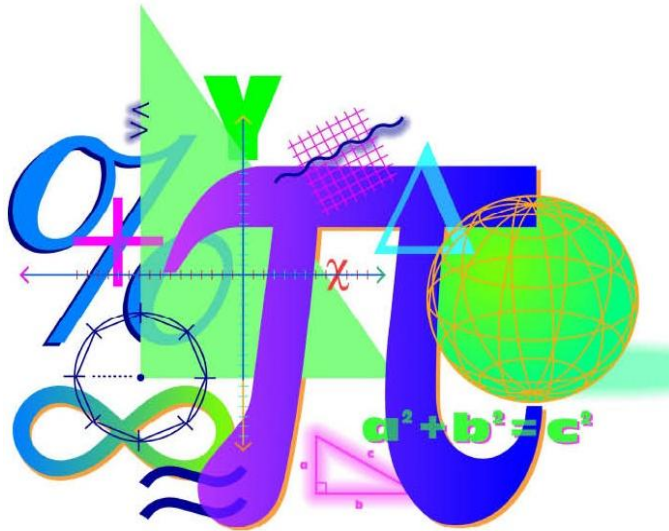


# Spring 2012 Student Performance Analysis



## Algebra II Standards of Learning

Presentation may be paused and resumed  
using the arrow keys or the mouse.

# Simplify Rational Expressions

## SOL AII.1

The student, given rational, radical, or polynomial expressions, will

- a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
- b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
- c) write radical expressions as expressions containing rational exponents and vice versa; and
- d) factor polynomials completely.

# Suggested Practice for SOL AII.1

Students need additional practice simplifying algebraic expressions, in particular, when the process includes factoring the numerator and/or denominator.

**Simplify each expression:**

$$\text{a) } \frac{x^2 + x - 6}{x^2 + 4x + 3} \cdot \frac{3x}{x^2 - x - 2}$$

$$\text{c) } \frac{2x + 1}{x - 1} - \frac{2x - 1}{x + 1}$$

$$\text{b) } \frac{15x^2 - 12x}{36x + 45} \div \frac{15x^2 + 3x - 12}{24x + 30}$$

$$\text{d) } \frac{12 - 3x^2}{x^2 - x - 6} - \frac{3x + 4}{x - 3}$$

$$\text{a) } \frac{3x}{(x + 1)^2}$$

$$\text{b) } \frac{2x}{3(x + 1)}$$

$$\text{c) } \frac{6x}{(x - 1)(x + 1)}$$

$$\text{d) } \frac{-2(3x - 1)}{(x - 3)}$$



# Suggested Practice for SOL AII.1

Students need additional practice factoring polynomials completely.

Simplify each expression:

a)  $24x^3 - 81$

$$3(2x - 3)(4x^2 + 6x + 9)$$

b)  $125x^3 + 729y^3$

$$(5x + 9y)(25x^2 - 45xy + 81y^2)$$

c)  $225x^6 - 81y^{10}$

$$9(5x^3 - 3y^5)(5x^3 + 3y^5)$$

d)  $3x^2y + 9xy - 54y$

$$3y(x + 6)(x - 3)$$

# Identify Field Properties

## SOL AII.3

The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of  $i$ , and **identify field properties that are valid for the complex numbers.**

## Suggested Practice for SOL AII.3

Students need additional practice identifying the field properties that are valid for complex numbers.

Identify the property used in each of the following:

- a)  $2i \cdot 1 = 2i$       **Identity Property of Multiplication**
- b)  $2i + 5i = (2 + 5)i$       **Distributive Property**
- c)  $2i + (5i + 9) = (2i + 5i) + 9$       **Associative Property of Addition**
- d) If  $2i + 5i = y$ , then  $y = 2i + 5i$       **Symmetric Property**
- e) If  $2i - 4 = x + 3$ , and  $x + 3 = y$ , then  $2i - 4 = y$   
**Transitive Property**

# Solving Quadratic Equations

## SOL AII.4

The student will solve, algebraically and graphically,

- a) absolute value equations and inequalities;
- b) quadratic equations over the set of complex numbers;
- c) equations containing rational algebraic expressions; and
- d) equations containing radical expressions.

## Suggested Practice for SOL AII.4

Students need additional practice solving quadratics over the set of complex numbers.

Solve these quadratic equations, and name both roots:

$$\text{a) } 3x^2 - 2x + 1 = 0 \quad \left\{ \frac{1 - i\sqrt{2}}{3}, \frac{1 + i\sqrt{2}}{3} \right\}$$

$$\text{b) } 6x^2 = x - 24 \quad x = \frac{1 \pm 5i\sqrt{23}}{12}$$

$$\text{c) } 3x(3x + 2) = 3x - 4 \quad x = \frac{-1}{6} \pm i\frac{\sqrt{15}}{6}$$



# Analyzing Functions

## SOL AII.7

The student will investigate and analyze functions algebraically and graphically. Key concepts include

- a) **domain and range**, including limited and discontinuous domains and ranges;
- b) **zeros**;
- c) **x- and y-intercepts**;
- d) **intervals in which a function is increasing or decreasing**;
- e) **asymptotes**;
- f) **end behavior**;
- g) **inverse of a function**; and
- h) **composition of multiple functions**.

## Suggested Practice for SOL AII.7

Students need additional practice determining domain and range, finding asymptotes, and determining end behavior.

Given the function:

$$f(x) = \frac{x^2 + 3x + 1}{4x^2 - 9}$$

a) What is the domain and range of this function?

The domain is all real numbers except  $\frac{3}{2}$  and  $-\frac{3}{2}$ , and the range is all real numbers.

b) What are the horizontal and vertical asymptotes of this function?

The vertical asymptotes are  $x = \frac{3}{2}$  and  $x = -\frac{3}{2}$ . The horizontal asymptote is  $y = \frac{1}{4}$ .

c) As  $x$  approaches positive infinity, what is the end behavior of this function? As  $x$  approaches positive infinity,  $f(x)$  approaches  $\frac{1}{4}$ .



## Suggested Practice for SOL AII.7

Students need additional practice determining domain and range, finding asymptotes, and determining end behavior.

Given the function:

$$f(x) = \log(x + 2)$$

a) What is the domain of this function?

$$\text{Domain: } \{x | x > -2\}$$

b) What is the vertical asymptote of this function?

$$x = -2$$

## Suggested Practice for SOL AII.7

Students need additional practice determining domain and range, finding asymptotes, and determining end behavior.

Given the function:

$$f(x) = 3 \cdot 2^{x-4}$$

a) What are the asymptotes of this function?

One horizontal asymptote at  $y = 0$

b) What is the domain and range of this function?

Domain:  $\{x | -\infty < x < \infty\}$  Range:  $\{y | y > 0\}$

c) What is the x-intercept and y-intercept of this function?

There is no  $x$  – intercept. The  $y$  – intercept is  $(0, \frac{3}{16})$ .

d) As  $x$  approaches negative infinity, what is the end behavior of this function?

As  $x$  approaches negative infinity,  $f(x)$  approaches 0.



## Suggested Practice for SOL AII.7

Students need additional practice determining domain and range, finding asymptotes, and determining end behavior.

Given the function:

$$f(x) = -(2x^2 - 4) + 2$$

a) What is the domain and range of this function?

Domain:  $\{x | -\infty < x < \infty\}$  Range:  $\{y | y \leq 6\}$

b) What are the horizontal and vertical asymptotes of this function? There are no horizontal or vertical asymptotes.

c) As  $x$  approaches positive infinity, what is the end behavior of this function?

As  $x$  approaches positive infinity,  $f(x)$  approaches negative infinity.



## Suggested Practice for SOL AII.7

Students need additional practice finding asymptotes.

For each function, determine all asymptotes.

a)  $f(x) = \frac{x^2 - 2x - 3}{x^2 - 9}$

c)  $f(x) = 2^{(x+7)}$

One vertical asymptote at  $x = -3$   
and one horizontal asymptote at  $y = 1$

One horizontal asymptote at  $y = 0$

b)  $f(x) = \sqrt[3]{x - 9}$

There are no asymptotes.

## Suggested Practice for SOL AII.7

Students need additional practice finding asymptotes.

Select the function(s) that have an asymptote at  $x = 3$ .

$$f(x) = \frac{2x^2 + 3}{4x - 12}$$

$$f(x) = 2^x - 3$$

$$f(x) = |x - 3|$$

$$f(x) = \log(x - 3)$$

## Suggested Practice for SOL AII.7

Students need additional practice finding the inverse of a function.

Find the inverse of each function:

a)  $f(x) = 8x^3 - 6$        $f^{-1}(x) = \frac{\sqrt[3]{x+6}}{2}$

b)  $f(x) = 16x^2 + 1$ , where  $x \geq 0$        $f^{-1}(x) = \frac{\sqrt{x-1}}{4}$  where  $x \geq 1$

c)  $f(x) = \frac{-5}{7}x + 2$        $f^{-1}(x) = \frac{-7x+14}{5}$

d)  $f(x) = \sqrt{x+3}$  where  $x \geq -3$   
 $f^{-1}(x) = x^2 - 3$  where  $x \geq 0$



# Describing the Solutions to a Polynomial Equation

## SOL AII.8

The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

## Suggested Practice for SOL AII.8

Students need additional practice describing the roots of an equation.

Choose one of these phrases to describe the roots of the equations shown:

Exactly 2 distinct real roots

Exactly 2 distinct imaginary roots

Exactly 3 distinct real roots

Exactly 1 real root and 2 distinct imaginary roots

a)  $x^2 - 1 = 0$

Exactly 2 distinct real roots

b)  $x^2 + 1 = 0$

Exactly 2 distinct imaginary roots

c)  $x^3 + x^2 - 2x = 0$

Exactly 3 distinct real roots

# Determining Curve of Best Fit

## SOL AII.9

The student will collect and analyze data, **determine the equation of the curve of best fit, make predictions,** and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential and logarithmic functions.

## Suggested Practice for SOL AII.9

Students need additional practice identifying the equation for the curve of best fit and making predictions using the curve of best fit.

A data set is displayed in this table. Using the exponential curve of best, what is the value of  $y$ , rounded to the nearest hundredth, when  $x = 5$ ?

|     |       |      |     |   |
|-----|-------|------|-----|---|
| $x$ | -3    | -2   | -1  | 0 |
| $y$ | 3.375 | 2.25 | 1.5 | 1 |

$$y = 0.13$$

# Solving Problems Involving Variation

## **SOL AII.10**

The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

## Suggested Practice for SOL AII.10

Students need additional practice identifying a variation equation that models a situation, and solving problems involving variation.

Given:  $x$  varies jointly with  $w$  and  $y$ .

If  $x = 20$  when  $w = \frac{2}{5}$  and  $y = 10$ , what is the value of  $x$  when

$w = 8$  and  $y = \frac{3}{15}$ ?

$$x = 8$$

## Suggested Practice for SOL AII.10

Which equations represent this situation?

A car's stopping distance,  $d$ , varies directly with the speed it travels,  $s$ , and inversely with the friction value of the road surface,  $f$ .

$$df = ks$$

$$dk = \frac{s}{f}$$

$$d = ksf$$

$$d = \frac{ks}{f}$$

$$ds = kf$$

$$k = \frac{df}{s}$$

## Suggested Practice for SOL AII.10

The amount of time required to stack boxes varies directly with the number of boxes and inversely with the number of people who are stacking them. If 2 people can stack 60 boxes in 10 minutes, how many minutes will be required for 6 people to stack 120 boxes?

6 minutes and 40 seconds



# Using Properties of the Normal Curve

## **SOL AII.11**

The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

## Suggested Practice for SOL AII.11

Students need additional practice using mean and standard deviation to find the area under a normal curve and apply properties of the normal distribution to solve problems.

The running times for a group of 200 runners to complete a one mile run are normally distributed with a mean of 6.5 minutes and a standard deviation of 1.5 minutes.

- a) Approximately how many of the runners have a time greater than 8 minutes?
- b) What percentage, rounded to the nearest tenth, of these runners can complete this run in less than 3.5 minutes?

32

2.3%

# Distinguishing Between Permutations and Combinations

**SOL AII.12**

The student will **compute and distinguish between permutations and combinations** and use technology for applications.

## Suggested Practice for SOL AII.12

Decide whether each of these can be answered using a permutation or a combination and then determine the answer.

a) Twenty horses competed in a race. In how many ways could the horses have finished in first place through third place?

6,840

b) A 10 person student council will be selected from 18 students at a school. How many possibilities are there for this student council?

43,758

# Practice Items

This concludes the student performance information for the spring 2012 Algebra II SOL test.

Additionally, test preparation practice items for Algebra II can be found on the Virginia Department of Education Web site at:

[http://www.doe.virginia.gov/testing/sol/practice\\_items/index.shtml#math](http://www.doe.virginia.gov/testing/sol/practice_items/index.shtml#math)