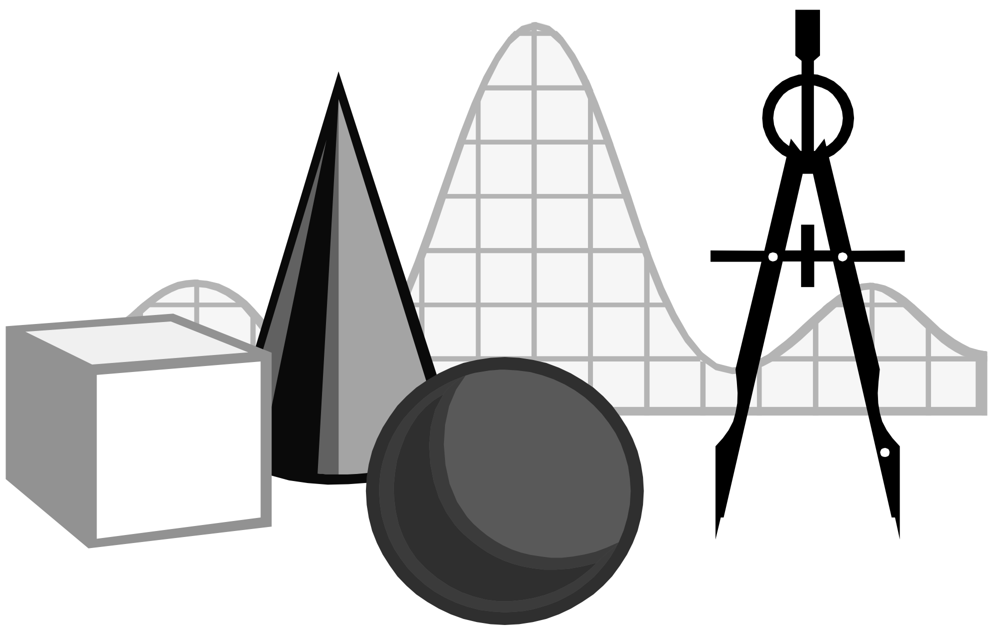
MATHEMATICS STANDARDS OF LEARNING ENHANCED SCOPE AND SEQUENCE

Algebra I



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Richmond, Virginia
2004

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Introduction

The *Mathematics Standards of Learning Enhanced Scope and Sequence* is a resource intended to help teachers align their classroom instruction with the Mathematics Standards of Learning that were adopted by the Board of Education in October 2001. The Mathematics Enhanced Scope and Sequence is organized by topics from the original Scope and Sequence document and includes the content of the Standards of Learning and the essential knowledge and skills from the Curriculum Framework. In addition, the Enhanced Scope and Sequence provides teachers with sample lesson plans that are aligned with the essential knowledge and skills in the Curriculum Framework.

School divisions and teachers can use the Enhanced Scope and Sequence as a resource for developing sound curricular and instructional programs. These materials are intended as examples of how the knowledge and skills might be presented to students in a sequence of lessons that has been aligned with the Standards of Learning. Teachers who use the Enhanced Scope and Sequence should correlate the essential knowledge and skills with available instructional resources as noted in the materials and determine the pacing of instruction as appropriate. This resource is not a complete curriculum and is neither required nor prescriptive, but it can be a valuable instructional tool.

The Enhanced Scope and Sequence contains the following:

- Units organized by topics from the original Mathematics Scope and Sequence
- Essential knowledge and skills from the Mathematics Standards of Learning Curriculum Framework
- Related Standards of Learning
- Sample lesson plans containing
  - Instructional activities
  - Sample assessments
  - Follow-up/extensions
  - Related resources
  - Related released SOL test items.

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Organizing Topic Using Algebraic Topics

Standards of Learning

A.2 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.

A.3 The student will justify steps used in simplifying expressions and solving equations and inequalities. Justifications will include the use of concrete objects; pictorial representations; and the properties of real numbers, equality, and inequality.

A.17 The student will compare and contrast multiple one-variable data sets, using statistical techniques that include measures of central tendency, range, and box-and-whisker graphs.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Organize a set of data.	____________________
• Calculate the mean, median, mode, and range of a set of data.	____________________
• Use the calculated values for mean, median, mode, and range, stem-and-leaf, and box-and-whisker plots to compare and contrast two sets of data.	____________________
• Describe the relationships between and among data sets.	____________________
• Use algebraic expressions to describe mathematical relationships.	____________________
• Evaluate algebraic expressions for a given replacement set, using order of operations.	____________________
• Create and interpret pictorial representations for simplifying expressions.	____________________
• Justify the steps in evaluating algebraic expressions, using the commutative, associative, and distributive properties.	____________________

Measures of Central Tendency

Organizing topic	Using Algebraic Topics
Overview	In contrast to computing measures of central tendency when given data, students must use the measures as given to determine a possible data set.
Related Standard of Learning	A.17

Objectives

- The student will organize a set of data
- The student will calculate the mean, median, mode, and range of a set of data.
- The student will describe the relationships between and among data sets.

Materials needed

- A copy of the "Measures of Central Tendency" handout for each student

Instructional activity

1. Distribute the handouts to the students.
2. Assign a given number of yard sale items for the sale. (Note: Because you may assign any number of yard sale items, this activity may be repeated a number of times with different numbers of items assigned.)
3. Have the students work alone or in pairs. Make sure they recognize the need to work backwards through the problem-solving process for measures of central tendency.
4. Give each pair of students a different scenario, and bring the group together at the end of the class to discuss each scenario; or set the room up in stations, and let students rotate through all three scenarios.

Sample assessment

- Have students write a journal entry about the activity.
- Compare and contrast the three scenarios by describing the role of the mean, median, mode, and/or range of the data in determining a set of prices to fit the scenario(s).

Follow-up/extension

- Have the students consider one of the scenarios. If the number of items available at the yard sale doubled, how would the set of prices that fits the scenario be affected?

Homework

- Have the students consider the other two scenarios. If the number of items available at the yard sale doubled, how would the set of prices that fits each scenario be affected? Then have them consider the situation in which someone donates a single high-end (or low-end) item. How would the set of prices be affected?

Measures of Central Tendency

Scenario 1: You have been given the dubious honor of chairing the annual CMS yard sale. One of your duties is to advertise in the *Free Lance Star.* The ad reads:

CMS YARD SALE
Items range in price from 20 cents to $4.80.
Median price of items is $2.10.
Mean price of items is $2.10.

1. Explain what this means to a potential customer.
2. Give an example of a set of prices that fits this scenario.

Scenario 2: You have been given the dubious honor of chairing the annual CMS yard sale. One of your duties is to advertise in the *Free Lance Star.* The ad reads:

CMS YARD SALE
Items range in price from 20 cents to $4.80.
Median price of items is $2.10.
Mean price of items is $2.20.

1. Explain what this means to a potential customer.
2. Give an example of a set of prices that fits this scenario.

Scenario 3: You have been given the dubious honor of chairing the annual CMS yard sale. One of your duties is to advertise in the *Free Lance Star.* The ad reads:

CMS YARD SALE
Items range in price from 20 cents to $4.80.
Median price of items is $2.10.
Mean price of items is $1.80.

1. Explain what this means to a potential customer.
2. Give an example of a set of prices that fits this scenario.

Box-and-Whisker Plots

Organizing topic	Using Algebraic Topics
Overview	The activity allows students to review box-and-whisker-plot basics and graphing calculator skills.
Related Standard of Learning	A.17

Objective

- The student will construct box-and-whisker plots, using paper and pencil and using the graphing calculator.

Prerequisite understandings/knowledge/skills

- The student should know how to enter data into LIST function of the graphing calculator.

Materials needed

- Graphing calculators and handouts for each student
- Graphing calculator and overhead view screen
- Overhead projector
- A copy of each of the three handouts for each student

Instructional activity

Note: Students may work alone or in pairs.

Sample assessment

- Included in activity handouts

Follow-up/extension

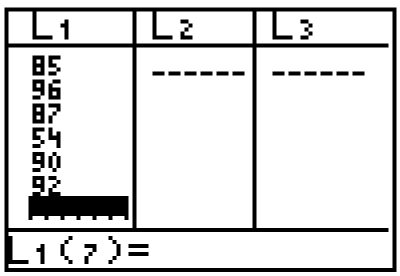
- Vashon-Maury Island Activity

Box-and-Whisker Plots 1

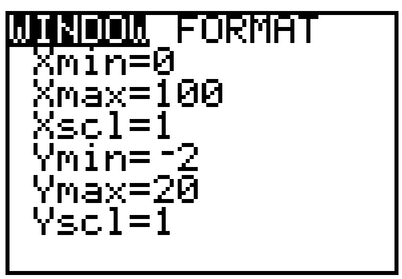
1. Manually make a box-and-whiskers plot of these scores on a Statistics exam:

   85 96 87 54 90 92

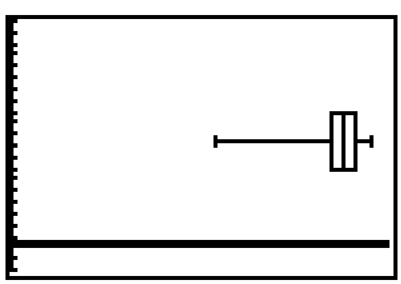
2. Now enter the scores into L_1:



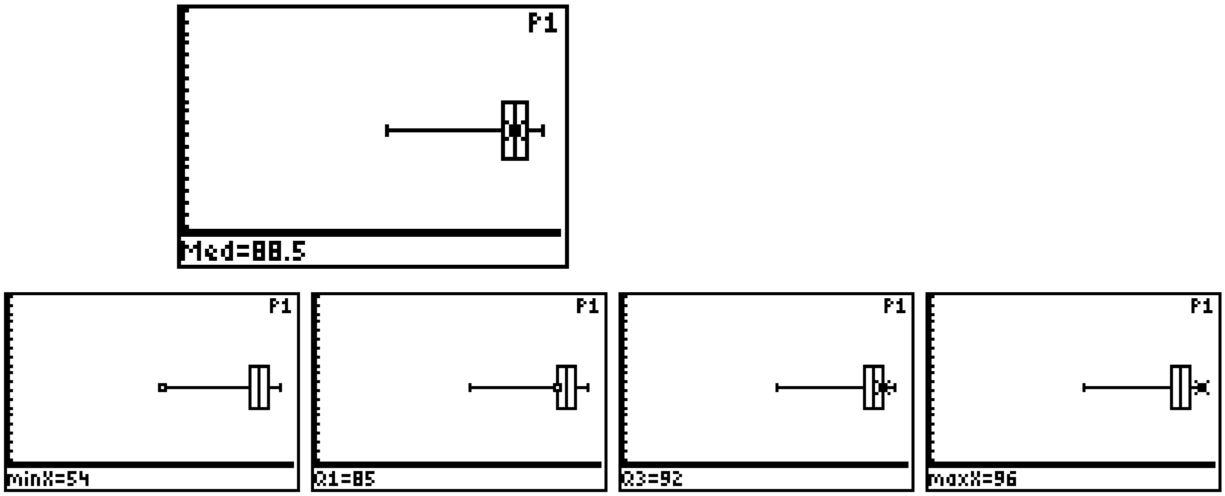
3. Choose an appropriate Window:



4. Results as follows:



5. Tracing:



6. Try different windows. See what happens.

Box-and-Whisker Plots 2

Scores on the first Physics test are as follows:

Class 1

Student	A	B	C	D	E	F	G	H	I	J
Score	55	64	83	92	100	77	86	95	80	98

Class 2

Student	A	B	C	D	E	F	G	H	I
Score	52	79	71	100	100	76	100	78	76

1. Make a box-and-whisker plot of each set of data on the same graphics screen. Note: You will have to use two different LISTS and two different STAT PLOTS.
2. Sketch each box-and-whisker plot, identifying the Min, Q1, Med, Q2, and Max of each.
3. Which class did better?
4. What is the average (mean or median?) score for each class?
5. Note: When you prompt the calculator to do one-variable statistics, you *must* follow the prompt with the "place" you stored the statistics, i.e., $L_{???}$.
6. Does this change your opinion about which class did better?
7. When statistics are "quoted," what words can be deceiving?
8. Can the wording affect how one perceives the overall picture of "which class did better?" If so, in what way?

Box-and-Whisker Plots 3

An experiment found a significant difference between men and women pertaining to their ability to identify unseen objects held in their left hands. The left hand is controlled by the right side of the brain, while the right hand is controlled by the left side of the brain. The test involved 20 small objects, which participants were not allowed to see. First they held 10 of the objects, one by one, in their left hands and guessed what they were. Then they held the other 10 objects, one by one, in their right hands and guessed what they were.

Correct Guesses

Women Left	Women Right	Men Left	Men Right
8	4	7	10
9	1	8	6
10	8	7	10
6	9	5	10
10	6	7	7
8	10	8	9
9	4	10	10
7	9	4	8
9	8	10	10
10	9	8	9

Make a box-and-whisker plot that will allow you to compare the data.

Vashon-Maury Island Soil Study

Organizing topic Using Algebraic Topics

Overview The Vashon-Maury Island soil study provides students with a real application of box-and-whisker plots in data analysis.

Related Standard of Learning A.17

Objective

- Using box-and-whisker plots generated during the Vashon-Maury Island soil study, the student will draw conclusions from the data.

Materials needed

- "Vashon-Maury Island Soil Study" handouts for each student (Sections 1.0, 6.0, and 6.3 of the Final Report, two maps, and two box-and-whisker plots)
- A transparency of each of the two maps (optional)

Instructional activity

Note: This activity is an excellent opportunity to address the question, "When am I ever going to use this?" As a result of this study, Vashon-Maury Island is now an EPA Superfund cleanup site.

1. Distribute copies of the handouts to each student. Have students read Section 1.0 of the Final Report. The maps will be helpful in following the article. You may wish to discuss the implications of lead contamination with students.
2. The two box-and-whisker plots address different elements: arsenic and lead. Divide students into small groups. Assign each group one of the box-and-whisker plots. Using the map and box-and-whisker plot, each group must determine where Zones 1, 2, and 3 are.
3. Have each group include a complete description of their discussions and findings. The reasoning/justification process is critical and must be detailed.

Sample assessment

- Have each group present its conclusions to the whole class.

Homework

- Have the students read Sections 6.0 and 6.3 of the Final Report. Does the information in Sections 6.0 and 6.3 support your group's conclusions? Why or why not?

Final Report of the 1999–2000 Vashon-Maury Island Soil Study

SECTION 1.0

Project description and objectives

In 1999 and 2000, Public Health – Seattle & King County (PHSKC) performed the first comprehensive survey of contamination by arsenic, lead, and cadmium in surficial soils on Vashon-Maury Island, Washington. This study also included an initial "pilot scale" evaluation of surficial soils in shoreline areas of the King County mainland, east of Vashon-Maury Island. The results of soil sampling and analysis in both areas are presented and evaluated in this report.

In this study, soils were sampled to a depth of 6 inches below the forest duff layer. The term "surficial" rather than the simpler term "surface" soils is used to emphasize that there can be important differences in potential exposures (e.g., frequency of contact) between soils within the top inch or so versus soils at depths of, for example, 4 to 6 inches. "Surficial" is in this sense used to refer to true surface soils as well as near-surface soils.

Previous studies

Since the early 1970s, more than a dozen studies have determined arsenic concentrations in surficial soils in selected areas of Vashon-Maury Island (see Sections 2.2 and 7.0, and the references in Section 9.0). Analyses in some of the studies included lead, with analyses of additional metals (e.g., cadmium, selenium, or antimony) occasionally reported. Those studies were motivated by concerns over the magnitude, extent, and fate of contamination resulting from operation of the former ASARCO Tacoma Smelter, located at Ruston on the shoreline of Commencement Bay, a few miles south/southwest of Vashon-Maury Island (see PSAPCA 1981a and 1981b for a general review of Tacoma Smelter operations and emissions). The Tacoma Smelter operated for over 90 years, closing smelting operations in 1985 and arsenic processing operations in 1986. For many years, the Tacoma Smelter was the sole domestic supplier of arsenic for the United States. Arsenic has generally been considered the best tracer element for evaluating smelter impacts (see, for example, Crecelius et al. 1974 and PSAPCA 1981a and 1981b), and it has also been the focus of concerns for potential human health risks from smelter emissions. The smelter site and surrounding areas (within an approximate one-mile radius) are subject to ongoing cleanup actions under EPA's Superfund program.

Soil studies over the past 30 years, performed both during smelter operations and after closure, have documented elevated levels of arsenic, lead, and other metals in surficial soils on Vashon-Maury Island. However, the previous studies in aggregate do not provide a comprehensive portrait of current soil contamination levels. Separate studies used different protocols for collection and laboratory analysis of soil samples, making comparisons among studies more difficult. The studies also targeted different types of land uses, such as residential yards and garden areas, parks and playgrounds, and relatively undeveloped and undisturbed areas. The degree of soil disturbance has been shown to be an important factor affecting the residual concentrations of air-deposited contaminants in surficial soils; the marked differences in past soil-disturbing activities among sampled land types is expected to contribute significantly to variability in results. The representativeness of studies performed up to 30 years ago for characterizing current soil contamination levels can be questioned, especially with respect to current depth profiles for contamination. Finally, the number (density) and spatial pattern of prior sampling locations are both limited; most of the previous studies have focused on Maury Island and south Vashon Island, with relatively few samples collected on north Vashon Island. Evaluation of all of these factors supported the conclusion that previous studies, considered cumulatively, could not provide a comprehensive description of current soil contamination levels on Vashon-Maury Island. Previous study results were used in developing the study design for the current PHSKC study (see Section 2.2).

In addition to soil sampling and analysis studies, several other types of investigations help to characterize the likely extent of soil contamination in the area downwind of the former Tacoma Smelter. Those additional studies included deposition modeling, plume tracking (opacity) studies, an extended series of precipitation chemistry monitoring studies, sediment core chemistry monitoring, vegetation sampling, and bee biomonitoring (see references listed in Section 9.0). Several of these studies (e.g., deposition modeling, precipitation chemistry studies, and bee biomonitoring) have provided "contour" mapping of potential impact areas. The resulting predicted pattern of soil contamination levels on Vashon-Maury Island had not been fully validated by soil sampling and analysis data prior to this PHSKC study. The "contouring" study results also indicated that the spatial scale for measurable smelter impacts may be on the scale of tens of miles, extending onto the mainland areas of King County.

Very few soil samples from such mainland areas are available from previous studies; nevertheless, some elevated soil arsenic results have been reported. The study of Vashon-Maury Island soils was extended to include an initial "pilot scale" evaluation of soils in mainland King County shoreline areas east of Vashon-Maury Island.

Project description and objectives

The ongoing Superfund cleanup of areas surrounding the Tacoma Smelter that were affected by air deposition from smelter emissions does not include any areas on Vashon-Maury Island (see USEPA, Region 10 1993). Residents of Vashon-Maury Island were active participants in several earlier processes involving Tacoma Smelter operations and impacts, including the PSAPCA Environmental Impact Statement process under SEPA and EPA's proposed rules for arsenic under the National Emissions Standards for Hazardous Air Pollutants (NESHAPS) provisions of the Clean Air Act. After EPA issued its Superfund Record of Decision for cleanup of residential areas in Ruston and North Tacoma, in 1993, soil contamination issues arose for a number of years primarily in connection with real estate transactions. In 1998 and 1999, a proposal to expand gravel mining operations at the Lone Star (now Glacier Northwest) Maury Island gravel mine (see King County DDES 1999) resulted in two independent soil studies on gravel mine property. Both studies showed significantly elevated soil arsenic concentrations, with maximum levels in relatively undisturbed areas exceeding 300 ppm. These findings resulted in heightened interest in soil contamination levels and impacts among some Vashon-Maury Island residents. In this same time frame, Ecology was also beginning to evaluate "area contamination" issues under the Model Toxics Control Act (MTCA), which represented areas of contamination far larger than the typical MTCA site. Widespread soil contamination by arsenic and other smelter-related metals was recognized by Ecology as one such "area contamination" problem.

A recently discovered study by Environment Canada comparing periods before and after smelter closure showed significant decreases in sulfate and arsenic levels in precipitation along the Canadian border coinciding with smelter closure. See D.A. Faulkner, "The Effect of a Major Emitter on the Rain Chemistry of Southwestern British Columbia — a Second Look," presented at the November 8–10, 1987 annual meeting of the Pacific Northwest International Section, Air Pollution Control Association, in Seattle, WA.

The current study was conceived in response to both community concerns specific to Vashon-Maury Island and Ecology interest in evaluating area contamination problems where threats to human health or the environment could occur over large areas. It is intended to provide a comprehensive survey of current surficial soil contamination over all areas of Vashon-Maury Island, as well as an initial evaluation of King County mainland areas to the east. Funding for the study was provided jointly by PHSKC and Ecology. PHSKC staff developed study plans (see PHSKC 1999), in collaboration with Ecology, and provided staff to perform all field sampling work. Laboratory analyses were performed by OnSite Environmental, Inc., Redmond, Washington under contract to PHSKC. The study design was developed through a multi-party Vashon-Maury Island Soils Work Group. PHSKC retained Gregory L. Glass as a consultant to work with the Soils Work Group in developing a study design; he also performed data evaluations and prepared the project report. PHSKC maintained the database of analytical results and performed data validation reviews. Ecology provided technical support and oversight of the study.

The primary objectives of the study are to document the current magnitude and large scale spatial patterns in soil contamination in relatively undisturbed King County areas downwind from the former Tacoma Smelter, including a comprehensive survey of all of Vashon-Maury Island and an initial exploration of King County mainland shoreline areas to the east. This regional-scale study provides information useful for focusing additional investigation and response actions by the agencies and the public. Several detailed data evaluation objectives were also identified and reflected in the study design:

1. to evaluate the vertical pattern of soil contaminant concentrations at different depths (depth profiles);
2. to assess the relationship (correlations) among different soil contaminants; and
3. to evaluate how soil contaminant concentrations vary within relatively small areas (variability and spatial scale)

Since all soil sampling occurred in relatively undisturbed areas, the results are believed to be biased toward upper bound contaminant concentrations. All data interpretations should take this study design feature into account. This study does not provide any information on the comparative magnitudes or patterns of contamination between relatively undisturbed and developed/disturbed properties.

The detailed study design and data evaluations are discussed in subsequent sections of this report. In brief, the study involved collection of soil samples at 0–2 inch and 2–6 inch depth intervals; analyses for arsenic, lead, and cadmium — three primary smelter-related contaminants; targeted sampling in forested areas believed to represent least disturbed soils where contaminant levels are likely to be highest; and collection of samples in an approximate grid pattern within forested areas, plus additional collection of closely spaced samples (in comparison to grid spacing) to look at local variability. In all, 436 soil samples were analyzed for arsenic and lead, and 338 for cadmium.

SECTION 6.0

Data evaluations

As described in Section 1.0, a set of data evaluation objectives was identified for this study. Each of the identified data evaluation objectives is discussed in its own subsection below, preceded by a brief discussion of the approach taken to data evaluations. The discussions that follow emphasize the primary data evaluation results; references are provided to more detailed statistical results and data plots that are included as Attachment D, for readers interested in such details.

SECTION 6.3

Spatial patterns

Arsenic and lead results were mapped using GIS software. Figures 7 and 8 show the resulting maps; spatial sampling locations are identified only at the level of the parcel sampled. Not all results (see Attachment B for a complete listing) are plotted on Figures 7 and 8; only the maximum concentration at any depth is plotted for a given sampling location. Where only one grid location at a parcel was sampled, only a single result is shown. Multiple results are shown when spatial scale sampling occurred or when more than one grid cell location within a single parcel was sampled. For example, at sampling location 113 in Zone 1, at the southern end of Maury Island, the three values of 250 ppm, 210 ppm, and 130 ppm represent the maximum arsenic concentrations regardless of depth at the three spatial scale sampling locations (i.e., B1-113-T, B1-113-TY, and B1-113-SZ). The sampling grid locations are color coded on Figures 7 and 8 to reflect ranges of maximum contaminant concentrations, making it easier to see spatial patterns in the results.

Inspection of Figures 7 and 8 reveals that substantial differences in contamination levels can occur within localized areas. This small scale spatial variability is discussed further in Section 6.6. A large scale spatial pattern is also visible; that large scale pattern is similar for arsenic, lead, and cadmium, indicating a high degree of co-occurrence and correlation among the three contaminants (discussed further in Section 6.4). The highest contaminant levels are generally found on Maury Island (Zone 1B), followed by South Vashon Island (Zone 1A) and the Mainland (Zone 2). The lowest concentrations are found on North Vashon Island (Zone 3). The degree of variability in results within zones is large enough that the distributions in these subareas are broadly overlapping rather than entirely dissimilar. Nevertheless, there is a clear and meaningful large scale spatial pattern evident in the data. (This large scale pattern would be particularly evident if, for example, the results were averaged over blocks of significant size, such as 1 to 2 square miles). As noted in Section 2.3, all mapped results are representative of relatively undisturbed areas and are very likely not generalizable to developed/ disturbed areas. Land use and the history of site disturbance are emphasized again as very important factors afffecting the current pattern (including depth of contamination) and magnitude of residual soil contamination. The best use of the maps of contamination magnitudes from this study is as likely upper bounds (or near upper bounds, because the number of samples is limited) for soil contamination across various types of land uses.

The dominant spatial pattern can be illustrated using the sampling zones to partition the data set (while recalling that the actual pattern is gradational rather than clearly associated with the chosen zone boundaries). Tables 4a, 4b, and 4c provide detailed comparisons of selected percentile values across all sampling zones, based on individual samples; Table 5 provides similar information for maximum location values. The ranking of subareas is shown clearly in these Tables. Bar charts showing the distributions for arsenic and lead in each sampling zone are included in Attachment D. Comparison of these bar charts across zones also reveals the large scale spatial pattern in the results.

Figures 9 through 10 show multiple box-and-whisker plots of arsenic and lead results (with not detected results assigned values of one-half the detection limit) for individual samples by sampling zone. These visual summaries of the statistical distributions by zones show that the median and extreme high values follow the ranking of Zones 1, 2, and 3, from highest to lowest levels; more detailed examination supports the finding that Zone 1B (Maury Island) shows generally higher contaminant levels than Zone 1A (South Vashon Island). [Note: the lead box and whisker plot for Zone 3 (see Figure 10) is likely biased high because there are comparatively fewer 2–6 inch analyses in Zone 3, and lead depth profiles typically show higher concentrations in the 0–2 inch interval. Thus, the actual differences between Zones 1 and 2 versus 3 for lead are probably understated. Additional box-and-whisker plots for arsenic and lead in the 0–2 inch samples only are provided in Attachment D, showing similar results.]

Box and whisker plots provide a visual summary of statistical data distributions. The box encloses the 25th to 75th percentiles of the data (i.e., the middle 50 percent), defining the size of the interquartile range. The median is shown as a line within this box. The straight line whiskers above and below the box extend to the values in the data set closest to, but not exceeding, 1.5 times the interquartile range above and below the 75th and 25th percentiles. Outlier values beyond the whiskers are plotted individually, with a special symbol if they are more than 3 times the interquartile range above or below the ends of the box. Multiple box and whisker plots allow data sets to be rapidly compared visually.

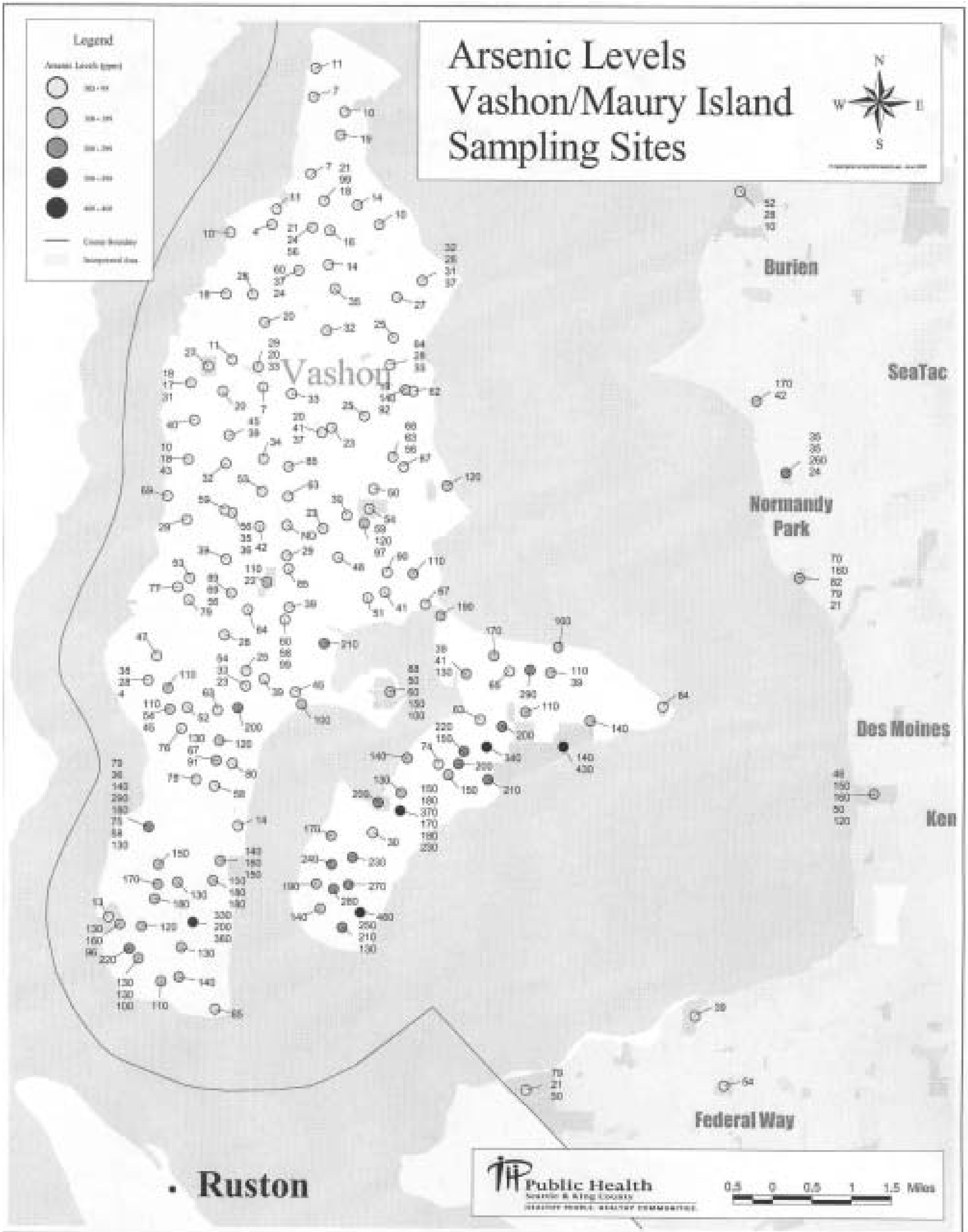
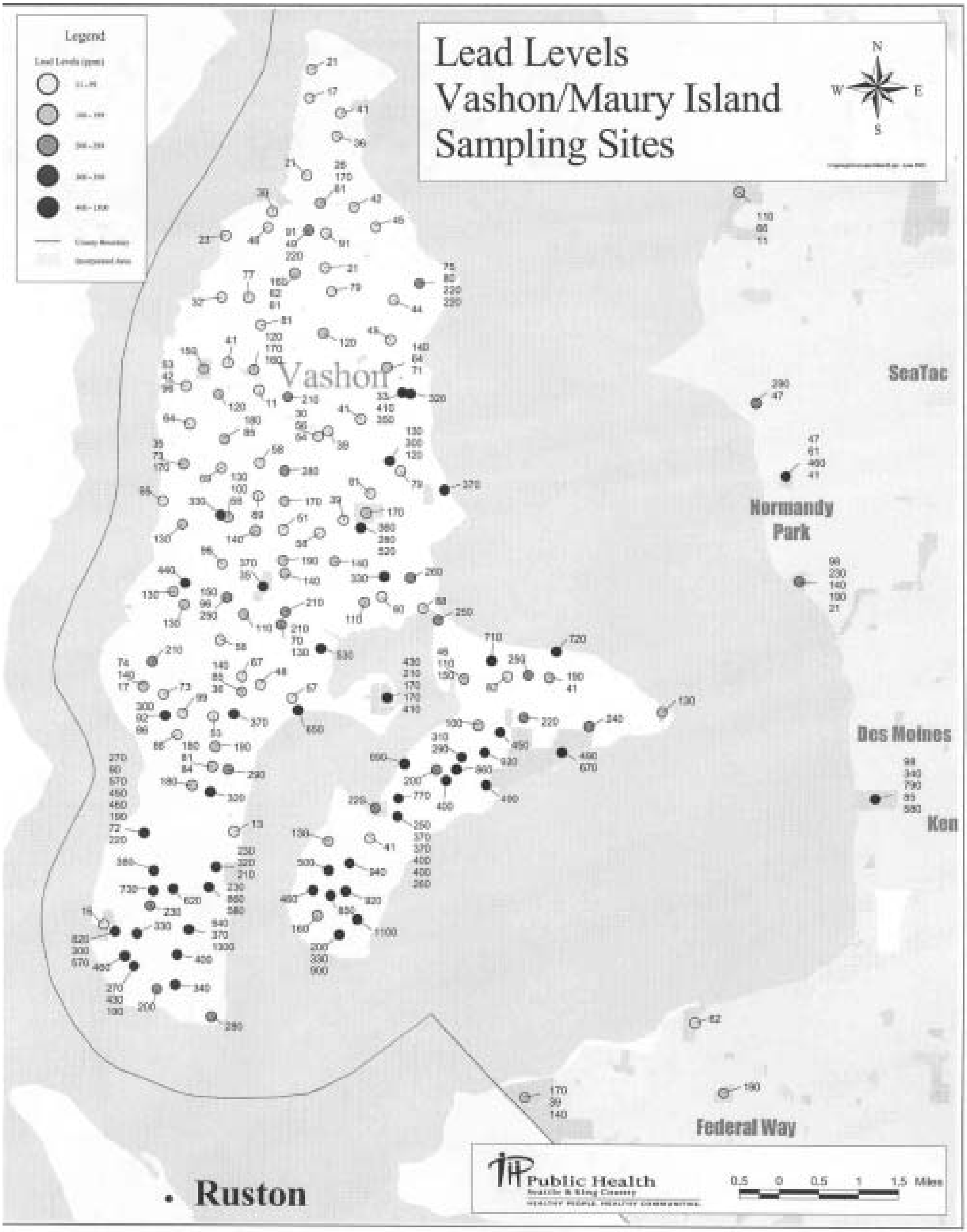

Legend
Arsenic Levels
Vashon/Maury Island
Sampling Sites
N
W
E
S
Vashon
Burien
SeaTac
Normandy
Park
Des Moines
Federal Way
Ruston
Public Health
Seattle & King County
0.5 0 0.5 1 1.5 Miles

Figure 7


Lead Levels
Vashon/Maury Island
Sampling Sites
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Legend
Vashon
SeaTac
Normandy Park
Des Moines
Federal Way
Ruston
Public Health
Seattle & King County
0.5 0 0.5 1 1.5 Miles

Figure 8

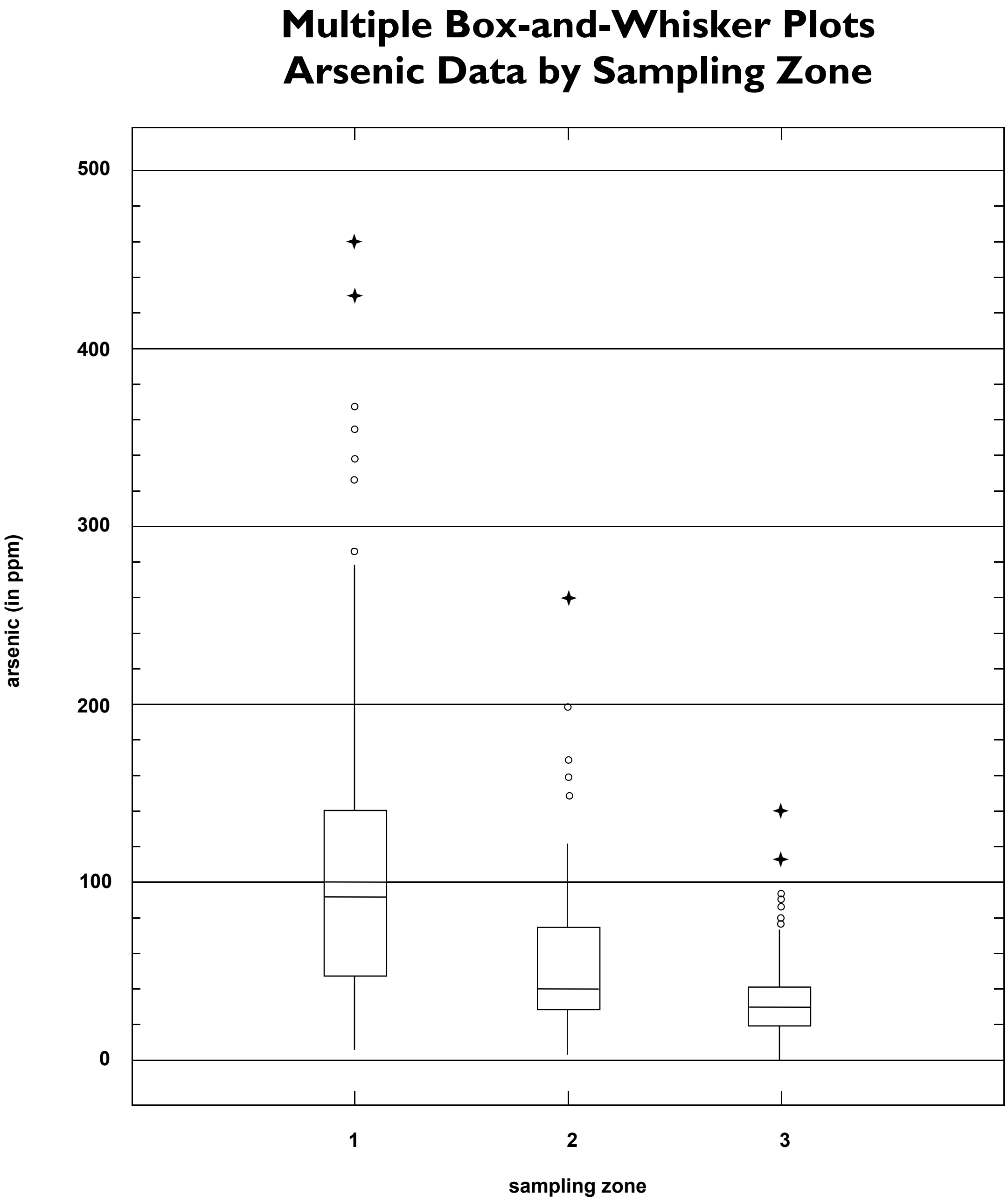

Multiple Box-and-Whisker Plots
Arsenic Data by Sampling Zone
500
400
300
200
100
0
arsenic (in ppm)
1
2
3
sampling zone

Figure 9

Multiple Box-and-Whisker Plots
Lead Data by Sampling Zone

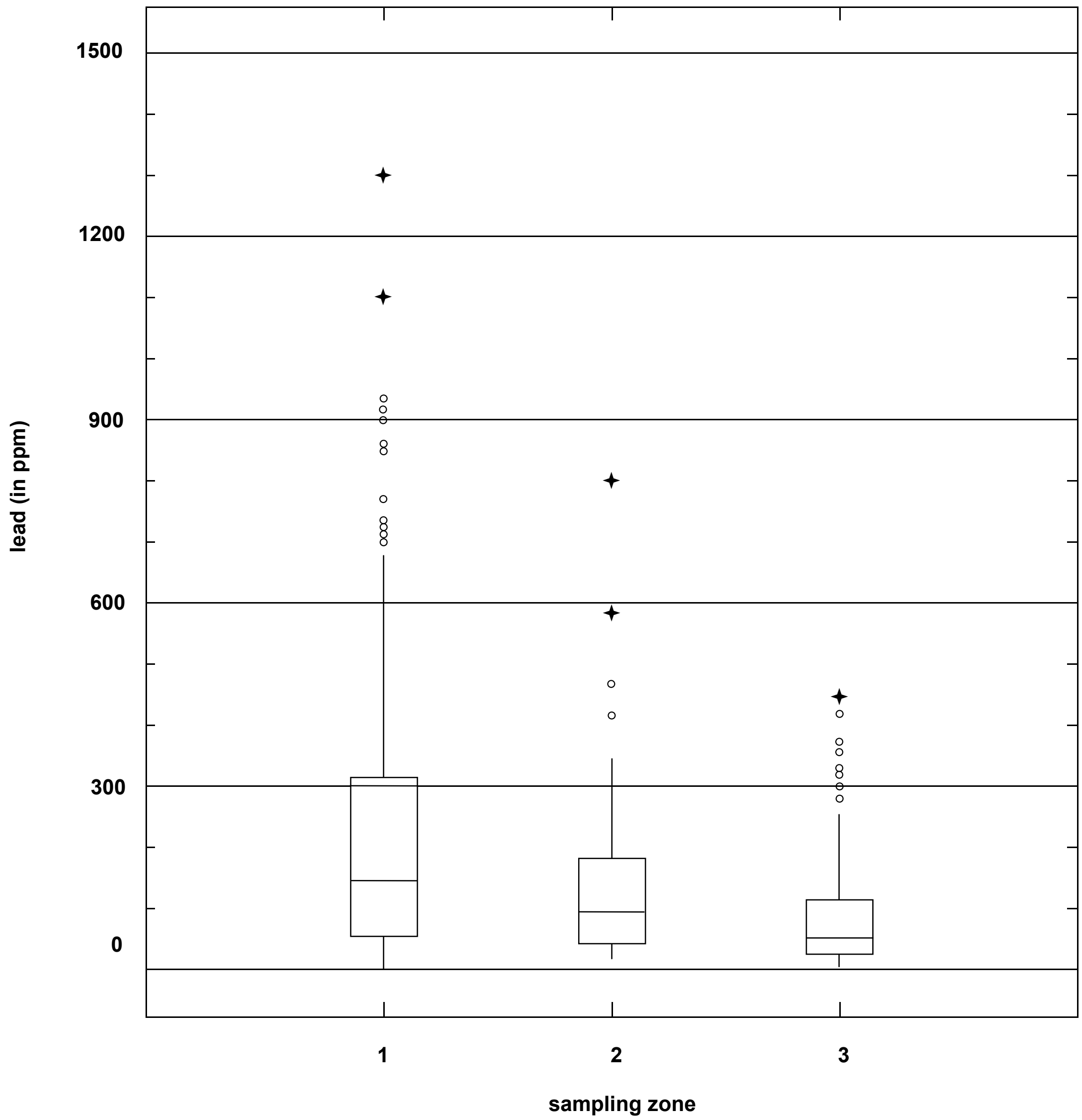


Figure 10

Traffic Jam

Organizing topic Using Algebraic Topics

Overview The activity allows students to engage in a large-movement experience, a simulation with manipulatives, and a technology application, each generating a different look at the same question.

Related Standard of Learning A.2

Objective

- The student will use algebraic expressions to describe mathematical relationships.

Prerequisite understandings/knowledge/skills

- The student should be able to recognize patterns, rethink patterns as relationships, and write simple algebraic expressions to represent the relationships.

Materials needed

- Seven sheets of paper for each group
- Six two-color counters for each group
- Computer with Internet access (optional but desirable)

Instructional activity

The Problem

There are seven stepping-stones in a straight row and six people. On the three left-hand stones, facing the center, stand three of the people. The other three people stand on the three right-hand stones, also facing the center. The center stone is not occupied.

The Challenge

Exchange places! Everyone must move so that the people standing originally on the right-hand stones are on the left-hand stones and those standing on the left-hand stones are on the right-hand stones, leaving the center stone again unoccupied.

The Rules

- Only one person may move at a time.
- After each move, each person must be standing on a stone.
- If you start on the left, you may move only to the right. If you start on the right, you may move only to the left.
- You may “jump” another person if the stone on the other side is empty. You may not “jump” more than one person.

The Activity

1. Begin with a large-movement activity. Give each group of six students seven sheets of paper to use as stepping-stones. Assign areas of the room to each group. Allow enough time for groups to try to find the minimum number of moves necessary to complete the task. Students should keep notes about the activity in their notebooks or math journals.

2. Give each group of three students six two-color counters. Have them draw a representation of the seven stepping-stones and use the two-color counters to simulate people moving in the first part of the activity. Again, students should try to minimize the number of moves necessary to complete the exchange. Allow enough time for groups to try to find the minimum number of moves necessary to complete the task.
3. To simulate the Traffic Jam, using technology, have students go to a Java applet on the Math Forum at Drexel University, http://mathforum.org/alejandre/frisbie/student.jam.html. Be sure students try the various options that are available, including level of difficulty, show history, and redraw history. Encourage students to look for a pattern.
4. Once more, have students go back in their groups of six and repeat the activity, using large movement and the paper stepping-stones.

Formalizing the Mathematics

5. Have students make a table in their notebook or math journal similar to the one below and complete the first two columns.

Number of Pairs	Number of People	Minimum Number of Moves
1	2	
2	4	
3	6	15
4	8	
5	10	
6	12	
7	14	
8	16	

6. In large group, encourage discussion that will complete the first three or four rows of the third column. Students may need to go back to the manipulative simulation with the two-color counters. Then have students talk out their thoughts about a pattern. Be patient and encouraging. Students will keep themselves on track if they continue to address the original problem.
7. Put students in groups of four, and ask the groups to organize a presentation that will account for the previous discussion about the chart. Listen to group discussions but do not lead them. Students should verbalize the pattern and then formalize it mathematically. Encourage the use of a variable to represent the number of pairs.

Sample assessment

- Teacher observation of individuals and groups
- Student notebooks or math journals

Homework

- State the algebraic expression generated in class that represents the minimum number of moves needed for n pairs of people to move from one side to the other.
- If $n = 11$, what will be the minimum number of moves? Show your work and explain your answer.
- Explain how the symbolic representation relates back to the six people moving on the stepping-stones.

Evaluating and Simplifying Expressions

Organizing topic Using Algebraic Topics

Overview The students use Algeblocks™ to develop a conceptual understanding of simplifying and evaluating algebraic expressions.

Related Standards of Learning A.2, A.3

Objectives

- The student will simplify algebraic expressions.
- The student will evaluate algebraic expressions.
- The student will use the commutative, associative, and distributive properties to justify steps in evaluating expressions.

Materials needed

- Algeblocks™
- Handouts from Algeblocks™ Unit 6 — “Thinking about Variables” (pages 56–60)
- Handout from Algeblocks™ Unit 7 — Lesson 7-1, “Like Terms” (page 66)

Instructional activity

Note: See the Teacher’s Resource Binder for Algeblocks™. Be sure to study the Teacher’s Notes for Units 6 and 7. Section 6.5 is optional but desirable.

1. Remember that students must not only manipulate the Algeblocks™ on the mats, but also draw the pictorial and symbolic representations.
2. Encourage students to use properties to justify their steps in evaluating and simplifying algebraic expressions.

Sample assessment

- Included with Algeblocks™ activities

Sample resources

Mathematics SOL Curriculum Framework
http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html

SOL Test Blueprints

Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html

Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra

NASA http://spacelink.nasa.gov/.index.html

The Math Forum http://forum.swarthmore.edu/

4teachers http://www.4teachers.org

Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm

Eisenhower National Clearinghouse http://www.enc.org/

Additional Sample assessment

Victor bought a computer for \$1,800. He made a down payment of \$200 and will pay the rest in 5 equal payments. If p represents the amount of each payment, which equation can be used to find this amount?

A $\$200p = \$1,800$

B $\$1,800 + 5p = \$200$

C $\$1,800 + \$200 = 5p$

D $\$1,800 = 5p + \$200$

What is the value of $x(5 + y)$ if $x = 4$ and $y = 2$?

F 18

G 22

H 28

J 36

Which is an example of the commutative property of addition?

F $3 + 5m = 3 + (1 + 4)m$

G $3 + 5m = 5m + 3$

H $3 + 5m = (3 + 5)m$

J $3 + 5m = 3m + 5$

What property of real numbers justifies the following statement?

$4x(y + 2) - 3y$ is equivalent to $4x(y) + 4x(2) - 3y$

A The associative property of multiplication

B The commutative property of multiplication

C The distributive property of multiplication over addition

D The closure property of multiplication

During a summer reading program, Mary read nine books. The books contained 217 pages, 138 pages, 159 pages, 356 pages, 270 pages, 112 pages, 138 pages, 210 pages, and 195 pages, respectively. What was the median number of pages of the nine books that Mary read during the summer reading program?

F 138

G 159

H 195

J 244

Jorge made the following stem-and-leaf diagram of the weights of the members of the football team he was coaching.

Stem	Leaf
10	9
11	
12	3, 8
13	2, 4, 4, 6, 8
14	1, 3, 5, 5, 9
15	2, 3, 7, 7, 9
16	1, 3, 7, 8, 8, 8, 9
17	3, 8

What was the mode of the weight of the players on the team?

F 145
G 150
H 152
J 168

In which data set is the median value equal to the mean value?

A {2, 4, 6, 7, 8}
B {12, 18, 20, 23, 24}
C {16, 17, 18, 19, 20}
D {50, 60, 65, 75, 85}

Organizing Topic Linear Equations and Inequalities

Standards of Learning

A.1 The student will solve multistep linear equations and inequalities in one variable, solve literal equations (formulas) for a given variable, and apply these skills to solve practical problems. Graphing calculators will be used to confirm algebraic solutions.

A.3 The student will justify steps used in simplifying expressions and solving equations and inequalities. Justifications will include the use of concrete objects; pictorial representations; and the properties of real numbers, equality, and inequality.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Introduce solving multistep linear equations by briefly reviewing one-step and two-step equation solving.	____________
• Create and interpret pictorial and concrete representations and justifications for solving equations.	____________
• Solve multistep equations by using	
◦ commutative properties	____________
◦ associative properties	____________
◦ distributive property	____________
◦ order of operations	____________
◦ addition and multiplication properties of equality	____________
◦ closure property	____________
◦ identity and inverse properties	____________
◦ reflexive, symmetric, and transitive properties of equality	____________
◦ substitution property of equality.	____________
• Use the properties listed above to justify steps in solving equations.	____________
• Solve literal equations (formulas) and implicit equations for a specified variable.	____________
• Confirm algebraic solutions to equations, using the functions of a graphing calculator.	____________
◦ In an implicit equation, graph each side of the equation separately but on the same set of axes, and find the coordinates of the point of intersection.	____________
◦ In an explicit equation, graph the equation, and find the x-intercept.	____________
• Solve linear inequalities in one variable.	____________
• Justify steps in solving linear inequalities, using properties of real numbers and order.	____________
• Graph the solution to a linear inequality in the coordinate plane.	____________

Cover-up Problems

Organizing topic Linear Equations and Inequalities

Overview This is a quick, intuitive lesson for students to derive a method of solving simple equations in one variable.

Related Standards of Learning A.1, A.3

Objectives

- The student will briefly review one-step and two-step equation solving.
- The student will derive a method to solve simple equations in one variable.
- The student will identify and discuss inverse operations.
- The student will create and interpret pictorial and concrete representations and justifications for solving equations.
- The student will solve equations, using the properties of real numbers and equality, and use these properties to justify steps in solving equations.

Materials needed

- A “Cover-up Problems” handout for each student

Instructional activity

1. Work through the equations, leading the students to solve the equations for *x*.
2. Conversation about *how* the students know their answers and *what operations* they performed to get them is important. Have students use the properties of real numbers and equality to justify steps in equation solutions.
3. Identify and discuss inverse operations.

Sample assessment

- Choose another “covered” equation to see whether the students can work through the problem and explain their thinking.

Follow-up/extension

- Have the students record their steps with one-variable equations.

Cover-up Problems

Equation 1: $4 * x = 12$

What is the value "under the hand"?

 = 12

Now what is the value "under the hand"?

4 *  = 12

Equation 2: $x + 7 = 43$

What is the value "under the hand"?

 = 43

Now what is the value "under the hand"?

 + 7 = 43

Equation 3: $x - 11 = 15$

What is the value "under the hand"?

 = 15

Now what is the value "under the hand"?

 – 11 = 15

Equation 4: $\frac{x}{5} = 10$

What is the value "under the hand"?

 = 10

Now what is the value "under the hand"?

 / 5 = 10

Equation 5: $4 * x + 8 = 12$

What is the value "under the hand"?

 = 12

Now what is the value "under the hand"?

 + 8 = 12

Now what is the value "under the hand"?

4 *  + 8 = 12

Equation 6: $\frac{x}{5} - 1 = 4$

What is the value "under the hand"?

 = 4

Now what is the value "under the hand"?

 – 1 = 4

Now what is the value "under the hand"?

 / 5 – 1 = 4

Equation 7: $\frac{2 * x}{3} + 7 = 13$

What is the value "under the hand"?

 + 7 = 13

Now what is the value "under the hand"?

 / 3 + 7 = 13

Now what is the value "under the hand"?

2 *  / 3 + 7 = 13

Algeblocks™ and Equation Solving

Organizing topic	Linear Equations and Inequalities
Overview	Using a manipulative allows students to build upon pre-algebra concepts in equation solving.
Related Standard of Learning	A.1

Objectives

- The student will create and interpret pictorial and concrete representations and justifications for solving equations.
- The student will solve multistep equations, using the field properties of real numbers and properties of equality, and using these properties to justify steps in solving equations.
- The student will solve linear inequalities in one variable.
- The student will justify steps in solving linear inequalities, using properties of real numbers and order.
- The student will graph the solution to a linear inequality in the coordinate plane.

Materials needed

- Algeblocks™ for each student or pair of students
- Handouts from Algeblocks™ Labs 10-2 through 10-8 for each student

Instructional activity

1. Use the Algeblocks™, Teacher's Resource Binder, Chapter 10, Labs 10-2, 10-3, 10-4, 10-5, 10-6, 10-7, and 10-8. Distribute handouts.
2. Emphasize the connection among representations (concrete, pictorial, and symbolic), and ensure that students are secure in their understanding of all three representations.
3. Reinforce the use of the field properties of real numbers and the properties of equality in justifying steps in solving equations.
4. Have the students confirm solutions to equations with the graphing calculator and by graphing the solutions in the coordinate plane on paper.

Sample assessment

- Included in the Algeblocks™ labs

Follow-up/extension

- Algeblocks™ Lab 10-10

Homework

- "Think about It" from the Algeblocks™ labs

Greetings

Organizing topic Linear Equations and Inequalities
Overview Students write a function describing the situation and then solve an inequality.
Related Standards of Learning A.1, A.3

Objective

- The student will solve a linear inequality in one variable and graph the solution in the coordinate plane.

Instructional activity

Problem Situation

The school choir purchased customized cards from a company that charges $100 for a set-up fee and $2 per box of cards. The choir members will sell the cards at $3 a box.

1. Introduce the problem situation, and have the students write a function describing the choir's profit, p dollars, for selling x boxes of cards.
2. Have the students answer the following questions:
   - What does the word *profit* mean?
   - What does the 3 in the expression $3x$ represent?
   - What does x represent?
   - What does p represent?
   - Why are there parentheses used in the function rule?
   - How can you use the distributive property to simplify the expression?
   - Which of the variables are you given in question 2?
   - Which of the variables represents $200 in question 3?
   - What does it mean to "break even"?
   - Describe how you might use a table to answer question 3.
   - Describe how you might use a graph to answer question 3.
   - What do the expressions "$3x$" and "$100 + 2x$" mean in this situation?
   - How much money will the choir make if they sell 200 boxes? What is your solution strategy?
   - How many boxes must the choir sell to make a $200 profit? Explain how you found your answer.
   - How many boxes must the choir sell to make a $500 profit? Use a different strategy to find the solution.
   - How many boxes will the choir have to sell to break even?
   - The choir will not consider this project unless they can raise at least $1,000. Write and solve an inequality that will help them determine if they should do this project. Graph the inequality in the coordinate plane, using paper and pencil and using the graphing calculator.

Sample assessment

- Embedded in the activity

Follow-up/extension

- What will happen in this situation if the $100 set-up fee is omitted?

Homework

- For another situation, the profit is represented by $p = 3x - (30 + 2.50x)$. Describe the cost and selling process for this situation. Under what conditions is the second situation better than the first?

Algeblocks™: Solving for y

Organizing topic	Linear Equations and Inequalities
Overview	The lesson allows students to use Algeblocks™ to conceptualize solving an implicit equation for a specific variable.
Related Standard of Learning	A.1

Objective

- The student will solve an implicit equation for a specified variable.

Materials needed

- Algeblocks™ for each student or pair of students.

Instructional activity

1. Use the Algeblocks™ Teacher's Resource Binder, Chapter 12, Lab 12-3, "Solving for *y*."
2. Emphasize the connection among representations (concrete, pictorial, and symbolic), and ensure that students are secure in their understanding of all three representations.
3. Reinforce the use of the field properties of real numbers and the properties of equality in justifying steps in solving equations.

Sample assessment

- Embedded in the Algeblocks™ labs

Homework

- "Think about It" from the Algeblocks™ labs

Algeblocks™: Solving Inequalities

Organizing topic Linear Equations and Inequalities

Overview The lesson allows students to use Algeblocks™ to conceptualize solving inequalities.

Related Standards of Learning A.1, A.3

Objective

- The student will solve linear inequalities in one variable and graph the solution in the coordinate plane.

Prerequisite understandings/knowledge/skills

- Students should be familiar with Algeblocks™.

Materials needed

- Algeblocks™ for each student or pair of students

Instructional activity

1. Use the Algeblocks™ Teacher's Resource Binder, Chapter 10, Lab 10-9, "Solving Inequalities."
2. Emphasize the connection among representations (concrete, pictorial, and symbolic), and ensure that students are secure in their understanding of all three representations.
3. Reinforce the use of the field properties of real numbers and the properties of order in justifying steps in solving inequalities.

Sample assessment

- Embedded in the Algeblocks™ labs

Homework

- "Think about It" from the Algeblocks™ labs

A Mystery to Solve

Organizing topic Linear Equations and Inequalities
Overview The students look at properties with an undefined operation.
Related Standard of Learning A.3

Objective

- The student will demonstrate knowledge of identity, inverse, and associative properties.

Instructional activity

1. This table shows how an operation, *, works with the set of numbers {1, 2, 3, 4, 5, 6}.

*	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	1	6	2	5
4	4	5	6	1	3	2
5	5	6	2	3	1	4
6	6	1	5	2	4	3

2. Does this set have an identity for this operation? If so, what is it, and how did you determine that it was an identity element?
3. Does any number in the set have an inverse for this operation? If so, identify the inverse for each number that has an inverse. How did you determine whether or not a number has an inverse?
4. Use the table to solve the equation $3 * x = 5$. Check your solution by finding $3 * x$ for your x.
5. Steve tried the following:
   $3 * x = 5$
   $3 * 3 * x = 3 * 5$
   $x = 2$
   Explain what Steve did at each step. Check his solution. Is it correct? What happened?

Sample assessment

- Group discussion

Follow-up/extension

- Have students look at a closed set {–1, 0, 1}. Which of the field properties of real numbers hold for the set? Have them provide work and written explanations to support their work.

Homework

- Is the set {–1, 0, 1} closed with respect to addition? Multiplication? Provide work and written explanations to support your work.

Solving Linear Equations

Organizing topic Linear Equations and Inequalities

Overview Students explore the functions of the TI-83 graphing calculator and solving equations.

Related Standard of Learning A.1

Objective

- The student will solve linear equations in one variable, using the graphing calculator.

Materials needed

- Graphing calculators

Instructional activity

1. Explain the following to the students: An *equation* is a statement that two expressions are equal. To solve an equation means to find those values, which when substituted for the variables, make the equation true. Such values are called *solutions*. For example, if x is any real number and the equation is $2x - 1 = 5$, the problem solving method, guess and check, can be used to see that 3 is a solution of this equation because $2(3) - 1 = 5$. However, how do we know that this is the only solution? How would you solve a more complicated equation? One method for solving equations on the TI-83 is discussed below.
2. Intersection method: Enter each member of the equation in the $Y =$ list. Graph each side and find the coordinates of the points of intersection. The x-coordinates will be solutions to the equation.
   - Example 1: Solve $3(x + 2) = x - 4$.
   - Solution: Enter $Y1 = 3(x + 2)$ and $Y2 = x - 4$ in the $Y =$ list, and graph the functions in the Zstandard window. Press 2cd [CALC] to display the Calc menu, and select [5: intersect].
   - Press ENTER twice to identify the two graphs. At the Guess? prompt, you have two options. Move the cursor to the point that is your guess as to the location of the intersection, or simply enter your guess directly from the keyboard. Then press ENTER a third time.
   - Since two lines intersect in no more than one point, the number $x = -5$ is the only solution to the equation $3(x + 2) = x - 4$.

Sample assessment

- Have the students solve two or three equations, using the intersection method on the graphing calculator.

Homework

- Have the students solve several equations algebraically and verify solutions, using the method described herein.
- Have the students solve several other equations, using the method described herein, and then verify solutions algebraically.

Sample resources

Mathematics SOL Curriculum Framework
http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html

SOL Test Blueprints
Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html
Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra
NASA http://spacelink.nasa.gov/.index.html
The Math Forum http://forum.swarthmore.edu/
4teachers http://www.4teachers.org
Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm
Eisenhower National Clearinghouse http://www.enc.org/

Organizing Topic Systems of Equations

Standards of Learning

A.4 The student will use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication. Data will arise from business, industrial, and consumer situations.

A.9 The student will solve systems of two linear equations in two variables both algebraically and graphically and apply these techniques to solve practical problems. Graphing calculators will be used both as a primary tool for solution and to confirm an algebraic solution.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Solve a system of linear equations algebraically to find the ordered pair that satisfies both equations simultaneously and solves the system. Use the substitution or elimination techniques to solve the system.	____________
• Solve a system of linear equations graphically by graphing the equations on the same set of axes and finding the coordinates of the point of intersection.	____________
• Decide if the solution to a system of equations is reasonable.	____________
• Interpret the solution to a system of equations that describes a practical situation in terms of the practical situation.	____________
• Determine if a system of linear equations has one solution, no solution, or infinitely many solutions.	____________
• Interpret the number of solutions to a system of equations in terms of the graph of the system and the geometry of the lines.	____________
• Use matrices to organize data arising from applications in business, industry, and consumerism.	____________
• Add and subtract matrices.	____________
• Multiply a matrix by a scalar factor.	____________
• Represent a system of linear equations as a matrix.	____________

Road Trip

Organizing topic Systems of Equations

Overview Students write linear equations to describe equations and functions from real-life situations. They generate tables and enter data into lists to create a STAT PLOT with the graphing calculator.

Related Standard of Learning A.9

Objectives

- The student will solve a system of linear equations algebraically to find the ordered pair that satisfies both equations simultaneously and solves the system.
- The student will solve a system of linear equations graphically by graphing the equations on the same set of axes and finding the coordinates of the point of intersection.
- The student will decide if the solution to a system of equations is reasonable.
- The student will interpret the solution to a system of equations that describes a practical situation in terms of the practical situation.

Materials needed

- Graphing calculators
- A copy of the "Road Trip," "Saving Money," and "Salaries" handouts for each student

Instructional activity

1. Distribute the handouts. Have students investigate the "Road Trip" problem and the cost for Prestige Auto. Have them work in pairs to complete the Prestige Auto chart, concentrating on the "independent" and "dependent" variable terminology. Have the students write an equation that represents this data.
2. Then have the students complete the same procedure for Getaway Auto, entering the data in the Getaway Auto chart and writing an equation to represent this data. Have them then compare the data in the table and answer the questions comparing the data and the range of values when it makes sense to use either auto rental company.
3. Have the students enter the two equations into the graphing calculator "$Y =$." Have them use the graphing calculator to graph the data and verify their answers in steps 1 and 2, based on the graphs.

Follow-up/extension

- Have the students follow the same procedures for the "Saving Money" problem.

Homework

- Have the students complete the "Salaries" problem.

Road Trip

You are planning a one-day road trip, but you don't own a car. You have investigated available rental car companies in the area and have decided to rent a car from either Prestige Auto or Getaway Auto.

1. Prestige Auto charges \$35 a day plus 24¢ per mile. Fill in data in the chart below to indicate the charges you would incur for rental from Prestige Auto.

Miles Driven	Start up Cost	Cost for Miles Driven	Total Cost of Trip
0			
20			
40			
60			
80			
100			
120			
140			
160			

- Which values change in this situation?
- What causes the values to change?
- What is the "independent" variable? (causes the change)
- What is the "dependent" variable? (is affected by the change)
- In words, write an equation to explain the situation.
- Using algebraic notation, write an equation to explain this situation.

2. Getaway Auto charges \$51 a day plus 16¢ per mile. Fill in data in the chart below to indicate the charges you would incur for rental from Getaway Auto.

Miles Driven	Start-up Cost	Cost for Miles Driven	Total Cost of Trip
0			
20			
40			
60			
80			
100			
120			
140			
160			

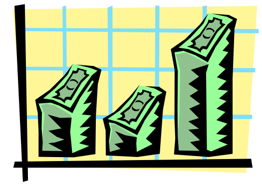
- Which values change in this situation?
- What causes the values to change?
- What is the "independent" variable? (causes the change)
- What is the "dependent" variable? (is affected by the change)
- In words, write an equation to explain the situation.
- Using algebraic notation, write an equation to explain this situation.

3. Complete the table below, using the equations you developed.

Miles Driven	Cost of Car from Prestige Auto	Cost of Car from Getaway Auto
50		
75		
100		
200		
250		
300		
325		

- Is there a particular number of miles driven at which the cost of using Prestige is the same as using Getaway?
- Is there a range of values of miles driven in which the cost of Prestige is less than using Getaway?
- When is it cheaper to use Getaway?

Extension: Saving Money



Nilda has $480 dollars in her sock drawer. She plans to save $30 per week from now on.

1. Complete the chart to show the amount of money Nilda has in her sock drawer.

No. of Weeks	Beginning Amount	Amount Added	Total Amount
0			
1			
2			
3			
4			
5			
6			

- Which "values" change in this situation?
- What "causes" the values to change?
- What is the "independent variable"? (causes the change)
- What is the "dependent variable"? (is affected by the change)
- In words, write an equation to explain this situation.
- Using algebraic notation, write an equation to explain this situation.
- At this rate, after how many weeks will Nilda have $690 in her sock drawer?
- After how many weeks will she have $2,040 in her sock drawer?
- If her mom had put money in her sock drawer at the same rate each week, how long had Nilda's mom been saving before Nilda took over?
- Put your equation in the $Y =$ screen of your graphing calculator. Do your answers match the graph? Do your answers match the table?

Homework: Salaries

Manny just graduated from high school and has been offered a job. He will start at $18,000 per year with a promise of a $500 raise per year. Sonny just graduated from college and has been offered a job. He has been offered $24,000 per year with a promise of a $300 raise per year.

1. Complete the following chart.

Manny		Sonny	
# Years Experience	**Salary**	**# Years Experience**	**Salary**
0		**0**	
1		**1**	
2		**2**	
3		**3**	
4		**4**	
5		**5**	

- What is the "independent" variable?
- What is the "dependent" variable?
- Enter the data into LIST on the graphing calculator, and create a scatterplot with STAT PLOT.
- What is the equation for each graph?
- Graph the equations on the same screen. Does Manny ever make more money than Sonny? If so, when? How do you know?
- You want to know after how many years Manny and Sonny will make the same amount of money. Since *Y* represents the amount of money each person earns, and you want to know when these two amounts are the same, what can you do algebraically to determine this equalization?

What's Your Call?

Organizing topic	Systems of Equations
Overview	Students are given two calling plans for cell phones. The first plan is given in the form of a table, the second in a graph format. Students determine a function rule for each plan, decide which is the better plan, and explain their reasoning.
Related Standard of Learning	A.9

Objectives

- The student will solve a system of linear equations algebraically to find the ordered pair that satisfies both equations simultaneously and solves the system.
- The student will solve a system of linear equations graphically by graphing the equations on the same set of axes and finding the coordinates of the point of intersection.
- The student will decide if the solution to a system of equations is reasonable.
- The student will interpret the solution to a system of equations that describes a practical situation in terms of the practical situation.

Materials needed

- Chart paper
- Markers
- Rulers or meter sticks
- Graphing calculators
- A copy of the "Cellulink Phones" and "Flight SST" handouts for each student

Instructional activity

1. Present students with the situation of choosing between two cell phone plans. Let the students briefly discuss the situation.

   Situation: Pam's mom has gotten a new job, and she needs to purchase a cell phone so that she can stay in contact with her family. Pam and her brother are trying to help her mom decide which plan makes the most sense and is the most economical for them. Both plans, Cellulinks Phones and Flight SST, offer 100 free anytime minutes. The plans have different ways of showing monthly expenses for the phone. Pam needs help analyzing the data and deciding which phone plan would work best.

2. Have students examine the data in the first plan, as shown in the Cellulinks Phones table on the handout. After identifying the basic information provided by the table, form groups of students and instruct them to analyze the plan and prepare to explain to the rest of the class how it works. Allow the groups time to interpret the plan. Provide chart paper and markers for each group to record the information they extract from the table. Direct them to articulate any patterns they find and try to generate a general rule (function rule) for the information.
3. Introduce the students to the second phone plan, which is presented in graphic form on the "Flight SST" handout. Direct students to interpret the second plan in their groups and find a general rule to represent the information.
4. Direct each group to choose a method to compare the two plans and prepare to explain their reasoning to the class.

5. When the groups are finished, display the charts and instruct each group to explain the strategies they used and their results. Compare the alternate forms of function rules, address any incorrect mathematics in the solutions, and encourage discussion if groups disagree on the better plan.
6. Ask students to use their rules to determine the monthly cost if they used the phone for an additional 150 anytime minutes under each plan. Ask them to address whether the results support their decision about which plan is better, and have them explain their reasoning.

Sample assessment

- Group presentations involving each plan
- Give students an additional situation, and ask them to compare the cost for each phone plan.

Follow-up/extension

- Have the students use graphing calculators to enter the rules for the two plans in the $Y =$ menu and view the results in a table and then on a graph. Have the students use the TRACE feature to pinpoint the intersection of the graphs. Hold a discussion about the meaning of that point.
- If students derive different versions of the rule for a particular plan, they can enter the various forms of the general rule in the $Y =$ menu and view the results in a table. Seeing that the expressions all produce the same values when using this quick, visual technique is a way to reinforce that all their expressions are equivalent.

CELLULINKS PHONES

Number One in Quality Phones

First 100 Minutes Free!	
Additional Anytime Minutes	**Cost**
0	**$19.95**
10	**22.45**
20	**24.95**
30	**27.45**
40	**29.95**
50	**32.45**
60	**34.95**
70	**37.45**
80	**39.95**
90	**42.45**
100	**44.95**
110	**47.45**
120	**49.95**
130	**52.45**
140	**54.95**
150	**57.45**
160	**57.45**
170	**57.45**
180	**57.45**
190	**57.45**
200	**57.45**

For $9.95 per month, you receive 100 free anytime minutes.

AND...

Unlimited nights and weekend minutes.

The chart shows you the charges for additional anytime minutes.

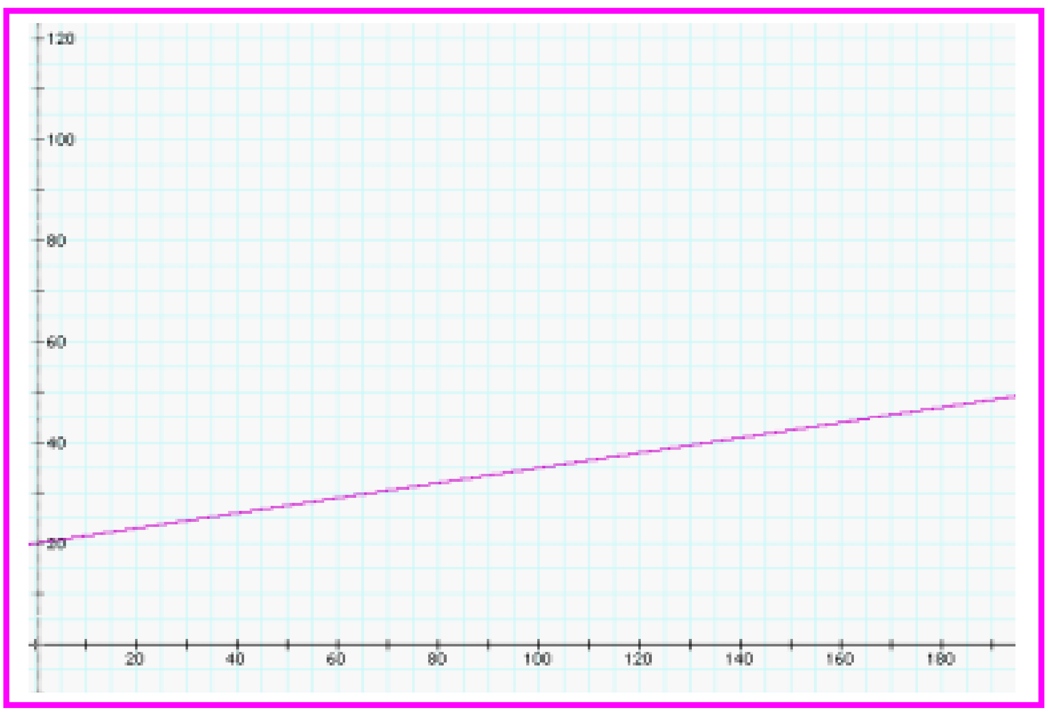
Can You Hear Me?

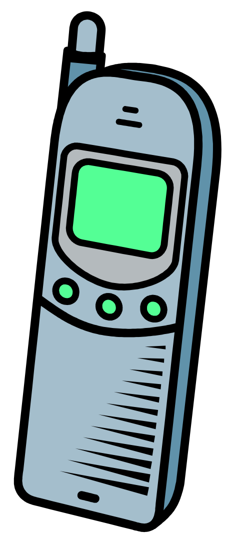
Flight SST

Quality so clear,
You can hear a pin drop!

100 Free Anytime Minutes for only $20 per month
Check our monthly charges below to see why *Flight SST*
is a great deal.

Your cost





Number of additional anytime minutes

Spring Fling Carnival

Organizing topic Systems of Equations

Overview Students use the graphing calculator to write equations and find the solution (intersection) graphically.

Related Standard of Learning A.9

Objectives

- The student will solve a system of linear equations graphically by graphing the equations on the same set of axes and finding the coordinates of the point of intersection.
- The student will interpret the solution to a system of equations that describes a practical situation in terms of the practical situation.
- The student will interpret the number of solutions to a system of equations in terms of the graph of the system and the geometry of the lines.

Materials needed

- Graphing calculators

Instructional activity

1. The students will read the given problem, formulate the equations, and find their intersection on the graphing calculator.

   Problem: The eighth grade needs to raise money for its end-of-the-year field trip. Team 8A wants to sell popcorn at the Spring Fling Carnival. Team 8B wants to sell cotton candy. The cost to rent the popcorn machine is $15.00. A cotton candy maker costs $25.00 to rent. The cost of additional supplies for the popcorn is $0.05 per bag. The additional cost for the cotton candy is $0.10 per stick. Team 8A will sell the bags of popcorn for $0.50 each. Team 8B will sell the cotton candy for $0.75 per stick. Each team needs to raise $100.00.
2. How many bags of popcorn will Team 8A need to sell to reach their goal?
3. How many sticks of cotton candy will Team 8B need to sell?
4. What is the least number of items each team would need to sell in order to avoid losing money on the sale?
5. At what point do both teams earn the same amount of profit? How do you know?

Follow-up/extension

- Change the selling price of popcorn to $.75 and the cotton candy to $.80. How does this change the answers above?

The Exercise Ring

Organizing topic Systems of Equations
Overview Students set up a system of equations as a matrix.
Related Standard of Learning A.9

Objectives

- The student will solve a system of linear equations algebraically to find the ordered pair that satisfies both equations simultaneously and solves the system.
- The student will decide if the solution to a system of equations is reasonable.
- The student will interpret the solution to a system of equations that describes a practical situation in terms of the practical situation.
- The student will determine if a system of linear equations has one solution, no solution, or infinitely many solutions.
- The student will interpret the number of solutions to a system of equations in terms of the graph of the system and the geometry of the lines.
- The student will represent a system of linear equations as a matrix.

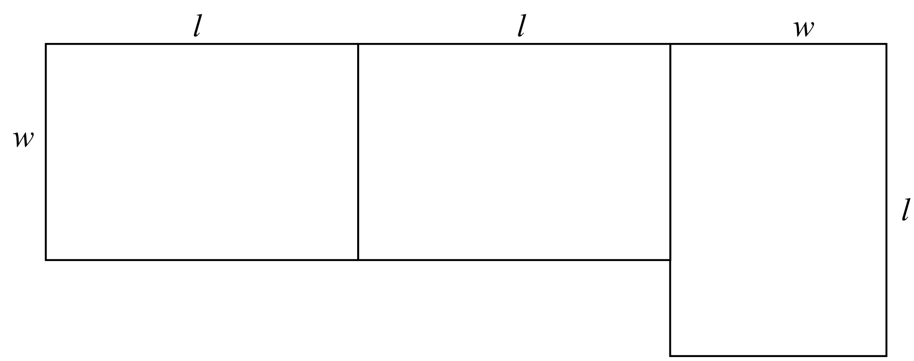
Materials needed

- Graphing calculators

Instructional activity

1. Present the following problem to the students:

   Problem: Sandra is planning to build exercise rings for her horses. She has land available and has decided to build 3 rectangular spaces as shown below.

   The three rings are the same size and shape. Sandra has been advised by other horse trainers that the perimeter of each of three rings should be 440 feet. She decides to use 1,200 feet of fence, because that is all she can afford.
2. Have the students use two different types of representation (table, symbol, or graph) to model the way in which Pam might build the rings.
3. Direct the students to write a system of equations to represent a solution to the problem. Ask them to use matrices to represent the system of equations.
4. Have the students enter the equations into the graphing calculator, letting the width be the y-value and the length be the x-value. Have them graph the system of equations and find the coordinates of the point of intersection.

Follow-up/extension

- Have the students explain what it would mean if the equations were multiples of each other.
- Have the students explain what it would mean if the lines did not intersect (were parallel).

Homework

- Have the students explain in their journal the meaning of the coordinates of the point of intersection in terms of the original problem.

Matrix Multiplying

Organizing topic	Systems of Equations
Overview	Students read about matrix multiplication on the Math Forum at Drexel University's Web site.
Related Standard of Learning	A.9

Objective

- The student will multiply a matrix by a scalar factor.

Materials needed

- A computer lab with a computer for each pair of students, or a computer with a projection device that can be seen by all students

Instructional activity

1. Have the students go to the Math Forum Web site (sponsored by Drexel University) at http://mathforum.org.
2. Have students find the "Ask Dr. Math" section of the Web site. Dr. Math answers questions for anyone who cares to submit.
3. Have students read and take notes on the article entitled "Multiplying a Matrix by a Scalar," http://mathforum.org/library/drmath/view/55489.html.
4. Have students read and take notes on the article entitled "Matrix Multiplication," http://mathforum.org/library/drmath/view/51458.html.

Sample assessment

- Provide opportunities for the students to practice scalar multiplication of a matrix.

Beginning Matrices

Organizing topic Systems of Equations
Overview Students use matrices to organize information.
Related Standard of Learning A.4

Objective

- The student will use matrices to organize data arising from applications in business, industry, and consumerism.

Materials needed

- Graphing calculators

Instructional activity

1. Introduce the following problem to the students:

   Problem: Gertrude, Matilda, and Homer love to eat at fast food restaurants. Generally, Gertrude orders 4 hamburgers, 4 sodas, and 4 fries. Matilda orders 6 hamburgers, 1 soda, and 1 fries. Homer orders 2 hamburgers, 10 sodas, and 2 fries. They shop around and find the following information for their favorite fast food restaurants. Wendy's™ charges \$1.69 for a hamburger, \$0.79 for a soda, and \$0.69 for fries. McDonald's™ charges \$1.85 for a hamburger, \$0.59 for a soda, and \$0.75 for fries. Burger King™ charges \$1.75 for a hamburger, \$0.65 for a soda, and \$0.59 for fries.
2. Have the students organize the information into matrices.
3. Direct the students to use the matrices to determine how much each person will spend at each fast food restaurant.
4. Have the students recommend a fast food restaurant for each character and explain their recommendations.

Matrix Multiplication

Organizing topic Systems of Equations
Overview Students use matrices to organize and manipulate data.
Related Standard of Learning A.4

Objective

- The student will use matrices to organize data arising from applications in business, industry, and consumerism.
- The student will multiply a matrix by a scale factor.

Materials needed

- Graphing calculators

Instructional activity

1. Introduce the following problem to the students:

   Problem: Jim, Kevin, Adam, and Dan are going shopping for Christmas presents. Jim wants to buy 3 CDs, 1 tape, and 8 videos. Kevin wants 6 CDs, 1 tape, and 3 videos. Adam wants 2 CDs, 7 tapes, and 1 video. Dan wants to buy 3 CDs, 2 tapes and 6 videos. Since they are wise consumers (who have only *one* car), they have researched the prices at two stores. The regular prices at Audio Attic are \$18.99 for CDs, \$13.99 for tapes, and \$25.99 for videos. Audio Attic is having a special sale where CDs are only \$14.99, tapes are \$10.99, and videos are \$21.99. Bargain Basement's regular price for CDs is \$16.99, for tapes is \$12.99, and for videos is \$29.99. They, too, are having a blowout sale where CDs are \$13.99, tapes are a rock bottom \$9.99 and videos are \$26.99.

2. Have the students set up a matrix to express the number of each item each person wants to purchase.

$$\begin{array}{cc} & \begin{array}{cccc} \text{Jim} & \text{Kevin} & \text{Adam} & \text{Dan} \end{array} \\ \begin{array}{l} \text{CDs} \\ \text{Tapes} \\ \text{Videos} \end{array} & \begin{bmatrix} \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \end{bmatrix} \end{array}$$

3. Now have them set up two matrices to express the regular and sale prices at each store.

$$\begin{array}{cc} & \begin{array}{ccc} \text{CDs} & \text{Tapes} & \text{Videos} \end{array} \\ \begin{array}{l} \text{Regular price} \\ \text{Sale price} \end{array} & \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{ccc} \text{CDs} & \text{Tapes} & \text{Videos} \end{array} \\ \begin{array}{l} \text{Regular price} \\ \text{Sale price} \end{array} & \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \end{array}$$

4. Which store should the group use to purchase their presents? (Remember that they only have *one* car so they must all go to the same store.) How much does each person spend? How much does each person save over the regular prices?

Follow-up/extension

- Suppose each person could go to the store that gave him the best price. Which store should each person go to, and how much would the group spend overall? How much does the group save?

Matrices

Organizing topic Systems of Equations
Overview Students organize data into a matrix and multiply by a scalar.
Related Standard of Learning A.4

Objectives
- The student will add and subtract matrices.
- The student will multiply a matrix by a scalar factor.

Materials needed
- Graphing calculators

Instructional activity
1. Have the students enter the following information in matrix form into Matrix A and Matrix B on their calculator:

$$\begin{array}{rc} & \begin{array}{ccccc} \text{shoes} & \text{socks} & \text{jackets} & \text{ties} & \text{rings} \end{array} \\ \begin{array}{r} \text{Matrix A} \\ \text{Matrix B} \end{array} & \begin{bmatrix} 11 & 14 & 3 & 7 & 2 \\ 15 & 18 & 6 & 4 & 20 \end{bmatrix} \end{array}$$

2. When they are finished, have them return to the HOME SCREEN.
3. Have the students find/explain the following:
   - [A] + [B] = . Explain a situation that describes this operation.
   - [A] – [B] = . Explain a situation that describes this operation.
   - 5[A] = . Explain a situation that describes this operation.
   - Explain the process necessary to multiply Matrix [A] by 11. What is the actual result?
   - Explain the process necessary to add Matrix [A] to Matrix [B] and multiply this result by 5. What is the actual result?
   - Explain the process necessary to add 5 times Matrix [B] to Matrix [A]. What is the actual result?
   - Explain the process necessary to multiply Matrix [A] by 11, multiply Matrix [B] by 4, and then add the results together. What is the actual result?
5. Have the students simplify the following algebraic expressions:
   - $5(x - 9) =$
   - $8(9 + 4x) =$
   - $x(y + 7) =$
   - $4(3x - 9r) =$
   - $6(5x + 3y) =$
   - $5x(7 - 3y) =$
   - $4a(5c + 2r) =$
   - $(10z - 9)5 =$
   - $(8 + 9T)6 =$

6. The measure of an angle is described as m∠ABC. If m∠ABC is also described as $4x + 19$, write the expression for 6 times m∠ABC.

Sample assessment

- Have the students solve the following two problems, using matrices:

About Sushi
(From *Problems with a Point.* May 23, 2001. EDC ©2001 by Mark Saul)

Two hamachi (yellowtail tuna), three anago (eels), and one sake (salmon) cost \$4. One hamachi, two anago, and three sake cost \$7.

Find the price of

1. 3 hamachi, 5 anago, and 4 sake
2. 4 hamachi, 6 anago, and 2 sake
3. 5 hamachi, 8 anago, and 5 sake
4. 3 hamachi, 5 anago, and 4 sake
5. 5 hamachi and 7 anago
6. 1 anago and 5 sake.

Bonnie's Dilemma
Bonnie and Carmen are lab partners in a Chemistry class. The experiment calls for a 5-ounce mixture that is 65% acid and 35% distilled water. There is no pure acid in the chemistry lab, but they did find two mixtures that are labeled as containing some acid. Mixture A contains 70% acid and 30% distilled water. The other mixture, B, contains 20% acid and 80% distilled water. How many ounces of each mixture should they use to make a 5-ounce mixture that is 65% acid and 35% distilled water? Justify your solution using symbols, tables, and graphs.
Hints:

- How much distilled water is in one ounce of Mixture A. Describe how you determine that amount.
- How many ounces of acid are there in 4 ounces of Mixture A?
- How many ounces of distilled water are there in 2 ounces of Mixture B?
- What are the variables in this situation?

Sample resources

Mathematics SOL Curriculum Framework
 http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html
SOL Test Blueprints
Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html
Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra
NASA http://spacelink.nasa.gov/.index.html
The Math Forum http://forum.swarthmore.edu/
4teachers http://www.4teachers.org
Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm
Eisenhower National Clearinghouse http://www.enc.org/

Additional sample assessment

Which is the solution to the system of equations shown below?

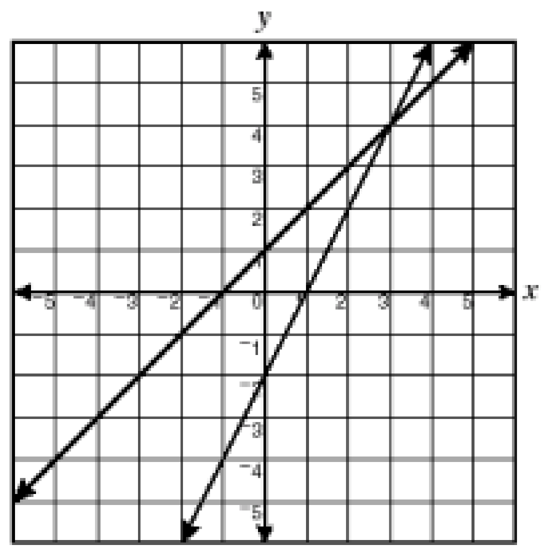
$$\begin{cases} 2x + y = 4 \\ 3x - y = -14 \end{cases}$$

F (–2, 8)
G (–2, 0)
H (2, 0)
J (0, –2)

What is the solution to this system of equations?

$$\begin{cases} 5x + 4y = 22 \\ 3x + 4y = 10 \end{cases}$$

F $x = 2,\ y = 2$
G $x = 2,\ y = 3$
H $x = 2,\ y = 1$
J $x = 6,\ y = -2$



Which is most likely the solution to the system of equations shown in the graph above?

F (4, 3)
G (–2, 0)
H (3, 4)
J (1, 0)

Matrix A shows the cost per pound of apples and oranges at three different markets during the first week of September.

$$\begin{array}{r c} & \begin{array}{ccc} \textit{GoGo} & \textit{Alto} & \textit{A\&B} \end{array} \\ \begin{array}{r} \textit{apples} \\ \textit{oranges} \end{array} & \begin{bmatrix} 1.09 & 1.11 & 0.89 \\ 1.15 & 1.11 & 0.79 \end{bmatrix} = A \end{array}$$

Matrix B shows the prices one week later at the same three markets.

$$\begin{array}{r c} & \begin{array}{ccc} \textit{GoGo} & \textit{Alto} & \textit{A\&B} \end{array} \\ \begin{array}{r} \textit{apples} \\ \textit{oranges} \end{array} & \begin{bmatrix} 1.09 & 1.14 & 0.49 \\ 1.19 & 1.14 & 0.89 \end{bmatrix} = B \end{array}$$

Which matrix correctly shows the difference in prices, $B - A$?

F $\begin{bmatrix} 0 & 0.03 & -0.40 \\ 0.04 & 0.03 & 0.10 \end{bmatrix}$

G $\begin{bmatrix} 0.06 & 0 & -0.10 \\ 0.10 & 0 & 0.40 \end{bmatrix}$

H $\begin{bmatrix} 0 & 0.03 & 0.40 \\ 0.04 & 0.03 & 0.10 \end{bmatrix}$

J $\begin{bmatrix} 2.18 & 2.25 & 1.38 \\ 2.34 & 2.25 & 1.68 \end{bmatrix}$

$$[G] = \begin{bmatrix} 4 & 3 \\ 2 & -1 \\ -2 & 1 \end{bmatrix}$$

$$[H] = \begin{bmatrix} 8 & 2 \\ 3 & -3 \\ 5 & 7 \end{bmatrix}$$

[G] + [H] = ?

A $\begin{bmatrix} 12 & 5 \\ 5 & -4 \\ 3 & 8 \end{bmatrix}$

B $\begin{bmatrix} 12 & 5 \\ 5 & 4 \\ -3 & 8 \end{bmatrix}$

C $\begin{bmatrix} 7 & 10 \\ 1 & 0 \\ -1 & 2 \end{bmatrix}$

D $\begin{bmatrix} -4 & 1 \\ -1 & 2 \\ -7 & -6 \end{bmatrix}$

$$[Q] = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 3 & 4 \end{bmatrix} \qquad [R] = \begin{bmatrix} -7 & 3 \\ -4 & 1 \\ 3 & -2 \end{bmatrix}$$

$[Q] - [R] = ?$

F $\begin{bmatrix} -2 & 9 \\ 0 & 3 \\ 6 & 9 \end{bmatrix}$

G $\begin{bmatrix} 9 & -2 \\ 3 & 0 \\ 0 & 6 \end{bmatrix}$

H $\begin{bmatrix} -5 & 2 \\ -5 & 0 \\ -9 & -6 \end{bmatrix}$

J $\begin{bmatrix} 7 \\ 3 \\ -11 \end{bmatrix}$

Which of the following operations would result in the matrix $\begin{bmatrix} -4 & 2 \\ 6 & 1 \end{bmatrix}$?

F $2\begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix}$

G $\frac{1}{2}\begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix}$

H $\begin{bmatrix} 5 & 5 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} -1 & -3 \\ -2 & -2 \end{bmatrix}$

J $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix} + \begin{bmatrix} -7 & 3 \\ 8 & -1 \end{bmatrix}$

Organizing Topic Relations and Functions: Linear

Standards of Learning

A.5 The student will create and use tabular, symbolic, graphical, verbal, and physical representations to analyze a given set of data for the existence of a pattern, determine the domain and range of relations, and identify the relations that are functions.

A.6 The student will select, justify, and apply an appropriate technique to graph linear functions and linear inequalities in two variables. Techniques will include slope-intercept, x- and y-intercepts, graphing by transformation, and the use of the graphing calculator.

A.7 The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined. The graphing calculator will be used to investigate the effect of changes in the slope on the graph of the line.

A.8 The student will write an equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

A.15 The student will, given a rule, find the values of a function for elements in its domain and locate the zeros of the function both algebraically and with a graphing calculator. The value of $f(x)$ will be related to the ordinate on the graph.

A.16 The student will, given a set of data points, write an equation for a line of best fit and use the equation to make predictions.

A.18 The student will analyze a relation to determine whether a direct variation exists and represent it algebraically and graphically, if possible.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Describe slope as a constant rate of change.	____________________
• Recognize and describe a line with a slope that is positive, negative, or zero.	____________________
• Using the line $y = x$ as a reference, generalize the effect of changes in the equation on the graph of the line. Use the graphing calculator to facilitate development of this concept.	____________________
• Characterize the changes in the graph of the line as translations, reflections, and dilations.	____________________
• Calculate the slope of a line, given	
° the coordinates of two points on the line	____________________
° the equation of the line	____________________
° the graph of the line.	____________________
• Recognize and describe a line with a slope that is undefined.	____________________

- Given the equation of a line, solve for the dependent variable and graph
  - using transformations ______________________
  - using the slope and *y*-intercept ______________________
  - using the *x*-intercept and *y*-intercept. ______________________
- Graph an inequality in two variables. ______________________
- Write the equation of a line, given
  - the coordinates of two points on the line ______________________
  - the slope and the *y*-intercept of the line ______________________
  - one point on a vertical line ______________________
  - one point on a horizontal line ______________________
  - the graph of the line. ______________________
- Write the equation of a line, given
  - the coordinates of two points on the line ______________________
  - the slope and the *y*-intercept of the line ______________________
  - one point on a vertical line ______________________
  - one point on a horizontal line ______________________
  - the graph of the line. ______________________
- Using the equation of the best fit line, make predictions about unknown outcomes. ______________________
- Recognize and describe direct variation from a graph or a table of values. ______________________
- Given a set of data, determine whether a direct relationship exists and write the equation for the variation, if one exists. ______________________
- Identify the domain and range for a linear relation when given a set of ordered pairs, a table, a mapping, or a graph. ______________________
- Identify whether or not a linear relation is a function and justify the classification. Data may be presented as a set of ordered pairs, a table, a mapping, or a graph. ______________________
- Compare and contrast relations and functions. ______________________
- Analyze a set of data for the existence of a pattern that defines the change relating input and output values. Represent the pattern symbolically and graphically and determine whether or not the relation is a function. ______________________
- For each value, x, in the domain of f, find $f(x)$. ______________________
- Identify the zeros of the function algebraically and graphically. ______________________

Slippery Slope

Organizing topic Relations and Functions: Linear
Overview Students develop with a manipulative activity the concept of slope.
Related Standards of Learning A.5, A.7

Objective

- The student will describe slope as a constant rate of change.
- The student will recognize and describe a line with a slope that is positive, negative, or zero.
- The student will calculate the slope of a line given
  - the coordinates of two points on the line
  - the equation of the line
  - the graph of the line.
- The student will recognize and describe a line with a slope that is undefined.
- The student will, given the equation of a line, solve for the dependent variable and graph
  - using transformations
  - using the slope and y-intercept
  - using the x-intercept and y-intercept.

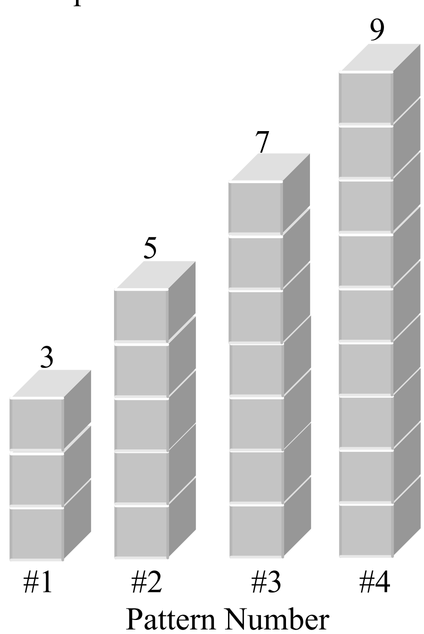
Materials needed

- Linking cubes

Instructional activity 1

Continuing a pattern

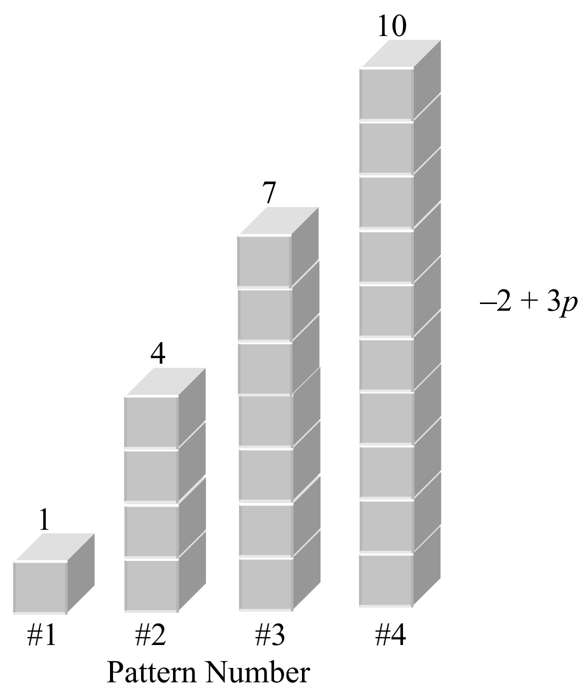
1. Present students with a sequence of towers as shown below, and ask them to draw the next 3 towers.



2. To learn whether they can generalize this activity, ask students to figure out how many cubes would be in the tenth tower *without* drawing all the towers in between. (Note: Some students will

reason that because the number of blocks increased by 2 from tower to tower and because 6 more towers were required to get to the 10^{th} tower, they just need to add 12 to the number of cubes in the 4^{th} tower. Other students will notice that the number of cubes in each tower is one more that twice the pattern number for the tower.)

3. Ask students how they could describe the pattern symbolically. (Note: This way of viewing the towers does not encourage thinking about the change from one tower to the next. You need to ask questions to help students focus on building the formula on the basis of change.
4. Ask, “How many cubes would be in the 0^{th} tower?”
5. Ask, “If you start with the 0^{th} tower, how many more cubes would you need to build the 3^{rd} tower?” (Note: This focuses on the additional sets of two blocks needed to build the 3^{rd} tower.)
6. Ask, “How many cubes would be in the 8^{th} tower?” (Note: Students would probably add 8 sets of 2 blocks to the number in the 0^{th} tower. This question helps students see the 2 in $2p + 1$ as a rate of change.)
7. Present the students with the following sequence of towers.

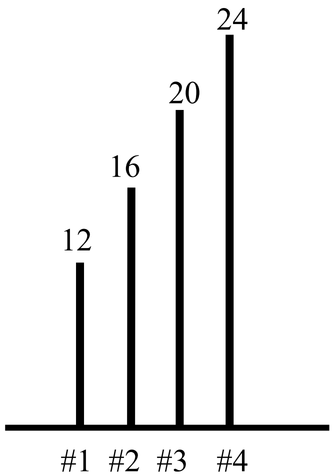


8. Ask the same set of questions as in the first example.
   - Draw the next three towers.
   - How many cubes are in the 15^{th} tower?
   - Draw the 0^{th} tower. Describe the 0^{th} tower. (Note: Here the 0^{th} floor is “underground” — i.e., two floors are in the basement.
   - What is the rate of change from tower to tower?
9. Reverse the order of the towers (put the 10-cube tower first; then the 7-cube tower, and so on). What is the rate of change now?

Instructional activity 2

Moving toward the Cartesian plane and the equation of a line

1. Because students have difficulty giving meaning to an ordered pair, capitalize on the towers model. Tower 1 has 12 cubes, tower 2 has 16 cubes, tower 3 has 20 cubes, and tower 4 has 24 cubes. Drawing or building all these towers is cumbersome. Therefore, introduce the idea of drawing each tower as a "stick" with the number of cubes it represents written at the top.

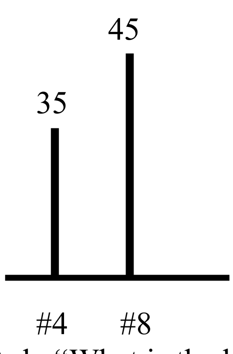


2. Ask, "What is the height of the 10^{th} building?"
3. Ask, "What is the height of the 25^{th} building?"
4. Ask, "What is the height of the 0^{th} building?"
5. Ask, "If you know the pattern number, can you write a formula that will give you the height of any tower or building?" (Note: Problem solving method: Determine the rate of change in the heights of the towers. Find the height of the 0^{th} tower. Put these elements together in a formula: height of the 0^{th} tower + (pattern #) * change = total number of cubes in the tower. This formula, which is a symbolic representation of the formula, grows out of student thinking about such different graphical representations as towers of cubes, sticks, and so on.
6. Have students answer the same questions about a sequence of the following towers: 11 8 5

Instructional activity 3

Moving toward a rate of change

1. Move to tasks in which students cannot figure out the rate of change by comparing the heights of consecutive towers. For example:



2. Ask, "What is the height of the 5^{th} tower?" (The change in the height of the towers is 10, and tower 8 is 4 towers beyond tower 4.)

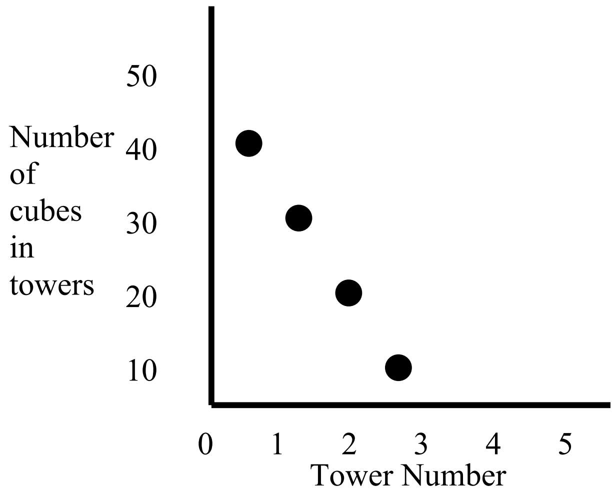
   $10 \div 4 = 2.5$ blocks per tower, which is the rate of change.

   $35 + 2.5 = 37.5$, which is the height of the 5^{th} tower
3. Ask, "What is the height of the 12^{th} tower?"

Instructional activity 4

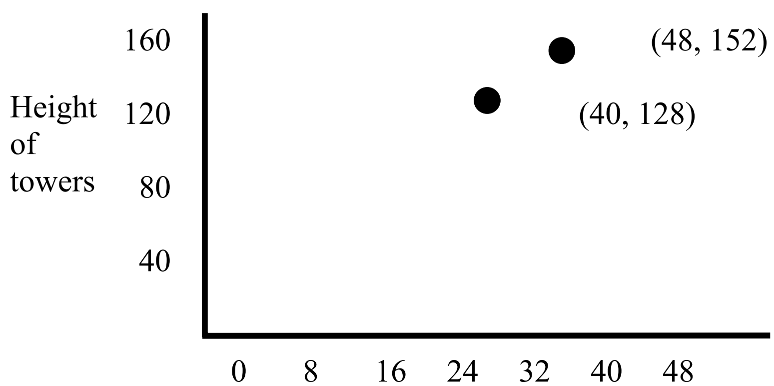
Interpreting points in the Cartesian plane

1. The next step is to interpret the towers as points.



Instead of drawing a stick to represent the height of the tower, show students how to use a grid to indicate the height of the tower and the pattern number.

2. Ask, "How can we discover a formula that relates the height of the tower and the pattern number?" (Note: Students may again use the strategy of first determining the change in building height and then working back to the height of the 0th tower by repeated addition. We want them to move beyond repeated addition to learn the height of the 0th tower. Give them patterns in which that method is too cumbersome in order to push them toward a different strategy.



The height of the 40th and 48th towers are given. The rate of change is 3.
(152 – 128) ÷ 8 = 24 ÷ 8 = 3.

3. Tell the students that to get back to the 0th building, they would have to subtract three 40 times! It is easier to subtract 3 * 40 or 120. Therefore, 128 – 120 = 8, which is the height of the 0th building.

Sally Snail

(From *Problems with a Point.* January 11, 2001. ©EDC 2000 by Mark Saul)

Organizing topic	Relations and Functions: Linear
Overview	Students develop the concept of slope as a constant rate of change.
Related Standard of Learning	A.7

Objective

- The student will describe slope as a constant rate of change.

Instructional activity

Traveling at constant rate problem

Sally Snail crawls at the rate of five inches per minute. She leaves her home at 3:00 pm, crawling along a straight line.

1. (a) How far away from home is she at 3:01?
   (b) How far away from home is she at 3:04?
2. (a) At what time will she be 30 inches from home?
   (b) At what time will she be 32 inches from home?
3. (a) At 3:00 we know Sally is at home. How long will it take her to crawl 15 inches from home?
   (b) At 3:15 Sally passes her friend Tilly's home. How long will it take her to crawl 15 inches from Tilly's home?
4. Draw a graph showing Sally's distance from home at each minute from 3:00 until 3:15.
5. Answer the following "true" or "false." Be prepared to justify your answer.
   (a) When Sally first leaves home, it will take her 3 minutes to crawl 15 inches. When she is way past Tilly's home, it will still take her 3 minutes to crawl 15 inches.
   (b) It takes Sally twice as long to crawl 20 inches as it does for her to crawl 10 inches.
   (c) It takes Sally twice as long to crawl 300 inches as it does for her to crawl 150 inches.
   (d) It takes Sally three times as long to crawl 20 inches as it does for her to crawl $6\frac{2}{3}$ inches.
   (e) Sally will crawl three times as far in 24 minutes as she will crawl in 8 minutes.
6. According to this little story,
   (a) Does Sally ever get tired of crawling?
   (b) Does she slow down or speed up?
   (c) Does she stop?

Homework

- Which of the problems, 1a–6c, could not be answered if Sally ever changed speed during her trip?

The Submarine

Organizing topic Relations and Functions: Linear

Overview Students develop the idea of slope as a rate of change.

Related Standard of Learning A.5

Objectives

- The student will describe slope as a constant rate of speed.
- The student will recognize and describe a line with a slope that is positive, negative, or zero.

Instructional activity

1. Introduce the following situation to the students:

   A submarine is cruising at 195 meters beneath the surface of the ocean and begins to surface at 12 meters per minute.

2. Have the students describe verbally, algebraically, and graphically a function relating the submarine's position and the amount of time it has been rising.
3. How long does it take the submarine to reach the ocean's surface? Explain.
4. Suppose the submarine started at its original depth of 195 meters and must reach the ocean's surface 5 minutes sooner than before. Describe how this will change the function and the graph of the original situation. What is the new function, and how did you determine it?

Transformationally Speaking

Organizing topic Relations and Functions: Linear
Overview Students identify patterns in families of graphs and their equations.
Related Standard of Learning A.6

Objectives

- The student will generalize the effect of changes in the equation on the graph of the line.
- The student will characterize the changes in the graph of the line as translations, reflections, and dilations.

Materials needed

- Graphing calculators

Instructional activity

1. Build a table of values for $y = x$, and graph the points on the calculator.
2. Graph $Y1 = x$, the parent graph.
3. Graph Set 1 on the graphing calculator. What do you observe?

$$\text{Set 1}\ \begin{cases} Y2 = x + 1 \\ Y3 = x + 2.4 \\ Y4 = x + 6 \end{cases}$$

4. Graph Set 2 on the graphing calculator. What do you observe?

$$\text{Set 2}\ \begin{cases} Y1 = x \\ Y2 = x - 0.6 \\ Y3 = x - 2.1 \\ Y4 = x - 9 \end{cases}$$

5. Consider the equation $y = x + c$. What is the effect of c on the graph of $y = x$? What kind of transformation is this?
6. Graph Set 3 on the graphing calculator. What do you observe?

$$\text{Set 3}\ \begin{cases} Y1 = x \\ Y2 = 1.6x \\ Y3 = 2x \\ Y4 = 3.4x \end{cases}$$

7. Graph Set 4 on the graphing calculator. What do you observe?

$$\text{Set 4}\ \begin{cases} Y1 = x \\ Y2 = 0.9x \\ Y3 = 0.5x \\ Y4 = 0.1x \end{cases}$$

8. Consider the equation $y = ax$. What is the effect of a on the graph of $y = x$? Which transformation is this?

Set 5 $\begin{cases} Y1 = x \\ Y2 = -x \end{cases}$

9. Graph Set 5 on the graphing calculator. What do you observe?

Set 6 $\begin{cases} Y1 = 0.5x \\ Y2 = -0.5x \end{cases}$

10. Graph Set 6 on the graphing calculator. What do you observe?

11. Consider the equation $y = -ax$ (a > 0). What is the effect of $-a$ on the graph of $y = x$? Which transformation is this?

Set 7 $\begin{cases} Y1 = 0.5x + 2 \\ Y2 = -0.5x + 2 \\ Y3 = -(0.5x + 2) \end{cases}$

12. Graph Set 7 on the graphing calculator. What do you observe?

13. Consider the equation $y = -a(x + b)(a > 0)$. What is the effect of b on the graph of $y = x$? Which transformation is this?

Transformation Investigation

Organizing topic Relations and Functions: Linear

Overview Students investigate the significance of the components of the equation of a line.

Related Standards of Learning A.6, A.7, A.8

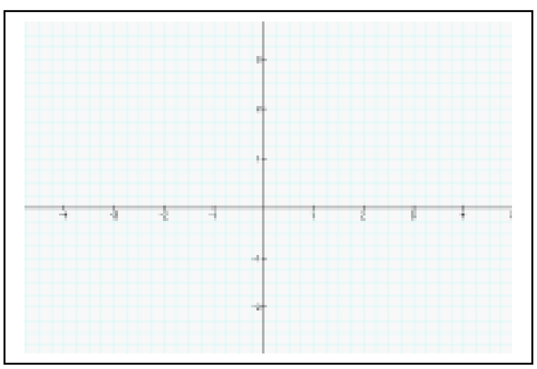
Objectives
- The student will use the line $y = x$ as a reference and generalize the effect of changes in the equation on the graph of the line.
- The student will characterize the changes in the graph of the line as translations, reflections, and dilations.

Materials needed
- Graphing calculators

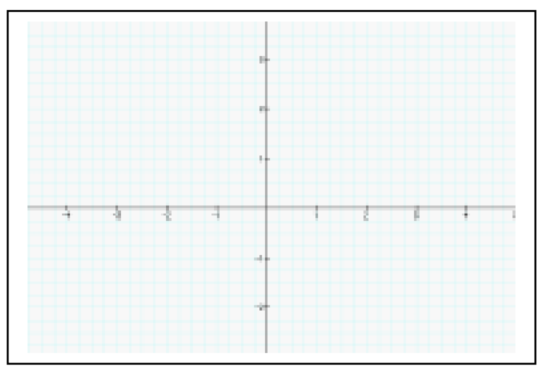
Instructional activity
1. Have students graph linear equations of the form $Y = A(X \pm B)$, when $A \neq 0$ and $B \geq 0$.
2. Have students set their calculator window to
   Xmin = –10
   Xmax = 10 Xsc1 = 1
   Ymin = –6
   Ymax = 6

Part I

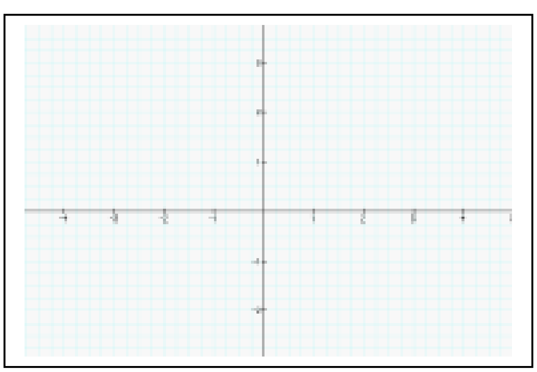
Basic function: $Y = 1(X + 0)$



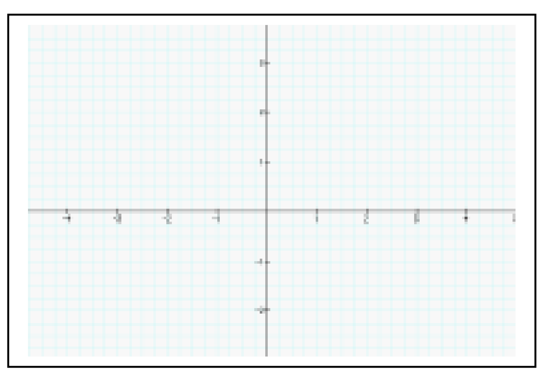
$Y_2 = 1X + 2$



$Y_3 = 1X + 4$



$Y_4 = 1X + 6$



Critical Statistics

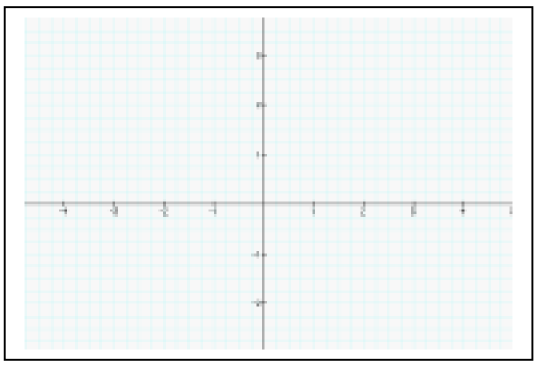
	Y	Y_1	Y_2	Y_3	Y_4
y-intercept					
x-intercept					
Slope					

1. What effect does "changing" B have on the basic function?
2. What generalization can you make about the change in the y-intercept if $B \geq 0$?
3. What generalization can you make about the change in the x-intercept if $B \geq 0$?

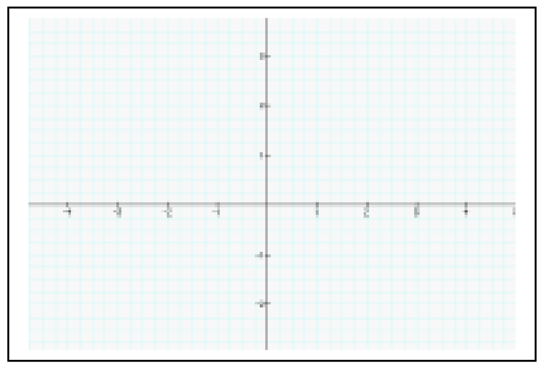
Part II

Basic function: $Y = 1(X + 0)$

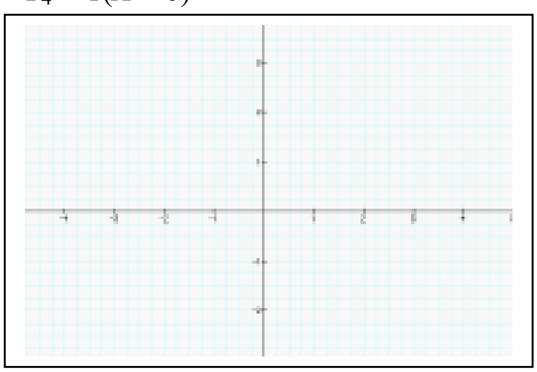
$Y_2 = 1(X - 2)$



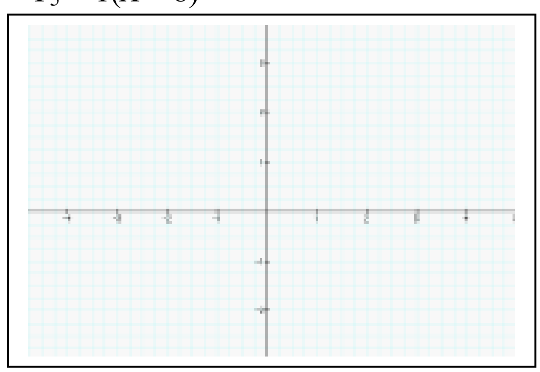
$Y_3 = 1(X - 4)$



$Y_4 = 1(X - 6)$



$Y_5 = 1(X - 8)$



Critical Statistics

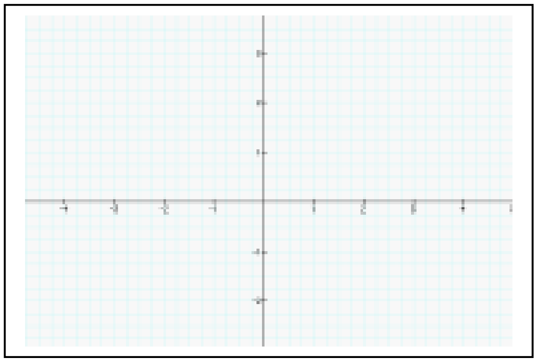
	Y	Y_2	Y_3	Y_4	Y_5
y-intercept					
x-intercept					
Slope					

1. What effect does "changing" B have on the basic function?
2. What generalization can you make about the change in the y-intercept if $B \geq 0$?
3. What generalization can you make about the change in the x-intercept if $B \geq 0$?
4. Does the slope have an effect on the way the graph changes?

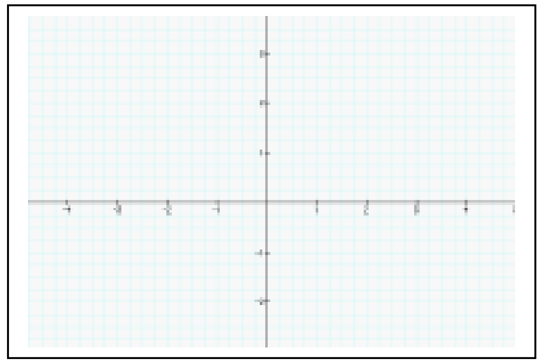
Part III

Sketch a graph for each of the following equations with a basic function: $Y = 2(X + B)$

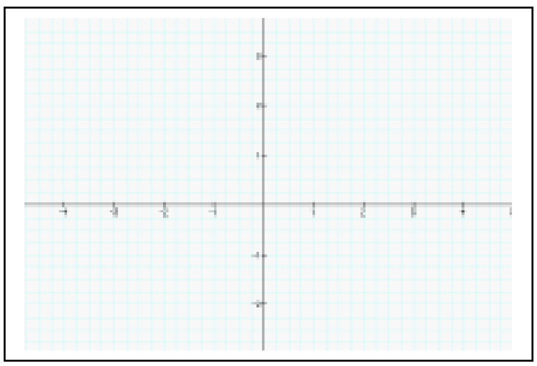
$Y_1 = 2(X + 0)$



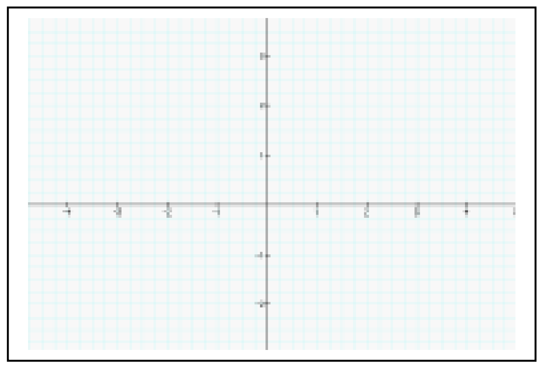
$Y_2 = 2(X + 2)$



$Y_3 = 2(X + 4)$



$Y_4 = 2(X + 3)$



Critical Statistics

	Y	Y_1	Y_2	Y_3	Y_4
y-intercept					
x-intercept					
Slope					

Compare the critical statistics of Y_1, Y_2, Y_3, Y_4 to the critical statistics of Y. What effect(s) does "changing" B have on the basic (parent) function?

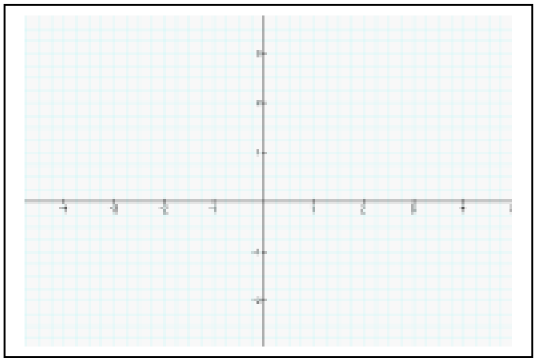
Generalizing: (if $B \geq 0$)

1. What was the slope in each of the problems above? Do you think that the slope has any effect on the graph?
2. Adding a value of B to the X in the previous problems resulted in a transformation of the x-intercept to the ________________
3. Adding a value of B to the X in the previous problems resulted in a transformation of the y-intercept to the _________.

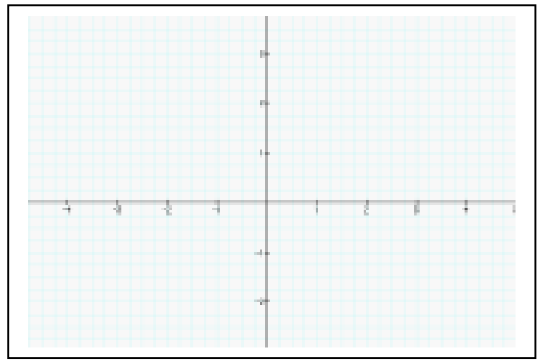
Part IV

Sketch a graph for each of the following equations with a basic function: $Y = 2(X - B)$

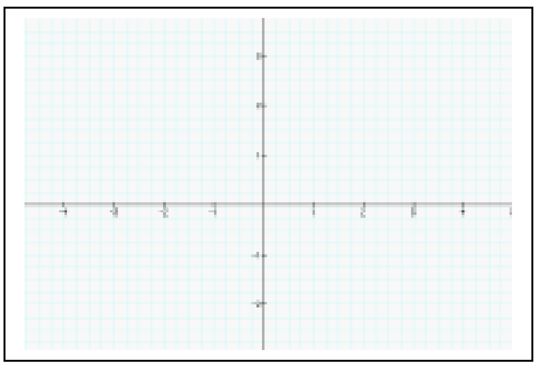
$Y = 2(X - 0)$



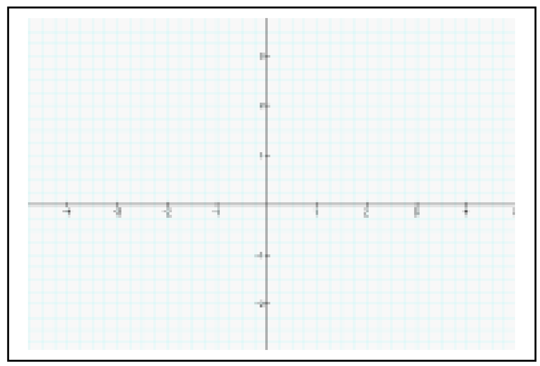
$Y_1 = 2(X - 2)$



$Y_2 = 2(X - 3)$



$Y_3 = 2(X - 4)$



Critical Statistics

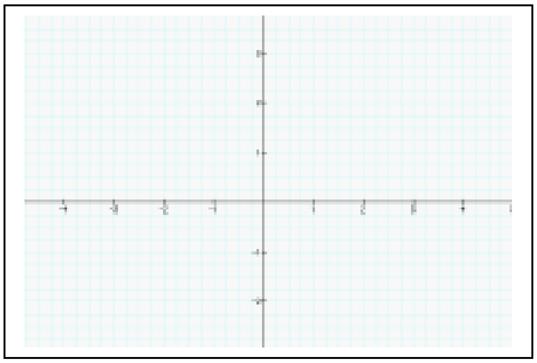
	Y	Y_1	Y_2	Y_3	Y_4
y-intercept					
x-intercept					
Slope					

1. What is the slope of each graph above?
2. What effect does subtracting a value of *B* have on the graph?
3. What is the effect of subtracting a value of *B* on the *x*-intercept? *y*-intercept?

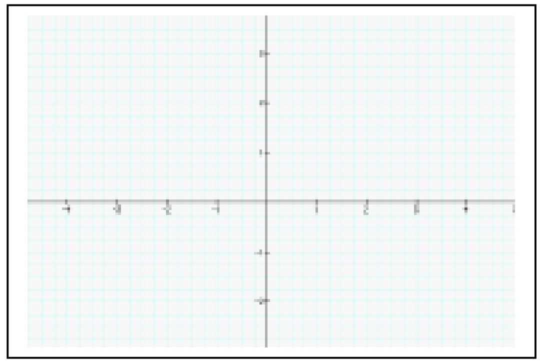
Part V

Sketch a graph for each of the following equations for the basic function

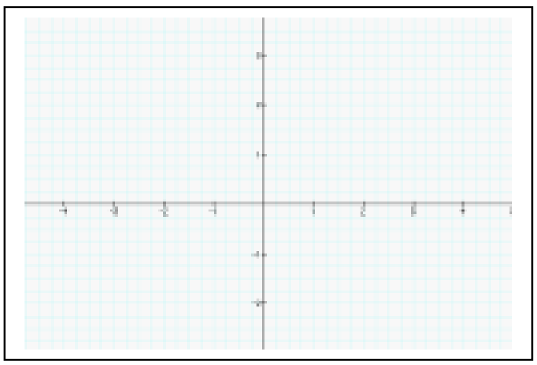
$Y = -1(X + B)$



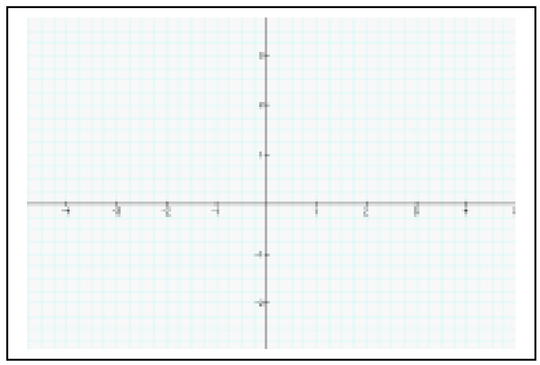
$Y_2 = -1(X + 2)$



$Y_3 = -1(X + 4)$



$Y_4 = -1(X - 4)$



Critical Statistics

	Y	Y_1	Y_2	Y_3	Y_4
y-intercept					
x-intercept					
Slope					

1. What is the slope of each graph above?
2. What effect does adding/subtracting a value of B have on changing the graph?
3. What is the effect of changing the slope on the x-intercept? y-intercept?

Generalizations for Parts I –V

1. When the slope of a line is a positive 1, adding a value of B to the X results in _________________.
2. When the slope of a line is a positive 1, subtracting a value of B from the X results in ___________.
3. When the slope of a line is A where $A > 0$, adding a value of B to the X results in _______________.
4. When the slope (A) of a line is a positive number, subtracting a value of B from the X results in _________________.
5. When the slope of a line is a negative 1, adding a value of B to the X results in ________________.
6. When the slope of a line is negative 1, subtracting a value of B from the X results in ____________.
7. When the slope of a line is negative ($A < 0$), adding a value of B to the X results in ______________.
8. When the slope of a line is negative ($A < 0$), subtracting a value of B to the X results in __________.

9. Given that the slope of a line is 2 and the x-intercept is 5, what is the y-intercept? __________ What would be an equation of this line?________________________

10. Given that the slope of a line is 2, the x-intercept is R, and $R > 0$, what is the y-intercept? ________ What would be an equation of this line?________________________

11. Given that the slope of a line is m, the x-intercept is R, and $R > 0$, what is the the y-intercept? _________ What would be an equation of this line?______________

12. Given that the slope of a line is 2 and the y-intercept is 6, then the x-intercept is ________. What would be an equation of this line?_____________________________

13. Given that the slope of a line is A and the y-intercept is B, then the x-intercept is_______. An equation of the line is ______________________.

Slope 2 Slope

Organizing topic Relations and Functions: Linear

Overview Students extend their intuitive definition of slope into horizontal and vertical lines and graphing a line.

Related Standard of Learning A.6

Objectives

- The student will calculate the slope of a line, given
  - the coordinates of two points on the line
  - the equation of the line
  - the graph of the line.
- The student will recognize and describe a line with a slope that is undefined.
- The student will, given the equation of a line, solve for the dependent variable and graph
  - using transformations
  - using the slope and x-intercept
  - using the x-intercept and y-intercept.

Materials needed

- Graphing calculators
- A "Slope-2-Slope Squares Game" handout for each student

Instructional activity

1. Before engaging in this activity, students should have completed the "Slippery Slope" activity to develop an intuitive formula for slope. The teacher should extend student definitions to a variety of plausible, contextual situations, including figuring out slope when given the coordinates of two points on the line, the equation of a line, and the graph of a line.
2. Discuss positive and negative slope. What is the effect on the graph of the line?
3. Allow students to experiment with the slopes of horizontal and vertical lines. Let them generalize their findings. Ask about equations for the x and y-axes.
4. Have students graph explicit equations, and discuss the transformation(s) necessary to produce the image graph from the parent graph, $y = x$. Have students describe the graph in terms of the slope and the intercepts. Have students discuss ways to figure out the slope and the intercepts when all they have is the equation. Reinforce the general coordinates of the intercepts $(x, 0)$ and $(0, y)$ by reviewing the equations of the x- and y-axes. Then reverse the process, and graph, using transformations, slope and y-intercept, and the x- and y-intercepts.

Sample assessment

- Have the students complete the "Slope-2-Slope Squares Game" handout.

Slope-2-Slope Squares Game
Standard to slope-intercept form — and back again!

1. Cut the squares apart.
2. Match equivalent expressions or sentences.
3. You should get a new four-by-four square.

Square	Top	Left	Right	Bottom
Row 1, Square 1	$y = 4$	$y = -2x - 2$	$3x + 4y = 24$	$2x - y = -7$
Row 1, Square 2	$y = 2x - 5$	$y = -x + 6$	$10x + 2y = 8$	$y = 4 - 2x$
Row 1, Square 3	$y = 2x - 4$	$y = x - 6$	$2x - 3y = 8$	$y = 1/3x - 3$
Row 1, Square 4	$8x - y = 2$	$y = 3x - 2$	$y = 2x + 5$	$x = 4$
Row 2, Square 1	$4y - 4 = 0$	$3x - y = 2$	$4x + y = -1$	$7x - 2y = -10$
Row 2, Square 2	$y = -4$	$y = -x + 6$	$y = 2/3x + 8/3$	$y = 3/4x + 6$
Row 2, Square 3	$2x + y = 4$	$3x - 2y = 6$	$4x - 2y = 8$	$x - y = 6$
Row 2, Square 4	$3y = x + 4$	$y = 4/3x - 6$	$3x + 4y = 20$	$x - 5y = 15$
Row 3, Square 1	$y = 2/3x - 12$	$4x - 3y = 27$	$y = x$	$3y = x + 12$
Row 3, Square 2	$x - y = 11$	$2x + y = -2$	$2x - 3y = 36$	$y = 2/3x - 8/3$
Row 3, Square 3	$y = x$	$y = 7/2x + 5$	$y = 2x - 4$	$y = 1/3x + 4$
Row 3, Square 4	$y = -5x + 4$	$x - 3y = -21$	$y = -3x$	$y = 8x - 2$
Row 4, Square 1	$3x - y = 0$	$x + y = 6x$	$y = 4x - 1$	$-x + y = 13$
Row 4, Square 2	$-y = 3x$	$y = x + 6$	$x + y = 6$	$x + \frac{1}{4} = y$
Row 4, Square 3	$4x - 4y = -1$	$2y = 7x + 5$	$y = 3/2x - 3$	$y = -3/4x + 5$
Row 4, Square 4	$x - y = 0$	$4x + 2y = -10$	$y = 3x$	$y = 4$

Inequalities

Organizing topic Relations and Functions: Linear

Overview Students consolidate the concepts of graphing a line with solving an inequality in one variable.

Related Standard of Learning A.6

Objective

- The student will graph an inequality in two variables.

Materials needed

- Graphing calculators
- A "Squares Game: Solving Inequalities" handout for each student

Instructional activity

1. Review solving inequalities in one variable and graphing the solution set.
2. Review graphing a line.
3. Compare the open and closed points used in graphing the solution of an inequality in one variable to the dashed and solid boundary lines used in graphing the solution to an inequality in two variables.
4. Compare shading in the plane to solution of an inequality in one variable.
5. Have the students play the "Squares Game: Solving Inequalities."

Sample assessment

- The teacher should provide opportunities for students to practice.

Squares Game: Solving Inequalities

1. Cut the squares apart.
2. Match equivalent expressions or sentences.
3. You should get a new four-by-four square.

Square	Top	Left	Right	Bottom
Row 1, Square 1	$x \geq 2$	$x \geq 6$	$x < 17$	$-5x < -35$
Row 1, Square 2	$x < 10$	$-4 < 12$	$x \geq -1$	$x > 5$
Row 1, Square 3	$x > 15$	$x > -8$	$8 > 5 - x$	$x \geq -2$
Row 1, Square 4	$5 - 2x \leq 3x - 15$	$-8x \leq 64$	$x > 5$	$x > 5$
Row 2, Square 1	$2x < 10$	$x \geq 5$	$-4x + 2 \leq -22$	$x \leq -6$
Row 2, Square 2	$x < 21$	$1 \leq x$	$x < -3$	$x - 3 < 12$
Row 2, Square 3	$x > -5$	$x < 18$	$x \geq 5$	$4x < -24$
Row 2, Square 4	$x + 1 < -8$	$x \leq 1$		$1 - x < 1$
Row 3, Square 1	$2/3\ x < 12$	$2x + 3 \geq 13$	$x < 10$	$5x - 8 < -2$
Row 3, Square 2	$x - 4 < 6$	$-3x > 12$	$x \geq 5$	$x < 23$
Row 3, Square 3	$x \geq 4$	$x \geq 2$	$x \leq 0$	$x + 4 < -9$
Row 3, Square 4	$4 - 2x \leq 8$	$2x - 7 \leq 9$	$-x/2 + 3 \geq 0$	$x < -4$
Row 4, Square 1	$x < 15$	$x \leq -12$	$x > 0$	$2/5x - 6 \leq -6$
Row 4, Square 2	$x \leq 1$	$x/4 - 1 > 5$	$x < 0$	$x > -3$
Row 4, Square 3	$11 - 2x \geq 3x + 16$	$x < 0$	$13 + 4x \geq 9$	$x \leq 6$
Row 4, Square 4	$x \leq -1$	$x \geq -8$	$x < 2$	$-3/4x \geq 9$

Equations of Lines

Organizing topic	Relations and Functions: Linear
Overview	Students practice writing the equations of lines.
Related Standard of Learning	A.8

Objective

- The student will write the equation of a line, given
  - the coordinates of two points on the line
  - the slope and the *y*-intercept of the line
  - one point on a vertical line
  - one point on a horizontal line
  - the graph of the line.

Materials needed

- A copy of each of the three “Squares Game” handouts for each student

Instructional activity

- Have the students play the three “Squares Games,” as shown on the handouts.

Squares Game 1: What is the equation of the line if…?

1. Cut the squares apart.
2. Match each equation to the corresponding solution.
3. You should get a new four-by-four square.

	$x = 5$			$y = -2/3x + 9$			$y = 4x + 7$			$y = -2/3x - 9$	
$y = x + 5$		$m = 2$ $b = -6$	$y = -x + 4$		$m = 3$ $b = -10$	$y = 2x - 4$		(3, 2) (6, 0)	$y = -3/4x + 7$		$m = -6$ $b = 2$
	$m = 4$ $b = 7$			$m = 3$ (–3,1)			$y = 7$			$m = -1$ (5,–2)	
	$y = 2x - 4$			$y = \frac{3}{4}x - 7$			$y = -x + 3$			$y = 4x + 7$	
$x = 3$		(5,–4) (–1,–4)	$y = \frac{1}{2}x + 9$	$b = -3$	m: undefined	$y = 3x - 10$	x–intercept: –3	m: undefined	$y = -2x - 8$	x–intercept: 6	$m = -2/3$
	(–4,0) (3,3)			$m = -2$ (1,4)			$m = 4$ (0,7)			(–2,1) (2,3)	
	$y = -3/2x + 9$			$y = 3x + 10$			$y = \frac{1}{2}x + 2$			$y = -2x + 6$	
$y = -2x - 3$		$m = -3/4$ $b = 7$	$y = 5$	x–intercept: –4	$m = -2$	$y = -4$		(4,3) (4,–1)	$y = 4$		$m = 4$ $b = 0$
	$m = -2/3$ (6,5)			(0,–4) (2,0)			(5,0) (10, –2)			m is undefined (5,8)	
	$y = \frac{1}{2}x + 4$			$y = 8$			$y = 1/3x + 10$			$y = 2/3x + 5$	
$x = -3$		$m = 0$ $b = 4$	$y = -2/3\,x + 4$		(5,0) (0,5)	$y = -6x + 2$		$m = \frac{1}{2}$ $b = 9$	$x = 4$		(0,–8) (4,0)
	$m = 0$ (5,8)			(3,7) (0,5)			$m = \frac{1}{2}$ (4,6)			(7,1) (7,6)	

Squares Game 2: Finding slope and *y*-intercept

1. Cut the squares apart.
2. Match each equation to its slope and *y*-intercept.
3. You should get a new four-by-four square.

Square	Top	Left	Right	Bottom
1	$y = -3/4x + 5$	m = 1 b = 1	$20x = 4y - 4$	$x - y = 6$
2	m = 1 b = 0	m = 5 b = 1	$6y - 3x = 12$	$-2x - y = -7$
3	$3x + 4y + 20$	m = ½ b = 2	$-7x + y = 8$	$y = 4 - 2x$
4	$y - 8 + 2x$	m = 7 b = 8	m = ½ b = –3	$2y = 7x + 10$
5	m = –1 b = –6	$y = 4/3x - 9$	$x + y - 5 = 0$	$-2x + y = -4$
6	m = –2 b = 7	m = –1 b = 5	$x + 4y - 4 = 0$	$y = -x + 6$
7	m = –2 b = 4	m = ¼ b = 1	$5x + y = 4$	$7x - y = 14$
8	m = 7/2 b = 5	m = –5 b = 4	$-x + y = 13$	$y = 2x + 7$
9	m = 2 b = –4	m = 10 b = 5	$4x + 8y = 24$	$3x - 2y = 6$
10	m = –1 b = 6	m = –½ b = 3	$y = 3x$	$x - 5y = 15$
11	m = 7 b = –14	m = 3 b = 0	$y = 2x - 3$	$3x + 4y = 24$
12	m = 2 b = 7	m = 2/5 b = –3/5	$x - y = 11$	$2x + y = -2$
13	m = 3/2 b = –3	$7x = 4y + 25$	$3x + 2y = 6$	$2x - y = 8$
14	m = 1/5 b = –3	m = –3/2 b = 3	$-2x - y = -7$	$y = 4x - 1$
15	m = –3/4 b = 6	m = –2 b = 7	$y = 3x - 2 = -2$	m = 6/5 b = 2
16	m = –2 b = –2	m = 3 b = –2	m = 5 b = ½	$-4x - y = 1$

Squares Game 3: Finding *x*- or *y*-intercepts

1. Cut the squares apart.
2. Match equivalent expressions or sentences.
3. You should get a new four-by-four square.

	$2y = 7x + 10$			$y = -3$			$x = -1/6$			$x + 2y = 8$	
$x = -11$		$3y - 5x = 4$	$x + y = -5$		$x = 6/5$	$y = 5$		$y = -x + 6$	$-2x + y = -4$		$x = 0$
	$y = 1/3$			$x + 5y = 15$			$x + 5y = 11$			$x = -4/5$	
	$y = 3$			$y = 1$			$5x - y = 6$			$-2x - y = -4$	
$x - 2y = 8$		$x = 5/6$	$x = 2$		$3x + 4y = 24$	$y = 3x - 4$		$y = 4$	$x = -5$		$x = 6$
	$y = ½$			$y = -1$			$y = 12/5$			$x = 8$	
	$-3x + 5y = 12$			$x = 4/3$			$y = 4 - 2x$			$x = -8$	
$y = -3x + 3$		$x = 6$	$-3x + y = 5$		$y = -4$	$2x - y = 8$		$x = 5$	$y = -4/5$		$y = 4$
	$y = -5$			$x = 2/3$			$x = 8$			$x = 21$	
	$x = 1$			$x = 2$			$x = 15$			$x + 4y = 4$	
$y = 0$		$x = 2$	$y - x = 6$		$x = 11$	$y = -1/3x + 7$		$y = 5$	$x - 5y = 15$		$y = -8$
	$4x - 2y = 8$			$y = 4 - 2x$			$y = 7$			$x = -6$	

Line of Best Fit

Organizing topic Relations and Functions: Linear
Overview Students develop a best fit line for data.
Related Standard of Learning A.8

Objective

- The student will, using the equation of the best fit line, make predictions about unknown outcomes.

Instructional activity

Year	Fare
1953	0.15
1966	0.20
1970	0.30
1972	0.35
1975	0.50
1981	0.75
1984	0.90
1986	1.00
1990	1.15
1992	1.25

1. The table at right shows the New York subway fare increases and the years in which the increases occurred. Graph the data (year, fare) as ordered pairs on the graphing calculator. Use only the last two digits of the year. You will need to adjust your viewing window.
2. Visualize a line that best fits the data.
3. Try to write an equation for a line that best describes the data. Enter your equation into $Y1$, and graph your line.
4. Make adjustments as necessary to your equation, and check your graph. Record your equations below.
5. Enter the year data into L1 and fare data into L2. Use only the last two digits of the years.
   - STAT Edit
   - 2nd $Y =$ Turn STAT PLOT 1 on
   - ENTER
   - Be sure scattergraph is selected; *X*list is L1; *Y*list is L2.
   - Use a window of [50, 100] with a scale of 10 and [0, 2.0] with a scale of 0.5.
   - Clear all equations from $Y =$ and GRAPH.

	Equation
$Y1$	
$Y2$	
$Y3$	
$Y4$	
$Y5$	
$Y6$	

6. To find an equation for a line of best fit, STAT CALC 4 2nd L1, 2nd L2 ENTER
7. On the screen is the equation $y = ax + b$ with values for a and b. Record the values to thousandths.
8. Graph the line of best fit $Y =$ VARS 5 EQ ENTER GRAPH
9. How does the calculator's best fit line compare with yours?

Sample assessment

- Use the median-median line of best fit method to find the equation of a best fit line. The method is on the STAT CALC menu. Find out how the median-median method works.

Follow-up/extension

- Compare and contrast the 3 equations you have generated (yours, the calculator linear regression, and the calculator median-median) and the graphs of the 3 equations.

Direct Variation

Organizing topic Relations and Functions: Linear
Overview Students are introduced to direct variation.
Related Standard of Learning A.18

Objectives

- The student will recognize and describe direct variation from a graph or a table of values.
- The student will, given a set of data, determine whether a direct relationship exists and write the equation for the variation, if one exists.

Instructional activity

1. A low flow showerhead uses 2.5 gallons of water per minute. Create a table of values for time (in minutes) spent in the shower and the amount of water used.

Time (in minutes)	Water Used (in gallons)
1	
2	
3	
4	
5	
6	
7	
8	

2. What is the amount of water used at time zero? How does this relate to the y-intercept of the graph of this function?
3. Write an equation that describes this relationship.
4. Which is the dependent variable? Which is the independent variable?
5. Translate your equation into x and y. What is the slope? What is the y-intercept?
6. What is the coefficient of x?
7. This is an example of a direct variation. The equation of any direct variation can be written as $y = kx$. What is the slope, and what is the y-intercept of this equation?

Functions I

Organizing topic Relations and Functions: Linear

Overview Students develop the concepts of function and domain and range.

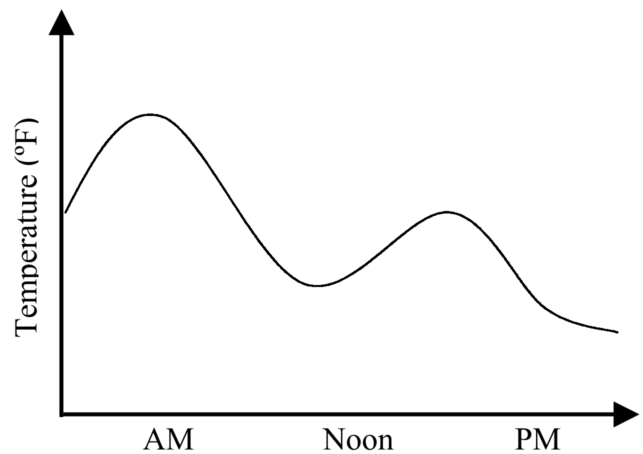
Related Standards of Learning A.5, A.15

Objectives

- The student will identify the domain and range for a linear relation when given a set of ordered pairs, a table, a mapping, or a graph.
- The student will identify whether or not a linear relation is a function and justify the classification.
- The student will compare and contrast relations and functions.

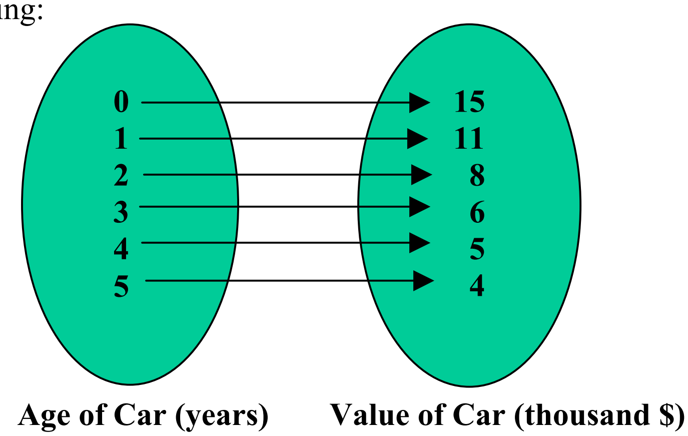
Instructional activity

1. Review domain and range from Grade 8. Relate domain and range to dependent and independent variables.
2. Present the students with this scenario: Eleanor stayed home from school yesterday because she had a fever. The graph at right charts Eleanor's temperature through the day.
3. Have the students write a story about the course of Eleanor's illness yesterday.
4. Have the students identify a reasonable domain and range for this function.

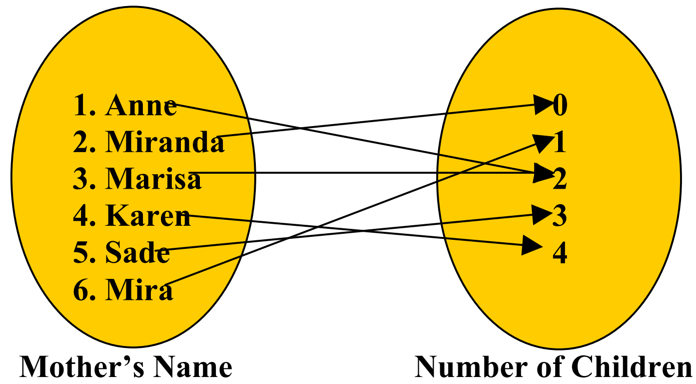


Age of Car (years)	Value of Car (dollars)
0	15,000
1	11,000
2	8,000
3	6,000
4	5,000
5	4,000

5. The value of a car decreases every year that it is used. For the data in the chart at left, what is the dependent variable? What is the independent variable?
6. Which variable represents the domain? Which variable represents the range?
7. Mapping:



8. Using the data at right showing that the number of children is a function of the who the mother is, map (1, 2), (2, 0), (3, 2), (4, 4), (5, 3), and (6, 1) as ordered pairs. What is the domain? What is the range? Which is the dependent variable? Which is the independent variable?



9. What is a *function* in mathematics? (A *function* is a relation in which each element of the domain is matched to exactly one element in the range.) Are all relationships mathematical functions?
10. Provide the students with examples of functions and non-function relations in a variety of formats (table, set of ordered pairs, mapping, graph), and have students identify those that are functions and those that are not. Have students explain their reasoning by referring to the definition of function.

Follow-up/extension

- Show students the vertical line test. Have them explain why it works.

Homework

- Compare and contrast relations and functions.

Square Patio

Organizing topic Relations and Functions: Linear

Overview Students construct square patios, collect the data, and analyze the data involving polynomial addition; domain, range and function; matrices and organization; finding the equation of a line; and matrix multiplication.

Related Standards of Learning A.5, A.15

Objectives

- The student will analyze a set of data for the existence of a pattern that defines the change relating input and output values.
- The student will represent the pattern symbolically and graphically.
- The student will determine whether or not the relation is a function.

Materials needed

- Graphing calculators
- Materials for building square patio models:
  - Corners: marshmallows
  - Border stabilizers: colored toothpicks
  - Frames: wooden toothpicks
  - Tiles: squares cut to the length of the wooden toothpicks
- A “Square Patio” handout for each student

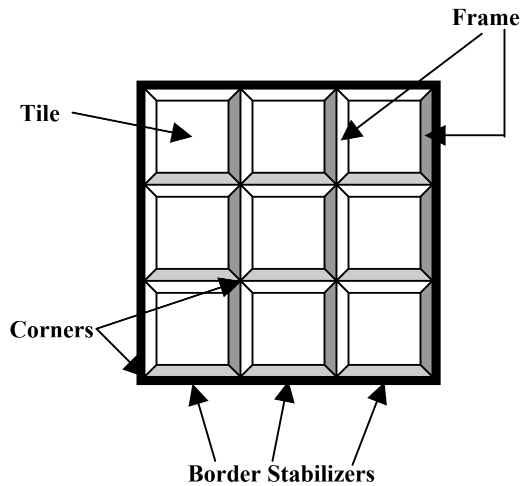
Instructional activity

1. Have students build square patios and record their data in the chart on the handout.
2. Ask students if they can see any patterns and predict the patterns for sizes they have not built in their models.
3. Work through the problems, and make sure students come to the point of being able to write an expression for the number of tiles needed for any given (n) dimension.
4. Have students investigate the relationship between the number of border stabilizers and the number of corners or the size of the patio. Make sure they are able to represent that relationship algebraically.
5. Move to the relationship between the number of corners and the number of frames, then to the relationship between the patio size and the number of frames, and then to the relationship between the patio size and the number of corners.
6. Help students develop prices for the tiles, corners, border stabilizers, and frames. Have students use these prices to set up matrices to determine the cost of building a “square patio” of any size.

Sample assessment

- Ask students to find the number of tiles, corners, frames, and border stabilizers needed for a 21-by-21 patio.

Square Patio



Patio Dimension	Tiles Needed	Frames Needed	Corners Needed	Border Stabilizers Needed
1-by-1				
2-by-2				
3-by-3				
4-by-4				
5-by-5				
6-by-6				
7-by-7				
8-by-8				

1. Build square patios, and record your data in the chart above.
2. Can you see any patterns and predict the patterns for sizes you have not built in your models?
3. How many tiles would you need for a 10-by-10 patio?
4. How many tiles would you need for a 20-by-20 patio?
5. How many tiles would you need for an n-by-n patio?
6. Write an expression for the number of tiles needed for any given dimension (n).
7. Does the number of border stabilizers depend on the number of corners or on the size of the patio?
8. What is the pattern that describes the relationship between the size of the patio and the number of border stabilizers needed?
9. What would be the rule (or formula) for that relationship?
10. Does it seem reasonable that the number of corners and the number of frames also *depend* on the patio size? Why?
11. Can you find a relationship between the patio size and the number of frames? (You may want to consider looking at the number of tiles for a hint.)
12. Can you find a relationship between the patio size and the number of corners?

Functions 2

Organizing topic Relations and Functions: Linear
Overview Students extend concepts associated with functions.
Related Standards of Learning A.5, A.15

Objectives

- The student will find $f(x)$ for each value x in the domain of f.
- The student will identify the zeros of the function algebraically and graphically.

Instructional activity

1. Equations that represent functions can be written in function notation, where x represents domain elements and $f(x)$ represents range elements:

Equation	Function Notation
$y = 3x - 7$	$f(x) = 3x - 7$

2. Consider the set of ordered pairs: (*, &) (=, *) (3, ÷) (x, =) (▲, *).
   - Identify the domain and range.
   - Is the relation a function?
   - Draw the function as a mapping.
   - What is the value of $f(*)$?
   - What is the value of $f(3)$?
3. Consider the function $f(x) = 3x - 7$.
   - Is the relation a function?
   - Identify the domain and range.
   - What is the value of $f(6)$?
   - Find $f(-2)$.
   - Find $f(0.31)$.
4. What is the equation of the x-axis? What do all the points on the x-axis have in common?
5. Solve $y = 3x - 7$. Then graph $Y1 = 3x - 7$. Trace to find the coordinates of the point where the line crosses the x-axis.
6. Solve $2x - 3.4y = 17$. Then solve the equation for y and graph the line. Trace to find the coordinates of the point where the line crosses the x-axis.
7. Create 5 equations for a partner to solve algebraically and graphically. The solutions are called zeros.

Follow-up/extension

- Why are the solutions of an equation called zeros?

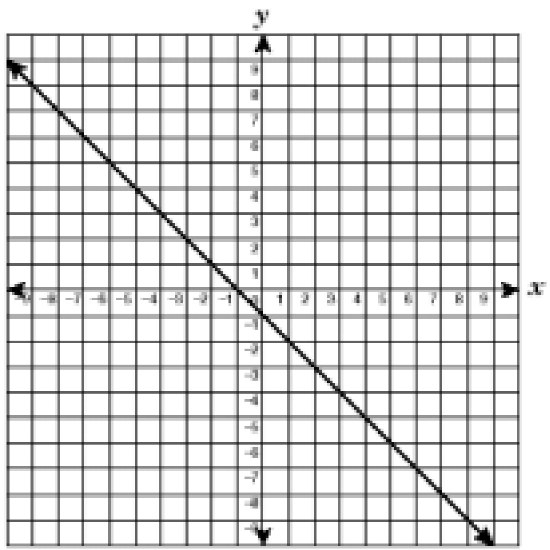
Sample resources

Mathematics SOL Curriculum Framework
http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html

SOL Test Blueprints

Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html
Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra
NASA http://spacelink.nasa.gov/.index.html
The Math Forum http://forum.swarthmore.edu/
4teachers http://www.4teachers.org
Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm
Eisenhower National Clearinghouse http://www.enc.org/

Sample assessment



The line on the grid is best described by the equation —

A $y = x + 1$
B $y = x - 1$
C $y = {}^-x + 1$
D $y = {}^-x - 1$

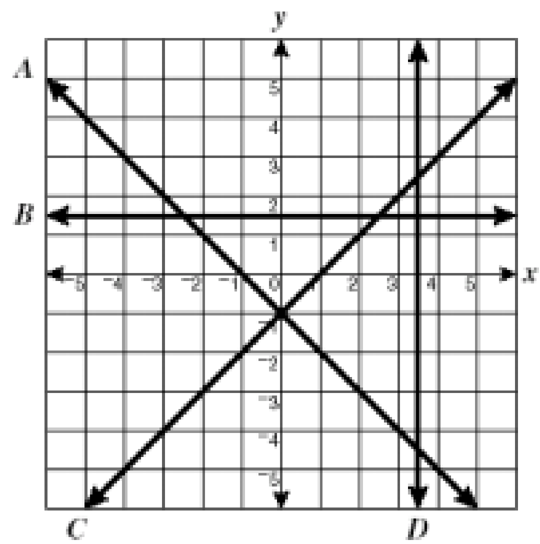
What is the slope of the line that contains (4, ⁻1) and (3, 3)?

F $^-4$

G $^-\frac{1}{2}$

H $^-\frac{1}{4}$

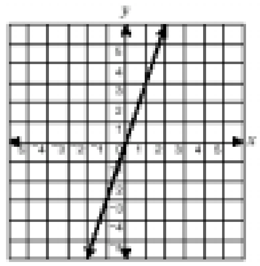
J 2



Which line on the graph has an undefined slope?

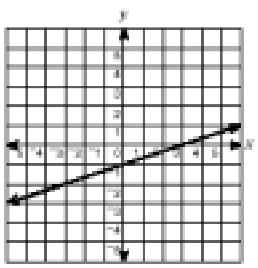
A *A*
B *B*
C *C*
D *D*

The graph below represents the equation $y = 3x$.

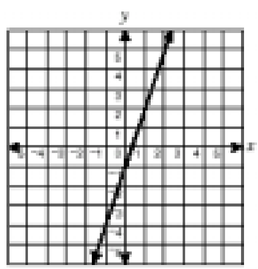


Which graph best represents $y = 3x - 1$?

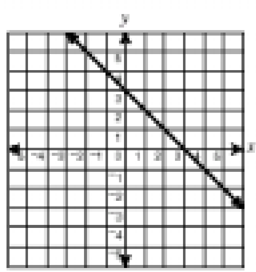
F



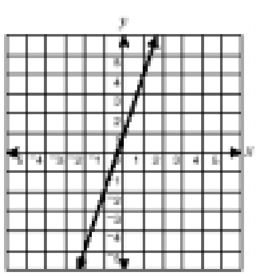
G

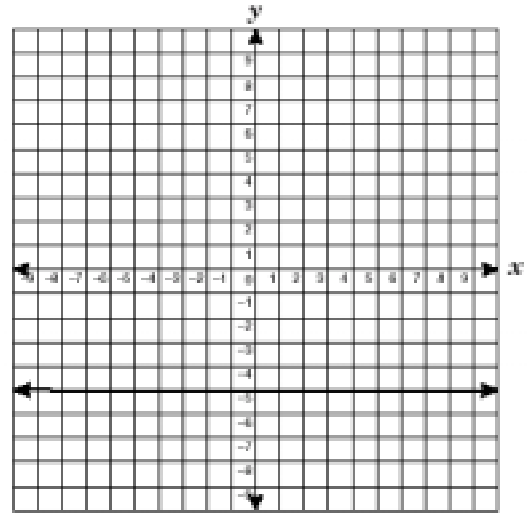


H



J





Which equation best describes this graph?

F $x = 5y$

G $x = {}^{-}5$

H $y = {}^{-}5x$

J $y = {}^{-}5$

Which is an equation for the line that contains the points (${}^{-}2$, 3) and (2, ${}^{-}1$)?

A $y = x + 5$

B $y = x - 3$

C $y = {}^{-}x + 1$

D $y = {}^{-}2x - 1$

x	y
0	4
3	1
6	${}^{-}2$

Which equation *most* likely describes the relation indicated by the table?

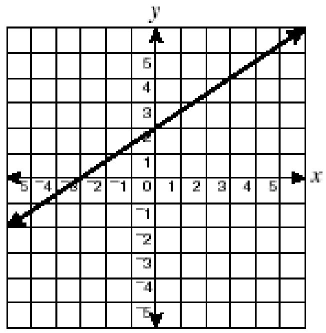
F $y = x + 4$

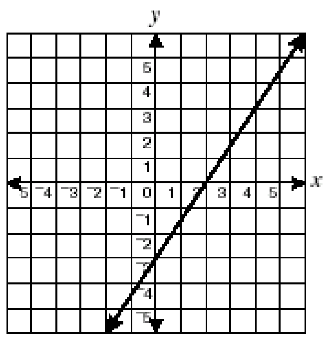
G $y = x - 2$

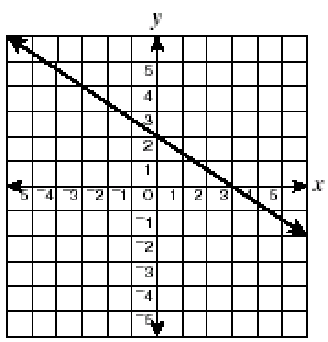
H $y = {}^{-}x + 4$

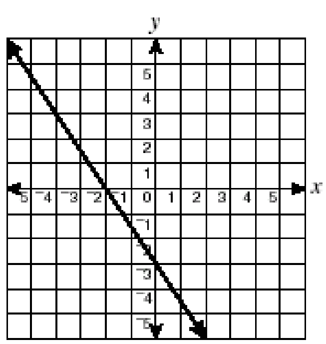
J $y = {}^{-}x - 8$

Which line has y-intercept $^{-}3$ and x-intercept 2?

A


B


C


D


The table shows the relationship between the cost, c, in dollars of a taxi ride and the number, t, of minutes the ride lasts.

t	5	10	15	20
c	4.75	6.5	8.25	10

Which equation algebraically represents this data?

A $c = 3 + 0.35t$

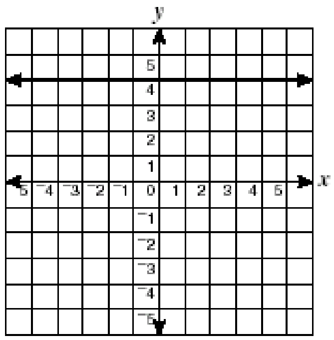
B $c = 2.75 + 0.5t$

C $c = t - 0.25$

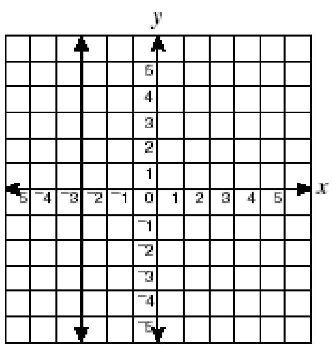
D $c = 4 + 0.15t$

In which graph is y a direct variation of x?

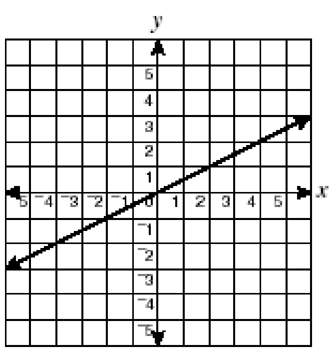
F



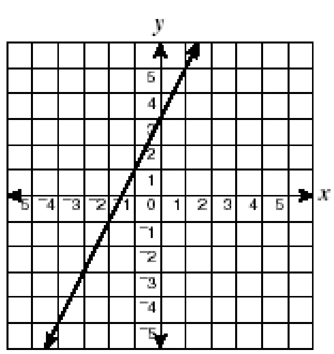
G



H



J



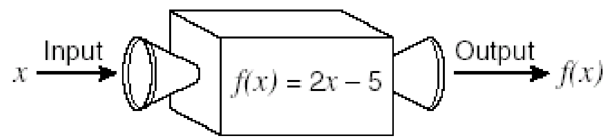
Which of the following sets of ordered pairs is a function?

A $\{(2, 1), (2, 2), (3, 4), (5, 6)\}$

B $\{(-2, -1), (1, 2), (3, 4), (1, 5)\}$

C $\{(1, 2), (2, 2), (3, 3), (2, 4)\}$

D $\{(1, 1), (2, 1), (3, 2), (4, 4)\}$



Using the function machine from the diagram, what is $f(10)$?

F 5

G 7.5

H 15

J 25

Which is a zero of the function $f(x) = x^2 + 3x - 4$?

A -4

B -1

C 3

D 4

What is the range of the function $f(x) = \frac{1}{2}x + 5$ when the domain is $\{2, 4, 6\}$?

F $\{-6, -2, 2\}$

G $\{6, 7, 8\}$

H $\{2, 4, 6\}$

J $\{1, 3, 5\}$

Organizing Topic Relations and Functions: Quadratic

Standards of Learning

A.5 The student will create and use tabular, symbolic, graphical, verbal, and physical representations to analyze a given set of data for the existence of a pattern, determine the domain and range of relations, and identify the relations that are functions.

A.10 The student will apply the laws of exponents to perform operations on expressions with integral exponents, using scientific notation when appropriate.

A.11 The student will add, subtract, and multiply polynomials and divide polynomials with monomial divisors, using concrete objects, pictorial and area representations, and algebraic manipulations.

A.12 The student will factor completely first- and second-degree binomials and trinomials in one or two variables. The graphing calculator will be used as a tool for factoring and for confirming algebraic factorizations.

A.13 The student will express the square root of a whole number in simplest radical form and approximate square roots to the nearest tenth.

A.14 The student will solve quadratic equations in one variable both algebraically and graphically. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.15 The student will, given a rule, find the values of a function for elements in its domain and locate the zeros of the function both algebraically and with a graphing calculator. The value of $f(x)$ will be related to the ordinate on the graph.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Identify the base, exponent, and coefficient of a monomial.	____________________
• Recognize polynomials and classify them according to the number of terms they have or by the degree of the polynomial.	____________________
• Develop the laws of exponents using the graphing calculator and pattern recognition. Generalize the patterns and represent the patterns symbolically.	____________________
• Express very large numbers and very small numbers using scientific notation.	____________________
• Add, subtract, multiply, and divide numbers written in scientific notation.	____________________
• Use pattern recognition and algebra manipulatives to develop strategies for adding, subtracting, and multiplying monomials, binomials, and polynomials.	____________________
• Find the quotient of polynomials.	____________________
• Simplify expressions involving polynomials.	____________________

- Identify common factors in polynomials expressions and use the distributive property to rewrite the expressions in factored form. ________________________
- Use the patterns recognized and generalized in multiplying polynomials to develop patterns in reverse for factoring. ________________________
- Factor first and second degree polynomials completely. ________________________
- Solve quadratic equations graphically by graphing the equation and finding the x-intercepts of the graph. The x-intercepts are the real roots or solutions of the equation. ________________________
- Solve quadratic equations algebraically by factoring (zero product property) or by using the quadratic formula. Factors may be found algebraically and confirmed graphically or may be found graphically and confirmed algebraically. ________________________
- Estimate the square root of a number to the nearest tenth. ________________________
- Write the square root of a whole number in simplest radical form. ________________________
- Use a calculator to find decimal approximations of radical expressions. ________________________
- Determine the domain and range of quadratic relations. ________________________
- For values of x in the domain of a function, find the associated values of f(x) in the range of the function. ________________________

Mo and Poly Nomial

Organizing topic	Relations and Functions: Quadratic
Overview	Students review mathematical terms associated with polynomials.
Related Standard of Learning	A.10

Objectives

- The student will identify the base, exponent, and coefficient of a monomial.
- The student will recognize polynomials and classify them according to the number of terms they have or by the degree of the polynomial.

Materials needed

- Computer lab with a computer for each pair of students or a computer with a projection system

Instructional activity

1. Have the students go to the Math Forum Web site (sponsored by Drexel University) at http://mathforum.org.
2. Have them find the "Ask Dr. Math" section of the Web site. Dr. Math answers questions for anyone who cares to submit.
3. Ask them to read and take notes on the articles entitled "Monomials, Polynomials," http://mathforum.org/library/drmath/view/57441.html and "Polynomials: Terms, Exponents, Degrees," http://mathforum.org/library/drmath/view/60414.html.
4. Lead a discussion of the students' notes, and provide opportunities for students to practice with their definitions.

Exponents

Organizing topic Relations and Functions: Quadratic
Overview Students develop concepts of exponents.
Related Standard of Learning A.10

Objectives

- The student will develop the laws of exponents, using the graphing calculator and pattern recognition.
- The student will generalize the patterns and represent the patterns symbolically.

Materials needed

- Graphing calculators
- Computer for each group of students or a computer with a projection device

Instructional activity

1. Review the definition of *exponent* from middle school.
2. Using the definition of *exponent*, allow students to explore multiplying monomials with exponents. Use the table below as a model.

$2^3 * 2^5$	2 * 2 * 2 * 2 * 2 * 2 * 2 * 2	$2^8 = 256$	8 * 32 = 256
$4^5 * 4^2$			
$3^1 * 3^6$			
$x^4 * x^7$			

3. Assessment: What pattern do you see? Write it using algebraic symbols only.

4. Now explore another “law of exponents.”

$(2^3)^4$	(2 * 2 * 2) * (2 * 2 * 2) * (2 * 2 * 2) * (2 * 2 * 2)	$(2^3)^4 = 4{,}096$	$(8)^4 = 4{,}096$
$(4^5)^2$			
$(3^1)^7$			
$(x^4)^9$			

5. Assessment: What pattern do you see? Write it using algebraic symbols only.
6. What happens when monomials raised to a power are *divided*?
7. Work with a partner to design an experiment similar to the two you have already done. Execute your experiment and write up your results. Be prepared to share your work with the class.
8. What does it mean when exponents are *negative*?
9. Work with a partner to design an experiment similar to the three you have already done. Be sure that the exponents in the denominator are greater than the exponents in the numerator. Execute your experiment and write up your results. Be prepared to share your work with the class.

Follow-up/extension

- Have the students go to the Math Forum Web site (sponsored by Drexel University) at http://mathforum.org/library/drmath/view/58201.html to read the article entitled “Explaining Exponents” and take notes.
- Have the students also read and take notes on the following articles:
  - “Exponents in the Real World,” http://mathforum.org/library/drmath/view/58203.html
  - “Exponent Divided by Another Exponent,” http://mathforum.org/library/drmath/view/58224.html
  - “Are Negative Exponents Like Other Exponents?” http://mathforum.org/library/drmath/view/61088.html.

Scientifically Speaking

Organizing topic	Relations and Functions: Quadratic
Overview	Students review basics of scientific notation and use numbers written in scientific notation in binary operations.
Related Standard of Learning	A.10

Objectives

- The student will express very large numbers and very small numbers, using scientific notation.
- The student will add, subtract, multiply and divide number written in scientific notation.

Materials needed

- Computer for each group of students or a computer with a projection device

Instructional activity 1

1. Have the students go to the Math Forum Web site (sponsored by Drexel University) at http://mathforum.org
2. Have them find the "Ask Dr. Math" section of the Web site. Dr. Math answers questions for anyone who cares to submit.
3. Ask them to read the article entitled "Scientific Notation," http://mathforum.org/library/drmath/view/58207.html, and take notes.
4. Have them also read and make notes on the following articles:
   - "Explaining Scientific Notation," http://mathforum.org/library/drmath/view/61563.html
   - "Scientific Notation in Everyday Life," http://www.math.toronto.edu/mathnet/questionCorner/scinot.html
5. Scientific notation is a case of exponents in action or an application of exponents. All the laws of exponents that you know still apply in scientific notation.

Instructional activity 2

Extreme Multiplication (From *Problems with a Point.* April 26, 2002. EDC ©2002)

Multiplying extremely large and small numbers together by hand doesn't necessarily require a lot of steps. As you work through the following problems, look for clever shortcuts. *Do not use a calculator for these problems.*

These first numbers aren't very large or small, but they should help you handle the next problem.

1. a. $223 \times 100 =$
   b. $223 \times 10{,}000 =$
   c. $223 \times 0.01 =$
   d. $223 \times 0.00001 =$

2. a. $223 \times 400 =$
   b. $223 \times 40{,}000 =$
   c. $223 \times 0.04 =$
   d. $223 \times 0.00004 =$

3. a. $2.23 \times 100 \times 400 =$
   b. $2.23 \times 100 \times 40{,}000 =$
   c. $2.23 \times 100 \times 0.04 =$
   d. $2.23 \times 100 \times 0.00004 =$

   Compare each of your answers to the answer for the previous problem. How are the answers similar? Why?
4. Rewrite problems 3a through 3d in the form $2.23 \times 10 \times 4 \times 10 =$.
5. Rewrite the answers to problems 3a through 3d in scientific notation.
6. Compare problems 3 through 5. Find a shortcut for multiplying two numbers written in scientific notation, and explain why the shortcut works.
7. Consider $3.23 \times 1{,}012 \times 4 \times 10^{-3} =$.
   a. Use your shortcut to find this product.
   b. Write the product in scientific notation, if necessary.
   c. If the product was not in scientific notation after you used your shortcut, why wasn't it?
8. Find the following products, giving the answers in scientific notation:
   a. $39{,}200{,}000 \times 720{,}000 =$
   b. $3.92 \times 10^{5} \times 0.0072 =$
   c. $3.92 \times 10^{-33} \times 7.2 \times 10^{-23} =$ (Take a little extra care with this one.)
   d. $7.2 \times 10^{14} \times 3.92 \times 10^{32} =$

Follow-up/extension

The Size of an Atom

(From *Problems with a Point.* April 24, 2002. EDC © 2001)

The smallest unit of an element, such as hydrogen, oxygen, or gold, is the atom. Each atom is made up of electrons, protons, and neutrons. Because these particles are so small, scientific notation is helpful when talking about chemistry. Write all your answers for the following problems in scientific notation, unless the directions say otherwise.

1. A proton has a mass of about 1.67252×10^{-24} grams. A single electron has about 9.1091×10^{-28} grams of mass.
   a. How many electrons does it take to equal the mass as one proton?
   b. A hydrogen atom has one proton and one electron. Find the mass of a hydrogen atom.
2. Oxygen atoms have 8 protons and 8 electrons.
   a. Find the mass of 8 protons.
   b. The most common type of oxygen atom has 8 neutrons. A neutron has mass of 1.67482×10^{-24} grams. Find the mass of 8 neutrons.
   c. Find the total mass of this type of oxygen atom.
   d. Oxygen is how many times more massive than hydrogen? Answer using standard notation, not scientific notation.
3. Water consists of small particles called molecules. One molecule of water contains two hydrogen atoms and one oxygen atom.
   a. Find the mass of one molecule of water.
   b. Water is how many times more massive than hydrogen? Answer using standard notation, not scientific notation.
   c. Compare your answer to part b with your answer to problem 2d.
4. One gram of hydrogen has about 6×10^{23} atoms. To help you understand just how much this is, consider that a small paper clip has about a gram of mass, which weighs about 2.205×10^{-3} pounds. For this problem, write your answers using both scientific notation and standard notation.

a. Some adults weigh around 150 pounds. About how many paper clips would it take to weigh that much?

b. A car might weigh around 3,000 pounds. How many times heavier is a car than a 150-pound person?

c. About how many paper clips are needed to weigh as much as a 3,000 pound car?

d. About how many 3,000-pound cars would it take to weigh as much as 6×10^{23} paper clips? Answer in scientific notation only.

d. There are almost 300 million people in the United States, including children.
   i. About how many cars (on average) would each person in the United States need to own for the combined weight to be as much as the weight of 6×10^{23} paper clips?
   ii. Compare your answer to question 4.d.i. to the number of people in the United States.

e. There are about 6 billion people in the world.
   i. About how many cars (on average) would each of those people need to own for the combined weight to be as much as the weight of 6×10^{23} paper clips? Remember, 6×10^{23} hydrogen atoms have about the same mass as only one paper clip.
   ii. Compare your answer to question 4.e.i. to the number of people in the United States.

Old Polly

Organizing topic	Relations and Functions: Quadratic
Overview	Students develop concepts involving polynomials by using Algeblocks™.
Related Standard of Learning	A.11

Objectives

- The student will use pattern recognition and algebra manipulatives to develop strategies for adding, subtracting, and multiplying monomials, binomials, and trinomials.
- The student will simplify expressions involving polynomials.

Materials needed

- Algeblocks™
- Handouts from Algeblocks™ Unit 7 — "Addition and Subtraction of Polynomials," Lessons 7-1, 7-2, 7-4, 7-5, 7-6
- Handouts from Algeblocks™ Unit 8 — "Multiplying with Variables," Lessons 8-1 through 8-7
- Handouts from Algeblocks™ Unit 9 — "Dividing with Variables," Lessons 9-1 through 9-3

Instructional activity

1. See the Teacher's Resource Binder for Algeblocks™. Be sure to study the Teacher's Notes for Units 7, 8, and 9.
2. Remember that students not only must manipulate the Algeblocks™ on the mats but also must draw the pictorial and symbolic representations.
3. Encourage students to use properties to justify their work whenever possible.

Sample assessment

- Included with Algeblocks™ activities.

Follow-up/extension

- Lessons 7-7, 8-7, and 9-4

Functionality

Organizing topic Relations and Functions: Quadratic

Overview Students reverse polynomial multiplication to develop patterns to use in factoring.

Related Standard of Learning A.12

Objectives

- The student will identify common factors in polynomial expressions and use the distributive property to rewrite the expressions in factored form.
- The student will use the patterns recognized and generalized in multiplying polynomials to develop patterns in reverse for factoring.
- The student will factor first and second degree polynomials completely.

Materials needed

- Graphing calculators
- Algeblocks™
- Handouts from Algeblocks™ Unit 9.5–9.11 — "Factoring Trinomials"
- Copies of the "Squares Game: Factoring" handout for each student

Instructional activity 1

1. See the Teacher's Resource Binder for Algeblocks™ . Review with students the beginning of Unit 9.
2. Remember that students need to work directly with the blocks. Students also need to draw the pictorial representations and write them in symbolic form. Since this a follow-up for the work that students have done with the Algeblocks™ before, much direct demonstration by the teacher should not be necessary. Students should be ready to work on their own, following a short review.

Follow-up/extension

- Create a matching activity on multiplying binomials and factoring trinomials for students to use for practice.

Instructional activity 2

1. Students should preview this activity by solving several linear equations. They may use Algeblocks™, the graphing calculator, and/or symbolic manipulation. The teacher should emphasize using the graphing calculator, locating the x-intercepts of the graph, and identifying the x-coordinate as the zero of the function.
2. Discuss the Fundamental Theorem of Algebra with students. Show students the graphs of a wide variety of polynomial functions, and have them discuss the possible number of zeros.
3. Allow students to graph polynomial functions and identify the zeros, using the functions of the graphing calculator.

Follow-up/extension

- Have the students play the "Squares Game: Factoring" on the handout.

Squares Game: Factoring

1. Cut the squares apart.
2. Match each equation to the corresponding solution.
3. You should get a new four-by-four square.

	$(x-2)(x+2)$			$(4x-1)^2$			$(6x+1)(x-2)$			$(x+1)(x-1)$	
$(5x-4)^2$		$x^2-4x-12$	$(x+2)(x-6)$		x^2-16	$(x+4)(x-4)$		$6x^2+13x+6$	$(3x+2)(2x+3)$		$x^2-14x+24$
	x^2+6x+9			$x^2-10x+24$			$25x^2-16$			$6x^2+41x+30$	
	$(x+3)^2$			$(x-4)(x-6)$			$(5x-4)(5x+4)$			$(x+6)(6x+5)$	
$(x-2)(x-9)$		$x^2+3x-18$	$(x-3)(x+6)$		$x^2+6x-16$	$(x-2)(x+8)$		$9x^2-12x+4$	$(3x+2)^2$		$x^2+7x-18$
	$4x^2-25$			x^2-9			$16x^2-1$			$x^2-7x+12$	
	$(2x+5)(2x-5)$			$(x+3)(x-3)$			$(4x-1)(4x+1)$			$(x-4)(x-3)$	
$(x+2)(x+5)$		$(x^2-6x-16)$	$(x+2)(x-8)$		$x^2-2x-15$	$(x+3)(x-5)$		$4x^2+x-5$	$(4x+5)(x-1)$		$6x^2-x-2$
	x^2+4x+3			$7x^2-19x+10$			$9x^2-4$			$x^2-8x+16$	
	$(x+3)(x+1)$			$(7x-5)(x-2)$			$(3x-2)(3x+2)$			$(x-4)^2$	
$(2x-5)(2x+1)$		$4x^2+20x+25$	$(2x+5)^2$		$3x^2+2x-1$	$(3x-1)(x+1)$		x^2-x-12	$(x+3)(x-4)$		x^2+16
	$25x^2+20x+4$			x^2+9			$x^2+3x-10$			x^2-15	

Factoring for Zeros

Organizing topic	Relations and Functions: Quadratic
Overview	Students identify the relationship between the roots or zeros of quadratics and the graph of the equation.
Related Standard of Learning	A.14

Objective

- The student will solve quadratic equations graphically by graphing the equation and finding the *x*-intercepts of the graph. The *x*-intercepts are the real roots or solutions of the equations.

Materials needed

- Graphing calculators
- A "Factoring for Zeros" handout for each student

Instructional activity

1. Distribute the handouts. Have students adjust their windows in the calculator as suggested for the beginning set up.
2. Have students work in pairs to investigate the graphs and the tables of the functions.
3. After the first 5 problems, discuss the questions that follow.
4. Continue with problems 6 through10.
5. Answer the questions that follow.
6. Repeat the process once again for questions 11 through 15 and for questions 17 through 20.

Factoring for Zeros

Set your viewing window on the calculator and table function as follows:

Window: [-8, 8] by [-8, 8] Table: TblStart = –8 ΔTbl = 1

Note: You *may* have to ADJUST your window in order to see the complete graph.

Part I

		GRAPH	TABLE
1.	Enter:	$Y_1 = x^2 + 4x + 3$	$Y_2 = (X + 1)(X + 3)$
2.	Enter:	$Y_1 = x^2 + 7x + 10$	$Y_2 = (X + 2)(X + 5)$
3.	Enter:	$Y_1 = x^2 + 7x + 6$	$Y_2 = (X + 6)(X + 1)$
4.	Enter:	$Y_1 = x^2 + 8x + 7$	$Y_2 = (X + 7)(X + 1)$
5.	Enter:	$Y_1 = x^2 + 10x + 9$	$Y_2 = (X + 9)(X + 1)$

In how many "places" did these graphs cross (intersect) the x-axis
What is the y-coordinate of a point on the x-axis?
When the function crosses the x-axis, what is the value of y?
What is the significance of the x-intercepts of the graphs?

Part II

		GRAPH	TABLE
6.	Enter:	$Y_1 = x^2 – 5x + 4$	$Y_2 = (X – 4)(X – 1)$
7.	Enter:	$Y_1 = x^2 – 8x + 15$	$Y_2 = (X – 5)(X – 3)$
8.	Enter:	$Y_1 = x^2 – 8x + 12$	$Y_2 = (X – 6)(X – 2)$
9.	Enter:	$Y_1 = x^2 – 9x + 14$	$Y_2 = (X – 7)(X – 2)$
10.	Enter:	$Y_1 = x^2 – 13x + 30$	$Y_2 = (X – 10)(X – 3)$

In how many "places" did these graphs cross (intersect) the x-axis?
What is the y-coordinate of a point on the x-axis?
When the function crosses the x-axis, what is the value of y?
What is the significance of the x-intercepts of the graphs?

Part III

		GRAPH	TABLE
11.	Enter:	$Y_1 = x^2 - 1$	$Y_2 = (X + 1)(X - 1)$
12.	Enter:	$Y_1 = x^2 - 4$	$Y_2 = (X + 2)(X - 2)$
13.	Enter:	$Y_1 = x^2 - 9$	$Y_2 = (X + 3)(X - 3)$
14.	Enter:	$Y_1 = x^2 - 25$	$Y_2 = (X + 5)(X - 5)$
15.	Enter:	$Y_1 = x^2 - 100$	$Y_2 = (X + 10)(X - 10)$

In how many "places" did these graphs cross (intersect) the x-axis?

What is the y-coordinate of a point on the x-axis?

When the function crosses the x-axis, what is the value of y?
What is the significance of the x-intercepts of the graphs?

Part IV

		GRAPH	TABLE
16.	Enter:	$Y_1 = x^2 - x - 2$	$Y_2 =$
17.	Enter:	$Y_1 = x^2 + x - 2$	$Y_2 =$
18.	Enter:	$Y_1 = x^2 - x - 6$	$Y_2 =$
19.	Enter:	$Y_1 = x^2 + x - 6$	$Y_2 =$
20.	Enter:	$Y_1 = x^2 - 3x - 10$	$Y_2 =$

In how many "places" did these graphs cross (intersect) the x-axis?

What is the y-coordinate of a point on the x-axis?

When the function crosses the x-axis, what is the value of y?

What is the significance of the x-intercepts of the graphs?

Journal: For all odd numbered questions, factor the quadratic expression completely. Compare the factors to the x-intercepts of the graph of the equation. What connection do you see?

Follow-up/extension

- If you know the x-intercepts of a quadratic graph, how would you use that information to write the factors of the equation?
- How do you use the factors to write the equation of the quadratic?

Estimating Square Roots

Organizing topic Relations and Functions: Quadratic

Overview Students use the graphing calculator to practice the inverse operations of squaring, and taking the square root of a number.

Related Standard of Learning A.13

Objective

- The student will estimate the square root of a number to the nearest tenth.

Materials needed

- Graphing calculators

Instructional activity

1. Have students estimate the square roots for the following and complete the table.

X	$\sqrt{X}$
17	
48	
85	

2. After students have taken the square roots of the numbers "manually," have them set up a table, using the List capability of the calculator. <ENTER>

L_1	L_2	L_3
L_1 (1) =		

3. Enter Xs in L_1

L_1	L_2	L_3
1 2 3 4 5 6 7		
L_2 (1) =		

4. Enter $\sqrt{L_1}$ into L_2

5. Correct results are as follows:

L_1	L_2	L_3
1 2 3 4 5 6 7	1 1.4142 1.7321 2 2.2361 2.4495 2.6458	
L_3 (1) =		

6. Ask students to use L_3 to "re-create" L_1 using L_2. Students should enter the data so that $L_3 = (L_2)^2$. If they do not, work through estimating square roots with which they are familiar (roots of 4, 9, 16, etc.), and have them try to enter these through the list function. Then move to those shown.
7. Have students then put the L_1 values in L_2, and have them estimate the results for L_1.

L_1	L_2	L_3
	1 2 3 4 5 6 7	
$L_1 = (L_2)^2$		

L_1	L_2	L_3
1 4 9 16 25 36 49	1 2 3 4 5 6 7	
L_1 (1) = 1		

Simplifying Square Roots

Organizing topic	Relations and Functions: Quadratic
Overview	Students simplify square roots.
Related Standard of Learning	A.13

Objectives

- The student will write the square root of a number in simplest radical form.
- The student will use a calculator to find decimal approximations of radical expressions.

Materials needed

- Graphing calculators
- Computer for each group of students or a computer with a projection device

Instructional activity

1. Have the students go to the Math Forum Web site (sponsored by Drexel University) at http://mathforum.org.
2. Have them find the "Ask Dr. Math" section of the Web site. Dr. Math answers questions for anyone who cares to submit.
3. Ask them to read the article entitled "Why Simplify Square Roots," http://mathforum.org/library/drmath/view/52294.html, and take notes.
4. Have them also read and take notes on the following articles:
   - "Simplifying Square Roots," http://mathforum.org/library/drmath/view/52617.html
   - "Simplifying Expressions with Radicals," http://mathforum.org/library/drmath/view/52641.html.
5. Lead a discussion of the articles that students have read, and allow students to synthesize a process for finding the square root of a number. Different groups of students may invent different methods for finding the square root of a number. As long as a method is mathematically correct, it should be considered valid by the teacher.
6. Lead students in extending the process they discovered, and have them include expressions with radicals as well.

Sample assessment

- Provide opportunities for students to practice finding the square root of a number. The graphing calculator should be used to verify results.

Follow-up/extension

- Have the students discover a method for finding the cube root of a number, and have them describe their method in detail. Have them give at least five examples of their method and verify their results with a calculator.

Domain and Range — Quadratics

Organizing topic Relations and Functions: Quadratic

Overview Students review domain and range of linear functions and extend their knowledge to quadratic functions.

Related Standards of Learning A.5, A.15

Objectives

- The student will determine the domain and range of quadratic relations.
- The student will, for values of x in the domain of the function, find the associated values of $f(x)$ in the range of the function.

Materials needed

- Graphing calculators

Instructional activity 1

Domain and range review

1. When a submarine descends into the ocean, the hull pressure increases, as shown in the table below. Have the students describe a function that relates the depth of the submarine and the pressure on its hull. They should provide their descriptions in words and in symbols.

Depth	0	300	600	900	1,200	1,500
Pressure	0	32	64	96	128	160

2. Have the students graph the function.
3. Have the students respond to the following questions:
   - What are the domain and range of the function?
   - How will the situation restrict the domain and range of the function?
   - When the submarine is submerged at 1,575 meters, what will be the pressure on the hull?
   - If the hull pressure is $240\dfrac{kg}{cm^2}$, at what depth is the submarine?
   - How does the pressure change as the submarine goes deeper?
   - What is the independent variable, and what does it represent?
   - What is the dependent variable, and what does it represent?
   - What is the rate of change of the hull pressure?

Instructional activity 2

1. Have the students go to the Math Forum Web site (sponsored by Drexel University) at http://mathforum.org.
2. Have them find the "Ask Dr. Math" section of the Web site. Dr. Math answers questions for anyone who cares to submit.

3. Ask them to read the article entitled “Domain and Range,” http://mathforum.org/library/drmath/view/54551.html, and take notes.
4. Have them also read and take notes on the following articles:
   - “Relations versus Functions,” http://mathforum.org/library/drmath/view/53154.html
   - “Functions and Equations,” http://mathforum.org/library/drmath/view/53236.html.

Instructional activity 3

- Provide opportunities for students to determine the domain and range of quadratic functions.

Follow-up/extension

- Extend the types of functions to which students are exposed. Include transcendental functions and higher order functions.

Sample resources

Mathematics SOL Curriculum Framework
 http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html

SOL Test Blueprints

Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html

Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra

NASA http://spacelink.nasa.gov/.index.html

The Math Forum http://forum.swarthmore.edu/

4teachers http://www.4teachers.org

Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm

Eisenhower National Clearinghouse http://www.enc.org/

Sample assessment

When completely factored, $3x^2 - 48$ equals —

A $3(x^2 - 48)$
B $3(x^2 + 16)$
C $3(x - 4)(x + 4)$
D $(3x - 16)(x + 3)$

When completely factored, $x^2 + x - 12$ equals —

A $(x + 3)(x - 4)$
B $(x + 4)(x - 3)$
C $(x + 7)(x - 5)$
D $(x + 12)(x - 1)$

The population of Asia is about 3.4×10^9. The population of Africa is about 7×10^8. About how many more people live in Asia than live in Africa?

F 27,000,000
G 270,000,000
H 360,000,000
J 2,700,000,000

Which expression is equivalent to $\frac{8x^4 - 2x^2}{2x^2}$?

F $4x^2$
G $6x^2$
H $4x^2 - 1$
J $6x^2 - 1$

Which is closest to the value of $3\sqrt{5}$?

A 3.9
B 6.7
C 7.5
D 8.7

Which is equivalent to $\frac{b^6}{b^2}$?

A $\frac{1}{b^3}$
B b^3
C b^4
D b^8

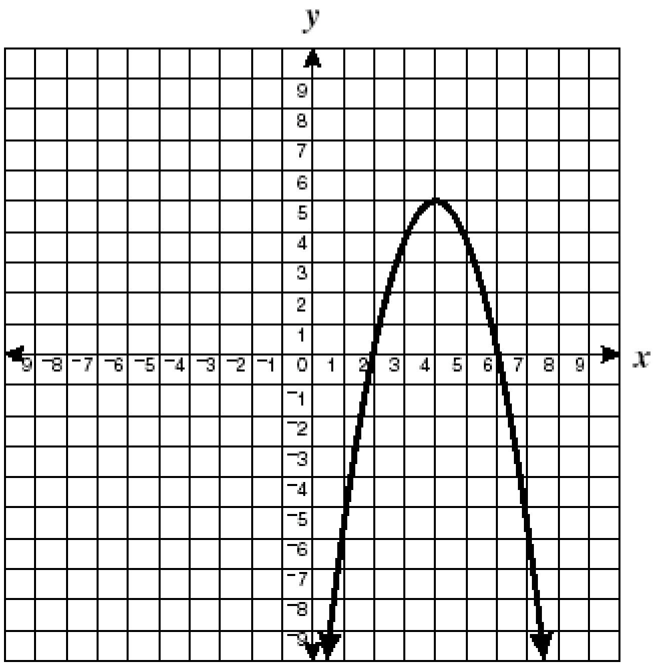
Which is closest to the value of $\sqrt{12} \cdot \sqrt{15}$?

F 52.0
G 13.5
H 13.4
J 6.7

Which is equivalent to $(5x^2 + 4x + 1) + ({}^-7x + 2)$?

A ${}^-2x^2 + 6x + 1$
B $5x^2 - 3x - 1$
C $5x^2 - 3x + 3$
D $5x^2 + 11x + 3$

The graph shows part of a function *f*.



What is the range of the function?

A All real numbers

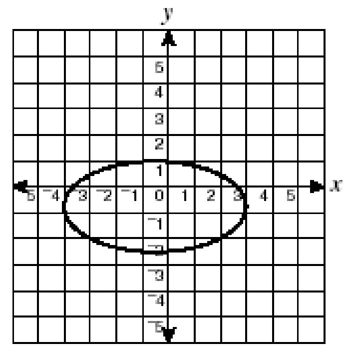
B All real numbers less than or equal to five

C All real numbers greater than zero

D All real numbers between 2 and 6

Which of the following represents the graph of a function?

F



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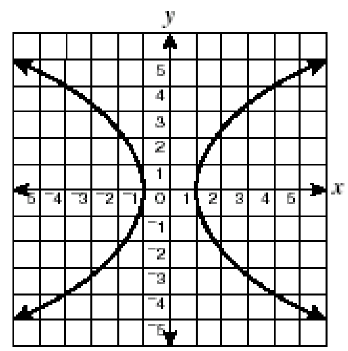

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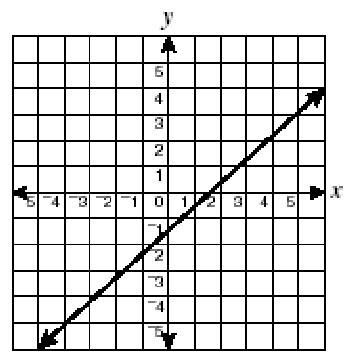

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