggested Practice for SOL A.2a

Students need additional practice applying the laws of exponents to simplify expressions.

Simplify:

a. $y^3(2y)^4$  
   $16y^7$

b. $\left(\frac{30x^5}{6x^3}\right)^2$  
   $25x^4$

c. $\frac{(12x^{-2})^0}{6x^4y^{-3}}$  
   $\frac{y^3}{6x^4}$

d. $\frac{(a^{-2})^{-3}}{(a^5)^{-1}}$  
   $a^{11}$
Suggested Practice for SOL A.2b

Students need additional practice dividing polynomials.

a. Find the quotient of $\left(6x^2 - 19x + 15\right)$ and $(2x - 3)$. $(3x - 5)$

b. Simplify the following expression. Assume the denominator does not equal zero.

$$\frac{7x^2 + 12x + 5}{x + 1} \quad (7x + 5)$$

c. Simplify: $(x^2 - 4x + 3) \div (x - 1) \quad (x - 3)$
Students need additional practice completely factoring polynomials, particularly when there is a greatest common factor.

Identify all of the factors of $8x^2 - 18$ when it is completely factored.

$$\begin{align*}
x & \quad 2 \\
8x^2 & \quad (2x - 3) \quad (2x - 9) \\
(2x + 3) & \quad (x + 2) \quad (4x^2 - 9)
\end{align*}$$
Suggested Practice for SOL A.2c

Students need additional practice using the \( x \)-intercepts from the graphical representation of the polynomial to determine and confirm its factors.

Using two of the factors shown, create a possible equation for the graphed relation.

\[
y = (x - 2)(x + 3)
\]
Express Square Roots and Cube Roots in Simplest Radical Form

SOL A.3
The student will express the square roots and cube roots of whole numbers and the square root of a monomial algebraic expression in simplest radical form.
Suggested Practice for SOL A.3

Students need additional practice simplifying the square root of a monomial algebraic expression.

Write each expression in simplest radical form.

a. $\sqrt{108x^9} = 6x^4\sqrt{3x}$

b. $\sqrt{50x^{12}y^{15}} = 5x^6y^7\sqrt{2y}$

c. $\sqrt{48x^8y^{16}} = 4x^4y^8\sqrt{3}$
SOL A.4

The student will solve multistep linear and quadratic equations in two variables, including

a) solving literal equations (formulas) for a given variable;

b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;

c) solving quadratic equations algebraically and graphically;

d) solving multistep linear equations algebraically and graphically;

e) solving systems of two linear equations in two variables algebraically and graphically; and

f) solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.
Students need additional practice solving literal equations.

The formula for the surface area \( S \) of a triangular prism is

\[
S = hp + 2B
\]

where \( h \) is the height of the prism, \( p \) is the perimeter of the base, and \( B \) is the area of the base. Solve the equation for the given variable:

a. Solve for \( h \):

\[
h = \frac{S - 2B}{p}
\]

b. Solve for \( B \):

\[
B = \frac{S - hp}{2}
\]
Suggested Practice for SOL A.4c

Students need additional practice finding solutions to quadratic equations presented algebraically and graphically.

Identify the solutions to the equation: \(5x^2 - 7x - 6 = 0\)

a. \(\left\{ \frac{6}{5}, 1 \right\}\)

b. \(\left\{ -\frac{6}{5}, -1 \right\}\)

c. \(\left\{ \frac{3}{5}, -2 \right\}\)

d. \(\left\{ -\frac{3}{5}, 2 \right\}\)
Suggested Practice for SOL A.4c

The graph of $y = -x^2 + x + 12$ is shown.
Plot the solutions to $-x^2 + x + 12 = 0$.
Suggested Practice for SOL A.4d

Students need additional practice describing solutions to equations that have the following solutions: \( x=0 \), an infinite number of real solutions, and no real solutions.

Describe the solution to each equation.

a. \( 11 + 3(x - 2) = 3x - 1 \) \hspace{1cm} No real solutions

b. \( 2(x + 8) = 4x + 16 \) \hspace{1cm} \( x=0 \)

c. \( 2(x - 5) = 4x - (10 + 2x) \) \hspace{1cm} An infinite number of real solutions
Students need additional practice finding solutions to systems of equations presented algebraically.

1. Which system of equations has no real solution?

   a. \[
   \begin{align*}
   6x + 4y &= -24 \\
   -2x + 3y &= 36
   \end{align*}
   \]
   b. \[
   \begin{align*}
   6x + 4y &= -24 \\
   -5x + 3y &= 36
   \end{align*}
   \]
   c. \[
   \begin{align*}
   -4x + 6y &= -24 \\
   -2x + 3y &= 36
   \end{align*}
   \]
   d. \[
   \begin{align*}
   -4x + 6y &= -24 \\
   -5x + 3y &= 36
   \end{align*}
   \]

2. What is \(x\)-value of the solution to this system of equations?

   \[
   \begin{align*}
   5x + 2y &= 12 \\
   6x + 3y &= 9
   \end{align*}
   \]

   \(x = 6\)
SOL A.5
The student will solve multistep linear inequalities in two variables, including
a) solving multistep linear inequalities algebraically and graphically;
b) justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets;
c) solving real-world problems involving inequalities; and
d) solving systems of inequalities.
Students need additional practice solving multistep inequalities.

An inequality is solved as shown. Between which two steps is an error made? Explain the error.

Step 1: \(-3(x + 2) \geq 8\)
Step 2: \(-3x + 6 \geq 8\)
Step 3: \(-3x + 6 - 6 \geq 8 - 6\)
Step 4: \(-3x + 0 \geq 8 - 6\)
Step 5: \(-3x \geq 8 - 6\)
Step 6: \(-3x \geq 2\)
Step 7: \(\frac{-3}{-3}x \leq \frac{2}{-3}\)
Step 8: \(1x \leq -\frac{2}{3}\)
Step 9: \(x \leq -\frac{2}{3}\)

The -3 was not distributed properly to the second term.
Students need additional practice identifying properties of inequality.

Given: \( 3x + 6 \geq 7x - 4 \)

Using the given inequality, select all that illustrate the application of the subtraction property of inequality.

\[
\begin{align*}
3x + 6 - 7x & \geq 7x - 4 - 7x \\
\frac{1}{3} (3x + 6) & \geq \frac{1}{3} (7x - 4) \\
3x + 6 - 6 & \geq 7x - 4 - 6 \\
\frac{(3x + 6)}{7} & \geq \frac{(7x - 4)}{7}
\end{align*}
\]
Students need additional practice identifying ordered pairs that are solutions to a system of inequalities.

Which ordered pairs are solutions to this system of inequalities?

\[
\begin{align*}
y & \leq 5x - 4 \\
y & \geq -2x - 1
\end{align*}
\]

\((-4, -3)\quad (0, -4)\quad (1, 1)\quad (1, 3)\quad (3, -1)\quad (4, 3)\quad (6, 0)\)
Determining Slope of a Line

SOL A.6
The student will graph linear equations and linear inequalities in two variables, including

a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and

b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.
Students need additional practice finding slope.

Find the slope, \( m \), of the line represented by the given equation.

a. \( 3x + 2y = 36 \) \( m = \frac{-3}{2} \)

b. \( 3y = 4x + 6 \) \( m = \frac{4}{3} \)

c. \( y = 2(x - 4) + 3 \) \( m = 2 \)
Suggested Practice for SOL A.6a

Students need additional practice finding slope.

a. Find the slope of the line passing through the points (6,3) and (4,2).

\[ m = \frac{1}{2} \]

b. Find the slope of the line passing through the point (5,1) with an x-intercept of 4.

\[ m = 1 \]
Suggested Practice for SOL A.6

Given: \( 8x + 4y = 20 \)

a. What is the slope of the line represented by this equation? \(-2\)
b. What is the \(y\)-intercept of the line represented by this equation (SOL A.7)? \((0,5)\)
c. Graph the line represented by this equation.
Suggested Practice for SOL A.6b

Graph the inequality $8x + 4y < 20$. 
SOL A.7
The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including

a) determining whether a relation is a function;
b) domain and range;
c) zeros of a function;
d) x- and y-intercepts;
e) finding the values of a function for elements in its domain; and
f) making connections between and among multiple representations of
g) functions including concrete, verbal, numeric, graphic, and algebraic.
Suggested Practice for SOL A.7b

Students need additional practice identifying the domain and range from a graph.

What appears to be the range of the relation shown?

a. $\{x| -5 \leq x \leq 6\}$

b. $\{y| -6 \leq y \leq 7\}$

c. $\{y| y = -6, -3, 1, 3, 7\}$

d. $\{x| x = -5, -2, 2, 4, 6\}$
Suggested Practice for SOL A.7b

What appears to be the domain of the relation shown?

- a. All real numbers greater than -1
- b. All real numbers greater than 2
- c. All real numbers less than 10
- d. All real numbers
**Suggested Practice for SOL A.7c**

Students need additional practice finding zeros of linear functions.

Graph each function and then plot a point at the location of the zero.

a. \( f(x) = 5x - 10 \)
   
   The zero is 2, located at \((2, 0)\).

b. \( f(x) = -x + 6 \)
   
   The zero is 6, located at \((6, 0)\).

c. \(-4x + 2y = 8\)
   
   The zero is \(-2\), located at \((-2, 0)\).
Suggested Practice for SOL A.7c

Students need additional practice finding zeros of quadratic functions presented algebraically.

a. What are the zeros of the function \( f(x) = 6x^2 - x - 2 \) ?

\[
\left\{ -\frac{1}{2}, \frac{2}{3} \right\}
\]

b. What are the zeros of the function \( f(x) = 25x^2 - 16 \) ?

\[
\left\{ -\frac{4}{5}, \frac{4}{5} \right\}
\]
Suggested Practice for SOL A.7d

Students need additional practice identifying the $x$- and $y$-intercepts of a function.

What are the $x$- and $y$- intercepts of the function $f(x) = 3x + 8$?

a. $x$- intercept of $(0, \frac{8}{3})$ and $y$- intercept of $(-8, 0)$

b. $x$- intercept of $(0, -\frac{8}{3})$ and $y$- intercept of $(8, 0)$

c. $x$- intercept of $(\frac{8}{3}, 0)$ and $y$- intercept of $(0, -8)$

d. $x$- intercept of $(-\frac{8}{3}, 0)$ and $y$- intercept of $(0, 8)$
Suggested Practice for SOL A.7d

Plot the $x$- and $y$-intercepts of the relation shown on the graph.

The $x$-intercepts are located at $(-2,0)$ and $(6,0)$ and the $y$-intercept is located at $(0,-3)$. 
Analyzing Direct and Inverse Variations

SOL A.8
The student, given a situation, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.
Students need additional practice selecting ordered pairs from a list to make a relation that is a direct or inverse variation.

Given this set of ordered pairs:

\[\{(−4, 3), (−6, 2), (4, 16), (3, 12), (−2, −8), (3, 9), (4, 4), (−1, 12)\}\]

a. Select three points that will create a direct variation relation.

\[\{(4, 16)(3, 12)(−2, −8)\}\]

since this set of ordered pairs can be represented by the equation \(\frac{y}{x} = 4\).

b. Select three points that will create an inverse variation relation.

\[\{(−4, 3)(−6, 2)(−1, 12)\}\]

since this set of ordered pairs can be represented by the equation \(xy = −12\).
Students need additional practice identifying a direct variation equation algebraically and graphically.

Identify the equations that represent a direct variation.

\[ y = x^2 + 4 \quad y = 4x + 1 \quad y = \frac{1}{5} \, x \]

\[ y = \frac{3}{x} \quad y = 3x \quad y = -0.5x \]

\[ 1500 = \frac{y}{x} \quad 1000 = xy \]
Suggested Practice for SOL A.8

Identify the graph of a direct variation.
SOL A.9
The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.
Suggested Practice for SOL A.9

Students need additional practice performing calculations with statistical information.

a. A data set has a mean of 55 and a standard deviation of 3.5. The z-score for a data point is -1.2. What is the data point?

50.8

b. A data set has a standard deviation of 3. The element 16 is an element of a data set, with a z-score of 2.4. What is the mean of the data set?

8.8
Students need additional practice performing calculations with statistical information.

The number of minutes book club students read on Monday night is displayed by the graph. The mean number of minutes for this data set is 21.18, and the standard deviation of the data set is 6.5. The z-score for the data point representing the number of minutes Tim read is 1.25. In which interval does this data point lie? The interval 25 to 30 minutes.
SOL A.10
The student will **compare and contrast** multiple univariate data sets, using **box-and-whisker plots**.
Students need additional practice interpreting data plotted in box-and-whisker plots.

Each of these box-and-whisker plots contain 15 unique elements.

1. Write a statement comparing the range of both plots.
   
   The value of the range for both plots is equal to 17.

2. Which box-and-whisker plot has the greater interquartile range?
   
   Plot A. The value of the interquartile range for Plot A is 11, and the value of the interquartile range for Plot B is 10.

3. Which data set has more elements with a value of 11 or greater?
   
   Plot B. There are 12 elements in Plot B with a value of 11 and above and 8 elements in Plot A with a value of 11 and above.
Suggested Practice for SOL A.10

Songs Downloaded

Plot A

Plot B

Number of Songs

- Plot A represents the total number of songs downloaded by each of 15 students in Mr. Archer’s class during October. Each student in Mr. Archer’s class downloaded a different number of songs from the others.
- Plot B represents the total number of songs downloaded by each of 20 students in Mrs. Baker’s class during October. Each student in Mrs. Baker’s class downloaded a different number of songs from the others.

During the month of October, what is the difference between the number of students who downloaded more than 6 songs in Mrs. Baker’s class and the number of students who downloaded more than 6 songs in Mr. Archer’s class?

The difference is 4. There were 15 students who downloaded more than 6 songs in Mrs. Baker’s class, and 11 students who downloaded more than 6 songs in Mr. Archer’s class.
Suggested Practice for SOL A.10

This box-and-whisker plot summarizes the number of pieces of pizza each of ten volunteers served at a concession stand one night.

Another volunteer served 16 pieces of pizza that night, and 16 is added to the original data set. A new box-and-whisker plot is drawn. Which two statements comparing the new box-and-whisker plot to the original box-and-whisker plot must be true?

- The interquartile range of the box-and-whisker plot increases.
- The range of the box-and-whisker plot increases.
- The value of the upper extreme increases.
- The value of the median increases.
Using the Curve of Best Fit

SOL A.11
The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions.
Suggested Practice for SOL A.11

Students need additional practice making predictions using the linear or quadratic curve of best fit.

This set of ordered pairs shows a relationship between $x$ and $y$.

$$\{(-6, 88), (-4, 32), (-2, 0), (-1, -7), (0, -8), (3, 25), (5, 77), (6, 112)\}$$

a. What is the equation for the quadratic curve of best fit for this set of data?

$$y = 3x^2 + 2x - 8$$

b. Predict the value of $y$ when $x = 8$. 200
This table shows the value, $v$, of an account at the end of $m$ months. There was an initial deposit of $50 and no other deposits were made.

<table>
<thead>
<tr>
<th>$m$, time in months</th>
<th>$v$, value in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>129</td>
</tr>
<tr>
<td>3</td>
<td>299</td>
</tr>
<tr>
<td>5</td>
<td>485</td>
</tr>
<tr>
<td>7</td>
<td>687</td>
</tr>
<tr>
<td>9</td>
<td>905</td>
</tr>
</tbody>
</table>

If the value of the account continues to increase in the same way, predict the value of the account at the end of 13 months. Use the quadratic curve of best fit to make the prediction.

$1,389.00$
The data in the table shows the average United States farm size, in acres, for the years 2000-2007.

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Acres Per Farm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>434</td>
</tr>
<tr>
<td>2001</td>
<td>437</td>
</tr>
<tr>
<td>2002</td>
<td>436</td>
</tr>
<tr>
<td>2003</td>
<td>441</td>
</tr>
<tr>
<td>2004</td>
<td>443</td>
</tr>
<tr>
<td>2005</td>
<td>444</td>
</tr>
<tr>
<td>2006</td>
<td>446</td>
</tr>
<tr>
<td>2007</td>
<td>449</td>
</tr>
</tbody>
</table>

Using the line of best fit for the data shown in the table, what is the best prediction of the average farm size in the year 2014?

a. 437 acres  b. 441 acres  c. 447 acres  d. 463 acres
This concludes the student performance information for the spring 2013 Algebra I SOL test.

Additionally, test preparation **practice items** for Algebra I can be found on the Virginia Department of Education Web site at:

Contact Information

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