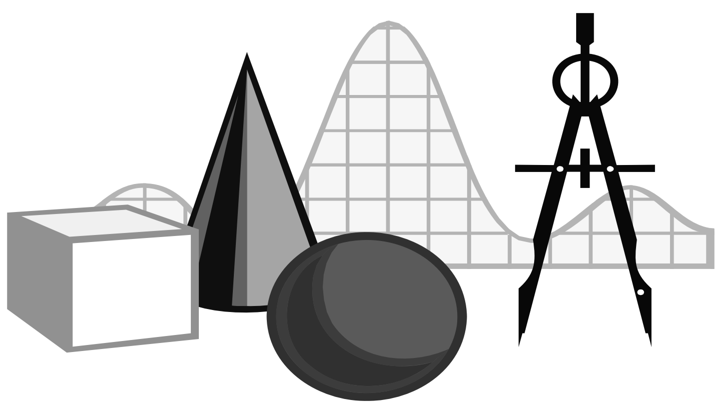
MATHEMATICS STANDARDS OF LEARNING ENHANCED SCOPE AND SEQUENCE

Algebra, Functions, and Data Analysis



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Introduction

The *Mathematics Standards of Learning Enhanced Scope and Sequence* is a resource intended to help teachers align their classroom instruction with the Mathematics Standards of Learning that were adopted by the Board of Education in October 2001 (January 2008 for Algebra, Functions, and Data Analysis). The Mathematics Enhanced Scope and Sequence is organized by topics from and includes the content of the Standards of Learning and the essential knowledge and skills from the Curriculum Framework. In addition, the Enhanced Scope and Sequence provides teachers with sample lesson plans that are aligned with the essential knowledge and skills in the Curriculum Framework.

School divisions and teachers can use the Enhanced Scope and Sequence as a resource for developing sound curricular and instructional programs. These materials are intended as examples of how the knowledge and skills might be presented to students in a sequence of lessons that has been aligned with the Standards of Learning. Teachers who use the Enhanced Scope and Sequence should correlate the essential knowledge and skills with available instructional resources as noted in the materials and determine the pacing of instruction as appropriate. This resource is not a complete curriculum and is neither required nor prescriptive, but it can be a valuable instructional tool.

The Enhanced Scope and Sequence contains the following:

- Essential knowledge and skills from the Mathematics Standards of Learning Curriculum Framework
- Related Standards of Learning
- Sample lesson plans containing
  - instructional activities;
  - follow-up/extensions; and
  - related resources.

Acknowledgments

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Organizing Topic: Linear Modeling

Mathematical Goals:

- Students will model linear relationships using a CBR™ (Calculator Based Ranger).
- Students will explore positive, negative and zero slopes as rates of change.
- Students will make predictions using the line (curve) of best fit for the modeled data as well as evaluating the mathematical model for specific values in the domain of the data.

Standards Addressed: AFDA.1; AFDA.2; AFDA.3; AFDA.4

Data Used: Data obtained using the CBR™, Internet Web sites, surveys, experiments and data provided in tables.

Materials:

- CBR™
- Applications: **EasyData™** and **Transformation™** Application
- Graphing calculator and links
- Handout – Up to Speed
- Handout – Investigating Slope, Equations, and Tables using the CBR™
- Handout – See Starbuck Run
- Handout – White Water Rafting on Silly Creek
- May also need graph paper

Instructional Activities:

I. Introduction--Up to Speed

Students will acknowledge the average rate of change by discussing speed as miles per hour. If a car is set on cruise control, how far will it travel in 1 hour, 2 hours, 5 hours?

Concepts covered include:

- domain and range;
- scatter plots;
- continuity;
- function;
- evaluation of a function for domain values;
- independent and dependent variables;
- rate of change (slope);
- slope-intercept form of an equation;
- linear regression;
- transformations; and
- direct variation.

II. Investigating Slope, Equations, and Tables using the CBR™

Students will physically model positive and negative lines by using the CBR™ and walking toward or away from the CBR™. Extension to the activity would be to model horizontal lines and have students attempt to model vertical lines.
Concepts covered include:

- scatter plots;
- domain and range;
- continuity;
- function;
- evaluation of a function for domain values;
- independent and dependent variables;
- rate of change (slope);
- slope-intercept form of an equation;
- linear regression;
- transformations; and
- direct variation.

III. See Starbuck Run

Students will be given data in a table that represent the results recorded as a horse runs. The data represent a positive rate of change but do not have a correlation coefficient of 1.
Concepts covered include:

- scatter plots;
- domain and range;
- continuity;
- function;
- evaluation of a function for domain values;
- independent and dependent variables;
- rate of change (slope);
- slope-intercept form of an equation;
- linear regression;
- transformations; and
- direct variation.

IV. White Water Rafting on Silly Creek

Students will be given data in a table that represent recorded distances and elevations of a group of white water rafting enthusiasts on their spring trip. The data represent a negative rate of change and do not have a correlation coefficient of 1.
Concepts covered include:

- scatter plots;
- domain and range;
- continuity;
- function;
- evaluation of a function for domain values;
- independent and dependent variables;
- rate of change (slope);

- slope-intercept form of an equation;
- linear regression;
- transformations; and
- direct variation.

Activity I: Teacher Notes—Up To Speed

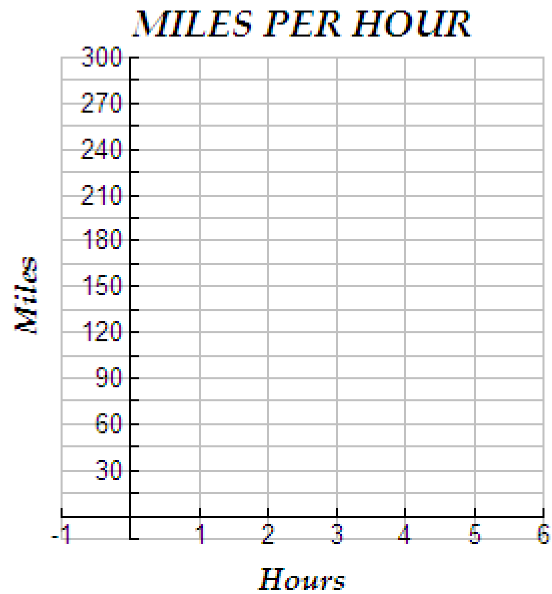
Up To Speed builds understanding of the mathematical model of a vehicle on cruise control and the distance traversed (at a constant pace.) Students will plot the data, (hours, distance) and discuss which function family the data most resembles (linear); whether the data are discrete or continuous; whether the relation is a function; the domain and range of the data; and the independent and dependent variables.

Students will enter the data in the graphing calculator and determine the equation of the line of best fit using the **Transformation™** Application. Students will also discuss the meaning of the y-intercept and the slope in the context of **Up To Speed**.
Students may also use the equation of the line of best fit to determine how far a car will travel in **n** hours.

Up to Speed

What does it mean when we say "60 miles per hour"?

Hours	Miles
0	
1	
2	
3	
4	
5	



1. Create a scatter plot using the data and coordinate plane above.
    a) What is the independent variable?
    b) What is the dependent variable?

2. Using the graphing calculator, enter data into the lists (*hours* into **L1** and *miles* into **L2**). Graph the scatter plot in a "friendly" window.
    a) What is the domain of the relation?
    b) What is the range of the relation?
    c) Is the relation continuous?
    d) Is the relation a function?
    e) What family of functions does the data most resemble?

    f) Write the general form of the equation that would represent the data.

3. What is the average rate of change in miles per hour for the entire trip?
    a) Show computations.

    b) Turn on the **Transformation™** Application and determine the equation of the line of best fit using the general form of the equation representing the data. Record the equation for the line of best fit.

    $y =$ ______________________________

    c) What do you notice about your answers in parts 3a and 3b?

4. What is the rate of change in miles per hour when driving, according to the table, from time = 0 to the end of the first hour?
    a) Show computations.

    b) Enter data into the graphing calculator for the indicated hours and distance in **L3** and **L4**. Readjust the window and graph. Use the **Transformation™** Application and determine the equation of the line of best fit using the general form of the equation.
    Record the equation for the line of best fit.

    $y =$ ________________________

    c) What do you notice about your responses to parts 4a and 4b?

5. What is the rate of change in speed when driving, according to the table, from the 3rd to the 4th hour?
    a) Show computations.

    b) Enter data into the graphing calculator for the indicated hours and distance in **L3** and **L4**. Readjust window and graph. Use the **Transformation™** Application and determine the equation of the line of best fit using the general form of the equation representing the data.
    Record the equation for the line of best fit.

    $y =$ ________________________

    c) What do you notice about your responses to parts 5a and 5b?

6. Compare each of the answers in 3c, 4c, and 5c. Explain your response.

7. Using linear regression, determine the equation that best models the data for the entire trip. Record the equation generated by the linear regression.

8. How does the equation from the linear regression compare to the equations of the lines of best fit determined using the **Transformation™** Application?

9. What do you think the average rate of change in miles per hour was 2 hours after the start of the trip?

10. Calculate the distance traveled at

    a) 3 hours

    b) 7 hours

    c) 1.5 hours

Fitting the Equation

11. The graphing calculator will automatically store the residuals, **RESID**, under **2nd [Stat].** Residual = Actual – Fitted values. The closer the sum of the residuals is to zero, the better the fit. At this time**, L1** has *time* data and **L2** has the *distance traveled*. You have already determined the equation of the curve of best fit, recorded in question 7. **Stat Plot 1** will activate the scatter plot for (time, distance) graph. Press **2nd [Graph]** to view the table for the line (curve) of best fit. Complete the table below.

TIME L1	**ACTUAL DISTANCE L2**	**FITTED DISTANCE L3**	**ACTUAL – FITTED L4 = L2 – L3**	**(ACTUAL – FITTED)^2 L5 = (L4)^2**
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

12. Turn on **Stat Plot 2** to activate another scatter plot of (time, RESID). See above to enter **RESID** for the **Ylist**. If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?

How does the above table help us determine if the line of best fit is actually the best?

Activity II: Teacher Notes--Investigating Slope, Equations, and Tables Using the CBR™

Students need to work in groups of two or three. Groups will be instructed to either walk toward or away from the CBR™. The idea is for the "walker" to walk at a steady rate so that the data recorded will stretch across the screen. One calculator and one CBR™ unit should be issued to each group for the initial portion of the activity.

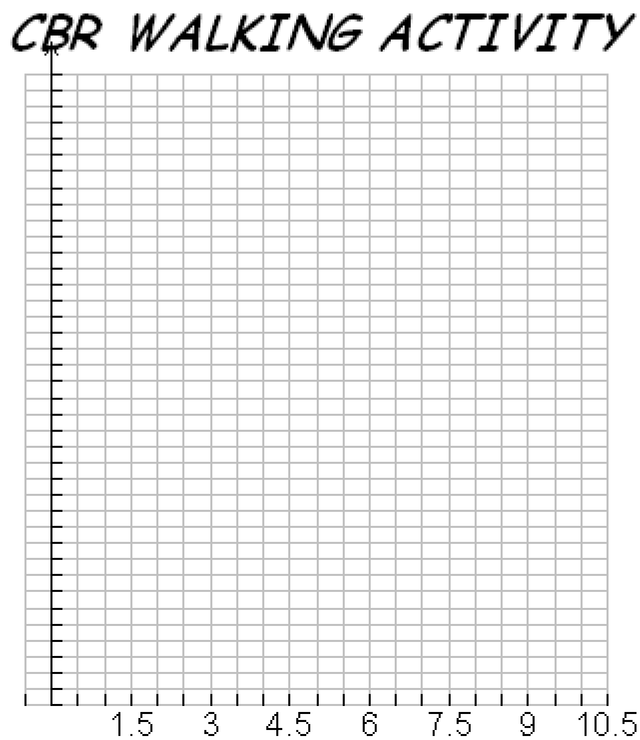
Set up the activity with an interval of 0.5 seconds and number of samples as 20. (Your experiment length will compute automatically.) Students need to position themselves so that nothing will come between the walker and the CBR™ as data are recorded. Students may accept the graph they obtain or use the "Select Region" function available in the program to isolate a section of the graph, eliminating areas not needed. Students will complete the table and graph on the handout with the data obtained from the "walk".

Discussion may include domain; range; definition of a function; evaluation of a function at a given domain value; slope (positive, negative, zero, undefined); relating the table to the points on the graph; relating what the "walker" was doing at specific points in time or over a time interval; independent and dependent variables; and continuity. Remaining members of the groups need to obtain a calculator for their own use and link to obtain the recorded data.

Investigating Slope, Equations, and Tables Using the CBR™

1. My group is walking away from/walking toward the CBR™. (Circle the correct response for your group.)
2. Using one calculator and one CBR™ unit for your group, set up the activity using the **EasyData™** Application with an interval of 0.5 seconds and number of samples to be taken by the CBR™ as 20. (Your experiment length will compute automatically.)
3. Position the CBR™ so that nothing will interfere or come between the walker and the CBR™ as data are recorded.
4. All members of the group need to complete the table and graph below with the data obtained from the "walk". Be sure to label the axes with a title and values.

Time	Distance
0.5	
1	
1.5	
2	
2.5	
3	
3.5	
4	
4.5	
5	
5.5	
6	
6.5	
7	
7.5	
8	
8.5	
9	
9.5	
10	



5. Remaining members of the groups need to obtain a calculator for their own use and link to obtain the recorded data.
6. Graph the data on the calculators in a friendly window.
   a) What is the independent variable?
   b) What is the dependent variable?

7. Determine the following about the relation that is the data obtained from your "walk".
   a) Domain? b) Range?

   c) Continuous? d) Function?

   e) What type of graph (from the families of functions) does the data most resemble?

f) Write the general form of the equation that would represent the data.

g) What unit of measure would be appropriate for the average rate of change in feet over a given time?

8. Determine the rate of change between the following selected points in time:
   a) t = 0.5 and t = 1

   b) t = 3 and t = 4

   c) t = 8.5 and t = 9.5

   d) t = 0 and t = 10

9. Turn on the **Transformation™** Application and enter the general form of the equation representing the data. Manipulate the values to determine the line of best fit.
   Record the equation for the line of best fit.

   y = ________________________________

10. How does the equation of the line of best fit (#9) relate to the results in #8?

11. How does the appearance of the data in this activity compare with the data in **Up to Speed**?

12. Determine the equation generated by the linear regression for the entire walk. Record the equation from the linear regression.

    y = ________________________________

13. How does the equation from the linear regression compare to the equations of the lines of best fit determined using the **Transformation™** Application?

14. What was the rate of change of the speed of the walker in feet per second 2.5 seconds after the start of the walk?

15. Determine the total distance traveled at:

a) 5 seconds

b) 7.8 seconds

16. If the walk was continued, where would the walker be after 18 seconds?

Fitting the Equation

17. The graphing calculator will automatically store the residuals, **RESID**, under **[2nd] [Stat]**. The closer the sum of the residuals is to zero, the better the fit. Why? At this time, **L1** is time and **L2** is the distance traveled. You have already determined the curve of best fit, recorded in question 13. Turn on **Stat Plot 1** to activate the scatter plot for (time, distance) graph. Press **[2nd] [Graph]** to view the table for the curve of best fit. Complete the table below.

TIME L1	**ACTUAL DISTANCE L2**	**FITTED DISTANCE L3**	**ACTUAL – FITTED L4 = L2 – L3**	**(ACTUAL – FITTED)^2 L5 = (L4)^2**
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

Turn on **Stat Plot 2** to activate another scatter plot of (time, RESID). See above to enter **RESID** for the **Ylist**.

If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?

18. How does the above table help us determine if the line of best fit is actually the best?

Activity III: Teacher Notes--See Starbuck Run

Students may participate in the activity individually or in small groups.

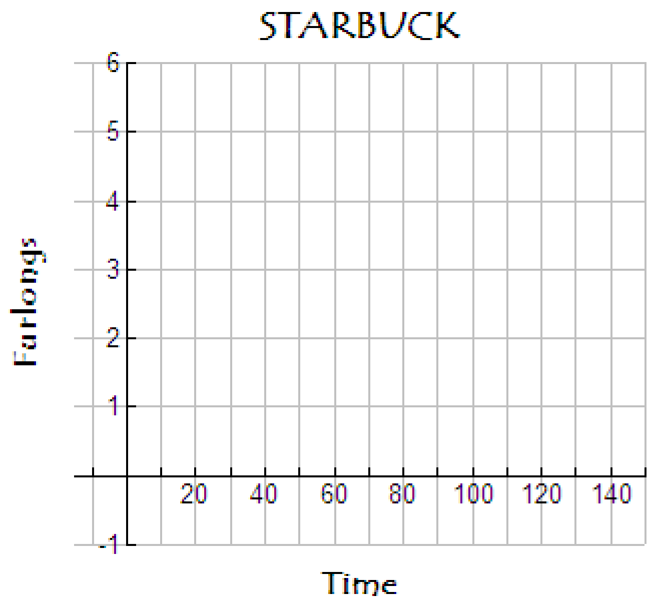
Students will continue to determine the average rate of change of data showing a positive correlation.

Students will need to recognize that the data in the table are not in the order typically written; therefore, they must identify the independent and dependent variables. Computations to calculate average rate of change for various intervals will result in different values in this activity. Discussions may include the average rate of change for the entire steeplechase, the average rate of change for each time and/or distance interval; whether the data are continuous; whether the data represents a function; the meaning of the y-intercept and the slope within the context of the problem; and whether we can determine the location of the horse at time equals 150 seconds.

See Starbuck Run

Mike Millionaire is watching a 10-furlong steeplechase near Charles Town, WV. He is doing some research during the steeplechase in anticipation of attending to watch his favorite horse, Starbuck, at some future date. As Starbuck passes a furlong (F) marker, Mike Millionaire records the time (t) elapsed in seconds since the beginning of the steeplechase. The data are shown in the table below.

Furlong	Time
0	0
1	23
2	33
3	46
4	59
5	73
6	86
7	100
8	112
9	124
10	135



Looking at the Data

1. Create a scatter plot using the data and coordinate plane above.
   a) What is the independent variable?
   b) What is the dependent variable?

2. Using the graphing calculator, enter data into the lists (**L1** and **L2**) and graph in a friendly window.
   a) Domain? b) Range?

   c) Continuous? d) Function?

   e) What type of graph (from the families of functions) does the data most resemble?

   f) Write the general form of the equation that would represent the data.

   g) What unit of measure would be appropriate for the average rate of change in furlongs over a given time?

3. How fast is Starbuck running from the start to the very end of his event?

4. How fast is Starbuck running from the exact moment he passes the 4th furlong marker to the moment he passes the 5th furlong marker?

5. How fast is Starbuck running from the moment he passes the 6th furlong marker to the moment he passes the 7th furlong marker?

6. How fast is Starbuck running during the last furlong?

7. How long does it take for Starbuck to finish the event?

8. Turn on the **Transformation™** Application. Use the general form of the equation and manipulate the ***a*** and ***b*** to determine the equation of the line of best fit for the given data.
   Record the equation.

   y = ______________________________

   a) What does the ***a*** in the equation of the line of best fit represent?

   b) What does the ***b*** in the equation of the line of best fit represent?

9. How does the appearance of the data in this activity compare with **Up to Speed**?

10. How does the equation of the line of best fit relate to the outcomes of questions 3 through 6? Explain.

11. Using the calculator, determine the equation from linear regression for the entire event.
    Record the equation from the linear regression.

    y = ______________________________

12. How does the equation from linear regression compare to the equations of the lines of best fit determined using the **Transformation™** Application?

13. What is the average rate of change when the horse was 46 seconds from the start of the steeplechase?

14. Calculate the total distance traveled at the end of:
    a. 46 seconds

    b. 150 seconds

    c. 90 seconds

Investigation

15. Between which two furlong markers is Starbuck running the fastest? Show your computations and explain in writing.

16. Compare the values recorded in the table with the graph, the average rate of change and what is happening in the steeplechase.

Fitting the Equation

17. The graphing calculator will automatically store the residuals, **RESID**, under **[2nd] [Stat]**. At this time, **L1** is *time* and **L2** is the *distance traveled*. You have already determined the equation of the curve of best fit, recorded in #11. Turn on **Stat Plot 1** to activate the scatter plot for (time, distance) graph. Press **[2nd] [Graph]** to view the table for the curve of best fit. Complete the table below.

TIME L1	**ACTUAL DISTANCE L2**	**FITTED DISTANCE L3**	**ACTUAL – FITTED L4 = L2 – L3**	**(ACTUAL – FITTED)^2 L5 = (L4)^2**
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

Turn on **Stat Plot 2** to activate another scatter plot of (time, RESID). See above to enter **RESID** for the **Ylist**.

If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?

How does the above table help us determine if the line of best fit is actually the best?

Activity IV: Teacher Notes--White Water Rafting on Silly Creek

Students may participate individually or in small groups. Students will continue to determine an average rate of change but this activity is an example of a negative correlation. The average rate of change will relate a distance measure to another distance measure.

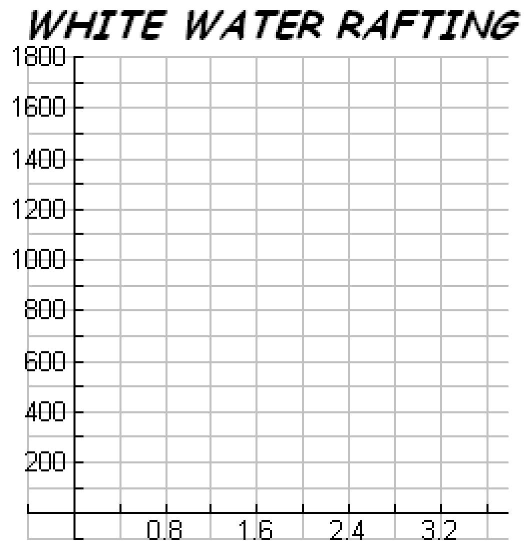
Computations to calculate average rate of change for various intervals will result in different values in this activity. Discussions will include the average rate of change for the entire trip; the rate of change for each interval; whether the data is continuous; whether the data represents a function; and the meaning of the y-intercept and the slope in the context of the problem. Students are also asked to determine the most dangerous section that the white water rafters will experience on their trip.

White Water Rafting on Silly Creek

White water rafting enthusiasts in West Virginia enjoy a stretch of 3.60 miles on Silly Creek. A contour map of a section of the river shows that there is a drop in elevation of more than 770 feet. Estimations of the elevations in feet (y) at various distances in miles down the creek (x) from the start of the rafting trip are shown in the table below.

Displaying the Data

Distance	Elevation
0	1575
0.55	1515
0.91	1445
1.2	1385
1.3	1321
1.42	1261
1.63	1199
1.73	1139
1.97	1041
2.24	997
2.45	931
2.78	862
3.6	801



1. Create a scatter plot using the above data and coordinate plane.
   a) What is the independent variable?
   b) What is the dependent variable?

2. Using the graphing calculator, enter data into the lists (**L1** and **L2**) and graph in a friendly window.
   a) Domain?
   b) Range?
   c) Continuous?
   d) Function?
   e) What type of graph (from the families of functions) does the data look like?
   f) Write the general form of the equation that would represent the data.
   g) What unit of measure would be appropriate for the average rate of change in elevation over a given distance?

3. Distance is measured in ______________.

4. Elevation is measured in ____________________.

5. What unit of measure would be appropriate for the average rate of change in elevation for a given distance?

6. Calculate the total distance traveled. Show calculations.

7. Calculate the total change in elevation.

8. What is the average rate of change in the elevation over the distance traveled from the start to the very end of the trip for the white water rafter?

9. Explain how your answers to questions 6 and 7 relate to your findings in question 8?

10. What is the rate of change in the elevation over the distance traveled from the start of the trip, 0 miles, to the time when the white water rafter passes the 0.55 mile marker?

11. What is the rate of change in the elevation over the distance traveled from the time when the rafter passes the 1.73 mile marker to the time when she passes the 1.97 mile marker?

12. Turn on the **Transformation™** Application. Use the general form of the equation and manipulate ***a*** and ***b*** to determine the equation of the line of best fit for the given data.

    Record the equation that represents the line of best fit.

    y = ________________________

    a) What does ***a*** in the equation of the line of best fit represent? How is ***a*** related to white water rafting on Silly Creek?

    b) What does the ***b*** in the equation of the line of best fit represent? How is ***b*** related to white water rafting on Silly Creek?

13. How does the data in this activity compare with **Up to Speed** and **See Starbuck Run?**

14. Compare the equation of the line of best fit (question 12) to your results in questions 10 and 11. Explain.

15. Using linear regression, determine the equation of the line of best fit for the entire white water rafting trip. Record the equation for the linear regression.

16. How does the linear regression equation compare to the equations of the lines of best fit determined using the **Transformation™** Application?

17. Determine the average rate of change in the elevation over the distance traveled when the white water rafters pass the 1.42 mile marker.

18. Calculate the elevation at the specified distances from the beginning of the trip:

    a) 1.3 miles

    b) 1.8 miles

    c) 3.75 miles

Investigation

19. White water rafting on Silly Creek will be the most dangerous between which two points? Explain your reasoning.

20. Relate the values recorded in the table with the graph, the average rate of change in elevation over the distance traveled, and what the white water rafters are experiencing during their trip.

Fitting the Equation

21. The graphing calculator will automatically store the residuals, **RESID**, under **[2nd] [Stat]**. The closer the sum of the residuals is to zero, the better the fit.
At this time, **L1** is elevation and **L2** is the distance traveled. You have already determined the equation of the curve of best fit, recorded in #15.
Stat Plot 1 will activate the scatter plot for (elevation, distance) graph.
Press **2nd [Graph]** to view the table for the curve of best fit.
Complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

Turn on **Stat Plot 2** to activate another scatter plot of (elevation, RESID). See above to enter **RESID** for the **Ylist**.
If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?

22. How does the above table help us determine if the line of best fit is actually the best?

Sample Resources:

http://mathforum.org/workshops/usi/dataproject/usi.genwebsites.html
-- The Data Library

http://education.ti.com Activities Exchange

- Crime Scene Investigations – Stride Pattern Analysis with CBR2™
- Bungee Jumper
- Distance vs. Time to school
- Do You Have a Temperature? TI-83
- Home for the Holidays
- USA Today
    - ⇒ Math Today: More Students Apply Early
    - ⇒ Wildfire Deaths
    - ⇒ Population On The Move

http://gapminder.org

http://statmaster.sdu.dk/courses/st111/data/index.html#children

http://www.ilovemath.org

- Lesson Plans & Activities

http://panther.moundsparkacademy.org/~dethier/activities.htm

Organizing Topic: Quadratic Modeling

Mathematical Goals:

- Students will model quadratic relationships using data.
- Students will analyze quadratic functions.
- Students will transform the quadratic parent function, making relationships to real-life situations.
- Students will be able to relate graphs to the corresponding tables and quadratic function.
- Students will experiment with changing the elements of quadratic functions and will predict the changes in the outcome of the function that includes obtaining different roots and different maximum or minimum points.
- Students will state the number of roots, the coordinates of the vertex, and how the parabola opens when given an equation in vertex form.
- Students will state any limitations to the domain and range of a quadratic function and how it relates to real-life situations.
- Students will make predictions using the curve of best fit for the modeled data as well as evaluating for specific values in the domain of the data.

Standards Addressed: AFDA.1; AFDA.2; AFDA.3; AFDA.4

Data Used: Data obtained using Web sites, surveys, experiments, and textbook

Materials:

- Graphing calculator applications:
  - **EasyData/Ranger**™
  - **Transformation™**
- CBR™ or CBR2™
- Graphing calculator
- Handout – Quadratic CBR™ Exploration
- Handout – Solutions, Factors, Line Of Reflection, …
- Handout – Quadratic Investigation Vertex Form
- Handout – Triangular Numbers
- Handout – Terry's Time
- Handout – Quadratics Playground Exploration
- Handout – Quadratics Web Page Exploration
- May also need graph paper and concrete objects such as two-color chips or snap blocks

Instructional Activities:

I. Quadratic CBR™ Exploration

The exploration encourages students to relate four different physical movements to a graph that is illustrated using a parabola.

Concepts will include:

- domain and range;
- scatter plots;

- distance-time graphs;
- curve of best fit; and
- relationship between movements and the graphs produced.

II. Solutions, Factors, Line Of Symmetry

The activity relates the factors of *c*, the constant value of a trinomial, to the solution(s) of the equation and the factors of the trinomial. The relationship between the linear coefficient and the line of symmetry will be explored. Lastly, the relationship between minimum points of the family of parabolas with solutions that are factors of *c* are explored.

Concepts will include:

- domain and range;
- intercepts;
- zeros of the function;
- solutions of the equation;
- line of symmetry;
- maximum and minimum points;
- relationship between zeros, factors, and the equation; and
- curve of best fit.

III. Quadratic Investigation Vertex Form

Students will investigate each type of transformation on the parent quadratic graph.

Concepts will include:

- domain and range;
- intercepts;
- zeros;
- line of symmetry;
- intervals in which the graph is increasing and decreasing;
- maxima and minima;
- (h, k) or vertex form of the equation;
- how the graph opens;
- translations;
- reflections;
- dilations; and
- rotations.

IV. Triangular Numbers

Students will explore the quadratic equation that represents the sequence commonly known as ***triangular numbers***.

Concepts will include:

- domain and range;
- scatter plots;
- patterns;
- relationship between the numerical differences in the values of the independent variable and the equation representing the data;
- relationship between written expressions and symbols; and
- systems of equations.

V. Terry's Time

Terry is running a relay race as her coach records her position at various times. Students will explore the rate of change at various time intervals; relate what is happening physically to the graph and the equation that best represents Terry's movement. Graphical analysis, the equation of the curve of best fit, and the associated table of values will be explored.

Concepts will include:

- domain and range;
- rate of change;
- scatter plots;
- maximum and minimum points;
- distance-time graphs;
- curve of best fit;
- line of symmetry;
- intervals in which the function is increasing and decreasing; and
- relating real world situations to graphs.

VI. Quadratics Playground Exploration

Given a specific amount of fencing, students will determine the size of a playground that will maximize the usable area. Analysis of the graph, the curve of best fit, and the table of values representing the situation will be explored.

Concepts will include:

- domain and range;
- rate of change;
- scatter plots;
- area-width and length-width graphs;
- maximum and minimum points (optimization);
- curve of best fit;
- line of symmetry;
- intervals in which the function is increasing and decreasing;
- relating real-life situation to graphs;
- relating graphs of linear data to quadratic data; and
- using data to make decisions.

VII. Quadratics Web Page Exploration

A picture is placed on a Web site, starting as a dot and growing as time elapses. Relationships are made with length versus time and area versus time when the picture is in the shape of a square. The challenge increases as the picture is inserted as a rectangle. Relationships between time and length, time and height, time and area, and length and area are explored.

Concepts will include:

- domain and range;
- rate of change;
- scatter plots;
- area-time and length-time graphs;
- maximum and minimum points (optimization);
- curve of best fit;

- line of symmetry;
- intervals in which the function is increasing or decreasing;
- relating real-life situation to graphs;
- relating graphs of linear data to quadratic data; and
- using data to make decisions.

Activity I: Teacher Notes--Quadratic CBR™ Exploration

Assign students to small groups of two to four. The settings on the CBR™ should be in feet and set to gather a maximum of 20 pieces of data taken in 0.5 second intervals. There are four different activities.

1. Students will hold the CBR™ unit parallel to the floor and as high as they can safely. Two movements may be assigned to each group or different groups can be responsible for one movement and then all groups can compare results.
   Using a hand or a book, students will move the object, starting near the unit and moving up and then back toward the unit. Then the student will reverse the movements, starting high, moving downward toward the unit, then back up again. Students will record data in the table of values and graph the results. Questions are on the handout regarding object movements and the equation that represents the scenario.
2. Students will place the CBR™ unit on the floor facing up. Two movements may be assigned to each group or different groups can be responsible for one movement and then all can compare their results. Using their hand or a book, students will move the object, starting near the unit and moving upward and then back toward the unit. Then the student may reverse the movements, starting high, moving downward toward the unit, then back up again. Students will record data in their table of values and graph their results. Questions are on the handout regarding their movements and the equation that represents the scenario.
3. One student will hold the CBR™ perpendicular to the floor facing another student. Nothing should be in the path between the CBR™ and the student that might interfere with the data recording. Two movements may be assigned to each group or

different groups may be responsible for one movement and then all can compare their results. The student will stand near the unit, walk away, and then walk back again. The walker does not physically turn around. It is best if students get to a point and then walk backwards toward the unit. The student will stand ***x*** feet away from the unit and, when the data begins to record, will walk toward the unit and then walk back again. The walker does not physically turn around. It is best if they get to a point and then walk backwards toward the unit. Students will record data in their table of values and graph the results. Questions on the handout address student movements and the equation that represents the scenario.

4. The walker will cross the room while the holder of the unit stays stationary, pivoting slowly so that the unit follows the walker. Students will record data in a table of values and graph the results. Questions are on the handout regarding their movements and the equation that represents the scenario.

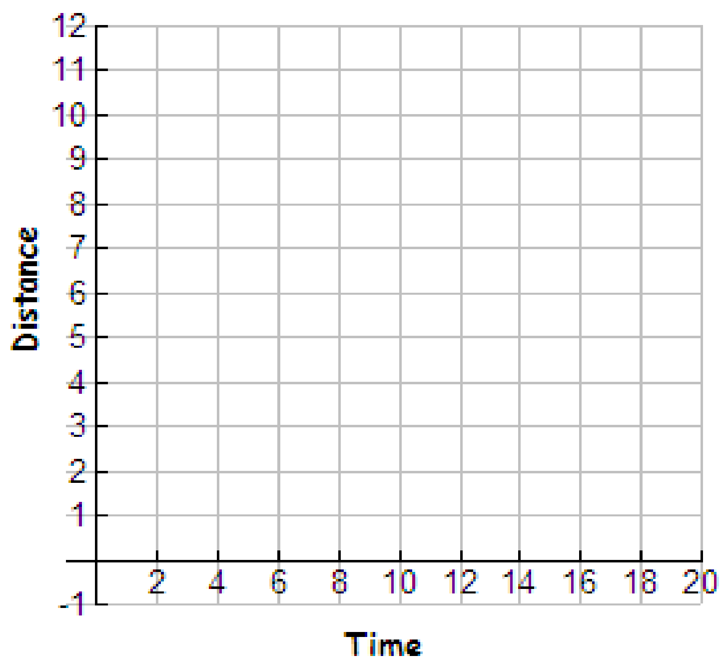
Quadratic CBR™ Exploration

Connect the CBR™ to the graphing calculator and begin the **Ranger**™ program or **EasyData**™. In the setup menu, change the units to feet.

Collecting the data.

1. Aim the CBR™ downward toward the floor.
   - Begin with a ball, a book, or even your hand, on the floor and slowly raise it upward toward the CBR™ and then back again to the floor OR begin with your hand or object close to the CBR™, lower to the floor and then back up again.
   - Attempt to maintain the same speed of movement going downward and upward.
   - Record the values below and plot the points on the graph. Be sure to select points that will enable you to illustrate the curve (points while moving upward and points while moving downward).

Time	Distance



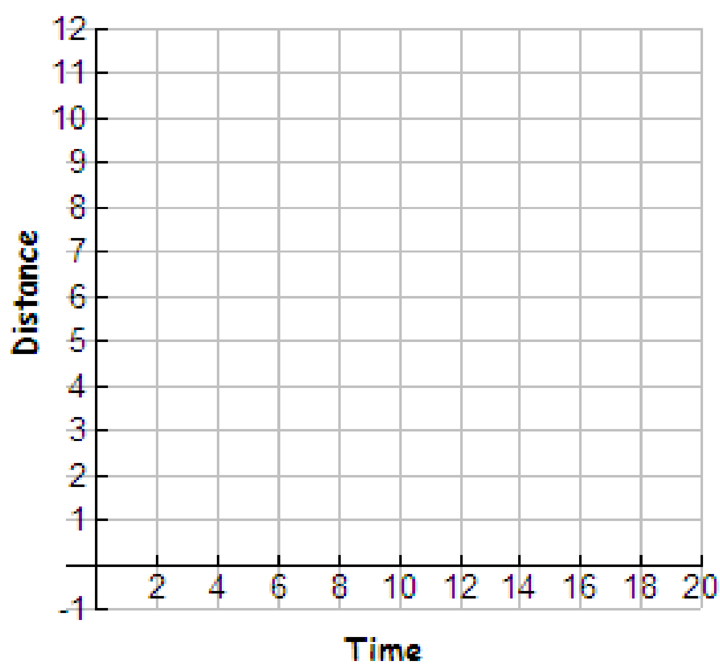
2. Describe the graph created by your movements. Relate movements to the points and the overall shape.

3. Determine the equation of the curve of best fit using **Transformation™** Application.

4. Determine the equation of the curve of best fit using regression on your calculator.

5. Place the CBR™ on the floor facing upward. Begin with a ball, a book, or even your hand, several feet above the CBR™.
    - Lower the object or your hand toward the CBR™ and then back up again OR
    - Begin at the unit and raise the object or your hand and then lower it back toward the unit.
    - Attempt to maintain the same speed of movement going downward and upward.
    - Record the values below and plot the points on the graph. Be sure to select points that will enable you to illustrate the curve (points while moving upward and points while moving downward).

Time	Distance



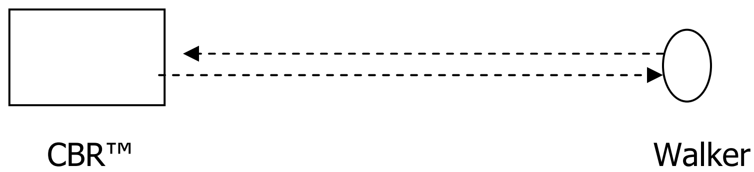
6. Describe the graph created by your movements. Relate movements to the points and the overall shape.

7. Determine the equation of the curve of best fit using the **Transformation™** Application.

8. Determine the equation of the curve of best fit using regression on your calculator.

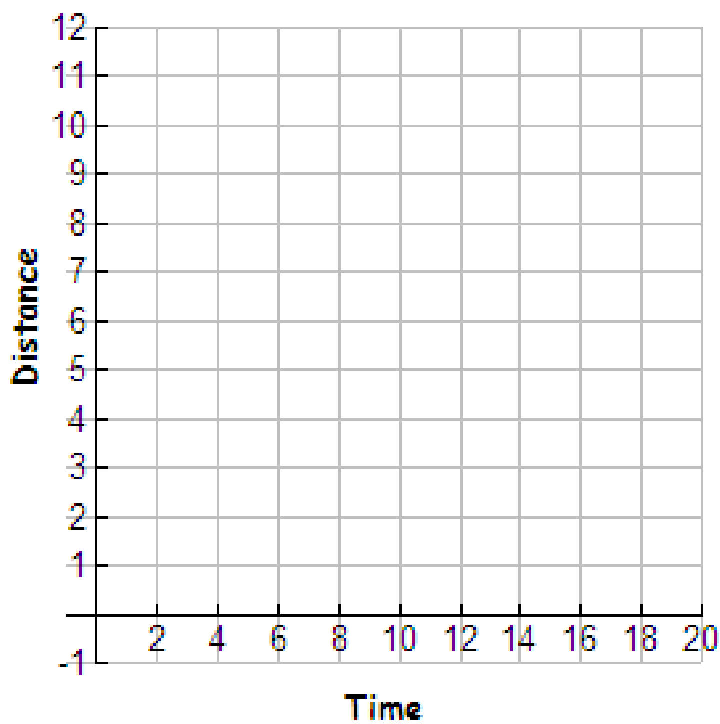
9. Compare the results from questions 1-4 and 5-8.

Collecting the Data



This exploration involves the CBR™ being pointed at a walker. The walker will either move away from the CBR™ and then back toward it or the reverse. When the walker begins moving he/she needs to begin slowing their pace as they get near the point where their moving direction will change and then speed up slowly the farther they get away from the turning point on the return trip. The walker's body does not turn – he/she needs to walk facing the same direction at all times and walk backwards when needed. Walking movements may also be reversed.

Time	Distance

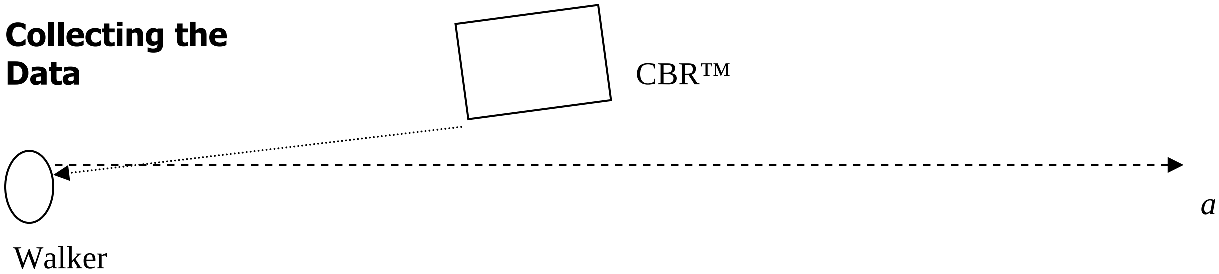


10. Describe the graph created by your movements. Relate your movements to the points and the overall shape.

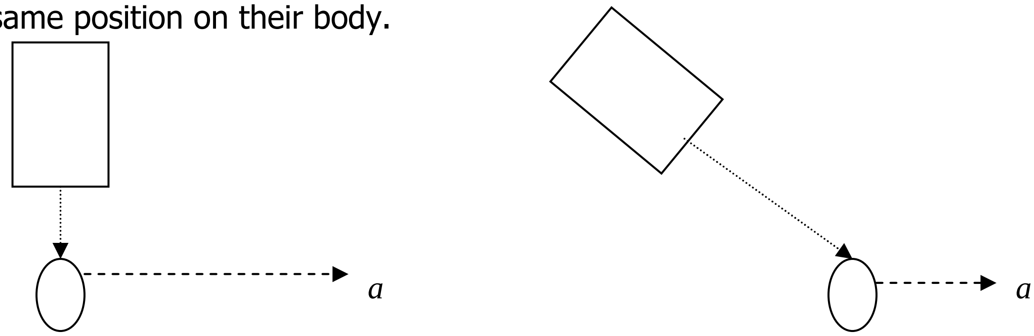
11. Determine the equation of the curve of best fit using **Transformation**™ Application.

12. Determine the equation of the curve of best fit using regression on your calculator.

Collecting the Data

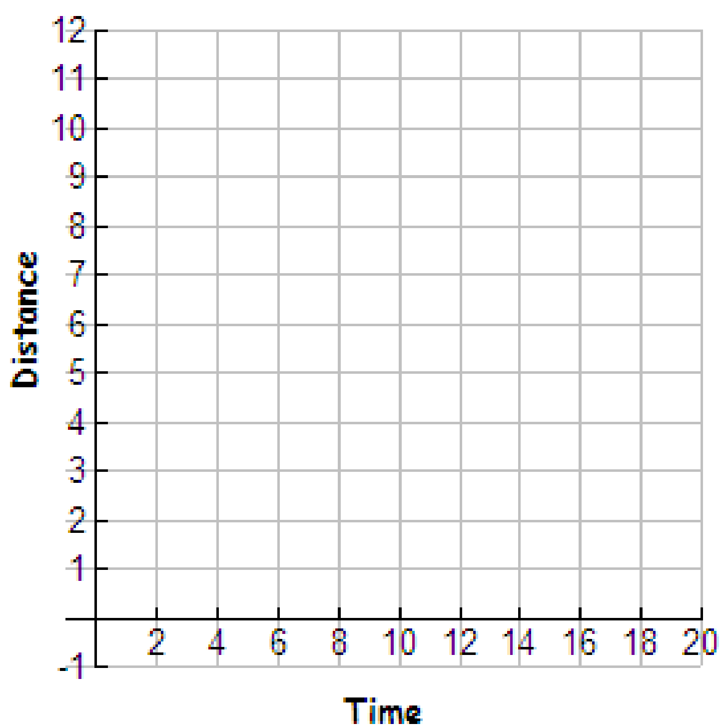


During this exploration an individual will be walking along line *a*. As the person gets closer to being directly in front of the CBR™ their speed should decrease and then increase as they continue on their path. At the same time the person holding the CBR™ will continually rotate so that the CBR™ will always be pointing at the walker at about the same position on their body.



13. Record the values and graph the data.

Time	Distance



14. Describe the graph created by your movements. Relate movements to the points and the overall shape.

15. Determine the equation of the curve of best fit using **Transformation**™ Application.

16. Determine the equation of the curve of best fit using regression on your calculator.

17. Compare your results.

Fitting the Equation

18. Select one or more of the above explorations. Record the data in **L1** and **L2** under **[Stat] [Edit]**. The graphing calculator will automatically store the residuals, **RESID**, under **[2nd] [Stat]** (**Residual** = **Actual** – **Fitted** values). The closer the sum of the residuals is to zero, the better the fit.
You have already determined the equation of the curve of best fit. See the specific exploration for the regression equation. Turn on **Stat Plot 1** to activate the scatter plot for graph. Press **[2nd][Graph]** to view the table of values for the curve of best fit and complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	$(\text{ACTUAL – FITTED})^2$ L5 = $(L4)^2$
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total $(\text{Actual – Fitted})^2$ [Sum(L5)]				

Turn on **Stat Plot 2** to activate another scatter plot showing the residuals.
If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis.   Why?

Use your calculator and, using linear regression, determine the equation of the curve of best fit for the given data. Repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	$(\text{ACTUAL – FITTED})^2$ L5 = $(L4)^2$
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

Compare and contrast the data in the two tables.

Activity II: Teacher Notes--Solutions, Factors, Lines of Reflection

Students will relate solutions of quadratic equations to factors of quadratic expressions as part of quadratic functions, and zeros of quadratic functions. The solutions are all factors of a specific integer. Students will discover the relationship between average value of the solutions to the optimal point and line of reflection. Lastly, students will relate all the quadratic equations that have solutions that are factors of the same integer by plotting all the graphs on the same coordinate plane. Their minimum points will be plotted for each and the equation determined that contain these elements.

Solutions, Factors, Lines of Reflection

Factors

1. Name all the factors of -36.

2. What is the equation of the line that is the x-axis?

3. Using the graphing calculator, graph the following trinomial given in question 1 as **y1** and your response to question 2 as **y2**.

   a) $\mathbf{y = x^2 - 35x + 36}$

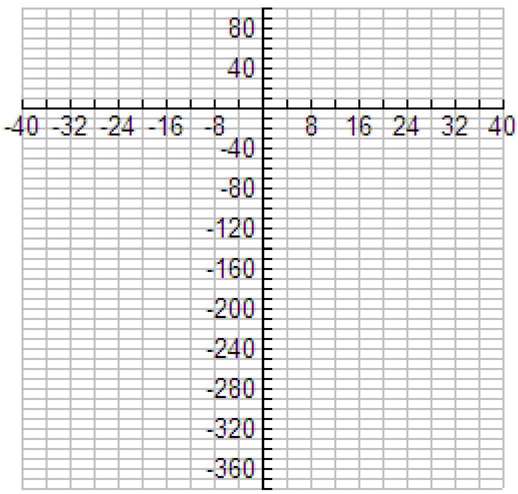
   b) Determine the point abscissas where the graph crosses the x-axis.

      ______ and ______

   c) Use the graphing calculator to determine the coordinates of the minimum point for the quadratic recorded in **y1**.

      (,   )

   d) Graph **y1** on the coordinate plane provided using questions 2 and 3 as an aid.



y = x^2 - 35x -36

   e) Set each of the values found in question 2 equal to ***x***. These ***x*** values are called ***solutions***.

      ____________________ and ____________________

   f) Solve the equation implied in part **e**.

   g) Determine the mean of the two ***x*** values given in part **e**.

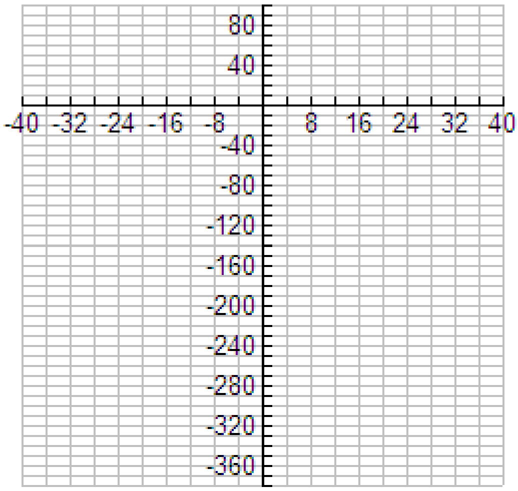
   h) Each of the expressions that are set equal to zero in part **f** are called **factors** of the original graphed polynomial. Multiply the two factors from part **f**.

   i) Recall that the general form of the trinomial is $\mathbf{y} = \boldsymbol{a}\mathbf{x}^2 + \boldsymbol{b}\mathbf{x} + \boldsymbol{c}$ where **a**, **b**, and **c** are real values. Determine the value of the linear coefficient, ***b***, and substitute into the equation $x = \frac{-b}{2}$.

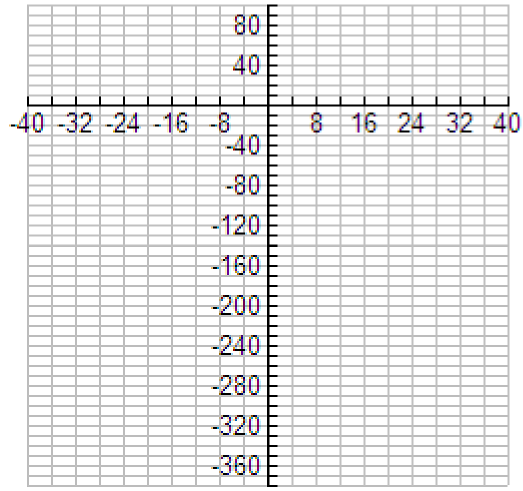
   j) On the graph, draw a dotted line that represents the equation in part **i**.

4. Repeat the activity in question 3 for each of the following by completing the chart and graphing each on the next page.

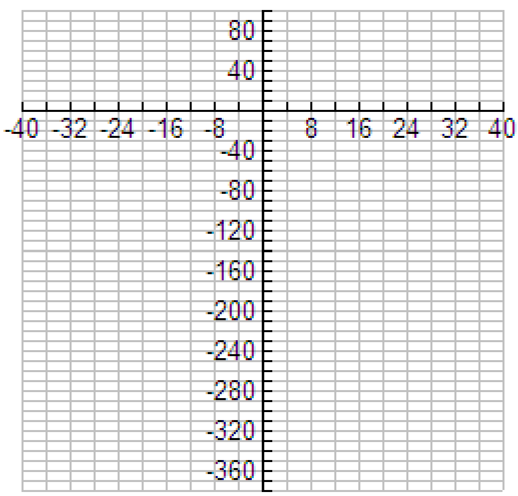
	Trinomial - A	**B x - values**	**C Minimum Point**	**D See next page**	**E Solutions**	**F Set = 0**	**G Multiply Factors**	**H** x = $\frac{-b}{2}$
1.	**y = x^2 -35x - 36**							
2.	**y = x^2 -16x - 36**							
3.	**y = x^2 - 9x - 36**							
4.	**y = x^2 - 5x - 36**							
5.	**y = x^2 – 36**							
6.	**y = x^2 + 35x – 36**							
7.	**y = x^2 + 16x - 36**							
8.	**y = x^2 + 9x - 36**							
9.	**y = x^2 + 5x - 36**							



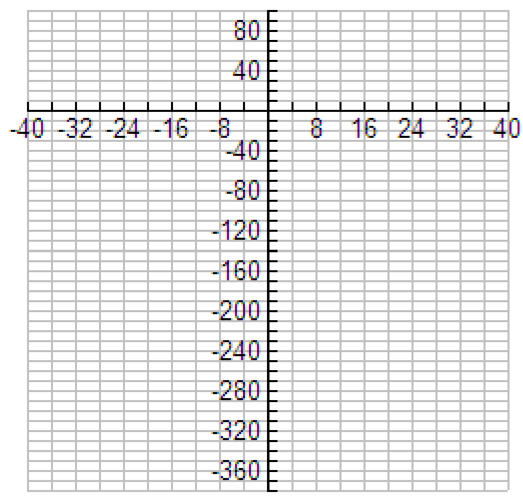
y = x^2 - 16x -36



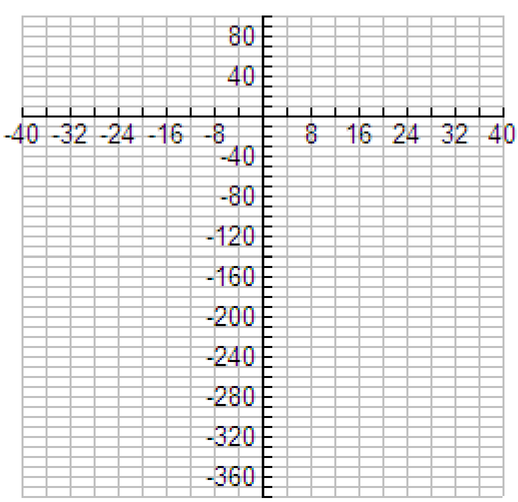
y = x^2 - 9x -36



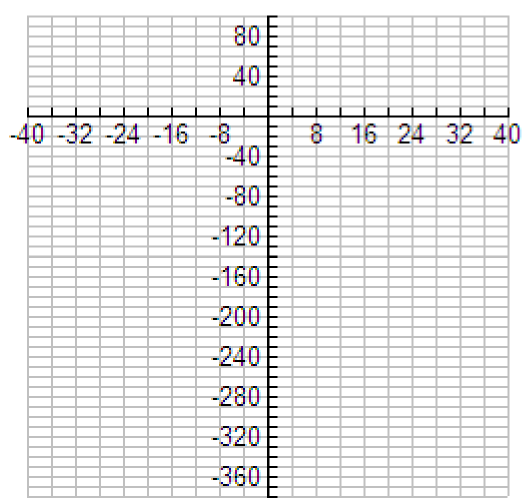
y = x^2 - 5x -36



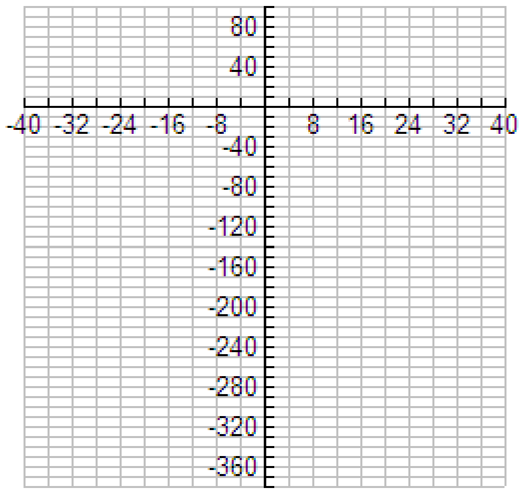
y = x^2+16x -36



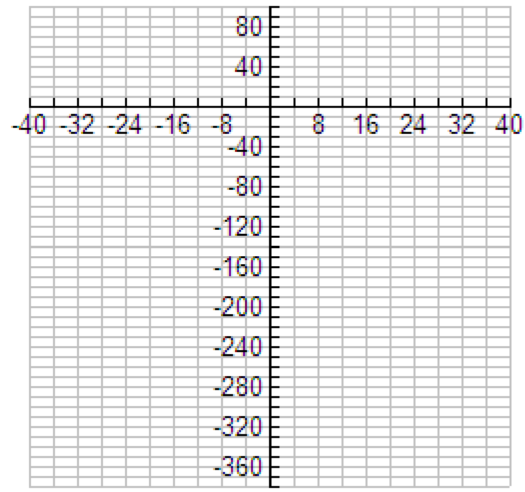
y = x^2+35x -36



y = x^2 -36



y = x^2+9x -36



y = x^2 +5x -36

Information from Graphs

5. Discuss the answers to part **b** and any relationships to other parts in the exploration.

6. Compare the results and relationships between parts **b, c, e, f,** and **h**.

7. Compare the results to part **c** and their relationship to part **i**.

8. Discuss the relationship between parts **g**, **i,** and **j**.

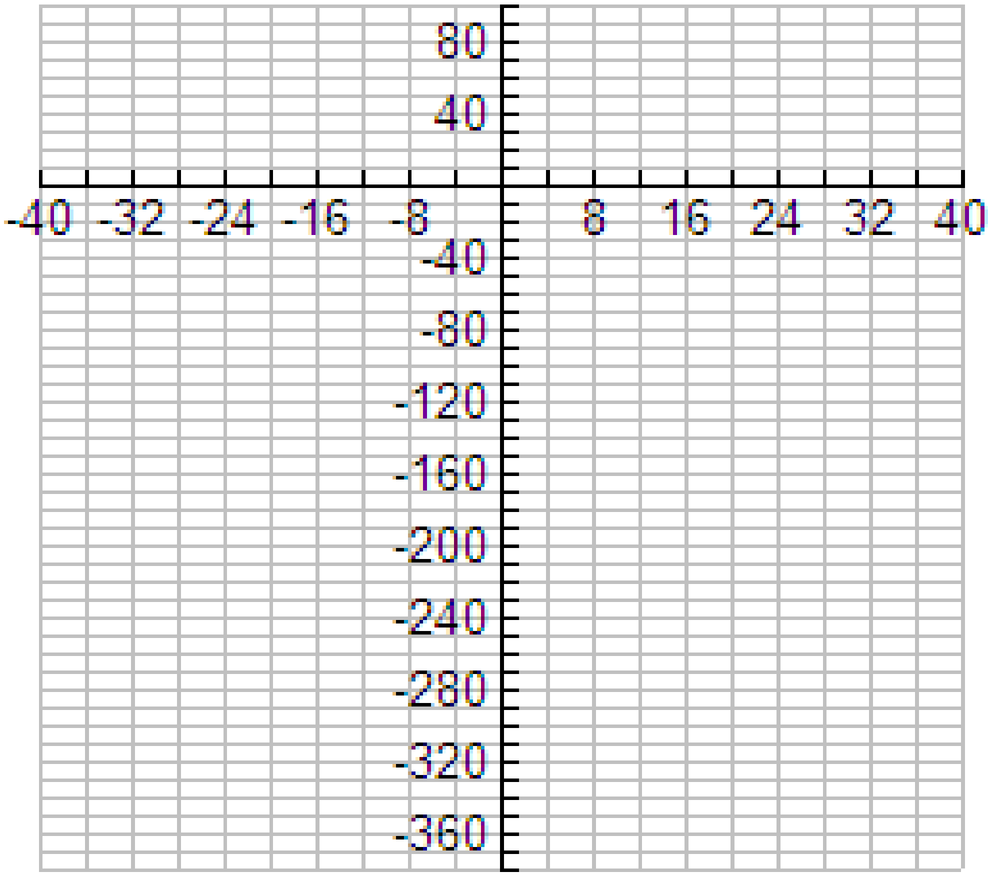
More graphing

9. Draw each of the graphs on the same coordinate plane below.

10. On each of the graphs, plot the minimum point and record the coordinates of the minimum points in the table.

11. Draw the curve that connects the minimum points of the graph.

Minx	Miny



Parent Functions and Minima

12. Using the graphing calculator, record each of the minimum points using the **List** function.

    L1 = minimum x value, **L2** = minimum y value

13. What type of parent function does the curved line formed by connecting the minimum points illustrate?

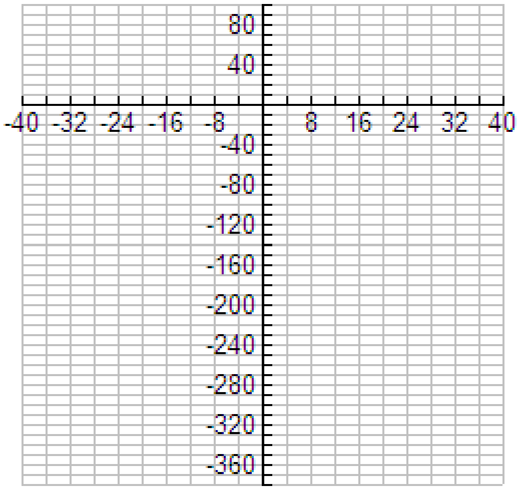
14. Use the **Transformation™** Application and the general equation of a trinomial to determine the best fitting curve that contains all of the minimum points.

    Record the equation of the curve of best fit. y = ___________________________

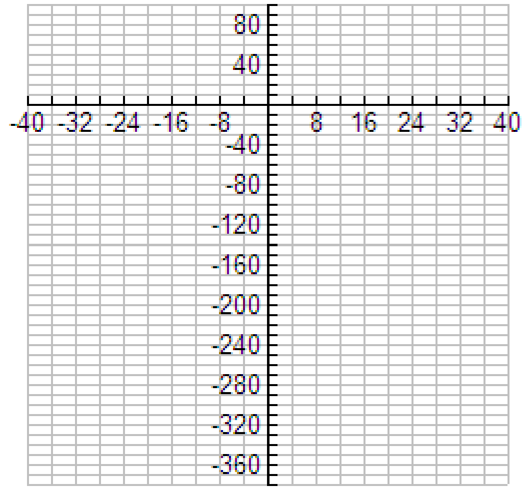
    Summarize this exploration in your own words. Be sure to think about each section and how it might relate to other sections.

15. Repeat the entire exploration using the factors of positive 36.

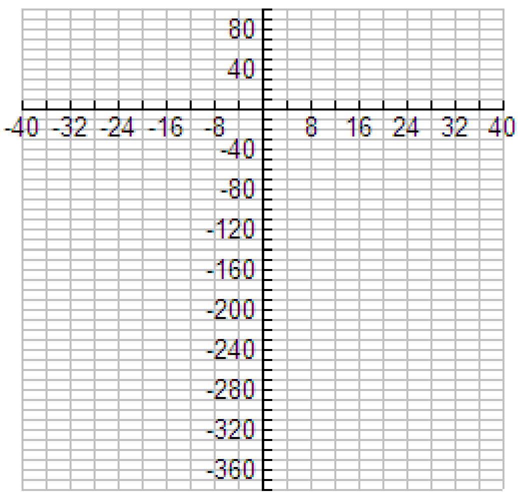
	Trinomial - A	B x - values	C Minimum Pt.	D See next page	E Solutions	F Set = 0	G Multiply Factors	H x = -b/2
1.	$y = x^2 -37x + 36$							
2.	$y = x^2 -20x + 36$							
3.	$y = x^2 - 15x + 36$							
4.	$y = x^2 - 13x +36$							
5.	$y = x^2 - 12x + 36$							
6.	$y = x^2 + 37x + 36$							
7.	$y = x^2 + 20x + 36$							
8.	$y = x^2 + 15x + 36$							
9.	$y = x^2 + 13x + 36$							
10.	$y = x^2 + 12x + 36$							



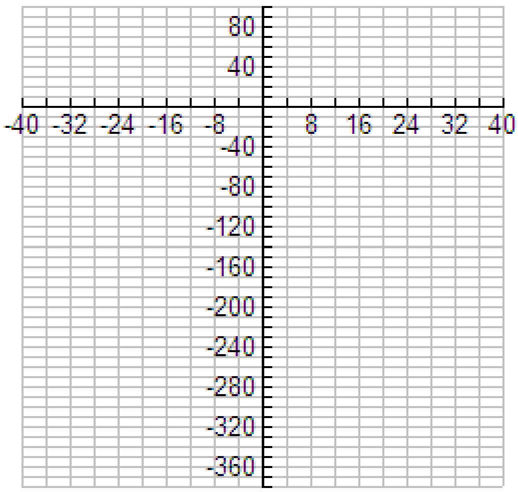
y = x^2 - 37x + 36



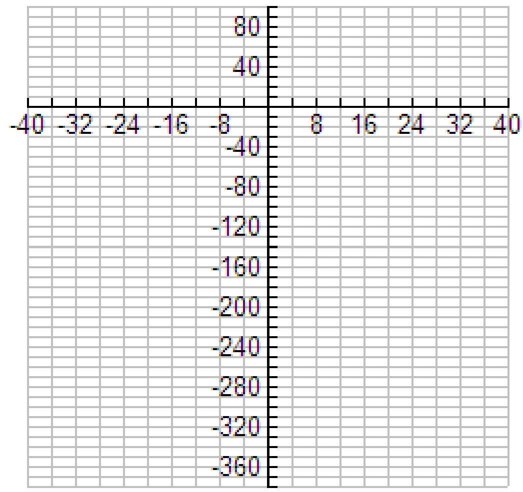
y = x^2 - 20x + 36



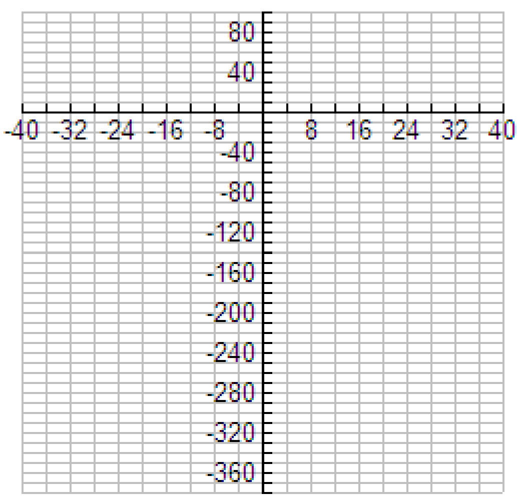
y = x^2 - 15x + 36



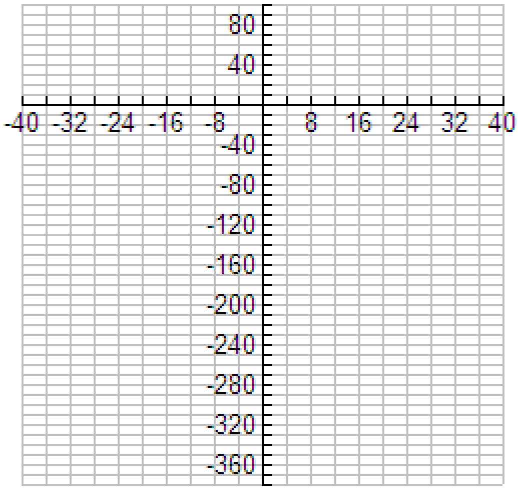
$y = x^2 - 13x + 36$



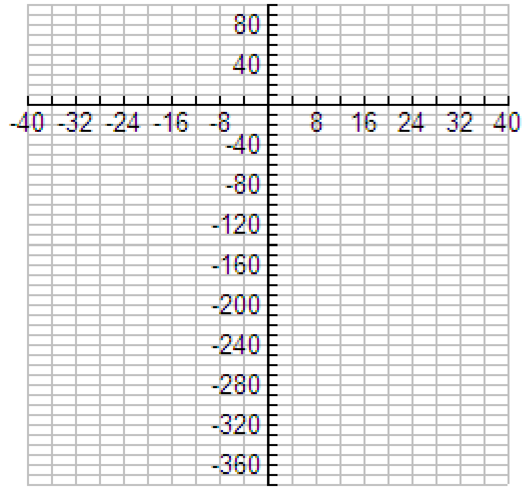
$y = x^2 - 12x + 36$



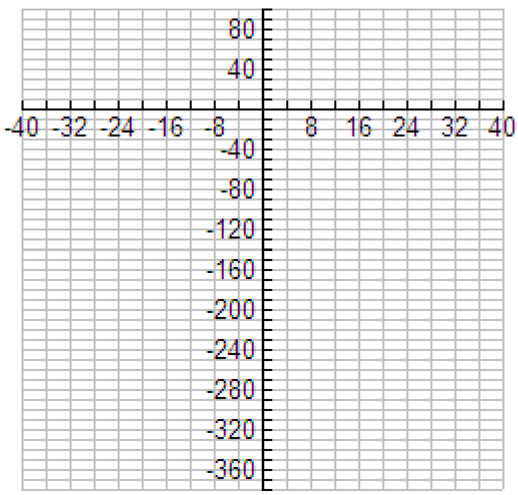
$y = x^2 + 37x + 36$



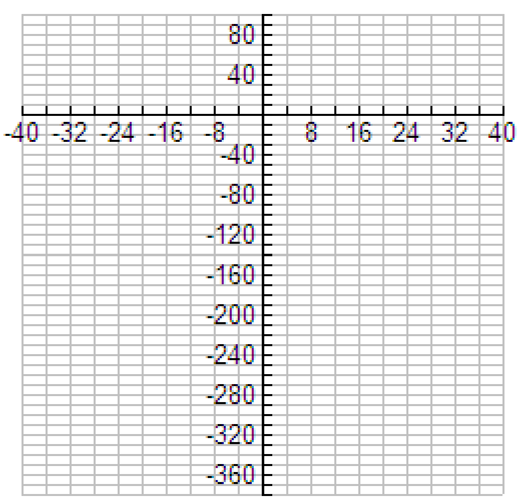
y = x^2 + 20x + 36



y = x^2 + 15x + 36



y = x^2 + 13x + 36



y = x^2 + 12x + 36

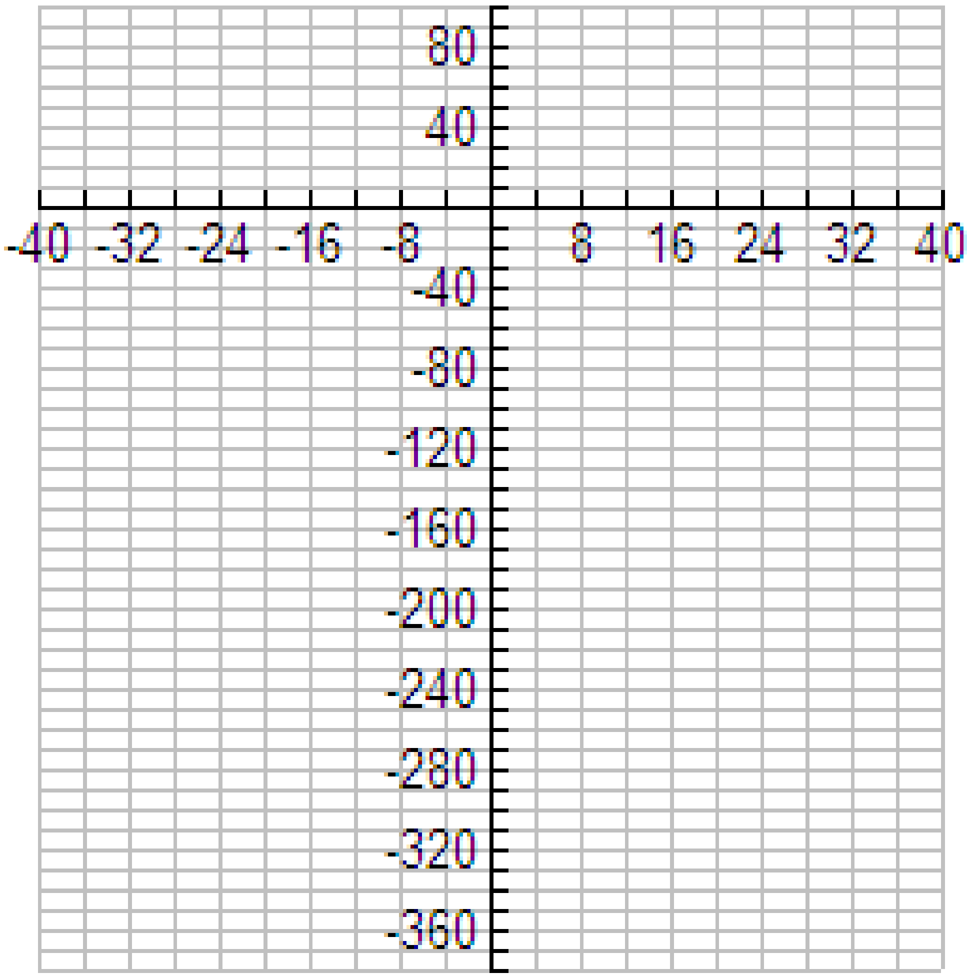
16. Discuss the answers to part **b** and any relationships to other parts in the exploration.

17. Compare the results to part **c** and their relationship to part **i**.

18. Discuss the relationship between parts **g**, **i,** and **j**.

Analysis of Graphs

19. Draw each of the graphs on the same coordinate plane below. On each of the graphs, plot the minimum point and record the minimum points on the table.



Minx	Miny

21. Draw the curved line that connects the minimum points of each graph.

More Analysis of Graphing

22. Using the graphing calculator, record each of the minimum points using the list function.
    L1 = minimum x-value, **L2** = minimum y-value

23. What type of parent function does the curve formed by connecting the minimum points illustrate?

24. Use the **Transformation**™ Application and the general equation of a trinomial to determine the best fitting curve that contains all of the minimum points. Record the equation of the curve of best fit. y = ________________________

25. Summarize this exploration in your own words. Be sure to think about each phase and how it might relate to other sections.

Activity III: Teacher Notes--Quadratic Investigation Vertex Form

Students will graph the parent quadratic equation in vertex form. Then, taking the ***a*, *h*,** and ***k*** (one at a time) and changing the value, the student will re-graph the transformed equation. The new graph will be explored and related to the parent function.

Quadratic Investigation Vertex Form

Graphing
Turn on the **Transformation™** Application on the graphing calculator. In **y1,** type in $\boldsymbol{a}\,(\mathbf{x} - \boldsymbol{b})^2 + \boldsymbol{c}$.

I.

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{1}\,(x - \boldsymbol{0})^2 + \boldsymbol{0}$									
$y = \boldsymbol{2}\,(x - \boldsymbol{0})^2 + \boldsymbol{0}$									
$y = \boldsymbol{5}\,(x - \boldsymbol{0})^2 + \boldsymbol{0}$									

1. Describe what is happening to the graph when $\boldsymbol{a} > 1$.

2. Describe the graphs by comparing and contrasting the items analyzed above.

Quadratic Equation	Domain	Range	Zeros	x-int.	y-int.	Line of Sym.	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{-1}(x - \boldsymbol{0})^2 + \boldsymbol{0}$									
$y = \boldsymbol{-2}(x - \boldsymbol{0})^2 + \boldsymbol{0}$									
$y = \boldsymbol{-5}(x - \boldsymbol{0})^2 + \boldsymbol{0}$									

3. Describe what is happening to the graph when ***a*** **< 0**.

4. Describe the graphs by comparing and contrasting the items analyzed above.

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y = \frac{1}{2}(x-0)^2 + 0$									
$y = \frac{1}{3}(x-0)^2 + 0$									
$y = \frac{1}{7}(x-0)^2 + 0$									

5. Describe what is happening to the graph when $\mathbf{0 < \boldsymbol{a} < 1}$.

6. Describe the graphs by comparing and contrasting the items analyzed above.

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{1}(x - \boldsymbol{1})^2 + \boldsymbol{0}$									
$y = \boldsymbol{1}(x - \boldsymbol{2})^2 + \boldsymbol{0}$									
$y = \boldsymbol{1}(x - \boldsymbol{5})^2 + \boldsymbol{0}$									

7. Describe what is happening to the graph when $\boldsymbol{b > 0}$.

8. Describe the graphs by comparing and contrasting the items analyzed above.

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{1}\,(x - {}^{-}\mathbf{1})^2 + \boldsymbol{0}$									
$y = \boldsymbol{1}\,(x - {}^{-}\mathbf{2})^2 + \boldsymbol{0}$									
$y = \boldsymbol{1}\,(x - {}^{-}\mathbf{5})^2 + \boldsymbol{0}$									

9. Describe what is happening to the graph when $\boldsymbol{b < 0}$.

10. Describe the graphs by comparing and contrasting the items analyzed above.

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{1}\,(x - \mathbf{0})^2 + \boldsymbol{1}$									
$y = \boldsymbol{1}\,(x - \mathbf{0})^2 + \boldsymbol{3}$									
$y = \boldsymbol{1}\,(x - \mathbf{0})^2 + \boldsymbol{7}$									

11. Describe what is happening to the graph when $\boldsymbol{c > 0}$.

12. Describe the graphs by comparing and contrasting the items analyzed above.

Quadratic Equation	Domain	Range	Zeros	x-intercept	y-intercept	Line of Symmetry	Max/Min Point	Increasing	Decreasing
$y = \boldsymbol{1}\,(x - \mathbf{0})^2 + {}^{-}\boldsymbol{1}$									
$y = \boldsymbol{1}\,(x - \mathbf{0})^2 + {}^{-}\boldsymbol{3}$									
$y = \boldsymbol{1}\,(x - \mathbf{0})^2 + {}^{-}\boldsymbol{7}$									

13. Describe what is happening to the graph when $\boldsymbol{c < 0}$.

14. Describe the graphs by comparing and contrasting the items analyzed above.

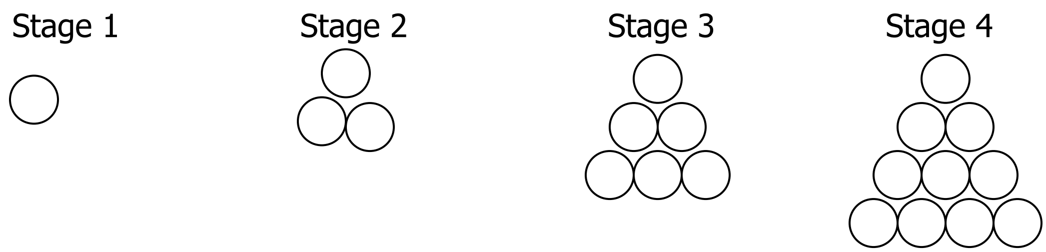
Activity IV: Teacher Notes--Triangular Numbers

Students may either draw or use concrete objects to create representations of triangular numbers through the fourth stage (iteration). Working in groups or individually, students will attempt to determine the number of objects at each stage (iteration). Students will then discover that the difference between the dependent values is not constant. By determining the differences of the last values, students will be able to acknowledge that there are two stages of differences to obtain the constant; therefore, the data will follow a quadratic model.

For enrichment, students may use three of the stage (iteration) values and the general form of the quadratic equation to determine the equation of the curve of best fit using systems of equations.

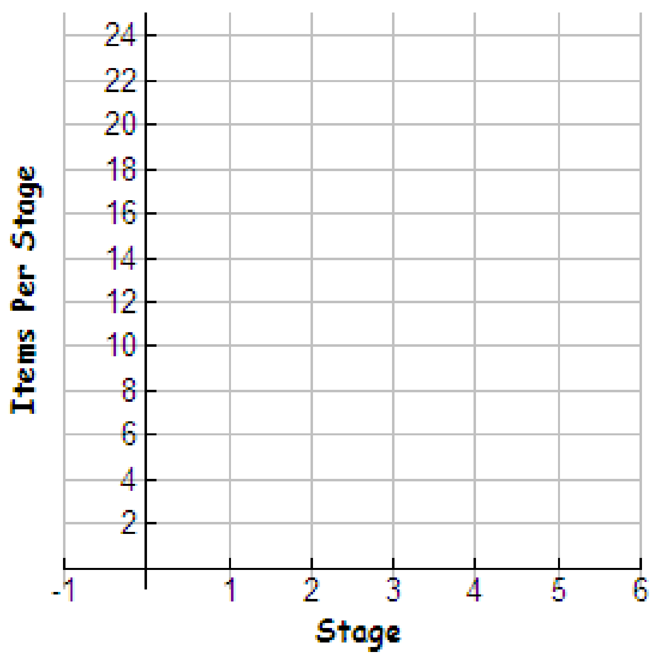
Triangular Numbers

Given the following pattern:



Collecting the Data
Complete the table below and then plot the points on the coordinate plane.

STAGE	ITEMS PER STAGE
1	
2	
3	
4	
n	



Writing About the Pattern

1. Use Stage 4 and fill in the following chart by brainstorming methods to determine the number of items at that stage. Each method must be based on that specific stage. Use the illustration to help you in your efforts.

Written explanation	Symbols	Simplify expression

STAGE	ITEMS PER STAGE
1	
2	
3	
4	
n	

2. Determine the difference in the values in the "Items Per Stage". Are they the same?

______

______

______

3. If not, continue finding the difference of the last results until the values are constant. How many times did you have to find the difference in values before those values were constant?

Graphing the Data

4. Use the **List** function on the graphing calculator and enter the data (Stage, Items Per Stage). Enter into lists as follows:
   L1 = Stage, **L2** = Items Per Stage.

5. Turn on the **[Stat Plot]** and use the **Transformation**™ Application to determine the curve of best fit.

   Record the equation of the curve of best fit y=______________________

Using the Data to Predict

6. How many items will there be at the following stages?

Stage	**Items Per Stage**
5	
10	
15	
55	
n	

The general form of the equation that represents ***triangular numbers*** is $\boldsymbol{y} = \boldsymbol{a}x^2 + \boldsymbol{b}x + \boldsymbol{c}$. The value of **x** stands for the stage and the **y**-value stands for the number of items in that stage.

7. Using the stages and information gathered in the table, substitute a stage for all the x-values and its corresponding y-values (the number of items).

   Stage 2 ____________________

   Stage 3 ____________________

   Stage 4 ____________________

8. Solve the system of equations. Use the simplified equations for stage 2 and stage 3. Eliminate the variable ***c***. Solve another system of equations. Use the equations from #3 and #4 and eliminate the variable ***b***, solving for ***a***.

9. Now that you know ***a***, use one of the equations from #3 or #4 and substitute in ***a***, solve for ***b***.

10. Now you know the values for ***a*** and ***b***. Select one of the original simplified equations from #2 and solve for ***c***.

11. Substitute the values found by substituting into the systems into the equation

    $y = a x^2 + b x + c.$ ***y =*** ______________________

12. Compare this result to the curve of best fit found using the **Transformation™** Application.

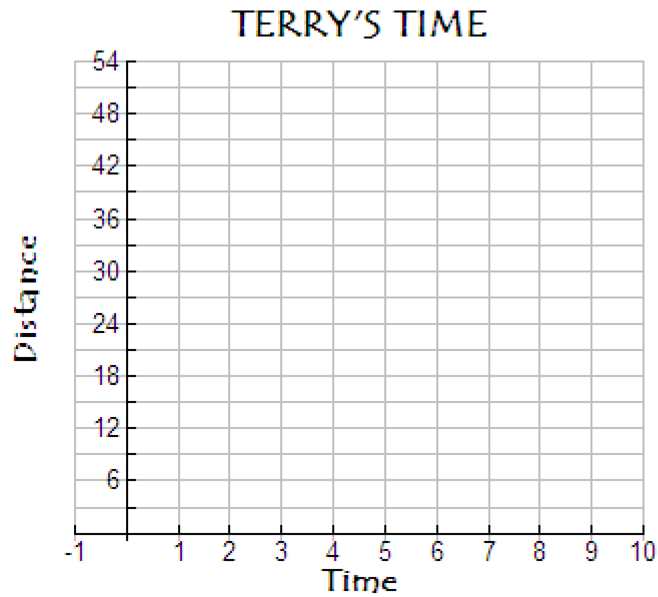
Activity V: Teacher's Notes--Terry's Time

Terry's coach has timed Terry as she runs. The coach has taken time readings at various distances from the starting line as Terry has practiced. Students will graph the data and begin analyzing the data from the table, the graph, and the actual race. The analysis will include acknowledging that the rate of change is not the same in different time intervals. Forward and backward movements and how they relate to the graph will be explored along with what is actually happening when Terry is halfway through the race.

Terry's Time

The coach at the local high school decides to videotape the practice race and time the various members of the team so they can discuss techniques. Terry's time is recorded in seconds for the 50 yard relay.

Time	Distance
0	0
1	19.753
2	34.568
3	44.444
4	49.383
5	48.383
6	44.444
7	34.568
8	19.753
9	0



Collecting and Describing the Data

1. Graph data from the table onto the coordinate plane above.

2. What would be the unit of measure for the average rate of change during the relay race?

3. How fast is Terry running between the 1st and 2nd seconds of the race?

4. How fast is Terry running between the 4th and 5th seconds of the race?

5. How fast is Terry running as she passes the 44.444 yard mark?

6. How fast is Terry running midway into the relay race?

7. When is Terry running away from the starting position?

8. When is Terry running toward the starting position?

9. When is Terry's rate of change the greatest?

10. When is Terry's rate of change the least?

11. What is happening at exactly 4.5 seconds?

12. Determine the following for the given scenario.

    Domain   Range

13. Determine an equation that will best fit the data using the **Transformation™** Application and the vertex form of the parent function typed in **y1**.

    y = ______________________________________

    Let the calculator determine the regression equation. y = ___________

14. What is the equation of the line of symmetry for the graph illustrating the scenario?

15. Describe the relationships between the data represented in the table, on the scatter plot, and as elements of the function.

16. Describe in words Terry's race using the average rate of change for the various times above and how they relate to the race and the graph.

Fitting the Equation

17. The graphing calculator will automatically store the residuals, **RESID**, under **[2nd] [Stat].** The closer the sum of the residuals is to zero, the better the fit. At this time **L1** is time and **L2** is the distance traveled. You have already determined the equation of the curve of best fit, recorded in #13. Turn on **Stat Plot 1** to activate the scatter plot for (time, distance) graph. Press **[2nd] [Graph]** to view the table for the curve of best fit. Complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	$(\text{ACTUAL – FITTED})^2$ L5 = $(L4)^2$
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total $(\text{Actual – Fitted})^2$ [Sum(L5)]				

18. Turn on **Stat Plot 2** to activate another scatter plot of (time, **RESID**). See above to enter **RESID** for the **Ylist**. If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?
Use your calculator and determine the regression for the given data and repeat the process in #17, recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	$(\text{ACTUAL – FITTED})^2$ $L5 = (L4)^2$
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total $(\text{Actual – Fitted})^2$ [Sum(L5)]				

19. Compare and contrast the data in the two tables.

Activity VI: Teacher Notes--Quadratics Playground Exploration

Students are given a scenario in which they are to create a playground by using a specific length of fencing. Students are to determine the dimensions of the sides of the playground when one of the sides is the house so that the play area is maximized. Students will relate the various widths to lengths and the various widths to area graphically and will explain the relationships. An analysis of the graph and the equation representing the data will include domain, range, intervals in which the graph is increasing or decreasing, and the line of symmetry.

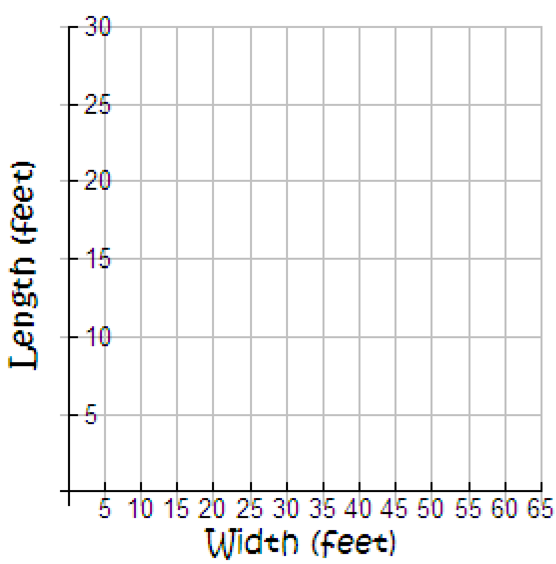
Quadratics Playground Exploration

Mary Ellen wants to add a playground to the backyard of her home for her small children. For convenience and safety, she wants the playground to be enclosed with a fence but one side of the play area will be bounded by the house. She went to the store and got a great deal on 60 feet of fencing. She has been trying to determine the best dimensions for the playground but is frustrated. So she decides to create a chart with the data she has already derived. Using the chart and the corresponding graph will enable Mary Ellen to make a wise decision.

Collecting the Data

1. Complete the following table:

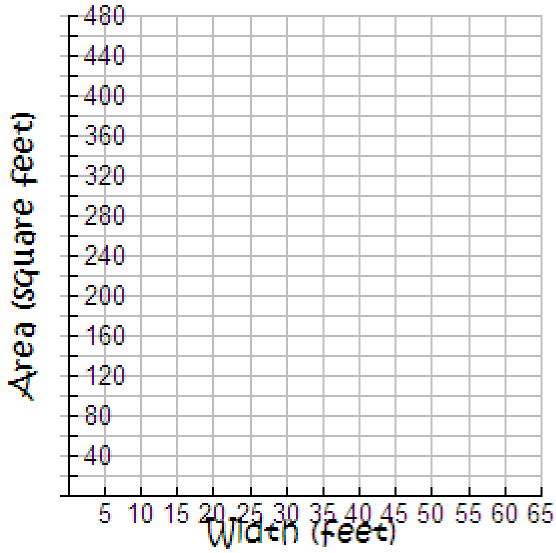
WIDTH	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**
LENGTH											
PERIMETER											
AREA											



2.

a. Graph the relationship between the width in feet and the length in feet of the playground.

b. Describe the change in the playground by comparing the width in feet to the length in feet.



3. Graph the relationship between width in feet of the playground to its overall area in square feet.

4. Describe the change in the playground by comparing the width in feet to the overall area in square feet.

5. What type of function models the curve relating width to the length of the playground? Explain your reasoning.

6. Determine the function that describes the curve relating width to length.
   Note: Use the **Transformation**™ Application on the calculator to help you.

   y = ____

7. In your own words, describe how the curve relates width to length.

8. What type of function models the curve relating width to the area of the playground? Explain your reasoning.

9. Determine the function that describes the curve relating width to the area of the playground.
   Note: Use the **Transformation™** Application to help you.

   y = ____

10. In your own words, describe how the curve relates width to the area of the playground.

11. You have made your decision about the best length and width of the playground.

    The maximum area will be ____________________________,

    which is created using the following dimensions: ________________________.

12. Describe how you might determine the answers to maximize the area and determine the dimensions using the graphs.

13. Determine the following for the given scenario.
    a. Domain   Range

    b. Explain and state the equation written in (*h,k*) form that represents the data.

    c. What is the equation for the line of symmetry for the graph illustrating the scenario?

    d. Describe what is happening in the real life situation when the graph is increasing and/or decreasing.

    e. Describe the relationships between the data represented in the table, on the scatter plot, and as elements of the function.

Fitting the Equation

14. Determine the quadratic regression for the (width, area) data. The graphing calculator will automatically store the residuals, RESID, under **[2nd] [Stat].** (**Residual** = **Actual** – **Fitted**) values. The closer the sum of the residuals is to zero, the better the fit. At this time, **L1** is width and **L2** is the area of the playground. Turn on **Stat Plot 1** to activate the scatter plot for (width, area) graph. Press **[2nd] [Graph]** to view the table of values for the curve of best fit and complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

15. Turn on **Stat Plot 2** to activate another scatter plot of (width, **RESID**).
If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?

Use your calculator and determine the linear regression for the given data and repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

16. Compare and contrast the data in the two tables.

Activity VII: Teacher's Notes--Quadratics Web Page Exploration

This activity works well with individuals or with students in small groups. A student is going to insert a picture of a car on a Web site he/she is creating. There are two different scenarios, one with the picture inserted as a square and the next time as a rectangle.

Square: The picture becomes larger (both length and width increase at a constant rate of change). Area for each set of dimensions is determined. Students will graph time versus height, determine the curve of best fit using the **Transformation™** Application by first recognizing the parent function. Comparisons between the graphs are discussed and the domain and range for this scenario is determined.

Rectangle: The picture becomes larger (both length and height increase at a rate in which the length is always twice the height). Area for each set of dimensions is determined. Students will graph time versus height and time versus length; determine the equation of the curve of best fit using the **Transformation™** Application by first recognizing the parent function. Comparisons between the graphs are discussed. Graphs will be created illustrating time versus area and length versus area. The equation of the curve of best fit will be determined using the **Transformation™** Application by first recognizing the parent function. Comparisons between the graphs are discussed. The domain and range for this scenario are determined.

Finally, given a specific size for a computer monitor, the student will determine which quadrilateral (square or rectangle) to use (the maximum dimensions of the quadrilateral that will fill most of the monitor screen).

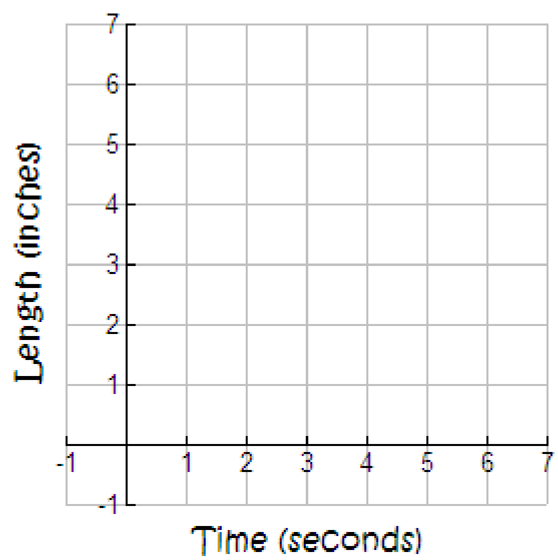
Quadratics Web Page Exploration

You are creating your own Web page and do not want it to be boring. To add some interest you decide to insert a picture of your car but it was to appear and become larger the longer it was seen on the screen. A square would be easier to work with numerically and geometrically. When your Web page comes up, you do not see the car (or the square where it will be). After the first second, the square is visible and for each additional second, the side lengths of the square will increase by an inch.

Complete the following table.

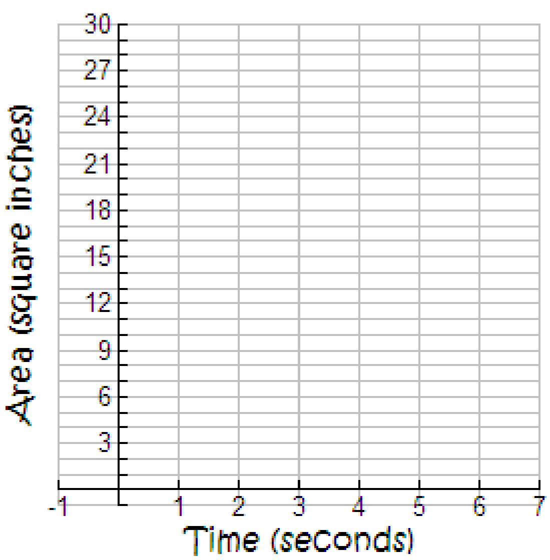
TIME	SIDE LENGTH OF IMAGE	AREA OF PICTURE
0		
1		
2		
3		
4		
5		
6		
Expression		

1. Graph the relationship between the time in seconds and the length in inches.



2. Describe the change in the picture size by comparing the time to the length.

3. Graph the relationship between time in seconds and the area in square inches.



4. Describe the change in the picture by comparing the time to the area.

5. What type of function models the curve relating time to side length?

6. Determine the function that describes the curve relating time to side length.

7. In your own words, describe how the curve relates time to side length.

8. What type of function models the curve relating time to the area of the picture?

9. Determine the function that best describes the curve relating time to the area of the picture.

10. In your own words, describe how the curve relates time to the area of the picture.

11. You love the Web page but decide to make one more change. The monitor screen is 12 inches by 9 inches and you want the picture to take up as much of the screen as possible.

    The maximum side length is ____________________________

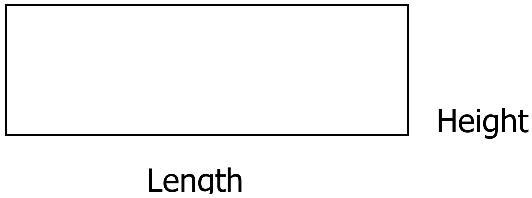
    The maximum area will be _____________________________

12. Describe how you might determine the above answers to maximum side length and maximum area using the graphs.

13. Discuss the possible values for the domain and for the range.

14. You still are not satisfied and decide to take it up another level. To add some interest, you decide to insert the same picture of your car but it is to appear and become larger the longer it is seen on the screen. The length will always be twice as long as its height. The rectangle is similar to the original shape of the picture and, therefore, will create a better picture on the screen. When your Web page

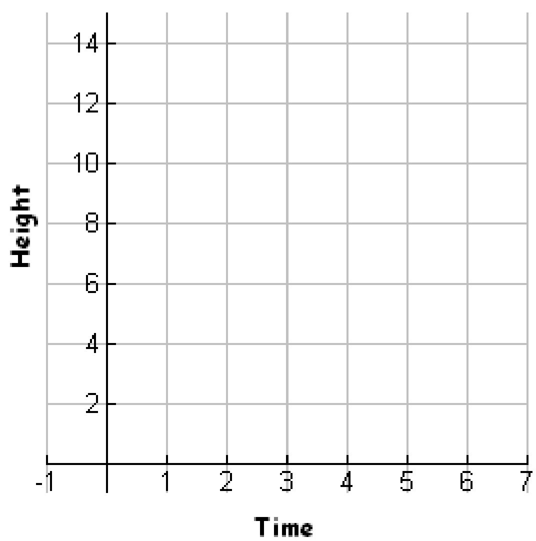
comes up, you do not see the car (or the rectangle where it will be). After the first second the square is visible and, for each additional second, the height of the rectangle will increase by an inch.



Complete the following table:

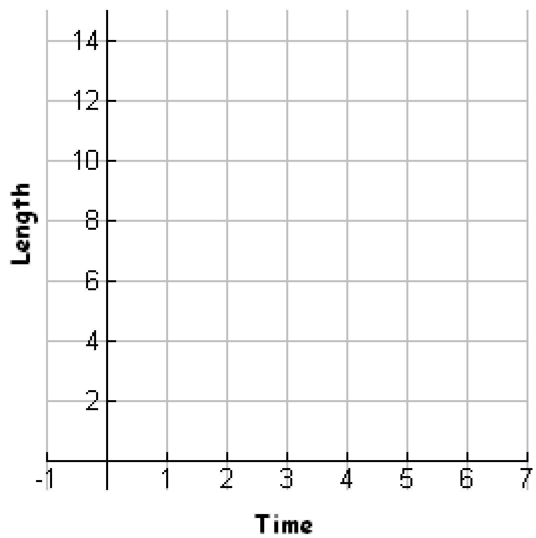
TIME	HEIGHT	LENGTH	AREA OF PICTURE
0			
1			
2			
3			
4			
5			
6			
Expression			

15. Graph the relationship between the time in seconds and the height in inches.



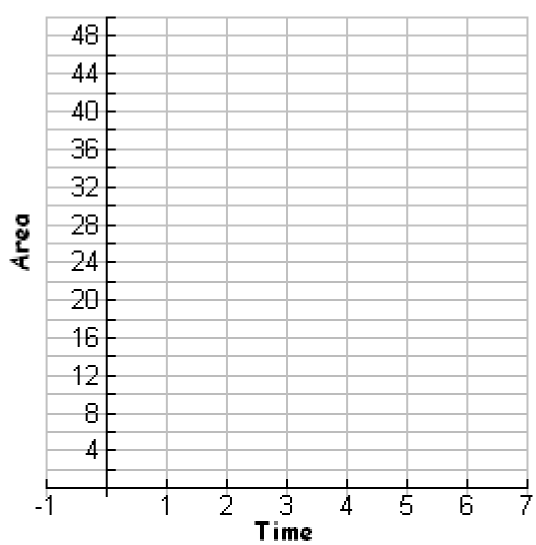
16. Describe the change in the picture size by comparing the time to the height.

17. Graph the relationship between the time in seconds and the length in inches.



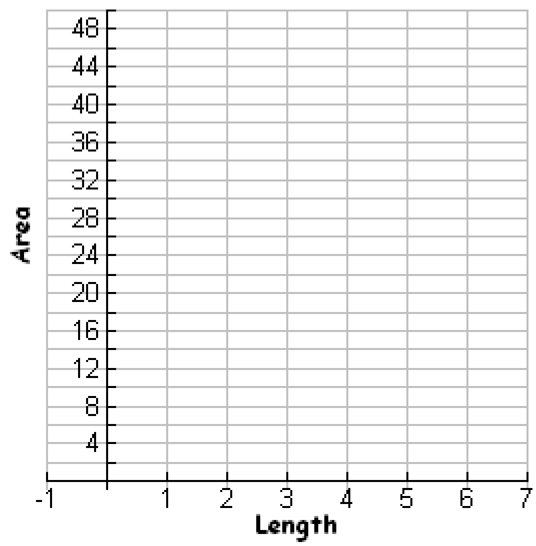
18. Describe the change in the picture size by comparing the time to the length.

19. Graph the relationship between time in seconds and the area in square inches.



20. Describe the change in the picture by comparing the time to the area.

21. Graph the relationship between length and the area in square inches.



22. Describe the change in the picture by comparing the length to the area.

23. What type of function models the curve, relating time to height?

24. Determine the function that describes the curve relating time to height.

25. What type of function models the curve relating time to length?

26. Determine the function that describes the curve relating time to length.

27. In your own words, describe how the curve relating time to height compares to the curve relating time to length.

28. What type of function models the curve relating time to the area of the picture?

29. Determine the function that describes the curve relating time to the area of the picture.

30. What type of function models the curve relating length to the area of the picture?

31. Determine the function that describes the curve relating length to the area of the picture.

32. In your own words, describe how the curve relating time to the area of the picture compares to the curve relating length to the area of the picture.

33. You love the Web page but need to put a limit on how big the picture will get. The monitor screen is 12 inches by 15 inches and you want the picture to take up as much of the screen as possible.

34. Would you use the square or the rectangle to create the picture that best adapts to the screen? Why?

    The maximum length is ________________________________

    The maximum height is ________________________________

    The maximum area will be ______________________________

    Describe how you might determine the above answers to maximum side length and maximum area using the graphs.

35. Discuss the possible values for the domain and for the range.

Fitting the Equation

36. Select either the data from time versus area or length versus area. The graphing calculator will automatically store the residuals, **RESID,** under **[2nd] [Stat].** The closer the sum of the residuals is to zero, the better the fit. At this time, **L1** is time/length and **L2** is the area. You have already determined the equation of the curve of best fit, recorded in #7 or #9.Turn on **Stat Plot 1** to activate the scatter plot for (time/length, area) graph. Press **[2nd] [Graph]** to view the table for the curve of best fit. Complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

37. Turn on **Stat Plot 2** to activate another scatter plot of (time, RESID). See above to enter **RESID** for the **Ylist**. If this was a perfect fit, **Stat Plot 2** values would all be on the x-axis. Why?

38. Use your calculator and determine the linear regression equation for the given data and repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED $L4 = L2 – L3$	$(ACTUAL – FITTED)^2$ $L5 = (L4)^2$
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total $(Actual – Fitted)^2$ [Sum(L5)]				

39. Compare and contrast the two tables.

Sample Resources:

http://mathforum.org/workshops/usi/dataproject/usi.genwebsites.html

- The Data Library

http://education.ti.com

- Activities Exchange

http://hintsandthings.com/garage/stopmph.htm

http://csgnetwork.com/stopdistinfo.html

- Braking and stopping distance

http://www.childstats.gov/americaschildren/tables.asp

- Family stats (ELL, Adolescent births, foreign born parents, …)

http://stats.bls.gov/cpi/home.htm

- Create customized tables using the Consumer Price Index – Average Price Data

http://factfinder.census.gov/home/saff/main.html?_lang=en

- Geographic and population data

http://gapminder.org

http://statmaster.sdu.dk/courses/st111/data/index.html#children

http://www.ilovemath.org

- Lesson Plans & Activities

Organizing Topic: Exponential Modeling

Mathematical Goals:

- Students will model exponential relationships from data gathered during activities and from Internet database sources.
- Students will investigate and analyze key characteristics of exponential functions including domain, range, asymptotes, increasing/decreasing behavior, and end behavior.
- Students will make predictions using exponential curve-fitting and evaluating the model at specific domain values outside the given data set.

Standards Addressed: AFDA.1; AFDA.2; AFDA.3; AFDA.4

Data Used:

- Data obtained from observation/measurement in activities
- Data imported from Internet databases

Materials:

- Applications: **EasyData™** and **Transformation™** Application
- Graphing calculator and links
- Handout – Who Wants to be a Millionaire?
- Handout – Paper Folding
- Handout – M&M™ Decay
- M&M™'s Fun Size, cups, paper towels
- Handout – Decaying Dice Game
- Handout – Population Growth
- Handout – Baseball Players' Salaries
- Graph paper

Instructional Activities

I. Who Wants to be a Millionaire?

Students will investigate the similarities and differences between and among constant, linear, and exponential functions. Students will gain familiarity with the graphing calculator (how to use the **Stat** functions to put data in lists, find regression equations, use **StatPlot, ZoomStat**, **Table**, and **Tblset functions**.)

Concepts covered include:

- scatter plots;
- domain and range;
- continuity;
- linear and exponential functions;
- evaluating a function for a given domain element;
- independent and dependent variables;
- slope;
- slope-intercept form of an equation;
- linear and exponential regression; and
- transformations.

II. Paper Folding Activity

Students will model exponential growth and exponential decay functions by folding paper. Students will investigate how quickly an exponential function increases/decreases.

Concepts covered include:

- scatter plots;
- domain and range;
- continuity;
- linear and exponential functions;
- evaluating a function for a given domain element;
- independent and dependent variables;
- slope;
- slope-intercept form of an equation;
- exponential regression;
- transformations;
- function;
- exponential growth/decay;
- asymptotes; and
- end behavior.

III. M&M™ Decay

Students will collect experimental data from trials. Data will vary. An exponential decay model will best represent the data, with a rate of decay close to 0.5.

Concepts covered include:

- scatter plots;
- domain and range;
- continuity;
- linear and exponential functions;
- evaluating a function for a given domain element;
- independent and dependent variables;
- slope;
- slope-intercept form of an equation;
- exponential regression;
- transformations;
- function;
- exponential decay;
- asymptotes;
- end behavior; and
- theoretical and experimental probability.

IV. Decaying Dice Game (optional)

Students will play a game to reinforce the concepts of exponential decay and probability of events.

Concepts covered include:

- domain and range;
- continuity;
- independent and dependent variables;
- transformations;
- function;
- exponential decay;
- end behavior; and
- theoretical and experimental probability.

V. Population Growth

Students will use the Internet to collect data on average salaries of baseball players from 1975 to present. Students will analyze data and model with an appropriate function.

Concepts covered include:

- scatter plots;
- domain and range;
- continuity;
- linear and exponential functions;
- evaluating a function for a given domain element;
- independent and dependent variables;
- exponential regression;
- transformations;
- function;
- asymptotes; and
- end behavior.

VI. Baseball Players Salaries

Students will use the Internet to collect data on average salaries of baseball players from 1975 to present. Students will analyze data and model with an appropriate function.

Concepts covered include:

- scatter plots;
- domain and range;
- continuity;
- linear and exponential functions;
- evaluating a function for a given domain element;
- independent and dependent variables;
- exponential regression;
- transformations;
- function;
- asymptotes; and
- end behavior.

Activity I: Teacher Notes--Who Wants to Be a Millionaire?

Students should recognize that Options 1 and 2 are both linear and that Option 3 is not linear. Intuitively, students may feel that either Option 1 or 2 is the best until they realize how fast the value of the pennies is increasing additively, or better yet, increasing multiplicatively. This warm-up takes students from a function with which they are very familiar and compares it with the exponential function.

Have students plot the data, (days, salary) and discuss what the data look like (linear for Options 1 and 2, exponential for 3). Discuss whether the data are discrete or continuous; whether or not the relation is a function; finding the domain and range; and identifying the independent and dependent variables. Have students enter the data in the graphing calculator and determine the curve of best fit using the **Transformation™** Application or **LinReg/ExpReg™**. Discuss the meaning of the y-intercepts and the rate of change. Guide students in a discussion of the differences between linear functions and exponential functions. Have students identify key characteristics of exponential functions and define the function of *a* and *b* in $y=a(b)^x$.

Who Wants to be a Millionaire?

You are sitting in mathematics class, and the famous billionaire, Bill Buffett Jobs, offers you the job of a lifetime. You would only need to work for one month (30 days) and could become a millionaire. But there is a catch! He offers you three payment options and to show yourself worthy, you must pick the best option and explain your choice.

Option 1: You earn $1,000,000, evenly distributed over the 30 day period.

Option 2: You earn $3,000 the first day, then for each following day an additional $3,000 will be added to the previous day's salary for the 30 days.

Option 3: You earn one cent the first day, two cents the second, and double your salary each day thereafter for 30 days.

Collecting Data

1. Which option should you choose? Explain your reasoning:

   Complete the following tables for each of the 3 options.

Option 1	
Day	**Salary**
1	
2	
3	
4	
5	
6	
7	

Option 2	
Day	**Salary**
1	
2	
3	
4	
5	
6	
7	

Option 3	
Day	**Salary**
1	
2	
3	
4	
5	
6	
7	

We will now enter each set of data into the calculator to determine the mathematical model for each and decide which payment option is wisest.

2. Option 1 and Option 2 can both be modeled by a linear regression equation ***y = mx+b***.

3. Option 1 can be modeled by the equation: y = ___________________
   Give an explanation for the slope:

4. Option 2 can be modeled by the equation: y = ___________________
   Give an explanation for the slope:

5. Does option 3 seem to follow a linear model? Explain your reasoning.

6. For Option 3, find an equation for the curve using regression (use **ExpReg** under **Stat:Calc** menu).

7. An equation for an exponential curve is of the form $y = a(b)^x$. Option 3 can be modeled by the equation: y = _______________ What is the significance of ***b*** in this equation?

8. Graph the three equations representing the three options simultaneously in y_1, y_2, y_3. Press the **Window** button and type in these settings to properly view all three graphs **Xmin** = 0, **Xmax** = 30, **Xscl** = 1, **Ymin** = 0, **Ymax** = 1,000,000, **Yscl**=1
What observations can you make about the three graphs?

   What do you notice about the Option 3 graph?

9. Using the **Table** function, evaluate how much your salary would be on Day 30 using Option 2 and then Option 3.

10. If you were to find the sum of all the payments under Option 3, it would be over $10,000,000! That's a lot of pennies!

Fitting the Equation

11. Using Option 3, enter the data into **[Stat] [Edit]** and perform the appropriate regression to determine the equation of the curve of best fit. Activate **Stat Plot 1** for the scatter plot (time, growth) graph. Activate **Stat Plot 2** for the scatter plot (time, **RESID**) graph. Complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	$(\text{ACTUAL} - \text{FITTED})^2$ $L5 = (L4)^2$
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total $(\text{Actual} - \text{Fitted})^2$ [Sum(L5)]				

12. Use your calculator and determine another type of regression for the given data and repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	$(\text{ACTUAL} - \text{FITTED})^2$ $L5 = (L4)^2$
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total $(\text{Actual} - \text{Fitted})^2$ [Sum(L5)]				

13. Compare and contrast the data in the two tables.

Activity II. Teacher's Notes: Paper Folding Activity

The instructor may choose to introduce this activity by choosing a student and challenging him/her, "How many times do you think you can fold this sheet of paper in half?"

Have the student demonstrate how difficult the task becomes after only 6 folds.
This may be done individually or in small groups. Students will follow directions on worksheet and complete.

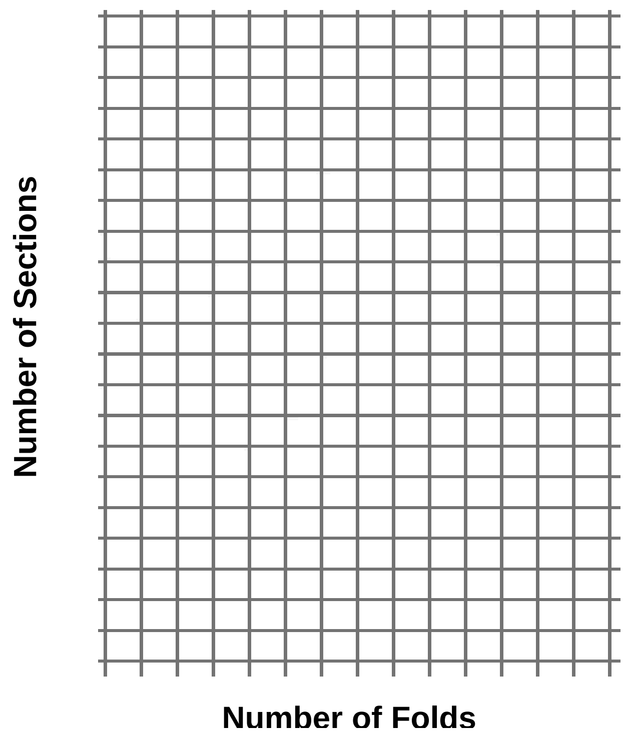
Activity II is a very hands-on, concrete example of both exponential growth and exponential decay and makes it clear what the values of ***a*** and ***b*** in the exponential equation mean in terms of their effects on the equation and in the context of the problem.

Paper Folding Activity

Number of Sections: How many times do you think you can fold a piece of paper in half?

1. Fold an 8.5″ x 11″ sheet of paper in half and determine the number of sections the paper has after each fold.
2. Record your data in the table below and continue folding in half until it becomes too hard to fold the paper.
3. Then make a scatter plot of your data.

Number of Sections	
Number of Folds	**Number of Sections**
0	
1	
2	
3	
4	
5	
6	
7	
8	



4. Using your calculator, determine the mathematical model that represents this data:
   y = ____________

5. Explain in words what the mathematical model means.

6. What might be different if you tried this experiment with wax paper or tissue paper?

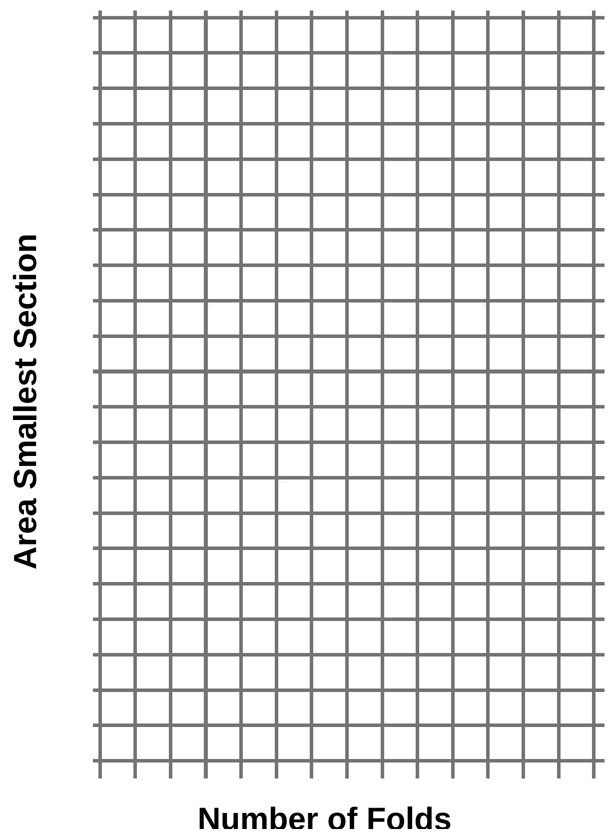
This is an example of exponential growth. The thickness of the paper grows very rapidly with each fold. To get an idea of this incredible growth, consider:

At 7 folds, it is as thick as a notebook.
At 17 folds, it would be taller than the average house.
At 20 folds, the sheet of paper is thick enough to extend a quarter of the way up the Sears Tower in Chicago.
At 30 folds, it has crossed the outer limits of the atmosphere.

Area of Smallest Section

7. Again, fold a piece of paper in half and determine the area of the smallest section after you have made a fold. What is the original area of the sheet of paper?
8. Record your data in the table below.
9. Then make a scatter plot of your data.

Number of Sections	
Number of Folds	**Area of Smallest Section**
0	
1	
2	
3	
4	
5	
6	
7	
8	



10. Using your calculator, determine the mathematical model that represents this data:

    y = ______________.

    This is an example of exponential decay.

11. Explain what each part of the mathematical model means.

12. What would be the area of the smallest section of the piece of paper, if you were able to fold it 10 times?

Fitting the Equation

13. Enter the data into **[Stat] [Edit]** and perform the appropriate regression to determine the equation of the curve of best fit.
    Activate **Stat Plot 1** for the scatter plot (time, growth/decay) graph.
    Activate **Stat Plot 2** for the scatter plot (time, **RESID)** graph.
    Complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

14. Use your calculator and determine another type of regression for the given data and repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

15. Compare and contrast the data in the two tables.

Activity III: Teacher Notes--M&M™ Decay

M&M™ Decay is a group activity (3-4 students in a group). Students will perform the experiment and collect their own data from a series of trials.

Students will use a cup to mix M&M™'s in a variety of colors and place them on a paper towel on the desk. It is important that students do not use a value of 0 for ***Number of M&M'™s Remaining*** so that they can perform an **ExpReg™** on their data.

This activity provides a concrete example of exponential decay. Through this activity, students should begin to understand how exponential growth and exponential decay can occur as natural phenomena.

Extension is optional but provides further investigation into exponential functions.

M&M™ DECAY

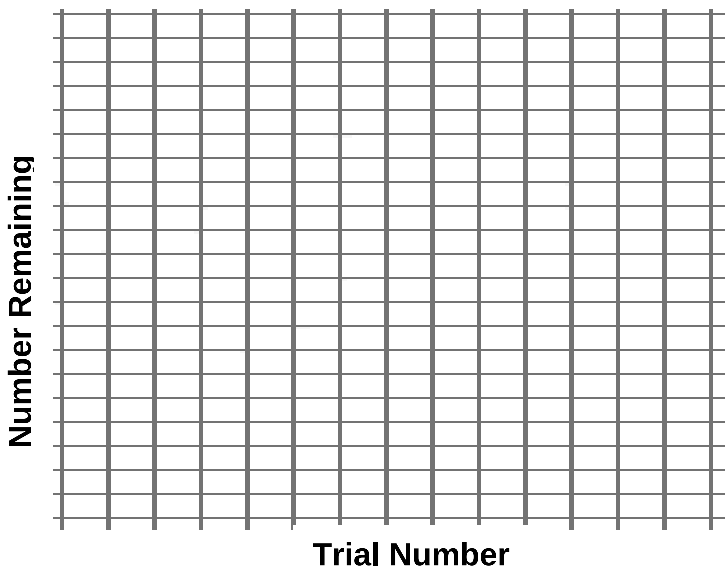
Collecting Data

Empty your bag of M&M™'s onto the table and count the M&M™'s. Then place the M&M™'s in a cup and mix them well. Pour them out on the desk, count the number that show an "m," and place them back in the cup. The others may be eaten or removed. Record the number of M&M™'s that show an "m" in your data table. Then repeat the procedure. Continue until the number of M&M™'s remaining is less than 5, but greater than 0.

Number of M&M's	
Trial Number	**Number of M&M's Remaining**
0 (initial amount)	
1	
2	
3	
4	
5	
6	
7	
8	

Graphing and Determining the Model

Use a graphing calculator to make a scatter plot of your data. Copy your scatter plot onto the grid below. Then use the graphing calculator to find the curve of best fit, and graph the equation. Sketch in the curve and write your equation.



Equation:________________

Interpreting the Data

1. In your model, $y = a(b)^x$, what value do you have for ***a***? To what does ***a*** seem to relate when you consider your data? When x = 0, what is your function value? Compare this to the values in your data table.

2. What is the theoretical probability that any single M&M™ will be removed in a trial?

3. What is the value for ***b*** in your exponential model? Explain the significance of this value and how it relates to your data.

4. If you started with 40 M&M™'s, how many trials do you think it would take before the number of M&M™'s was between five and zero? What equation would model this new, initial value?

5. How does the M&M™ experiment compare with the paper-folding activity? How are they alike and how are they different?

Extension—Beyond M&M™ Decay

6. What other objects could be used that would follow the same exponential model as in the previous experiment? What objects could you use to change the value of ***b***?

7. How could you use M&M™'s to model exponential growth instead of exponential decay?

8. Use the Internet to research real-life phenomena that follow either an exponential growth model or an exponential decay model.

Fitting the Equation

9. Enter the data above that illustrates exponential decay in to **[Stat] [edit]** and perform the appropriate regression to determine the curve of best fit. Activate **Stat Plot 1** for the scatter plot (time, decay) graph. Activate **Stat Plot 2** for the scatter plot (time, RESID) graph. Complete the table below.

TIME L1	**ACTUAL DISTANCE L2**	**FITTED DISTANCE L3**	**ACTUAL – FITTED L4 = L2 – L3**	**(ACTUAL – FITTED)^2 L5 = (L4)^2**
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

10. Use your calculator and determine another type of regression for the given data and repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

11. Compare and contrast the data in the two tables.

Activity IV: Teacher Notes--Decaying Dice Game (optional)

This is a group activity (2 to 4 students per group). Students may play the game in a small group to reinforce the concepts in the **M&M™ Decay** exploration.

This activity can be used if some groups finish the **M&M™ Decay** activity early.

Students will have to calculate probabilities, and make predictions, based on a new exponential decay model.

Decaying Dice Game

Objective of the game: To predict as closely as possible how long your dice will "stay alive".

Instructions: Before a player rolls the dice, the player makes a prediction of how many throws of the dice they will have until all dice are removed. Dice that come up "6" are removed, and the remaining dice are thrown. Repeat the process and keep count of how many rolls it takes to remove all the dice. In the subsequent rounds, remove one die at the beginning of the game.

Scoring: 50 points, if the number of actual rolls matches the player's prediction
25 points, if it is 1 number above or below the prediction
10 points, if it is 2 away from the prediction.

Alternatives: Choose multiple numbers to be removed or choose a number that "reproduces" another die to be added.

Player	**Prediction**	**Actual**	**Score**
Round 1			
Round 2			
Round 3			
Round 4			
Round 5			
Round 6			
Total Score			

Player	**Prediction**	**Actual**	**Score**
Round 1			
Round 2			
Round 3			
Round 4			
Round 5			
Round 6			
Total Score			

Player	**Prediction**	**Actual**	**Score**
Round 1			
Round 2			
Round 3			
Round 4			
Round 5			
Round 6			
Total Score			

Player	**Prediction**	**Actual**	**Score**
Round 1			
Round 2			
Round 3			
Round 4			
Round 5			
Round 6			
Total Score			

Activity V: Teacher Notes--Population Growth

Population Growth can be used as an individual or group activity and will require Internet access. Students will collect data from **U.S. Census Bureau International Database** to compare population growth in three countries. We suggest that students use Ethiopia, U.S., and China as the three countries and these three display very different characteristics. Ethiopia seems to follow a strong exponential model, the U.S. appears very linear, and China can best be described as logistic or S-shaped. If other countries are chosen, we suggest that the countries be located in very different geographical regions.

The instructor might choose to have students complete part or all of this activity outside of the classroom as a special project. The instructor might also choose to expand upon this activity and collaborate with a history/social studies teacher and ask students to go into more depth by describing the similarities and differences of the countries and the factors that may influence human population growth.

This activity might also be modified to look at animal populations of three different species in a specific region.

Population Growth

Using the Internet to access the U.S. Census Bureau's International Database, you will collect population data from three countries. You will analyze the data and decide which type of model best represents the data.
The Internet site: http://www.census.gov/ipc/www/idb/

Collect the Data

1. From this site, go to the "Tables" link. We will be looking at total midyear population. Go to the bottom of the screen and hit "Submit Query." On the next screen, scroll down to the list of countries. Holding down the Control key, select China, Ethiopia, and the United States (other countries may be used with teacher approval). Scroll further down to the Year Selection and type in From: 1950 To: 2005 By: 5. Then select Go. The next screen should give you a table of populations for the selected countries.

Country:		**Country:**		**Country:**	
Year	**Population**	**Year**	**Population**	**Year**	**Population**
1950		**1950**		**1950**	
1955		**1955**		**1955**	
1960		**1960**		**1960**	
1965		**1965**		**1965**	
1970		**1970**		**1970**	
1975		**1975**		**1975**	
1980		**1980**		**1980**	
1985		**1985**		**1985**	
1990		**1990**		**1990**	
1995		**1995**		**1995**	
2000		**2000**		**2000**	
2005		**2005**		**2005**	

Graph and Determine the Model

2. With a graphing utility, create a scatter plot by entering data in the tables above into your calculator (**L1** = year, **L2** = population of country 1, **L3** = population of country 2, and **L4** = population of country 3). Examine the data and determine what mathematical model you think best represents the data (it may be linear, exponential or perhaps something else!) Draw your graphs on a separate piece of graph paper.

   Country 1:________________Regression Equation: y = ____________________

   Country 2:________________Regression Equation: y = ____________________

   Country .3:_______________ Regression Equation: y = ____________________

Interpret the Data

3. Compare and contrast your observations of the scatter plots and graphs.

4. Discuss the domain, range, shape, and end behavior of each.

5. Which country has the fastest growth rate? Which has the smallest growth rate?

6. Based on your models, make a prediction for the future populations of each country in ten years.

7. Compare your predictions to the predictions of the U.S. Census Bureau by revisiting the Web site and selecting the countries and designated year. How do they compare?

Extension

8. Discuss several factors that influence human population growth.

Want to see how the world's population is growing right now?
Go to: http://www.worldometers.info/

Fitting the Equation

9. Select one of the three countries and use the regression model that applies to the population data for that country and was found in Part 2. Complete the table below. In the column representing year, let 0 represent 1950; represent 1951, and so on.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

Enter the data above into **L1, L2, L3**.
Activate **Stat Plot 1** for the scatter plot (time, population) (**L1, L2**) graph.
Activate **Stat Plot 2** for the scatter plot (time, residual) (**L3, L4**) graph.

10. Select one of your other choices and determine the regression for the given data. Repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

11. Compare and contrast the data in the two tables.

Activity VI: Teacher Notes--Baseball Players' Salaries

Baseball Players' Salaries is an individual activity and requires Internet access for each participant. Students must complete the handout. We suggest that students do their own search.

One Web site that works: http://eh.net/encyclopedia/article/haupert.mlb

Baseball Players' Salaries

Collecting Data

1. Use the Internet to find the average salaries of major league baseball players for selected years 1975 to the present. Write the URL you used below. Put your data into tabular form.

2. Using a graphing utility, draw a scatter plot for this data comparing the average salaries of major league players over the years.

3. Find the equation that best fits the data: y = ______________

4. On graph paper, draw the scatter plot and regression curve.

5. What is the correlation coefficient for this data? (Use the **DiagonisticOn** function on the calculator and the correlation coefficient is ***r***.)

6. Extrapolate from the data (make a prediction) what the average baseball player's salary will be in 5 years; in 10 years.

7. Does this model seem reasonable to you? Why do you think baseball players' salaries have followed this model? Do you think other occupations' salaries would follow a similar model?

Fitting the Equation

8. Enter the baseball salary data in to **[Stat] [Edit]** and perform the appropriate regression to determine the curve of best fit. Activate **Stat Plot 1** for the scatter plot (time, salary) graph. Activate **Stat Plot 2** for the scatter plot (time, **RESID)** graph. Complete the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

9. Use your calculator and determine another type of regression for the given data. Repeat the above process recording the values in the table below.

TIME L1	ACTUAL DISTANCE L2	FITTED DISTANCE L3	ACTUAL – FITTED L4 = L2 – L3	(ACTUAL – FITTED)^2 L5 = (L4)^2
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

10. Compare and contrast the data in the two tables.

Other Topics for Exploration in Exponential Growth and Decay

Calculating compound interest

Population growth of species

Limitations of exponential models and logistic curves

Exponential stories: Rice on a chessboard; Waterlilies

Cell growth in biology

Radioactive decay

Processing power of computers and Moore's Law

Resources:

http://mathforum.org/workshops/usi/dataproject/usi.genwebsites.html
- The Data Library

http://www.census.gov/ipc/www/idb/
- U.S. Census Bureau International Database

http://mathforum.org/mathtools/sitemap2/a2/
- Math Topics for Algebra II

http://eh.net/encyclopedia/article/haupert.mlb
- Average Salaries of Baseball players

http://www.worldometers.info/

Worldometers world statistics updated in real time

Organizing Topic: Logarithmic Modeling

Mathematical Goals:

- Students will model logarithmic relationships from data gathered during activities and from Internet database sources.
- Students will investigate and analyze key characteristics of logarithmic functions including domain, range, asymptotes, increasing/decreasing behavior, and end behavior.
- Students will use knowledge of transformations to write an equation given the graph of function and graph a function, given its equation.
- Students will make predictions using logarithmic curve-fitting and evaluation of the model at specific domain values outside the given data set.

Standards Addressed: AFDA.1; AFDA.2; AFDA.3; AFDA.4

Data Used: Data obtained from observation/measurement in activities and imported from Internet databases.

Materials:

- Applications: **EasyData™** and **Transformation**™, **TI-Interactive™**
- Computer lab
- Graphing calculator and links
- Patty paper
- Graph paper
- Handout – Switch It Up
- Handout – Transforming Functions
- Handout – High/Low Guessing Game
- Handout – Life Expectancy

Instructional Activities:

I. Introduction to the unit – Switch It Up

Students will investigate the inverses of functions as an introduction to the logarithmic function as the inverse of the exponential. Students will be able to identify and explain what inverses and logarithms are both algebraically and graphically.

Concepts covered include:

- domain and range;
- continuity;
- linear functions;
- quadratic functions;
- exponential functions;
- logarithmic functions;
- evaluating a function for a value in the domain;

- independent and dependent variables;
- inverses;
- zeros; and
- intercepts.

II. Transforming Functions

Students will investigate the transformations of quadratic, exponential, and logarithmic functions. Students will identify reflections, translations, and dilations of known parent functions. Students will match equations to graphs using a transformational approach.

Concepts covered include:

- parent functions (linear, quadratic, exponential, logarithmic);
- transformations (translations, reflections, dilations);
- domain and range;
- zeros; and
- intercepts.

III. High/Low Guessing Game

Students will play the game and develop an optimal strategy which decreases the number of possibilities by half. Students will be able to identify this as an exponential decay function. Students will then investigate a logarithmic function which reverses the process, solving an exponential equation.

Concepts covered include:

- exponential decay;
- logarithmic functions;
- properties of logarithms;
- change of base formula; and
- domain and range.

IV. Life Expectancy

Students will use the Internet to collect data from the National Center for Health Statistics Web site. Students will analyze data and model various functions.

Concepts covered include:

- scatter plots;
- domain and range;
- continuity;
- function;
- evaluate a function;
- independent and dependent variables;
- linear regression;
- quadratic regression;
- exponential regression;
- logarithmic regression;
- correlation coefficient;
- transformations;
- asymptotes; and
- end behavior.

Activity I: Teacher Notes--Switch It Up

The logarithmic function is introduced as the inverse of the exponential function.

The concept of an inverse function should be reinforced by students' prior knowledge of inverse functions as reflections across y = x. The teacher should reinforce previously learned concepts of linear and quadratic functions, domain, and range.

The teacher may choose to incorporate more examples of inverse functions or having students graph more pairs of inverses.

Switch It Up

What is the inverse of a function?

The inverse reverses or undoes the previous action. If f(x) represents the original function then $\mathbf{f^{-1}(x)}$ is the inverse function.

Look at the function $\mathbf{f(x) = x + 5}$.
This function adds 5 to a number, so what is the reverse operation or inverse function?
The inverse is to ___________________________
$\mathbf{f^{-1}(x)}$ = _____________

Look at the function $\mathbf{g(x) = 2x}$.
This function multiplies a number by 2, so what would be the inverse function?
The inverse is to ___________________________
$\mathbf{g^{-1}(x)}$ = _____________

Look at the function $\mathbf{h(x) = x^2}$.
What does this function do? _________________________
The inverse is to ___________________________________
$\mathbf{h^{-1}(x)}$ = ______________

1. Fill in the tables for **h(x)** and $\mathbf{h^{-1}(x)}$

x	h(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	

x	$h^{-1}(x)$
0	
1	
4	
	9

On graph paper, plot the points from the tables and connect the points with a smooth line.

What do you notice about the graphs?

The graphs are mirror images or reflections across the line: y = ___________________

What do you notice about the domain and range of each?

2. Now we will investigate the inverse of an exponential function $\mathbf{f(x) = 3^x}$.
   Fill in the table below. Then sketch a graph of the function.

x	**- 3**	**- 2**	**-1**	**0**	**1**	**2**	**3**	**4**
f(x)								

   What is the domain of the exponential function?

   What is the range of the exponential function?

3. Trace the x-axis, y-axis, and the graph of **f(x)** onto a sheet of patty paper. Be sure to label the axes. Reflect the graph of f(x) across the line **y = x** by holding the top-right and bottom left corners of the patty paper in each hand and flipping the sheet of patty paper over. Sketch what you see.

   Is this new graph a function? Explain how you know.

   What is the domain?

   What is the range?

4. Fill in the table below for the new graph.

x								
g(x)	**- 3**	**- 2**	**-1**	**0**	**1**	**2**	**3**	**4**

5. The inverse of an exponential function has a special name called a **logarithmic** function.
   If $\mathbf{b^y = x}$ then we can write $\mathbf{\log_b x = y}$.
   Ex. $\mathbf{5^3 = 125}$ so $\mathbf{\log_5 125 = 3}$
   Fill in the table completely, creating two of your own problems.

Exponential Form	**Logarithmic Form**
$4^3 = 64$	$\log_4 64 =$
$5^4 =$	$\log_5 625 = 4$
$3^4 = 81$	
	$\log_2 128 = 7$
$6^3 =$	
	$\log_2 64 =$

On the calculator, you will notice the two logarithmic functions most commonly used, "log" which is base 10 and "ln" which is base *e,* an irrational number.

6. Summarize what you have learned.

How would you explain the inverse of a function to someone else?

How would you explain logarithms to someone else?

Activity II: Teacher Notes--Transforming Functions

Students will investigate a transformational approach to graphing functions using a graphing technology. The **Transformation™** Application may be used. A TI-interactive could be used in a computer laboratory for a hands-on activity that explores transformational graphing techniques. This activity builds upon students' prior knowledge, reinforces previous concepts, while it develops new understanding of logarithmic functions. Transformation cut-outs can be used as an individual or group activity for enrichment or review.

Transforming Functions

We have encountered several basic or parent functions such as linear, quadratic, exponential, and logarithmic functions. By using these basic or parent functions, we can build new functions by several types of movements such as dilating, translating, and reflecting the original parent function. We will now investigate these transformations.

For each part, make a sketch of each original function and its transformations on one grid of graph paper. Use different colored pencils to represent each transformation.

A **reflection** is a movement where a graph "flips" over an axis. It is called a **reflection** because it will be a mirror image of the original. We will graph f(x), - f(x), and f(-x)

1. Sketch the graph of each function.
   a. $y = x^2$ b. $y = - x^2$ c. $y = (-x)^2$

2. Sketch the graph of each function.
   a. $y = e^x$ b. $y = -e^x$ c. $y = e^{-x}$

3. Sketch the graph of each function.
   a. $y = \ln x$ b. $y = - \ln x$ c. $y = \ln (- x)$

What do you notice about the graphs of – f(x), in each b above?

What do you notice about the graphs of f(- x), in each c above?

A **dilation** is a transformation that enlarges or shrinks a graph.

4. Sketch the graph of each function.
   a. $y = x^2$ b. $y = 2x^2$ c. $y = 0.5x^2$

5. Sketch the graph of each function.
   a. $y = e^x$ b. $y = 2e^x$ c. $y = 0.5e^x$

6. Sketch the graph of each function.
   a. $y = \ln x$ b. $y = 2 \ln x$ c. $y = 0.5 \ln x$

What do you notice about the graphs of **a f(x)** when a > 1, in each b above?

What do you notice about the graphs of **a f(x)** when a < 1, in each c above?

A **translation** is a transformation that involves sliding a graph vertically or horizontally.

7. Sketch the graph of each function.
   a. $y = x^2$ b. $y = x^2 + 3$ c. $y = x^2 - 3$

   d.= $(x - 3)^2$ e. $y = (x + 3)^2$

8. Sketch the graph of each function.
   a. $y = e^x$ b. $y = e^x + 3$ c. $y = e^x - 3$

   d. $y = e^{x - 3}$ e. $y = e^{x + 3}$

9. Sketch the graph of each function.
   a. $y = \ln x$ b. $y = \ln (x) + 3$ c. $y = \ln (x) - 3$

   d. $y = \ln (x - 3)$ e. $y = \ln (x + 3)$

What do you notice about the graphs of f(x) + k , in B and C above?

What do you notice about the graphs of f(x – h), in D and E above?

Combining Transformations

10. Sketch the graph of $\mathbf{y = (x - 3)^2 + 4}$ and describe the transformations of the parent graph.

11. Sketch the graph of $\mathbf{y = 2e^{x + 4}}$ and describe the transformations of the parent graph.

12. Sketch the graph of $\mathbf{y = -\ln(x - 3) + 1}$ and describe the parent graph.

13. Sketch the graph of **$y = \ln(-x) - 4$** and describe the transformations of the parent graph.

14. Sketch the graph of **$y = 0.5(x + 4)^2 - 2$** and describe the transformations of the parent graph.

15. Sketch the graph of **$y = -x + 3$** and describe the transformations of the parent graph.

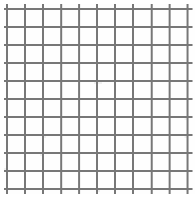
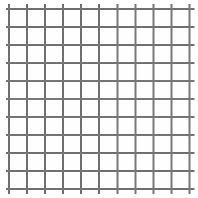
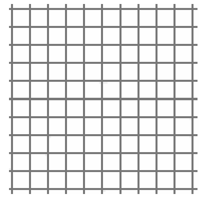
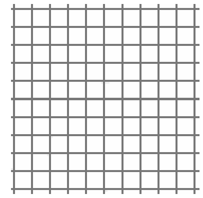
16. Write the equation of an exponential function that is reflected across the y-axis and translated up 3.

17. Write the equation of a quadratic function that is reflected across the x-axis, translated right 2 and down 4.

18. Write the equation of a logarithmic function that is dilated by a factor of 2 and translated left 1 and up 5.

19. Write the equation of a linear function that is translated to the right 2.

Quadratic Reflect x-axis Translate up 2	Exponential Reflect y-axis Translate up 3	Logarithmic Translate up 2 Translate right 3	Quadratic Parent function
Exponential Parent function	Logarithmic Parent function	Quadratic Dilate by 2 Translate left 3	Exponential Translate down 3 Translate left 2
Logarithmic Reflect x-axis Translate down 2	Quadratic Reflect x-axis Translate down 3 Translate left 2	Exponential Dilate by 3 Translate right 2 Translate up 4	Logarithmic Translate left 2 Translate down 4
$y = -(x+2)^2 - 3$	$y = e^{x+2} - 3$	$y = \ln(x-3) + 2$	$y = x^2$
$y = e^x$	$y = -\ln(x) - 2$	$y = -x^2 + 2$	$y = e^{-x} + 3$
$y = \ln(x+2) - 4$	$y = 2(x+3)^2$	$y = 3e^{x-2} + 4$	$y = \ln x$

Transformation Cut-outs

Activity III: Teacher Notes--High/Low Guessing Game

Students should work with a partner for this activity. Use the activity as a warm-up or introduction to lessons on properties of logarithms, solving exponential equations, or using a change of base formula.

Students will team up with a partner to play the game and determine an optimal strategy, which reduces the possibilities in half each time. This is actually an exponential decay process.

Students will solve an exponential equation by taking the log of both sides. Students may wonder how this formula was derived which leads into a properties of logarithms lesson. The formula is a change of base formula which can be used to find logarithm with any base, $\log_b x = \frac{\log x}{\log b}$.

High/Low Guessing Game

The High/Low Guessing Game requires that one try to determine by means of ***yes*** or ***no*** questions a number chosen between two given numbers.

With a partner, play the game. Start by having one person pick a counting number between one and twenty. The other person must determine strategically what the number is in as few guesses as possible. After the number is guessed, switch roles. How many guesses did it take?

Describe the best strategy to pick the number correctly with the fewest number of guesses.

Now try your strategy by picking a number between one and forty. Work with a partner again.
How many guesses did it take?

How many guesses do you think it would take to pick the correct number if it is between one and one million? Explain your reasoning.

One and One Million!

Each question can cut the number of possibilities in half. Thus the first question, ***Is it less than or equal to 500,000?*** leaves 1,000,000 (0.5) possibilities. The second question leaves $1{,}000{,}000(0.5)^2$ possibilities and so forth.

To determine number of questions needed to determine a number between 1 and n, we need to solve the equation $\mathbf{2^x = n}$, where x is the number of equations. The left member of the equation is an exponential expression, but the equation can be solved using logarithms. There is no base 2 logarithm key on the calculator, so using properties of logarithms and the change of base formula we can obtain:

$$x = \frac{\log n}{\log 2}$$

To solve for the number of questions to determine a number between 1 and 1,000,000, use the above formula and let n = 1,000,000.
How many questions does it take?

Use the formula to determine how many guesses it would take to determine a number chosen between one and one billion.

Activity IV: Teacher Notes--Life Expectancy

Life Expectancy may be an individual or group activity and requires Internet access. Students will collect data from **National Center for Health Statistics** to find life expectancy at birth depending on year born. Students will discover that a logarithmic function best models the given data overall, but that this model has limitations for extrapolating into the distant past or distant future.

Life Expectancy

What do you think is the average life expectancy for someone born today and living in the United States all of his or her life?

What do you think was the average life expectancy for someone born in the United States in the year 1900?

What would you predict about the average life expectancy of someone who will be born 20 years from now?

Collecting Data

1. Use the Internet to find the **National Center for Health Statistics** Web site. We want to research the data topic of **Aging** and then go to **Trends in Health and Aging**. We want to examine the data tables for **Life Expectancy**. Collect data for life expectancy at time of birth. Complete the table below.

Year	Life Expectancy	Year	Life Expectancy
1900		**1995**	
1920		**1996**	
1950		**1997**	
1960		**1998**	
1970		**1999**	
1975		**2000**	
1980		**2001**	
1985		**2002**	
1990		**2003**	
1994		**2004**	

Graphing and Determining the Best Model

2. Use the graphing calculator and enter the data above into the **STAT** lists. Display the scatter plot of the data. Use the calculator to determine what equation best models the given data. Round all coefficient values in the equations to the nearest hundredth. For each regression, record the correlation coefficient, ***r***, rounded to nearest ten-thousandth. Make sure that you have **DiagnosticOn**, which can be found by pressing the **2nd** and then the **0** key and scrolling down.

Linear Regression Equation:________________________ r = __________

Quadratic Regression Equation:____________________ R^2 = __________

Exponential Regression Equation:__________________ r = __________

Logarithmic Regression Equation:__________________ r = __________

Use **ZoomStat** to set initial viewing screen. Then graph the four models together with the scatter plot. What observations can you make about each model?

What similarities do you notice about the graphs of the functions?

What differences do you notice about the graphs of the functions?

Which model seems to best fit the data?

We want to make sure our model fits the real-world over an extended period of time. Use **ZoomOut** to observe overall behavior and end behavior of the graph.

What do you notice about the quadratic function?

What do you notice about the values of the correlation coefficients and how the graph fits the data?

Which function do you think would best fit the given data?

Extrapolating Beyond the Given Data

3. Using the logarithmic model, extrapolate, or predict beyond the data, what the average life expectancy would be for each birth year.

Year	**Life Expectancy**
2015	
2020	
2030	
3000	
1800	
1700	
1600	

What can you say about the model's ability to determine what the life expectancy will be in the future? Do these results make sense?

What can you say about the model's ability to determine what the life expectancy was in the past, before the given data? Do these results make sense?

Fitting the Equation

4. Select one of the regression models you performed on the life expectancy data and enter it in to **[Stat] [Edit]** and perform the appropriate regression to determine the curve of best fit. Activate **Stat Plot 1** for the scatter plot (year, life expectancy) graph. Activate **Stat Plot 2** for the scatter plot (year, RESID) graph. Complete the table below.

TIME L1	**ACTUAL DISTANCE L2**	**FITTED DISTANCE L3**	**ACTUAL – FITTED L4 = L2 – L3**	**(ACTUAL – FITTED)^2 L5 = (L4)^2**
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

5. Select one of your other choices and determine the regression equation for the given data. Repeat the above process recording the values in the table below.

TIME L1	**ACTUAL DISTANCE L2**	**FITTED DISTANCE L3**	**ACTUAL – FITTED L4 = L2 – L3**	**(ACTUAL – FITTED)^2 L5 = (L4)^2**
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
Total Actual – Fitted [Sum(L4)]				
Total (Actual – Fitted)^2 [Sum(L5)]				

6. Compare and contrast the two tables.

Other Topics for Exploration in Logarithmic Functions

- Earthquakes and Richter Scale R = log I
- Decibel and Sound Intensity $B = 10 \text{ Log } (I/I_0)$
- Acidity $pH = - \log[H^+]$
- Time it takes to double one's money in an investment
- Linearization of Data and use of logarithmic graph paper
- Logarithmic spirals

Resources:

http://mathforum.org/workshops/usi/dataproject/usi.genwebsites.html
- The Data Library

http://www.census.gov/ipc/www/idb/
- U.S. Census Bureau International Database

http://mathforum.org/mathtools/sitemap2/a2/
- Math Topics for Algebra II

http://www.worldometers.info/
- Worldometers world statistics updated in real time

http://www.cdc.gov/nchs/
- National Center for Health Statistics

http://en.wikipedia.org/wiki/Logarithm
- Wikipedia Uses of Logarithms

http://www.algebralab.org
- Algebra Lab

http://mathbits.com/MathBits/TISection/Statistics2/logarithmic.htm
- Math Bits, A logarithmic model for tree growth

Organizing Topic: Probability

Mathematical Goals:

- Students will identify examples of complementary, dependent, independent, and mutually exclusive events.
- Students will use the addition rule for calculating the probabilities of mutually exclusive events.
- Students will determine probabilities from Venn diagrams.
- Students will calculate the number of possible events using the concept of permutations.
- Students recognize that as an experiment is repeated again and again, the relative frequency probability tends to approach the actual probability.

Standards Addressed: AFDA.6

Data Used: Data obtained from experiments and given data

Materials:

- Applications: NCTM Illuminations
- Graphing Calculator: **Prob Sim™**
- Handout – Name the Event
- Handout – Venn Diagrams
- Handout – Permutations
- Handout – Law of Large Numbers

Instructional Activities:

I. Introduction to Types of Events – Name the Event

Students will determine whether two events are complementary, dependent, independent, and/or mutually exclusive. Students will interact with a partner to analyze various event pairings.

Concepts will include:

- complementary events;
- dependent events;
- independent events; and
- mutually exclusive events.

II. Venn Diagram Analysis – Students with an Earring; Band and Choir

Students will calculate probabilities using hypothetical Venn diagrams. Students will begin by finding simple probabilities which lead into probabilities of independent and dependent events.

Concepts will include:

- Venn diagrams;
- probability notation;
- sample space;

- complements;
- conditional probability; and
- independent events.

III. Permutations

Students will calculate the number of permutations of a given number of objects. Students should already be familiar with the Fundamental Counting Principle.
Concepts will include:

- permutations; and
- factorials and factorial notation.

IV. Law of Large Numbers

Students will use a simulation to investigate the values of the relative frequency probability. As an experiment is repeated again and again, the relative frequency probability tends to approach the actual probability.
Concepts will include:

- probability based on relative frequency;
- actual (experimental) probability; and
- Law of Large Numbers.

Activity I: Teacher's Notes--Name the Event

Students need to understand the difference between *independent, dependent, mutually exclusive,* and *complementary events.* Two events, *A* and *B*, are *independent* if the occurrence of one does not affect the probability of the occurrence of the other. If events *A* and *B* are **not** *independent*, then they are said to be *dependent.* Therefore, two events are *dependent* if the outcome of the first affects the outcome of the second.

Mutually exclusive events cannot occur simultaneously. The *complement* of Event *A* consists of **all** outcomes in which event *A* does not occur. *Complementary events* are always *mutually exclusive*, but *mutually exclusive events* are not necessarily complementary. Given an experiment involving rolling two dice, the event of the dice dots having a sum of six and the event of the dice dots having a sum of eight are *mutually exclusive*. In that same experiment, the event of the dice dots having an even sum is the *complement* of the event of the dice dots having an odd sum.

Given another experiment involving two dice, the probability of the first die showing six dots does not affect the probability of the second die showing six dots. Therefore, the rolls of each die are independent; one roll does not affect the other.

Given a third experiment involving two dice, the probability of rolling a sum greater than ten greatly improves when the first roll produces a six. Therefore, the probability of rolling a sum greater than ten depends on the number from the first roll.

Students may need additional examples to understand the difference between these four types of events. Once students grasp the differences, they should be able to complete the first page of **Name the Event**. Review these answers before students begin creating their own event pairings. Sports, cars, course schedules, and characteristics of students are topics in which students could find event pairings. Finally, students need partners to share their event pairings. They should avoid reading them in order to make the challenge of identifying the relationship more realistic for their partners. Each partnership then selects one of each kind of relationship to add to a class list. Discuss the nuances of each event pairing that places it in a distinct category.

Name the Event

Given the following events, identify each as *dependent, independent, or mutually exclusive*. If they are *mutually exclusive*, say whether or not they are complementary.

1. A National Football Conference team wins the Super Bowl and an American Football Conference team wins the Super Bowl.

2. The Washington Redskins win the Super Bowl and the Washington Wizards win the National Basketball Association championship.

3. The Washington Redskins win the Super Bowl and Washington D.C. has a parade.

4. The first two numbers of your Pick 3 lottery ticket match the winning numbers and the third number does not match.

5. Gus takes the bus to school and he receives a speeding ticket on his way to school.

6. Caitlin sings in the school choir and she is on her school's soccer team.

7. Hope sings in the school choir and she sings in a concert.

8. Alex is a waiter at Olive Garden and he is a chef at Olive Garden.

9. Karissa drives at night and she hits a deer.

10. Lance's girlfriend has a cold sore and Lance has a cold sore.

11. Create two different situations that illustrate independent events.
    a.

    b.

12. Create two different situations that illustrate dependent events.
    a.

    b.

13. Create two different situations that illustrate mutually exclusive events.
    a.

    b.

14. With a partner, take turns reading your examples while the other person determines whether each example illustrates **independent** events, **dependent** events, or **mutually exclusive** events.

    Record your guesses:
    a.

    b.

    c.

    d.

    e.

    f.

15. With your partner, pick one of each type to share with the whole class.
    a. Independent:

    b. Dependent:

    c. Mutually Exclusive:

Activity II: Teacher Notes--Students with an Earring; Band and Choir (Venn Diagram Analysis)

Students should already have a basic understanding of probability from earlier math courses. This activity will guide students into calculating the probabilities of conditional events. Introduce the probability notation ***P* (*E*)**, which refers to the probability that the event E occurs.

Students should already know that $P(\text{event}) = \dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

Set theory is used to represent relationships among events. In general, if ***A*** and ***B*** are two events in the sample space, ***S***, then

$A \cup B$ is said "*A* union *B*" which means that either *A* or *B* occurs or both occur

$A \cap B$ is said "*A* intersection *B*" which means that both *A* and *B* occur

A' is said "the complement of *A*" which means that *A* does not occur

If *A* and *B* are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.

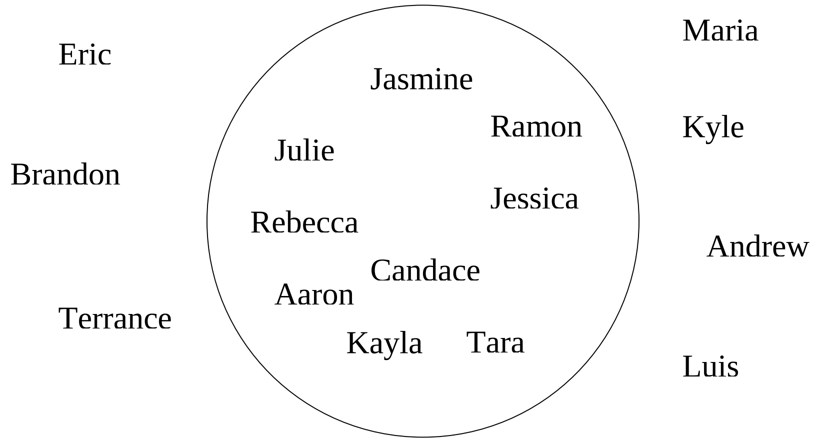
If *B* is A', then $P(A \cup A') = P(A) + P(A') = 1$.

If *A* and *B* are independent events, then $P(A \cap B) = P(A)P(B)$.

If *B* is dependent on *A*, then $P(B \mid A)$ is the probability that *B* will occur given that *A* has already occurred. $P(B \mid A)$ is called the conditional probability of *B* given *A*.

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \quad \Rightarrow \quad P(A \cap B) = P(A)P(B \mid A)$$

Students with an Earring



Answer the following questions using the Venn Diagram.

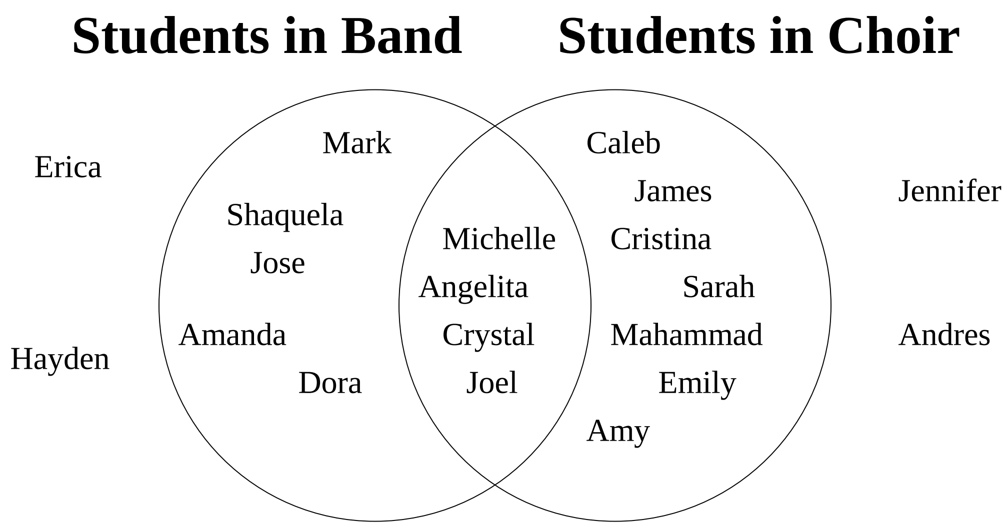
1. How many students are in the class?

2. How many students have earrings?

3. How many students do not have earrings?

If one student is picked at random, find the following probabilities.

4. P(girl)
5. P(not a girl)
6. P(boy)
7. P(student with earring)
8. P(student without earring)
9. P(boy with earring)
10. P(girl without earring)
11. P(girl with earring)
12. P(boy with earring $\cup$ girl with name starting with J)
13. P(name starting with A $\cup$ name starting with T)
14. Write a probability question which has an answer of $\frac{5}{8}$.



15. How many students are in the sample?

16. How many students are only in Band?

17. How many students are in Band and in Choir?

If one student is picked at random, find the following probabilities.

18. P(student in choir)

19. P(student in neither band nor choir)

20. P(boy in band)

21. P(girl in band and choir)

22. P(boy in band or choir)

23. P(name starts with C and in choir)

24. P(student in band | boy)

25. P(girl | student in band)

26. P(boy | student in both band and choir)

27. Write a probability question which has an answer of $\frac{1}{2}$.

28. Write a probability question which has an answer of $\frac{4}{7}$.

Activity III: Teacher's Notes--Permutations

Students may need a review of the Fundamental Counting Principle. The handout will lead students to a discussion of factorials. Following the introduction to factorials, students will explore permutations, including the formula ${}_nP_r = \frac{n!}{(n-r)!}$. Emphasize the need for the objects to be arranged, as opposed to grouped.

Permutations

Mr. Vandergoobergooten has decided to put Adam, Brianna, Chase, Destiny, and Eduardo in the front row of his classroom. He has five desks in the front row but is undecided about which student should go in which desk.

1. How many different students could he place in the rightmost desk?

2. After selecting a student for the rightmost desk, how many different students could he place in the next desk?

3. After selecting a student for the first two desks, how many different students could he place in the next desk?

4. After selecting a student for the first three desks, how many different students could he place in the next desk?

5. After selecting a student for the right four desks, how many different students could he place in the leftmost desk?

6. Using the Fundamental Counting Principle, write an expression that would find the number of different ways Mr. Vandergoobergooten could choose to arrange these five students.

7. How many different ways can he arrange the students?

8. If Mr. Vandergoobergooten had six desks in the front row and wanted to put Felicia in the front row too, write an expression that would find the number of different arrangements that are now possible.

9. How many different arrangements are now possible?

The process of multiplying every counting number from 1 to n can be abbreviated as n! which is said "n factorial." Therefore $3 \cdot 2 \cdot 1 = 3! = 6$, $4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$, and $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8! = 40{,}320$.

10. Write 2! as a multiplication problem and then find the value of 2!.

11. Write 7! as a multiplication problem and then find the value of 7!.

Since $2! = 2$ and $3! = 6$, then $3! = 3 \cdot 2! = 3 \cdot 2$
Since $3! = 6$ and $4! = 24$, then $4! = 4 \cdot 3! = 4 \cdot 6$
Since $4! = 24$ and $5! = 120$, then $5! = 5 \cdot 4! = 5 \cdot 24$

12. Since $6! =$ __________ and $7! =$ __________, then $7! =$ _____________ = ___________

13. Since $10! = 3{,}628{,}800$ and $11! = 39{,}916{,}800$, then $11! =$ _____________ $\cdot 10!$

14. Mr. Vandergoobergooten has 25 students and 25 desks in his class. Using factorials, how many different ways could he create a seating chart?

15. Mr. Vandergoobergooten has 12 poster-size pictures of his cat that he wants to line up on the wall above his chalkboard. Using factorials, how many different ways could he hang them on the wall?

16. Mr. Vandergoobergooten discovered that his cat posters are too big to fit all 12 on the wall. Instead, he will only be able to hang 4 of his posters.
    a) How many different posters can he select for the first position on the wall?

    b) After selecting the first poster, how many different posters can he select for the second position?

    c) After selecting the first two posters, how many different posters can he select for the third position?

    d) After selecting the first three posters, how many different posters can he select for the fourth and final position?

    e) Using the Fundamental Counting Principle, write an expression for the number of ways Mr. Vandergoobergooten can hang four cat posters.

    f) How many different ways can Mr. Vandergoobergooten hang four of these posters?

17. For the back wall, Mr. Vandergoobergooten has room for three of his seven poster-size pictures of his goldfish. How many different ways can Mr. V hang three of these posters?

18. How many goldfish posters did Mr. Vandergoobergooten not use?

19. How many ways can seven posters be arranged?

20. How many ways can the posters that were not used be arranged?

21. What do you find if you divide your answer from 19 by your answer from 20?

22. Using this idea, what would be another way to calculate 16f (Mr. Vandergoobergooten's cat posters)?

Permutations are used to calculate the number of possible arrangements of *n* objects taken *r* at a time. For example, 12 posters are taken 4 at a time or ${}_{12}P_4$.

$${}_nP_r = \frac{n!}{(n-r)!}$$

For example, ${}_{12}P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8!} = 12 \cdot 11 \cdot 10 \cdot 9 = 11{,}880$

23. Mr. Vandergoobergooten has five desks in the front row of this classroom and 25 students in the class. How many different ways can he seat students in that first row so that every seat is filled?

24. Ms. Pi has eight seats in the first row of her classroom and she has 14 students in class. How many different ways can she seat students in that first row so that every seat is filled?

25. Joseph has six classes he still has to take to graduate. He will take four of them in the fall semester and then the other two plus two electives in the spring. How many different schedules could Joseph have in the fall semester?

Activity VI: Teacher's Notes--Law of Large Numbers

Students need access to Probability Simulation™ on the TI-83+ or TI-84+ calculators, which can be downloaded from http://education.ti.com/, or a very similar, and easier to use, simulation is available at http://illuminations.nctm.org/ActivityDetail.aspx?ID=79.

On the calculator, **Prob Sim™** is found on the Apps menu. Students will need **Spin Spinner**. Select **Set**. **Trial Set** should start at 1, but this value will gradually increase. Sections should be at 5. Graph should be on **Prob**. On the **Advanced Menu (ADV)**, students should first change one of the **Prob** values to 0.5; observe how the other **Prob** values change; select **ok** and **ok** again; and observe the new appearance of the spinner. Students should return to the **Advanced Menu** and select new values for all five sections.

Spin will perform the **Trial Set** number of experiments. After spinning with **Trial Set** as one, use **Esc** to return to the previous menu without losing the data. The arrow keys will show the relative frequency or experimental probability of each section on the bar graph. Students should write the probabilities as percentages to the nearest hundredth of a percent.

At the end of the investigation, discuss students' conclusions from the last step. Be sure that everyone understands that as a procedure is repeated again and again, the relative frequency probability of an event **tends** to **approach** the actual probability.

Sample Resource:

http://www.valottery.com

- The Virginia Lottery

Law of Large Numbers

Use a spinner simulator on a calculator or computer. Set the number of sections on the spinner to 5. Change one of the actual (theoretical) values to 50%.

1. What has to be true of the other four actual (theoretical) values?

2. What happened to the appearance of the spinner?

Select any five probability values that you would like. Record these actual (theoretical) probability values as percentages.

	Section: 1	2	3	4	5
3. Theoretical Values	_____	_____	_____	_____	_____

Using the calculator, graph the experimental probability values.

4. Experimental Probability (Relative Frequency) as a percentage:

after 1 spin	_____	_____	_____	_____	_____
after 2 spins	_____	_____	_____	_____	_____
after 5 spins	_____	_____	_____	_____	_____
after 10 spins	_____	_____	_____	_____	_____

Change the Trial Set or Number of Spins to 10.

after 20 spins	_____	_____	_____	_____	_____

5. Compare the relative frequency (experimental) probabilities in #4 with the actual (theoretical) probabilities from #3. Are you surprised by your results? Why or why not?

6. How many spins do you think it will take for the two types of probabilities to be equal?

7. Probability from Relative Frequency

after 30 spins ______ ______ ______ ______ ______

after 40 spins ______ ______ ______ ______ ______

after 50 spins ______ ______ ______ ______ ______

after 100 spins ______ ______ ______ ______ ______

8. Change the Trial Set or Number of Spins to 100.

after 200 spins ______ ______ ______ ______ ______

after 300 spins ______ ______ ______ ______ ______

after 400 spins ______ ______ ______ ______ ______

after 500 spins ______ ______ ______ ______ ______

9. Change the Trial Set or Number of Spins to 500.

after 1000 spins ______ ______ ______ ______ ______

after 2000 spins ______ ______ ______ ______ ______

after 3000 spins ______ ______ ______ ______ ______

10. Round each of your relative frequency (experimental) probabilities from 3000 spins to the nearest whole percentage.

______ ______ ______ ______ ______

11. Copy your actual (theoretical) probabilities from step 3.

______ ______ ______ ______ ______

12. Compare the relative frequency (experimental) probabilities in step 23 with the actual (theoretical) probabilities from step 24.

13. Look at the results from three other people. What conclusion can you make about the relationship between relative frequency probability and actual probability as the number of experiments increases?

Organizing Topic: Data Analysis

Mathematical Goals:

- Students will analyze and interpret univariate data using measures of central tendency and dispersion.
- Students will calculate the z-scores for data.
- Students will investigate and compare data using z-scores.
- Students will explore normally distributed data, the normal curve and calculate probabilities using the probability density function and z-scores.
- Students will determine percentiles using z-scores.
- Students will design and conduct surveys and experiments using a variety of sampling techniques.
- Students will investigate and describe bias.

Standards Addressed: AFDA.7; AFDA.8

Data Used: Data will be obtained from sampling, and observations, measurement, experiments and Internet sources.

Materials:

Graphing calculator
Handouts
Graph paper
Computer with Internet access, spreadsheet software, and graphing software

Instructional Activities:

1. **z-scores and NFL Quarterback Salaries – Introduction to z-scores**
   Students will analyze 2007 football quarterback salaries by examining means, variances, standard deviations, and z-scores. The activity is designed to be an example of a comprehensive analysis that can be done with a data set.
   Concepts covered include:
   - measures of variability;
   - average deviation;
   - standard deviation;
   - variance; and
   - analysis of data variability;

2. **z-scores and Major League Baseball Salaries**
   Students will compare a team's total payroll with the number of wins each team had and determine the z-scores for each value in the two data sets. Students will discuss the relative inequities in the salaries and whether or not that is reflected in the number of wins. Students will also represent and analyze the data graphically.
   Concepts covered include:
   - z-score transformations; and
   - normal distribution.

3. **What is a Normal Curve?**
In this activity, students will conduct a coin flipping experiment that should yield data in a normal distribution. Students will describe attributes of normal curves and investigate the effect on the curve of changing the mean and/or standard deviation.
Concepts covered include:
- normal distribution;
- graph of normally distributed data;
- attributes of normal curves; and
- effect of mean and standard deviation on the normal curve.

4. **Standard Normal Distributions**
Students will investigate the properties of a normal curve and the standard normal distribution. They will transform normally distributed data to standard normal scores using z-scores transformations in two problem contexts. The activity also introduces how percentiles are determined using a standard normal table or by using a calculator. The associated height project provides an opportunity to discuss appropriate sampling techniques, the formula for the sample standard deviation, and a context where a set of normally distributed data is analyzed through the lens of the standard normal distribution.
Concepts covered include:
- properties of a normal curve;
- the standard normal distribution;
- z-score transformations;
- percentiles;
- area under the normal curve;
- sample standard deviation; and
- population parameters and sample statistics.

5. **Probability Density**
Students will investigate how the area between the standard normal curve and the x-axis can be used to calculate probabilities. Students will connect the ideas of probability and percentile. Students will also link the concept of area under the curve with probability and learn to shade the appropriate region under a standard normal curve to represent a certain probability.
Concepts covered include:
- area under a normal curve;
- standard normal distribution;
- probabilities associated with areas under the normal curve; and
- percentiles.

6. **Sampling: Politics and Opinions**
Students will investigate how political polling takes place and will construct their own public opinion poll using one of the sampling techniques learned in this course.

Concepts covered include:

- sampling techniques; and
- identifying sources of bias.

Activity 1--z-scores and NFL Quarterback Salaries—Introduction to z-scores

In this activity, students will analyze 2007 football quarterback salaries by examining means, variances, standard deviations, and z-scores. This activity is designed to be a comprehensive example of analysis that can be done with a data set.

Extension:
Repeat the process with salaries of other position players or for quarterbacks in another given year.

Questions to ask might be:

Which is a "better" salary for a quarterback, $8,000,000 in 2007 or $850,000 in 1991?

Which position player has a better relative 2007 salary, a punter making $500,000 or a quarterback $6,000,000?

Ask students to pretend that a typo was made When entering the Rams' quarterback salary. Instead of $17,502,040 it should have been $1,750,204.

Have students gather longitudinal data for a specific position and find an appropriate regression curve.

Answers:

a. N=32

b. $\mu = 5286688$

d. $\sigma = 3959516$

e.

Salary	Team	z-score
$1,147,000	Browns	-1.05
$5,003,360	Buccaneers	-0.07
$10,339,920	Cardinals	1.28
$5,005,760	Chargers	-0.07
$3,753,600	Chiefs	-0.39
$11,003,840	Colts	1.44
$1,506,000	Cowboys	-0.95
$6,000,000	Dolphins	0.18
$5,504,080	Eagles	0.05
$3,503,720	Falcons	-0.45
$1,675,000	Forty-niners	-0.91
$6,450,000	Giants	0.29
$1,105,280	Jaguars	-1.06
$4,000,000	Jets	-0.32
$1,500,000	Lions	-0.96
$11,000,480	Packers	1.44
$7,400,000	Panthers	0.53
$6,005,160	Patriots	0.18
$3,200,000	Raiders	-0.53
$17,502,040	Rams	3.09
$4,001,200	Ravens	-0.32
$578,020	Redskins	-1.19
$3,000,000	Saints	-0.58
$6,004,320	Seahawks	0.18
$1,009,840	Steelers	-1.08
$8,000,000	Texans	0.69
$13,143,000	Titans	1.98
$1,485,000	Vikings	-0.96

z-scores and NFL Quarterback Salaries

Data from http://www.theredzone.org/2007/salaries/showposition.asp?Position=QB

1. Using the quarterback salary data in the table provided,

   a. The number of teams, N, is equal to ___________

   b. Find the sum of all of the quarterback salaries, $\sum x$, and fill in that number in your table.

   c. Calculate the mean $\left(\mu = \frac{\Sigma x}{N}\right)$:_________________

      - What information does the mean provide?

      - How many salaries are above the mean? Below the mean? _____

      - Was that what you expected? Explain.

   d. Use the mean to fill in the next two columns in the table, the deviation from the mean and the square of that number for each datum.

      - What is the sum of the "Deviation from the Mean" column?________

      - Explain why that makes sense. What led statisticians to use the sum of the squares of the deviations?

   e. Calculate the variance $\left(\sigma^2 = \frac{\sum (x-\mu)^2}{N}\right)$:______________________

   f. Calculate the standard deviation: $(\sigma = \sqrt{\sigma^2})$ _____________________

      - Which quarterback salaries are within 1 standard deviation of the mean?

      - What percentage of salaries is that?_____

- Which quarterback salaries are **not** within 2 standard deviations of the mean?

g. Calculate the z-score for each salary $z - score = \frac{x - \mu}{\sigma}$ and complete that column of the table:

- Find which team's quarterback has a salary farthest from the mean. What was the corresponding z-score?

- Find which team's quarterback has a salary closest to the mean. What was the corresponding z-score?

- What information does each z-score provide?

- Why are some z-scores negative numbers?

- Determine how many of the z-scores are positive and how many are negative and explain why that makes sense.

- What is the sum of all of the z-scores? Explain why this makes sense.

Quarterback Salary Data 2007 Table

Data from http://www.theredzone.org/2007/salaries/showposition.asp?Position=QB				
Team Name	Quarterback Salary	Deviation from the Mean	Square each Deviation from the mean	z- score
	x	$x-\mu$	$(x-\mu)^2$	$(x-\mu)/\sigma$
Bears	$2,039,800			
Bengals	$7,250,000			
Bills	$1,804,560			
Broncos	$8,253,020			
Browns	$1,147,000			
Buccaneers	$5,003,360			
Cardinals	$10,339,920			
Chargers	$5,005,760			
Chiefs	$3,753,600			
Colts	$11,003,840			
Cowboys	$1,506,000			
Dolphins	$6,000,000			
Eagles	$5,504,080			
Falcons	$3,503,720			
Forty-niners	$1,675,000			
Giants	$6,450,000			
Jaguars	$1,105,280			
Jets	$4,000,000			
Lions	$1,500,000			
Packers	$11,000,480			
Panthers	$7,400,000			
Patriots	$6,005,160			
Raiders	$3,200,000			
Rams	$17,502,040			
Ravens	$4,001,200			
Redskins	$578,020			
Saints	$3,000,000			
Seahawks	$6,004,320			
Steelers	$1,009,840			
Texans	$8,000,000			
Titans	$13,143,000			
Vikings	$1,485,000			
TOTALS	$\sum x =$	$\sum(x-\mu) =$	$\sum(x-\mu)^2 =$	$\sum(x-\mu)/\sigma =$

Activity 2:

Teacher's Notes--Data Set Comparison, z-scores, and Major League Baseball (MLB) Payroll Analysis

In this activity, students will compare a team's total payroll with the number of wins each team had. To do that, they will determine the z-scores for each value in the two data sets and discuss the relative inequities in the salaries and whether or not that is reflected in the number of wins. Students will also represent and analyze the data graphically.

z-score Table Completed

Team	Payroll	z-score	Wins	z-score
New York Yankees	$189,639,045	3.21	94	1.42
Boston Red Sox	$143,026,214	1.81	96	1.64
New York Mets	$115,231,663	0.98	88	0.76
Los Angeles Angels	$109,251,333	0.80	94	1.42
Chicago White Sox	$108,671,833	0.78	72	-0.99
Los Angeles Dodgers	$108,454,524	0.77	82	0.11
Seattle Mariners	$106,460,833	0.71	88	0.76
Chicago Cubs	$99,670,332	0.51	85	0.43
Detroit Tigers	$95,180,369	0.38	88	0.76
Baltimore Orioles	$93,554,808	0.33	69	-1.32
St. Louis Cardinals	$90,286,823	0.23	78	-0.33
San Francisco Giants	$90,219,056	0.23	71	-1.10
Philadelphia Phillies	$89,428,213	0.20	89	0.87
Houston Astros	$87,759,000	0.15	73	-0.88
Atlanta Braves	$87,290,833	0.14	84	0.32
Toronto Blue Jays	$81,942,800	-0.02	83	0.22
Oakland Athletics	$79,366,940	-0.10	76	-0.55
Minnesota Twins	$71,439,500	-0.34	79	-0.22
Milwaukee Brewers	$70,986,500	-0.35	83	0.22
Cincinnati Reds	$68,904,980	-0.41	72	-0.99
Texas Rangers	$68,318,675	-0.43	75	-0.66
Kansas City Royals	$67,116,500	-0.47	69	-1.32
Cleveland Indians	$61,673,267	-0.63	96	1.64
San Diego Padres	$58,110,567	-0.74	89	0.87
Colorado Rockies	$54,424,000	-0.85	90	0.98
Arizona Diamondbacks	$52,067,546	-0.92	90	0.98
Pittsburgh Pirates	$38,537,833	-1.32	68	-1.43
Washington Nationals	$37,347,500	-1.36	73	-0.88
Florida Marlins	$30,507,000	-1.56	71	-1.10
Tampa Bay Devil Rays	$24,123,500	-1.75	66	-1.65
	Mean $82,633,066		Mean $81	
	Standard Deviation 33352233.38		Standard Deviation 9.137772644	

Extensions:

1. What happens to the z-score if the New York Yankees are removed from the data set for payroll? Additionally, what happens when the Boston Red Sox are removed as well?
2. Have students research to see how MLB actually does try to maintain equity among the teams?
3. Have students research ticket prices for the different MLB teams and compare those to payroll using z-scores.
4. Using the z-scores obtained in this activity, determine how strong of a correlation exists between payroll and wins.

Data Set Comparison, z-scores, and Major League Baseball Payroll Analysis

Using the 2007 total team payroll and records for each team in Major League Baseball on the table on the next page:

a. Use a spreadsheet or graphing calculator to find the mean and standard deviation for the payroll data as well as the z-score for each team.

b. Are there any teams' payrolls more than two standard deviations away from the mean?

c. Some people say that it is not fair for the Yankees to pay so much and the Devil Rays to pay so little. Do you think it is fair? If you do think it is fair, explain why. If not, how could you make it fairer? What should Major League Baseball do to try to maintain equity in competition?

d. Find the mean and standard deviation for the number of wins data and z-score for each team. How do the z-scores from that data set compare to the payroll z-scores? Does a team that pays more in salary lead to more wins in a season? Explain your reasoning.

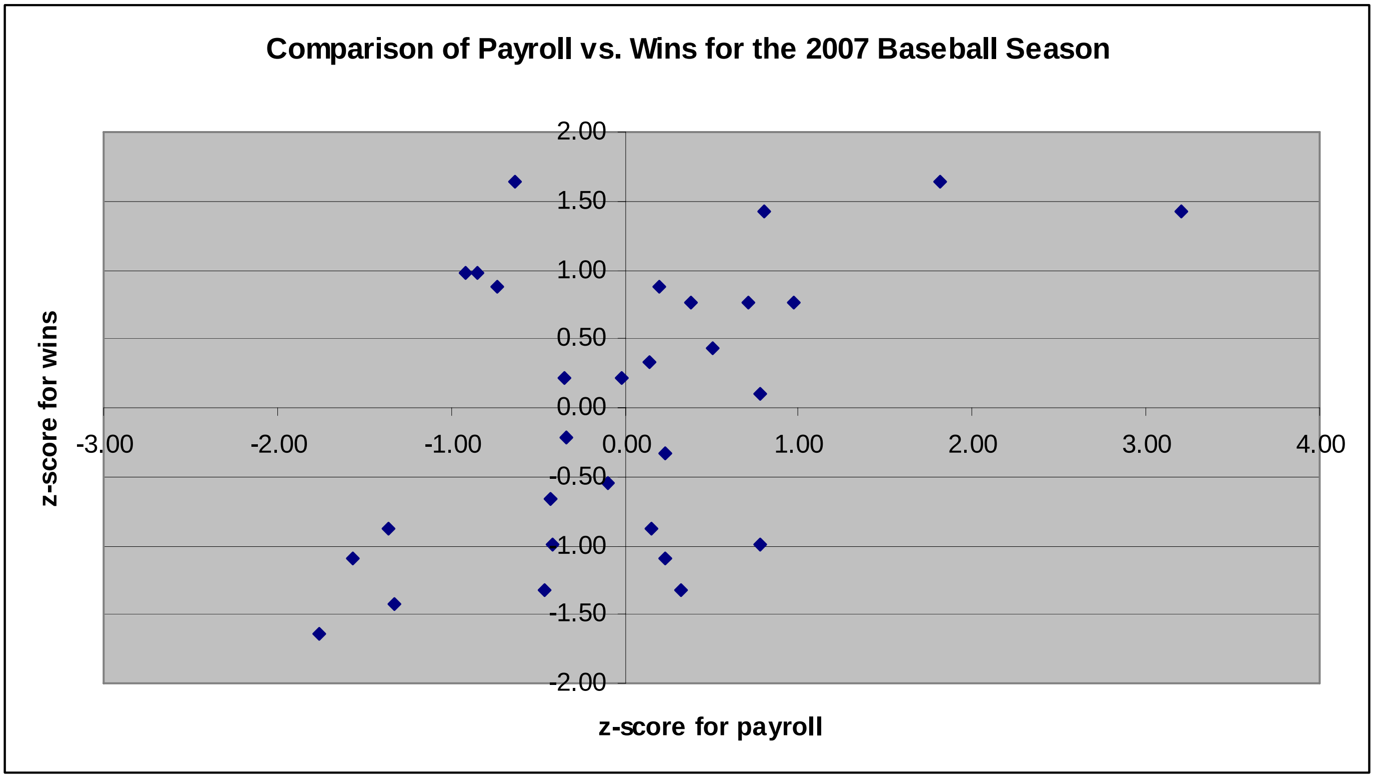
e. Look at the data and determine which team seemed to get the most "bang for the buck" – justify your answer.

Baseball 2007 Payroll and Wins Data Table

Data from: http://asp.usatoday.com/sports/baseball/salaries/default.aspx
http://www.baseball-reference.com

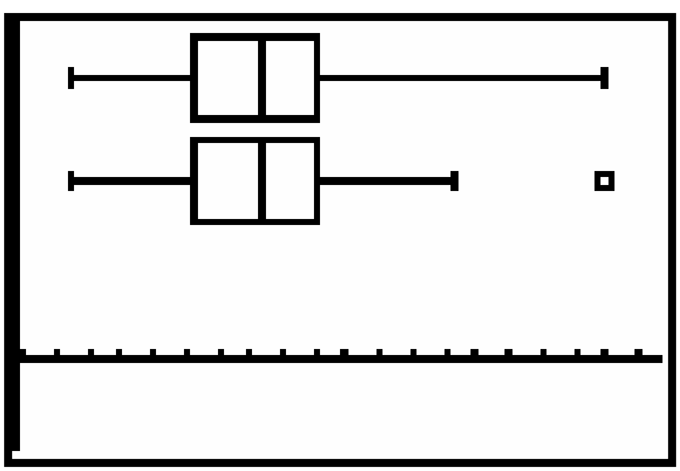
Team	Payroll	z-score	Wins	z-score
New York Yankees	$189,639,045		94	
Boston Red Sox	$143,026,214		96	
New York Mets	$115,231,663		88	
Los Angeles Angels	$109,251,333		94	
Chicago White Sox	$108,671,833		72	
Los Angeles Dodgers	$108,454,524		82	
Seattle Mariners	$106,460,833		88	
Chicago Cubs	$99,670,332		85	
Detroit Tigers	$95,180,369		88	
Baltimore Orioles	$93,554,808		69	
St. Louis Cardinals	$90,286,823		78	
San Francisco Giants	$90,219,056		71	
Philadelphia Phillies	$89,428,213		89	
Houston Astros	$87,759,000		73	
Atlanta Braves	$87,290,833		84	
Toronto Blue Jays	$81,942,800		83	
Oakland Athletics	$79,366,940		76	
Minnesota Twins	$71,439,500		79	
Milwaukee Brewers	$70,986,500		83	
Cincinnati Reds	$68,904,980		72	
Texas Rangers	$68,318,675		75	
Kansas City Royals	$67,116,500		69	
Cleveland Indians	$61,673,267		96	
San Diego Padres	$58,110,567		89	
Colorado Rockies	$54,424,000		90	
Arizona Diamondbacks	$52,067,546		90	
Pittsburgh Pirates	$38,537,833		68	
Washington Nationals	$37,347,500		73	
Florida Marlins	$30,507,000		71	
Tampa Bay Devil Rays	$24,123,500		66	
	Mean Standard Deviation		Mean Standard Deviation	

f. Using the scatter plot below and the z-scores you calculated, identify the data point for six of the teams. If you were an owner of a baseball team, in which quadrant would you prefer your team's data point to be located?



g. Below are two box-and-whisker plots of the team payroll data – one includes all of the data and the other displays the "outlier" separate. From the data, determine and label the Minimum, 1st Quartile, Median, 3rd Quartile and Maximum. Label those values on the graphs below.

Baseball 2007 Team Salary Data



h. Use your graphing calculator to help make a box-and-whisker plot for the wins data. Sketch the plot below. Label Min., Q1, Median, Q3, and Max.

Baseball 2007 Wins Data Box and Whisker Plot

i. List the teams between Q1 and Q2 in terms of number of wins. Which teams also appear in the list of teams between Q1 and Q2 in payroll?

j. Is the median for the wins data equal to the mean? What would need to be true for this to happen? If this is not the case, why not?

Activity 3: Teacher's Notes--What is a Normal Curve?

In this activity, students will conduct a coin flipping experiment that should yield data in a normal distribution. Students will describe attributes of normal curves and investigate the effect of changing the mean and/or standard deviation.

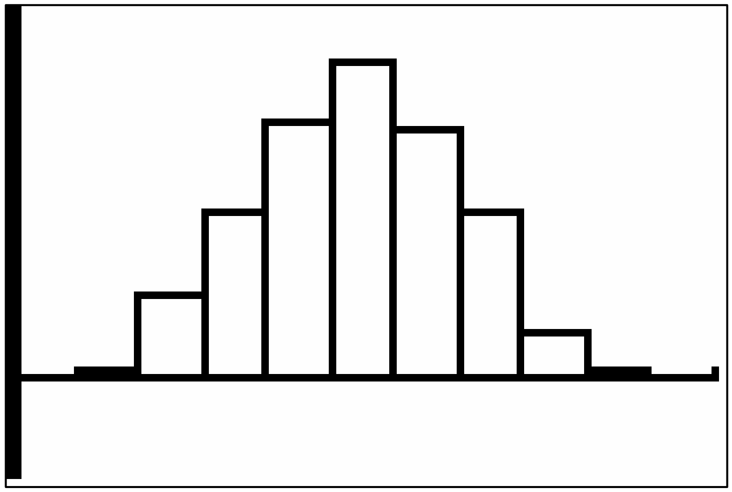
In addition to considering the theoretical probabilities and how they lead to a discussion of a normal distribution, students will benefit from simulating the ten coin experiment. Students can simulate the experiment a number of ways, including but not limited to actually flipping pennies.

Another possibility is to do a computer simulation. A short program that "flips" ten coins is easy to write. The program below simulates the flipping of ten coins by generating 0's and 1's randomly in groups of ten. By assigning a 1 to be a "heads" and a 0 to be "tails", finding the sum for each ten indicates the number of heads in ten flips (each trial).

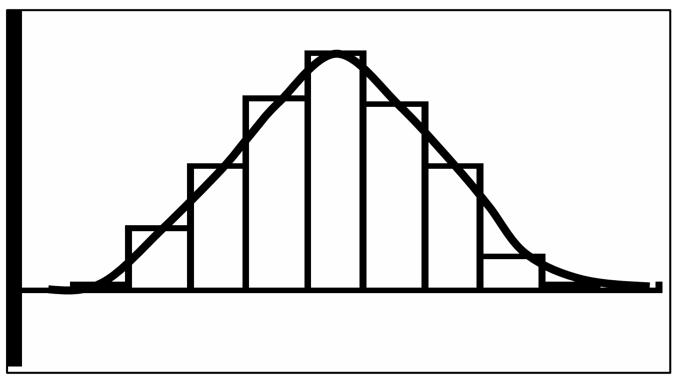
```
PROGRAM:FLIP
:ClrAllLists
:For(N,1,500)
:sum(randInt(0,1
,10)→L₁(N)
:End
:█
```

Once the data is stored in a list, it can be graphed in a histogram. While the experimental probability likely deviates from the theoretical probability, it can validate the theoretical results and appear to fit a normal curve. One set of results that were graphed looked like:

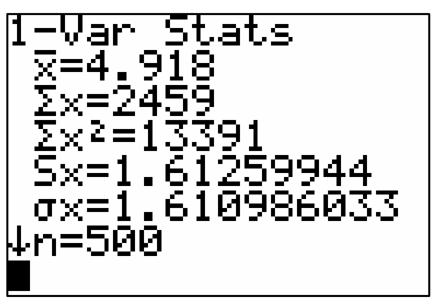
No of Heads	Frequency
0	1
1	3
2	32
3	63
4	98
5	121
6	95
7	65
8	18
9	3
10	1



Students can connect the center points at the top of each bar to reveal the shape of the distribution:

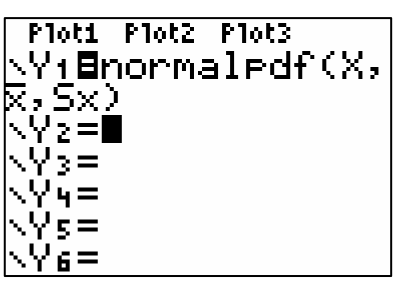


From this data, students can calculate the mean and standard deviation:

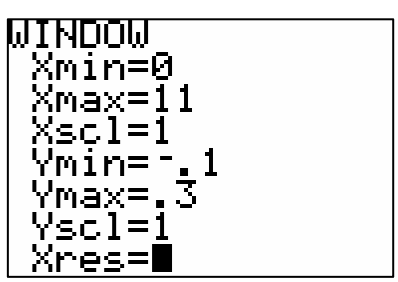


Then graph the corresponding normal curve as follows:

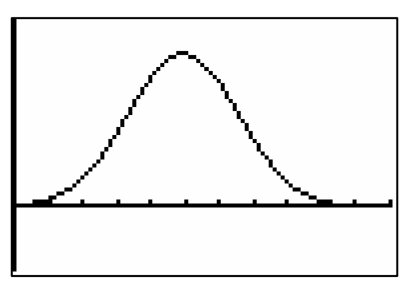
1. Enter formula



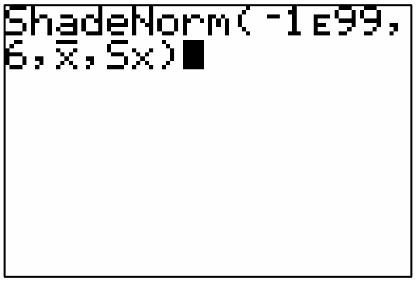
2. Set Window Dimensions



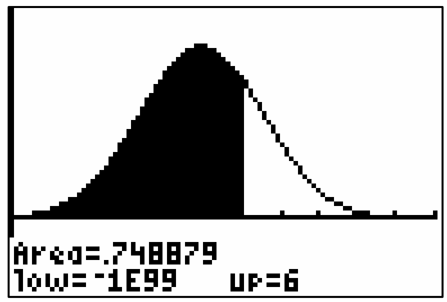
3. Graph



After an introduction to the normal probability density function, a student can use the defined function to evaluate probabilities. For instance, to find the probability that the number of heads is 6 or fewer, a student can enter:



With the result:



What is a Normal Curve?

Experiment – Flip ten coins 1024 times and count the number of heads in each trial. First focus on the theoretical probabilities.

Using *combinations*, we can obtain the expected values and theoretical probabilities in the table below:

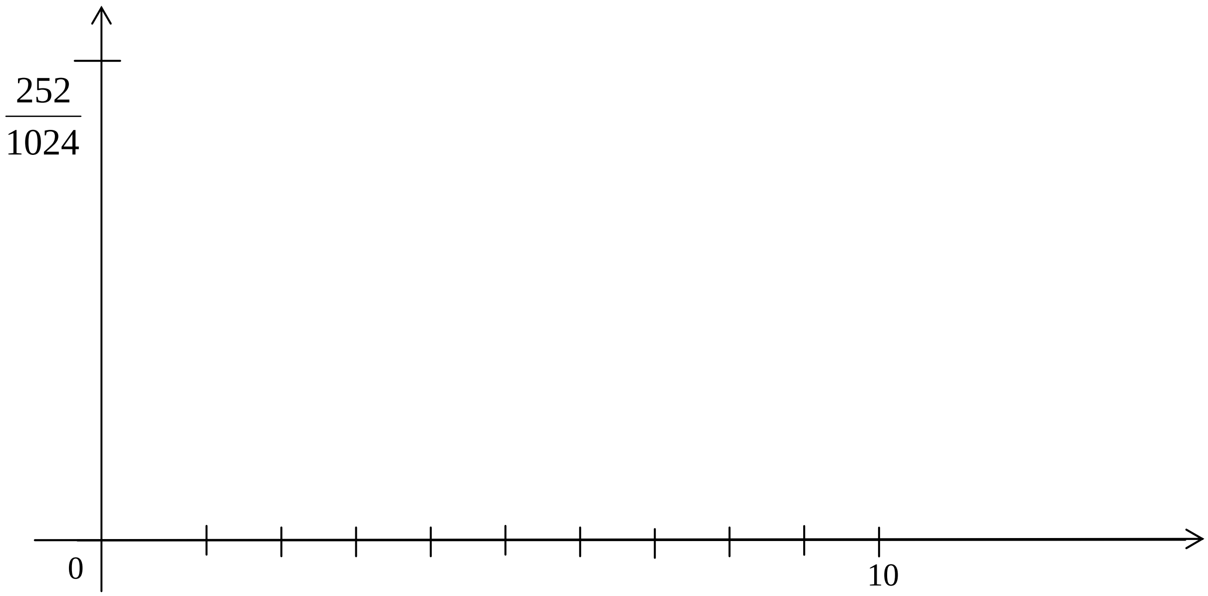
Number of Heads	Expected Frequency value out of 1024	Theoretical Probability	Percent Likelihood
0	1	$\frac{1}{1024}$	
1	10	$\frac{10}{1024}$	
2	45	$\frac{45}{1024}$	
3	120	$\frac{120}{1024}$	
4	210	$\frac{210}{1024}$	
5	252	$\frac{252}{1024}$	
6	210	$\frac{210}{1024}$	
7	120	$\frac{120}{1024}$	
8	45	$\frac{45}{1024}$	
9	10	$\frac{10}{1024}$	
10	1	$\frac{1}{1024}$	

1. What is the sum of all the probabilities?________

2. Complete the ***percent likelihood*** column by converting the theoretical probability to a decimal and then to a percent.

3. What observations can you make about the data in the table thus far?

4. On the axis provided, plot the theoretical probability for each number of heads.



5. Connect the data points with a smooth curve – the result is a **normal curve**.

6. What do you observe about the graph's:

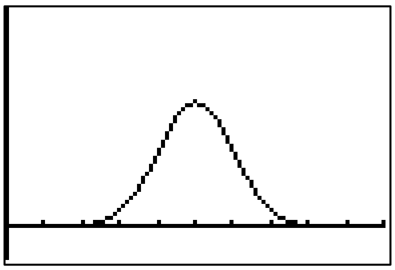
    - shape?
    - symmetry?
    - highest point?
    - mean/median/mode probability?

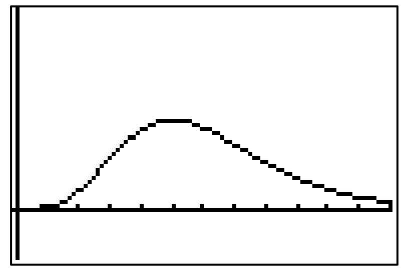
7. Read about the characteristics of a normal curve below and discuss how the curve you drew compares to the characteristics.

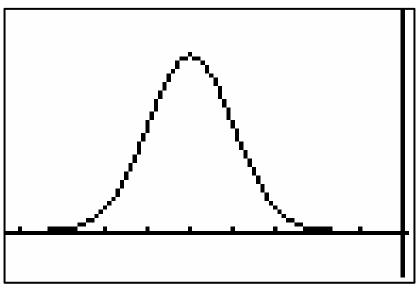
The Graph of a Normal Distribution is a Normal Curve and every normal curve has the following characteristics:

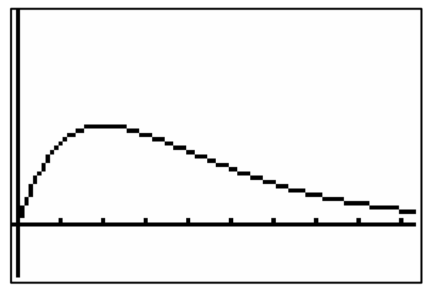
- The mean, median and mode are equal.
- They are bell-shaped and symmetrical about the mean.
- The curve never touches, but it becomes closer and closer to the y-axis as it gets farther from the mean.
- The total area under the curve is equal to 1.

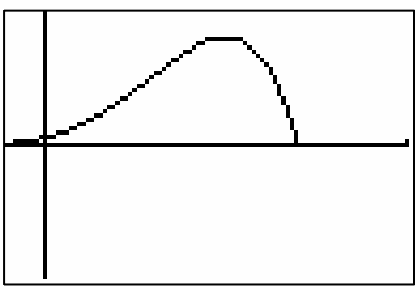
8. Which of the following appear to be normal curves?

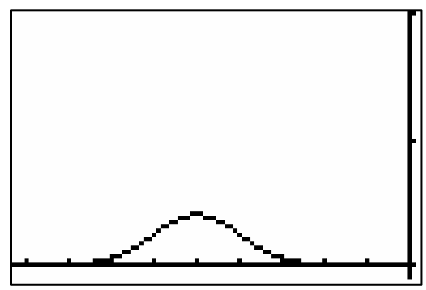






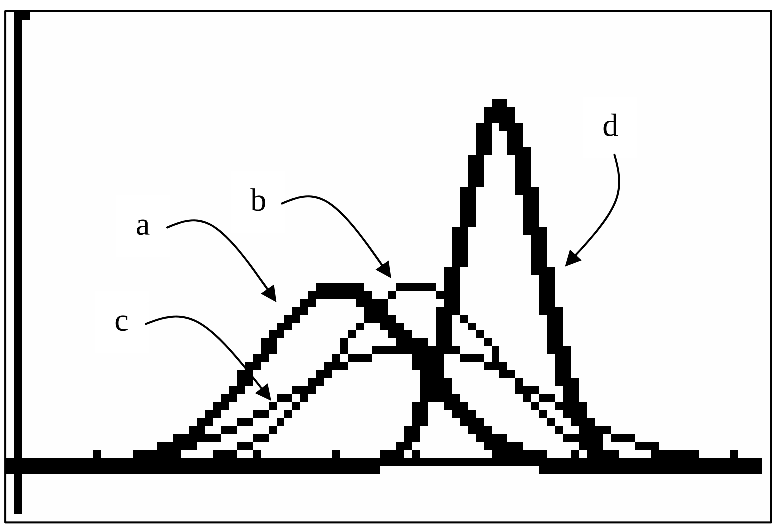






9. Each graph in the figure to the right is a normal curve. Match the curve to the corresponding information below.

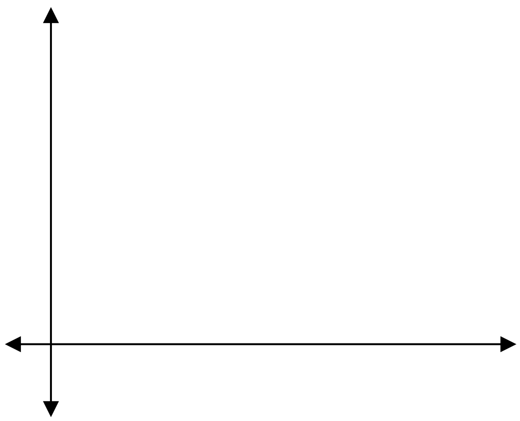
Graph letter	
______	a. Mean = 5 Standard Deviation =1
______	b. Mean = 4 Standard Deviation = 1
______	c. Mean = 6 Standard Deviation = .5
______	d. Mean = 5 Standard Deviation = 1.5

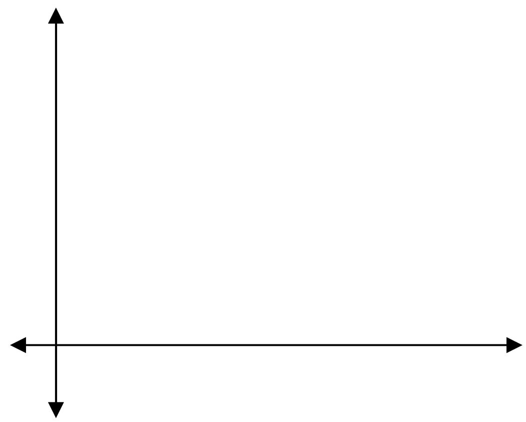


10. On each of the following axes, you will draw two normal curves with the information given:

a. Same mean and different standard deviation

b. Different mean and same standard deviation





11. Conduct research in texts and on the Internet to find ten examples of data that are normally distributed.

a.______________________________ b.______________________________

c.______________________________ d.______________________________

e.______________________________ f.______________________________

g.______________________________ h.______________________________

i.______________________________ j.______________________________

12. Conduct research in texts and on the Internet to find ten examples of data that are NOT normally distributed.

a.______________________________ b.______________________________

c.______________________________ d.______________________________

e.______________________________ f.______________________________

g.______________________________ h.______________________________

i.______________________________ j.______________________________

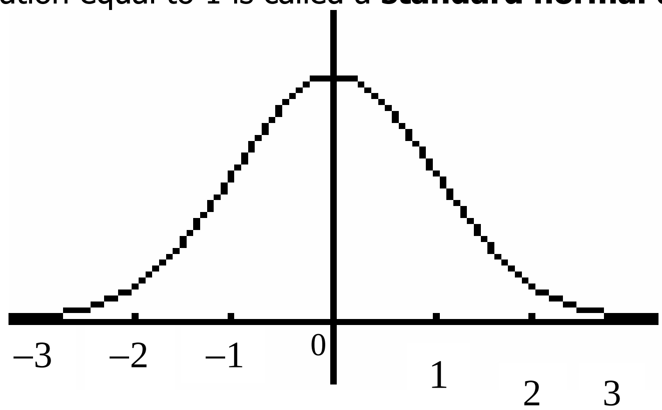
Activity 4: Teacher's Notes--Standard Normal Distributions, Percentiles and Heights

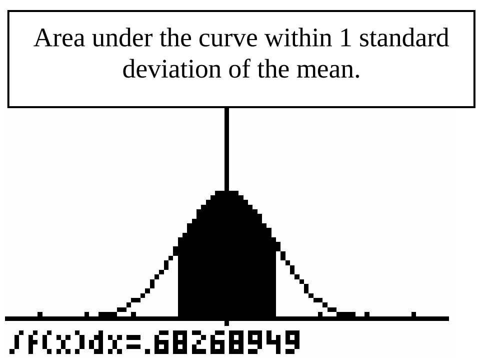
In this activity, students will investigate the properties of a normal curve, particularly the standard normal distribution and will transform normally distributed data to a standard normal curve using z-scores in two problem contexts. The activity introduces how percentiles are determined using a standard normal table or by using a calculator. The associated height project provides an opportunity to discuss appropriate sampling techniques, the formula for the sample standard deviation and a context where a set of normally distributed data is analyzed.

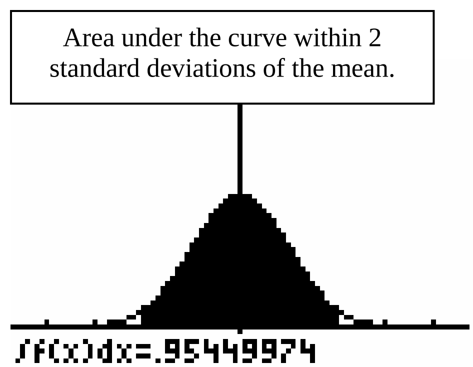
It is important for students to understand the distinction between population parameters and sample statistics. Understanding why ***n-1*** is used for the sample standard deviation while ***n*** is used for the population standard deviation is a topic that is technically beyond the scope of this course, but students should be given the opportunity to develop an intuitive understanding of degrees of freedom. Graphing calculators and spreadsheets have the capacity to produce both. In fact, when the TI-83/84 does its single variable statistics for a list, both sample and population statistics are generated. There are other methods than those mentioned in the Sampling Techniques box in the activity. The goal when sampling is to reduce bias. Students should be expected to construct a method for gathering data that reduces bias.

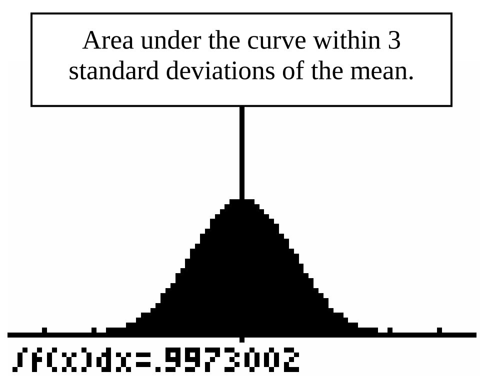
Standard Normal Distributions, Percentiles, and Height

Background Information: A Normal Distribution with mean equal to 0 and standard deviation equal to 1 is called a **standard normal distribution**.









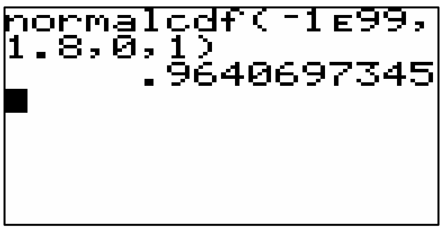
Sometimes this is called the 68-95-99.7 rule

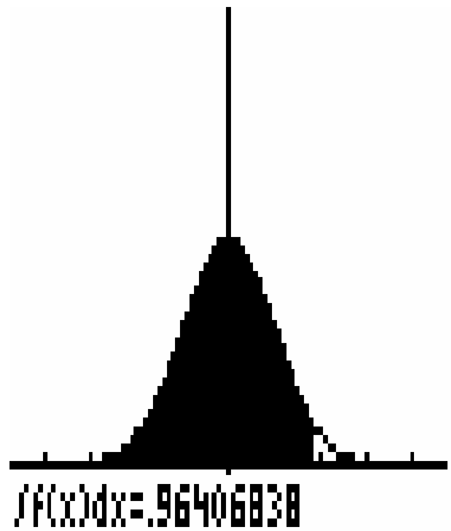
Because the total area under a standard normal curve is 1, these numbers represent the percentage of area in the bounded (shaded) region. They can also be used in the calculation of probability.

To find out the **percentile** (percent of data **below** a given value in a data set) for a given data value, we can look at the accumulated area up to that value. That **area** can be found using calculus, a standard normal table or a calculator command.

Example: Find the percentile for a z-score of 1.8. We can see that the accumulated area under the curve is about .96407, so 1.8 falls approximately at the 96th percentile.

Use the normalcdf command on your TI Calculator to find the percentile of 1.8 on the standard normal distribution.





Note: Normal distributions can be transformed to standard normal curves. It is possible to use z-scores and the standard normal curve to find areas under any normal curve.

1. The College of Knowledge gives an admission qualifying exam. The results are normally distributed with a mean of 500 and a standard deviation of 100. The admissions department would only like to accept students who score in the 65th percentile or better.

   Determine the z-scores and the percentiles of the following student's scores using the **standard normal table** on the next page (once the z-score is calculated to three decimal places, look for the first two decimal places on the left hand column of the table and the third across the top, then find the corresponding value in the row and column of the two). Which students qualify for admission?

Student Score	**Z-Score**	**Percentile**
530		
570		
650		
800		
540		

2. The MP3 Player made by Mango Corporation, aPod, has an average battery life of 400 hours. Battery life for the aPod is normally distributed with a standard deviation of 25 hours. The MP3 player made by Pineapple, Incorporated, the PeaPod, has an average battery life of 390 hours. The distribution for its battery life is also normally distributed with a standard deviation of 30 hours.

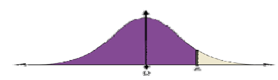
   a. Find the z- scores for each battery with lives of 250, 350, 410, and 450 hours.

   b. Which battery lasting 410 hours performed better?

   c. What percent of the aPod batteries last **between** 375 and 410 hours?

   d. What percent of PeaPod batteries last **more** than 370 hours?

Standard Normal Table using z-scores



z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0
–3.0	0.001	0.001	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
–2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
–2.8	0.0019	0.002	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
–2.7	0.0026	0.0027	0.0028	0.0029	0.003	0.0031	0.0032	0.0033	0.0034	0.0035
–2.6	0.0036	0.0037	0.0038	0.0039	0.004	0.0041	0.0043	0.0044	0.0045	0.0047
–2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.006	0.0062
–2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.008	0.0082
–2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
–2.2	0.011	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
–2.1	0.0143	0.0146	0.015	0.0154	0.0158	0.0162	0.0166	0.017	0.0174	0.0179
–2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
–1.9	0.0233	0.0239	0.0244	0.025	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
–1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
–1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
–1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
–1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.063	0.0643	0.0655	0.0668
–1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
–1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
–1.2	0.0985	0.1003	0.102	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
–1.1	0.117	0.119	0.121	0.123	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
–1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
–0.9	0.1611	0.1635	0.166	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
–0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.209	0.2119
–0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.242
–0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
–0.5	0.2776	0.281	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.305	0.3085
–0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.33	0.3336	0.3372	0.3409	0.3446
–0.3	0.3483	0.352	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
–0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.409	0.4129	0.4168	0.4207
–0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
–0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.484	0.488	0.492	0.496	0.5
z	**0**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0	0.5	0.504	0.508	0.512	0.516	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.591	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.648	0.6517

0.4	0.6554	0.6591	0.6628	0.6664	0.67	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.695	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.719	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.758	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.791	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.834	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.877	0.879	0.881	0.883
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.898	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.975	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.992	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.996	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.997	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.998	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Height Project

Research Question: Can we assume the heights of 15-year-olds are normally distributed?

1. Random Sample Design: Determine a method for collecting a truly random sampling of **thirty** 15-year-olds in school to find their heights in inches. Does it make sense to use a stratified sample? a cluster sample? a systematic sample? (see below)
   Explain:

2. Gather the data and then enter it into a spreadsheet or a graphing calculator.

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Height															

Student	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Height															

3. Determine the sample mean, $\bar{x} = \frac{\sum x}{N}$

4. Determine sample standard deviation, $s = \sqrt{\frac{\sum (x - \bar{x})^2}{N - 1}}$

5. Calculate each student's z-score and use a standard normal curve based upon your sample statistics to determine how many students lie within 1, 2 and 3 standard deviations of the mean.

6. If a 15-year-old is 6 feet tall, what percent of students are shorter than 6 feet?

7. A 15-year-old student is considered "tall" if he/she falls at or above the 75th percentile. What is the minimum height for a student to be considered "tall"?

8. How does your analysis compare with that of other groups in your class?

9. Gather the data and do the same analysis with a random sampling of an additional 20 or 30 students.

10. How do the results in question 9 compare with the results you obtained with your sample of 30?

Sampling Techniques

Stratified Sample – sample members from each "strata" (gender, income, age, ethnicity, etc.) to make sure each segment of the population is fairly represented.

Cluster Sample – sample members of selected clusters where each cluster has similar characteristics.

Systematic Sample – sample members chosen by assigning each member of a population a number and selecting a random starting point and choosing every n^{th} member.

Activity 5: Teacher's Notes--Probability Density

In this activity, students will investigate how a standard normal curve can be used to calculate probabilities. Students will connect the ideas of probability and percentile. Students will also link the concept of area under the curve with probability and learn to shade the appropriate region under a standard normal curve to represent a certain probability.

This use of the normal probability density function can be used with several activities and problems where the mean and standard deviation are calculated or where those statistics are provided.

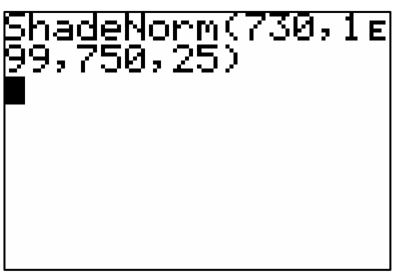
Solutions:

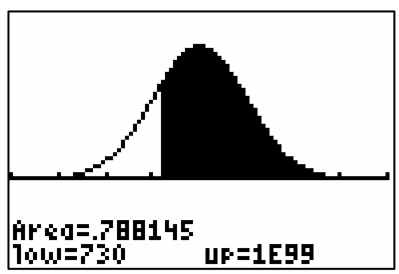
1. This can be done by finding the z-score for 730 $z = \frac{730 - 750}{25} = -.8$

   Find the corresponding value on the Standard Normal Table, **0.2119**
   That represents the area up to 730 and the probability, P(x < 730). Since we want the area above 730 and since the total area under the curve is 1, that area is

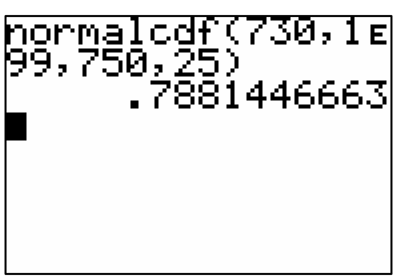
   P(x > 730) =1 - 0.2119 = **0.7881**

   or using the calculator, the area/probability can be shaded if the probability density function is in the calculator.



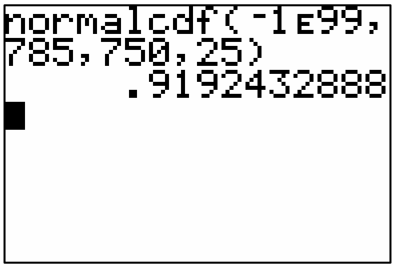


   If the probability function is not in Y_1, the calculator can find it by:



Below are some possible approaches for the other three problems:

1.



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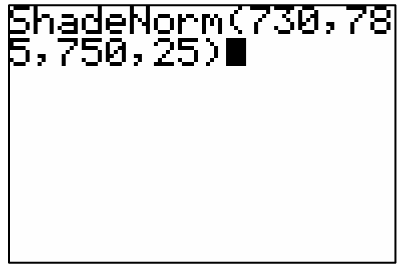

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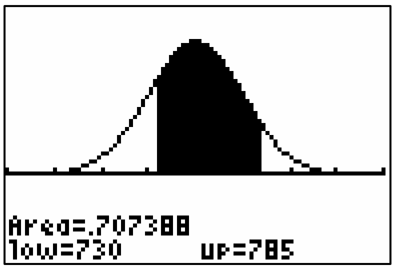

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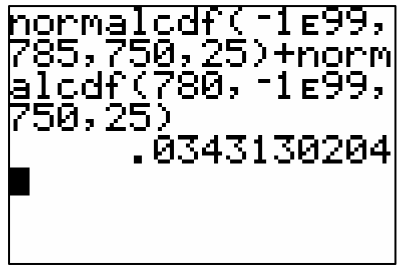
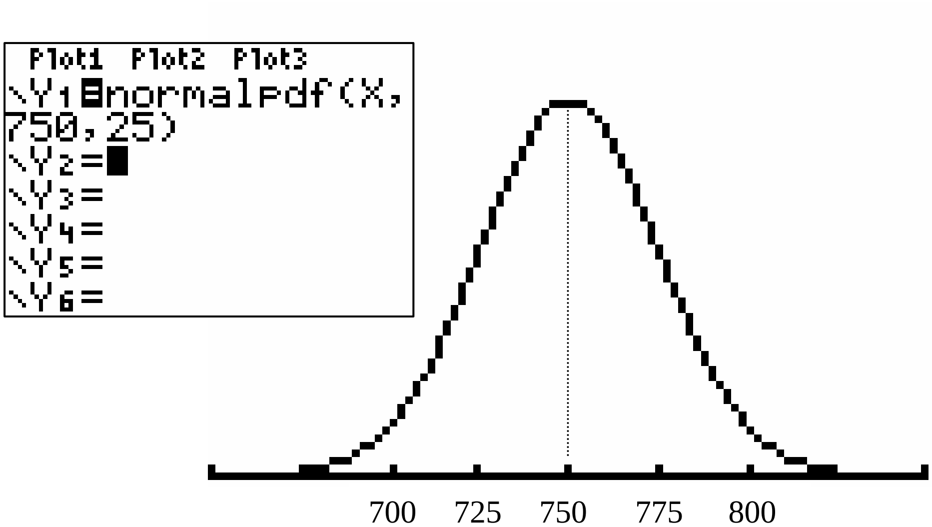


Probability Density

If an experiment generates outcomes that are normally distributed, the standard normal curve can be used to calculate probabilities of specific outcomes.

Example: The life span of a particular machine gasket is normally distributed with a mean of 750 hours with a standard deviation of 25 hours. The graph of the distribution is below:



The accumulated area up to 775 represents the **probability** that the gasket will last 775 or fewer hours.

The shaded area below represents the **probability** that the gasket will last 775 or fewer hours. The corresponding numerical probability is determined using a z-score.

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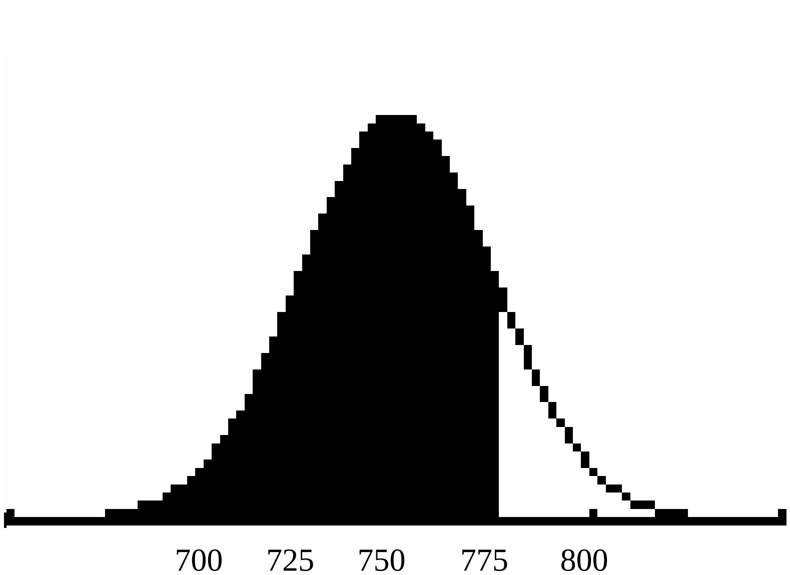
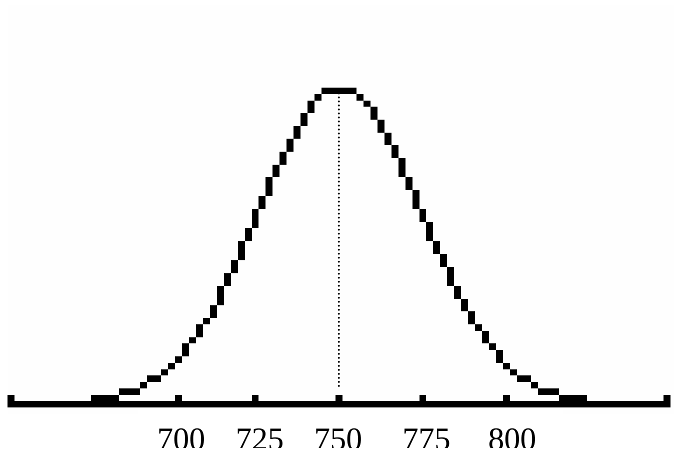


$z = \frac{775 - 750}{25} = 1$ and using the Standard Normal Table or a Calculator we can see

P(x<775) = 0.841

Using the same machine gasket example, **shade** the region representing the given probability and then determine the numerical probability using the table or a calculator.



1. The probability the gasket will last **more** than 730 hours.

   P(x > 730)

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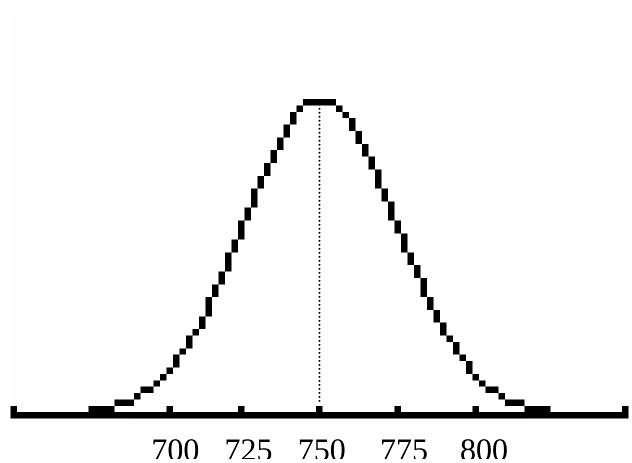
2. The probability the gasket lasts **less** than 785 hours.

   P(x < 785)



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3. The probability the gasket lasts **between** 730 and 785 hours.

P(730 < x < 785)

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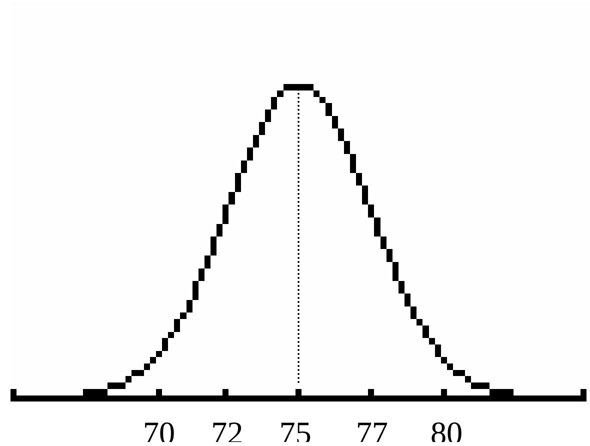

4. The probability that the gasket lasts **more** than 780 hours **or less** than 720 hours.

P(780 < x or x < 725)

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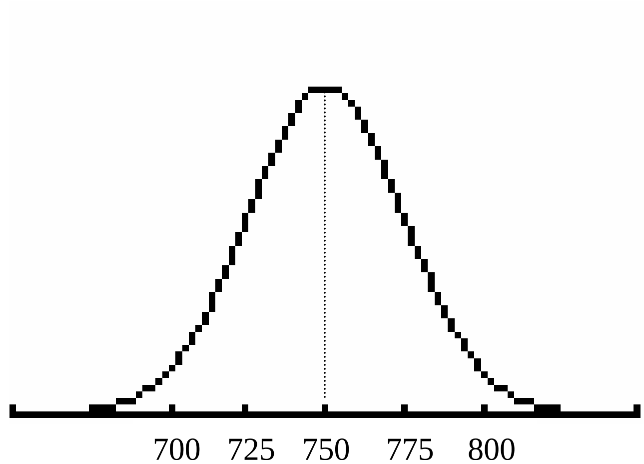

Activity 6: Teacher's Notes--Sampling: Politics and Opinions

Students will investigate how political polling takes place and will construct their own public opinion poll using one of the sampling techniques learned in this course. This sample survey example is one of many that can be done that will be of interest to students. The emphasis should be on survey design and constructing a sample process that minimizes experimental error and bias.

The **Wikipedia** links mentioned at the beginning of this organizing topic provide an avenue for students to explore the sampling techniques mentioned. Take the opportunity to have students research the evolution of political polling from a historical and methodological perspective. Discuss the reliability of subjective versus objective data. Have students investigate the Nielsen television ratings, how the research is conducted, and how the results impact the cost of commercial time.

Sampling: Politics and Opinions

You have been hired by the School Board to answer this question about **ALL** of the students/teachers in your school: How much homework do teachers assign and/or do students do? Unfortunately, you cannot possibly get that from everyone, so you decide to take a sample and use the data from your sample to make conclusions about the entire school population.

Initial Questions

1. What kinds of quantitative questions do you think need to be asked?

2. What qualitative questions should be asked?

3. What kind demographic information might be useful to the School Board?

4. Since you are going to use a sample, which kind of sample strategy will get you the most accurate information? How many students need to be included in the sample?

5. What kinds of things could bias your sample? What can you do to help to eliminate bias?

6. Construct a survey with 5-8 questions containing both quantitative and qualitative questions. Be sure to include the personal data you want as well.

Homework Survey - The purpose of the survey is to find out information requested by the school board. 1. 2. 3. 4. 5. 6. 7. 8.

7. Describe the plan you have to gather the sample data. Include the number in your sample, your strategy, and timeline.

Sample Description

8. In a spreadsheet, organize your data and determine what statistics you will need to use in your presentation to the Board.

9. For your numeric data (i.e., minutes of homework done daily), calculate the mean and sample standard deviation. Assuming that is a normal distribution, determine the likelihood that a student in the school population would study more than 60 minutes.

10. Anticipate what information the School Board might want to know about the study and how the research was conducted. Prepare a 5-10 minute presentation to explain how you conducted the survey and your results.

Resources:

http://asp.usatoday.com/sports/baseball/salaries/default.aspx

- Baseball Payroll information

http://www.baseball-reference.com

- Baseball Statistics

http://www.theredzone.org/2007/salaries/showposition.asp?Position=QB

- Source for football salaries

http://en.wikipedia.org/wiki/Cluster_Sample

http://en.wikipedia.org/wiki/Stratified_Sample

http://en.wikipedia.org/wiki/Systematic_Sample

- Resources with links to additional information about sampling

http://exploringdata.cqu.edu.au