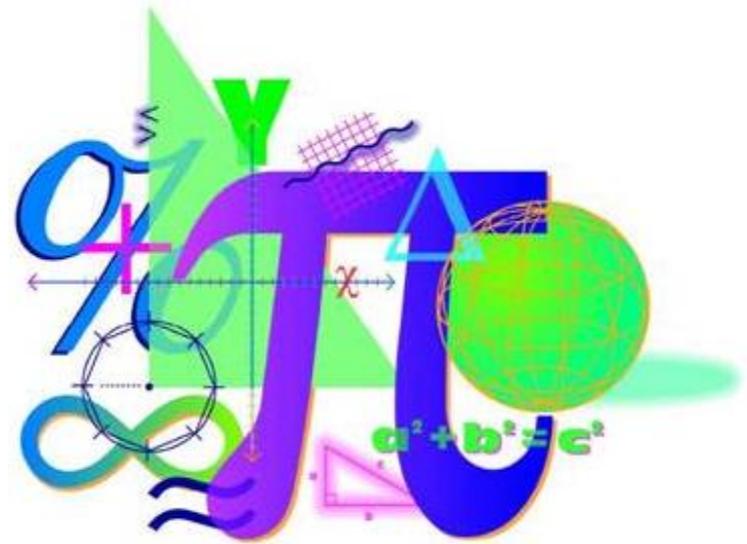


Spring 2014 Student Performance Analysis

Algebra II Standards of Learning



Presentation may be paused and resumed
using the arrow keys or the mouse.

Operations on Rational Expressions and Factoring Polynomial Expressions

SOL AII.1

The student, given rational, radical, or polynomial expressions, will

- a) add, subtract, multiply, divide, and simplify rational algebraic expressions;**
- b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;**
- c) write radical expressions as expressions containing rational exponents and vice versa; and**
- d) factor polynomials completely.**

Suggested Practice for SOL All.1a

Students need additional practice performing operations on rational expressions.

Assuming no denominator equals zero, completely simplify each expression.

a.
$$\frac{16 - 9x^2}{12x^2 + 10x - 8} \cdot \frac{6x^2 + 3x - 3}{3x^2 + 2x - 8} = \frac{-3(x + 1)}{2(x + 2)}$$

b.
$$\frac{\frac{5 - m}{42m^2}}{\frac{m - 5}{7m^6}} = \frac{-m^4}{6}$$

c.
$$\frac{4d^4 - 4d^2}{d^3 - 7d^2 - 8d} = \frac{4d(d - 1)}{d - 8}$$

Suggested Practice for SOL All.1a

Assuming the denominator does not equal zero, completely simplify the expression shown.

$$\frac{16}{x^2 - 16} + \frac{2}{x + 4}$$

A

$$\frac{2}{x - 4}$$

C

$$\frac{18}{(x^2 - 16)(x + 4)}$$

B

$$\frac{2}{x - 3}$$

D

$$\frac{18}{x^2 + x - 12}$$

Suggested Practice for SOL AII.1b

Students need additional practice simplifying and performing operations on radical expressions.

Simplify the expression for positive x and y values.

a. $\sqrt{45x^9y^{36}}$ $3x^4y^{18}\sqrt{5x}$

b. $\sqrt[3]{6x^5} \cdot \sqrt[3]{9x^2}$ $3x^2\sqrt[3]{2x}$

Suggested Practice for SOL All.1c

Students need additional practice writing radical expressions as expressions containing rational exponents and vice versa.

- a. Write an expression in radical form equivalent to $8^{\frac{1}{6}}x^{\frac{5}{6}}y^{\frac{23}{6}}$.

$$y^3 \sqrt[6]{8x^5y^5}$$

- b. Write an expression containing rational exponents equivalent to $\sqrt{144x^{49}y^9}$.

$$12x^{\frac{49}{2}}y^{\frac{9}{2}}$$

Finding the Sum of a Geometric Series

SOL AII.2

The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the n^{th} term, and **evaluating summation formulas**. Notation will include \sum and a_n .

Suggested Practice for SOL AII.2

Students need additional practice finding the sum of a geometric series, particularly when the common ratio is negative.

Find the sum of the infinite geometric series:

a. $-\frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} + \dots = -\frac{1}{12}$

b. $\sum_{n=0}^{\infty} 4 \left(\frac{1}{4}\right)^n = \frac{16}{3}$

Complex Numbers

SOL AII.3

The student will perform **operations on complex numbers**, express the results in simplest form using patterns of the powers of i , and **identify field properties that are valid for the complex numbers**.

Suggested Practice for SOL AII.3

Students need additional practice performing operations on complex numbers involving radicals.

What number is equivalent to-

$$\sqrt{-11} \cdot \sqrt{-44}$$

- A 22
- B $2\sqrt{11}$
- C $-2\sqrt{11}$
- D -22

Suggested Practice for SOL AII.3

a. $\sqrt{18} \cdot \sqrt{-50}$ $30i$

b. $\sqrt{-27} \cdot \sqrt{-32}$ $-12\sqrt{6}$

Suggested Practice for SOL AII.3

Students need additional practice identifying the field properties that are valid for complex numbers.

Identify the property that justifies each step of the simplification shown.

STEPS

JUSTIFICATIONS

$$5i\sqrt{3} + 2(6z + i\sqrt{3})$$

Given Expression

$$2(6z + i\sqrt{3}) + 5i\sqrt{3}$$

Commutative Property of Addition

$$(2 \cdot 6z + 2 \cdot i\sqrt{3}) + 5i\sqrt{3}$$

Distributive Property

$$(12z + 2i\sqrt{3}) + 5i\sqrt{3}$$

Substitution Property

$$12z + (2i\sqrt{3} + 5i\sqrt{3})$$

Associative Property of Addition

$$12z + 7i\sqrt{3}$$

Substitution Property

Solving Equations

SOL AII.4

The student will solve, algebraically and graphically,

- a) absolute value equations and inequalities;
- b) quadratic equations over the set of complex numbers;
- c) equations containing rational algebraic expressions; and
- d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions.

Absolute Value Equations

Suggested Practice for SOL All.4a

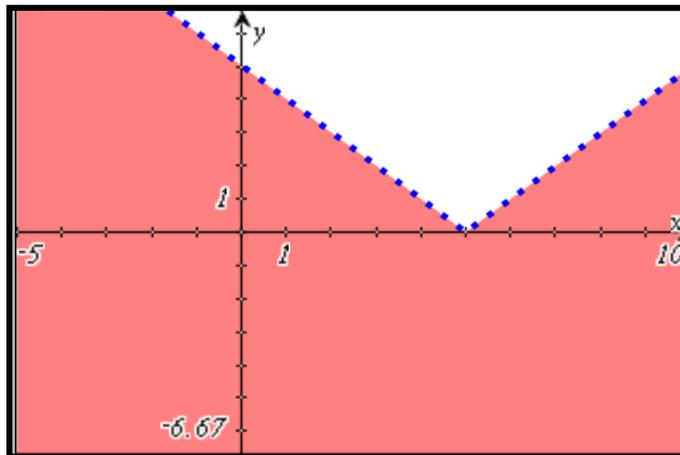
Students need additional practice solving absolute value equations and inequalities algebraically and graphically.

a. Find the solution to:

$$|22 - (4 - 3x)| < 30 \quad -16 < x < 4$$

b. Determine an ordered pair that is a solution to:

$$y < |x - 5|$$



Suggested Practice for SOL AII.4a

How many values of x will satisfy the absolute value equation?

c. $-7|2x - 1| = 0$ **one** $x = \frac{1}{2}$

d. $10 - 4|3x + 2| = 0$ **two** $x = \frac{1}{6}$ and $x = -\frac{3}{2}$

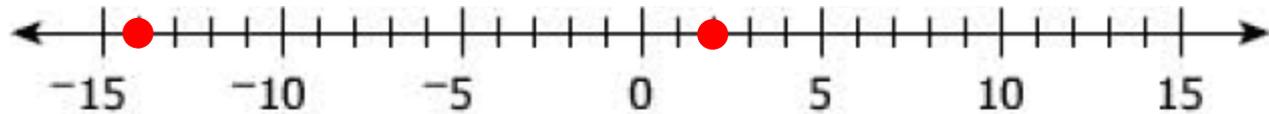
e. $-5|-6 + 4x| = 15$ **zero** \emptyset

Extension: What value(s) of x would be the solutions to the equations?

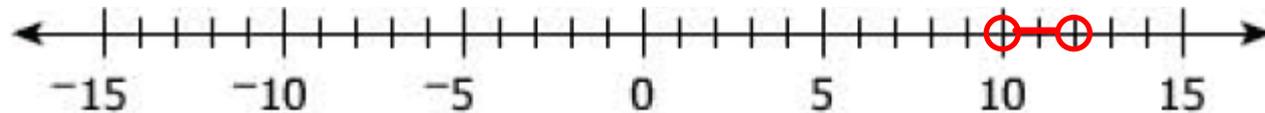
Suggested Practice for SOL All.4a

Graph the solutions to the problems shown.

f. $\left| \frac{1}{2}x + 3 \right| = 4$



g. $|x - 11| < 1$



Quadratic Equations

Suggested Practice for SOL AII.4b

Students need additional practice solving quadratic equations over the set of complex numbers.

What are the solutions or roots for each equation shown?

a. $3x^2 = 2x - 5$ $x = \frac{1 \pm i\sqrt{14}}{3}$

b. $7x^2 = -6x - 1$ -0.631 and -0.227

c. $15x^2 + 7x - 4 = 0$ $\left\{ -\frac{4}{5}, \frac{1}{3} \right\}$

Equations Containing Rational Expressions

Suggested Practice for SOL All.4c

Students need additional practice solving equations containing rational expressions.

If no denominator is equal to zero, what is the solution set to the equation?

a. $\frac{x+1}{x} + \frac{x-3}{2x} = \frac{2x+7}{3x}$ $x = \frac{17}{5}$

b. $\frac{5x-2}{x^2} = \frac{3}{4x}$ $\left\{ \frac{8}{17} \right\}$

c. $\frac{1}{n^2} = \frac{1}{24n} + \frac{1}{48}$ $\{-8, 6\}$

Determining Solutions for a System of Equations

SOL AII.5

The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

Suggested Practice for SOL AII.5

Students need additional practice finding the solutions of a system of linear-quadratic equations when the equations are given symbolically.

What are the x -values of the solutions for the systems?

a.
$$\begin{cases} y = x^2 + 2x - 3 \\ y = 2x + 6 \end{cases}$$

$$x = -3 \text{ and } x = 3$$

b.
$$\begin{cases} y = 2x - 3 \\ y = x^2 - 5x \end{cases}$$

$$x = \frac{7 \pm \sqrt{37}}{2}$$

Extension: How many real solutions exist for the systems shown?

a. **2**

b. **2**

Shapes of Functions

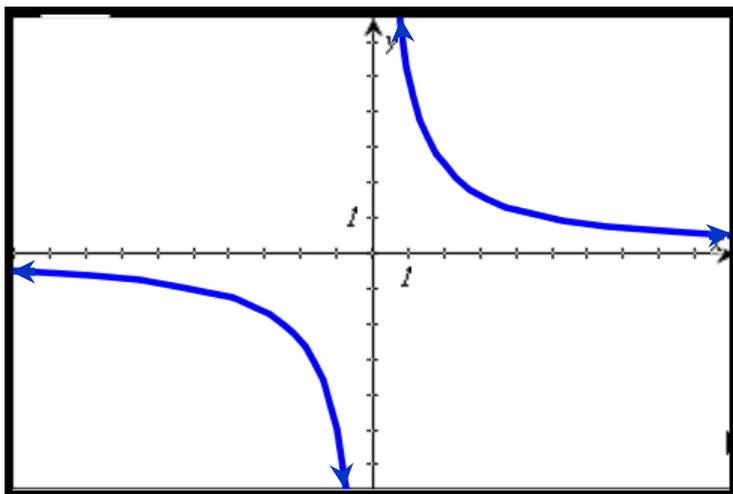
SOL AII.6

The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. **A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.**

Suggested Practice for SOL AII.6

Students need additional practice identifying equations that belong to the same parent function.

The graph of a parent function is shown. Identify each function which belongs to this same family.



$$f(x) = \frac{-2}{x^2}$$

$$h(x) = \frac{x^2 - 4}{x + 2}$$

$$g(x) = \frac{4}{x - 2}$$

$$j(x) = \frac{x - 4}{x}$$

Analyzing Functions

SOL AII.7

The student will investigate and analyze functions algebraically and graphically. Key concepts include

- a) domain and range, including limited and discontinuous domains and ranges;
- b) zeros;
- c) x - and y -intercepts;
- d) intervals in which a function is increasing or decreasing;
- e) asymptotes;
- f) end behavior;
- g) inverse of a function; and
- h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.

Suggested Practice for SOL AII.7a

Students need additional practice finding values that are not in the domain of a rational function, particularly when they have to select from a list of possible values.

Select all of the following that are NOT in the domain of $f(x)$.

$$f(x) = \frac{x - 3}{x^2 + 16x + 63}$$

-9	-7	-3
3	7	9

Suggested Practice for SOL AII.7a

Students need additional practice finding values for the domain and range in a real-world context.

A steel ball is dropped from a television tower and falls 1600 feet to the ground below. The relationship between the height of the ball, in feet, and the time the ball is in the air, in seconds, can be represented by the function $h(t) = 1600 - 16t^2$.

Which of these values for the domain and range are reasonable for this real-world situation?

$$-10 \leq t \leq 10$$

$$0 \leq t \leq 10$$

$$-\infty < h < 1600$$

$$0 \leq h \leq 1600$$

Domain:

$$0 \leq t \leq 10$$

Range:

$$0 \leq h \leq 1600$$

Suggested Practice for SOL AII.7b

Students need additional practice determining the zeros of a function.

Which is one of the zeros of $x^2 - 5x + 14$?

A 7

Incorrect answer if students factor the expression as $(x - 7)(x - 2)$

B -2

Incorrect answer if students factor the expression as $(x + 7)(x + 2)$

C $\frac{5 + 9i}{2}$

Incorrect answer if students find -5^2 rather than $(-5)^2$

D $\frac{5 + i\sqrt{31}}{2}$

Suggested Practice for SOL AII.7b

Students need additional practice finding the zeros of rational and logarithmic functions.

What is the zero of the function?

a. $f(x) = \frac{2x + 7}{x + 7}$ $x = -\frac{7}{2}$

b. $h(x) = \log(4 - 2x)$ $x = \frac{3}{2}$

Suggested Practice for SOL All.7c

Students need additional practice determining the x - and y -intercepts of a function.

Which is an x -intercept of the graph of $y = 5 + 2x - 3x^2$?

A $(0, -1)$

B $(1, 0)$

C $(0, 5)$

D $(1\frac{2}{3}, 0)$

Suggested Practice for SOL All.7c

Students need additional practice describing the x - and y -intercepts of the graph of a function.

Which statement describes the graph of $g(x) = \frac{x + 2}{x^2}$?

- A The graph has one x -intercept and one y -intercept.
- B The graph has one x -intercept and no y -intercept.**
- C The graph has no x -intercept and one y -intercept.
- D The graph has two x -intercepts and no y -intercept.

Suggested Practice for SOL AII.7d

Students need additional practice identifying the intervals throughout which a function is increasing or decreasing.

The function $f(x) = (3 - x)^2 - 2$ is decreasing throughout the interval —

A $-2 < x < \infty$

B $-\infty < x < 3$

C $-\infty < x < \infty$

D $2 < x < 4$

Suggested Practice for SOL AII.7d

The function $g(x) = x^3 + 4x^2 + 1$ is only increasing throughout which of these intervals?

$-\infty < x < -2.7$	$-2.7 < x < 1.8$	$-\infty < x < 0$
$1.8 < x < \infty$	$0 < x < \infty$	$4 < x < 0$

Suggested Practice for SOL All.7e

Students need additional practice finding asymptotes when the function is represented symbolically.

Which list contains only the equations of the asymptotes of the graph of the following function?

$$f(x) = \frac{x - 5}{x^2 - 25}$$

- A $x = -5$ and $y = 0$
- B $x = -5$ and $y = 5$
- C $x = 5$ and $y = 0$
- D $x = -5, x = 5,$ and $y = 0$

Suggested Practice for SOL AII.7f

Students need additional practice determining the end behavior of a function.

Which of these describes the end behavior of $f(x) = 3\left(\frac{5}{6}\right)^x$ as x approaches positive infinity?

- A $f(x)$ approaches negative infinity
- B $f(x)$ approaches positive infinity
- C $f(x)$ approaches $\frac{5}{6}$
- D $f(x)$ approaches 0

Suggested Practice for SOL AII.7f

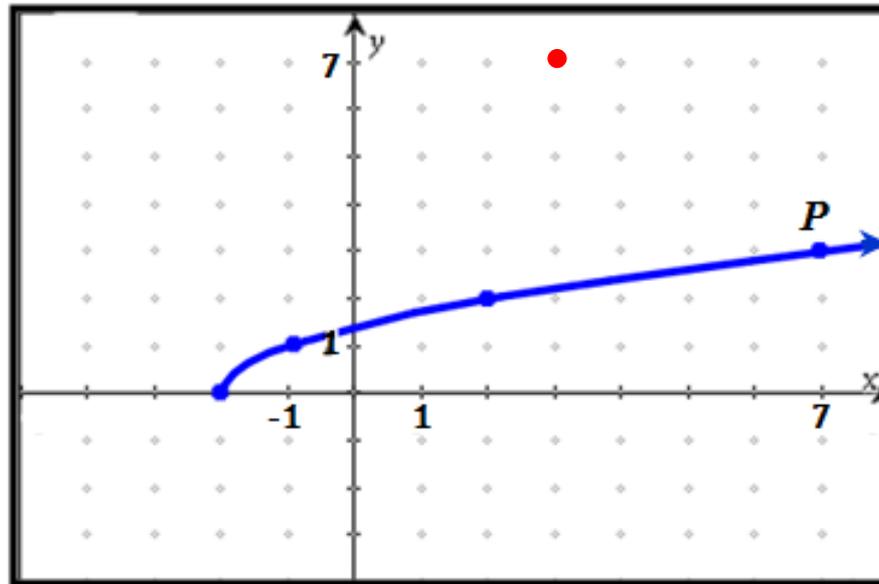
As x approaches negative infinity, which of the following describes the end behavior of $f(x) = -x^5 + bx^3 + c$?

- A $f(x)$ approaches c
- B $f(x)$ approaches 0
- C $f(x)$ approaches positive infinity
- D $f(x)$ approaches negative infinity

Suggested Practice for SOL All.7g

Students need additional practice determining the inverse of a function represented graphically.

Point P lies on the graph of $f(x) = \sqrt{x+2}$. Locate the image of point P that lies on the graph of $f^{-1}(x)$.



Suggested Practice for SOL All.7h

Students need additional practice finding the composition of functions when one or both functions are not linear.

a. Given $f(x) = 3x^3 - 2$ and $g(x) = 4x + 1$

What is the value of $g(f(x))$?

$$12x^3 - 7$$

b. Given $h(x) = x^2 + 1$ and $j(x) = x^2 - 3$

What is the value of $j(h(x))$?

$$x^4 + 2x^2 - 2$$

Relationships among the Zeros, Solutions, x -Intercepts, and Factors of a Function

SOL AII.8

The student will investigate and describe the **relationships among solutions of an equation, zeros of a function, x -intercepts of a graph, and factors of a polynomial expression.**

Suggested Practice for SOL AII.8

Students need additional practice making the connection between a zero, a solution, and the x -intercept.

If 5 is a zero of the polynomial function h , which statement must be true?

- A The y -intercept for the graph of $h(x)$ is $(0, 5)$.
- B The solution of $h(x) = 0$ is $x = -5$.
- C A factor of $h(x)$ is $(x + 5)$.
- D A factor of $h(x)$ is $(x - 5)$.

Making Predictions with Curves of Best Fit

SOL AII.9

The student will collect and analyze data, **determine the equation of the curve of best fit, make predictions,** and solve real-world problems, using mathematical models.

Mathematical models will include polynomial, exponential, and logarithmic functions.

Suggested Practice for SOL AII.9

Students need additional practice finding the curve of best fit for a set of data and making predictions using this curve.

- a. Which type of equation would best model the data in this table?

x	y
30	2
60	4
90	8
120	16
150	32
180	64

- A Exponential** B Linear C Logarithmic D Quadratic

- b. Using the equation of best fit from the data in the table, what would be the value of y if $x = 300$? **1024**

Solving Problems Involving Variation

SOL AII.10

The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

Suggested Practice for SOL AII.10

Students need additional practice finding the constant of proportionality involving a combination of direct and inverse variations.

If y varies directly with the square of x and inversely with the cube root of t , what is the constant of proportionality if $x = 4$, $y = 3$, and $t = 8$?

$$y = \frac{kx^2}{\sqrt[3]{t}}$$

$$3 = \frac{k(4)^2}{\sqrt[3]{8}}$$

$$\frac{3}{8} = k$$

Suggested Practice for SOL AII.10

Students need additional practice finding the constant of proportionality and solving real-world problems involving a combination of direct and inverse variations.

Assume that wind resistance varies jointly as an object's surface area and velocity. If a ball with a surface area of 25 square feet traveling at a velocity of 40 miles per hour experiences a wind resistance of 225 Newtons, what velocity must a ball with 40 square feet of surface area have in order to experience a wind resistance of 270 Newtons?

$$R = kAV$$

Where:

R = wind resistance (Newtons)

A = surface area (square feet)

V = velocity (miles per hour)

Students must calculate the constant of proportionality to find the answer. The constant of proportionality (k) = 0.225.

$$270 = 0.225(40)V$$

$$30 \text{ miles per hour} = V$$



Applying Properties of a Normal Distribution to Solve Problems

SOL AII.11

The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

Suggested Practice for SOL All.11

Students need additional practice in recognizing the properties of a normal distribution.

Which description of a normal distribution is most likely NOT true?

- A Approximately 99.7% of the data will fall within three standard deviations of the mean.
- B Approximately 95% of the data will fall within two standard deviations of the mean.
- C Approximately 68% of the data will fall within one standard deviation of the mean.
- D Approximately 34% of the data will fall within one standard deviation of the mean.

Suggested Practice for SOL AII.11

Students need additional practice using properties of the normal distribution curve to find the probability of an event, the percent of data that falls within a specified interval, and the number of expected values that fall within a specified interval.

- a. A normally distributed data set has a mean of 0 and a standard deviation of 0.75. What percent of the data would be expected to be between -1.5 and 1.5? **95%**

- b. The scores of a college history test were normally distributed with a mean of 75 and a standard deviation of 6. What is the probability of a student's score being an 80 or lower? **80%**

Suggested Practice for SOL AII.11

Bayside Elementary School is visiting a local amusement park. One of the amusement park's attractions requires that children must be at least 44 inches tall to ride. The heights of children at Bayside Elementary are normally distributed with a mean of 43 inches and a standard deviation of 3.4 inches. What is the probability rounded to the nearest tenth that a child selected at random does NOT meet the height requirement for the amusement park attraction?

Approximately 61.6%

Practice Items

This concludes the student performance information for the spring 2014 Algebra II SOL test.

Additionally, test preparation practice items for Algebra II can be found on the Virginia Department of Education Web site at:

http://www.doe.virginia.gov/testing/sol/practice_items/index.shtml#math

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