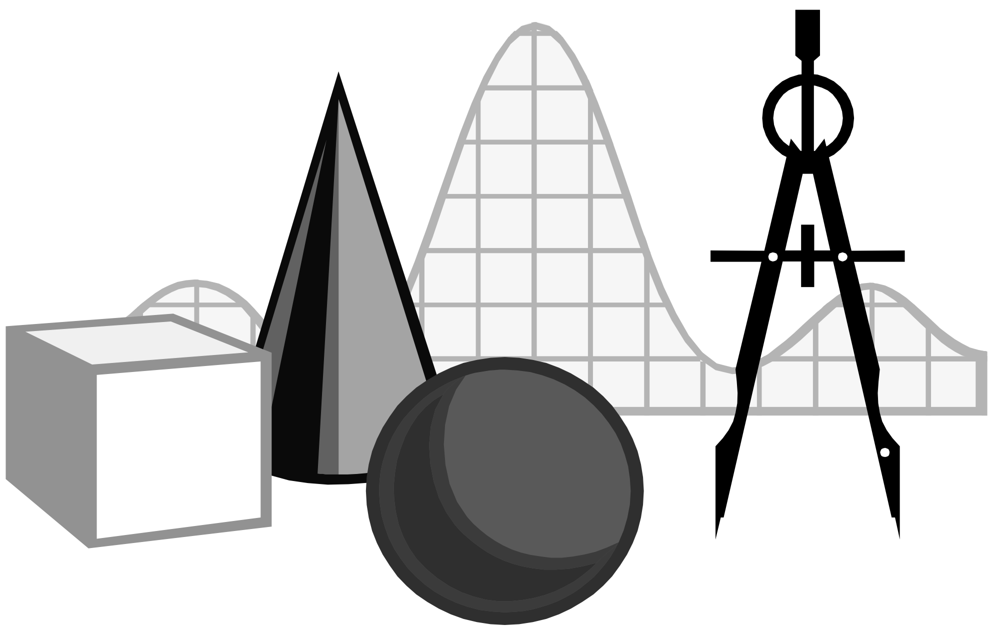
MATHEMATICS STANDARDS OF LEARNING ENHANCED SCOPE AND SEQUENCE

Geometry



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Introduction

The *Mathematics Standards of Learning Enhanced Scope and Sequence* is a resource intended to help teachers align their classroom instruction with the Mathematics Standards of Learning that were adopted by the Board of Education in October 2001. The Mathematics Enhanced Scope and Sequence is organized by topics from the original Scope and Sequence document and includes the content of the Standards of Learning and the essential knowledge and skills from the Curriculum Framework. In addition, the Enhanced Scope and Sequence provides teachers with sample lesson plans that are aligned with the essential knowledge and skills in the Curriculum Framework.

School divisions and teachers can use the Enhanced Scope and Sequence as a resource for developing sound curricular and instructional programs. These materials are intended as examples of how the knowledge and skills might be presented to students in a sequence of lessons that has been aligned with the Standards of Learning. Teachers who use the Enhanced Scope and Sequence should correlate the essential knowledge and skills with available instructional resources as noted in the materials and determine the pacing of instruction as appropriate. This resource is not a complete curriculum and is neither required nor prescriptive, but it can be a valuable instructional tool.

The Enhanced Scope and Sequence contains the following:

- Units organized by topics from the original Mathematics Scope and Sequence
- Essential knowledge and skills from the Mathematics Standards of Learning Curriculum Framework
- Related Standards of Learning
- Sample lesson plans containing
    - Instructional activities
    - Sample assessments
    - Follow-up/extensions
    - Related resources
    - Related released SOL test items.

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Organizing Topic Reasoning and Proof

Standard of Learning

G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
a) identifying the converse, inverse, and contrapositive of a conditional statement;
b) translating a short verbal argument into symbolic form;
c) using Venn diagrams to represent set relationships; and
d) using deductive reasoning, including the law of syllogism.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
• Use inductive reasoning to make conjectures.	________________________
• Write a conditional statement in if-then form.	________________________
• Given a conditional statement,	
° identify the hypothesis and conclusion	________________________
° write the converse, inverse, and contrapositive.	________________________
• Translate short verbal arguments into symbolic form ($p \rightarrow q$ and $\sim p \rightarrow \sim q$).	________________________
• Use valid logical arguments to prove or disprove conjectures.	________________________
• Use the law of syllogism and the law of detachment in deductive arguments.	________________________
• Solve linear equations and write them in if-then form (if $2x + 9 = 17$, then $x = 4$).	________________________
• Justify each step in solving a linear equation with a field property of real numbers or a property of equality.	________________________
• Present solving linear equations as a form of deductive proof.	________________________

Inductive and Deductive Reasoning

Organizing topic	Reasoning and Proof
Overview	Students practice inductive and deductive reasoning strategies.
Related Standard of Learning	G.1

Objectives

- The student will use inductive reasoning to make conjectures.
- The student will use logical arguments to prove or disprove conjectures.
- The student will justify steps while solving linear equations, using properties of real numbers and properties of equality.
- The student will solve linear equations as a form of deductive proof.

Instructional activity

1. Review the basic vocabulary included on the activity sheets.
2. Have students work in pairs or small groups to complete the activity sheets.
3. Use the algebraic properties of equality (shown on Activity Sheet 3) for matching, concentration, or filling in the steps of a proof in addition to writing.

Follow-up/extension

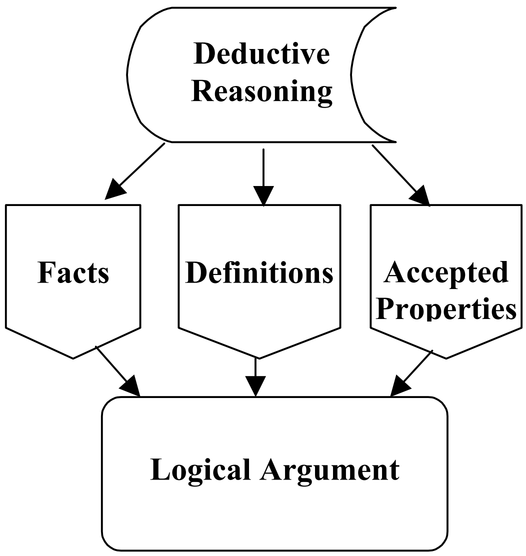
- Have students investigate practical problems involving inductive or deductive reasoning.
- Have students create their own conjectures to prove or disprove.

Sample assessment

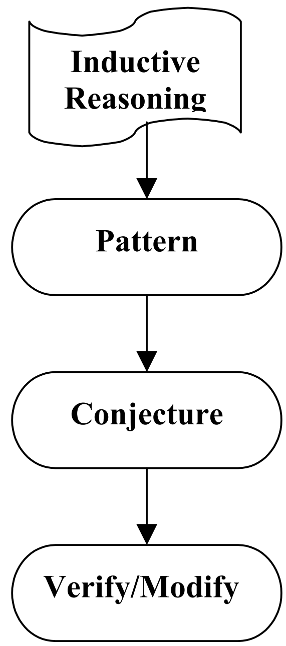
- Have students work in pairs to evaluate strategies.
- Use activity sheets to help assess student understanding.
- Have students complete a journal entry comparing and contrasting inductive and deductive reasoning strategies.

Activity Sheet 1: Inductive and Deductive Reasoning

Deductive reasoning works from the more general to the more specific.



Inductive reasoning works from the more specific observations to broader generalizations.



Example of Deductive Reasoning

- Tom knows that if he misses the practice the day before a game, then he will not be a starting player in the game.
- Tom misses practice on Tuesday.
- *Conclusion:* He will not be able to start in the game on Wednesday.

Example of Inductive Reasoning

- *Observation:* Mia came to class late this morning.
- *Observation:* Mia's hair was uncombed.
- *Prior Experience:* Mia is very fussy about her hair.
- *Conclusion:* Mia overslept.

Complete the following conjectures based on the pattern you observe in specific cases:

1 + 1 = 2	7 + 11 = 18
1 + 3 = 4	13 + 19 = 32
3 + 5 = 8	201 + 305 = 506

Conjecture: The sum of any two odd numbers is ________.

Conjecture: The product of any two odd numbers is ________.

Conjecture: The product of a number (n – 1) and the number (n + 1) is always equal to ________.

Prove or disprove the following conjecture:
Conjecture: For all real numbers x, the expression x^2 is greater than or equal to x.

Activity Sheet 2: Inductive and Deductive Reasoning

1. John always listens to his favorite radio station, an oldies station, when he drives his car. Every morning he listens to his radio on the way to work. On Monday when he turns on his car radio, it is playing country music. Make a list of valid conjectures to explain why his radio is playing different music.

2. ∠M is obtuse. Make a list of conjectures based on that information.

3. Based on the table to the right, Marina concluded that when one of the two addends is negative, the sum is always negative. Write a counterexample for her conjecture.

Addends		Sum
–8	–10	–18
–17	–5	–22
15	–23	–8
–26	22	–4

Statement	Reason
$5x - 18 = 3x + 2$	Given
$2x - 18 = 2$	Subtraction Property of Equality
$2x = 20$	Addition Property of Equality
$x = 10$	Division Property of Equality

The Algebraic Properties of Equality, as shown on Activity Sheet 3, can be used to solve $5x - 18 = 3x + 2$ and to write a reason for each step, as shown in the table on the left.

Using a table like this one, solve each of the following equations, and state a reason for each step.

4. $-2(-w + 3) = 15$
5. $p - 1 = 6$
6. $2r - 7 = 9$
7. $3(2t + 9) = 30$
8. Given $3(4v - 1) - 8\text{v} = 17$, prove $v = 5$.

Match each of the following conditional statements with a property:

A. Multiplication Property
B. Substitution Property
C. Transitive Property
D. Addition Property
E. Symmetric Property
F. Reflexive Property
G. Distributive Property
H. Subtraction Property
I. Division Property

9. If JK = PQ and PQ = ST, then JK = ST. _____
10. If m ∠S = 30°, then 5° + m∠S = 35°. _____
11. If ST = 2 and SU = ST + 3, then SU = 5. _____
12. If m ∠K = 45°, then 3(m∠K) = 135°. _____
13. If m ∠P = m ∠Q, then m ∠Q = m ∠P. _____

Activity Sheet 3: Algebraic Properties of Equality

a, b, and c are real numbers

Addition Property	**If $a = b$, then $a + c = b + c$**
Subtraction Property	**If $a = b$, then $a - c = b - c$**
Multiplication Property	**If $a = b$, then $ac = bc$**
Division Property	**If $a = b$ and $c \neq 0$, then $a \div c = b \div c$**
Reflexive Property	**$a = a$**
Symmetric Property	**If $a = b$, then $b = a$**
Transitive Property	**If $a = b$ and $b = c$, then $a = c$**
Substitution Property	**If $a = b$, then a can be substituted for b in any equation or expression.**
Distributive Property	**$a(b + c) = ab + ac$**

Logic and Conditional Statements

Organizing topic	Reasoning and Proof
Overview	Students investigate symbolic form while working with conditional statements.
Related Standard of Learning	G.1

Objectives

- The student will identify the hypothesis and conclusion of a conditional statement.
- The student will write the converse, inverse, and contrapositive of a conditional statement.
- The student will translate short verbal arguments into symbolic form.
- The student will use the law of syllogism and the law of detachment in deductive arguments.
- The student will diagram logical arguments, using Venn diagrams.

Materials needed

- Activity sheets 1 and 2 and handout for each student. (Activity sheets can be used as study tools or flash cards for group work.)

Instructional activity

1. Review the basic vocabulary included on the handout.
2. Have students work in pairs or small groups to complete the activity sheets.

Sample assessment

- Have students work in pairs to evaluate strategies.
- Use activity sheets to help assess student understanding.
- Have students complete a journal entry summarizing inductive and deductive reasoning strategies.

Follow-up/extension

- Have students investigate practical problems involving deductive reasoning.
- Have students create their own conjectures to prove or disprove.
- Have students investigate more truth tables and in-depth logic.

Sample resources

Mathematics SOL Curriculum Framework
 http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html
SOL Test Blueprints
Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html
Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra
NASA http://spacelink.nasa.gov/.index.html
The Math Forum http://forum.swarthmore.edu/
4teachers http://www.4teachers.org
Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm
Eisenhower National Clearinghouse http://www.enc.org/

Activity Sheet 1: Logic and Conditional Statements

Conditional Statement

If **Hypothesis** **then** **Conclusion**

If **p** **then** **q**

p → q

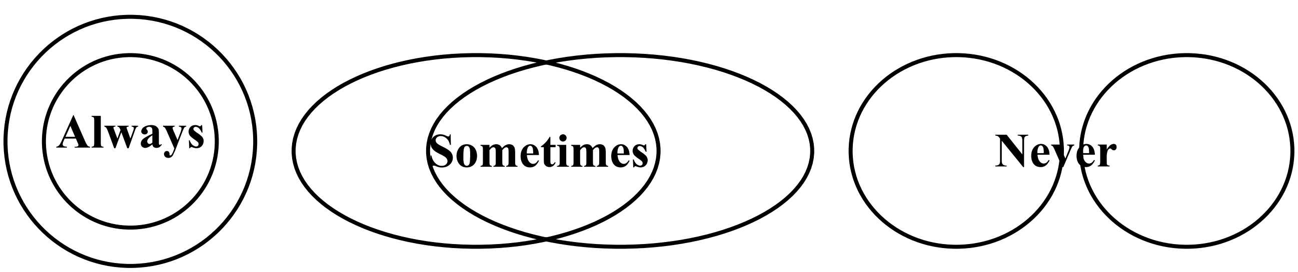
or **p implies q**

~p **is read** **"not p"** **and means** **the opposite of p**

Converse
p → q
"Switch"
q → p

Inverse
p → q
"Negate"
~p → ~q

Contrapositive
p → q
"Switch and Negate"
~q → ~p



Activity Sheet 2: Logic and Conditional Statements

Write each of the following statements as a conditional statement:

1. Mark Twain wrote, “ If you tell the truth, you don’t have to remember anything.”
2. Helen Keller wrote, “One can never consent to creep when one feels the impulse to soar.”
3. Mahatma Ghandi wrote, “Freedom is not worth having if it does not include the freedom to make mistakes.
4. Benjamin Franklin wrote, “Early to bed and early to rise, makes a man healthy, wealthy, and wise.”

Identify the hypothesis and conclusion for each conditional statement:

5. If two lines intersect, then their intersection is one point.
6. If two points lie in a plane, then the line containing them lies in the plane.
7. If a cactus is of the cereus variety, then its flowers open at night.

Write the converse, inverse, and contrapositive for each of the following conditional statements. Determine if each is true or false.

8. If three points are collinear, then they lie in the same plane.
9. If two segments are congruent, then they have the same length.

10. By the Law of Syllogism, which statement follows from statements 1 and 2?

    <u>Statement 1</u>: If two adjacent angles form a linear pair, then the sum of the measures of the angles is180°.

    <u>Statement 2</u>: If the sum of the measures of two angles is 180°, then the angles are supplementary.

    a. If the sum of the measures of two angles is 180°, then the angles form a linear pair.
    b. If two adjacent angles form a linear pair, then the sum of the measures of the angles is 180°.
    c. If two adjacent angles form a linear pair, then the angles are supplementary.
    d. If two angles are supplementary, then the sum of the measures of the angles is 180°.

11. “If it is raining, then Sam and Sarah will not go to the football game.” This is a true conditional, and it is raining. Use the Law of Detachment to reach a logical conclusion.

Let p: you see lightning and q: you hear thunder. Write each of the following in symbolic form:

12. If you see lightning, then you hear thunder.
13. If you hear thunder, then you see lightning.
14. If you don’t see lightning, then you don’t hear thunder.
15. If you don’t hear thunder, then you don’t see lightning.

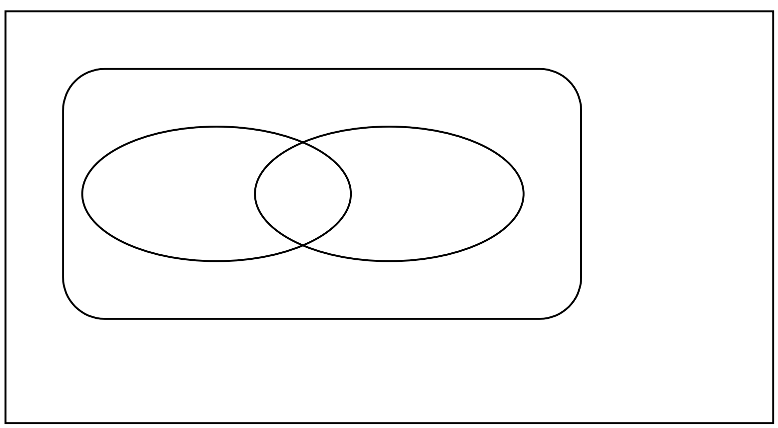
Let p: two planes intersect and q: the intersection is a line. Write each of the following in "If...Then" form:

16. $p \rightarrow q$
17. $\sim p \rightarrow q$
18. $q \rightarrow p$
19. $\sim q \rightarrow p$
20. $\sim p \rightarrow \sim q$
21. $\sim q \rightarrow \sim p$
22. $p \rightarrow \sim q$
23. $q \rightarrow \sim p$

Draw a Venn Diagram for each of the following statements:

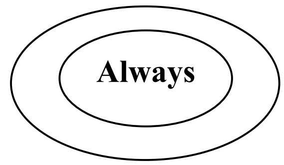
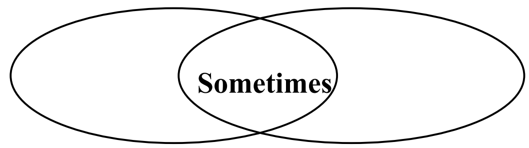
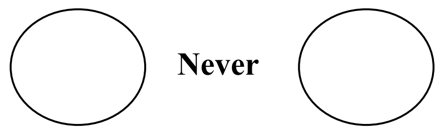
24. All squares are rhombi.
25. Some rectangles are squares.
26. No trapezoids are parallelograms.
27. Some quadrilaterals are parallelograms.
28. All kites are quadrilaterals.
29. No rhombi are trapezoids.

30. Complete the Venn Diagram for the list of terms to the right.



quadrilateral
parallelogram
rectangle
square
rhombus
trapezoid
kite

Logic and Conditional Statements

Conditional Statement	**p implies q**
Hypothesis	
Conclusion	
If	
Then	**$p \to q$**
Not	**~**
Converse	**"Switch"**
Inverse	**"Negate"**
Contrapositive	**"Switch and Negate"**

Sample assessment

Which conclusion logically follows these true statements? "If negotiations fail, the baseball strike will not end." "If the baseball strike does not end, the World Series will not be played."

F If the baseball strike ends, the World Series will be played.

G If negotiations do not fail, the baseball strike will not end.

H If negotiations fail, the World Series will not be played.

J If negotiations fail, the World Series will be played.

Let *a* represent "*x* is an odd number." Let *b* represent "*x* is a multiple of 3." When *x* is 7, which of the following is true?

A $a \wedge b$

B $a \wedge {\sim}b$

C ${\sim}a \wedge b$

D ${\sim}a \wedge {\sim}b$

Which of the following groups of statements represents a valid argument?

F Given: All quadrilaterals have four sides.
All squares have four sides.
Conclusion: All quadrilaterals are squares.

G Given: All squares have congruent sides.
All rhombuses have congruent sides.
Conclusion: All rhombuses are squares.

H Given: All four sided figures are quadrilaterals.
All parallelograms have four sides.
Conclusion: All parallelograms are quadrilaterals.

J Given: All rectangles have angles.
All squares have angles.
Conclusion: All rectangles are squares.

Which is the contrapositive of the statement, "If I am in Richmond, then I am in Virginia"?

A If I am in Virginia, then I am in Richmond.

B If I am not in Richmond, then I am not in Virginia.

C If I am not in Virginia, then I am not in Richmond.

D If I am not in Virginia, then I am in Richmond.

Which is the inverse of the sentence, "If Sam leaves, then I will stay"?

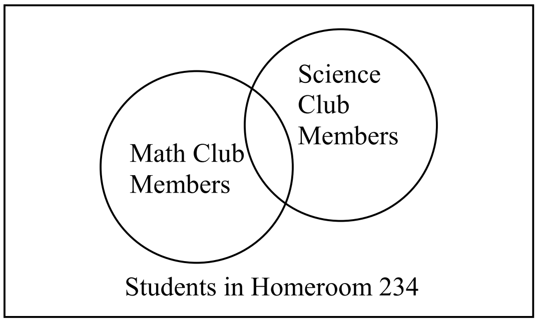
F If I stay, then Sam will leave.

G If Sam does not leave, then I will not stay.

H If Sam leaves, then I will not stay.

J If I do not stay, then Sam will not leave.

According to the diagram, which of the following is true?



A All students in Homeroom 234 belong to either the Math Club or the Science Club.

B All students in Homeroom 234 belong to both the Math Club and the Science Club.

C No student in Homeroom 234 belongs to both the Math Club and the Science Club.

D Some students in Homeroom 234 belong to both the Math Club and the Science Club.

Organizing Topic Lines and Angles

Standards of Learning

G.2 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
a) investigating and using formulas for finding distance, midpoint, and slope;
b) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
c) determining whether a figure has been translated, reflected, or rotated.

G.3 The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, angles formed when parallel lines are cut by a transversal, and angles in polygons.

G.4 The student will use the relationships between angles formed by two lines cut by a transversal to determine if two lines are parallel and verify, using algebraic and coordinate methods as well as deductive proofs.

G.11 The student will construct a line segment congruent to a given line segment, the bisector of a line segment, a perpendicular to a given line from a point not on the line, a perpendicular to a given line at a point on the line, the bisector of a given angle, and an angle congruent to a given angle.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Identify types of angle pairs:	
◦ complementary angles	____________
◦ supplementary angles	____________
◦ vertical angles	____________
◦ linear pairs of angles	____________
◦ alternate interior angles	____________
◦ consecutive interior angles	____________
◦ corresponding angles.	____________
• Use inductive reasoning to determine the relationship between complementary angles, supplementary angles, vertical angles, and linear pairs of angles.	____________
• Define and identify parallel lines.	____________
• Find the slope of a line given the graph of the line, the equation of the line, or the coordinates of two points on the line.	____________
• Investigate the relationship between the slopes of parallel lines.	____________
• Explore the relationship between alternate interior angles, consecutive interior angles, and corresponding angles when they occur as a result of parallel lines being cut by a transversal.	____________
• State these angle relationships as conditional statements.	____________

- Solve practical problems involving these angle relationships. ____________________
- Use the converses of the conditional statements about the angles associated with two parallel lines cut by a transversal to show necessary and sufficient conditions for parallel lines. ____________________
- Verify the converses using deductive arguments, coordinate, and algebraic methods. ____________________
- Using a compass and straightedge only, construct the following:
  - a line segment congruent to a given segment ____________________
  - an angle congruent to a given angle ____________________
  - the bisector of a given angle ____________________
  - a perpendicular to a given line from a point not on the given line ____________________
  - a perpendicular to a given line at a point on the given line. ____________________

Investigating Lines and Angles

Organizing topic Lines and Angles

Overview Students investigate parallel lines and their relationship to special angles.

Related Standards of Learning G.2, G.3, G.4

Objectives

- The student will define and identify parallel lines.
- The student will find the slope of a line.
- The student will investigate the relationship between the slopes of parallel lines.
- The student will investigate types of angle pairs.
- The student will use inductive reasoning to determine relationships between angle pairs.
- The student will explore the relationship between angle pairs and parallel lines.
- The student will state angle relationships as conditional statements.
- The student will verify the converses of the conditional statements about angle pairs.
- The student will prove lines are parallel.

Materials needed

- "Investigating Lines and Angles" activity sheets 1 and 2 for each student
- Dynamic geometry software (e.g., Geometer's Sketchpad™)

Sample assessment

- Have students work in pairs to evaluate strategies.
- Use activity sheets to help assess student understanding.
- Have students complete a journal entry summarizing their conclusions about special pairs of angles.

Follow-up/extension

- Have students investigate practical problems involving parallel lines.
- Have students complete creative diagrams, using parallel lines and special angles.

Activity Sheet 1: Investigation of Lines and Angles

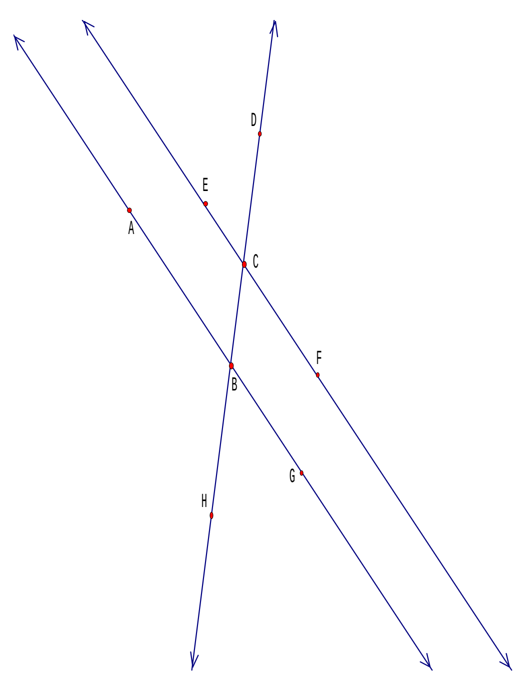
Let's investigate slope, using $y = 4x - 3$ as an example. (These directions are for the dynamic geometry software, Geometer's Sketchpad™, but are applicable to other packages.) Follow the steps below:

1. Go to Graph, Grid Form, Square.
2. Go to Graph, New Function.
3. Type the expression to the right of the equal sign. $f(x) = 4x - 3$ shows up.
4. Go to Graph, Plot Function.
5. The graph should come up on the coordinate plane.
6. From the graph, draw two points on the line. For example, (0, –3) and (1, 1).
7. Count the vertical and horizontal change from (0, –3) to (1, 1).
8. What is the slope? Remembering that slope = rise ÷ run.
9. Highlight the two points you have drawn on your line and use the straightedge tool to draw a segment from one point to the other.
10. Go to Measure, Slope.
11. Were your calculations correct?
12. Now, draw a point not on the line.
13. Highlight that point and the original line.
14. Go to Construct, Parallel Line.
15. A line parallel to our original line is drawn on the screen.
16. Measure the slope of the new line. What do you discover? What generalization can you make about the slopes of parallel lines?

Now, go through the same process with the line $y = (1/2)x$

We're now going to explore pairs of angles. Start a new sketch in Geometer's Sketchpad™, following these steps:

1. Draw two points. Label them A and B.
2. Go to Construct, Line.
3. Draw a point not on that line. Label it C.
4. Highlight point C and the line containing A and B.
5. Go to Construct, Parallel Line
6. Highlight points B and C
7. Go to Construct, Line.
8. Draw the line containing B and C.
9. Draw and label points D, E, F, G, and H, as shown on the diagram at the right.



Using your diagram, find the measure of the following angles:

- ∠ABH
- ∠ABC
- ∠ECB
- ∠ECD
- ∠HBG
- ∠BCF
- ∠GBC
- ∠FCD

List the pairs of angles whose measures add up to 90°.

List the pairs of angles whose measures add up to 180°.

List the pairs of angles that are congruent.

List angles that form a linear pair.

List angles that are complementary.

List angles that are supplementary.

List all corresponding angles

List all consecutive interior angles.

List all vertical angles.

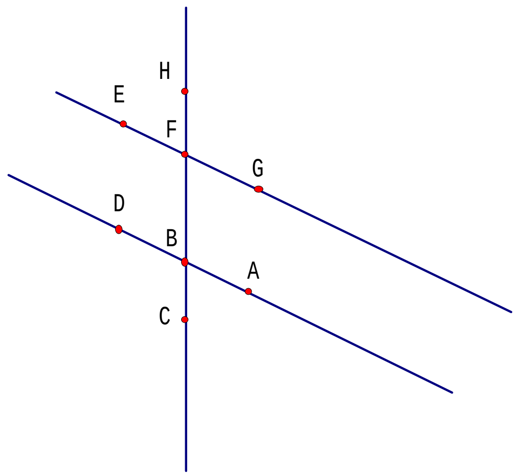
List all alternate exterior angles.

List all alternate interior angles.

What generalizations can you make from your lists?

Write each conclusion in “If...Then” form. For example, “If two parallel lines are cut by a transversal, then corresponding angles are congruent.” “If the measures of two angles add up to 90°, then those angles are complementary.”

Activity Sheet 2: Investigation of Lines and Angles



1. If you are told that ∠EFH ≅ ∠DBF, what conclusion can you make?
2. If you are told that ∠EFB ≅ ∠HFG, what conclusion can you make?
3. If you are told that m∠CBD + m∠DBF = 180°, what conclusion can you make?
4. If you are told that m∠GFB + m∠ABF = 180°, what conclusion can you make?
5. Name four conditions that involve angles and that are sufficient to prove that two lines are parallel.

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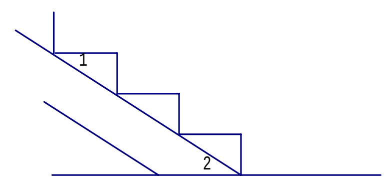
6. One way to build stairs is to attach triangular blocks to an angled support, as shown on the right. If the support makes a 32° angle with the floor (m∠2), what must m∠1 be so the step will be parallel to the floor? The sides of the angled support are parallel.



7. The white lines along the long edges of a football field are called sidelines. Yard lines are perpendicular to the sidelines and cross the field every five yards. Explain why you can conclude that the yard lines are parallel.

8. When you hang wallpaper, you use a tool called a *plumb line* to make sure one edge of the first strip of wallpaper is vertical. If the edges of each strip of wallpaper are parallel and there are no gaps between the strips, how do you know that the rest of the strips of wallpaper will be parallel to the first?

Constructions

Organizing topic	Lines and Angles
Overview	Students use a compass and straightedge to complete constructions.
Related Standard of Learning	G.11

Objectives

- The student will construct a line segment congruent to a given segment.
- The student will construct an angle congruent to a given angle.
- The student will construct the bisector of a given angle.
- The student will construct a line perpendicular to a given line from a point not on the given line.
- The student will construct a line perpendicular to a given line from a point on the given line.

Materials needed

- "Constructions" activity sheets 1 and 2 for each student
- Straightedge
- Compass

Instructional activity

- Have students complete the activity sheets. It may be helpful for them to work in pairs.

Sample assessment

- Have students work in pairs to evaluate strategies.
- Use activity sheets to help assess student understanding.
- Have students complete a journal entry summarizing steps for each construction.

Follow-up/extension

- Have students investigate practical problems involving constructions.
- Have students complete creative diagrams, using combined constructions.

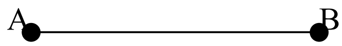
Sample resources

Mathematics SOL Curriculum Framework
 http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html
SOL Test Blueprints
Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html
Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra
NASA http://spacelink.nasa.gov/.index.html
The Math Forum http://forum.swarthmore.edu/
4teachers http://www.4teachers.org
Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm
Eisenhower National Clearinghouse http://www.enc.org/

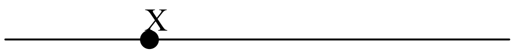
Activity Sheet 1: Constructions

Constructing a line segment congruent to a given line segment

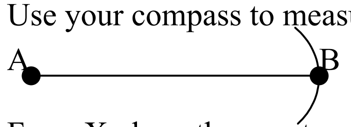
Given a line segment, AB,



- Use a straightedge to draw a line, choose a point on the line, and label it X.



- Use your compass to measure the length of segment AB, drawing an arc as you measure.



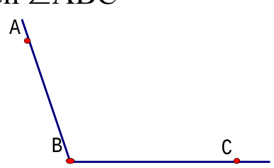
- From X, draw the exact arc that was drawn on segment AB.

$\overline{XY} \cong \overline{AB}$ Justification

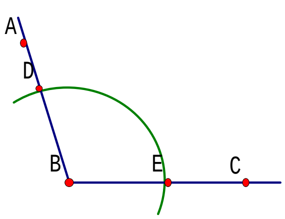
We use A as the center of a circle and B as a point on that circle. We keep the same radius and draw a congruent circle from point X so that Y is a point on that circle. Since segments AB and XY are radii of congruent circles, the segments are congruent.

Constructing an angle congruent to a given angle

Given ∠ABC

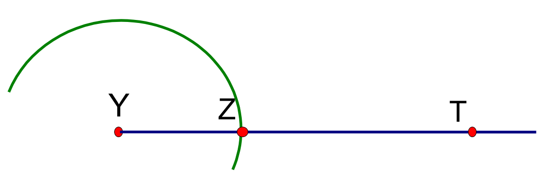


- From point B, use your compass to draw an arc that intersects ray BA and ray BC.

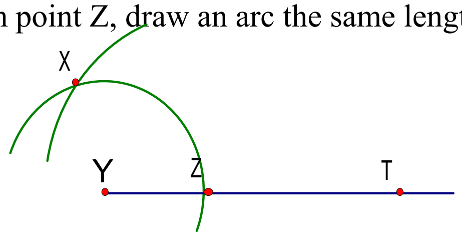


- Draw a ray, and label it $\overrightarrow{YT}$.

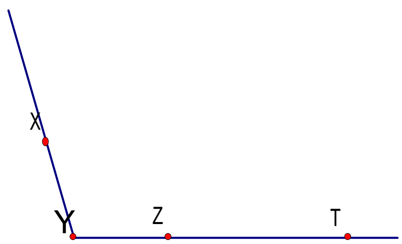
- From point Y, draw the exact arc that was drawn on ∠ABC. Label the point of intersection Z.



From point Z, draw an arc the same length as DE.



- Draw ray YX.

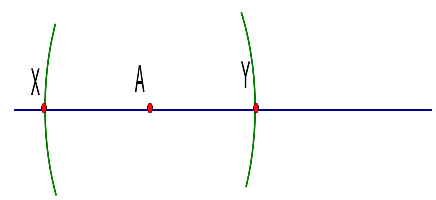


- Now, ∠XYZ ≅ ∠ABC.

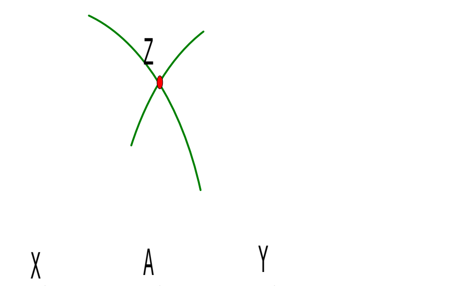
Constructing a perpendicular to a given line at a point on the line

Given line l and point A on l,

- From A, draw two arcs the same distance from A and intersecting line l and label the points of intersection X and Y.

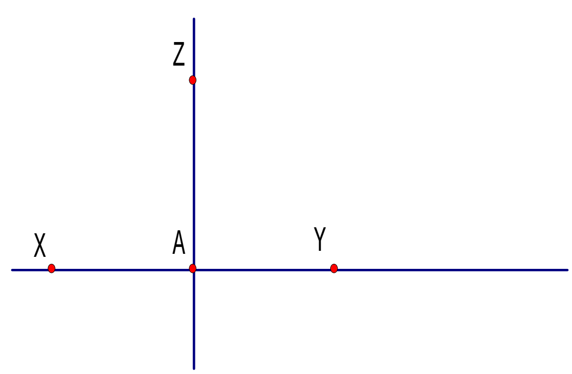


- From X, draw an arc that intersects line l past A. Then, draw the same arc from point Y.



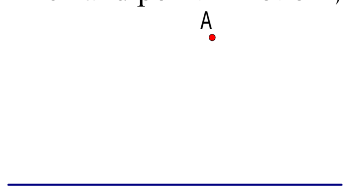
- Draw the line that passes through points A and Z.

- Line $\overleftrightarrow{AZ}$ is ⊥ line l at A.

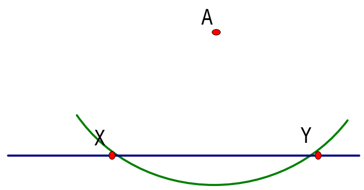


Constructing a perpendicular to a given line from a point not on the line

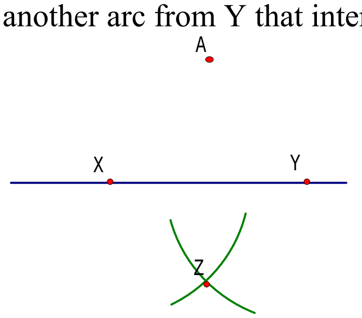
Given line *l* and point A not on *l*.



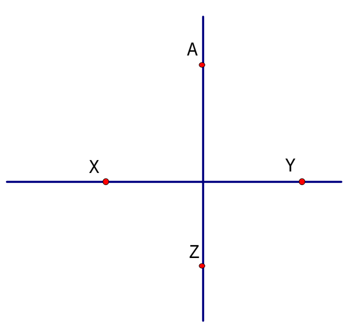
- From point A, draw an arc that intersects line *l* in two points. Call these points X and Y.



- From X, draw an arc that is more than half the length to point Y. Using the same arc length, draw another arc from Y that intersects the first arc.



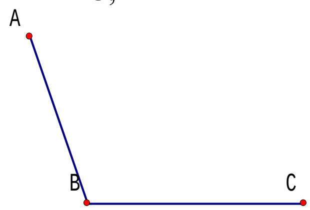
- Draw the straight line through points A and Z.



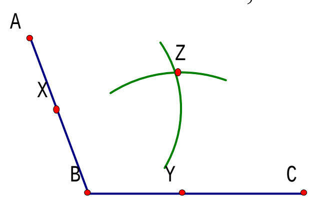
- Line $\overleftrightarrow{AZ}$ is ⊥ line *l*.

Constructing the bisector of a given angle

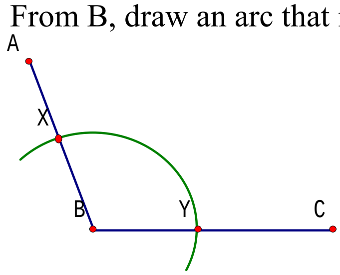
Given $\angle ABC$,



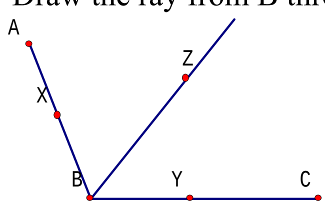
- From X, draw an arc that is large enough to reach past B. Using the same compass opening and Y as the circle center, draw another arc that intersects the first arc.



- From B, draw an arc that intersects $\overrightarrow{BA}$ at X and $\overrightarrow{BC}$ at Y.



- Draw the ray from B through Z. Ray $\overrightarrow{BZ}$ is the angle bisector of $\angle ABC$.



- $\overrightarrow{BZ}$ bisects $\angle ABC$.

Activity Sheet 2: Constructions

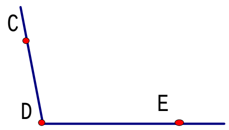
Construct a line segment congruent to each given line segment.

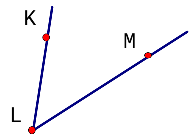
1. 

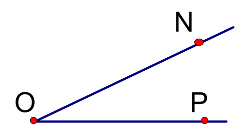
2. 

3. 

Construct an angle congruent to each given angle.

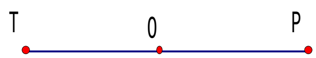
4. 

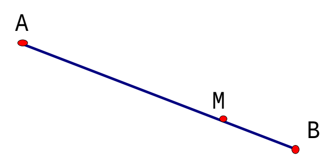
5. 

6. 

Construct a line perpendicular to each given line through the given point on the line.

7. 

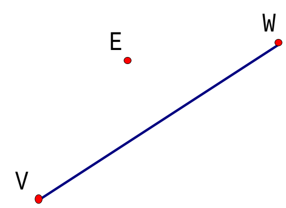
8. 

9. 

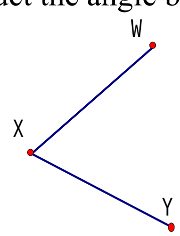
Construct a line perpendicular to each given line through the given point not on the line.

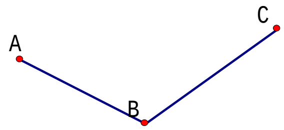
10. 

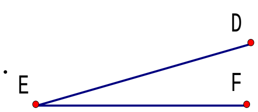
11. 

12. 

Construct the angle bisector of each given angle.

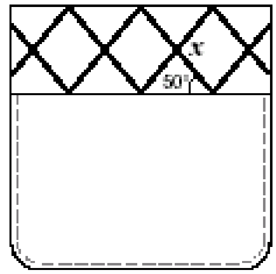
13. 

14. 

15. 

Sample assessment

A design made with parallel lines is sewn on a pocket of a shirt, as shown. What is the value of x?



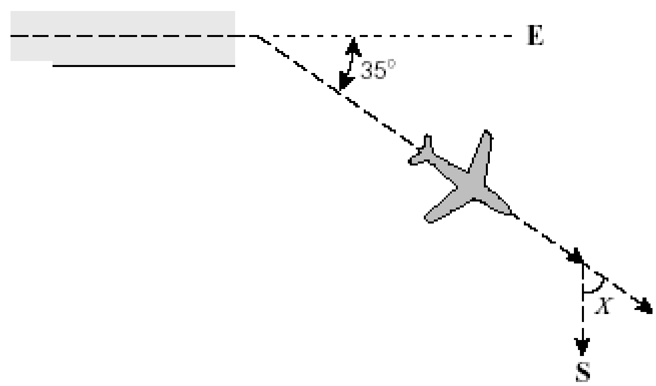
A 50°

B 80°

C 100°

D 130°

An airplane leaves a runway heading due east then turns 35° to the right, as shown in the figure. How much more will the airplane have to turn to be heading due south?



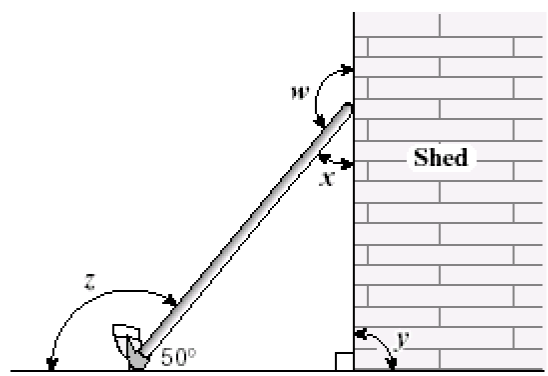
A 10°

B 45°

C 55°

D 65°

A gardener rests his hoe against a shed. The hoe makes a 50° angle with the ground, as shown in the diagram below. Which represents the supplement to the 50° angle?



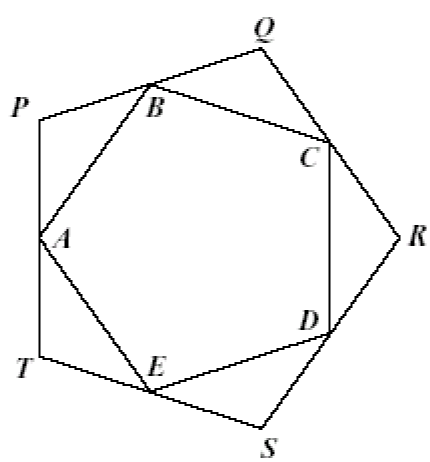
F w

G x

H y

J z

Regular pentagon *ABCDE* is formed by joining the midpoints of the sides of regular pentagon *PQRST*. What is the measure of $\angle PAB$?



F 30°

G 36°

H 60°

J 72°

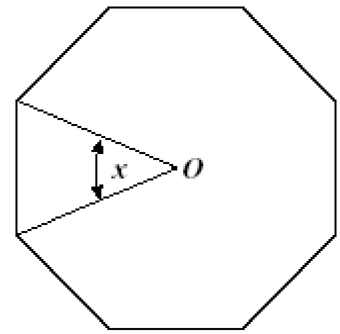
The polygon in the drawing on the right is a regular octagon with O as its center. What is the value of *x*?

A 30°

B 45°

C 60°

D 72°



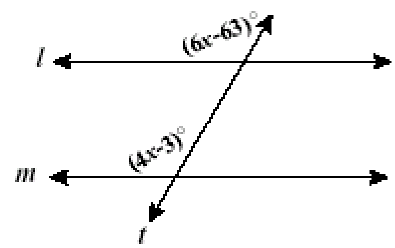
Line *l* is parallel to line *m* when the value of *x* is

F 3

G 12

H 30

J 38



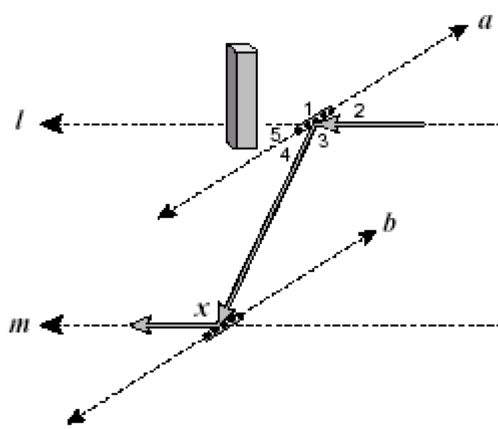
The drawing on the right shows an apparatus designed to divert light rays around an obstacle. Lines *a* and *b* are parallel, and angles 2 and 4 each measure 32°. If lines *l* and *m* need to be parallel, what must be the value of *x*?

F 32°

G 64°

H 116°

J 148°



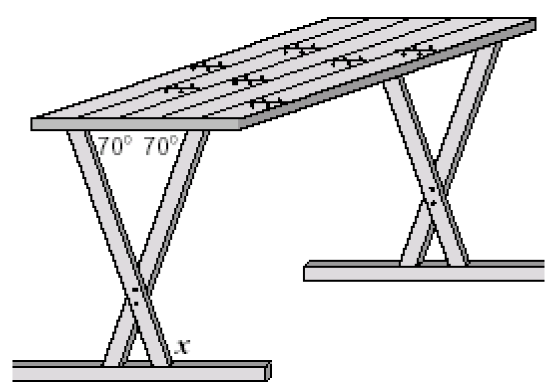
The diagram on the right shows a table under construction. If each leg piece forms a 70° angle with the top of the table, what must be the value of *x* so that the top of the table is parallel to the floor?

A 40°

B 70°

C 90°

D 110°



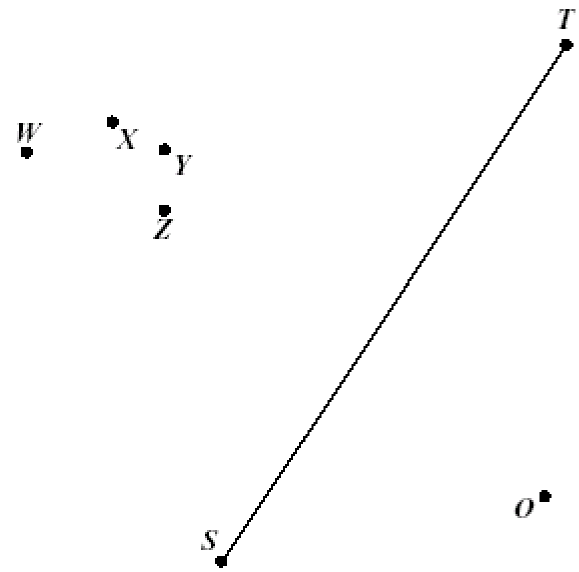
Use your compass and straightedge to construct a line that is perpendicular to $\overrightarrow{ST}$ and passes through point *O*. Which other point lies on this perpendicular?

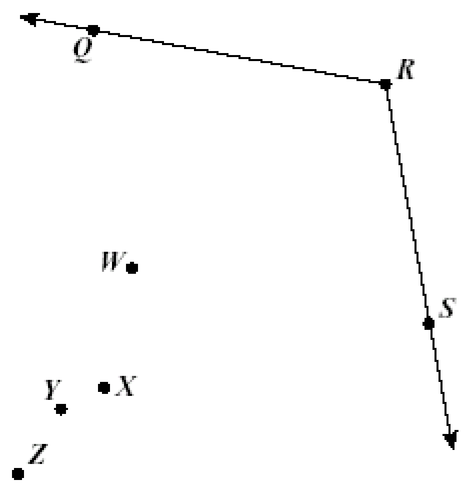
A *W*

B *X*

C *Y*

D *Z*





Use your compass and straightedge to construct the bisector of ∠*QRS*, shown on the left. Which point lies on this bisector?

F *W*

G X

H Y

J Z

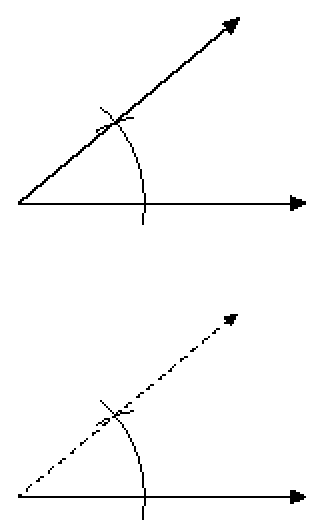
The drawing on the right shows a compass and straightedge construction of

A a line segment congruent to a given line segment

B the bisector of a line segment

C the bisector of a given angle

D an angle congruent to a given angle



What is the slope of the line through (–2, 3) and (1, 1)?

F $\frac{-3}{2}$

G $\frac{-2}{3}$

H $\frac{1}{2}$

J 2

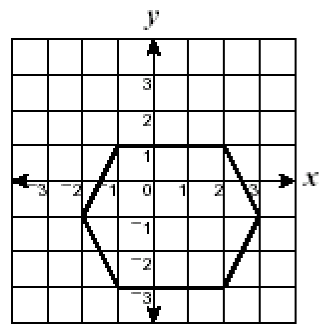
The hexagon in the drawing has a line of symmetry through

A (–1, –3) and (2, 1)

B (1, 1) and (1, –3)

C (2, 3) and (2, –3)

D (–2, –1) and (3, –1)



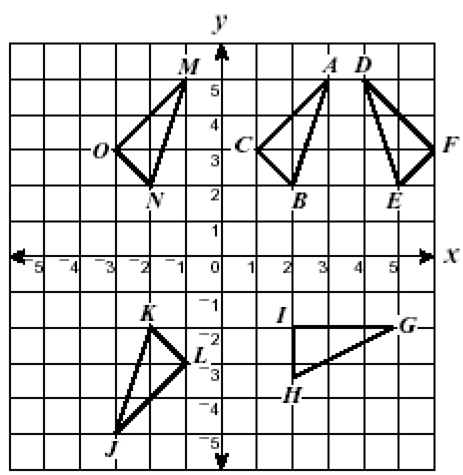
Which triangle is a 180° rotation about the origin of triangle *ABC*?

F Δ*DEF*

G Δ*GHI*

H Δ*JKL*

J Δ*MNO*



Organizing Topic Triangles

Standards of Learning

G.2 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
- a) investigating and using formulas for finding distance, midpoint, and slope;
- b) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
- c) determining whether a figure has been translated, reflected, or rotated.

G.5 The student will
- a) investigate and identify congruence and similarity relationships between triangles; and
- b) prove two triangles are congruent or similar, given information in the form of a figure or statement, using algebraic and coordinate as well as deductive proofs.

G.6 The student, given information concerning the lengths of sides and/or measures of angles, will apply the triangle inequality properties to determine whether a triangle exists and to order sides and angles. These concepts will be considered in the context of practical situations.

G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Investigate and identify congruent figures.	________________
• Define congruent figures.	________________
• Map corresponding parts (angles and sides) of congruent figures onto each other.	________________
• Discuss applications of congruence such as rubber stamps, manufacturing, and patterns.	________________
• Understand the structure of Euclidean geometry:	
° undefined terms	________________
° defined terms	________________
° postulates	________________
° theorems.	________________

- Verify that triangles are congruent using the following postulates:
    - side-angle-side (SAS) ____________
    - angle-side-angle (ASA) ____________
    - side-side-side (SSS) ____________
    - angle-angle-side (AAS) ____________
    - hypotenuse-leg (HL). ____________
- Plan proofs. ____________
- Write deductive arguments as well as coordinate and algebraic demonstrations that triangles are congruent. ____________
- Use the definition of congruent triangles (corresponding parts of congruent triangles are congruent) to plan and write proofs. ____________
- Explore the constraints on the lengths of the sides of a triangle to develop the triangle inequality. ____________
- Use the triangle inequality to determine if three given segment lengths will form a triangle. ____________
- Explore the relationship between the angle measures in triangles and the lengths of the sides opposite those angles. ____________
- Given side lengths in a triangle, identify the angles in order from largest to smallest or vice versa. ____________
- Given angle measures in a triangle, identify the sides in order from largest to smallest or vice versa. ____________
- Use indirect proof (proof by contradiction) to argue that all but one possible case in a given situation is impossible. ____________
- Use properties of proportions to solve practical problems.
- Investigate and identify similar polygons. ____________
- Define similar polygons. ____________
- Use the following postulates to verify that triangles are similar. Deductive arguments as well as algebraic and coordinate methods may be used.
    - angle-angle (AA) ____________
    - side-angle-side (SAS) ____________
    - side-side-side (SSS). ____________
- Find the coordinates of the midpoint of a line segment. ____________
- Solve practical problems involving the Pythagorean Theorem and its converse. Use a calculator to find decimal approximations of solutions. ____________
- Use the Pythagorean Theorem to derive the distance formula. ____________
- Use the distance formula to find the length of line segments when given the coordinates of the endpoints. ____________

- Investigate the side lengths of isosceles right triangles and 30-60-90 triangles. Use inductive reasoning to conjecture about the relationships among the side lengths. ____________________
- Use the properties of special right triangles to solve practical problems. Use a calculator to find decimal approximations of solutions. ____________________
- Define sine, cosine, and tangent as trigonometric ratios in a right triangle. ____________________
- Discuss exact values for trigonometric ratios and decimal approximations. ____________________
- Use right triangle trigonometry to solve right triangles. ____________________
- Use right triangle trigonometry to solve practical problems. Use a calculator to find decimal approximations of solutions. ____________________

Triangles from Midpoints

Organizing topic Triangles
Overview Students use dynamic geometry software to investigate triangles.
Related Standards of Learning G.2, G.5, G.6

Objectives
- The student will find the coordinates of the midpoint of a line segment.
- The student will investigate and identify congruent figures.
- The student will define congruent figures.
- The student will map corresponding parts of congruent figures onto each other.
- The student will discuss applications of congruence.
- The student will use the definition of congruent triangles to plan and write proofs.
- The student will use indirect proof.
- The student will write deductive arguments.
- The student will define and apply the following terms: *undefined terms, defined terms, postulates, theorems.*

Materials needed
- Geometer's Sketchpad™ or other dynamic geometry software package
- A "Triangles from Midpoints" activity sheet for each student

Instructional activity
1. Have students work in pairs to complete the activity sheet.
2. Each student should record his/her own findings.
3. Have students discuss findings with their partners.
4. Discuss findings as a whole group.

Sample assessment
- Have students complete a journal entry summarizing the activity.
- Have students complete the same activity for a different triangle.

Follow-up/extension
- Have students investigate patterns of congruence in the "real world."

Activity Sheet: Triangles from Midpoints

Use Geometer's Sketchpad™ or other dynamic geometry software package to complete the following tasks and questions:

1. Construct triangle ABC, and label the vertices A, B, and C.
2. Highlight each segment, one at a time, and go to Construct, Midpoint. Label the midpoint of segment AC, F. Label the midpoint of CB, D. Label the midpoint of AB, E.
3. Draw triangle DEF.
4. Measure each of the six angles A through F, and record the angle measures here.

5. What do you notice? Can you show that triangles ABC and DEF are similar? If no, what triangles can you show similar? Explain the process for planning your proof.

6. What postulates or theorems would you use to show that your triangles are similar?

7. Write a similarity statement, and map the corresponding angles and sides.

8. Your drawing includes four small triangles. Find the length of each segment, and investigate the relationship among these triangles. Record your findings here.

9. If you highlight the endpoints of the segment you want to measure and then go to Measure, Distance, you'll be given the length of the small segment.

10. Map all congruent angles and segments. Record your findings here.

11. What conclusions can you draw about these four small triangles?

12. How can you prove your conclusion? What postulates or theorems would you use?

13. Assuming this doesn't work for all triangles, find a counterexample.

14. Discuss where it might be helpful to have patterns such as this.

Instructor's Reference Sheet for Triangles from Midpoints

$m\angle CAE = 64.11^\circ$

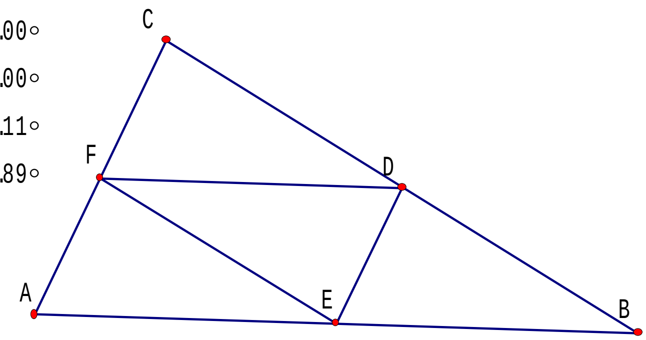
$m\angle ACB = 87.89^\circ$

$m\angle CBA = 28.00^\circ$

$m\angle DFE = 28.00^\circ$

$m\angle EDF = 64.11^\circ$

$m\angle FED = 87.89^\circ$



$m\overline{FD} = 3.65$ cm

$EA = 3.65$ cm

$EB = 3.65$ cm

$m\overline{FE} = 3.29$ cm

$DC = 3.29$ cm

$DB = 3.29$ cm

$ED = 1.72$ cm

$CF = 1.72$ cm

$FA = 1.72$ cm

How Many Triangles

Organizing topic Triangles

Overview Students use manipulatives to investigate the triangle inequality theorem.

Related Standards of Learning G.5, G.6

Objectives

- The student will explore the constraints on the lengths of the sides of a triangle to develop the triangle inequality theorem.
- The student will use the triangle inequality theorem to determine if three given segment lengths will form a triangle.
- The student will explore the relationship between the angle measures in triangles and the lengths of the sides opposite those angles.
- The student will identify the angles of a triangle in order from largest to smallest, when given side lengths for the triangle.
- The student will identify the angles of a triangle in order from smallest to largest, when given side lengths for the triangle.
- The student will identify the sides of a triangle in order from largest to smallest, when given the angle measures of the triangle.
- The student will identify the sides of a triangle in order from smallest to largest, when given the angle measures of the triangle.
- The student will define and apply the following terms: *undefined terms, defined terms, postulates, theorems.*

Materials needed

- A "How Many Triangles" activity sheet for each student
- Paper straw or narrow strip of paper 12 cm long for each student
- Metric ruler
- Marker
- Compass
- Protractor

Instructional activity

1. Have students work in small groups to complete the activity sheet.
2. Each student should record his/her own findings.
3. Have students discuss their findings in their group.
4. Discuss findings as a whole group.

Sample assessment

- Have each group present their findings to the class.
- Have students complete a journal entry summarizing the activity.

Follow-up/extension

- Have students investigate the Pythagorean Triples.
- Have students make generalizations about those lengths.

Activity Sheet: How Many Triangles?

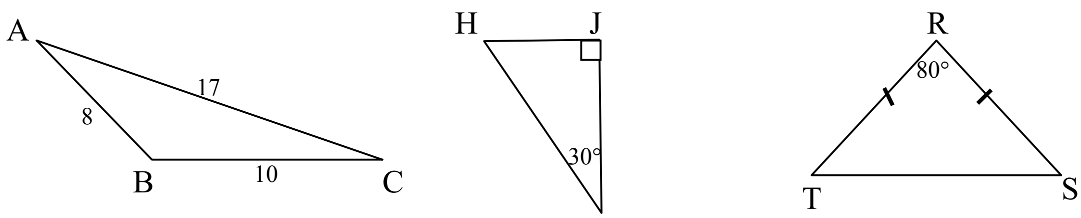
Trim your straw or strip to 12 cm and mark it at 1-cm intervals. Folding only at your marks, make as many different triangles as you can. Complete the table.

Side Lengths	Sketch	Triangle? Yes/No	Measure of each angle in each triangle formed

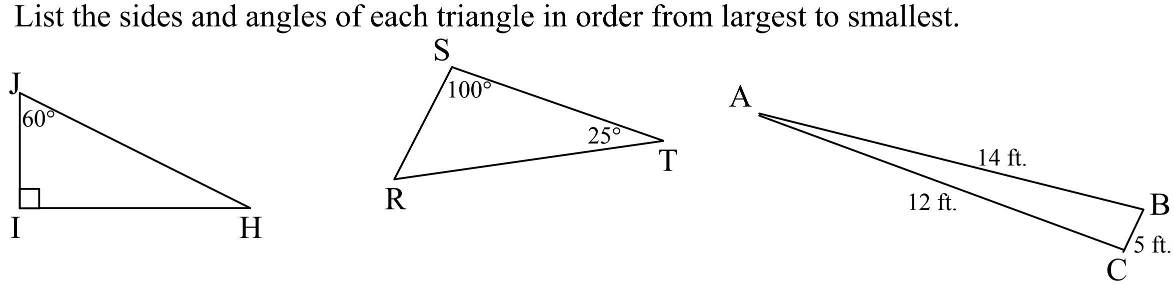
Practice these

1. Determine if the following lengths will form a triangle. Briefly explain each answer.
   a. 5 in., 2 in., 8 in. b. 6 cm, 18 cm, 15 cm c. 5 ft., 6 ft., 9 ft. d. 7 in., 7 in., 8 in.

2. List the sides and angles of each triangle in order from smallest to largest.



3. List the sides and angles of each triangle in order from largest to smallest.



4. ΔABC has side lengths of 1 inch, 1-7/8 inches, and 2-1/8 inches and angle measures of 90°, 28°, and 62°. Which side is opposite each angle?

Geoboard Exploration of the Pythagorean Relationship

Organizing topic Triangles

Overview Students use geoboard or dot paper to investigate the Pythagorean Theorem.

Related Standards of Learning G.2, G.5, G.7

Objectives

- The student will verify the Pythagorean Theorem and its converse, using deductive arguments.
- The student will verify the Pythagorean Theorem and its converse, using algebraic and coordinate methods.
- The student will solve practical problems involving the Pythagorean Theorem and its converse.
- The student will use the Pythagorean Theorem to derive the distance formula.
- The student will use the distance formula to find the length of line segments when given the coordinates of the endpoints.

Materials needed

- Eleven-pin geoboards or dot paper
- Overhead geoboard
- A copy of activity sheets 1, 2, and 3 for each student

Instructional activity

1. Have students complete activity sheet 1 in small groups and record his/her own findings.
2. Have students discuss their findings in their group.
3. Discuss findings as a whole group.
4. On a transparent geoboard on the overhead projector, construct a right triangle in which one leg is horizontal and the other is vertical.
5. Ask a student to construct a square on each leg and then on the hypotenuse of the triangle.
6. Ask students to find the area of each square. It may be difficult for some students to recognize a way to find the area of the square on the hypotenuse, so you may need to assist them.
7. Give students the activity sheet 2, "Geoboard Exploration of the Pythagorean Relationship." Have them fill in the data from the example done by the whole class.
8. Have students work to find several other examples and record them in the chart.
9. Have students present their findings to whole class.
10. Have students complete the remaining activity sheet in small groups and discuss findings as a whole class.

Sample assessment

- Have each group present their findings to class.
- Have students complete a journal entry summarizing the activity.

Follow-up/extension

- Have students investigate the Pythagorean Triples and make generalizations about those lengths.
- Have students find a proof of the Pythagorean Theorem other than the proofs investigated here. Have them present the proof to the class and/or write a journal entry about it.

Activity Sheet 1: Geoboard Exploration of Right Triangles

Make a right triangle on a large geoboard or dot paper. Construct a square on each side of the triangle. Label the shortest side a, the middle side b, and the longest side c. Complete the table.

Length of side a	Length of side b	Length of side c	Area of square on side a	Area of square on side b	Area of square on side c	$a^2 + b^2$

1. Across from what angle do you always find side c, the longest side?

2. What other patterns do you see?

3. Can you state the relationship in words? Using the letters a, b, and c?

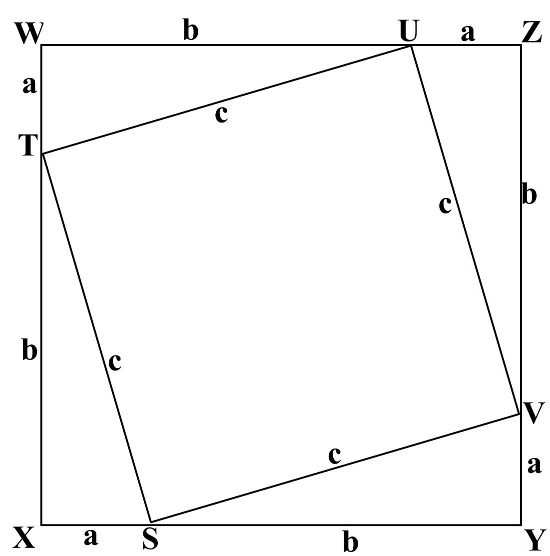
4. When do you think this would be true? Why?

An Algebraic Approach to the Pythagorean Theorem
Fill in expressions for each of the indicated areas.

1. Area of the large square WXYZ = (a + b)2 =
   (a + b)(a + b) = _________________

2. Area of the large square WXYZ =
   area of square STUV + 4(area of triangle XST) =
   ___________________ + ___________________

3. Set the expressions from #1 and #2 equal to each other and simplify. Where have you seen this before? Shade a right triangle in the drawing for which the relationship is true.



Activity Sheet 2 : Geoboard Exploration of the Pythagorean Relationship

Using your geoboard or dot paper, draw different types of triangles. Use a ruler to measure, if necessary. Complete the table.

Length of side a	**Length of side b**	**Length of side c**	**Sketch of Triangle**	**c^2**	**a^2**	**b^2**	**< $c^2 = a^2 + b^2$ >**	**Type of triangle: acute, right, or obtuse**

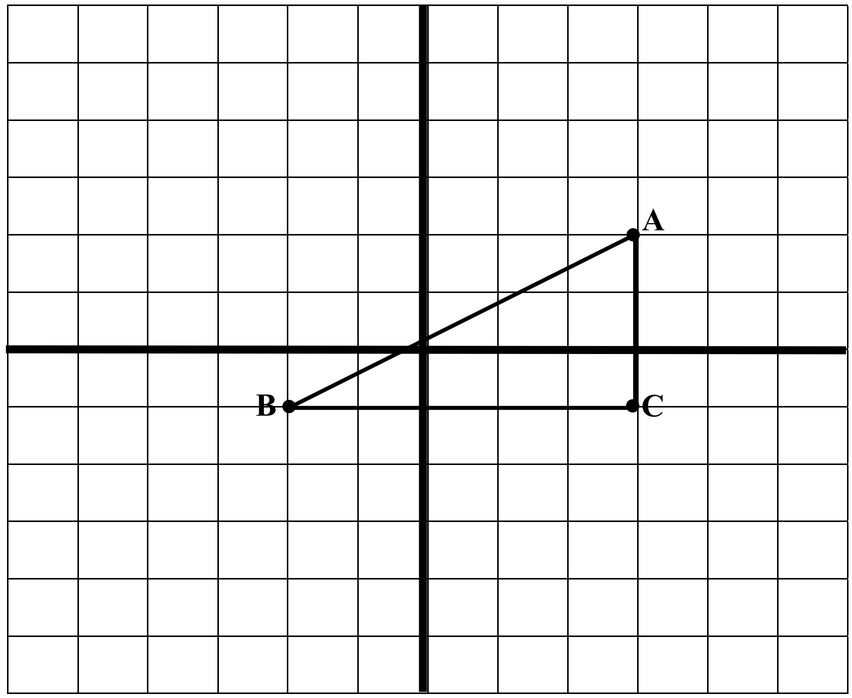
1. What patterns do you see emerging?

2. Can you state the relationship in words? Using the letters a, b, and c?

3. What generalizations can you make?

4. Decide whether the following numbers can represent the side lengths of a triangle. If they can, classify the triangle as right, acute, or obtuse.
   a) 20, 99, 101
   b) 21, 28, 35
   c) 2, 10, 12
   d) 2.2, 5, 5.5
   e) 10, 11, 14

Activity Sheet 3: Deriving the Distance Formula, Using the Pythagorean Theorem



1. Find the length of segment AB, as follows:
   a. Label point B as (x_1, y_1) and label point A as (x_2, y_2).
   b. Construct a square on each leg and then on the hypotenuse of the triangle.
   c. Find the area of each square.
   d. Use the Pythagorean Theorem to set up an algebraic expression.
   e. Solve for the length of the hypotenuse.
2. What is the expression you ended up with?
3. What is the name of this formula?
4. When do we use this formula?
5. Find the length of the requested segment, using the formula you've just found.
   a. AB, if A(3, 2) and B(2, –1)
   b. CD, if C(–1, –3) and D(9, 5)

Special Right Triangles and Right Triangle Trigonometry

Organizing topic	Triangles
Overview	Students use dynamic geometry software to investigate right triangles.
Related Standards of Learning	G.5, G.6, G.7

Objectives

- The student will investigate the side lengths of isosceles right triangles.
- The student will investigate 30-60-90 right triangles.
- The student will use inductive reasoning to conjecture about the relationships among the side lengths.
- The student will use properties of special right triangles to solve practical problems.
- The student will define sine, cosine, and tangent as trigonometric ratios in a right triangle.
- The student will discuss exact values for trigonometric ratios.
- The student will find decimal approximations of solutions.
- The student will define and apply the following terms: *undefined terms, defined terms, postulates, theorems.*

Materials needed

- Geometer's Sketchpad™ or other dynamic geometry software package
- Calculator
- A "Special Right Triangles and Right Triangle Trigonometry" activity sheet for each student

Instructional activity

1. Have students work in pairs to complete the activity sheet.
2. Each student should record his/her own findings.
3. Have students discuss their findings with their partners.
4. Discuss findings as a whole group.

Sample assessment

- Have students complete a journal entry summarizing the activity.
- Have students complete the same activity for a different triangle.

Follow-up/extension

- Have students solve practical problems involving trigonometric ratios.

Sample resources

Mathematics SOL Curriculum Framework
 http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html

SOL Test Blueprints

Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html
Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra
NASA http://spacelink.nasa.gov/.index.html
The Math Forum http://forum.swarthmore.edu/
4teachers http://www.4teachers.org
Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm
Eisenhower National Clearinghouse http://www.enc.org/

Activity Sheet: Special Right Triangles and Right Triangle Trigonometry

30-60-90 right triangles

1. Draw and label a right triangle ABC. Mark the right angle as B.
2. Measure each angle.
3. Adjust segments points A and C, if necessary, to make angle A 60 degrees and angle C 30 degrees. Get as close as possible. Be sure to keep angle B as a 90-degree angle.
4. Measure each segment length.
5. Sketch your diagram here. Show all angle measures and segment lengths. Map each angle to the side opposite the angle. Write the names of angles and sides in order from smallest to largest.

6. What do you notice about the relationship between the angle measures and the side opposite the angles?

7. Can you find a relationship among the sides? Explain here.

8. What is the formal relationship among the sides of a 30-60-90 right triangle?

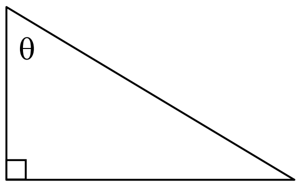
45-45-90 right triangles (Isosceles triangles)

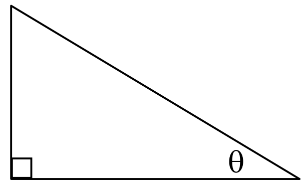
1. Draw and label a right triangle ABC. Mark the right angle as B.
2. Measure each angle.
3. Adjust points A and C so that angles A and C are each 45 degrees. Get as close as possible.
4. Measure each segment length.
5. Sketch your diagram here. Show all angle measures and segment lengths. Correspond each angle with the side opposite the angle. Write the names of angles and sides in order from smallest to largest.

6. Can you find a relationship among the sides? Explain here.

7. What is the formal relationship among the sides of a 45-45-90 right triangle?

Right triangle Trigonometry





1. From the given angle, label the opposite side, hypotenuse, and adjacent side for each right triangle. (Label on the triangle)

2. Define $\sin\theta$ | Define $\cos\theta$ | Define $\tan\theta$

$\sin\theta = \dfrac{\square}{\square}$ $\quad\cos\theta = \dfrac{\square}{\square}$ $\quad\tan\theta = \dfrac{\square}{\square}$

Use Geometer’s Sketchpad™ or other dynamic geometry software package to investigate sine, cosine, and tangent in right triangles.

1. Draw and label a right triangle ABC. Mark the right angle as B.
2. Measure the length of one side, any side, and the length of one angle (not the right angle).
3. Draw your diagram here, and record your measurements on the diagram.

4. From your diagram, label the measured side as opposite, hypotenuse, or adjacent to the angle you’ve measured.
5. Decide which length you would like to find next. Label that side as opposite, hypotenuse, or adjacent to the angle you’ve measured.
6. Using the definitions for sine, cosine, and tangent above, decide which trigonometric ratio uses the side you have and the side you’re looking for. Set up and write down your trigonometric ratio here.

7. Using your calculator, find the length of the segment of interest.

8. Once you have your value, check your work by measuring the segment on Geometer’s Sketchpad.

9. What did you find? Were you correct? Incorrect? Why?

10. Do this same process for the remaining side. Record your work here.

11. Discuss with your partner another method for finding the length of the third side when you have the lengths of two lengths of the triangle.

12. Draw a different triangle, and do this entire process again (steps 1–10).

Instructor's Reference Sheet for Special Right Triangles and Right Triangle Trigonometry

30-60-90 right triangles

$m\angle ABC = 90.00°$

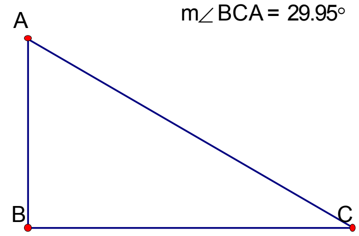
$m\angle CAB = 60.05°$

$m\angle BCA = 29.95°$

$m\overline{AC} = 5.41$ cm

$m\overline{BC} = 4.68$ cm

$m\overline{AB} = 2.70$ cm



45-45-90 right triangles

$m\angle ABC = 90.00°$

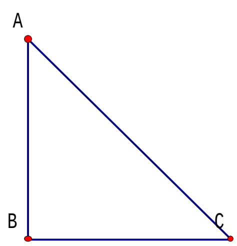
$m\angle ACB = 45.00°$

$m\angle BAC = 45.00°$

$m\overline{AC} = 4.04$ cm

$m\overline{AB} = 2.86$ cm

$m\overline{BC} = 2.86$ cm



Right triangle Trigonometry

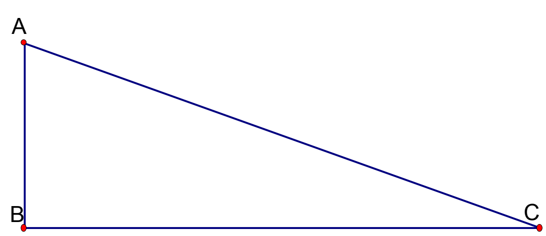
$m\angle ABC = 90.00°$

$m\angle CAB = 70.28°$

$m\overline{AB} = 2.65$ cm

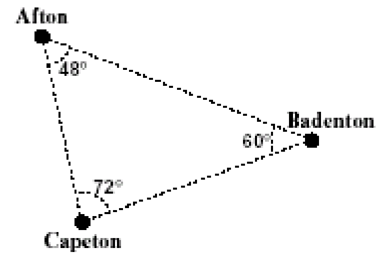
$m\overline{BC} = 7.38$ cm

$m\overline{AC} = 7.84$ cm



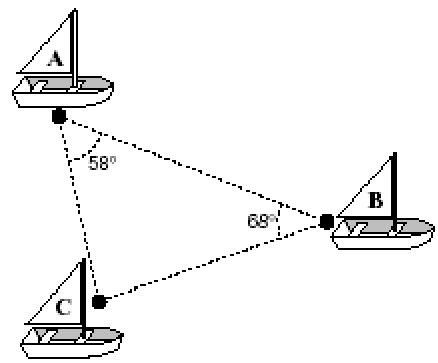
Sample assessment

Three towns form a triangle on a map. The angle formed at the point designating Afton is 48°, at Badenton 60°, and at Capeton 72°. Which lists the distances between towns in order, greatest to least?



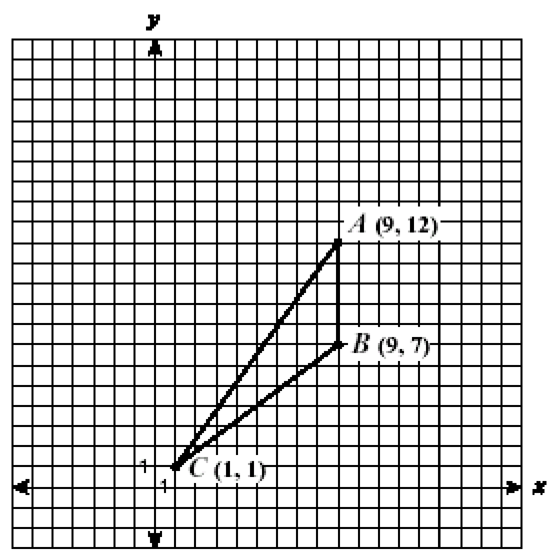
A Afton to Badenton, Badenton to Capeton, Afton to Capeton
B Afton to Badenton, Afton to Capeton, Badenton to Capeton
C Afton to Capeton, Afton to Badenton, Badenton to Capeton
D Badenton to Capeton, Afton to Badenton, Afton to Capeton

Three boats are anchored in a bay. Given the information in the diagram, which of the following statements concerning the distances between the boats is true?

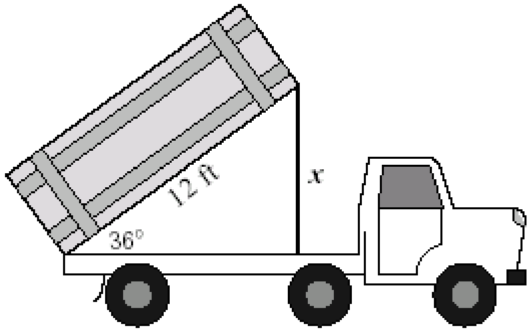


F $AC < AB$
G $BC < AB$
H $AB < AC$
J $AC < BC$

Erica plotted on a graph the 3 towns closest to her house, with town A at (9, 12), town B at (9, 7) and town C at (1, 1). She drew the triangle joining the 3 points. Which lists the angles formed in size, smallest to largest?



A $\angle A$, $\angle B$, $\angle C$
B $\angle B$, $\angle C$, $\angle A$
C $\angle C$, $\angle A$, $\angle B$
D $\angle B$, $\angle A$, $\angle C$

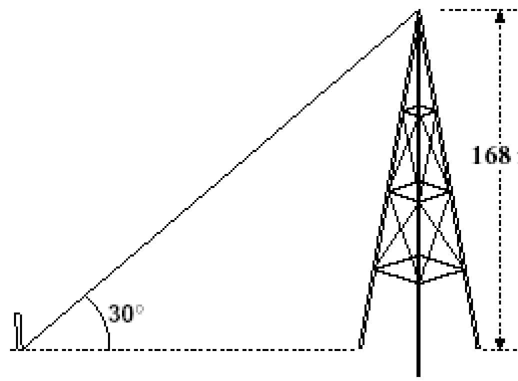


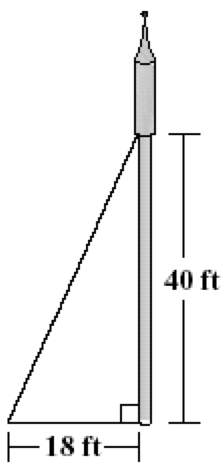
The 12-foot bed of a dump truck loaded with heavy stone must rise to an angle of 36° before the stone will spill out. Approximately how high must the front of the bed rise (x) to unload?

F 6 ft.
G 7 ft.
H 9 ft.
J 10 ft.

The angle from a point on the ground to the top of a 168-foot tower is 30°. About how long is a wire that reaches from the top of the tower to the point on the ground?

A 146 ft.
B 194 ft.
C 291 ft.
D 336 ft.



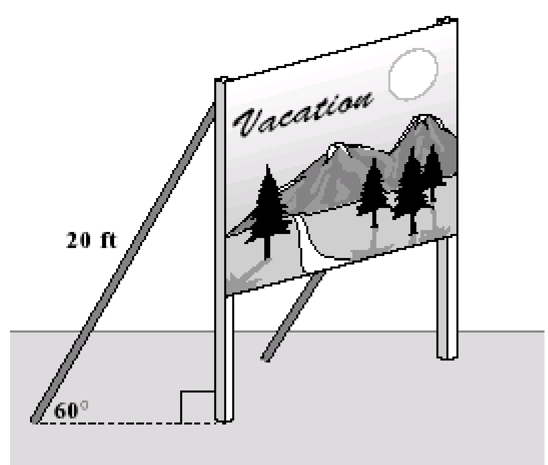


From a point 18 feet from the base of a tower, a wire is stretched to an attachment 40 feet up the tower. To the *nearest* foot, how long is the wire?

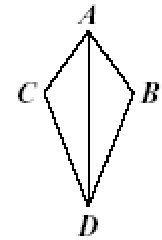
F 58 ft.
G 44 ft.
H 36 ft.
J 29 ft.

A billboard is supported by 20-foot lengths of tubing at an angle of 60°. How far from the base of the billboard is the bottom end of the brace?

A 5 ft.
B 8.7 ft.
C 10 ft.
D 17.3 ft.



Given: $AC = AB$ and $DC = DB$, which could be used to prove $\Delta ABD \cong \Delta ACD$?



F (SSS) If 3 sides of one triangle are congruent to 3 sides of another triangle, then the triangles are congruent.

G (SAS) If 2 sides and the angle between them of one triangle are congruent to 2 sides and the angle between them of another triangle, then the triangles are congruent.

H (ASA) If 2 angles and the side between them of one triangle are congruent to 2 angles and the side between them of another triangle, then the triangles are congruent.

J (AAS) If 2 angles and a side not between them of one triangle are congruent to 2 angles and a side not between them of another triangle, then the triangles are congruent.

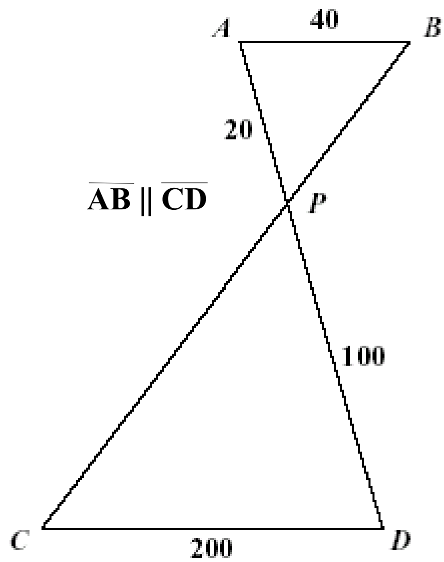
Which relationship is true about ΔAPB and ΔDPC?

F They are congruent.

G They are similar.

H They are equal in area.

J They are equal in perimeter.



Organizing Topic Other Polygons

Standards of Learning

G.2 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
- a) investigating and using formulas for finding distance, midpoint, and slope;
- b) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
- c) determining whether a figure has been translated, reflected, or rotated.

G.8 The student will
- a) investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
- b) prove these properties of quadrilaterals, using algebraic and coordinate methods as well as deductive reasoning; and
- c) use properties of quadrilaterals to solve practical problems.

G.9 The student will use measures of interior and exterior angles of polygons to solve problems. Tessellations and tiling problems will be used to make connections to art, construction, and nature.

G.14 The student will
- a) use proportional reasoning to solve practical problems, given similar geometric objects; and
- b) determine how changes in one dimension of an object affect area and/or volume of the object.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Identify, name, and classify polygons.	______________________
• Examine pre-image and image figures in the coordinate plane and determine whether a translation, reflection, or rotation has occurred.	______________________
• Use inductive reasoning to develop a formula for finding the sum of the measures of the interior angles of a convex polygon and the measure of each interior angle of a regular polygon.	______________________
• Investigate the sum of the measures of the exterior angles of any convex polygon and the measure of each exterior angle of a regular convex polygon.	______________________
• Use tessellations and tiling problems to make connections to art, architecture, construction, and the sciences.	______________________
• Investigate symmetry.	______________________
• Determine if a geometric figure has point symmetry, line symmetry, or no symmetry, and justify the conclusion.	______________________
• Define parallelogram.	______________________

- Use inductive reasoning to make conjectures about the properties of parallelograms. ______________________________
- Prove the properties of parallelograms using deductive arguments as well as algebraic or coordinate methods. ______________________________
- Verify the converses of the properties of parallelograms and use the converses to prove that a quadrilateral is a parallelogram. Coordinate methods may involve using slope to show that lines are parallel or perpendicular. ______________________________
- Identify rhombi, squares, and rectangles as special parallelograms and prove their properties using deductive arguments as well as algebraic and coordinate methods. ______________________________
- Identify trapezoids and isosceles trapezoids. Prove their properties using deductive arguments as well as algebraic and coordinate methods. ______________________________

Polygons

Organizing topic Other Polygons

Overview Students identify, name, and classify, polygons. Students also develop formulas for interior and exterior angles.

Related Standards of Learning G.2, G.9, G.14

Objectives

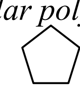
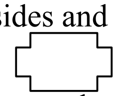
- The student will identify, name, and classify polygons.
- The student will identify translations, reflections, and rotations of polygons.
- The student will use inductive reasoning to develop a formula for finding the sum of the interior angles of a convex polygon.
- The student will use inductive reasoning to develop a formula for finding the measure of each interior angle of a regular polygon.
- The student will investigate the sum of the measures of the exterior angles of any convex polygon.
- The student will find the measure of each exterior angle of a regular polygon.
- The student will use polygons to investigate tessellations and their applications.
- The student will investigate similar polygons.
- The student will use similar polygons to solve practical problems.

Materials needed

- A copy of each of the four activity sheets for each student
- Dynamic geometry software package, such as Geometer's Sketchpad™ (optional to help with activity sheets 3 and 4)
- Patty paper (optional)

Instructional activity

1. Have students work in pairs to complete the activity.
2. Each student should record his/her own findings.
3. Have students discuss their findings with their partners.
4. Discuss findings as a whole group.
5. Explain/review the following:
    - *polygon:* a plane figure formed by three or more segments such that no two sides with a common endpoint are collinear. Each side intersects exactly two other sides, one at each endpoint. A *regular polygon* is one in which all of the sides and angles are equal.

      convex polygon:  *concave polygon:* 
    - *regular tessellation:* a tessellation made up of congruent regular polygons that cover the plane with a pattern in such a way as to leave no region uncovered. Only three regular polygons tessellate in the Euclidean plane: triangles, squares, and hexagons. Since the regular polygons in a tessellation must fill the plane at each vertex, the interior angle must be an exact divisor of 360 degrees.
    - *pre-image:* the original figure, which maps onto the new figure (*image*) in a transformation

- *translation:* the pre-image moved in the coordinate plane horizontally, vertically, or a combination of both
- *rotation:* the pre-image moved about a fixed point
- *reflection:* the pre-image reflected over a line.

Sample assessment

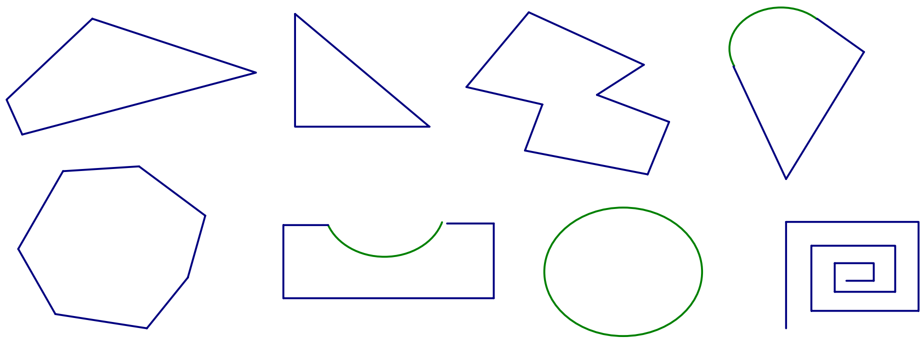
- Have students work in pairs to evaluate strategies.
- Use activity sheets to help assess student understanding.
- Have students complete a journal entry summarizing their investigations.

Follow-up/extension

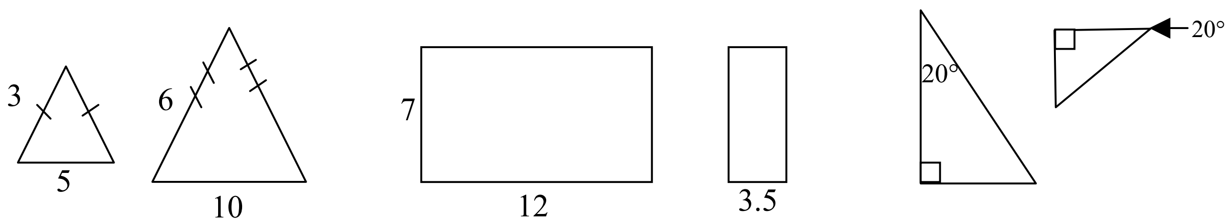
- Have students investigate irregular tessellations.
- Have students create their own tessellations.
- Have students investigate tessellations in art, construction, and science. Students could have a long-term project dealing with these investigations.

Activity Sheet 1: Polygons

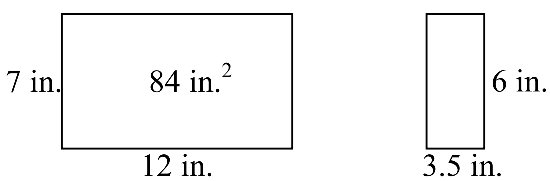
Identify which of the following figures are polygons. Name each polygon by the number of sides. Determine if each polygon is convex or concave.



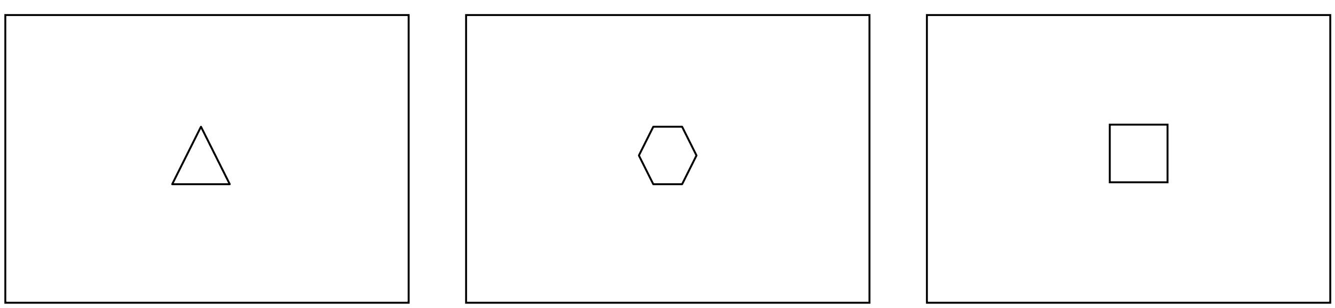
Determine if the following polygons are similar. Justify your decisions.



Given the following polygons, what happens to the area of each polygon when the height is increased by a factor of 1.5? Decreased by a factor of .75? Can you make a generalization about the changes?

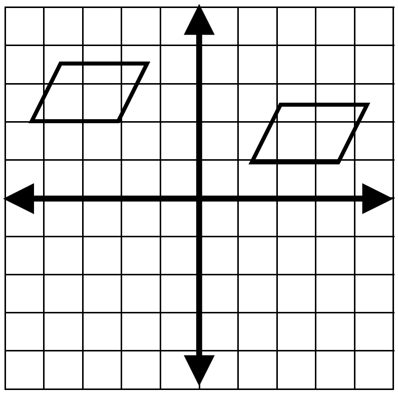


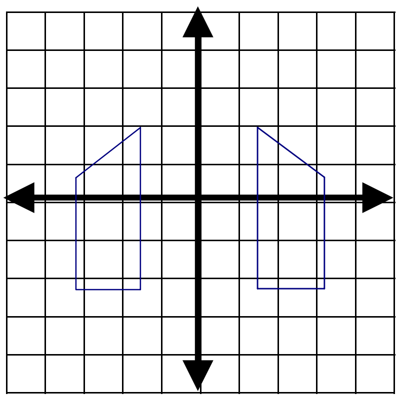
Given the following polygons, demonstrate how each would tessellate a plane. Explain why each does tessellate a plane.

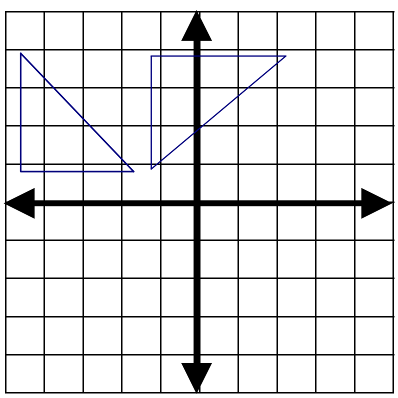


Activity Sheet 2: Polygons

Identify each transformation below as a translation, reflection, or rotation.

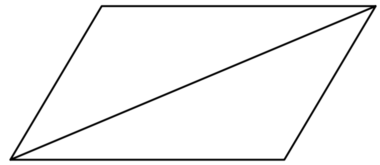






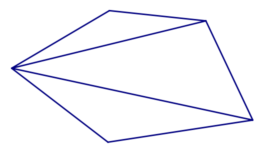
What is the sum of the measures of the interior angles of a triangle?

What is the sum of the measures of the interior angles of a quadrilateral?



From one vertex we draw a diagonal and form two triangles. Since the measures of the angles in each triangle add up to 180°, the quadrilateral has 2 × 180° = 360°.

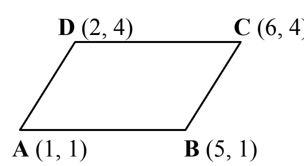
What is the sum of the measures of the interior angles of a pentagon (5 sided polygon)?



From one vertex we draw two diagonals and form three triangles. Since the measures of the angles in each triangle add up to 180°, the quadrilateral has 3 × 180° = 540°

Continue this process by completing the table shown on activity sheet 3. The goal is to figure out a formula that works for any n-gon (any polygon).

Draw on coordinate graph paper the parallelogram shown at right, but translate it six units right and six units up. Label the vertices of the translated image A′, B′, C′, and D′.
Rotate the shape 180° about the origin, and label the vertices.
Rotate the shape 90° about the point A′, and label the vertices.
Reflect the shape through the *x*-axis, and label the vertices.
Reflect the shape through the line $x = -2$, and label the vertices.



Activity Sheet 3: Polygons

Complete the following table.

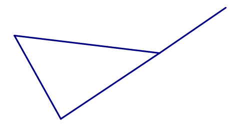
Type of regular polygon	Number of sides	Number of interior ∠s	Sketch	Number of diagonals	Number of ∆s made	Sum of measures of interior ∠s
Triangle						
Quadrilateral						
Pentagon						
Hexagon						
Heptagon						
Octagon						
Nonagon						
Decagon						
Dodecagon						
Icosagon						
n-gon						

Checkpoint

1. Check your formula by examining an octagon (8 sides, n = 8). Did the results agree with your table?
2. If not, check your computations, and try the formulas with a triangle and a quadrilateral where you are sure of what the answers should be.

Activity Sheet 4: Polygons

1. If the sum of the measures of the interior angles of a triangle is 180°, how large is each of the 3 congruent angles in a regular triangle?
2. How large is any exterior angle?
3. What is the sum of the measures of all the exterior angles?



Complete the following table.

Type of regular polygon	Number of interior ∠s	Sum of interior ∠s	Measure of each interior ∠	Measure of each exterior ∠	Sum of measures of exterior ∠s
Triangle					
Quadrilateral					
Pentagon					
Hexagon					
Heptagon					
Octagon					
Nonagon					
Decagon					
Dodecagon					
Icosagon					
n-gon					

Properties of Quadrilaterals

Organizing topic Other Polygons

Overview Students use a dynamic geometry software package to investigate the properties of quadrilaterals.

Related Standards of Learning G.2, G.8

Objectives

- The student will use quadrilaterals to investigate symmetry.
- The student will identify quadrilaterals.
- The student will prove properties of quadrilaterals, using deductive arguments.
- The student will prove that a quadrilateral is a parallelogram.
- The student will define *parallelogram*.
- The student will use properties of quadrilaterals to solve practical problems.
- The student will use inductive reasoning to make conjectures about the properties of parallelograms.

Materials needed

- Dynamic geometry software package, such as Geometer's Sketchpad™
- Patty paper (optional)
- A copy of each of the three activity sheets for each student

Instructional activity

1. Have students work in pairs to complete the activity sheets.
2. Each student should record his/her own findings.
3. Have students discuss their findings with their partners.
4. Discuss findings as a whole group.

Sample assessment

- Have students work in pairs to evaluate strategies.
- Use activity sheets to help assess student understanding.
- Have students complete a journal entry summarizing their investigations.

Follow-up/extension

- Have students complete a Venn Diagram showing the relationships among quadrilaterals.

Sample resources

Mathematics SOL Curriculum Framework
http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html

SOL Test Blueprints

Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html

Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra

NASA http://spacelink.nasa.gov/.index.html
The Math Forum http://forum.swarthmore.edu/
4teachers http://www.4teachers.org
Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm
Eisenhower National Clearinghouse http://www.enc.org/

Activity Sheet 1: Properties of Quadrilaterals

Complete each of the following tasks and questions.

1. Define *quadrilateral*. Make sketches of several types of quadrilaterals. Compare your answers with those of your partner.

2. Define *parallelogram*. Make sketches of several types of parallelograms. Compare your answers those of with your partner.

3. Explain the difference between a quadrilateral and a parallelogram.

4. Use Geometer's Sketchpad™ or similar dynamic geometry software package to draw a parallelogram.
   a. Draw a segment, and label the endpoints A and B.
   b. Draw a point not on the segment, and label the point C.
   c. Highlight segment AB and point C.
   d. Go to Construct, Parallel line.
   e. Draw segment AC.
   f. Highlight segment AC and point B.
   g. Go to Construct, Parallel line.
   h. Label the point where the two lines intersect as D.
   i. You have now formed parallelogram ABCD. Gently move one of the points and notice how the parallelogram changes.
5. Measure the length of each segment and record your findings here.

6. What do you notice? Now, gently move one of the points and notice what happens to the segment lengths. What generalization can you make about the sides of a parallelogram?

7. Measure each angle, and record your findings here.

8. What do you notice? Now, gently move one of the points and notice what happens to the angle measures. What generalization can you make about the angles of a parallelogram?

9. Draw and measure the diagonals. Label their point of intersection as E. Measure the length of segments AE and ED. What do you notice? Measure the length of segments BE and EC. What do you notice?

10. What generalizations can you make about the properties of a parallelogram?

11. A *square* is a parallelogram with four right angles and four congruent sides. How would you prove that a parallelogram is a square? Draw a square, using Geometer's Sketchpad™ or similar dynamic geometry software package, and explain how you prove that it is a square. Record your findings here. Measure all angles, all sides, and all diagonals (and even the angles formed by the diagonals) to help with your reasoning.

12. A *rhombus* is a parallelogram with all sides congruent. How would you prove that a parallelogram is a rhombus? Draw a rhombus using Geometer's Sketchpad™ or similar dynamic geometry software package, and explain how you prove that it is a rhombus. Record your findings here. Measure all angles, all sides, and all diagonals (and even the angles formed by the diagonals) to help with your reasoning.

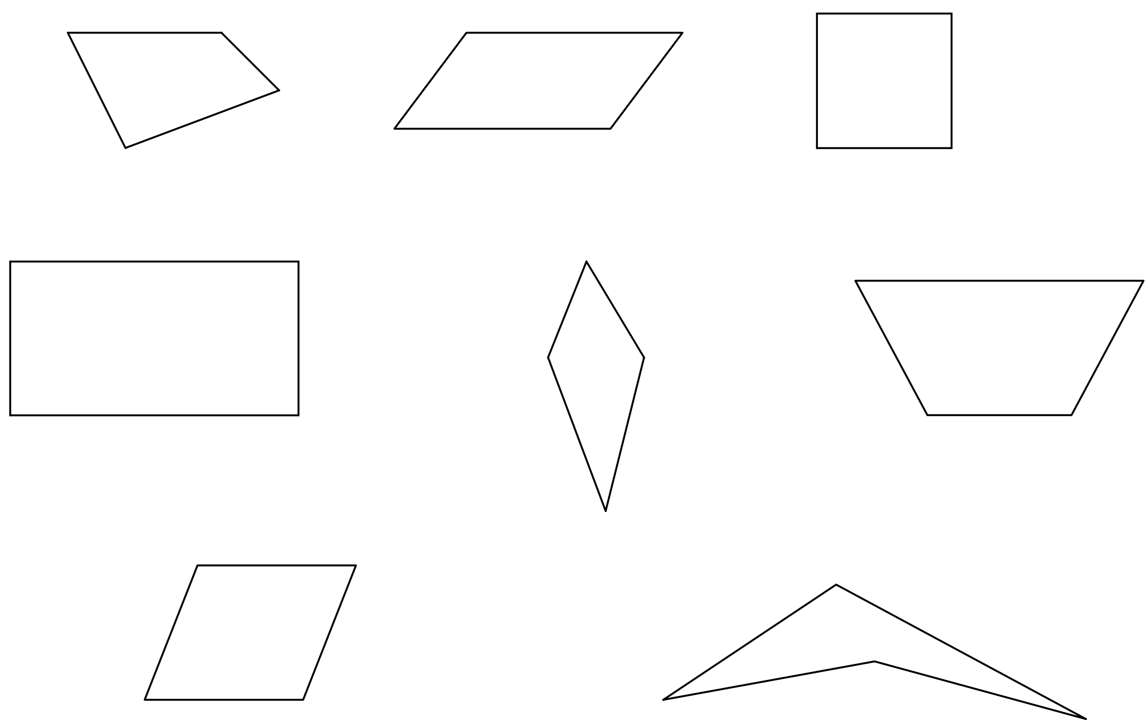
13. A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. Why can't you prove that a trapezoid is not a parallelogram? Draw a trapezoid, using Geometer's Sketchpad™ or similar dynamic geometry software package. Measure all sides and angles. What do you notice? How can you prove that a trapezoid is an isosceles trapezoid? What do you notice about angles, sides, and diagonals?

14. Using Geometer's Sketchpad™ or similar dynamic geometry software package, investigate the properties of the remaining two special quadrilaterals, rectangle and kite. Determine if each is a parallelogram. Be sure to investigate all sides, all angles (even the angles formed by the diagonals), and the diagonals.

15. After investigating the properties of quadrilaterals, parallelograms, rhombi, trapezoids, rectangles, and kites, use your information to identify and label each of the following quadrilaterals, using the list of properties below. More than one quadrilateral may have the stated properties.

This quadrilateral has…

1. four right angles
2. exactly one pair of parallel sides
3. two pair of opposite sides congruent
4. four congruent sides
5. two pair of opposite sides parallel
6. no sides congruent
7. two pair of adjacent sides congruent, but not all sides congruent
8. perpendicular diagonals
9. opposite angles congruent
10. diagonals bisect each other
11. four sides
12. four congruent angles
13. four congruent sides and four congruent angles.



16. Using your knowledge of quadrilaterals, investigate the types of symmetry (point symmetry, line symmetry, or no symmetry) the quadrilaterals shown above might have. Justify your answers.
Point symmetry: When a figure can be mapped onto itself by a rotation of 180°.
Line symmetry: When a figure can be mapped onto itself by a reflection over a line.

Activity Sheet 2: Properties of Quadrilaterals

Complete the following table.

Properties	Quadrilateral	Parallelogram	Rhombus	Square	Rectangle	Trapezoid	Kite
Both pairs of opposite sides must be | |.							
Both pairs of opposite sides must be ≅.							
All sides must be ≅.							
Both pairs of opposite ∠s must be ≅.							
All ∠s must be ≅.							
All ∠s must be right ∠s.							
Diagonals must be ≅.							
Diagonals must bisect each other.							
Diagonals must be ⊥.							
All sides may be ≅.							
Both pairs of opposite ∠s may be ≅.							
All ∠s may be ≅.							
All ∠s may be right ∠s.							
Diagonals may be ≅.							
Diagonals may bisect each other.							
Diagonals may be ⊥.							

Summarize five ways to prove that a quadrilateral is a parallelogram.

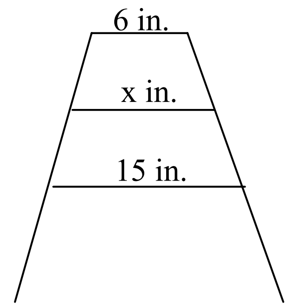
Summarize two ways to prove that a parallelogram is a rectangle.

Summarize two ways to prove that a parallelogram is a rhombus.

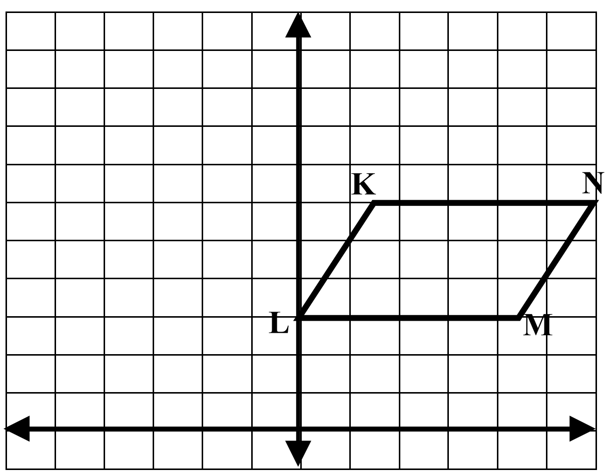
Activity Sheet 3: Properties of Quadrilaterals

Use your knowledge of quadrilaterals to solve the following problems.

1. You want to build a plant stand with three equally spaced circular shelves. You want the top shelf to have a diameter of 6 inches and the bottom shelf to have a diameter of 15 inches. The diagram at the right shows a vertical cross section of the plant stand. What is the diameter of the middle shelf?

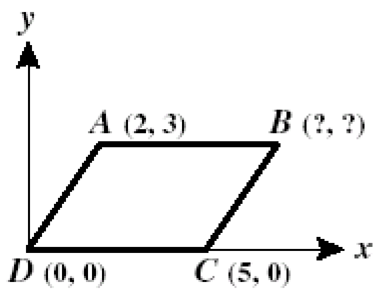


2. Prove the quadrilateral shown below on the grid is a rhombus. Show all work.



K (1.5, 6)
L (0, 3)
M (4.5, 3)
N (6, 6)

Sample assessment



If *ABCD* is a parallelogram, what are the coordinates of *B*?

F (3, 7)

G (5, 5)

H (7, 8)

J (7, 3)

Which of the following quadrilaterals could have diagonals that are congruent but do *not* bisect each other?

A A rhombus

B A rectangle

C A parallelogram

D A trapezoid

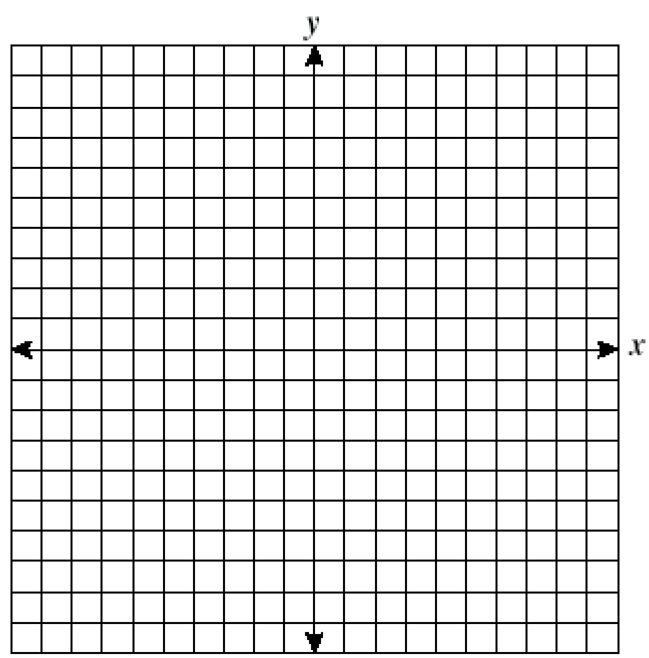
Three vertices of a square have coordinates (5, 1), (2, –2), and (–1, 1). You may want to plot the points on this grid. What are the coordinates of the fourth vertex?

F (–2, 2)

G (2, –2)

H (2, 4)

J (4, 2)



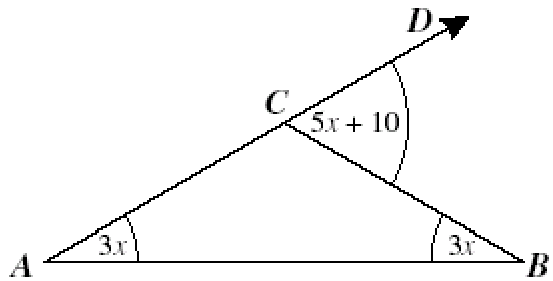
The figure at right has angle measures as shown. What is the measure of $\angle BCD$?

A 120°

B 80°

C 60°

D 30°



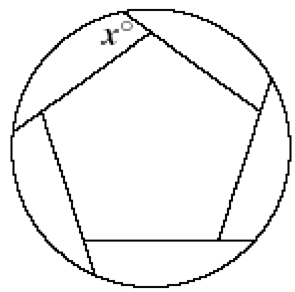
A floor tile is designed with a regular pentagon in the center of the tile with its sides extended. What is the value of x?

F 72°

G 90°

H 110°

J 120°



Each exterior angle of a certain regular polygon measures 30°. How many sides does the polygon have?

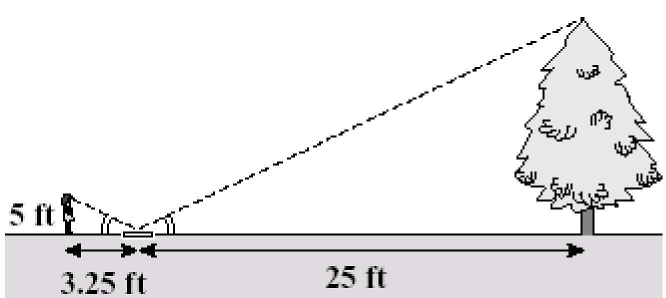
A 6

B 9

C 10

D 12

In order to determine the height of a tree, Maria places a mirror flat on the ground 25 feet from the base. After backing up 3.25 feet, she can just see the top of the tree in the mirror. Maria knows that her eyes are exactly 5 feet above ground level and that the angle between her eyes, the mirror, and the ground is the same as the angle between the treetop, the mirror, and the ground. Which is closest to the height of the tree?



A 24 ft.

B 28 ft. 4 in.

C 38 ft. 6 in.

D 40 ft.

Organizing Topic Circles

Standard of Learning

G.10 The student will investigate and solve practical problems involving circles, using properties of angles, arcs, chords, tangents, and secants. Problems will include finding arc length and the area of a sector, and may be drawn from applications of architecture, art, and construction.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Define the term *circle*.	____________________________
• Differentiate between a circle and a circular region.	____________________________
• Use the vocabulary associated with circles.	____________________________
• Explore and state properties of tangents.	____________________________
• Measure central angles, inscribed angles, and arcs of circles directly and indirectly.	____________________________
• Generalize the relationship between angle measure and arc measure.	____________________________
• Investigate properties of chords and arcs of circles.	____________________________
• Find the area of a sector and the area of a segment of a circle.	____________________________
• Use the properties of angles, arcs, segments, and lines associated with circles to solve practical problems involving circles. Look at applications in art, construction, architecture, and the sciences.	____________________________

Circles and Cylinders

Organizing topic Circles

Overview Students are presented with a problem, which challenges their intuition about measurements of the circumference of a tennis ball can and the height of the can.

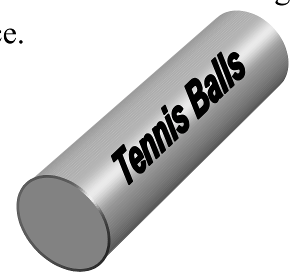
Related Standard of Learning G.10

Objectives

- The student will define *circle.*
- The student will differentiate between a circle and a circular region.
- The student will investigate circumference.

Materials needed

- Tennis ball cans
- String



Instructional activity

1. Have students bring a tennis ball can to class.
2. Have students give their definition of a circle. Discuss as necessary.
3. Have students analyze their tennis ball can. What shapes are the bases of the can? Have students differentiate between a circle and a circular region.
4. Ask students which they think is greater — the height of the can or the circumference of the can. Have them write down their answer and include supporting reasons.
5. Put the students into groups, and have each group present their conjecture to the class.
6. Have students wrap a string around their tennis ball can and compare the length of this string to the height of the can.
7. Have students discuss their findings within their group.
8. Discuss findings as a whole class.

Sample assessment

- Use the group presentations to assess comprehension.
- Have students complete a journal entry summarizing the activity.

Follow-up/extension

- Have students investigate circumference/perimeter and height of other three-dimensional objects.

Homework

- Have students prepare to present findings of their investigation to entire class or small group.

Cake Problem

Organizing topic Circles

Overview Students are presented with a problem that requires them to apply their knowledge of area of circles.

Related Standard of Learning G.10

Objectives

- The student will define a *sector* of a circle.
- The student will define a *segment* of a circle.
- The student will find the area of a sector of a circle.
- The student will find the area of a segment of a circle.
- The student will solve practical problems involving circles.
- The student will use vocabulary associated with circles.
- The student will differentiate between a circle and a circular region.

Materials needed

- An activity sheet for each student
- Calculators

Instructional activity

1. Explain the cake problem, as shown on the activity sheet, to the students without handing out the sheets. Make sure everyone understands the problem.
2. Put a 10-inch diameter circle on the overhead projector to model the cake, and ask a volunteer to make an estimate of the placement of the cut that solves the problem.
3. Hand out the activity sheet, and have students work in small groups to solve the problems. Each student should record the solutions on his/her own activity sheet.
4. Have each group present their solutions to the class. Discuss the variations of solutions.
5. Return to the transparency of the cake, and draw the correct solution. How close was the initial estimate?

Sample assessment

- Use the group presentations to assess comprehension.
- Have students complete a journal entry summarizing the activity.

Follow-up/extension

- Have students investigate how the problem changes if a different number of slices are requested.

Activity Sheet: Cake Problem

You have a cake that is 10 inches in diameter. You expect 12 people to share it, so you cut it into 12 equal pieces (Figure A).

1. Find the area of each slice of cake.

Before you get a chance to serve the cake, 12 more people arrive! So, you decide to cut a concentric circle in the cake so that you will have 24 pieces (Figure B).

2. How far from the center of the cake should the circle cut be made so that all 24 people get the same amount of cake?

3. What is the area of each segment of cake? How much cake will each person receive?

Figure A

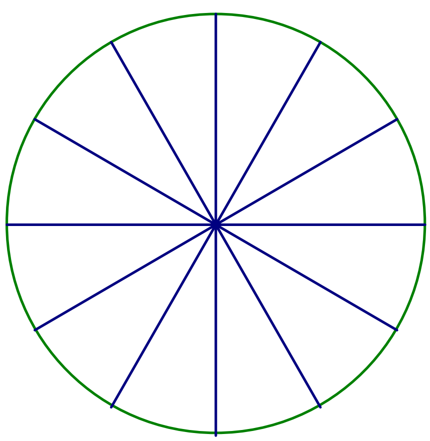
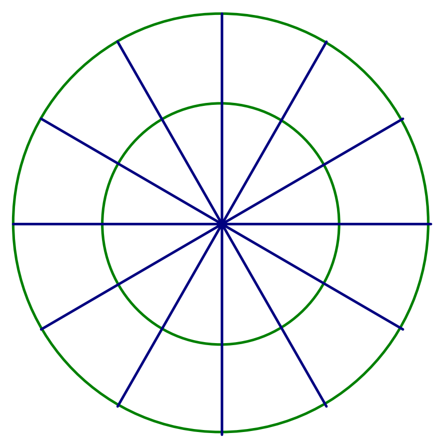


Figure B



Problem Solving with Circles

Organizing topic Circles
Overview Students solve practical problems involving circles.
Related Standard of Learning G.10

Objective
- The student will solve problems involving circles.

Materials needed
- Calculator
- Can of pineapple juice (46-fl.oz)
- Roll of duct tape
- A copy of the "Problem Solving with Circles" activity sheet for each student

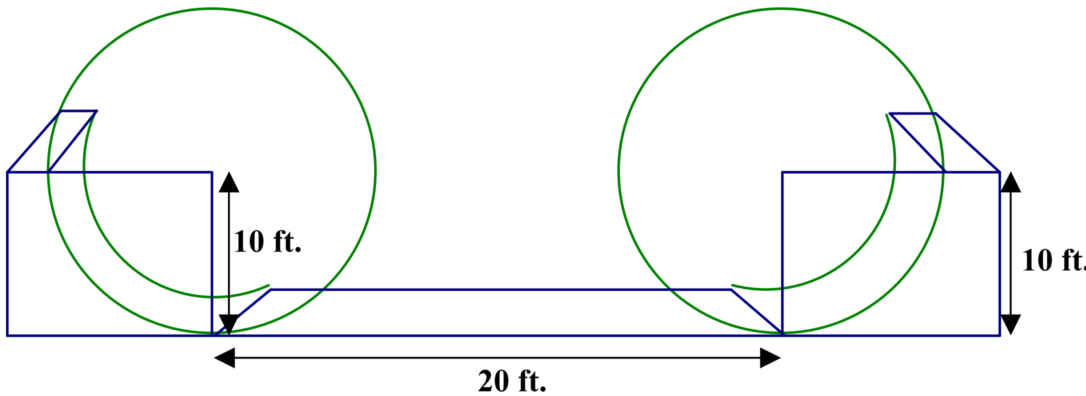
Instructional activity
1. Have students use their calculator to find the lengths of three different ramps that in-line skaters might use.
2. Review the formula for finding the circumference of a circle, if necessary.
3. Discuss how to find the lengths of various arcs of a circle.
4. Have students use the Pythagorean Theorem to find the length of the third ramp.
5. Hold a class discussion on the following questions:
   - What makes one ramp better than another?
   - Which ramp is safest? Why?
   - Which construction is more challenging? Why?

Sample assessment
- Have students work in small groups and present findings to whole class.

Follow-up/extension
- Have students complete "Thinking Cap" on the activity sheet.
- Have students find the steepness of the various ramps and use this data to determine the level of difficulty of each ramp.

Activity Sheet: Problem Solving with Circles

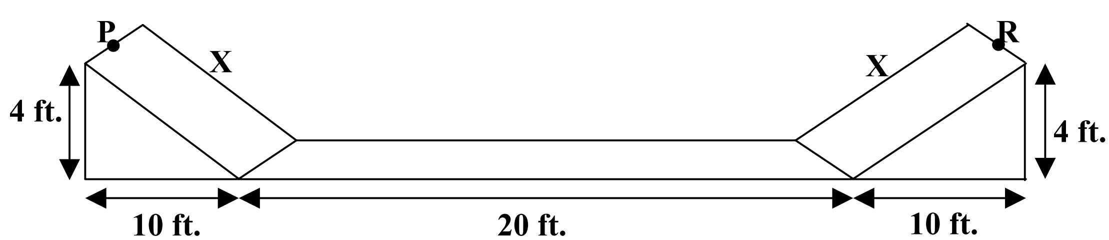
Skateboarding has become a popular sport. The parks department is thinking of constructing ramps in some of the local playgrounds. A “half-pipe” ramp is formed by two quarter-circle ramps, each of which is 10 feet high, plus a flat space 20 feet long between the diameters.



1. Find the distance a skater travels from the top of one ramp to the top of the other.

2. Another launch ramp is formed by 2 arcs, each with a central angle of 60 degrees and a radius of 10 ft. Find the length from the top of one ramp to the top of the other. (Hint: What fractional part of the circle is each arc?)

3. A third ramp has two straight ramps, each of which is 4 ft. high and 10 ft. long, with a flat space of 20 ft. in between. Find the distance a skater travels from the top of one ramp to the top of the other — from point P to point R. (Hint: Use the Pythagorean Theorem.)



Thinking Cap

A school track is formed by two straight segments joined by two semicircles. Each straight segment is l long, and each semicircle is d in length. Write a formula for finding the distance, D, around the track.

Soda Straws

How many straws full of pineapple juice can be taken from a 46-fl.-oz. can of juice that is filled to the top?

1. Measure to find the following:

   Diameter of the can: _________ Height of the can: ___________

   Diameter of the straw: _________ Length of the straw: ___________

2. Guess and check.

3. Create an algebraic expression, and use tables.

Investigation of Secants and Tangents

Organizing topic Circles

Overview Students use a dynamic geometry software package to investigate angles and theorems of circles.

Related Standard of Learning G.10

Objectives

- The student will use vocabulary associated with circles.
- The student will explore and state properties of tangents.
- The student will measure central angles, inscribed angles, and arcs of circles directly and indirectly.
- The student will generalize the relationship between angle measure and arc measure.
- The student will investigate properties of chords and arcs of circles.
- The student will measure angles formed by tangents, chords, and secants directly and indirectly.
- The student will generalize the relationship between angle measure and arc measure.

Materials needed

- Dynamic geometry software package like Geometer's Sketchpad™

Instructional activity

1. Have students complete the activity sheet.
2. Have students discuss their findings after each item is completed. Have them record these findings on their own activity sheet.
3. Discuss findings as a whole class.
4. If you wish, extend the activity to include exploration of segment length and relationships.

Sample assessment

- Have each group present their findings to the class.
- Have students complete a journal entry summarizing the activity.

Follow-up/extension

- Have students use Geometer's Sketchpad™ or similar dynamic geometry software package to investigate the following relationship: When two secant segments are drawn to a circle from an external point, the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

Sample resources

Mathematics SOL Curriculum Framework
 http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html

SOL Test Blueprints

Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html

Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra

NASA http://spacelink.nasa.gov/.index.html
The Math Forum http://forum.swarthmore.edu/
4teachers http://www.4teachers.org
Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm
Eisenhower National Clearinghouse http://www.enc.org/

Activity Sheet: Investigation of Secants and Tangents

Complete the following activities, using a dynamic geometry software package like Geometer's Sketchpad™.

Central Angle

1. Draw a circle with a radius that is about one inch long. Label the center A and a point on the circle B.
2. Draw a second point on the circle, and label it C.
3. Construct segment AB.
4. Construct segment AC.
5. While holding down the shift key, highlight A, B, and C.
6. Go to Measure, Angle. The angle measure will come up on the left of the screen.
7. While holding down the shift key, highlight A, B, and C, in counterclockwise order.
8. Go to Construct, Arc on circle.
9. Go to Display, Show label.
10. Go to Measure, Arc angle.
11. What conclusion can you make?

12. Gently click and hold one of the points on the circle and notice what happens to the measure of the central angle and the measure of its intercepted arc. Describe your findings.

Inscribed Angle

1. Draw a circle with a radius about one inch long. Label the center A and a point on the circle B.
2. Draw a second point on the circle, and label it C.
3. Draw a third point on the circle, and label it D. Place D so that neither segment BD nor CD would go through the center of the circle.
4. Construct segment BD and segment CD.
5. While holding down the shift key, highlight B, D, and C.
6. Go to Measure, Angle. The angle measure will come up in the top left of the screen.
7. While holding down the shift key, highlight A, C, and B (in counterclockwise order).
8. Go to Construct, Arc on circle.
9. Go to Display, Show label.
10. Go to Measure, Arc angle.
11. Go to Measure, Calculate.
12. Type in an expression so that the expression and the measure of angle BDC have the same measure.

13. What expression did you figure out? Write the expression here. What conclusions can you make about angle BDC and arc CB?

14. Draw another point on the circle, and label it E.
15. Construct segment EB and segment EC.
16. Highlight points B, E, and C.
17. Go to measure, Angle. The measure of angle BEC will come up in the top left of the screen.
18. What do you notice about the measure of the angle? What can you conclude?

19. Measure the length of segment EB and segment DC. Highlight the points and move them so that the lengths are congruent. Go through the process of measuring arcs EB and DC. What do you then notice about the measures of arcs EB and DC? What generalization can you make?

Tangents

1. Draw a circle with a radius about one inch long. Label the center A and a point on the circle B.
2. Draw another point on the circle, and label it C.
3. Draw a point not on the circle, and label it D.
4. Highlight points C and D. Go to Construct, Line.
5. Highlight points B and D. Go to Construct, Line.
6. Move points C and B until the line containing each point is tangent.
7. Highlight points C, D, and B. Go to Measure, Angle. The measure of angle CDB will come up in the top left of the screen.
8. After making sure nothing is selected, hold down the shift key while highlighting points A, B, and C, in that order.
9. Go to Construct, Arc on circle.
10. Go to Display, Show label.
11. Go to Measure, Arc angle. The measure of small arc BC will show up on the screen.
12. While holding down the shift key, highlight points A, C, and B, in that order.
13. Go to Construct, Arc on circle.
14. Go to Display, Show label.
15. Go to Measure, Arc angle. The measure of large arc CB will show up on the screen.
16. Go to Measure, Calculate.

17. Find an expression so that the measure of angle CDB equals the measure of the expression. You may need to adjust points C and B.

18. What expression did you figure out? Write the expression here. What conclusions can you make about the measure of angle CDB and large arc CB and small arc BC?

Secants

1. Draw a circle with a radius about one inch long. Label the center A and a point on the circle B.
2. Draw a second point on the circle, and label it C.
3. Draw a point outside the circle, and label it D.
4. Construct segment BD and segment CD. If these segments are not secants, move point D and point C until BD and CD are secants.
5. Mark the point on segment CD where the secant intersects the circle, and label it E.
6. Mark the point on segment BD where the secant intersects the circle, and label it F.
7. While holding down the shift key, highlight points A, F, and E, in that order.
8. Go to Construct, Arc on circle.
9. Go to Display, Show label.
10. Go to Measure, Arc angle. The measure of arc FE will show up in the top left of the screen.
11. While holding down the shift key, highlight points A, C, and B, in that order.
12. Go to Construct, Arc on circle.
13. Go to Display, Show label.
14. Go to Measure, Arc angle. The measure of arc CB will show up in the top left of the screen.
15. While holding down the shift key, highlight points E, D, and F.
16. Go to Measure, Angle. The measure of angle EDF will show up in the top left of the screen.
17. Go to Measure, Calculate. Using arcs FE and CB, figure out an expression so that the measure of angle EDF will be the same measure of the expression.
18. What expression did you figure out? Write the expression here. What conclusions can you make about the measure of angle EDF and arcs FE and CB?

19. Gently move point D. What do you notice? Does it matter how far point D is from the circle?

Circles

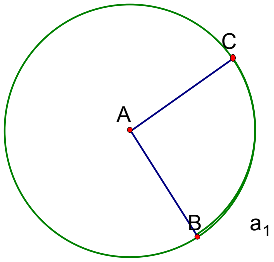
Draw two circles. Investigate the different ways two circles can intersect. Draw the diagrams here.

Instructor's Reference Sheet

Central Angle

m∠CAD = 92.47°

a_1 = 92.47°



Inscribed Angles

m∠BDC = 35.50°

a_1 = 71.01°

$\frac{a_1}{2}$ = 35.50°

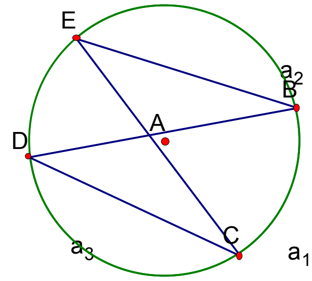
m∠BEC = 35.50°

m$\overline{BE}$ = 3.12 cm

m$\overline{DC}$ = 3.12 cm

a_2 = 116.56°

a_3 = 116.30°



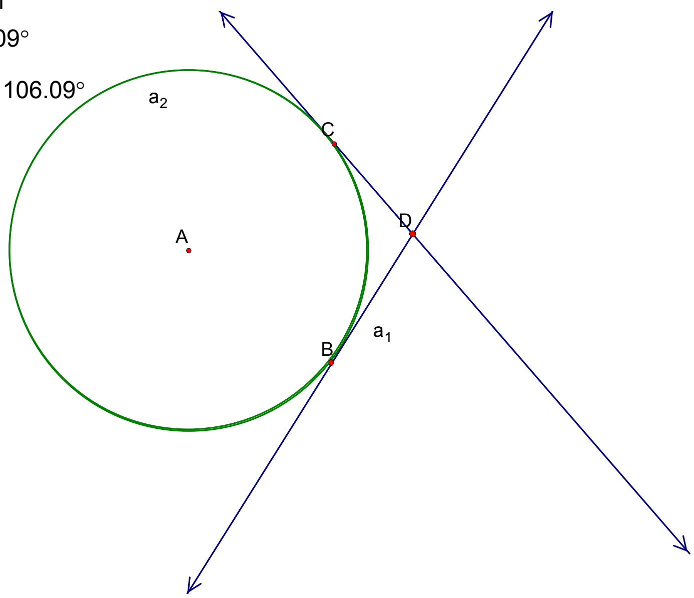
Tangents

m∠CDB = 106.77°

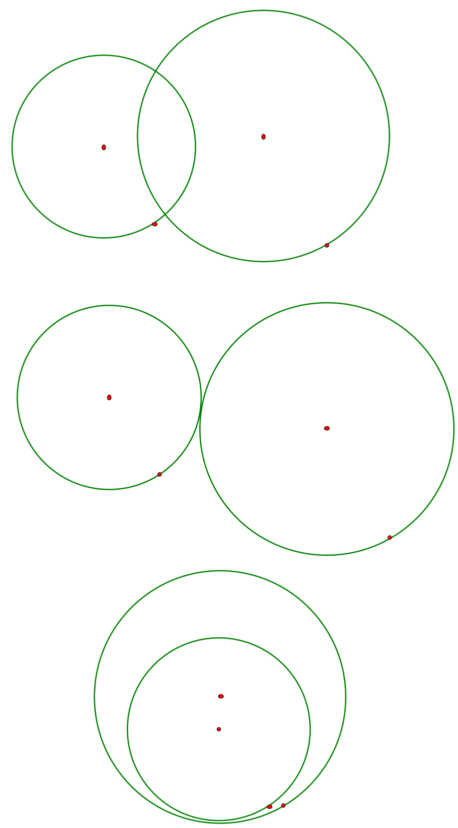
a_1 = 73.91°

a_2 = 286.09°

$\frac{a_2 - a_1}{2}$ = 106.09°



Circle Intersections



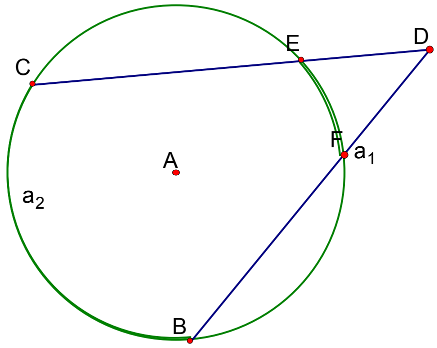
Secants

a_1 = 35.41°

a_2 = 127.06°

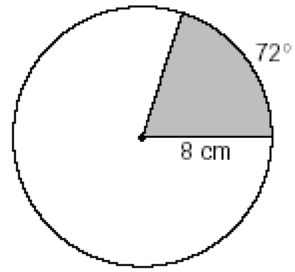
m∠EDF = 45.83°

$\frac{a_2 - a_1}{2}$ = 45.83°



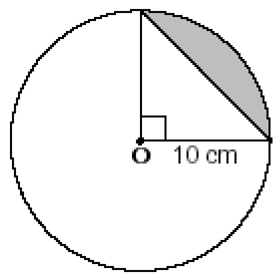
Sample assessment

A circle has a radius of 8 centimeters. The measure of the arc of the shaded section is 72°. Which is *closest* to the area of the shaded section of the circle?



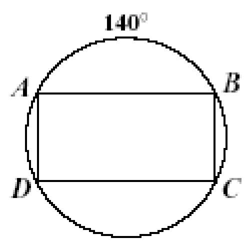
F 10.1 cm^2
G 40.2 cm^2
H 50.3 cm^2
J 160.8 cm^2

Point **O** is the center of the circle. What is the area of the shaded portion of the circle?



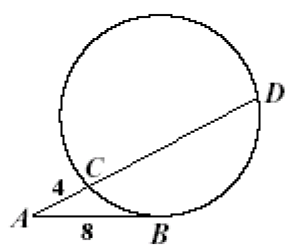
A 28.5 cm^2
B 34.2 cm^2
C 50 cm^2
D 78.5 cm^2

Rectangle *ABCD* is inscribed in a circle. If the measure of arc *AB* is 140°, what is the measure of arc *BC?*



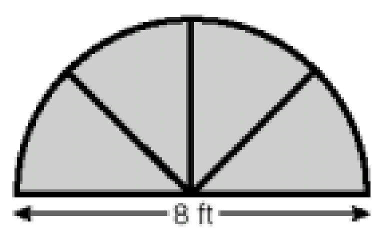
F 30°
G 40°
H 60°
J 80°

In the drawing, *A*, *C*, and *D* are collinear and *AB* is tangent to the circle at *B*. Using the values shown, what is the measure of *CD*?



A 16
B 12
C 10
D 8

This is a sketch of a stained-glass window in the shape of a semicircle. Ignoring the seams, how much glass is needed for the window?



A 4π sq. ft.
B 8π sq. ft.
C 12π sq. ft.
D 16π sq. ft.

Organizing Topic Three-Dimensional Geometry

Standards of Learning

G.12 The student will make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object. Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations.

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

G.14 The student will
a) use proportional reasoning to solve practical problems, given similar geometric objects; and
b) determine how changes in one dimension of an object affect area and/or volume of the object.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
• Use properties of three-dimensional objects to make models.	____________________
• Make a model of a three-dimensional figure from a two-dimensional drawing.	____________________
• Make a two-dimensional representation of a three-dimensional object.	____________________
• Use scale drawings, perspective drawings, blueprints, or computer drawings as models of three-dimensional objects to solve problems.	____________________
• Identify a three-dimensional object from different positions such as the top view, side view, and front view.	____________________
• Use the appropriate formulas to find the surface area of cylinders, prisms, pyramids, cones, and spheres.	____________________
• Use the appropriate formulas to calculate the volume of cylinders, prisms, pyramids, cones, and spheres.	____________________
• Solve practical problems involving surface area and volume of cylinders, prisms, pyramids, cones, and spheres as well as combinations of three-dimensional figures.	____________________
• Use proportions to compare surface area and volumes of three-dimensional geometric figures.	____________________
• Describe how a change in one measure affects other measures of an object.	____________________
• Solve practical problems involving similar objects.	____________________

The Globe Theater

Organizing topic Three-Dimensional Geometry

Overview Students investigate the geometric properties associated with the shape of the Globe Theater and use these properties to draw and/or construct scale models.

Related Standard of Learning G.12

Objectives

- The student will identify geometric properties associated with the shape of the Globe Theater.
- The student will investigate the angles of a regular icosagon.
- The student will draw and/or build a scale model of the Globe Theater.
- The student will use scale drawings to solve practical problems related to the Globe Theater.
- The student will solve practical problems involving similar objects.

Materials needed

- Compass
- Straightedge
- Graph paper
- Calculators
- Popsicle sticks (optional)

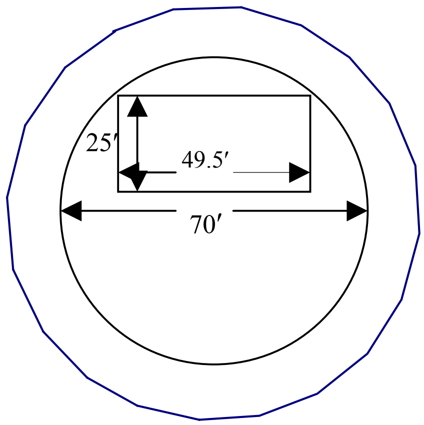
Instructional activity

The Globe Theater opened in London in 1599 and was the site of performances of many plays written by William Shakespeare. The original Globe no longer exists, and historical records of its measurements disagree slightly. Specific dimensions of the Globe Theater, as it is generally understood today, can be found at the Shakespeare Globe Centre site www.sgc.umd.edu/model.htm.

1. When viewed from above, the basic shape of the Globe Theater is a regular icosagon (a 20-sided polygon). Have students investigate the angles of a regular icosagon. Students can draw or construct this polyhedron, using a variety of materials such as popsicle sticks or compass and straightedge.
2. The Globe Theater could seat several hundred people in sheltered levels surrounding an approximately circular inner region. About 700 additional people could watch the plays in this inner region or field, which was located around and in front of the stage. This inner region was 70 feet in diameter, and the stage that jutted into this field was 49.5 feet × 25 feet. If the region were filled to capacity, about how many square feet of space in the field would each person occupy? Have the class mark on the floor with masking tape contiguous squares that comprise this size so that students can get a sense of how closely the audience would have to crowd together in the field for a performance.

Teacher's Notes

- Each exterior angle is 18 degrees; each interior angle is 162 degrees. Students should notice that there are many different kinds of icosagons, but that the regular icosagon has some unique properties.
- The area of this inner region can be modeled using a circle.



- The approximate audience region is the area of the inner circle minus the area of the rectangular stage (about 2,611 square feet). Given a full capacity of 700 people, each person would occupy about 3.7 square feet, or a "region" of about 2 feet on a side ($\sqrt{3.7} \approx 1.9$).
- An icosagonal region instead of a circular model can be used to obtain closer estimates.

Sample assessment

- Assess student understanding, using scale drawing, area problem, and ability to discuss the activity.

Follow-up/extension

- If this is completed as a class activity, have students build a scale model of The Globe Theater.
- Have students also read a play by William Shakespeare and either write a summary or actually act parts of the play.

Gulliver's Travels *and Proportional Reasoning*

Organizing topic Three-Dimensional Geometry

Overview Students use Jonathan Swift's *Gulliver's Travels* to investigate proportional reasoning.

Related Standards of Learning G.13, G.14

Objectives

- The student will read an excerpt from *Gulliver's Travels*.
- The student will use objects found in Gulliver's pockets to estimate the size of the Lilliputians.
- The student will use proportions to compare surface areas and volumes of three-dimensional figures.
- The student will investigate the relationship between changes of side length/diameter and surface area and volume.

Materials needed

- A copy of the excerpt from *Gulliver's Travels* for each student
- Calculators
- Objects like those in Gulliver's pockets (handkerchief, comb, folded and tied paper, snuff box, pocket watch, knife, and other objects described)
- Unit cubes

Instructional activity

1. Have the students read the excerpt from *Gulliver's Travels*. Take time to discuss the objects that are described there. Help students identify at least five of the objects described.
2. Using the objects like those found in Gulliver's pockets, have the students estimate the size of the Lilliputians. Have the students measure the comb, handkerchief, and other objects, and help them to investigate the proportional relationships.
3. At a point in the story before that in the excerpt, the Lilliputians feed Gulliver and are amazed by the amount of food he consumes. This is related to his volume and overall size as much as to his height. While the volume of a human is difficult to calculate, the volume of cubes and spheres is not. Use these shapes to investigate the relationship between changes of side length/diameter and surface area and volume.
4. Begin investigations with a cube. Using unit cubes, build at least three cubes with side lengths 1, 2, and 3. Use these models to review the formulas for surface area and volume in terms of side length. ($V = l^3$, $SA = 6l^2$)
5. For 10 to12 cubes of steadily increasing side length, have students enter three lists of data into the calculator: side length, surface area, and volume. The lists can be entered numerically or through formulas, depending on student facility with the calculations and the calculator.
6. Have the students use the graphing function of the calculator to plot length vs. surface area and length vs. volume, being careful to use different symbols for each plot.
7. Review the formulas for calculating surface area and volume of spheres ($V = (4/3)\pi r^3$, $SA = 4\pi r^2$). Have the students enter data for 10 to12 spheres of increasing diameter, and graph the relationships.

8. Discuss the data lists and graphs, noting the increasing rate of change for area/surface area and volume when compared to a steady rate of change for length.
9. Students should recognize the exponential growth of area and volume measures when compared to the linear growth of length/diameter.

Sample assessment

- Have students complete a journal entry describing the activity and summarizing conclusions. Have them describe how they might find the volume of a human.

Follow-up/extension

- Have students apply this information to such situations as
    - unit conversion (e.g., If there are 100 centimeters in a meter, how many square centimeters are there in a square meter?)
    - the estimation of surface area or volume, given length/diameter
    - the interpretation of scale drawings.

Excerpt from *Gulliver's Travels*

By Jonathan Swift

Gulliver's ship was caught in a storm. He swam to safety on a mysterious island called Lilliput. He was captured and bound, take to the Emperor's castle, and presented to the Emperor and Empress. When he was released from his rope bindings, the Lilliputians wanted to ensure that he had no weapons. This is the inventory of the contents of his pockets. (From part I, chapter II)

Imprimis. In the right coat-pocket of the Great Man-Mountain (the Lilliputian's name for Gulliver) after the strictest search, we found only one great piece of course cloth, large enough to be a foot-cloth for your Majesty's chief room of state. In the left pocket, we saw a huge silver chest, with a cover of the same metal, which we, the searchers, were not able to lift. We desired it should be opened, and one of us, stepping into it, found himself up to the midleg in a sort of dust, some part whereof, flying up to our faces, set us both a sneezing for several times together. In his right waistcoat-pocket, we found a prodigious bundle of white thin substances, folded one over another, about the bigness of three men, tied with a strong cable and marked with black figures; which we humbly conceive to be writings, every letter almost half as large as the palm of our hand. In the left, there was a sort of engine (a term used for any mechanical device), from the back of which were extended twenty long poles, resembling the palisades before your Majesty's court; wherewith we conjecture the Man-Mountain combs his head, for we did not always trouble him with questions, because we found it a great difficulty to make him understand us. In the large pocket on the right side of his middle cover (so I translate the word ranfu-lo, by which they meant breeches) we saw a hollow pillar of iron, about the length of a man, fastened to a strong piece of timber, larger than the pillar; and upon one side of the pillar were huge pieces of iron sticking out, cut into strange figures, which we know not what to make of. In the left pocket, another engine of the same kind. In the smaller pocket on the right side, were several round flat pieces of white and red metal, of different bulk; some of the white, which seemed to be silver, were so large and heavy, that my comrade and I could hardly lift them. In the left pocket were two black pillars, irregularly shaped: we could not, without difficulty, reach the top of them as we stood at the bottom of his pocket. One of them was covered, and seemed all of a piece; but at the upper end of the other, there appeared a white round substance, about twice the bigness of our heads. Within each of these were enclosed a prodigious plate of steel; which, by our orders, we obliged him to show us, because we apprehended they might be dangerous engines. He took them out of their cases, and told us, that in his own country his practice was to shave his beard with one of these, and to cut his meat with the other. There were two pockets which we could not enter: these he called his fobs; there were two large slits cut into the top of his middle cover, but squeezed closed by the pressure of his belly. Out of the right fob hung a great silver chain, with a wonderful kind of engine at the bottom. We directed him to draw out whatever was at the end of that chain; which appeared to be a globe, half silver, and half of some transparent metal: for on the transparent side we saw strange figures circularly drawn, and thought we could touch them, until we found our fingers stopped with that lucid substance. He put this engine to our ears, which made an incessant noise like that of a watermill. And we conjecture it is either some unknown animal, or the god that he worships; but we are more inclined to the latter opinion, because he assured us (if we understood him right, for he expressed himself very imperfectly), that he seldom did anything without consulting it. He called it his oracle, and said that it pointed out the time for every action of his life. From the left fob he took out a net almost large enough for a fisherman, but contrived to open and shut like a purse, and served him for the same use: we found therein several massy pieces of yellow metal, which, if they be of real gold, must be of immense value.

Having thus, in obedience to your Majesty's commands, diligently searched all his pockets, we observed a girdle about his waist made of the hide of some prodigious animal; from which, on the left side, hung a sword of the length of five men, and on the right, a bag or pouch divided into two cells, each cell capable of holding three of your Majesty's subjects. In one of these cells were several gloves or balls of a most ponderous metal, about the bigness of our heads, and required a strong hand to lift them: the other cell contained a heap of certain black grains, but of no great bulk or weight, for we could hold above fifty of them in the palms of our hands.

This is an exact inventory of what we found about the body of the Man-Mountain, who used us with great civility, and due respect to your Majesty's commission. Signed and sealed on the fourth day of the eighty-ninth moon of you Majesty's auspicious reign.

CLEFREN FRELOCK, MARSI FRELOCK

Introduction to the Soma Cube: Constructing the Soma Pieces

Organizing topic	Three-Dimensional Geometry
Overview	Students build, draw, and analyze Soma Pieces and apply that knowledge to investigate surface area and volume.
Related Standard of Learning	G.12

Objectives

- The student will build the seven Soma Cube pieces.
- The student will draw the seven Soma Cube pieces.
- The student will make a two-dimensional representation of a three-dimensional object.
- The student will use properties of three-dimensional objects to make models.
- The student will identify a three-dimensional object from different positions, such as top view, side view, and front view.
- The student will discover the surface area of each of the seven pieces.
- The student will discover the volume of each of the seven pieces.

Materials needed

- Twenty-seven one-inch wooden cubes, sugar cubes, or Snap cubes for each student
- Glue
- Permanent markers
- A "Soma Cube" handout for each student

Instructional activity

The students need to know the basic history of the Soma Cube. The Soma Cube was invented in Denmark by Piet Hein. He was listening to a lecture on quantum physics when the speaker talked about slicing up space into cubes. Hein then thought about all the irregular shapes that could be formed by combining no more than four cubes all the same size and joined at their faces. In his head he figured out what these would be and that it would take 27 cubes to build them all. From there he showed that the pieces could form a 3´3´3 cube. In fact, there are 1,105,920 different ways to assemble the cube.

1. Give 27 cubes to each student, or small group of students, if materials are limited.
2. Introduce the students to the following vocabulary: *tricube* — solid made of three unit cubes joined face-to-face, and *tetracube* — solid made of four unit cubes joined face-to-face.
3. Present students with the following problem: How many different ways can you join three cubes face-to-face?
4. Explain to students that if a tricube can be flipped, reflected, or rotated to be made to look like another tricube, then it is not considered different and is, therefore, not counted. Students should find that there are only two different tricubes, but for this activity, students will only use the non-rectangular tricube. Have students glue (or snap) the cubes together and number the completed pieces according to the Instructor Reference Sheet.
5. Present students with the following problem: How many different ways can you join four cubes face-to-face?

6. Following the same process as with the tricubes, help them find all six possible non-rectangular tetracubes.
7. Discuss the nature of the pieces:
   - Are any of the pieces reflections of each other? (pieces 5 and 6)
   - Which pieces can be placed so that they are only one unit high? (pieces 1, 2, 3, 4)
   - Which pieces must occupy space that is two units high? (pieces 5, 6, 7)
   - Which piece has a line of symmetry on a given face? (piece 3)
8. Have students draw diagrams of their pieces first on blank paper and then on isometric dot paper. Demonstrate on the overhead projector how to draw a single unit cube on isometric dot paper. (The easiest way is by drawing a Y in the center and then circumscribing a hexagon around it.)
9. Discuss:
   - How many dimensions are represented in the Soma Cube?
   - How many dimensions are represented in the solution sheet?
   - What are some strategies that might help in transferring the three-dimensional information onto the two-dimensional representation?
10. Review the concept of volume by identifying one cube as the unit cube. Explain the face of the unit cube as the unit of area. Have students find the surface area and volume of each of the numbered solids 1 through 7 and complete the following table.

Surface Area and Volume of the Soma Pieces

Piece #	Surface Area	Volume
1		
2		
3		
4		
5		
6		
7		

11. Discuss student findings:
    - What is the surface area of each of the solids?
    - How can you tell the surface area of a solid by only studying the picture and not actually building it?
    - What is the volume of each of the solids?
    - How can you tell the volume of a solid by only studying the picture and not actually building it?
    - Did the pieces with the same volume have the same surface area?
    - Did the pieces with the same surface area have the same volume?

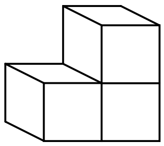
Sample assessment

- Assess student understanding by using the completed surface area and volume table and the diagrams drawn by students.

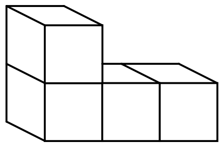
Follow-up/extension

- Have students build and sketch all pentacubes that can be made with five cubes each.

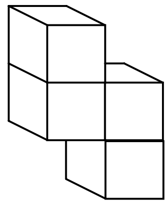
Constructing the Soma Pieces: Instructor's Reference Sheet



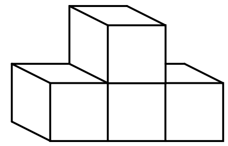
1. Tricube



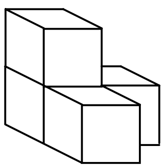
2. Tetracube A



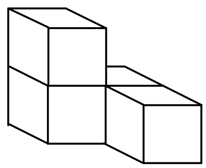
3. Tetracube B



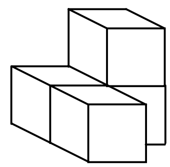
4. Tetracube C



5. Tetracube D

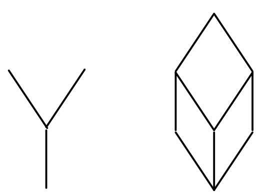


6. Tetracube E

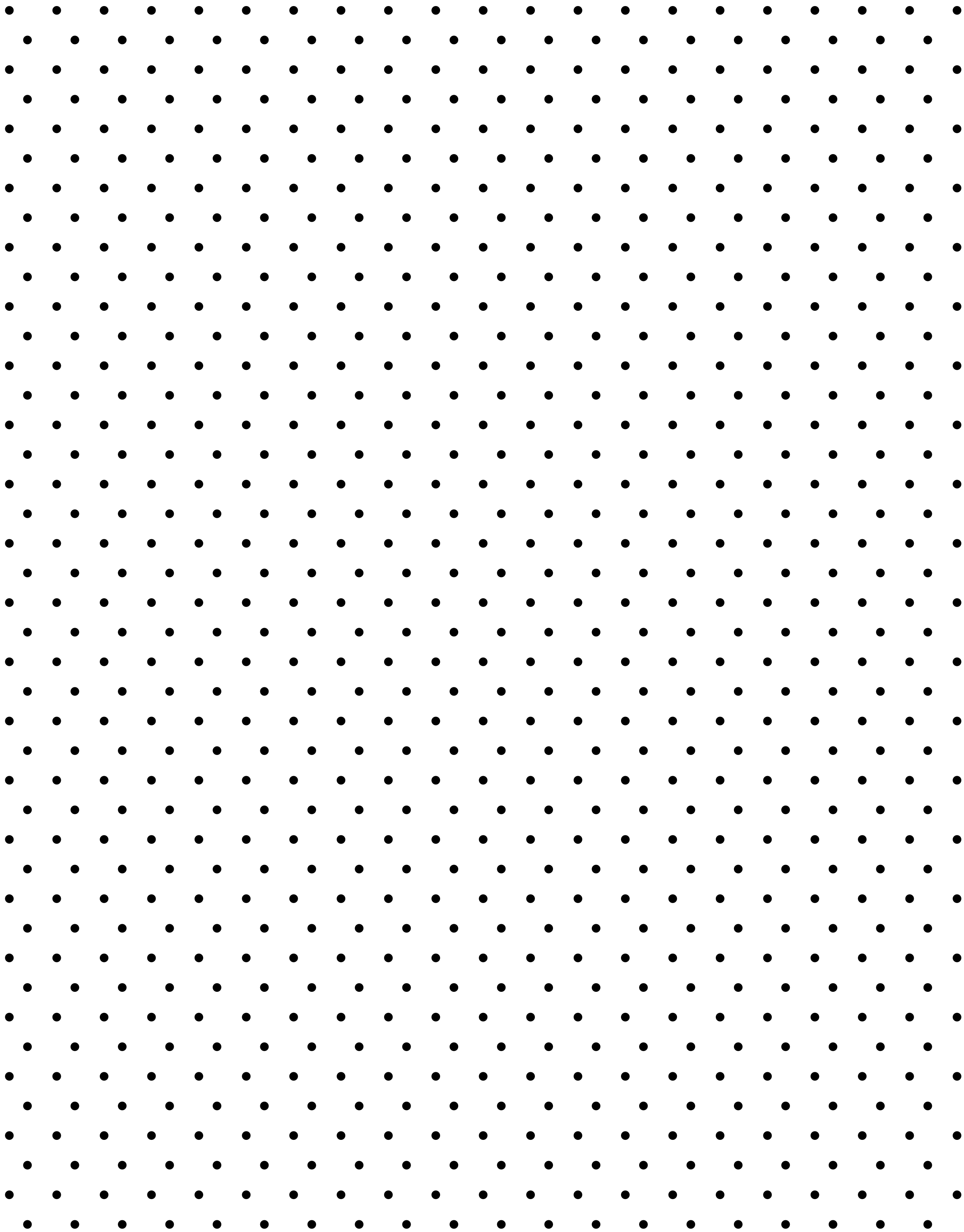


7. Tetracube F

Hint for drawing a cube on isometric dot paper:



Isometric Dot Paper



Exploring Surface Area and Volume

Organizing topic Three-Dimensional Geometry

Overview Students derive formulas for the surface area and volume of a rectangular prism, a cylinder, and a sphere.

Related Standard of Learning G.13

Objectives

- The student will derive the formulas for surface area and volume of a rectangular prism.
- The student will derive the formulas for surface area and volume of a cylinder.
- The student will derive the formulas for surface area and volume of a sphere.
- The student will use the appropriate formulas to find the surface area of cylinders, rectangular prisms, and spheres.
- The student will use the appropriate formulas to find the volume of cylinders, rectangular prisms, and spheres.
- The student will draw a net of a rectangular prism.
- The student will draw a net of a cylinder.
- The student will draw a net of a pyramid.
- The student will solve practical problems involving three-dimensional figures.

Materials needed

- A copy of each of the four activity sheets for each student
- Calculators
- Cans
- Scissors
- Boxes made of light cardboard
- Oranges
- Wax paper
- Knife (to cut orange)
- Unit cubes
- Milk cartons or other boxes of the same shape but different sizes
- Hollow, solid figures (power solids)
- Sand, rice, or water
- Rulers
- Graduated cylinders

Instructional activity

Part One

1. Discuss with the students: What are the formulas for determining surface area of solid figures? How can these formulas be used to solve problems?
2. Have students work with partners to complete the “Finding Formulas” activity sheet to derive the formulas for surface area and volume of a rectangular prism, a cylinder, and a sphere.

3. Have the students complete the second activity, "Making Nets," to extend their knowledge of surface area of a variety of solid shapes.
4. Have the students use the newly learned information to complete the "Solving Problems" activity sheet.
5. Discuss the relationship of surface area to two-dimensional perimeter and area.
6. Distribute milk cartons with the tops cut off, or open boxes of other kinds, along with unit cubes. Ask students to fill the boxes with cubes as completely as they can and estimate the volume of the box based on counting the cubes used to fill the box.
7. Ask students to develop a quicker method than filling and counting for figuring out how many whole cubes will fill the box. (They should decide on length × width × height.)
8. Have students generalize this finding to other prisms/cylinders by relating the area of the base to the height. Use a can as a model of a right circular cylinder. Students should calculate the area of the base and multiply by the height to find the volume. Since 1 cm^3 = 1 ml, a graduated cylinder can be used to compare the estimated volume with the actual volume.
9. Have students explore the relationship between the area of a pyramid and the area of a cube or prism with the same base and height, using the power solids and sand, water, or rice.
10. Have students complete the "Exploring Volume" activity sheet.

Sample assessment

- Use the completed activity sheets to assess student understanding.

Follow-up/extension

- Discuss the relationship of surface area and volume of three-dimensional solids and their relationship to area and perimeter of two-dimensional figures.

Sample resources

Mathematics SOL Curriculum Framework
http://www.pen.k12.va.us/VDOE/Instruction/Math/math_framework.html

SOL Test Blueprints

Released SOL Test Items http://www.pen.k12.va.us/VDOE/Assessment/Release2003/index.html

Virginia Algebra Resource Center http://curry.edschool.virginia.edu/k12/algebra

NASA http://spacelink.nasa.gov/.index.html

The Math Forum http://forum.swarthmore.edu/

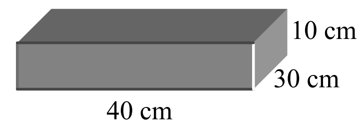
4teachers http://www.4teachers.org

Appalachia Educational Laboratory (AEL) http://www.ael.org/pnp/index.htm

Eisenhower National Clearinghouse http://www.enc.org/

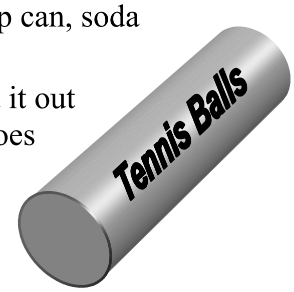
Exploring Surface Area and Volume
Activity Sheet 1: Finding Formulas

1. After you purchase a gift for a friend, you decide to cover the sides and bottom of the gift box with wrapping paper. A diagram of the box with its dimensions appears below.
   - How much wrapping paper will you need to cover the sides and bottom of the box?
   - Your gift box is called an open box because it has no top surface. If this were a closed box with a top surface, how much additional paper would be required to cover the top surface? How much total paper would be required?



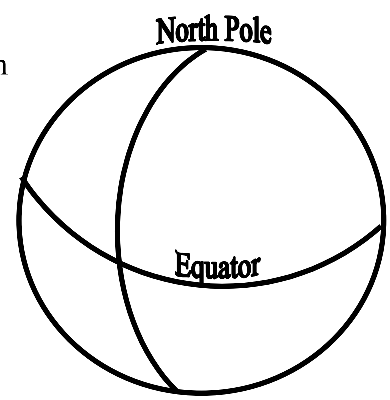
   - How can you generalize the process you used to find the surface area of the closed box?
   - Let l = length, w = width, and h = height of the box.
   - Compare the formula you and your partner developed to that of another group. Did you have the same result? You should be able to justify your formula to your classmates.

2. If your gift were a can of tennis balls, the surface area would be the surface of the cylinder (the lateral area) plus the areas of the top and bottom (the bases). Use a can (soup can, soda can, tennis ball can) for this activity.



   - Wrap a piece of paper around the can, trim it to fit exactly, and spread it out flat. What shape is it? How can you find its area? What relationship does the length of the label have to the can? The height of the label?
   - What shape are the bases of the can? Are the two bases congruent? What is the area of each base?
   - The *surface area of the can = the lateral area + the area of the two bases*. For your can, what is the surface area? Use your calculator to find decimal approximations to the nearest tenth.

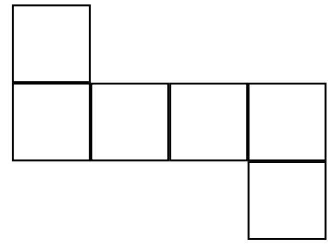
3. The surface area of a sphere is more difficult to figure out. On a globe, a *great circle* is a circle drawn so that when the sphere is cut along the line, the cut passes through the center of the sphere. The equator is a great circle on a globe.



   - Draw a great circle on an orange, and carefully cut the orange in half along the line of the great circle. Trace five cut halves on a piece of waxed paper.
   - Carefully peel both halves of the orange, and fill in as many circles as you can with the peel. How many circles did your group fill? How does this compare with the findings of other groups? What is the class estimate for the number of great circles that can be filled by the peel?
   - Using one of your great circle tracings, find the radius of your orange and the area of one great circle.
   - Given the area of one great circle and your estimate of the number of circles that can be filled by the peel, what is the surface area of the orange?
   - What is the general formula for the surface area of a sphere in terms of its radius?

Exploring Surface Area and Volume
Activity Sheet 2: Making Nets

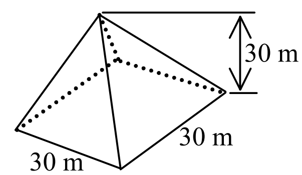
1. A *net* is a flattened paper model of a solid shape. For example, the net shown to the right, when folded, makes a cube. Can you draw a different net which, when folded, will also make a cube? If so, draw it, cut it out, and fold it to test your drawing.



2. A net is helpful because it represents the surface area of a shape. Take a box and cut it into a net. Note whether your box is open or closed. Sketch your box and its net. Use the formula you derived in "Finding Formulas" problem 1 to find the surface area of your box. Explain to a classmate how your net relates to your formula.

3. Now sketch a net of the can you used in "Finding Formulas" problem 2. How does this net relate to the surface area formula you found?

4. Sketch a net of the pyramid shown to the right. Use your net to find the surface area of the pyramid.



Scale: 1 cm = 15 m

Exploring Surface Area and Volume
Activity Sheet 3: Solving Problems

1. Two cylindrical lampshades 40 centimeter in diameter and 40 centimeter high are to be covered with new fabric. The fabric chosen is 1 meter wide. If you purchase a 1.5-meter length of this fabric, will you have enough to cover both lampshades? Justify your answer.

2. An umbrella designer has created a new model for an umbrella that when opened has the form of a hemisphere with a diameter of 1 meter. If a dozen sample models are to be made using a special waterproof material, approximately how much waterproof fabric will be needed, allowing 0.5 meter for seams and waste for each model? Explain your plan, strategies, and how you solved the problem.

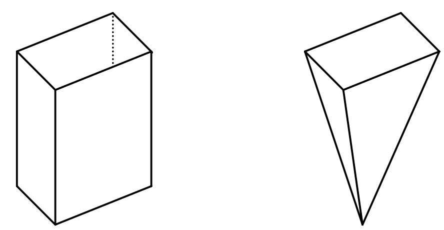
Exploring Surface Area and Volume
Activity Sheet 4: Exploring Volume

1. Which will carry the most water in a given length — two pipes with one having a 3 dm radius and the other a 4 dm radius, or one pipe with a 5 dm radius? Explain.

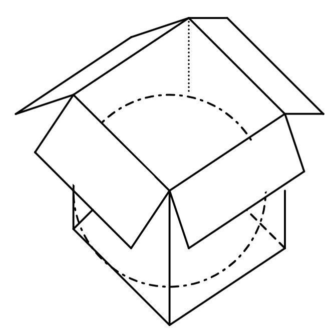
2. A company delivers 36 cartons of paper to your school. Each carton measures 40 cm × 30 cm × 25 cm. Is it possible to fit all cartons in an empty storage closet 1 m × 1 m × 2 m? Justify your conclusion with a visual explanation.

3. You have studied the pyramids and want to make a scale model of a pyramid with a square base and sides that are isosceles triangles. How much clay is required if the base of the actual pyramid is 30 m on each side and the height of the pyramid is 30 m? Your scale is 1 cm = 15 m.

4. A movie theater decides to change the shape of its popcorn holder from a rectangular box to a pyramidal box. The tops of both boxes are the same and the height remains the same. If the rectangular bag of popcorn cost $4.00, what is a fair price for the new box?



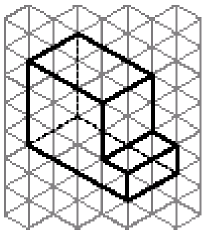
5. A manufacturer of globes that are approximately 1 m in diameter packs the globes in 1-cubic-meter boxes for shipping. How much packing material (Styrofoam peanuts) is needed for a shipment of 100 globes?



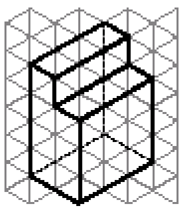
6. Take two sheets of paper the same size. Roll one sheet vertically and tape to form a right circular cylinder. Roll the second sheet horizontally and tape to form a second right circular cylinder. Tape each cylinder so that there is no overlap of paper — i.e., the edges should meet exactly. If each cylinder were filled with popcorn, would they contain the same amount? Explain and justify your answer.

Sample assessment

This is one view of a 3-dimensional object:



Which is a different view of the same object?

F


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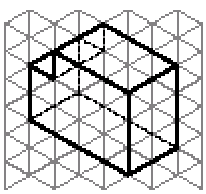

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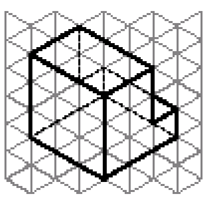

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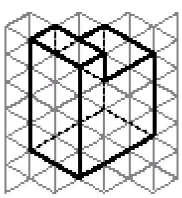

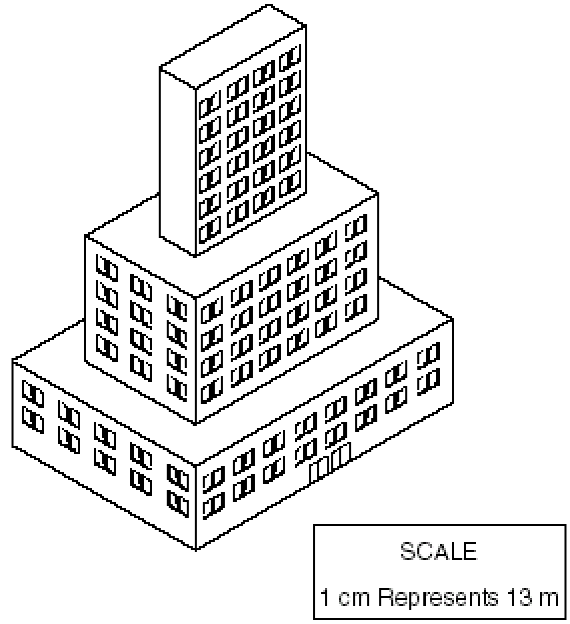

This is a scale drawing of a building. What is the actual height of the building?

A 58.5 m

B 71.5 m

C 78 m

D 84.5 m

What is the volume in cubic feet of a refrigerator whose interior is 4.5 feet tall, 2.5 feet wide, and 2 feet deep?

F 15 cu ft.

G 19 cu ft.

H 22.5 cu ft.

J 25 cu ft.

Rounded to the nearest hundred cubic meters, what is the total capacity (cone and cylinder) of the storage container at right?

A 1,400

B 2,000

C 5,700

D 8,100