

Spring 2013 Student Performance Analysis

Algebra II Standards of Learning



Presentation may be paused and resumed
using the arrow keys or the mouse.

Operations on Rational Expressions and Factoring Polynomial Expressions

SOL AII.1

The student, given rational, radical, or polynomial expressions, will

- a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
- b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
- c) write radical expressions as expressions containing rational exponents and vice versa; and
- d) factor polynomials completely.

Suggested Practice for SOL AII.1a

Students need additional practice performing operations on rational expressions.

Assuming no denominator equals zero, completely simplify each expression.

a. $\frac{x^2 + 8x + 15}{2x^2 + 5x - 3} \div \frac{-9x - 45}{1 - 2x} = \frac{1}{9}$

b. $\frac{4x^2 + 15x + 9}{8x^2 + 10x + 3} \cdot \frac{2x + 1}{x^2 + 4x} = \frac{x + 3}{x(x + 4)}$

Suggested Practice for SOL All.1a

Assuming no denominator equals zero, completely simplify each expression.

c.
$$\frac{5x^2 - 245}{2x^2 - 11x - 21} - \frac{4x + 35}{2x + 3} = \frac{x}{2x + 3}$$

d.
$$\frac{2}{x^2 - x - 20} + \frac{3}{x^2 + 7x + 12} = \frac{5x - 9}{(x + 4)(x - 5)(x + 3)}$$

Suggested Practice for SOL AII.1d

Students need additional practice factoring polynomials completely.

Factor each polynomial completely.

a. $729 - x^3$ $(9 - x)(81 + 9x + x^2)$

b. $16x^2y^2 - 25$ $(4xy + 5)(4xy - 5)$

Suggested Practice for SOL AII.1d

When completely factored, which is one factor of this polynomial?

$$y^5 + 64y^2$$

a. $8y$

b. $y + 4$

c. $y^2 + 4y + 16$

d. $y - 4$

Finding the n^{th} Term of a Sequence or Series

SOL AII.2

The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n .

Suggested Practice for SOL AII.2

Students need additional practice finding the n^{th} term of a sequence when a written description of the sequence is given.

What is the seventh term of the geometric sequence with a first term of 729 and a common ratio of $\frac{1}{3}$?

$$a_7 = 729 \left(\frac{1}{3}\right)^6$$

$$a_7 = 1$$

Suggested Practice for SOL AII.2

Students need additional practice finding the sum of a geometric series, particularly when the common ratio is negative.

Find the sum of this series.

$$-20 + 10 - 5 + \frac{5}{2} - \frac{5}{4} \dots$$

$$S_{\infty} = \frac{-20}{\left(1 - \left(-\frac{1}{2}\right)\right)}$$

$$S_{\infty} = -\frac{40}{3}$$

Identifying the Field Properties of Complex Numbers

SOL AII.3

The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i , and **identify field properties that are valid for the complex numbers.**

Suggested Practice for SOL AII.3

Students need additional practice identifying the field properties that are valid for complex numbers.

Identify the property represented in each example.

a. $3i \cdot \frac{1}{3i} = 1$ **Inverse Property of Multiplication**

b. $3i + 2i + 8 = 3i + 8 + 2i$ **Commutative Property of Addition**

c. If $3i + 2i = 5i$, and $5i = 11i - 6i$, then $3i + 2i = 11i - 6i$. **Transitive Property**

d. $3i(2i + 4) = -6 + 12i$ **Distributive Property**

e. If $3i - 8 = w$, then $w = 3i - 8$. **Symmetric Property**

Suggested Practice for SOL AII.3

Identify the property used between each step:

Step 1: $6i + 4 + 2(i + 3)$ Given

Step 2: $6i + 4 + 2i + 6$ **Distributive Property**

Step 3: $6i + 2i + 4 + 6$ **Commutative Property of Addition**

Step 4: $8i + 4 + 6$ **Substitution Property**

Step 5: $8i + 10$ **Substitution Property**

Solving Absolute Value Equations

SOL AII.4

The student will solve, algebraically and graphically,

- a) absolute value equations and inequalities;**
- b) quadratic equations over the set of complex numbers;**
- c) equations containing rational algebraic expressions; and**
- d) equations containing radical expressions.**

Graphing calculators will be used for solving and confirming the algebraic solutions.

Suggested Practice for SOL AII.4b

Students need additional practice solving quadratic equations over the set of complex numbers.

What is the solution set for each equation shown?

a. $4x^2 + 4x = -17$ $\left\{-\frac{1}{2} + 2i, -\frac{1}{2} - 2i\right\}$

b. $\frac{1}{3}x(15x + 12) = -8$ $\left\{-\frac{2}{5} \pm \frac{6}{5}i\right\}$

c. $(x - 3)^2 + 3 = 0$ $\{3 + i\sqrt{3}, 3 - i\sqrt{3}\}$

Determining Solutions for a System of Equations

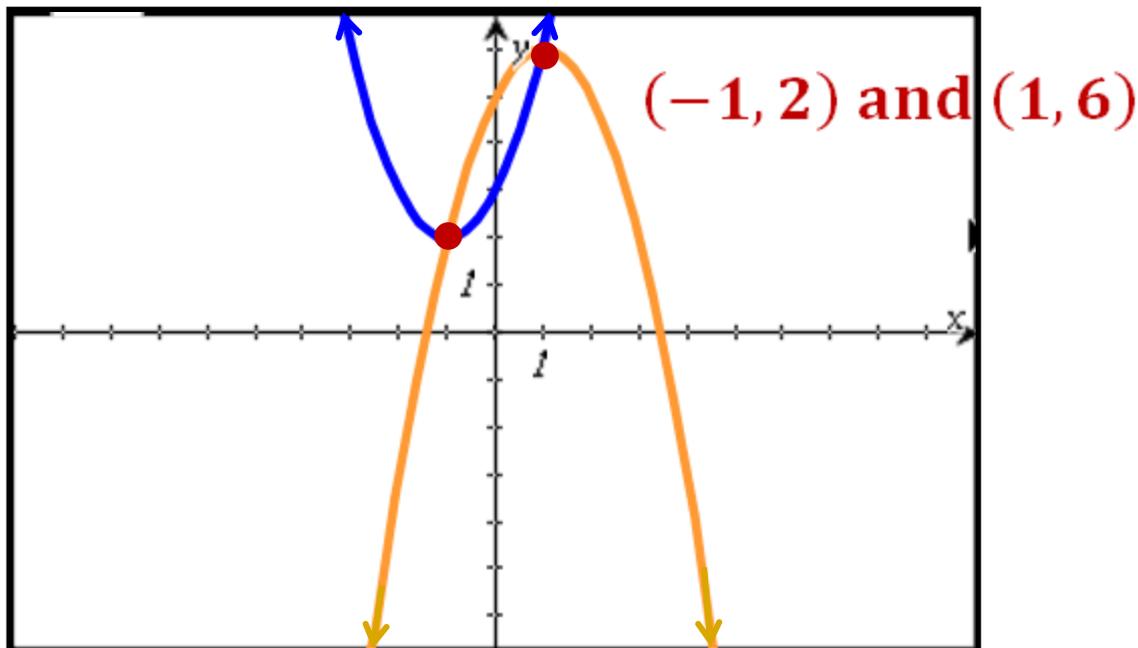
SOL AII.5

The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

Suggested Practice for SOL AII.5

Students need additional practice plotting the solution to a system of equations graphed on the coordinate plane.

Plot the apparent solutions to the system of equations shown.



Suggested Practice for SOL AII.5

Students need additional practice finding the solutions of a system of linear-quadratic equations when the equations are given symbolically.

1. What is the value of the y -coordinate of the solution to this system of equations?

$$\begin{cases} y = x^2 - 3x + 5 \\ y - 1 = x \end{cases} \quad y = 3$$

2. What are the values of the x -coordinates of the solutions to this system of equations?

$$\begin{cases} y - 7x^2 = 2 \\ x = \frac{y - 2}{7} \end{cases} \quad x = 0 \text{ and } x = 1$$

Analyzing Functions

SOL AII.7

The student will investigate and analyze functions algebraically and graphically. Key concepts include

- a) domain and range, including limited and discontinuous domains and ranges;
- b) zeros;
- c) x - and y -intercepts;
- d) intervals in which a function is increasing or decreasing;
- e) asymptotes;
- f) end behavior;
- g) inverse of a function; and
- h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.



Suggested Practice for SOL AII.7a

Students need additional practice finding values that are not in the domain of a rational function, particularly when they have to select from a list of possible values.

Select the values that are NOT in the domain of the function shown.

$$f(x) = \frac{x - 5}{x^2 - x - 12}$$

–5 –4 **–3** 0 3 **4** 5

Suggested Practice for SOL AII.7a

Students need additional practice finding the range of functions.

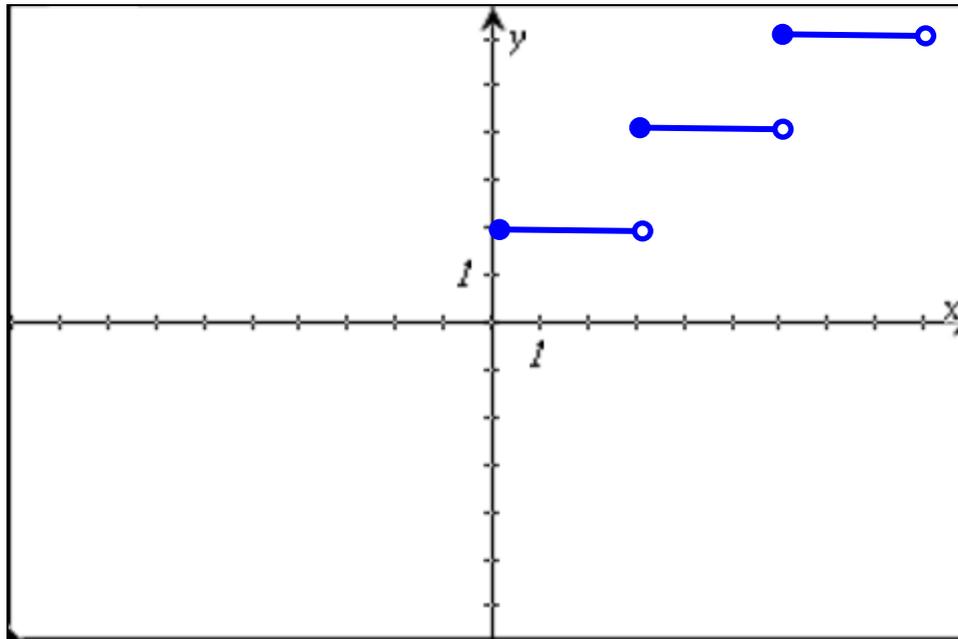
Select the best description for the range of the function shown.

$$f(x) = \frac{x^2 - 3x - 10}{x - 2}$$

- a. All real numbers
- b. All real numbers except for 2
- c. All real numbers except for -2 and 5
- d. All real numbers between -2 and 5

Suggested Practice for SOL AII.7a

What is the apparent range of the function shown on the coordinate plane?



- a. $\{y \mid 0 < y < 9\}$ b. $\{y \mid 2 < y < 6\}$ **c. $\{y \mid y = 2, 4, 6\}$** d. $\{y \mid y = 0, 3, 6, 9\}$

Extension: Name the apparent domain of this graph. $\{x \mid 0 \leq x < 9\}$



Suggested Practice for SOL AII.7b

Students need additional practice finding the zero of a logarithmic function.

Which value is a zero of $f(x) = \log(3x - 8)$?

a. $\frac{3}{8}$

b. $\frac{1}{3}$

c. 3

d. 6

Suggested Practice for SOL AII.7c

Students need additional practice determining the x - and y -intercepts of a function.

Select all of the x - and y -intercepts of the function shown by circling the x -intercept(s) and boxing the y -intercept(s).

$$f(x) = x^3 - 2x^2 - 5x + 6$$

$(0, 3)$ $(1, 0)$ $(3, 0)$ $(6, 0)$
 $(-2, 0)$ $(0, -2)$ $(0, 1)$ $(0, 6)$

Suggested Practice for SOL AII.7d

Students need additional practice identifying the intervals throughout which a function is increasing or decreasing.

Select the intervals throughout which the function shown is only increasing.

$$f(x) = 3x^3 - 9x$$

$$x < -1$$

$$x > -6$$

$$x > 1$$

$$-1 < x < 1$$

$$-6 < x < 6$$

Suggested Practice for SOL AII.7e

Students need additional practice finding asymptotes when the function is represented symbolically.

What are the asymptotes of the function shown?

$$f(x) = \frac{x - 6}{x^2 - 4x - 12}$$

- a. $x = 6, x = -2$ and $y = 0$
- b. $x = -6, x = 2$ and $y = 0$
- c. $x = 6$ and $x = -2$
- d. $x = -2$ and $y = 0$**

Suggested Practice for SOL All.7e

Name the horizontal asymptote of the function shown.

$$f(x) = \left(\frac{1}{8}\right)^{x-4}$$

Common error

$$f(x) = \left(\frac{1}{8}\right)^x - 4$$

a. $y = -4$

b. $y = -2$

c. $y = \frac{1}{8}$

d. $y = 0$

Suggested Practice for SOL AII.7f

Students need additional practice determining the end behavior of a function.

Select two true statements about the graph of $f(x) = -x^4 + bx^3 + c$.

As x approaches positive infinity, $f(x)$ approaches positive infinity.

As x approaches negative infinity, $f(x)$ approaches negative infinity.

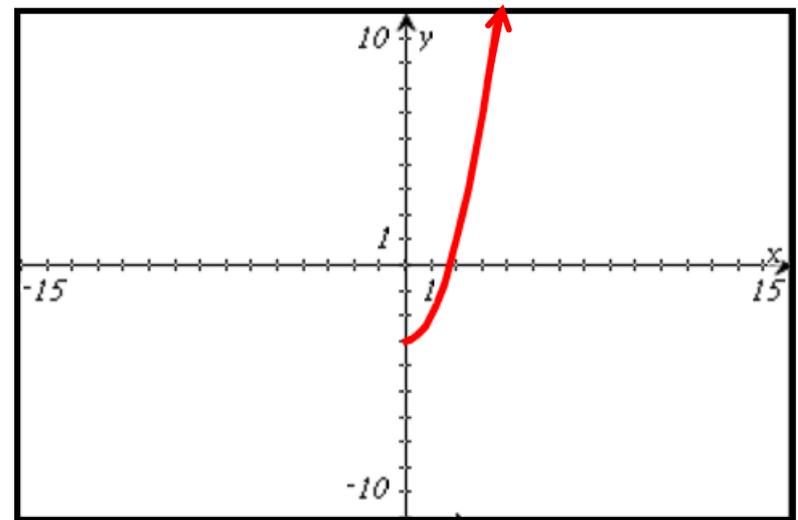
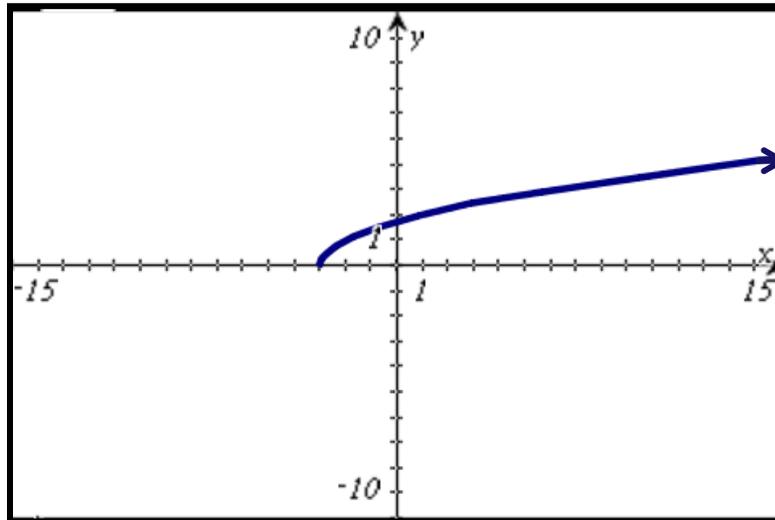
As x approaches negative infinity, $f(x)$ approaches positive infinity.

As x approaches positive infinity, $f(x)$ approaches negative infinity.

Suggested Practice for SOL AII.7g

Students need additional practice determining the inverse of a function represented graphically.

The function $f(x) = \sqrt{x+3}$ is shown on the coordinate plane. Graph the inverse of this function.



Suggested Practice for SOL All.7h

Students need additional practice finding the composition of functions when both functions are quadratic.

If $f(x) = x^2$ and $g(x) = x^2 - 5x + 6$, what is $g(f(x))$?

a. $2x^2 - 5x + 6$

b. $(x^2 - 5x + 6)^2$

c. $x^4 - 5x^2 + 6$

d. $x^4 - 5x^3 + 6x^2$

Recognizing the Relationship Among the Solution, Zero, and x -Intercept of a Graph

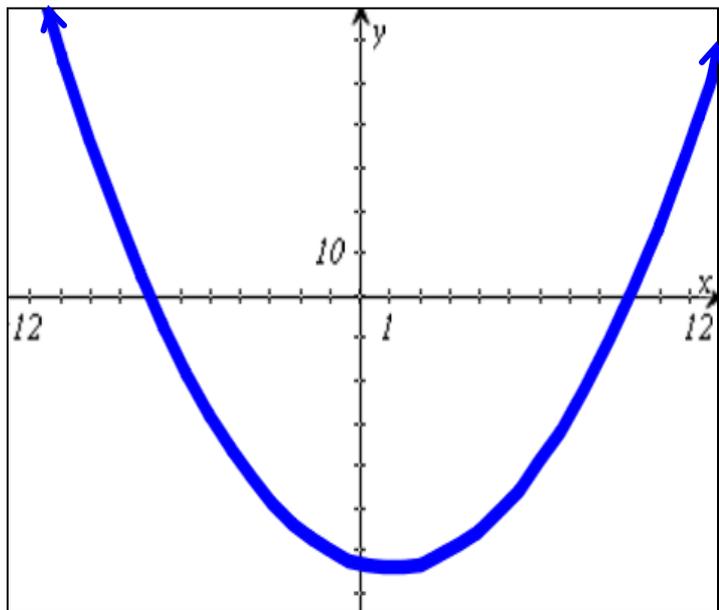
SOL AII.8

The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x -intercepts of a graph, and factors of a polynomial expression.

Suggested Practice for SOL AII.8

Students need additional practice making the connection between a zero, a solution, and the graphical representation of the x -intercept.

Multiple ways that these mathematical terms can be used to ask questions



- a. What appear to be the zeros of the function shown on the coordinate plane?

The zeros are -7 and 9 , located at $(-7, 0)$ and $(9, 0)$ respectively.

- b. What appear to be the solutions to the equation represented by the graph when $f(x) = 0$?

$x = -7$ and $x = 9$

- c. What are the apparent x -intercepts of the function shown on the graph?

$(-7, 0)$ and $(9, 0)$

Making Predictions with Curves of Best Fit

SOL AII.9

The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models.

Mathematical models will include polynomial, exponential, and logarithmic functions.

Suggested Practice for SOL AII.9

Students need additional practice finding the exponential curve of best fit for a set of data and making predictions using this curve.

The table provides the value of an account over time that earned annual compound interest. There was an initial deposit of \$1,500 into the account, and no other deposits were made.

Value of Account Over Time

Time in years, x	0	5	10	15	20	25	30
Value in dollars	1,500.00	1,914.42	2,443.34	3,118.39	3,979.95	5,079.53	6,482.91

Assuming the account continues to grow in the same way, use the exponential curve of best fit to find the value of the account at the end of 40 years, rounded to the nearest dollar.

\$10,560



Solving Problems Involving Variation

SOL AII.10

The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

Suggested Practice for SOL AII.10

Students need additional practice finding the constant of proportionality and solving real-world problems involving a combination of direct and inverse variations.

- a. If y varies inversely with the square of x , what is the constant of proportionality when $y = 10$ and $x = 5$?

250

- b. Body mass index (BMI) is directly proportional to a person's weight in pounds and inversely proportional to the square of a person's height in inches. A person with a BMI of 23.91 has a weight of 135 pounds and a height of 63 inches. Rounded to the nearest hundredth, what is the BMI of a person with a weight of 145 pounds and a height of 65 inches?
- Answers may vary depending on how the constant was rounded. BMI should be approximately 24.13.**



Applying Properties of the Normal Distribution to Solve Problems

SOL AII.11

The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

Suggested Practice for SOL AII.11

For assistance with this content, or to find suggestions on types of questions to present to students, please refer to the *Technical Assistance Document* located at the Virginia Department of Education Web site at:

http://www.doe.virginia.gov/instruction/mathematics/high/technical_assistance_algebra2_aii_11.pdf

Suggested Practice for SOL AII.11

Students need additional practice using properties of the normal distribution curve to find the probability of an event, the percent of data that falls within a specified interval, and the number of expected values that fall within a specified interval.

A population of adult males had their heights measured. The heights were normally distributed. Approximately what percentage of the heights, rounded to the nearest whole number, are within one standard deviation of the mean?

a. 34 %

c. 68%

b. 95%

d. 99.7%

Suggested Practice for SOL AII.11

At a company, the data set containing the ages of applicants for a particular job was normally distributed. The mean age of the applicants was 30 years old, and the standard deviation of the data set was 3.5 years. Which is closest to the percent of applicants that were 21 years old or younger?

a. 0.5 %

b. 2.0 %

c. 2.57 %

d. 9.0 %

Suggested Practice for SOL AII.11

A normally distributed data set of 500 values has a mean of 35 and a standard deviation of 7. Which is closest to the probability that a value in the data set will fall between 42 and 46?

a. 0.04

b. 0.10

c. 10

d. 50

Suggested Practice for SOL AII.11

A normally distributed data set of 600 values has a mean of 18.5 and a standard deviation of 3.25.

1. What is the approximate number of values in the data set expected to be 22 or greater?

Acceptable answers: 84 or 85

2. What is the approximate number of values in the data set expected to be 16 or fewer?

Acceptable answers: 132 or 133

3. Which is closest to the expected number of values in the data set that lie between 21 and 27?

- a. 6 b. 22 **c. 130** d. 467

Practice Items

This concludes the student performance information for the spring 2013 Algebra II SOL test.

Additionally, test preparation practice items for Algebra II can be found on the Virginia Department of Education Web site at:

http://www.doe.virginia.gov/testing/sol/practice_items/index.shtml#math

Contact Information

For assessment questions, please contact
Student_assessment@doe.virginia.gov

For instruction questions, please contact
Michael.Bolling@doe.virginia.gov