

Spring 2013 Student Performance Analysis

Geometry Standards of Learning



Presentation may be paused and resumed
using the arrow keys or the mouse.

Judging the Validity of a Logical Argument

SOL G.1

The student will construct and **judge the validity of a logical argument** consisting of a set of premises and a conclusion. **This will include:**

- a) identifying the converse, inverse, and contrapositive of a conditional statement;
- b) translating a short verbal argument into symbolic form;
- c) using Venn diagrams to represent set relationships; and
- d) **using deductive reasoning.**

Suggested Practice for SOL G.1

Students need additional practice judging the validity of a logical argument and using valid forms of deductive reasoning, including the law of syllogism, the law of contrapositive, the law of detachment, and counterexamples.

Example of the law of detachment:

Let p = “a dog eats bread”

Let q = “the dog gains weight”

$p \rightarrow q$, “If a dog eats bread, then the dog gains weight” is a true statement.

John’s dog eats bread. What can be concluded?

John’s dog gains weight.

Suggested Practice for SOL G.1

Example of the law of contrapositive:

Let p = “a dog eats bread”

Let q = “the dog gains weight”

$p \rightarrow q$ means “If a dog eats bread, then the dog gains weight.”

$p \rightarrow q$ is a true statement.

What is the contrapositive of this statement? Is it a true statement?

“If the dog does not gain weight, then the dog does not eat bread” is a true statement.

Suggested Practice for SOL G.1

Example of the law of syllogism:

Let p = “a dog eats bread”

Let q = “the dog gains weight”

Let r = “the dog gets fat”

$p \rightarrow q$ means “If a dog eats bread, then the dog gains weight.”

$q \rightarrow r$ means “If the dog gains weight, then the dog gets fat.”

Using the law of syllogism, what is the logical conclusion?

The law of syllogism concludes:

$p \rightarrow r$, which translates to: “If a dog eats bread, then the dog gets fat.”

Suggested Practice for SOL G.1

Example of a counterexample:

Let $p = "n \text{ is an integer and } n^2 \text{ is divisible by } 4"$

Let $q = "n \text{ is divisible by } 4"$

$p \rightarrow q$ means "If n is an integer and n^2 is divisible by 4, then n is divisible by 4."

A counterexample is any example that will prove a statement false. In this case, a number for which p is true and q is false is a counterexample.

What is a counterexample to $p \rightarrow q$?

In this case, $n = 10$ is a counterexample because $10^2 = 100$ and 100 is divisible by 4, but 10 is not divisible by 4.

Other possible answers: 2, 6, 14, 18, etc.

Determining and Verifying Parallelism

SOL G.2

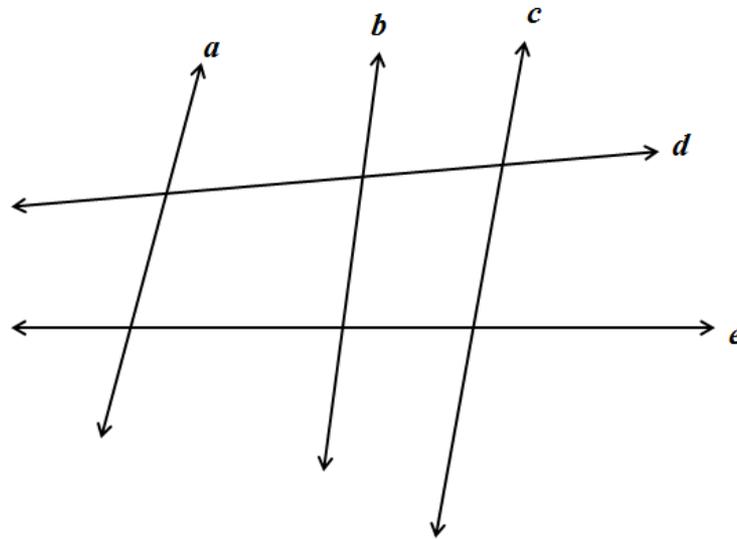
The student will use the relationships between angles formed by two lines cut by a transversal to

- a) **determine whether two lines are parallel;**
- b) **verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and**
- c) **solve real-world problems involving angles formed when parallel lines are cut by a transversal.**

Suggested Practice for SOL G.2a

Students need additional practice determining parallelism in complex figures.

This is an example of a complex figure with more than one transversal.



Given information about a figure, determine parallel lines or congruent angles.

Suggested Practice for SOL G.2a

Given: $a \parallel c$

What is the value of x ?

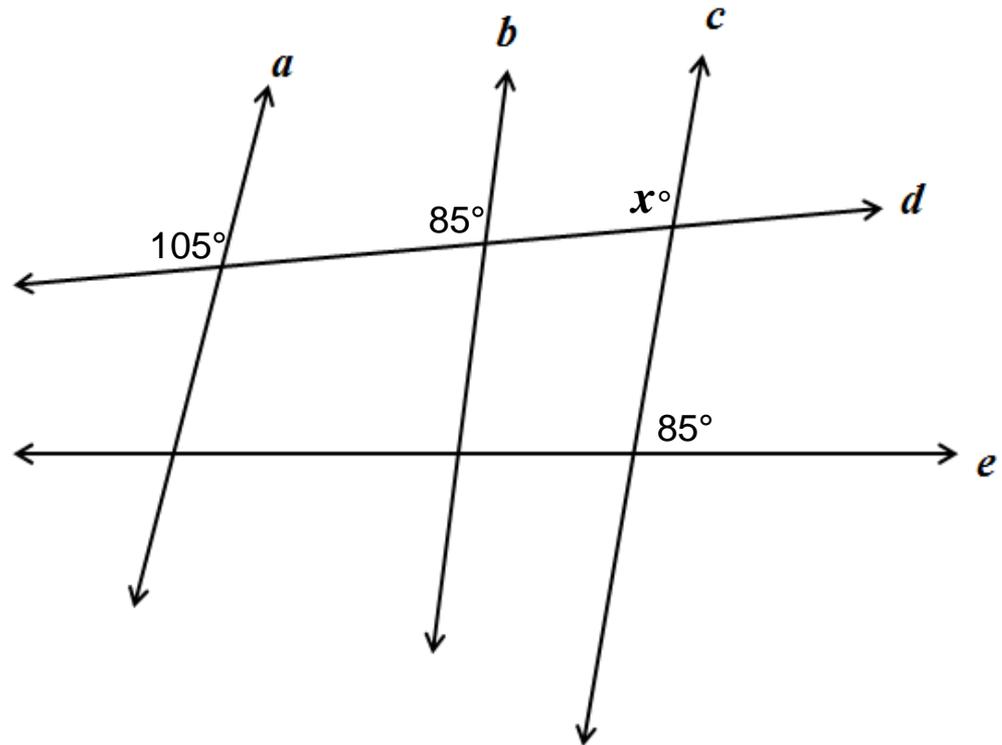
$x = 105$

Is $b \parallel c$? Why or why not?

No, because there are no congruent corresponding, alternate interior, or alternate exterior angles.

Is $d \parallel e$? Why or why not?

No, because there are no congruent corresponding, alternate interior, or alternate exterior angles.



Suggested Practice for SOL G.2a

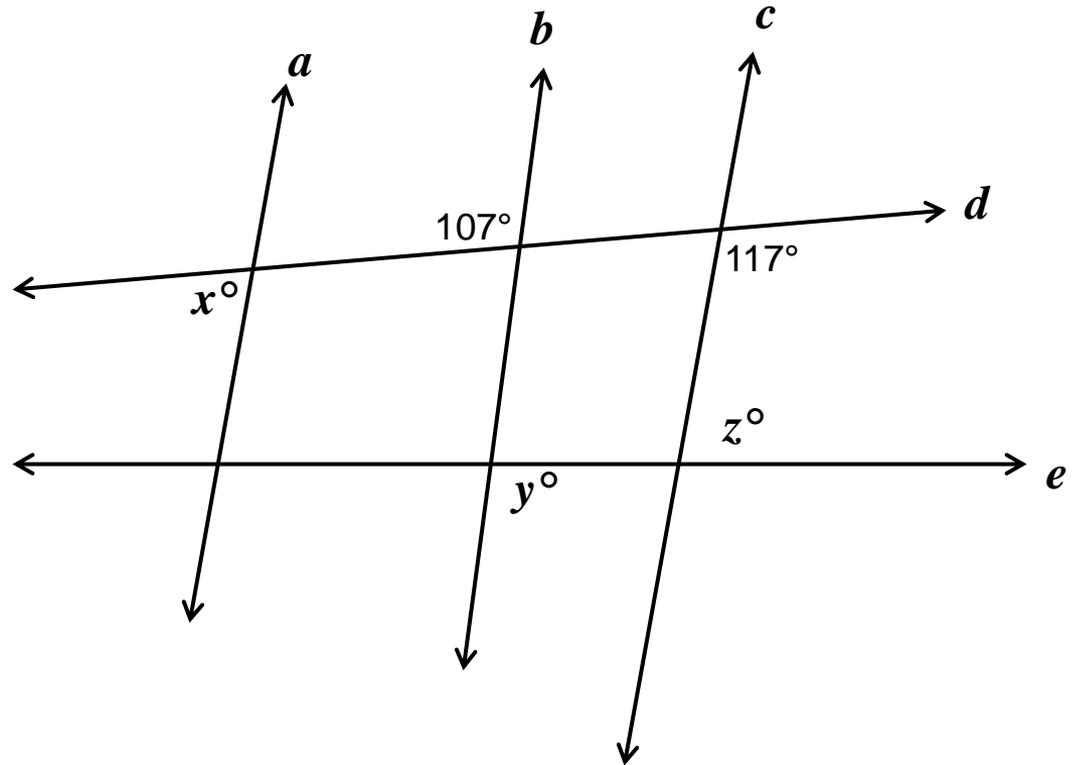
Given: $a \parallel b$ and $d \parallel e$

Find the values of x , y , and z .

$$x = 73$$

$$y = 107$$

$$z = 63$$



Identifying a Parallel or Perpendicular Line to a Given Line

SOL G.3

The student will use pictorial representations, including computer software, constructions, and **coordinate methods**, to solve problems involving symmetry and transformation. This will include

- a) **investigating** and using formulas for finding distance, midpoint, and **slope**;
- b) **applying slope to verify and determine whether lines are parallel or perpendicular**;
- c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
- d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

Suggested Practice for SOL G.3

Students need additional practice determining parallel and perpendicular lines on a coordinate grid.

Find the coordinates of a point that lies on a line which passes through point P and is parallel to line c .

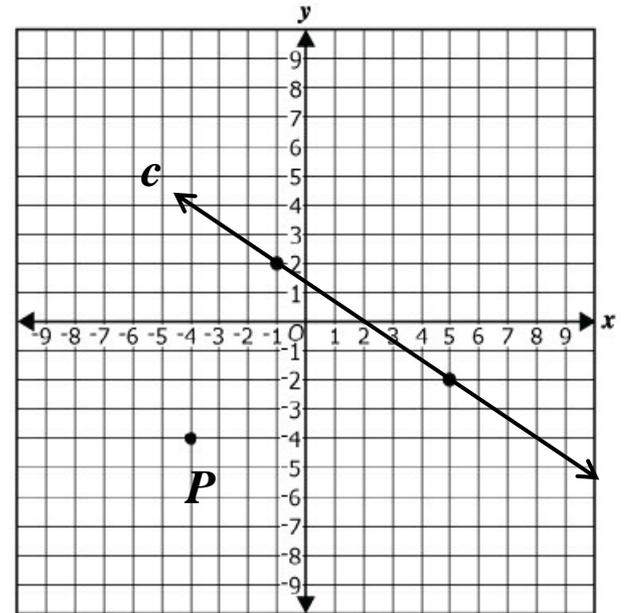
Possible answers:

$(-10,0)$, $(-7,-2)$, $(-1,-6)$, $(2,-8)$, $(5,-10)$

Find the coordinates of a point that lies on a line which passes through point P and is perpendicular to line c .

Possible answers:

$(-6,-7)$, $(-2,-1)$, $(0, 2)$, $(2, 5)$, $(4, 8)$



Justifying Constructions

SOL G.4

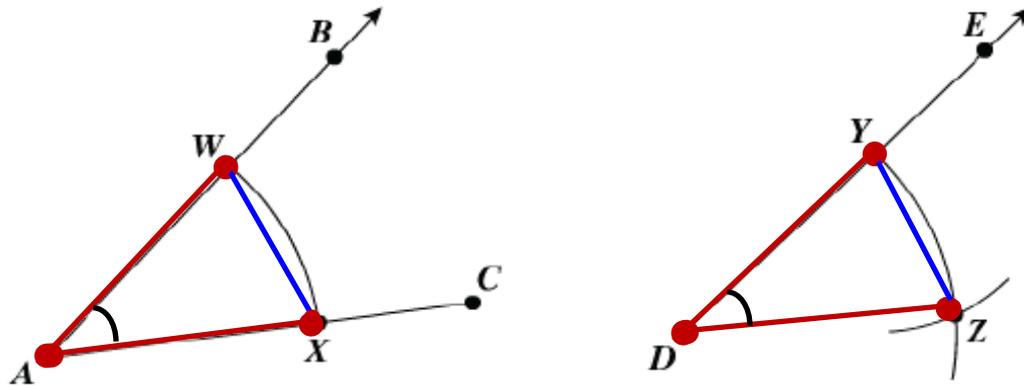
The student will construct and **justify the constructions** of

- a) a line segment congruent to a given line segment;
- b) the perpendicular bisector of a line segment;
- c) a perpendicular to a given line from a point not on the line;
- d) a perpendicular to a given line at a point on the line;
- e) the bisector of a given angle;
- f) a line parallel to a given line through a point not on the given line.

Suggested Practice for SOL G.4

Students need additional practice justifying constructions.

Arcs WX and YZ are drawn using the same radius. An arc with radius WX is centered on point Y and intersects arc YZ at point Z . A line segment will connect points D and Z .



Which statement justifies this construction?

- A. $\angle A \cong \angle D$ because they are corresponding parts of congruent triangles.
- B. $\angle A \cong \angle D$ because they are alternate interior angles.
- C. $\angle A \cong \angle D$ because $AB = DE$.

Proving Triangles Congruent

SOL G.6

The student, given information in the form of a figure or statement, will **prove two triangles are congruent**, using algebraic and coordinate methods as well as **deductive proofs**.

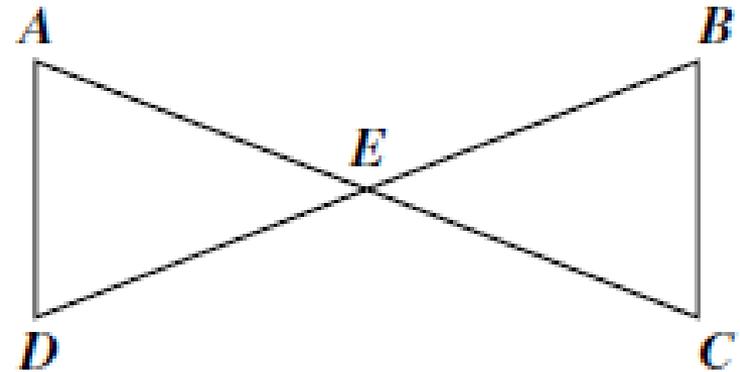
Suggested Practice for SOL G.6

Students need additional practice completing the steps and reasons in two-column deductive proofs that prove triangles congruent.

Given: In the figure, line segments AC and BD bisect each other at point E .

Prove: $\triangle AED \cong \triangle CEB$

Possible Answer:



<i>Statements</i>	<i>Reasons</i>
1. AC and BD bisect each other at point E .	1. Given
2. $AE=EC$ and $DE=EB$	2. Definition of bisector
3. $\overline{AE} \cong \overline{EC}$ and $\overline{DE} \cong \overline{EB}$	3. Definition of congruence
4. $\angle AED \cong \angle CEB$	4. Vertical angles are congruent.
5. $\triangle AED \cong \triangle CEB$	5. SAS (Side-Angle-Side) Theorem

Proving Triangles Similar

SOL G.7

The student, given information in the form of a figure or statement, will **prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.**

Suggested Practice for SOL G.7

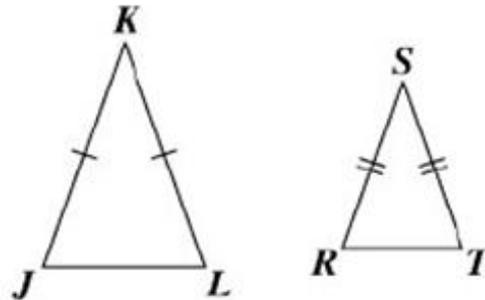
Complete the proof.

Given: $\triangle JKL$ and $\triangle RST$

$$\angle JKL \cong \angle RST$$

$$\overline{JK} \cong \overline{KL} \text{ and } \overline{RS} \cong \overline{ST}$$

Prove: $\triangle JKL \sim \triangle RST$



Statements	Reasons
1. $\angle JKL \cong \angle RST$	1. Given
2. $\overline{JK} \cong \overline{KL}$ and $\overline{RS} \cong \overline{ST}$	2. Given
3. $JK = KL; RS = ST$	3. Definition of congruence
4. $\frac{JK}{RS} = \frac{KL}{ST}$	4. Division property of equality
5. $\triangle JKL \sim \triangle RST$	5. SAS Similarity

Finding Arc Lengths and Areas of Sectors

SOL G.11

The student will use angles, arcs, chords, tangents, and secants to

- a) investigate, verify, and apply properties of circles;
- b) solve real-world problems involving properties of circles; and
- c) find arc lengths and areas of sectors in circles.

Suggested Practice for SOL G.11

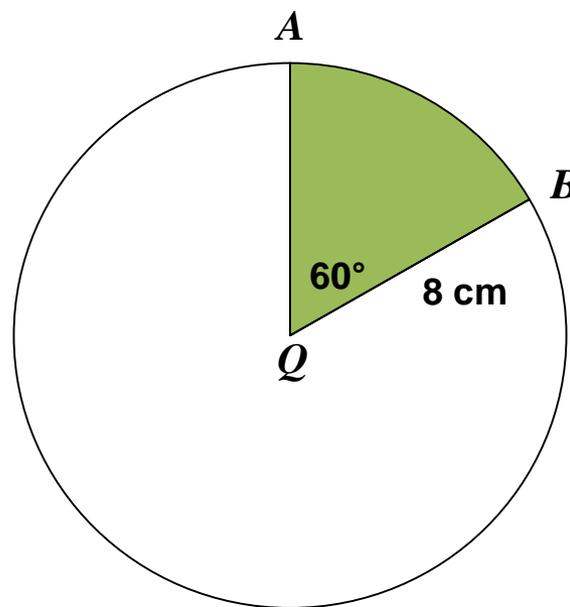
Students need additional practice finding the area of a sector or arc length when given the radius or diameter and a central angle of a circle.

Find the area of the shaded sector of Circle Q .

33.5 cm²

Find the length of arc AB .

8.4 cm



Identifying Points on a Circle

SOL G.12

The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

Students should have experiences that require them to determine any of the following from given information:

- the coordinates of the center;
- the length of a radius;
- coordinate endpoints of a radius;
- the length of a diameter;
- coordinate endpoints of a diameter;
- **the coordinates of a point on the circle;** and/or
- the equation of a circle.

Suggested Practice for SOL G.12

Students need additional practice identifying the coordinates of points that lie on a circle when given sufficient information to write the equation of the circle.

A circle has a center at $(-1, 4)$ and a diameter of 20.

Select each pair of coordinates that represent a point on this circle.

$(-7, -4)$

$(11, 20)$

$(19, 16)$

$(5, 12)$

$(-9, -2)$

$(-1, 4)$

Reasoning Using Similar Geometric Objects

SOL G.14

The student will use similar geometric objects in two- or three-dimensions to

- a) compare ratios between side lengths, perimeters, areas, and volumes;
- b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
- c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
- d) solve real-world problems about similar geometric objects.

Suggested Practice for SOL G.14

Students need additional practice determining how changes in dimensions of a geometric object affect surface area and volume, and how changes in surface area and volume affect its dimensions. Students also need additional practice determining these relationships for similar geometric objects.

Example Using Ratios:

Two similar rectangular prisms have side lengths with a ratio of 1:3.

What is the ratio of their surface areas?

1:9

What is the ratio of their volumes?

1:27

Suggested Practice for SOL G.14

Example Using Scale Factors:

The dimensions of a triangular prism with a surface area of 51.46 cm^2 are multiplied by a scale factor of 2.5 to create a similar triangular prism.

What is the surface area of the new triangular prism?

$$(2.5)^2(51.46) = (6.25)(51.46) = 321.625 \text{ cm}^2$$

Extension: What is the relationship between the volume of the original prism and the volume of the new prism?

The new volume is 15.625 times the volume of the original prism.

Suggested Practice for SOL G.14

Relationship Between Volume and Dimensions:

Cathy has two cylinders that are the same height, but the volume of the second cylinder is 16 times the volume of the first cylinder. The radius of the second cylinder is –

- A. 2 times the radius of the first cylinder.
- B. 4 times the radius of the first cylinder.
- C. 8 times the radius of the first cylinder.
- D. 16 times the radius of the first cylinder.

Suggested Practice for SOL G.14

Relationship Between Volume and Dimensions:

Cathy has two cylinders with radii the same length, but the volume of the second cylinder is 16 times the volume of the first cylinder. The height of the second cylinder is –

- A. 2 times the height of the first cylinder.
- B. 4 times the height of the first cylinder.
- C. 8 times the height of the first cylinder.
- D. 16 times the height of the first cylinder.**

Practice Items and Tools Practice

This concludes the student performance information for the Geometry tests administered during the spring 2013 test administration.

Additionally, test preparation **practice items** for Geometry can be found on the Virginia Department of Education Web site at:

http://www.doe.virginia.gov/testing/sol/practice_items/index.shtml#math

There is also an **End-of-Course Tools Practice** located at the same URL that provides students the opportunity to practice with the online tools and provides an interface for students to practice constructions using the online compass.

Contact Information

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