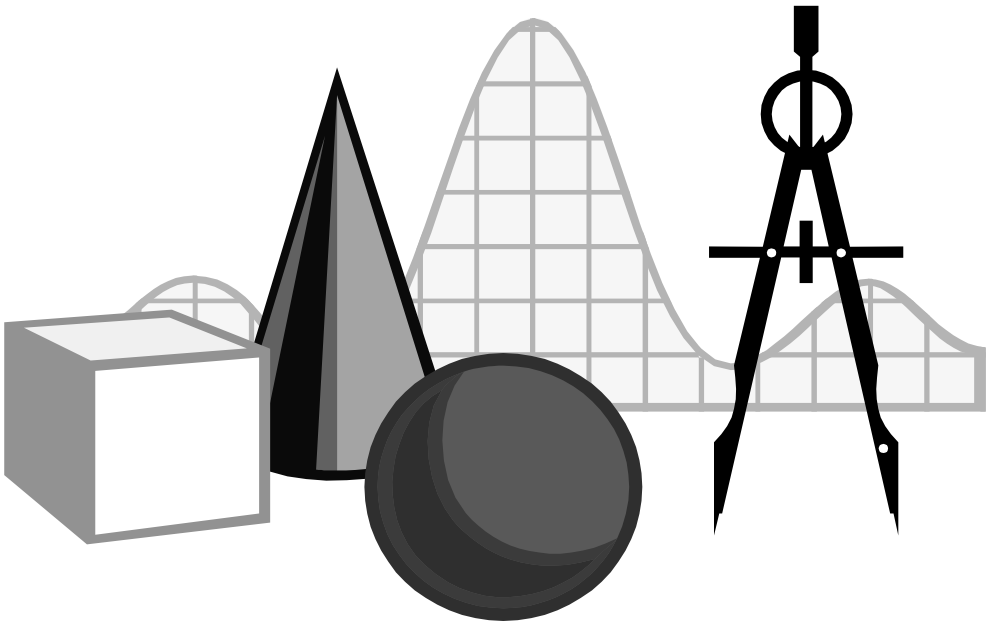
MATHEMATICS STANDARDS OF LEARNING ENHANCED SCOPE AND SEQUENCE

Grade 6



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Department of Education
Richmond, Virginia
2004

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The CTE Resource Center is a Virginia Department of Education grant project administered by the Henrico County Public Schools.

Introduction

The *Mathematics Standards of Learning Enhanced Scope and Sequence* is a resource intended to help teachers align their classroom instruction with the Mathematics Standards of Learning that were adopted by the Board of Education in October 2001. The Mathematics Enhanced Scope and Sequence is organized by topics from the original Scope and Sequence document and includes the content of the Standards of Learning and the essential knowledge and skills from the Curriculum Framework. In addition, the Enhanced Scope and Sequence provides teachers with sample lesson plans that are aligned with the essential knowledge and skills in the Curriculum Framework.

School divisions and teachers can use the Enhanced Scope and Sequence as a resource for developing sound curricular and instructional programs. These materials are intended as examples of how the knowledge and skills might be presented to students in a sequence of lessons that has been aligned with the Standards of Learning. Teachers who use the Enhanced Scope and Sequence should correlate the essential knowledge and skills with available instructional resources as noted in the materials and determine the pacing of instruction as appropriate. This resource is not a complete curriculum and is neither required nor prescriptive, but it can be a valuable instructional tool.

The Enhanced Scope and Sequence contains the following:

- Units organized by topics from the original Mathematics Scope and Sequence
- Essential knowledge and skills from the Mathematics Standards of Learning Curriculum Framework
- Related Standards of Learning
- Sample lesson plans containing
    - Instructional activities
    - Sample assessments
    - Follow-up/extensions
    - Related resources
    - Related released SOL test items.

Acknowledgments

Marcie Alexander
Chesterfield County

Melinda Batalias
Chesterfield County

Susan Birnie
Alexandria City

Rachael Cofer
Mecklenburg County

Elyse Coleman
Spotsylvania County

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Organizing Topic Rational Numbers

Standards of Learning

6.1 The student will identify representations of a given percent and describe orally and in writing the equivalence relationships among fractions, decimals, and percents.

6.4 The student will compare and order whole numbers, fractions, and decimals, using concrete materials, drawings or pictures, and mathematical symbols.

6.5 The student will identify, represent, order, and compare integers**.**

6.6 The student will solve problems that involve addition, subtraction, multiplication, and/or division with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less, and express their answers in simplest form; and find the quotient, given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero.

6.7 The student will use estimation strategies to solve multistep practical problems involving whole numbers, decimals, and fractions (rational numbers).

6.8 The student will solve multistep consumer-application problems involving fractions and decimals and present data and conclusions in paragraphs, tables, or graphs. Planning a budget will be included.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Recognize that *percent* means "out of 100" or *hundredths*, using the percent symbol (%).	_____
• Identify the decimal and percent equivalents for halves, thirds, fourths, fifths, and tenths.	_____
• Describe orally and in writing the equivalent relationship among decimals, percents, and fractions that have denominators that are factors of 100.	_____
• Draw a shaded region on a 10-by-10 grid to represent a given percent.	_____
• Represent in decimal, fraction, and/or percent form a given shaded region of a 10-by-10 grid.	_____
• Compare two whole numbers by representing the numbers with concrete objects or picture representations or by using the symbols <, >, or =.	_____
• Compare two fractions with denominators of 12 or less by representing the fractions with fraction manipulatives or picture representations or by using the symbols <, >, or =.	_____
• Identify an integer represented by a point on a number line.	_____

- Represent an integer on a number line. _____
- Compare and order integers, using a number line. _____
- Compare integers, using the mathematical symbols <, >, and =. _____
- Convert fractions to equivalent forms to perform the operations of addition and subtraction. _____
- Simplify fractional answers to simplest form. _____
- Solve problems that involve addition and/or subtraction with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less, and express answers in simplest form. _____
- Solve problems that involve multiplication and/or division with fractions and mixed numbers that include denominators of 12 or less, and express answers in simplest form. _____
- Solve multistep practical problems involving whole number and fractions by using estimation strategies and checking for the reasonableness of results. _____
- Solve multi-step consumer application problems involving fractions with denominators not greater than 12, where solutions require at least a 2-step process. _____

Around the World

Reporting categories Number and Number Sense, Computation and Estimation

Overview Students determine what percent of the time their right index finger will come to rest on land or water when catching a tossed globe. Students develop percent number sense by first estimating and then actually catching the globe and recording where the right index finger lands each time.

Related Standards of Learning 6.1, 6.7

Objectives

- The student will develop decimal number sense and make connections among different representations — fractions, decimals, and percents — for the same amount.
- The student will use estimation to relate percents to land and water areas around the world.

Materials needed

- One inflatable plastic globe, one for each of three groups of students
- "Around-the-World Recording Sheet 1," one copy for each student
- "Around-the-World Recording Sheet 2," one copy for each student for extension activity
- "Area of Land and Water around the World," one copy for each student for extension activity

Instructional activity

1. *Initiating Activity:* Have students predict the percent of land versus water found on the globe. Record a few of these predictions on the board for future reference.
2. Explain to students that they will be collecting data to verify their predictions. Organize students into three groups, and give each group a plastic globe and a set of recording sheets.
3. Prior to beginning the tosses, have the students record their individual predictions on their recording sheet. The prediction is a guess as to how many times out of 100 catches a student's right index finger will touch land and how many times it will touch water.
4. Explain the data-collecting process to the students. Have each group form a circle and toss the globe back and forth among the members for a total of 100 tosses. For each toss, have one student in each group record the data by putting a tally mark in the appropriate space on his/her recording sheet.
5. After the 100 tosses and all data collecting are completed, have the students in each group transcribe the total of the tally marks onto their own recording sheets. Then show them how to convert the data into the correct percents. Model the conversion with the following example: 72 tosses came to rest on water; 28 tosses came to rest on land. Record the data as 72 out of 100 equals $\frac{72}{100}$, .72, and 72%; and 28 out of 100 equals $\frac{28}{100}$, .28, and 28%. Students may represent the fraction and decimal by using place value materials or 100 grids.
6. Have each group share their results, and display all the data in a class chart.
7. *Closing Activity:* Using the results on the class chart, have students compare the results to the original predictions. If large differences exist between the predictions and the actual results, discuss why the differences exist.

Sample assessment

- During the activity, observe students as you walk around the room and check for understanding.
- At the end of the activity, have students respond in their math journals to the following prompts:
  - How are fractions, decimals, and percents related?
  - What connections are there among fractions, decimals, and percents?

Follow-up/extension

- This activity can be extended so that students make predictions about the percentages of the Earth occupied by the individual continents and the percentages of the Earth occupied by the individual oceans. Refer to the chart "Area of Land and Water around the World," and have the students use the "Around-the-World Recording Sheet 2." The class can be divided into two groups, each of which makes 50 tosses, and then the results can be added together.

Around-the-World Recording Sheet 1

Name: _____ **Date:** _____

1. Estimate the number of times out of 100 catches that your right index finger will come to rest on water.
2. Estimate the number of times out of 100 catches that your right index finger will come to rest on land.
3. Toss and catch the globe 100 times.
4. Keep a tally, or actual count, and make a record of where your right index finger comes to rest each time.
5. Figure the fraction, decimal, and percent representations of the number of times your finger came to rest on land; then figure the same representations for water.

	Estimate	Actual	Fraction	Decimal	Percent
Land		**Tally marks:** _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ **Total:** _____			
Water		**Tally marks:** _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ **Total:** _____			
Total	**100**	**100**	$\frac{100}{100}$	**1.00**	**100%**

Around-the-World Recording Sheet 2

Name: _____ Date: _____

1. Estimate the number of times out of 100 catches that your right index finger will come to rest on each of the continents shown below.
2. Toss and catch the globe 100 times.
3. Keep a tally, or actual count, and make a record of where your right index finger comes to rest each time.
4. Figure the percent representations of the number of times your finger came to rest on each continent.
5. Explain any discrepancies between your estimated and actual counts.

	Estimate	Actual	Percent
Asia			
Africa			
Antarctica			
Australia			
Europe			
North America			
South America			
All Land			
All Water			

Area of Land and Water around the World

Total Square Miles on Earth = 195,331,609

Continent	Square Miles	Percent of Earth
Asia	16,957,000	9%
Africa	11,704,000	6%
Antarctica	5,100,000	3%
Australia	2,967,909	2%
Europe	4,063,000	2%
North America	9,416,000	5%
South America	6,888,000	4%
Land Total	57,095,909	31%

Water	Square Miles	Percent of Earth
Pacific Ocean	64,186,000	32%
Atlantic Ocean	33,420,000	17%
Indian Ocean	28,350,000	14%
Arctic Ocean	5,105,700	2%
Seas, Gulfs, and Bays	7,174,000	4%
Water Total	138,235,700	69%

Museum Walk

Reporting category Number and Number Sense

Overview Students discuss common uses of fractions, decimals, and percent and their meanings. Using a museum-walk model, groups rotate around the room and share findings with one another.

Related Standard of Learning 6.1

Objectives

- The student will discuss and interpret common uses and meanings of fractions, decimals, and percent.
- The student will develop an understanding that the same quantity can be expressed as a fraction, decimal, or percent.

Materials needed

- Chart paper
- Colored markers of 3 to 6 different colors.

Instructional activity

1. *Initiating Activity:* Make three charts, headed as follows: "Real-Life Uses of Fractions," "Real-Life Uses of Decimals," and "Real-Life Uses of Percents." Model the use of the charts by listing some examples of ways we use these kinds of numbers. Divide the students into three groups, and assign one group to each chart. If dividing the class into three groups would make the groups too large, then make two charts of each type and have six groups, two groups per each type of chart.
2. Give each group a different colored marker. Have the groups rotate to the three different charts to record their uses.
3. Give the students approximately five minutes at each station. Continue until the groups are back to their starting places.
4. After walking and recording, ask students to compare the lists. Ask questions such as: In what ways are fractions, decimals, and percents alike? In what ways are they different? Compare answers and lists.
5. *Closing Activity:* Write the fraction one-half on the board in the three notations: $\frac{1}{2}$, 0.5, 50%. Ask:
   - What do you know about these three numbers?
   - How are they the same?
   - How are they different?
   - Where do you see these numbers on our charts?

Sample assessment

- Listen to student discussions about the use of fractions, decimals, and percents. In certain cases, one representation may be more commonly used than others. For example, a hitter can have a ".265 batting average," but we do not say he has a "265 thousandth batting average." In some situations, one form may not make sense to us. For example, we say "$\frac{1}{2}$ inch" but not "50% of an

inch." Sale prices may be "25% off" or "$\frac{1}{4}$ off," but not "0.25 off." A jogger runs "3 and $\frac{1}{2}$ miles," but not "3 and 50% miles." The language of their answers and comparisons of different uses is an important discussion to assess understanding of the representations of rational numbers.

Follow-up/extension

- Students may make a collage of fractions, decimals, and percents that they find in print media. The "Real-Life Uses" may also be extended following future activities.

Percent Grid Patterns

Reporting category Number and Number Sense

Overview Students color 10-by-10 grids in different patterns to identify and represent equivalent fractions, decimals, and percents.

Related Standard of Learning 6.1

Objective

- The student will identify equivalent fractions, decimals, and percents.

Materials needed

- "Base-Ten Grids" transparencies, with and without shading
- "Base-Ten Grids," without shading, one copy for each student
- Colored pencils/crayons
- Overhead markers
- Base-ten 100 flat

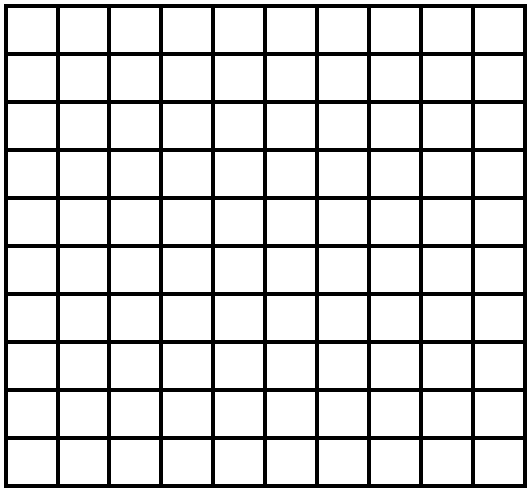
Instructional activity

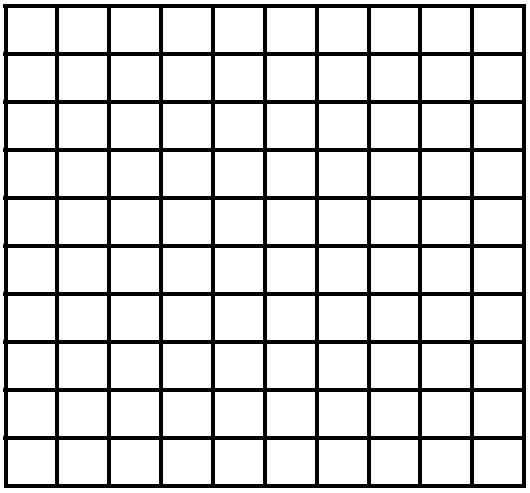
1. *Initiating Activity:* Discuss how many squares make up a 10-by-10-square grid. Present the 10-by-10 base-ten 100 flat and the 10-by-10-squares grid. Emphasize that the two are equal. For purposes of this activity, the students are going to use a representation of the base-ten 100 flat — i.e., a 10-by-10-squares grid. This is a representation that they could draw, but a handout has already been created that would save time. Remind the students that one grid represents 1 whole that has been divided into 100 equal-sized parts. (Note: We should be working from the concrete [base-ten flat] to representational [grid] to abstract.)
2. Show the base-ten transparency grid that has been shaded in and ask, "Who has a quick estimate of the percent of the squares that are shaded?" Record responses, and ask, "How did you determine how many squares? How did you estimate the percent of the whole that is shaded?"
3. Write below the grid on the transparency the percent and the equivalent fraction and decimal of the shaded portion. Ask volunteers to come up to the overhead and shade in a portion of the three remaining grids. Next, have the volunteers ask the class what percent of each grid is shaded.
4. It is important to write each decimal in "regular" fraction form, "expanded" base-ten fraction form, decimal form, and percent form — for example, $\frac{3}{8} = \frac{37}{100} + \frac{5}{1{,}000} = .375 = 37.5\%$. Continue to have volunteers participate in shading the transparency in order to produce a variety of fractions, decimals, and percents so that students have ample practice as an entire class before they move into individual work.
5. *Closing Activity:* Distribute colored pencils and a handout "Base-Ten Grids" to each student. Assign each student four *different* percents to color in their four grids and to label appropriately with the percent, the common and expanded base-ten fractions, and the decimal representation. For example, assign 2%, 5%, 7%, 13%, 25%, and 48% and common fractions such as thirds, fourths, fifths, sixths, and eighths. Grids should be displayed in the classroom.

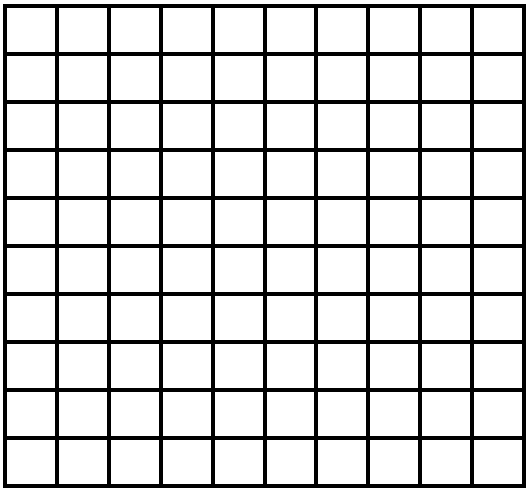
Sample assessment

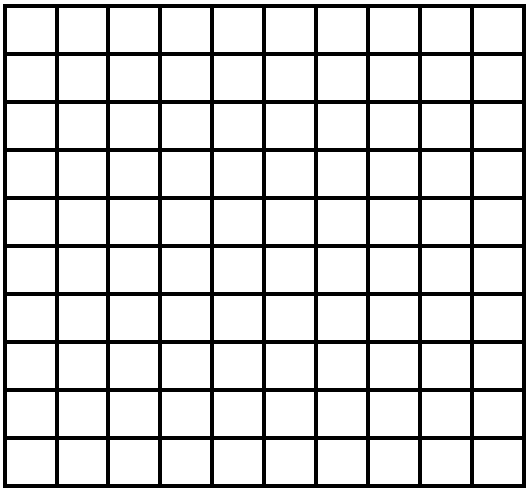
- During the activity, observe the students as they complete their grids, and ask them to explain the steps they are taking. Check for accurate expression of fractions, decimals, and percents, both in written form and in conversation.
- Encourage students to share their explanations with the rest of the class.

Base-Ten Grids









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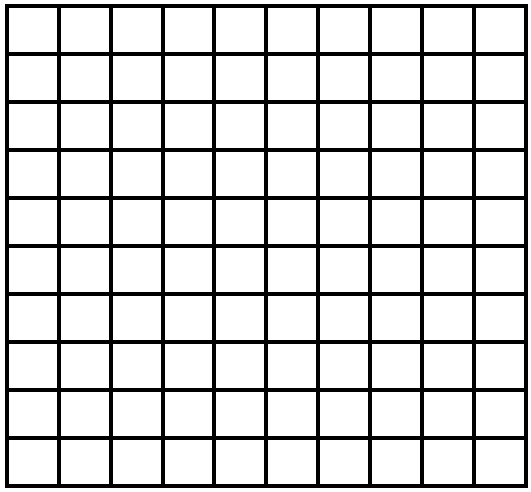
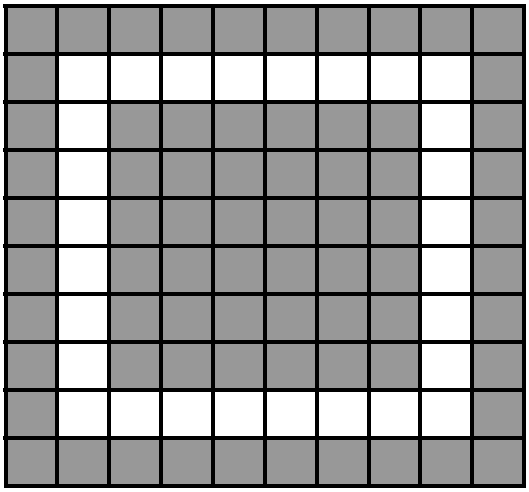
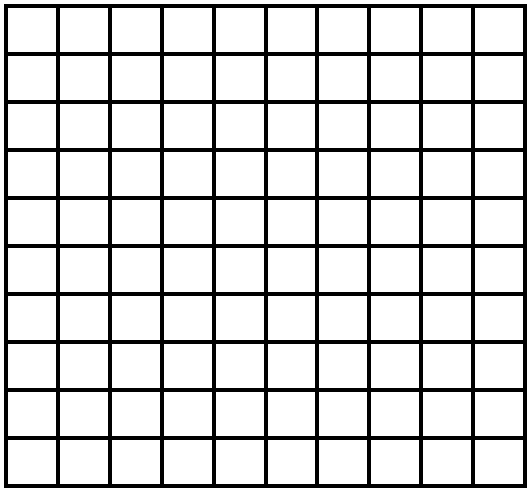
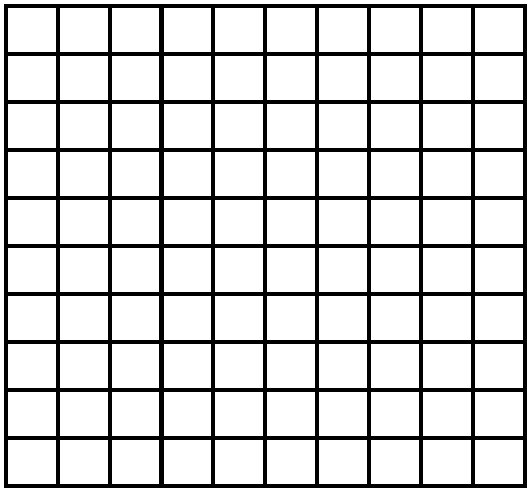








Who Has 100 Things?

Reporting category Number and Number Sense

Overview Students create a fraction number line and use it to find equivalent fractions, decimals, and percents. Students develop decimal number sense by forming different subsets of 100 objects. Students also make connections among different representations (fractions, decimals, and percents) for the same amount.

Related Standard of Learning 6.1

Objectives

- The student will find fractional parts and determine the equivalent decimal and percent.
- The student will develop decimal number sense and make connections among different representations (fractions, decimals, and percents) for the same amount.

Materials needed

- Rolls of adding machine tape
- One pre-cut meter strip of adding machine tape for each student
- Markers
- Scissors
- 100 similar objects packaged for each small group
- "Table of 100 Things," one copy for each small group
- A transparency of "Table of 100 Things" for each small group
- Yarn, string, or jump ropes; three pieces of string 2- to3-feet long
- Chart paper
- Protractors (optional)
- "Mosaic Art," one copy for each student for extension activity

Instructional activity

Part I

1. *Initiating Activity:* Have each student estimate a length of adding machine tape that is one meter long and cut that estimated length from a roll of tape. Have each student decorate his or her "meter" strip in some manner for later recognition. After the strips have been decorated, have the students tape their strips to a tape stripe on the wall. Strips should be taped from the shortest to the longest in bar-graph fashion. Measure the strips with a meter stick to determine which one is closest to one meter long. Now is a good time to talk about ways to estimate a meter: a meter is a little bit longer than a yard; it is about the distance from the floor to a doorknob; or it is the about the width of a twin bed.
2. After a brief discussion about a meter, distribute a pre-cut meter strip of tape to each student. Have the students place their strip horizontally on the desk or table in front of them.
3. Have the students write "0" on the left end of the strip and "1" on the right end of the strip. Discuss briefly that the strip now represents one unit. Model the labeling as you go.
4. Have the students fold the right end of their strip over to the left end and crease. Have them open their strip and observe that the crease makes it appear to be divided into two equal parts. Have the

students write "$\frac{0}{2}$" under the 0 on the left end, "$\frac{1}{2}$" on the crease, and "$\frac{2}{2}$" under the 1 on the right end. Model this as you go. After this step, discuss briefly with the students that they now have two ways to write the quantity zero — 0 and $\frac{0}{2}$ — and two ways to write the quantity one — 1 and $\frac{2}{2}$.

5. Have students fold the strip in half again and then fold it a second time and crease. Have them open their strip and observe that it appears to be divided into four equal parts.
6. Have the students write "$\frac{0}{4}$" under the 0 and $\frac{0}{2}$, "$\frac{1}{4}$" at the first crease, "$\frac{2}{4}$" under the $\frac{1}{2}$, "$\frac{3}{4}$" at the third crease, and "$\frac{4}{4}$" under the 1 and $\frac{2}{2}$. After this step, discuss briefly with the students that they now have three ways to write the quantity zero — 0, $\frac{0}{2}$, and $\frac{0}{4}$. Point out that they now have two ways to express the quantity one-half — $\frac{1}{2}$ and $\frac{2}{4}$ — and three ways to express the quantity one — 1, $\frac{2}{2}$, and $\frac{4}{4}$.
7. If you wish, the same procedure can be done for eighths. Have the students fold their strip in half, in half again, and in half a third time. The strip will now look like it has been divided into eight parts. Have them follow the same procedure for labeling. After labeling, point out equivalent fractions.
8. Folding into thirds is a bit tricky. Have the students think of the strip as a belt and lap the two ends toward the each other, adjusting until the folded lengths are of equal length. Model this folding for them. Have them crease and then label the three parts as they have done before. Be sure to point out that no thirds line up with halves, fourths, or eighths.
9. Have the students turn their strip over to the blank side, making sure that the 0 end is still to the left and the 1 end is to the right. Have them label the left end 0 and the right end 100. Now is the time to discuss the fact that one meter is equivalent to 100 centimeters and that is why 100 was chosen for the label.
10. Have the students fold their strip into two parts, open it, and label the parts. Have them write "$\frac{0}{100}$" under the 0 on the left end, "$\frac{50}{100}$" on the crease, and "$\frac{100}{100}$" under the 100 on the right end. To the side of each label, have them write the decimal represented: 0, .5, and 1. Remind them that percent literally means "out of a hundred" so that writing the percent from the fraction for each is easy. Directly under each label, have the students write the equivalent percent: 0%, 50%, and 100%. Now have students start the association with the first side of the strip: zero is zero as a fraction, decimal or percent. One-half can be expressed many ways: $\frac{1}{2}$ or $\frac{50}{100}$ or .5 or 50%.
11. Continue this process until the students have completed all the fractions on the first side of the strip. Explain to the students that they have now completed a fraction number line on one side of their meter strip and a decimal and percent equivalency line on the other side.

Part II

1. Organize the students into small groups, and give each group 100 similar objects, e.g. colored beads, an assortment of buttons, colored marshmallows, M & Ms, small screws and nuts, tricolor

dry pasta noodles, a collection of different coins, or different colored marbles. You might wish to place more than 100 objects on the table and ask the students to count out the 100 and store the rest.

2. Give each group a copy of the handout "Who Has 100 Things?" Ask each group to sort their 100 objects into at least three groups and then complete the chart and answer the question on the recording sheet. For example, one collection of 100 beads might consist of 25 red beads, 60 green beads, and 15 blue beads and should be recorded as follows:

	Title of Each Group	Number of Items in Group	Fraction	Decimal	Percent
Group #1	Red Beads	25	$\frac{25}{100}$	.25	25%
Group #2	Green Beads	60	$\frac{60}{100}$	.60	60%
Group #3	Blue Beads	15	$\frac{15}{100}$	.15	15%
	TOTAL	100	$\frac{100}{100}$	1.00	100%

3. Have each group present their data to the class on a transparency of the handout "Who Has 100 Things? Recording Sheet."
4. Encourage students to make the connection between representing their data in the chart or table and representing it graphically in a "rough" circle graph. As they do this, be certain that they understand that this is possible because they know what percent (or part) of the whole is represented by each group.
5. At each table, have the students estimate the location of the center of the circle. Using pieces of string or yarn, lay out the radii to the endpoint of each arc to form a pie-shaped wedge. Ask the students to outline the circumference of a circle with their 100 objects, making sure to keep each group of objects together. For example, using the 100 beads, they would begin the circle with the 25 red beads forming one arc of the circle, continue with the 60 green beads connected to the last red bead, and finish with the 15 blue beads completing the circle.
6. Have each group move from table to table to see the other groups' "circle graphs" that have been created. If the circle graphs have been made on a piece of chart paper, each group can label their graph appropriately and include a creative title.

Sample assessment

- As students work, circulate and watch carefully as they follow instructions.
- Answer any questions, and clarify any procedures that may be problematic.

Follow-up/extension for Part II

- Distribute copies of the handout "Mosaic Art," and assign the "Mosaic Art" project. Ask the students to display their mosaic art picture and chart at the next class session

Who Has 100 Things? Recording Sheet

Name: _____ Date: _____

1. Sort your 100 objects into at least three groups.
2. Fill in the chart, and answer the question.

	Title of Each Group	Number of Items in Group	Fraction	Decimal	Percent
Group #1					
Group #2					
Group #3					
Group #4					
Group #5					
Group #6					
Group #7					
TOTAL:					

3. How can you find the percent in group #2 without counting?

Mosaic Art

Name: _____ Date: _____

1. Make a picture that uses exactly 100 pieces of different colored paper. (The pieces of paper do not have to be the same size.)
2. Fill in the chart and attach it to your picture. If you use more colors, just add more rows to your chart.
3. Display your mosaic art at the next class meeting.

Color	Number of Pieces That Same Color	Fraction	Decimal	Percent

Investigating Integers

Reporting category Computation and Estimation
Overview Students explore various uses of integers.
Related Standard of Learning 6.5

Objectives

- The student will write about how integers can be used in real-life situations.
- The student will work with integers in various formats.

Materials needed

- Laminated number lines showing –20 to +20
- Game markers

Instructional activity

1. Prior to the beginning of the activity, have students write in their math journals ways in which they think integers can be applied in real-life situations. After a designated writing time, have students share the situations. Record on chart paper.
2. Have the students work in groups of four to investigate integers. Give each group a number line showing –20 to +20 and a deck of cards with the face cards removed. Each student places a different color marker on zero. As the student is dealt a card face up, the student moves that number of places; red is negative, black is positive. The first student to reach negative 20 or positive 20 wins the game.
3. Have students create a Celsius thermometer naming the temperature as positive and negative integers. Using diagrams of thermometers with missing negative and positive integers, have students will fill in the missing integers.
4. For a final activity, have students find examples of positive and negative numbers in newspapers or magazines. Next, have the students place the examples on the recording chart used in the beginning of the lesson.

Sample assessment

- Have students record in their journal a different situation in which positive and negative integers could be used.

Timeline

Reporting category Number and Number Sense

Overview Students build a timeline to further understand historical sequence and explore simultaneous developments in ancient cultures.

Related Standard of Learning 6.5

Objective

- The student will research developments in mathematics and develop a timeline.

Materials needed

- Wire or stiff string
- Clip-type clothespins or similar clips
- Felt-tip pen
- Tape
- Index cards

Instructional activity

1. Prepare for this activity by stringing wire across a wall or chalkboard. It should be reachable by the shortest student and fairly taut. Label index cards or small pieces of paper with the following dates: 6000 B.C., 5000 B.C., 4000 B.C., 3000 B.C., 2000 B.C., 1000 B.C., A.D. 1, A.D. 1000. Tape the dates in the correct order at even intervals along the wire or string to make a timeline.
2. If necessary, review the concept of a timeline and the definitions of A.D. and B.C.. If you wish, have volunteers locate events on the timeline, using the following dates:
   - 3100 B.C. – earliest Egyptian writing
   - 2500 B.C. – earliest writing in the Indus valley (India)
   - 1200 B.C. – first writing in China
   - 1000 B.C. – invention of the Phoenician alphabet
   - A.D. 1821 – Cherokee alphabet invented by Sequoya
   - A.D. 1867 – invention of the typewriter
3. When you are satisfied that students understand the concepts, divide the class into small groups. Assign an ancient civilization to each group. You may wish to assign any of the ancient civilizations in the following list:
   - Ancient Egypt
   - Ancient Israel
   - Ancient Greece
   - Ancient Mesopotamia
   - Ancient China
   - Ancient Nubia
   - Ancient India
   - Maya and Inca
   - Anasazi

- Nok People

4. Tell students that they are going to research their civilization and, as a group, choose what they think are the ten most significant dates in that people's history. They will then write the date of the event and a short description on an index card or small piece of paper. When all the cards are complete, have students tape or clip the event cards to the timeline in the correct places.
5. Have a class review of the timeline. Ask, How does a timeline help us to study events? Discuss with students the relationships among events. Are some of the simultaneous developments coincidences, or could there be cause-and-effect explanations? Elicit theories from students about the interaction of cultures and cause-and-effect relationships.
6. For example, a famine in one country could have caused it to start a war with a neighboring country. The famine might be the event mentioned in one culture, and the war in the other.

Teaching options

- You may wish to have students write short reports comparing events or explaining cause and effect relationships between certain events.
- Students might like to transfer the timeline, as a whole, to paper or create a Web site, adding illustrations and other additional information. Have students research the Gregorian calendar or other world calendars and report on how and by whom they were developed. Then, for students interested in ancient Rome, have them find out why 46 B.C. was called "the year of confusion." (Answer: In order to get his new calendar on track, Julius Caesar declared the year 46 B.C. to have 442 days.)

Area Model of Multiplication

Reporting categories Number and Number Sense, Computation and Estimation

Overview Students relate the area of a rectangle to the multiplication of fractions or decimals.

Related Standards of Learning 6.6, 6.7

Objectives

- The student will gain experience in visualizing the meaning of multiplication of fractions.
- The student will connect the meaning of multiplication of fractions to the multiplication of whole numbers.

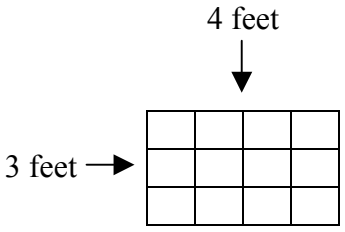
Materials needed

- Plain copy paper
- Pencils
- Colored pencils
- Rulers (optional)
- Blank transparencies
- Overhead and regular markers
- Chart paper
- Sticky notes

Note: It is important to script this activity carefully and thoroughly for the students. The value of this activity is dependent upon clearly relating the divisions of paper to the operation of multiplication. Each step of the operation should be fully discussed.

Instructional activity

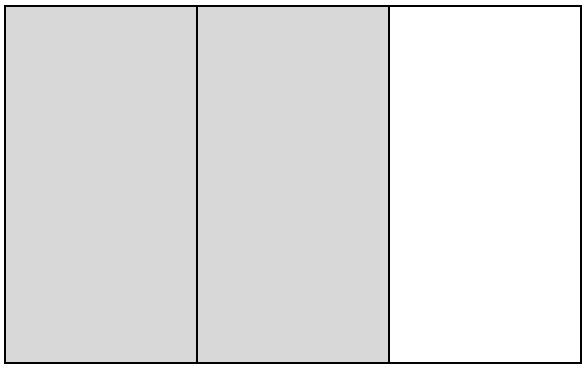
1. *Initiating Activity:* Lead the students in the following dialogue and steps: "We are using an area model for fractions. To understand this better, we should look at whole number multiplication as it relates to the area of a rectangle. How do we find the area of a rectangular piece of tile that is 3 feet wide by 4 feet long?" Demonstrate this on the board or overhead, as follows:



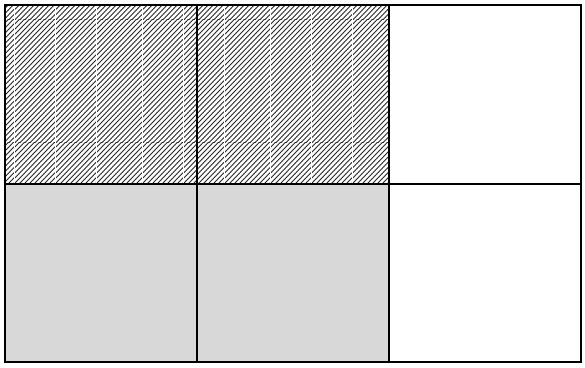
   Notice that the area contains 12 squares or 12 square feet. We multiplied 3 feet by 4 feet to get the total of 12 square feet.

2. "Consider the problem $\frac{1}{2}$ of $\frac{2}{3}$. We can think of this as finding the area of a rectangle with a height of $\frac{1}{2}$ and a width of $\frac{2}{3}$."

3. "Draw a rectangle. Separate it with two vertical lines into three equal-sized sections, and shade or color two adjacent sections."



4. "The shaded portion represents which fraction?" ($\frac{2}{3}$).
5. "We can represent the first fraction ($\frac{1}{2}$) by separating the original rectangle with a horizontal line into two equal-sized sections. In this case, however, we will use stripes to shade only that part of the picture that shows a fraction of the original fraction — that is $\frac{1}{2}$ of the $\frac{2}{3}$.



6. "What is the width of the rectangle that is striped?" ($\frac{2}{3}$ of a unit)
7. "What is the height of that rectangle?" ($\frac{1}{2}$ of a unit)
8. "What is the area of that rectangle?" "Count the equal portions of the original rectangle that are striped." (2 out of 6, or $\frac{2}{6}$, or $\frac{1}{3}$)
9. "How does this answer relate to the original problem $\frac{1}{2} \times \frac{2}{3}$?" (It is equivalent.)
10. Discuss the equivalence thoroughly. If necessary, model additional problems.

11. *Closing Activity:* Ask the participants to work in groups to draw the following problems on chart paper. (Write the problems on the overhead for groups to copy). Post students' work on the board, and allow everyone to walk around and view the work of other groups. Use sticky notes or markers to point out any discrepancies or disagreements. Hold a class discussion after everyone has had the opportunity to work all the problems.

$$\frac{1}{3} \times \frac{3}{4}$$

$$\frac{1}{2} \times \frac{1}{4}$$

$$\frac{1}{4} \times \frac{3}{5}$$

$$\frac{1}{2} \times \frac{5}{6}$$

Sample assessment

- Circulate around the classroom, observing students as they complete each part of the activity. Watch for understanding of each concept, pose questions about the activity, and encourage student explanations and questions. Note any misunderstandings and correct those immediately by engaging students in additional drawings and pictorial models.

Dividing Fractions, Using Pattern Blocks

Reporting category Computation and Estimation

Overview Students use pattern blocks to represent the whole and then determine a fractional amount of the whole.

Related Standard of Learning 6.6

Objective

- The student will be able to divide a whole number by a fraction.

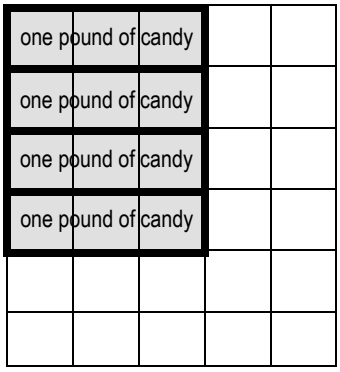
Materials needed

- Pattern blocks
- Graph/grid paper
- Colored pencils (optional)

Note: For easier management, put each pattern block set in a plastic storage bag for each student.

Instructional activity

1. *Initiating Activity:* Discuss with the class the following: What is division of whole numbers? What does 6 divided by 2 mean? What does 12 divided by 2 mean? What does 12 divided by 6 mean? In asking these questions, you are trying to encourage student understanding that division describes how many of a given divisor there are in a given dividend. Have students represent (sketch) 6 divided by 3 and make a story problem for 6 divided by 3. Have volunteers read their story problems.
2. Have students work in groups to model, using pattern blocks, the solution to the following problem: "Duncan has four pounds of candy and decides to use it all to make $\frac{1}{3}$-pound bags to give away. How many bags can he make with his four pounds of candy?"
   - Students may solve this problem by using four hexagons to model four pounds of candy. Using a rhombus to represent $\frac{1}{3}$ of a pound of candy, they will see that there are three rhombi in a hexagon and, therefore, twelve rhombi in four hexagons. Hence, one can make twelve
   - $\frac{1}{3}$-pound bags of candy out of four pounds of candy.
   - Another way students may solve this problem is to use graph paper to represent the problem. Since the problem uses division by thirds, discuss with the students why three equal-sized parts should be used to represent each pound of candy. Colored pencils may be used, if desired. Students should also write the problem as in the model at right:



$$4 \div \frac{1}{3} =$$

$$4 \bullet 3 = 12$$

3. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: "Susan had three blocks of candy. She wants to divide each block into

$\frac{1}{6}$-size pieces of candy. How many pieces of candy will she be able to make?" Students may solve this problem by using three hexagonal blocks to represent the three blocks of candy. They may then use a triangular block to represent $\frac{1}{6}$ of a block and find the solution. (18 pieces of candy)

4. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: "The Virginia Housing Company wants to divide five acres of land into $\frac{1}{2}$-acre lots. How many lots will there be?" Students may solve this problem by using five hexagonal blocks to represent the five acres of land. They may then use a trapezoidal block to represent a $\frac{1}{2}$-acre lot and find the solution. (10 lots)
5. Have students write a problem for 4 divided by $\frac{1}{6}$ and then solve, using pattern blocks and graph/grid paper.
6. Have students write a general rule for dividing a whole number by a unit fraction.

Note: If needed, the activity may be stopped here and briefly reviewed the next day before continuing.

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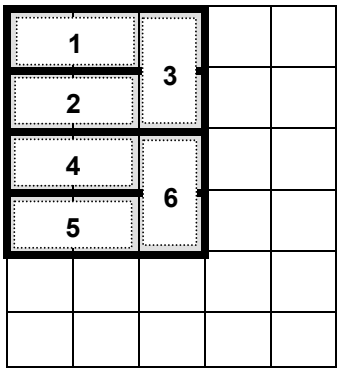
7. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: "Mark has four packs of paper and wants to repackage them for his Boy Scout project into packs that are each $\frac{2}{3}$ the size of each original pack. How many new packs will he have?"
   - Students may solve this problem by using four hexagonal blocks to represent the four original packs of paper. They may then use two rhombi to represent $\frac{2}{3}$ of a hexagonal block and find the solution. (six packs)
   - Students may represent the problem on graph paper as in the model at right:



$$4 \div \frac{2}{3} = 6$$

8. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: "For a science experiment, a class wants to cut six yards of yarn into $\frac{2}{3}$-yard pieces. How many pieces will they get?"
9. Have small groups of students write a problem for $8 \div \frac{2}{3}$, use pattern blocks to model it, and then represent it on graph paper to solve.
10. Have students write a general rule for dividing a whole number by $\frac{2}{3}$.

11. Have the students solve the following problems, using the procedures already established:
    a. $3 \div \frac{3}{4}$ b. $6 \div \frac{3}{4}$ c. $12 \div \frac{3}{4}$
12. Have students write a general rule for dividing a whole number by $\frac{3}{4}$. Ask, Why do you have to divide by three? Why multiply by four?
13. Have students develop a rule for dividing any whole number by any fraction that is less than one.

Sample assessment

- During the activity, observe students as you walk around the room and check for understanding.
- At the end of the activity, have students respond in their math journals to the following prompt: "Describe a rule for dividing a whole number by a fraction. Describe common circumstances in which people divide by fractions."

Follow-up/extension

- Encourage students to find examples of dividing by fractions in the real world. Another representation for pattern blocks is using available software. The following Web sites have "virtual" pattern blocks and activities:
    - http://www.arcytech.org/java/patterns/patterns_j.shtml
    - http://www.matti.usu.edu/nlvm/nav/frames_asid_169_g_1_t_2.html
    - http://step.k12.ca.us/community/fractions_institute/flash/pattern_block.html

Homework

- If this activity is used over two days, limit the first night's homework to problems involving the division of whole number by unit fractions. Answers should have whole number answers. Following completion of the activity, any problems assigned involving division of whole numbers by fractions less than one should result in whole-number answers.

Decimal Division

Reporting category Computation and Estimation

Overview Students explore division of decimals and gain understanding of decimal placement.

Related Standard of Learning 6.6

Objective

- The student will, given a dividend expressed as a decimal through thousandths, and a divisor expressed as a decimal to thousandths with exactly one non-zero digit, find the quotient.

Materials needed

- Decimal squares (easily made using Excel; create 10-by-10 squares)
- Scissors

Instructional activity

1. Review the parts of a division problem (quotient, dividend, divisor).
2. Ask students to share real-life situations in which knowledge of division of decimals is useful (e.g., sharing money and cutting objects into smaller pieces).
3. Explain to students that by shading and cutting apart decimal squares, that they can demonstrate division of decimals.
4. First, demonstrate a problem with a decimal dividend and a whole-number divisor. Give the problem 4.8 ÷ 4 by having students shade 4.8 decimal squares and then divide the squares into 4 equal groups. Ask them to tell how many are in each group (1.2). Explain that this is the quotient.
5. Repeat the process with several more examples, ensuring that students understand what is happening each time.
6. Ask students if they can also use this technique to solve problems that have decimal dividends and divisors.
7. Demonstrate the problem 7.8 ÷ 1.3. First, have students shade 7.8 decimal squares. Then have them divide the 7.8 into equal groups of 1.3. How many groups are there? (6 groups)
8. Ask students to compare this quotient with the quotient of 78 ÷ 13. (same) How do the divisor and dividend of each problem compare? (Second problem is 10 times larger than first.)
9. Repeat this process with several more examples.
10. Provide students with additional practice for homework.

Sample assessment

- Have students explain in their math journals the process of division of decimals, using the representations of the 10-by-10 grids.

Estimation Strategies

Reporting category Computation and Estimation

Overview Students explore various methods for estimation and understand when estimation is useful.

Related Standard of Learning 6.7

Objective

- The student will solve multi-step practical problems involving whole numbers and decimals by using estimation strategies and checking for reasonableness of results.

Materials needed

- "Lesson 2 Estimation Practice," one copy for each student

Instructional activity

1. Ask students to define *estimation* in their own words. Allow students to share their responses.
2. Ask students in what situations they might use estimation skills in the real world.
3. Give a variety of scenarios and ask students whether they would give an exact answer or an estimate (example scenario: the number of people attending a sporting event such as a pro football game.)
4. Place the following problem on the board or overhead and ask students to estimate the answer: 32,976 + 18,432. Allow students to share their methods of estimation and discuss the various methods (rounding to front number, front-end, compatible numbers, clustering, etc.)
5. Explain to students that there are several methods that can be used for estimation and that most methods are acceptable, although rounding is probably the most common method.
6. Review the various methods.
7. Complete several examples of estimation using a variety of methods. Allow students to compare the answers they found using the different methods. This can be done in small groups or whole group.
8. Discuss with students the benefits of estimating problems before solving them (allow for checking the reasonableness of an answer).
9. Find the actual answer of the problems demonstrated in class and compare with the estimates. Discuss how the answers are fairly close, so therefore, they are probably correct.

Sample assessment

- Distribute the "Lesson 2 Estimation Practice" worksheets to students, and have them complete the problems independently. Then have the students check with a partner and discuss answers. Caution students that if their methods of estimation were different, their answers may be slightly different.

Lesson 2 Estimation Practice

Name: _____ **Date:** _____

1. Estimate each of the following problems.
2. Indicate which method of estimation you used for each problem.
3. Solve each problem.
4. Compare your solutions with your estimations to check for reasonableness.

a. $42.7 + 68.9 =$

b. $3.001 + 0.957 =$

c. $3.02 - 1.99 =$

d. $112.25 - 36.49 =$

e. $28.5 \times 0.34 =$

f. $1.48 \times 0.55 =$

g. $78.6 \div .2 =$

h. $85.55 \div 2.5 =$

Estimation in Problem Solving

Reporting category Computation and Estimation
Overview Students use estimation strategies for problem solving.
Related Standard of Learning 6.7

Objective

- The student will solve multi-step practical problems involving whole numbers and decimals by using estimation strategies and checking for the reasonableness of results.

Materials needed

- Estimation problems on cards for groups of students
- "Lesson 3 Estimation Practice," one copy for each student
- "Lesson 4 Estimation Practice," one copy for each student

Instructional activity

1. Put each of the following estimation problems on a large index card to distribute to student groups:
   - Jeff made a total of $1,325 mowing lawns for 63 days this past summer. About how much money did he earn each day?
   - Each of the 47 students who enter the Blair County Math Competition needs to solve 32 different problems. About how many problems will the sponsors of the competition need to write for the competitors?
   - There are 187 seats in a local movie theater. The theater has been sold out for the past 18 shows. About how many people attended the shows?
   - Rhonda earned $371.09 this week. Her brother Rod earned $397.01. Who earned more money? How much more?
   - Ruby made a quilt using 552 squares. There were 24 rows of squares in the quilt. How many squares were in each row?
2. Review various methods for estimation.
3. Place students in groups of four, and assign each group an operation used to solve problems (addition, subtraction, multiplication, division). Provide each group with chart paper. Using the group's assigned operation, have them brainstorm key words found in word problems used to solve the problem.
4. Allow students to present their key words to the class.
5. Discuss the four steps for problem solving that should always be performed:
   1) Rephrase the question.
   2) List important facts.
   3) Solve the problem, explaining what you are doing and why.
   4) Check your final answer for reasonableness and accuracy.

   Stress the importance of using all steps in order to become a good problem solver.
6. Distribute to each group a multi-step practical problem or consumer application problem involving decimals. (Problems written on the index cards can be used.) Instruct each group to solve the problem on chart paper. Remind groups they should use the four-step process.

7. Once all groups have solutions to their problems, post around the room and conduct a walk-about. All groups should view all solutions and discuss.
8. Distribute the "Lesson 3 Estimation Practice" worksheets to students and have them complete independently using the four-step process. You may want to give them the four-step process on a worksheet to use for solving.

Lesson 3 Estimation Practice Key

Rudy had $64 to spend. He spent $18 on a haircut, $22 on shoes, and $7 for a movie ticket. About how much does Rudy have left?

Rephrase the question

- I need to know how much money Rudy has left.

Important facts

- Rudy started with $64.
- He spent $18, $22, and $7.
- I am estimating because the problem says *about.*

Solve it

- First I will round the numbers to the front place because I am estimating:
  - $64 = 60
  - $18 = 20
  - $22 = 20
  - $7 = 10
- Next I will add the amounts he spent because I need to find the total he spent:
  - $20 + $20 + $10 = $50
- Finally I will subtract the estimated amount he spent from the estimated amount he started with in order to find how much he has left:
  - $60 – $50 = $10.00
  - He has about $10 left.

Check it

- My answer is a reasonable answer.
- Check calculations by adding my answer to the estimated amounts he spent. They should equal the estimated total he started with:
  - $10 + $20 + $20 + $10 = $60

The theater of the Natural Science and History Museum was filled to capacity for its 276 shows this fall. The theater holds 35 people. About how many people attended the shows?

Rephrase the question

- I need to find out about how many people attended the shows.

Important facts

- There were 276 shows.
- The theater holds 35 people for each show.
- I am estimating because it says *about*.

Solve it

- First I will round the numbers to the front place because I am estimating:
  - 276 = 300
  - 35 = 40
- Next I will multiply because I am trying to find out how much is in 40 groups of 300:
  - 300 x 40 = 12,000
  - About 12,000 people attended the shows.

Check it

- I will check to see if my answer is reasonable. Yes, it is reasonable that 12,000 people would attend the theater.
- Next I will check my calculations by dividing my answer; 12,000 by 40, and it should equal 300:
  - 12,000 ÷ 40 = 300

Maria collected souvenirs from each place she visited on her trip. She collected 156 colored leaves and 124 rocks. About how many more colored leaves did she collect than rocks?

Rephrase the question

- I am trying to find out how many more colored leaves than rocks Maria collected.

Important facts

- Maria collected 156 leaves and 124 rocks.
- I will estimate because it says *about* how many more.

Solve it

- First I will round the numbers because it says *about* how many more:
  - 156 = 200
  - 124 = 100
- Next I will subtract because I am finding out how many more colored leaves than rocks were collected:
  - 200 – 100 = 100
  - About 100 more leaves than rocks were collected.

Check it

- First, I will ask myself if my answer makes sense. It is a little high but I did follow proper procedure so it makes sense.
- Next, I will check my calculations. I will add my answer to 100 and it should equal 200:
  - 100 + 100 = 200.

The Forest Service estimates that about 3,645 people visited the Cradle of Forestry in the past 31 days. About how many people visited the attraction each day?

Rephrase the question

- I am trying to find out about how many people visited Cradle of Forestry each day.

Important facts

- 3,645 people visited in 31 days.
- I will estimate because the problem says *about* how many people.

Solve it

- First, I will round the numbers to the front place because it says *about* how many people.
  - 3,645 = 4,000
  - 31 = 30
- Next, I will divide because I am breaking the large group of 30 days into smaller groups of 1 day.
  - 4,000 ÷ 30 = 133.333
- I will round my answer to the nearest one place because you can't have a part of a person.
- About 133 people visited each day.

Check it

- First, I will check to see if my answer is reasonable. Yes, it is reasonable that 133 people would visit each day.
- Next, I will check my calculations. I will multiply my answer by 30 and it should equal close to 4,000. It won't be exactly 4,000 because I rounded my final answer:
  - 133 x 30 = 3,990 (close to 4,000)

Lesson 4 Estimation Practice Key

The main span of the Greater New Orleans Bridge in Louisiana is 0.48 km long. Both approaches have a combined length of 3.613 km. The total length of the Astoria Bridge in Washington and Oregon is 6.613 km. Which bridge is longer? About how much longer?

Rephrase the question

- I need to know which bridge is longer and by about how much.

Important facts

- Greater New Orleans Bridge is 3.613 km.
- Astoria Bridge is 6.613 km.
- I will estimate because it says *about* how much longer.

Solve it

- First I will round the numbers to the front place because I am estimating:
  - 3.613 = 4 km
  - 6.613 = 7 km
- Next I will subtract the rounded amounts because I need to find the difference between the two:
  - 7 – 4 = 3 km
- The Astoria Bridge is longer by about 3 km.

Check it

- First, I will check to see if my answer is reasonable. Yes, I believe it is reasonable.
- Next I will check by adding my answer to the length of the shortest bridge and it should equal the length of the longest bridge:
  - 3 + 4 = 7 km

Sarah spent the following amounts at the candy store: $.38 on gum, $.78 on lifesavers, $.54 on jawbreakers, and $2.35 on candy bars. About how much money did Sarah spend?

Rephrase the question

- I need to know about how much money Sarah spent.

Important facts
- She spent the following amounts: $.38, $.78, $.54, and $2.35.
- I need to estimate because it said *about* how much money did she spend.

Solve it
- First I will round each amount
  - .38 = .40
  - .78 = .80
  - .54 = .50
  - 2.35 = 2.00
- Next, I will add the amounts because I need to know the total spent.
  - 2.00 + .50 + .80 + .40 = $3.70
- She spent $3.70

Check it
- I will decide if my answer is reasonable. It is.
- I will check by subtracting the amounts of each item from my answer. I should get zero:
  - $3.70 – $2.00 = $1.70
  - $1.70 - $.50 = $1.20
  - $1.20– $.80 = .40
  - $.40 – .40 = 0

Bobby bought the following items at the school store: 10 pencils for $.21 each, 8 pens for $0.45 each, and 2 posters for $0.55 each. About how much money did Bobby spend in all?

Rephrase the question
- I want to find out about how much money Bobby spent.

Important facts
- 10 pencils for .21 each
- 8 pens for .45 each
- 2 posters for .55 each
- I will estimate because it said *about* how much money did he spend.

Solve it
- First I will round each money amount to the nearest front number because I need to estimate:
  - .21 = .20
  - .45 = .50
  - .55 = .60
- Next I will find out how much was spent on each item by multiplying the rounded amount by the number of items bought for each amount:
  - .20 x 10 = $2.00
  - .50 x 8 = $4.00
  - .60 x 2 = $1.20
- Next I will add the above amounts to find the estimated total spent:
  - $2.00 + 4.00 + 1.20 = $7.20
- Bobby spent about $7.20.

Check it

- First, I will check to see if my answer is reasonable. It is.
- Then I will check my calculations. They are O.K.

Betsy made ribbons for school spirit day. Her roll of ribbon was 12 feet long. For each individual ribbon, she needs .65 ft. About how many ribbons can she make from her roll?

Rephrase the question

- I am trying to find out about how many ribbons Betsy can make.

Important facts

- A roll of ribbon is 12 feet.
- Each ribbon takes .65 ft.
- I will estimate because it says *about* how many ribbons.

Solve it

- First I will round the numbers to the front number because I am estimating:
  - 12 = 10
  - .65 = .7
- Next I will divide the total roll by the amount for each ribbon because I am cutting the roll into smaller pieces:
  - 10 ÷ .7 = 14.285714
- Betsy can make about 14 ribbons with one roll of ribbon. The decimal part doesn't count because I need whole pieces of ribbon.

Check it

- First, I will see if my answer is reasonable. Yes, it is.
- Next, I will check my calculations. I will multiply my answer times .7 and it should equal about 10 (not exactly because of the decimal answer):
  - 14 x .7 = 9.8 (close to 10)

Sample assessment

- Allow students to write problems for their partners to solve.

Follow-up/extension

- The "Lesson 4 Estimation Practice" worksheet can be used for additional follow-up to this lesson

Lesson 3 Estimation Practice

Name: _____ Date: _____

Solve each problem shown below. Be sure to show all steps: 1) rephrase the question in your own words, 2) list the important facts, 3) solve it (with explanations of what and why you did), and 4) check it. If you need to, show your work on a separate sheet.

1. Rudy had $64 to spend. He spent $18 on a haircut, $22 on shoes, and $7 for a movie ticket. About how much does Rudy have left?

2. The theater of the Natural Science and History Museum was filled to capacity for its 276 shows this fall. The theater holds 35 people. About how many people attended the shows?

3. Maria collected souvenirs from each place she visited on her trip. She collected 156 colored leaves and 124 rocks. About how many more colored leaves did she collect than rocks?

4. The Forest Service estimates that about 3,645 people visited the Cradle of Forestry in the past 31 days. About how many people visited the attraction each day?

Lesson 4 Estimation Practice

Name: _____ Date: _____

Solve each problem shown below. Be sure to show all steps: 1) rephrase the question in your own words, 2) list the important facts, 3) solve it (with explanations of what and why you did), and 4) check it. If you need to, show your work on a separate sheet.

1. The main span of the Greater New Orleans Bridge in Louisiana is 0.48 km long. Both approaches have a combined length of 3.613 km. The total length of the Astoria Bridge in Washington and Oregon is 6.613 km. Which bridge is longer? About how much longer is it?

2. Sarah spent the following amounts at the candy store: $0.38 on gum, $0.78 on lifesavers, $0.54 on jawbreakers, and $2.35 on candy bars. About how much money did Sarah spend?

3. Bobby bought the following items at the school store: 10 pencils for $.21 each, 8 pens for $0.45 each, and 2 posters for $0.55 each. About how much money did Bobby spend in all?

4. Betsy made ribbons for school spirit day. Her roll of ribbon was 12 feet long. For each individual ribbon she need .65 ft. About how many ribbons can she make from her roll?

Attacking Word Problems

Reporting category	Computation and Estimation
Overview	This lesson describes a brainstorming activity in which students identify words and/or phrases in word problems that allude to which operation is to be performed (addition, subtraction, multiplication, or division).
Related Standard of Learning	6.7

Objective

- The student will be able to work in groups to produce lists of words and/or phrases that indicate the operations of addition, subtraction, multiplication, and division.

Materials needed

- Classroom mathematics textbook(s)
- Paper and pencils
- "Attacking Word Problems," one copy for each student

Instructional activity

1. Divide students into groups of two to four.
2. Tell each group to select a word problem found in their math textbook or other math textbooks available in the room and to identify words and phrases that indicate they should add, subtract, multiply, or divide to find the answer. Model one example on the board to help students begin.
3. Although students are working together, have them record the group findings on their own handout for future reference.
4. Have the groups repeat the process with several other word problems.
5. After allowing students to brainstorm in their cooperative learning groups, have the whole class make a master list by having each group contribute their findings.
6. Follow up by repeating the activity the next day, adding additional contributions until the students have created the most complete list possible (optional).
7. Have students copy all words/phrases from the class master list and keep this personal master list for reference throughout the year.

Sample assessment

- Use the activity for a class work or homework grade. Make sure to return students' papers so that they can use the phrases as a reference tool all year long.

Sample resources

Solving Math Word Problems http://www.iss.stthomas.edu/studyguides/MATHPROBLEMS.htm

Key Words and Catch Phrases for Word Problems http://www.mathstories.com/key_words.htm

Attacking Word Problems

Name: _____ **Date:** _____

Words/Phrases

Add	Subtract	Multiply	Divide

Planning a Budget

Reporting category Computation & Estimation
Overview Students plan a budget, using estimation and computation skills.
Related Standards of Learning 6.7, 6.8

Objectives

- The student will use estimation strategies to solve multi-step practical problems involving whole numbers and decimals.
- The student will solve multi-step consumer application problems involving decimals and present data and conclusions in paragraphs, tables, or graphs. Planning a budget will be included.

Materials needed

- A variety of newspaper ads from several business
- Play money (If you create your own with pictures of teachers instead of presidents, students really love it.)

Instructional activity

1. Lead a discussion with students on money and the importance of knowing how to budget it.
2. Ask students to give examples of when they see their parents working on a budget.
3. Encourage students to share personal experiences about when they had money and had to plan how they would spend or save it. Ask students what they do when they want to make a big purchase and do not have enough money.
4. Create a shopping list, using ads from the newspaper, and distribute to students.
5. Place students into small groups of two to four, and give each group a certain amount of "money" based on the shopping list. It is important that the items on the shopping list be found in more than one ad so students will have to make choices.
6. Instruct students to go "shopping," using their ads and to purchase each item on the list. Their purchases must meet their budget restrictions — they may not spend more than they have. Encourage students to be wise shoppers by rewarding the group that has the most money left over after completing the shopping list.
7. Allow students to share the purchases they made. Remind them to discuss how they made their choices and when they had the most difficulty in finalizing their choices.
8. Review situations in which budgeting skills are needed in real life.

Sample assessment

- Have students record in their math journal the process of creating a budget and the benefits in having a budget.

Follow-up/extension

- Try including discount coupons and calculating tax in this activity to provide additional enrichment.

Pizza Your Way

Reporting category	Computation and Estimation
Overview	Students be able to use real-life situations not only to add, subtract, multiply, and divide decimals, but also to use percents and ratios by buying pizza and determining how much a pizza "really" costs.
Related Standards of Learning	6.6, 6.7

Objective

- The student will be able to use real-life situations not only to add, subtract, multiply, and divide decimals, but also to use percents and ratios.

Materials needed

- Internet connection
- Pencil
- Paper
- Supervised access to the Internet Pizza Server Web site http://www.ecst.csuchico.edu/%7Epizza/

Instructional activity

1. Give the following scenario to your students: "Now that you are a high school senior, your parents can go away for the weekend knowing it'll be okay if you have a *few* friends over for the evening. Your parents leave you only $30. Your mom doesn't have anything in the house to eat, so you need to get some food. You decide to order pizza. You want a mixture of two-toppings pizzas. Between your friends (10 of them), they can come up with only $10. Everyone is very, very hungry. What is the largest amount of two-or-more-toppings pizzas that you can order?"
2. Using the Pizza Menu on the Web site, have the students decide on the number of pizzas and toppings that they would be interested in ordering. Remember that each pizza has to have at least two toppings.
3. After they have decided how many pizzas to order and which toppings will be included, remind the students that they will have to include tax. Assume the tax rate is 8.25%. Find the tax on the pizzas.
4. List the subtotal and total on the pizzas.
5. The students can't drive, so they will have to give the delivery person a tip. We will assume that this is a set rate and it will be 9% of the total.
6. Now find out how much one piece of pizza "really" costs.
7. Asks questions such as, If Randy ate 7 pieces, what is the ratio of the number of pieces that he ate? What percent is this of the total? If Niles ate 34% of the pizza, how many pieces did he eat? Who ate the least/greatest amount of pizza? Write a ratio for the least/greatest amounts of pizzas eaten by one person. What percents did the person who ate the least/greatest amounts eat? Continue with the questions. You can now let your students "order" their pizza over the Internet. Each student can only order one pizza at a time. http://www.ecst.csuchico.edu/%7Epizza/ Order Up!!

Getting the Most for Your Money!

Reporting category Computation and Estimation

Overview Students create their own pizza, using choices of toppings. They "order" their creation from the Internet and see a digitalized version of their pizza. They use their order to calculate the area of various sizes of pizzas, determine the "better buy," and cost per topping. They use research skills to answer questions pertaining to the Internet Pizza Server Web site.

Related Standard of Learning 6.8

Objective

- The student will calculate the area of a pizza and determine the best buy and cost per topping.

Materials needed

- Supervised access to the Internet Pizza Server Web site http://www.ecst.csuchico.edu/%7Epizza/
- "Buying Pizza—Determine the Best Buy!" one copy for each student
- A metric ruler

Prior Knowledge

- Students must be able to calculate the area of a circle. In order to complete the research portion of the lesson, they should have a very basic understanding of how to "move" through the Internet. (Click once on colored text will send you to that site. To get back to where you were, click on the "back" button.)

Instructional activity

1. This lesson develops the idea of going to a pizza restaurant, ordering a pizza, and determining whether a small, medium, large, or family size pizza is a "better-buy." Students will first need to calculate the area of each of the pizzas. (This information is given in pixel measurements within the document, but I would suggest that the students merely measure each pizza on-screen.) Students will be asked to calculate the price/per/topping and determine whether this price is "fair."
2. Have students "search" the pizza servers' home page. They will need some time to experiment with various pizza toppings, and to read the directions on navigating through the steps. The directions are very well written. There is a considerable amount of text, so be sure to give the students enough time to read the material.
3. Ask students questions regarding their research, and have them calculate various costs regarding the pizza selections. I strongly recommend that their answers be supported with explanations on how they determined their answers. The students begin by going to *http://www.ecst.csuchico.edu/~pizza/* **(Note that this link contains access to sites that are open for free responses. The teacher should maintain close supervision.)**

Follow-up/extension

- Have the students write two or three additional questions that could be included in this lesson, including the answers to the questions and the URL they used to determine the answers.

Buying Pizza — Determine the Best Buy!

Problems and Questions

Note: Answers for some of the problems depend on individual screen size. All support work should be included with your calculations. You should include a copy of the pizza you "ordered," This must be printed from the Pizza Server Home Page. You may perform your calculations from the printed page if you choose.

1. Order a small, medium, large, and family-size pizza. Use a metric ruler and measure the diameter in centimeters of a small, medium, large, and family-size pizza. Calculate the area for each size pizza. Record this data.

2. What is the "base price" (price for pizza without any toppings) of a small, medium, large, and family-size pizza?

3. Calculate the price per topping. (Does this price change in relationship to the size of the pizza? How does this compare to real life?)

4. What size pizza with two toppings is the better buy? Explain your answer algebraically, and support with an explanatory paragraph.

5. What is the most expensive pizza that can be ordered? How did you determine your answer? How much would this same pizza cost if the only thing changed was the size to small?

6. What was your favorite topping?

7. When was this pizza page developed and what format was used? When was it posted on the World Wide Web?

8. How do you pay for pizza? What is the conversion rate between various countries? (Give at least two examples.)

Sample resources

Mathematics SOL *Curriculum Framework*

Thinking Rationally about Fractions, Decimals, and Percent: Instructional Activities for Grades 4 through 8 – Lesson plans available from VDOE at http://www.pen.k12.va.us/VDOE/Instruction/Math/FractionsDecimalsPercent.pdf.

NCTM. *Principles and Standards for School Mathematics*, pages 214–221.

http://www.iss.stthomas.edu/studyguides/MATHPROBLEMS.htm – *Solving Math Word Problems.* Strategies and key words that will help with solving word problems.

http://www.mathstories.com/key_words.htm – Math Stories.com. Key words and catch phrases for solving math problems.

http://math.rice.edu/~lanius/fractions/ – *Who Wants Pizza?* A fun way to learn about fractions.

http://illuminations.nctm.org/lessonplans/6-8/productgame/index.html – *The Product Game.*

http://mathforum.org/paths/fractions/m.fraclessons.html – Middle School Lessons and Materials for Teachers – Fractions.

http://www.mathguide.com/lessons/Integers.html – Operations with Integers from Mathguide.com.

Sample released test items

1. Which of the following is *not* true?

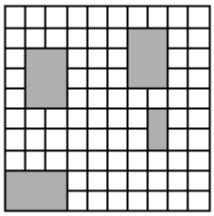
F $\frac{2}{5} = 0.4 = 40\%$

G $\frac{7}{10} = 0.7 = 70\%$

H $2\frac{1}{2} = 2.5 = 25\%$

J $3\frac{3}{8} = 3.375 = 337.5\%$

4. The shaded part of the square can be expressed by —



A 0.02

B 20%

C $\frac{1}{4}$

D $\frac{2}{5}$

6 Which is an integer?

F -19

G 1.5

H $\sqrt{2}$

J $\frac{1}{3}$

10 A recipe calls for $3\frac{1}{4}$ cups of flour and $1\frac{1}{2}$ cups of sugar. If the recipe is doubled, how much flour will be required?

F 3 cups

G $3\frac{1}{4}$ cups

H $4\frac{3}{4}$ cups

J $6\frac{1}{2}$ cups

12 Mark layered 3 pieces of wood to build the base for a lamp. The pieces were $\frac{1}{4}$ inch thick, $\frac{5}{8}$ inch thick, and $\frac{3}{16}$ inch thick. How thick was the base for the lamp?

F $\frac{7}{16}$ in.

G $\frac{9}{16}$ in.

H $\frac{15}{16}$ in.

J $1\frac{1}{16}$ in.

14 At the video rental shop, 5 movies can be rented for $6.99. Each additional movie rental is $1.99. What would be the total cost of renting 8 movies?

F $12.96

G $13.98

H $15.92

J $25.87

Organizing Topic Number Theory

Standard of Learning

6.3 The student will
a) find common multiples and factors, including least common multiple and greatest common factor;
b) identify and describe prime and composite numbers; and
c) identify and describe the characteristics of even and odd integers.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Identify common multiples and the least common multiple for up to three numbers less than or equal to 50.	
• Identify common factors and the greatest common factor for up to three numbers less than or equal to 50.	
• Identify which numbers are prime for numbers less than or equal to 50.	
• Identify which numbers are composite for numbers less than or equal to 50.	
• Explain orally and in writing why a number is prime or composite.	
• Explain orally and in writing why a given integer is even (divisible by two) or odd (not divisible by two).	

Partners and Leftovers

Reporting category Number and Number Sense
Overview Students investigate even and odd integers, using color tiles
Related Standard of Learning 6.3

Objective

- The student will describe characteristics of even and odd integers.

Materials needed

- Bags of color tiles, enough for each student to grab a handful

Instructional activity

1. Explain to students that they will be investigating characteristics of certain types of numbers. Ask students to grab a handful of color tiles from the bag. Begin the lesson by asking students to organize the tiles in an arrangement of two rows, matching each tile with another. Some students will have an extra tile. Make two columns on the board, one for numbers with partners, and one for numbers with a "leftover." Have students call out the number of tiles they grabbed from the bag and whether or not they had a leftover. Record the numbers on the board in the appropriate column.
2. Ask students if they notice any patterns in the numbers in the "partner" column. Explain to students that these numbers are referred to as even. Ask them if they can tell you what digits would be in the ones column in any even integer. (2, 4, 6, 8, 0) Have students examine the numbers in the "leftover" column and ask if they can describe the patterns in these numbers. Explain to students that these numbers are referred to as odd. Ask if they can figure out what digits would be in the ones column in any odd integer. (1, 3, 5, 7, 9).
3. Have students solve the following problems as they explore even and odd integers. Have students use the tiles and drawings to explain why these sums occur.
    - Is the sum of two even numbers even or odd?
    - Is the sum of two odd numbers even or odd?
    - Is the sum of one odd number and one even number even or odd?
    - Is the product of two even numbers even or odd?
    - Is the product of two odd numbers even or odd?
    - Is the product of one odd number and one even number even or odd?

Sample assessment

- Have students record in their journal the characteristics of even and odd integers, using drawings and symbols to explain their reasoning.

Sieve of Erastosthenes

Reporting category Number and Number Sense

Overview Students explore prime and composite numbers, using the Sieve of Erastosthenes

Related Standard of Learning 6.3

Objective

- The student will identify and describe prime and composite numbers.

Materials needed

- Grid paper
- Hundred number chart
- "Rectangles Chart," one copy for each student

Instructional activity

1. Explain to students that they will be investigating characteristics of certain types of numbers, using rectangles on their grid paper. Using an overhead sheet of grid paper, outline a 1-by-2 rectangle. Explain to students that this rectangle has been created with two square units, and has dimensions of 1 linear unit and 2 linear units. (Depending on the orientation of the rectangle, the dimensions could also be 2 and 1.) On the chart next to the number *2,* record the dimensions (1, 2) in the factors column. Explain to students that the whole number dimensions of this rectangle are also called "factors" of the number.
2. Have students create the rectangle with three square units. Ask what the dimensions of this rectangle are and record on the chart next to the *3.* Indicate that so far they have only been able to create one rectangle for each number and that the dimensions have been the number and 1. Practice with one more rectangle. Have students create the possible rectangles with four square units. Have students share the rectangles they created – one with dimensions of 4 and 1, and one with dimensions of 2 and 2.
3. Have groups of students complete the task by continuing to create rectangles, recording the dimensions and the number of rectangles that can be created for square units, for the numbers from 5 through 25. Each group can be assigned to work with a few numbers, and the data can be collected from each group once the task is completed.
4. Record all the data on the overhead chart, and have students discuss what they notice about the different numbers. They should notice that only one rectangle could be created with some of the numbers. Have students look for these numbers on the chart and discuss the characteristics of the factors of these numbers. Note that the factors of these numbers (dimensions of these rectangles) are only the number and one. Ask students if they know the name of these types of numbers. (prime). *Prime numbers* are defined as having exactly two unique factors. Explain that the other numbers, those that have more than two factors, are called *composite.*
5. Share with students that the number 1 has one factor (itself) and forms one rectangle (a 1 by 1 square); it is classified by mathematicians as a special number and is neither prime nor composite.
6. Have the students look at the rectangles that were made to represent 1, 4, 9, 16, and 25. Ask them to notice that one of the rectangles formed with the factors of these numbers is also a square.

7. For the second part of this lesson, tell students that Erastosthenes was an ancient Greek mathematician who studied prime and composite numbers. He used a method now called the Sieve of Erastosthenes. Distribute the hundreds chart to the students, and have them proceed through the following directions to locate the prime and composite numbers.
   - Since the number 1 is not prime, color it purple on the hundreds chart.
   - Circle the first prime number, 2, with your pencil. Color every multiple of 2 on your chart yellow. Do not color 2 itself.
   - Circle the next prime, 3. Color the multiples of 3 on your chart red. (Some multiples of 3, such as 6 and 12 may already be colored yellow. Ignore these and look for the uncolored multiples. When you finish, you should have 16 red squares.)
   - Circle the next prime, 5. Color any uncolored multiples of 5 blue. (You should have 6 blue squares.)
   - Circle the prime number, 7. Color any uncolored multiples of 7 green. (You should have only 3 green squares.)
   - Count the uncolored squares on your chart. Can you find 25 of them? If you can, then you have sifted out all the prime numbers under 100.

Sample assessment

- Have students record in their math journal the characteristics of prime and composite numbers and a description of the process used to find the primes, using the Sieve of Erastosthenes.

Rectangles Chart

Name: _____ **Date:** _____

Number	Factors	Number of Rectangles	Prime or Composite
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

Least Common Multiples and Greatest Common Factors

Reporting category Number and Number Sense

Overview Students explore volumes of prisms, using cubes, to discover least common multiples and greatest common factors.

Related Standard of Learning 6.3

Objectives

- The student will find common multiples and the least common multiple by constructing prisms and comparing their volumes.
- The student will find common factors and the greatest common factor by constructing rectangular prisms and comparing their dimensions

Materials needed

- 1-inch cubes, at least 40 per group of two students.
- "Greatest Common Factor and Least Common Multiple Recording Sheet," one copy for each student

Instructional activity

Part I

1. Distribute the cubes to each pair of students. Explain to students that they will be exploring volume of rectangular prisms. Review the concept of volume and rectangular prisms with students. Tell students to take 12 cubes and construct a rectangular prism. Have students share the various prisms that can be constructed; 12 cubes in 1 row, 6 cubes in 2 rows, 3 cubes in 4 rows, and 2 layers of cubes with 2 cubes in 3 rows. Record the various dimensions on an overhead chart. Explain that these dimensions correspond to the factors of 12: 1, 2, 3, 4, 6, and 12. Remind them that to find the volume of the figure, they multiply the height times the length times the width. For example, the prism created with 12 cubes in one row, has a volume of 12 (w) × 1 (l) × 1 (h).
2. Have students take 16 blocks for the next task. They are to construct all the rectangular prisms that have a volume of 16. Have students share the possible dimensions of the prisms they created. (1 × 1 × 16, 1 × 2 × 8, 1 × 4 × 4, and 2 × 2 × 4).
3. Remind students that for each number (12 and 16) they were able to create four possible rectangular prisms. Have students record the factors for each volume on the chart. Have students circle the greatest factor that appears in both lists of factors. Tell students that this number is called the greatest common factor.
4. Give students other pairs of numbers to find the greatest common factor, using the cubes.
5. Explain to students that the process that they have used to find the greatest common factor is helpful in simplifying fractions. Instead of using the dimensions of the prisms, students can determine the prime factors of both the numerator and denominator of the fraction, find the prime factors that are common to each and multiply them. This becomes the greatest common factor of the two numbers. In order to simplify the fraction, both the numerator and denominator are divided by the greatest common factor.

6. Have students record in their math journals the connections that can be made among the number of possible prisms created, the factors for the volume, and the concept of greatest common factor.

Part II

1. For the second part of the activity, students will explore the least common multiple of two numbers. Have the students begin with a layer of cubes that are 2×3, creating a volume of $1 \times 2 \times 3$. Have them compare this prism to one that has a layer of cubes that are 3×3, creating a volume of $1 \times 3 \times 3$. Ask the students what these two prisms have in common. They both are one layer high and have a common dimension of 3. Have students calculate the volume (or count the cubes) to discover that one has a volume of 6 and the other has a volume of 9. Have the students stack another layer of cubes on top of each prism. Ask them to record the new dimensions and the new volumes; $2 \times 2 \times 3$ with a volume of 12 and $2 \times 3 \times 3$ with a volume of 18. Have the students add one more layer to the smaller prism to make its dimensions $3 \times 2 \times 3$. They should also notice that both prisms now have equal volumes and they share the same dimensions. Tell students that 18, the volume, is the least common multiple of 6 and 9.
2. Have students follow the same process to find the least common multiple of 10 and 15. They will start with a layer of blocks 2×5 and 3×5 and continue to add layers to each prism until they discover the least common multiple.
3. Explain to students that the process they have used to find the least common multiple is helpful in determining a common denominator when adding fractions with unlike denominators. Instead of adding the layers of blocks to find the multiples, students can represent that step of the process by listing the multiples of the two denominators until they find a match, which is the least common multiple of the two numbers.

Sample assessment

- Have students describe in their math journals the process of using the volume of the prisms to find the least common multiple.

Greatest Common Factor and Least Common Multiple Recording Sheet

Name: _____ **Date:** _____

GREATEST COMMON FACTOR

Volume	Dimensions	Number of Prisms
12		
16		

Volume	Factors	Greatest Common Factor
12		
16		

LEAST COMMON MULTIPLE

	Prism 1	Prism 2
Dimensions of first layer		
Volume of first layer		
Volume after 2nd layer		
Volume after 3rd layer		
Dimensions of final prism		

Greatest Common Factor and Least Common Multiple Recording Sheet
Answer Key

GREATEST COMMON FACTOR

Volume	Dimensions	Number of Prisms
12	1 × 1 × 12 1 × 2 × 6 1 × 3 × 4 2 × 2 × 3	4
16	1 × 1 × 16 1 × 2 × 8 1 × 4 × 4 2 × 2 × 4	4

Volume	Factors	Greatest Common Factor
12	1 2 3 4 6 12	4
16	1 2 4 8 16	

LEAST COMMON MULTIPLE

	Prism 1	Prism 2
Dimensions of first layer	1 × 2 × 3	1 × 3 × 3
Volume of first layer	6	9
Volume after 2nd layer	12	18
Volume after 3rd layer	18	-
Dimensions of final prism	3 × 2 x 3	2 × 3 × 3

Sample resources

Mathematics SOL Curriculum Framework.

Thinking Rationally about Fractions, Decimals, and Percent: Instructional Activities for Grades 4 through 8 – Lesson plans available from VDOE at http://www.pen.k12.va.us/VDOE/Instruction/Math/FractionsDecimalsPercent.pdf.

NCTM. *Principles and Standards for School Mathematics,* pages 214-221.

http://illuminations.nctm.org/lessonplans/6-8/productgame/index.html – This Web site contains a game in which students start with factors and multiply to find the product, to develop understanding of factors and multiples. It contains a link to The Factor game in which students find the factors of numbers (http://illuminations.nctm.org/imath/6-8/FactorGame/index.html).

http://www.mathforum.org/alejandre/frisbie/locker.html – This Web site contains a fairly extensive group project in which students in groups use manipulatives and spreadsheets to help unravel a pattern based on multiples in opening and closing a thousand lockers. It also contains links to other "locker problems."

http://www.utm.edu/research/primes/index.html – The Prime Pages: Prime Number Research, Records, and Resources.

Sample released test items

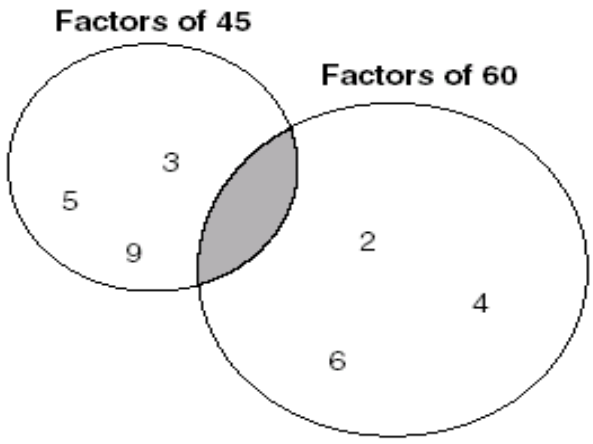
1. How many factors does a prime number have?

A 0
B 1
C 2
D 3

4. Which of the following is not a prime number?

F 2
G 5
H 17
J 121

5 Loretta is filling numbers in the Venn diagram. No number is to be entered more than once.



What is the greatest number that can be appropriately placed in the shaded area of the diagram?

A 5
B 10
C 15
D 180

Organizing Topic Proportional Reasoning

Standard of Learning

6.2 The student will describe and compare two sets of data, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a:b*.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Describe a relationship within a set by comparing part of the set to the entire set.	
• Describe a relationship between two sets by comparing part of one set to a corresponding part of the other set.	
• Describe a relationship between two sets by comparing all of one set to all of the other set.	
• Describe a relationship within a set by comparing one part of the set to another part of the same set.	
• Represent the relationship that makes a comparison by using the notations $\frac{a}{b}$, *a:b*, and *a* to *b*.	

Exploring Ratio

Reporting category Number and Number Sense
Overview Students explore various uses of ratios
Related Standard of Learning 6.2

Objectives

- The student will investigate various ways to record ratios
- The student will record describe and compare two sets of data using ratios.

Materials needed

- Two color counters, number cubes, people

Instructional activity

1. Use students in the classroom to collect and compare data. For example, compare the number of boys to girls, students to teacher, and colors of shoes.
2. Survey classmates to determine their favorite colors and write ratios from results.
3. Use ratio tables to solve word problems.
4. Have students, working in pairs, draw four cards. Use the numbers on cards to write two different ratios. Students will exchange ratios and find equivalent ratios. Repeat the activity, using different cards.

Sample assessment

- Have students record in their journals one situation in which they would use a ratio. Have them describe the situation and use a ratio to compare the data.

Paper Chains and Countries

Reporting category Number and Number Sense

Overview Students learn to represent comparisons of numbers as ratios by creating paper chains and dividing counters into groups.

Related Standard of Learning 6.2

Objectives

- The student will represent a relationship by comparing one set to another set.
- The student will represent the relationship by using the proper notations.

Materials needed

- 12 red paper strips for each student
- 12 blue paper strips for each student
- Tape or stapler for each student
- 24 buttons (or similar counters) for each student

Instructional activities

Preparation: Cut red and blue paper strips to be used as links for a chain to represent ratios. Determine a variety of ratios that can be made using a total of 12 red and 12 blue links.

Activity A

1. Draw three circles and four squares on the chalkboard.
2. Ask students what fraction of the shapes on the chalkboard are circles. ($\frac{3}{7}$)
3. Tell students they will be learning a new way to compare quantities (or numbers). Explain that a *ratio* compares the number or size of one item to the number or size of another related item.
4. Write *circles:squares* on the chalkboard. Tell students that this ratio compares circles to squares. Show students how to replace the words *circles* and *squares* with the actual number of each shape to create the ratio 3:4.
5. Tell students that the correct way to read the ratio is to say that the number of circles to squares is "3 to 4."
6. Discuss how the ratio is related to the fraction.
7. Discuss other quantities that are often represented as ratios, such as miles per hour, free throws hit to free throws missed, probabilities, etc.

Sample assessment

- When the students have completed the activity, ask the following questions:
  - How many total links would you need to represent a ratio of 4:7?
  - How did you determine your answer?
  - Could the ratio 4:7 be represented with another total number of links?
  - How would you determine another total number of links that could represent the ratio 4:7?
  - What strategies did you use to divide the buttons into groups to represent the given ratios?

Activity B

1. Tell students that they will be using links of a paper chain to represent different ratios and comparing them to the fractions that are associated with them.
2. Distribute the paper strips and stapler or tape to each student.
3. Tell students that you will demonstrate how to create a chain with the same ratio of red to blue links as the ratio of circles to squares on the chalk board. Write *red:blue* on the chalkboard and emphasize the importance of the order of the ratio. Create a chain with three red links and four blue links.
4. Add three more red links and four more blue links to the chain and ask students the ratio of the red: blue links. Tell students that the ratio can still be written as 3:4 because for every three red links there are exactly four blue links. Demonstrate by counting off in groups of three red links and four blue links.
5. Have students create chains to represent the ratios that you determined. After each chain is created, discuss how the ratio of red:blue compares to the fraction that represents the number of red links and the fraction that represents the number of blue links.
6. Give the students 24 buttons.
7. Tell students that they will be dividing the 24 buttons into two groups to represent ratios. Tell them that they should use all of the buttons to represent each ratio.
8. Tell them to divide the buttons into two groups to represent the ratio 1:3. Discuss the strategies used in dividing the buttons.
9. Repeat step 8, using the ratios 1:1, 2:4, 5:3, 7:5, and 1:2.

Animals Count

Reporting categories Number and Number Sense; Computation and Estimation; Patterns, Functions, and Algebra

Overview Students simulate a method employed by biologists to estimate animal populations, using beans to represent the animals. The method, called "capture-tag-recapture," uses red and white beans to represent tagged and untagged populations of deer. During the simulation, proportions are developed for use in determining the total number of the entire population.

Related Standard of Learning 6.2

Objective

- The student will use previously learned techniques for reasoning about ratios and proportions to estimate the size of a population when they cannot count the entire population.

Materials needed

- Small jar
- Large jar or other container with lid for each group of students
- Enough white beans to fill the jar of each group
- 100 red (kidney) beans for each group
- Paper cups or scoops for scooping beans
- "Animals-Count Recording Sheet," one copy for each group

Note: Two types (colors) of Goldfish™ crackers or other equal-sized objects can be substituted for beans.

Instructional activity

1. *Initiating Activity:* Discuss with the class: "How do wildlife officials determine how many sea otters there are in the United States?" "How do environmentalists decide whether an animal or plant should be on the endangered species list?" Answers will vary. Discuss scientific methodology for performing accurate counts of wildlife.
2. Demonstrate the experiment, using a small jar that is three-fourths full of white beans. Remove 10 beans and replace them with red beans. Cover the jar, shake, and remove a sample handful of beans.
3. Ask students to count the number of red beans and white beans in the sample. How can they set up a meaningful ratio using these numbers?

   $$\text{Ratio} = \frac{\text{number of red beans in sample}}{\text{total number of beans in sample}}$$

   Be sure the students recognize that at this point they know a) the number of marked (red) beans in the whole population and b) the number of marked (red) beans in the sample.
4. Ask the students to use what they know about making comparisons with ratios to estimate the total number of beans in the jar. Record their proportion and their estimate. Students should be able to record a proportion that shows the total number of red beans drawn in four trial samples related to

the total number of all beans drawn in the four samples. This ratio is equal to the ratio of the total number of red beans (100) to the total population, which is the number being sought, x.

$$\text{Proportion: } = \frac{\text{total number of red beans (100)}}{\text{total population } (x)}$$

5. Have a group count the number of beans in the jar to see how close the estimate was to the actual number of beans.
6. Discuss with students the methodology shown by the experiment, which models what really happens when the capture-tag-recapture method is used to estimate the number of deer in a large area. Some factors that must be considered are tagging deer from several places in the area under study, taking a sufficient sample for tagging, allowing the tagged animals time to mix thoroughly with the population, and taking the final samples from several places in the area.
7. *Simulation Activity:* Provide each group of three or four students with a jar with a lid, a large number of white beans, and an "Animals-Count Recording Sheet." Model the first trial of the deer-population experiment with the students, making sure that they know how to record their results.
8. Ask the groups of students: How are you mixing the beans? How are you keeping track of what you know? How might the number of samples you take affect your estimate?
9. Have each group remove 100 white beans from the jar and set them aside.
10. Have them place 100 red beans representing the tagged deer into the jar to replace the white beans removed.
11. Have them shake the jar to mix the beans and then scoop out a cupful of beans without looking at them.
12. Have them record on the recording sheet the number of red beans and the total number of beans in the sample.
13. Have them repeat this scoop-and-count procedure three more times. In each case, have them record on the recording sheet the number of red beans and the total number of beans in the sample.
14. *Closing Activity:* Have the groups study the data they collected and use the data to estimate the number of beans in the jar. Have each group report, explaining how they made their estimate and showing any calculations they made. The proportions should use the reasoning about the tagged deer and the total population as shown in the sample (see Initiating Activity, steps #3 and 4).

Sample assessment

- As student groups are working, circulate in the room to make sure they are accurately recording all needed information and to assess their understanding of ratios.
- As groups report, make sure their proportions make sense mathematically. Students should be able to articulate their answers and how they relate to the proportionality of the capture-tag-recapture method.

Follow-up/extension

- Additional problems using this method could be assigned for students struggling with forming their proportions. For instance, similar real-life problems might involve fish in a lake or beavers in a wooded area.

Animals-Count Recording Sheet

To simulate a method employed by biologists to estimate animal populations, we will perform an experiment using beans to represent the animals. The method is called "capture-tag-recapture." In this simulation, the red beans represent the deer that are tagged by the conservation officer and the white beans represent the deer that are not tagged. All the beans in the jar represent the total population of deer in a given region, which is the unknown quantity we seek to estimate.

Part One

1. Fill a jar with a large number of white beans.
2. Remove 100 white beans from the jar and set them aside.
3. Place 100 red beans into the jar to replace those white beans removed. These red beans represent the tagged deer.
4. Put a lid on the jar and shake it to mix the beans.
5. Without looking at the beans, scoop out a sample cupful of beans.
6. Count and record in the chart below the number of red beans and the total number of beans in your sample.
7. Return the sample to the jar.
8. Shake the jar again to mix the beans.
9. Repeat the scoop-and-count procedure three more times. For each trial, record the number of red beans and total number of beans in the sample.

Study the data you collected. Use the data to estimate the total number of beans (the total deer population) in your jar. How did you decide? Explain how you made your estimate, show your calculations, and explain your strategy.

	Sample 1	**Sample 2**	**Sample 3**	**Sample 4**
Number of Red Beans in Sample				
Total Number of Beans in Sample				
Ratio				

Gridlock

Reporting categories Number and Number Sense; Computation and Estimation; Patterns, Functions, and Algebra

Overview Students use proportional reasoning to apply their experience with unit ratio problems about population density to a new situation. They do this in order to create another example of unit ratio and use that ratio to create a proportional reasoning situation.

Related Standard of Learning 6.2

Objective

- The student will work with unit ratios and proportional reasoning to solve a problem.

Materials needed

- "Gridlock Recording Sheet," one copy for each student
- Calculators

Instructional activity

1. *Initiating Activity:* Distribute the "Gridlock Recording Sheet," and pose the following problem: "According to the *Guinness Book of Records*, Hong Kong is reported to have the highest traffic density in the world. In 1992, there were 418 registered cars and trucks per mile of road. Another way to represent this ratio would be to say there are about 12.63 feet of road per registered vehicle in Hong Kong."
2. Ask how the *Guinness Book of Records* determined these two different unit ratios.
3. Ask what the units are in each statement.
4. *Closing Activity:* Present the students with the following situation: "The city of Beetleville has 450,237 registered vehicles per 3,000 miles of road. What is the traffic density of Beetleville? Be prepared to explain your solution."
5. Ask the students to calculate both the number of vehicles per mile of road and the number of feet of road per vehicle.

Sample assessment

- Be certain that the students understand the solutions to the problems as follows:
  - **Hong Kong problem:** If there are 418 vehicles per mile of road, then the ratio is $\frac{418 \text{ cars}}{1 \text{ road mile}}$. If I want to convert to feet, I know that 1 mile = 5,280 ft. Therefore, $\frac{418 \text{ cars}}{5{,}280 \text{ road ft.}} = \frac{1 \text{ car}}{x \text{ road ft.}}$. So $x = \frac{5{,}280}{418} = 12.63$ road feet per vehicle.
  - **Beetleville problem:** The city of Beetleville has $\frac{450{,}237 \text{ vehicles}}{3{,}000 \text{ road miles}} = 150.1$ vehicles per road mile. Therefore, $\frac{150.1 \text{ vehicles}}{5{,}280 \text{ road ft.}} = \frac{1 \text{ vehicle}}{x \text{ road ft.}}$. So, $x = \frac{5{,}280}{150.1} = 35.18$ road feet per vehicle.

Gridlock Recording Sheet

Name: _____

Date: _____

1. According to the *Guinness Book of Records,* Hong Kong is reported to have the highest traffic density in the world. In 1992, there were 418 registered cars and trucks per mile of road. Another way to represent this ratio would be to say there are about 12.63 feet of road per registered vehicle in Hong Kong. How did the *Guinness Book of Records* determine these two different unit ratios? What are the units in each statement? (For this problem, remember that 1 mile = 5,280 ft.)

2. The city of Beetleville has 450,237 registered vehicles for 3,000 miles of road. What is the traffic density of Beetleville? Calculate both the number of vehicles per mile of road and the number of feet of road per vehicle. Be prepared to explain your method of calculation.

Sample resources

Mathematics SOL Curriculum Framework.

Thinking Rationally about Fractions, Decimals, and Percent: Instructional Activities for Grades 4 through 8 – Lesson plans available from VDOE at http://www.pen.k12.va.us/VDOE/Instruction/Math/FractionsDecimalsPercent.pdf.

NCTM. *Principles and Standards for School Mathematics, pages* 214–221.

http://www.aaamath.com/g62a-ratios.html – A Web site that explores ratios.

http://illuminations.nctm.org/lessonplans/6-8/billion/index.html – Students use a simulation to predict population growth.

Sample Assessment

- Linda is keeping a record of her scores and entered these statistics after her first basketball game:

Linda's Basketball Statistics

Field Goals Attempted	Field Goals Made	Points from Field Goals	Free Throws Attempted	Free Throws Made	Points from Free Throws	Total Points
12	8	16	6	4	4	20

What was the ratio of field goals made to field goals attempted?

A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D $\frac{3}{2}$

Organizing Topic Measurement

Standards of Learning

6.9 The student will compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system and estimate conversions between units in each system:
a) length — part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;
b) weight/mass — ounces, pounds, tons, grams, and kilograms;
c) liquid volume — cups, pints, quarts, gallons, milliliters, and liters; and
d) area — square units. *
The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.

6.10 The student will estimate and then determine length, weight/mass, area, and liquid volume/capacity, using standard and nonstandard units of measure.

6.11 The student will determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area and apply the appropriate formula.

6.12 The student will
a) solve problems involving the circumference and/or area of a circle when given the diameter or radius; and
b) derive approximations for pi (π) from measurements for circumference and diameter, using concrete materials or computer models.

6.13 The student will
a) estimate angle measures, using 45°, 90 °, and 180° as referents, and use the appropriate tools to measure the given angles; and
b) measure and draw right, acute, and obtuse angles and triangles.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system.	_____
• Estimate the conversion of units of length, area, weight/mass, and volume between the U.S. Customary system and the metric system by using ballpark comparisons.	_____
• Determine the most appropriate unit of measure for a given situation.	_____
• Estimate measurements by comparing the object to be measured against a benchmark.	_____
• Solve measurement problems by estimating and determining length, using standard and nonstandard units of measure.	_____

- Solve measurement problems by estimating and determining weight/mass, using standard and nonstandard units of measure. _____
- Solve measurement problems by estimating and determining area, using standard and nonstandard units of measure. _____
- Solve measurement problems by estimating and determining liquid volume/capacity, using standard and nonstandard units of measure. _____
- Determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area. _____
- Apply formulas to solve problems involving area and perimeter of triangles and rectangles. _____
- Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. _____
- Find the circumference of a circle by substituting a value for the diameter or the radius into the formula $C = \pi d$ or $C = 2\pi r$. _____
- Find the area of a circle by using the formula $A = \pi r^2$. _____
- Determine the circumference and/or area of a circle, using various tools. _____
- Create and solve problems that involve finding the circumference and/or area of a circle when given the diameter or radius. _____
- Estimate visually the angle measure of a given angle by using 45°, 90°, and 180° as referents, and use appropriate tools to check the reasonableness of the estimate. _____
- Draw and measure acute, right, and obtuse angles, using appropriate tools. _____
- Draw and measure acute, right, and obtuse triangles, using appropriate tools. _____

Measuring Mania

Reporting categories	Measurement, Geometry
Overview	Students compare and convert units of measure for length, area, weight/mass, and volume.
Related Standards of Learning	6.9, 6.10

Objectives

- The student will compare and convert units of measure within the U. S. Customary system and the metric system.
- The student will estimate the conversion of units between the two systems using ballpark comparisons.
- The student will determine the most appropriate unit of measure for a given situation.
- The student will estimate measurements by comparing the object to be measured against a benchmark.
- The student will solve measurement problems by estimating and determining length, area, weight/mass, and volume, using standard and nonstandard units.

Materials needed

Organize sets of materials for each group of four students.

- Rulers, yardsticks, centimeter tapes, meter sticks, meter trundle wheel
- Balance scale, weights
- Grid paper
- String
- Inch cubes, centimeter cubes
- Measuring cups, graduated cylinders
- Various empty containers, for measures of liquid volume, e.g., juice cans, juice boxes, condiment containers, milk, soda, etc.
- Various empty boxes for measures of measures of volume
- Access to www.mapquest.com or other map Web site to determine distance from school to various locations in the area
- "Measuring Mania Recording Sheet," one copy for each group

Instructional activity

1. Distribute sets of tools for each group of four students. Ask students to brainstorm a situation in which each tool would be the appropriate tool to use. Have students share Record the situations under the appropriate tool on a class chart. As various situations are presented, discuss ballpark comparisons that students could use to estimate various measurements. Be sure to include the following:
    - 1 inch is about 2.5 centimeters. (The diameter of a quarter is about 1 inch.)
    - 1 foot is about 30 centimeters.
    - 1 meter is a little longer than a yard, or about 40 inches. (A doorway is about a yard in width, and the doorknob is usually placed one meter from the floor on a door.)

- 1 mile is slightly farther than 1.5 kilometers.
- 1 kilometer is slightly farther than half a mile.
- 1 ounce is about 28 grams. (A slice of bread weighs about 1 ounce.)
- 1 nickel has the mass of about 5 grams. (1 gram is about the weight of a paper clip.)
- 1 kilogram is a little more than 2 pounds. (A textbook has a mass of about 1 kilogram.)
- 1 quart is a little less than 1 liter.
- 1 liter is a little more than 1 quart.

2. Distribute the Measuring Mania Chart and ask the groups to record an item for each measurement that could be found in the classroom. Have students estimate the measurement. Once all the items and the estimates are recorded, direct students to use the appropriate tools to measure the items in the classroom. If only one computer is available, have students rotate through the computer station to determine the distances to various locations in the area.

Sample assessment

- The Measuring Mania Chart can be used as an assessment.
- Observe students during the activity to verify appropriate use of measuring tools.
- Create a similar chart that students could use as a homework assignment. If some measuring tools are not available at home have students write estimates of the measurement. In their math journals, have students create measurement problems. Have students exchange problems and solve.

Measuring Mania

Name: _____ Date: _____

Unit of Measurement	Object in Classroom	Estimate	Actual
Part of an inch			
Inches			
Feet			
Yards			
Millimeters			
Centimeters			
Meters			
Ounces			
Pounds			
Grams			
Cups			
Pints			
Quarts			
Gallons			
Milliliters			
Liters			
Square Units			
To be completed at the computer station, using a map Web site			
Miles	**Determine a Location in the City/County**	**Estimate the Distance to the Location (in miles)**	**Actual Distance (using map Web site)**
Kilometers	**Determine a Location in the City/County**	**Estimate the Distance to the Location (in kilometers)**	**Actual Distance (using map Web site)**

Areas with Pentominoes/Graph Paper/ Geoboards

Reporting Category

Overview

Related Standard of Learning 6.11

Objectives

- The student will determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area.
- The student will apply formulas to solve problems involving area and perimeter.

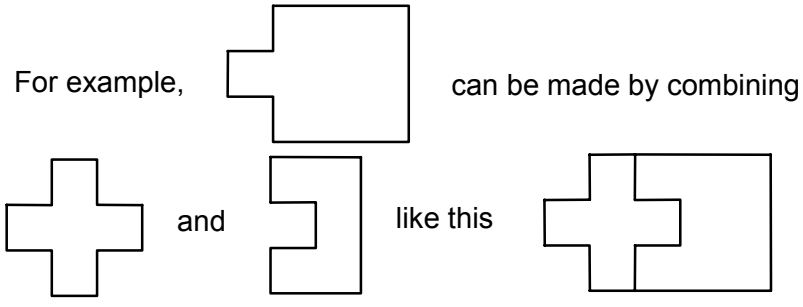
Activity A: Areas with Pentominoes

Materials needed

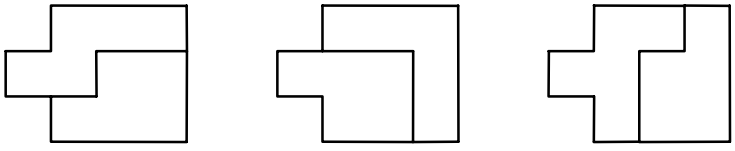
- Scissors
- Sets of pentominoes in an envelope or plastic bag (see "Patterns for Pentominoes" below)
- "Patterns for Pentominoes," (two pages), one copy for each student
- "3-by-5 Puzzle Solutions," one copy for each student
- "4-by-5 Puzzle Solutions" one copy for each student

Instructional activity

1. Students may be in small groups or in a large group. Ask students to carefully cut out a set of pentominoes from the "Patterns for Pentominoes" activity sheets.
2. Invite students to take some of their pentominoes and see if they fit together like jigsaw puzzle pieces. Point out that there are usually several ways to combine pentominoes to fill a given area.



or in several other ways.



3. Have students fill in the 3-by-5 and 4-by-5 areas with pentominoes. Ask them to keep a record of their work on a separate piece of graph paper. Have them compare their results with the 3-by-5 and 4-by-5 Puzzle Solutions.
4. Ask students how many square units there are in each pentomino. Ask them to find the pentomino with the largest perimeter and the pentomino with the smallest perimeter. Discuss how shapes with the same area can have different perimeters.

Sample assessment

- Ask students how many square units there are in all 12 pentominoes combined. (60) Have them design a symmetrical shape with an area equal to 60 square units and then see if it can be completely filled in with all twelve pentominoes.

Activity B: The Perimeter Is 24 Inches. What Is the Area?

Materials needed

- 24-inch paper strip (collar)
- 38 1-inch cubes
- 1-inch block graph paper
- $\frac{1}{2}$-inch block graph paper

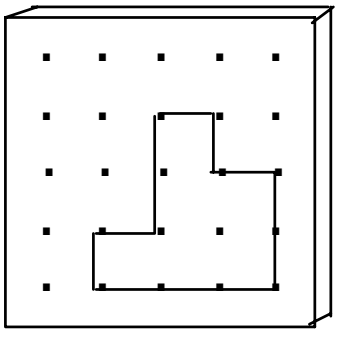
Instructional activity

1. Organize the students into teams of two or three.
2. Ask the teams to find out how many different rectangular arrays they can make that have a perimeter of 24 inches. Have them make the arrays with the cubes and have them check the perimeter with a 24-inch collar. What is the area of each array?
3. Once they find a rectangular array, have them draw its representation on the graph paper and write the perimeter and area for each array.

Activity C: Change The Area

Materials needed

- Square geoboards,
- Rubber bands



Instructional activity

1. Organize students into small groups.
2. Have students copy this figure on the geoboard and onto dot paper, labeling its area and perimeter.
3. Have students change the figure to make another shape that has the same area and a larger perimeter, recording it on dot paper with its area and perimeter.

4. Have the participants change the figure to make another shape that has the same area and a smaller perimeter, recording it on dot paper with its area and perimeter.

Sample assessment

- Have the students make three more shapes that have different perimeters but the same area, recording them on dot paper.
- Have them record their observations in their math journal.

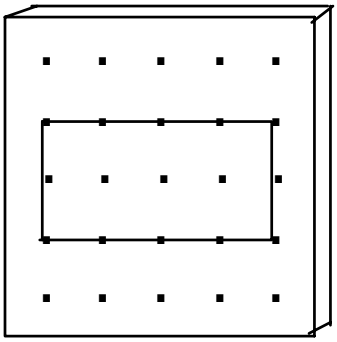
Activity D: Areas of Polygons on the Geoboard

Materials needed

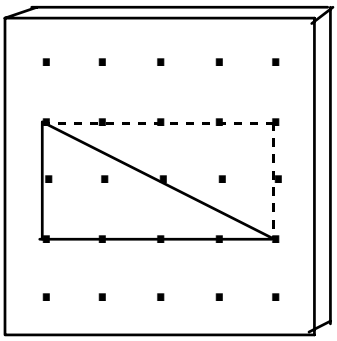
- Square geoboard and rubber bands, one set for each student

Instructional activity

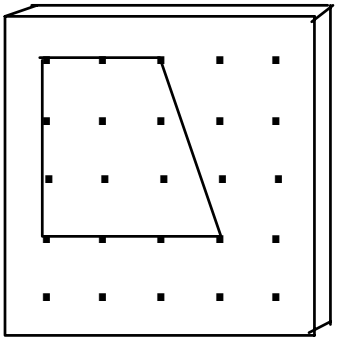
1. Have students construct this rectangle on their geoboards and find its area. They should record it and its area on the geoboard dot paper.



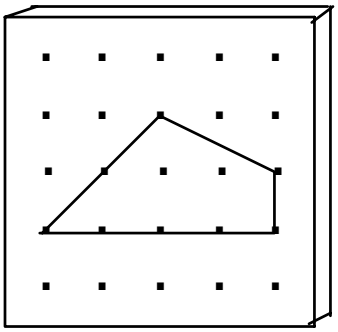
2. Have students construct this right triangle and find its area by first counting squares in the corresponding rectangle. Record.



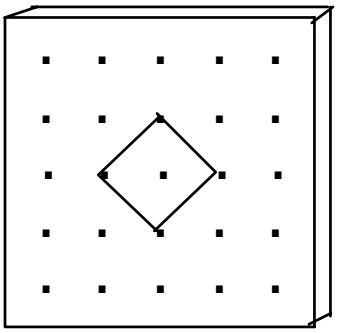
3. Have students construct this figure and find its area by counting squares.



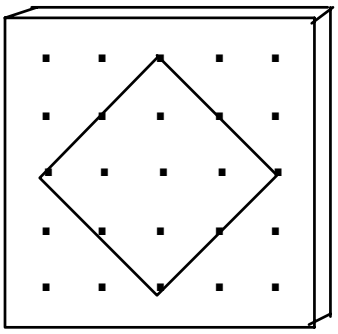
4. Have students form a 2 by 4 rectangle and construct this figure within it. They should then find its area from the area of the rectangle and record.



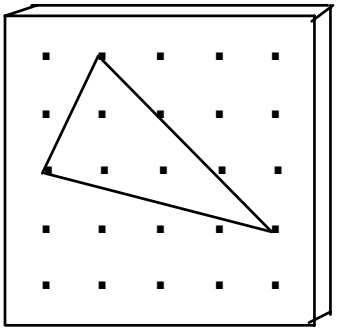
5. Have students construct this square on their geoboards, find its area, and record.



6. The students should next construct this square on their geoboards, find its area, and record.

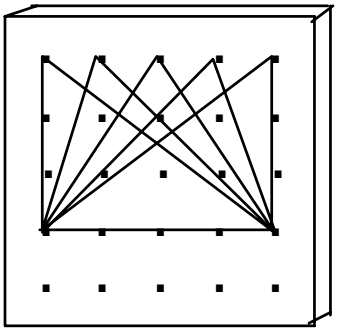


7. Have the participants construct this triangle, find its area, and record.

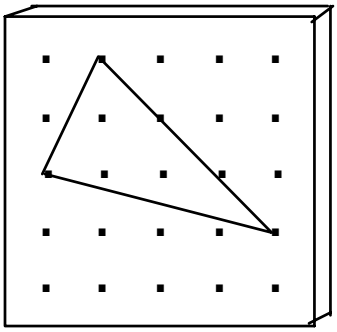


Follow-up/extension

- Have students find the area of these triangles using their geoboards. Discuss what they discovered.



- Next have students construct this triangle on their geoboard and find its area. Hint: They should start with the area of the rectangle enclosing it.



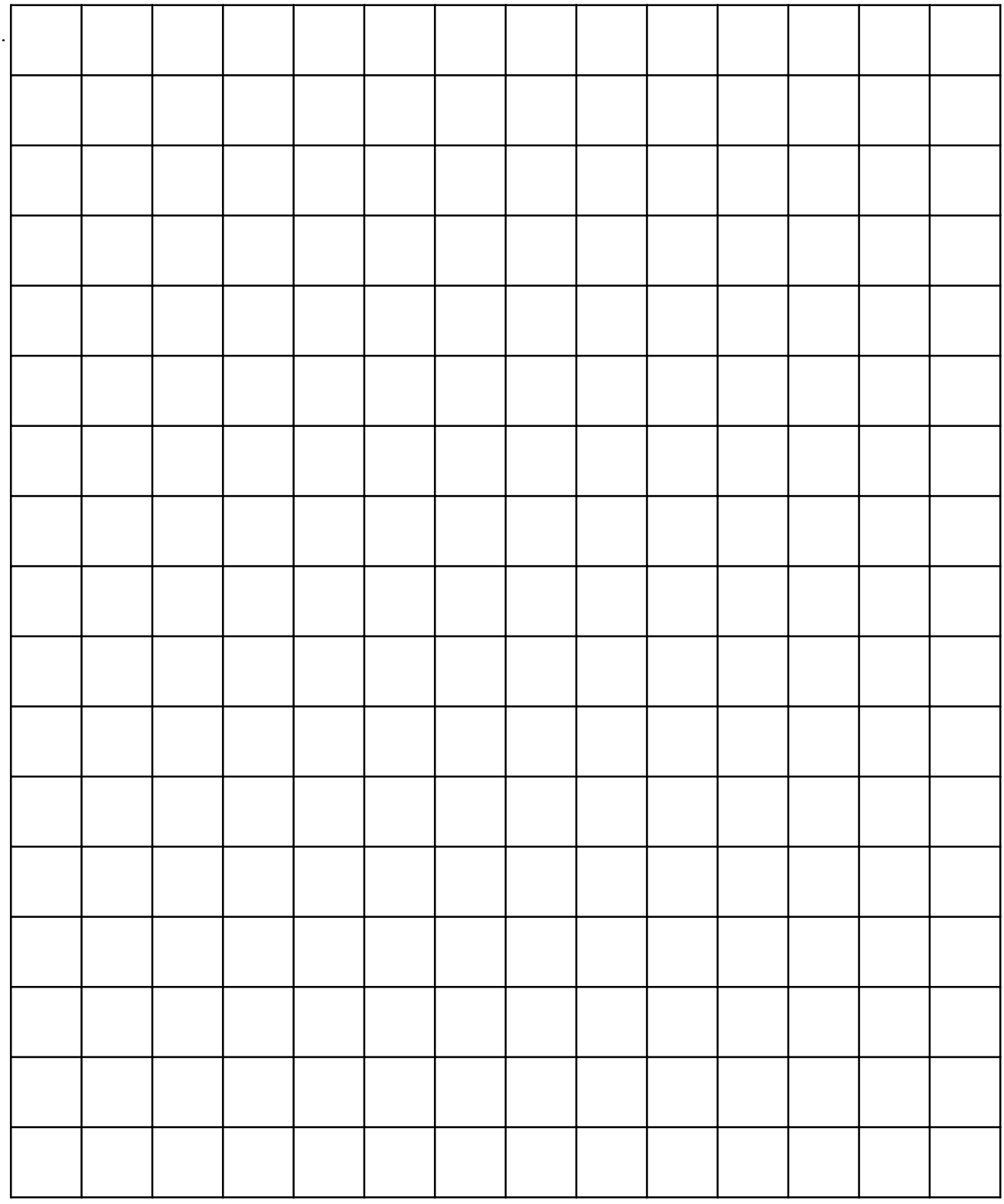
- The area of the previous triangle was $4\frac{1}{2}$ square units. Have students construct as many other triangles as they can with the same area, but with different shapes. They should copy each one on their geoboard dot paper.

Homework

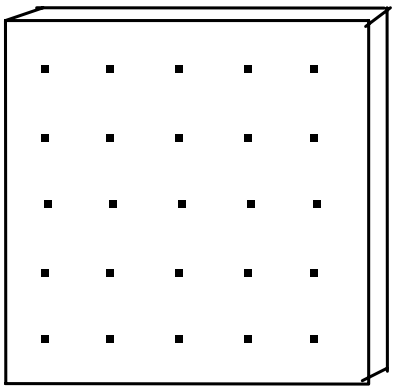
- Have each student make a complicated polygon on his/her geoboard and find its area. Then they should give it to another student to find the area.

One-Inch Graph Paper

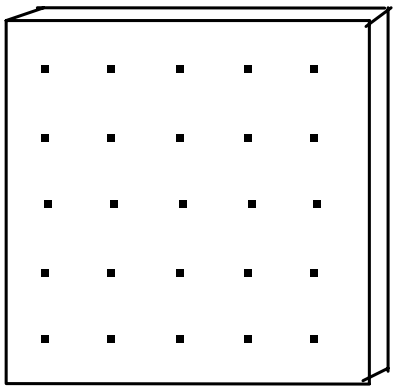
One-Half-Inch Graph Paper



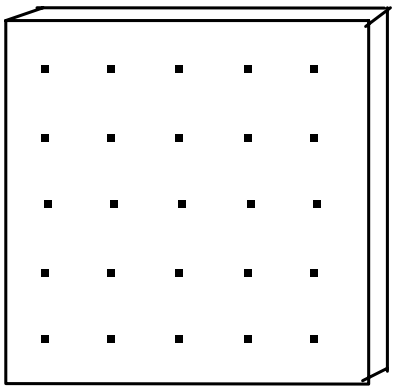
Geoboard Dot Paper



Perimeter: _____
Area: _____



Perimeter: _____
Area: _____



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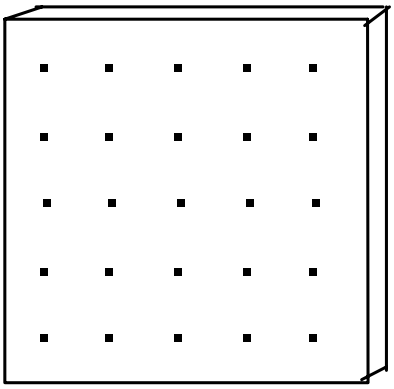
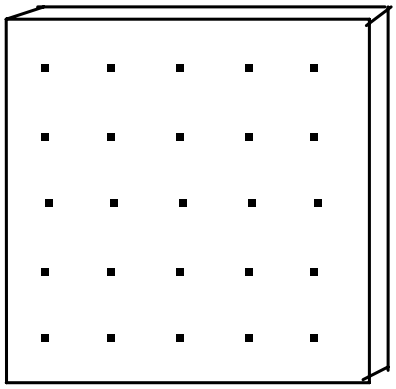
Area: _____



Perimeter: _____
Area: _____



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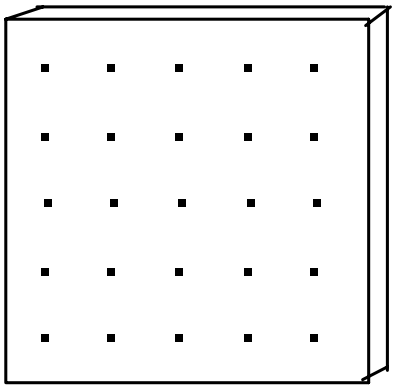

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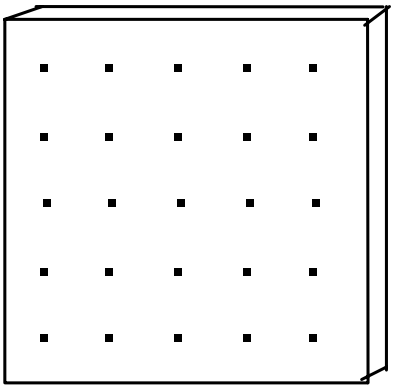

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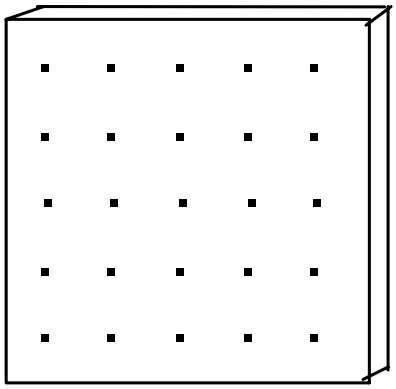
Area: _____



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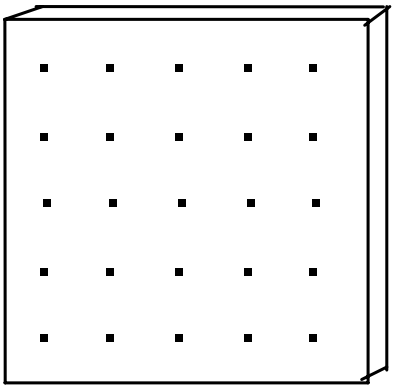
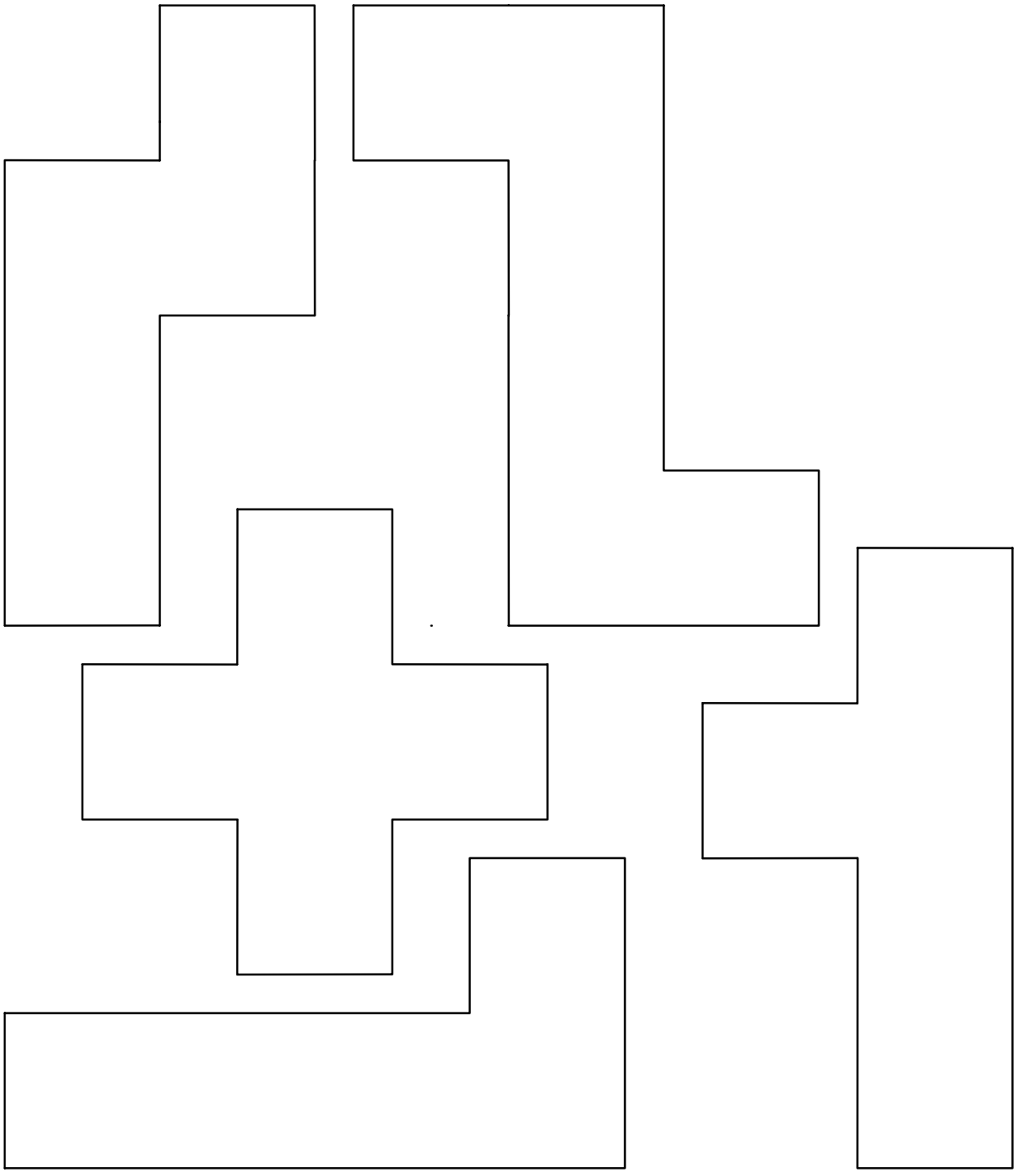
Area: _____

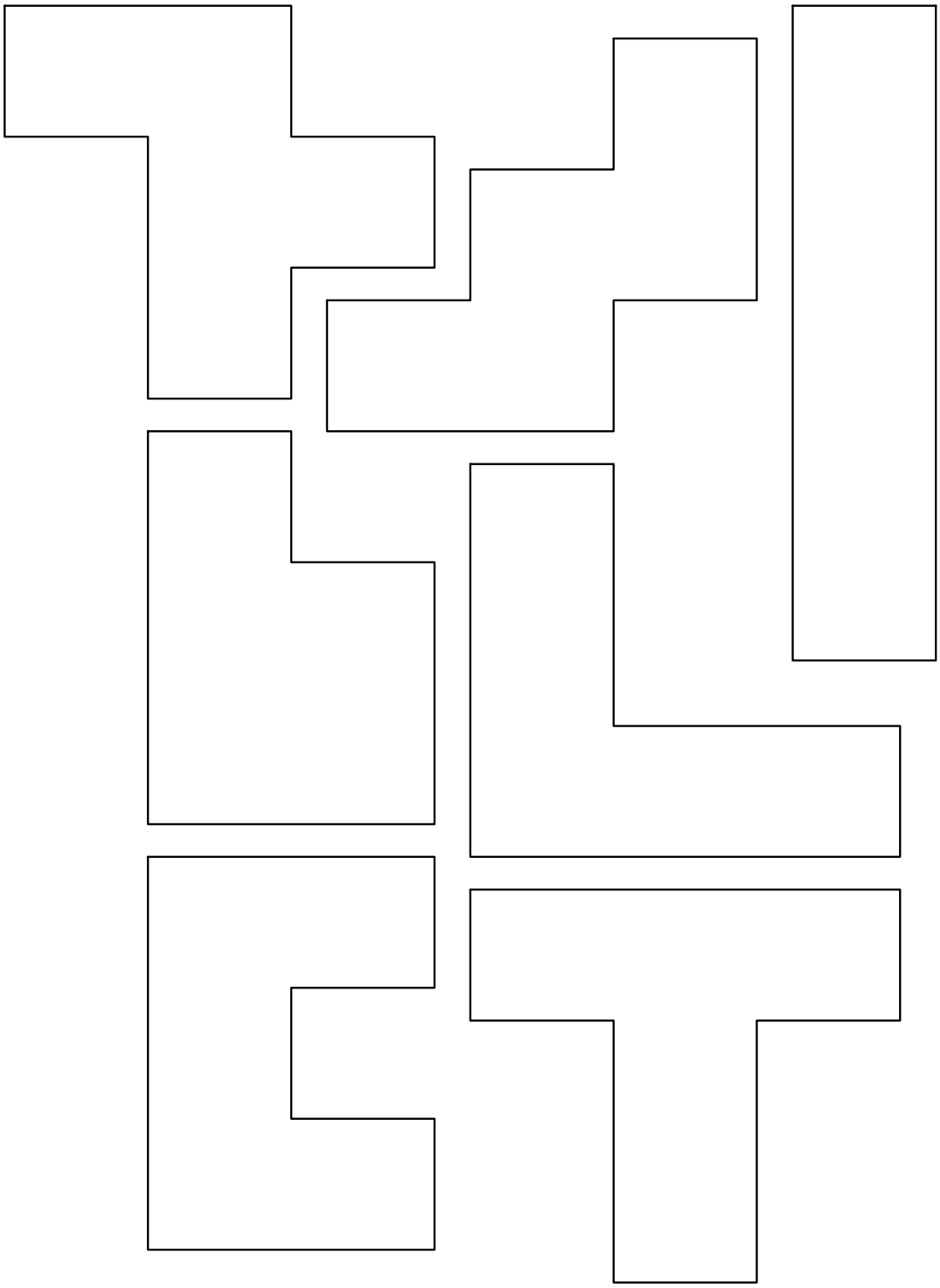


Perimeter: _____
Area: _____

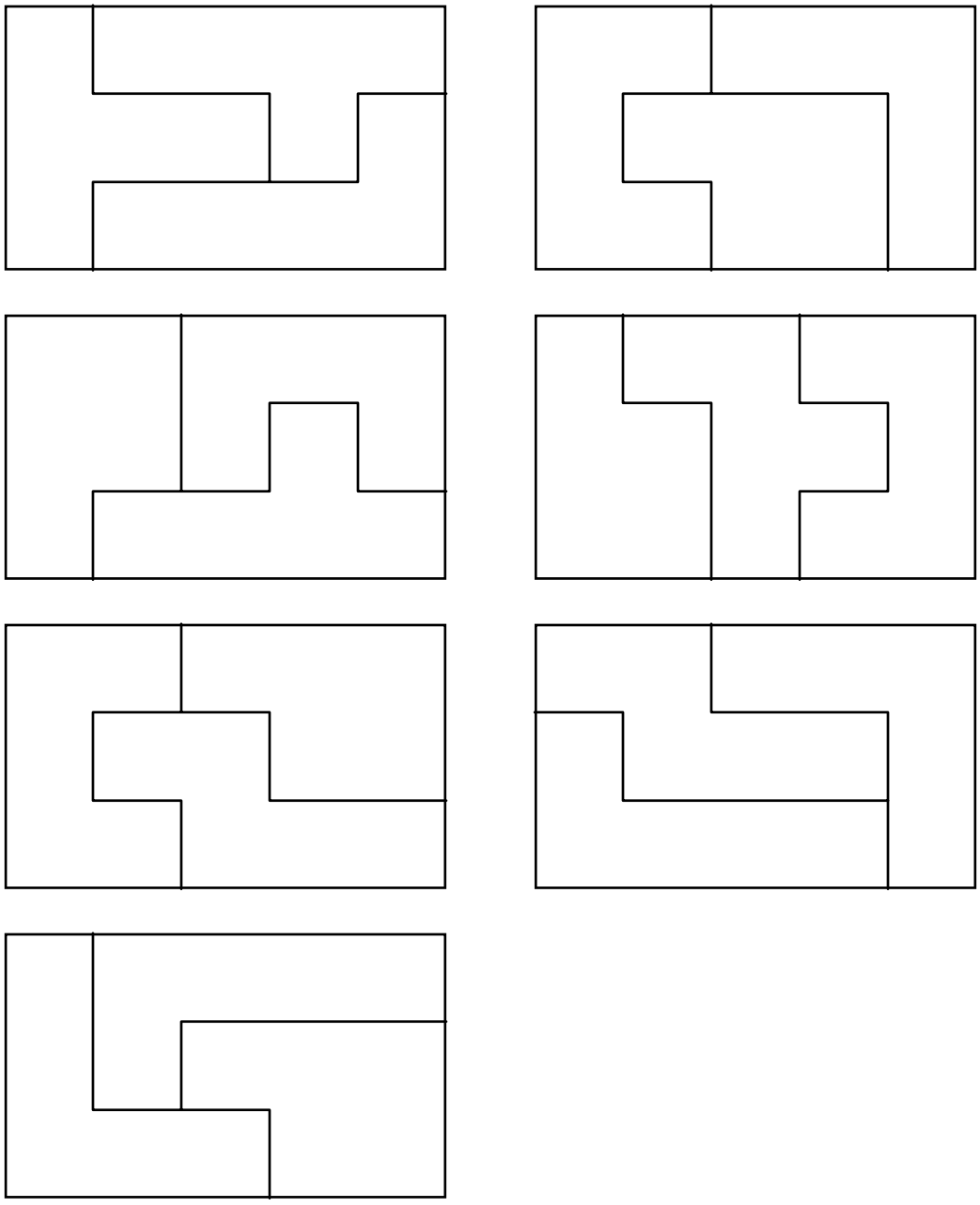
Patterns for Pentominoes



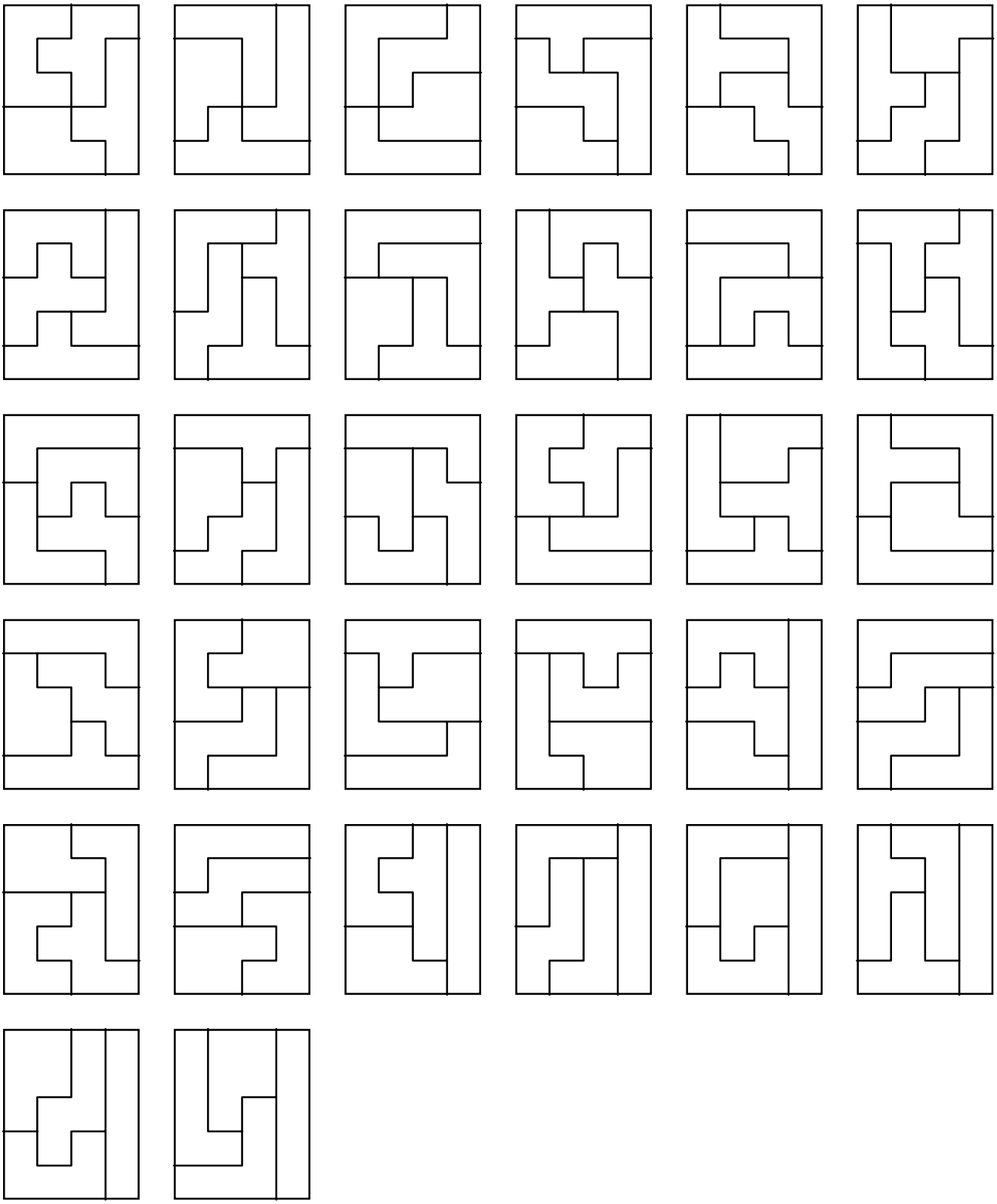
Patterns for Pentominoes



3-by-5 Puzzle Solutions



4-by-5 Puzzle Solutions



Circumference vs. Diameter

Reporting Category Measurement

Overview Students are presented problems that challenge their intuition about measurements of the circumference of a can and the height. They will apply their knowledge about the area of circles.

Related Standard of Learning 6.12

Objective

- The student will solve problems involving the circumference and/or area of a circle when given the diameter.

Materials needed

- A tennis ball can or chips can and a piece of string for each small group
- "Cake Problem," one copy for each student

Instructional activity

Activity A: Circumference vs. Diameter

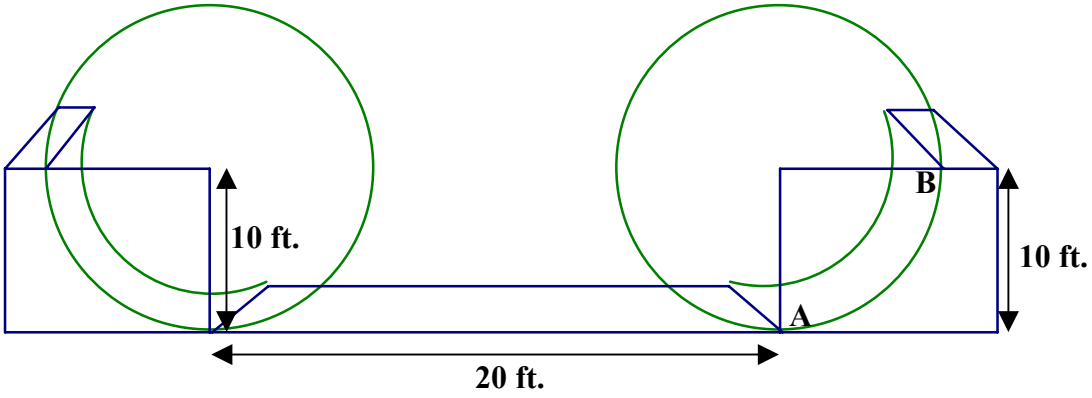
1. Provide a variety of cylindrical containers for each small group. Ask students which is greater: the diameter of the cylinder or the circumference of the cylinder? For each cylinder, have each student write down his/her answer supported with reasons. Then they should discuss it in groups before having a whole class discussion.
2. Have students wrap a string around each cylinder and compare this to the diameter of the cylinder. Students should record their observations.

Activity B: Cake Problem

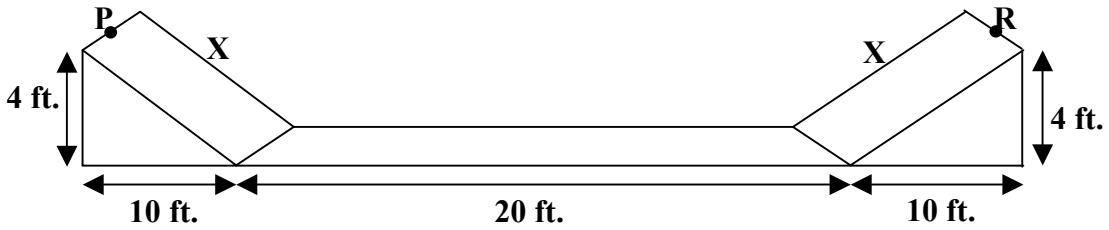
1. Present the cake problem to the students. Make sure everyone understands the problem. Put a 10-inch diameter circle on the overhead projector to model the cake and ask a volunteer to make an estimate of the placement of the cut that solves the problem.
2. Give out the handout and have students work in small groups to solve it. Have each student write up his/her solution.
3. Ask for volunteers to share solutions. Discuss variations on solutions.
4. Return to the transparency of the cake and draw the correct solution. How close did the estimate come?

Activity C: Exploring Circumference and Perimeter

Skateboarding has become a popular sport. The parks department is thinking of constructing ramps in some of the local playgrounds. A "half-pipe" ramp is formed by two quarter-circle ramps, each of which is 10 feet high, plus a flat space 20 feet long between the centers of the circles from which the two quarter-circle ramps are formed.



1. Find the distance a skateboarder travels from the top of one ramp to the top of the other. (Hint: What is the length of arc AB?)
2. Another launch ramp is formed by 2 arcs, each with a central angle of 60 degrees and a radius of 10 ft. Find the length from the top of one ramp to the top of the other. (Hint: What fractional part of the circle is each arc?)
3. A third ramp is a straight ramp 4 ft. high and 10 ft. long with a flat space of 20 ft. Find the length of the ramps from point P to point R. (Hint: Use the Pythagorean Theorem)



Sample assessment

- A school track is formed by 2 straight segments joined by 2 half circles. Each segment is *l* long and each half-circle diameter is *d* in length. Write a formula for finding the distance, *D*, around the track.

Follow-up/extension

Soda Straws

How many straws full of pineapple juice can be taken from a 46-fl.-oz. can of juice that is filled to the top?

1. Measure to find the following:

   Diameter of the can: _____ Height of the can: _____

   Diameter of the straw: _____ Length of the straw: _____

2. Explain how you determined your answers.

Cake Problem

Name: _____ Date: _____

You have a cake that is 10 inches in diameter. You expect 12 people to share it, so you cut it into 12 equal pieces (Figure A).

Before you get a chance to serve the cake, 12 more people arrive! Therefore, you decide to cut a concentric circle in the cake so that you will have 24 pieces (Figure B).

How far from the center of the cake should the circle cut be made so that all 24 people get the same amount of cake?

Figure A

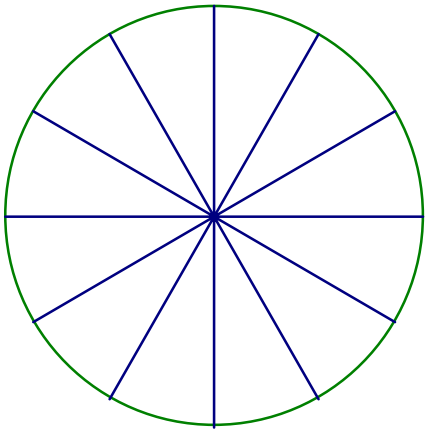
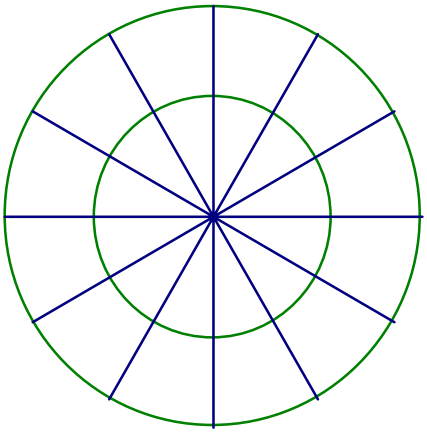


Figure B



Finding Pi

Reporting category Measurement
Overview Students will explore concepts associated with circles.
Related Standard of Learning 6.12

Objectives

- The student will derive an approximation for pi by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models.
- The student will find the circumference of a circle by substituting a value for the diameter or the radius into the formula $C = \pi d$ or $C = \pi r$.
- The student will find the area of a circle by using the formula $A = \pi r^2$.
- The student will determine the circumference and/or area of a circle, using various tools.
- The student will create and solve problems that involve finding the circumference and/or area of a circle when given the diameter or radius.

Materials needed

- Per small group: several round objects such as jars or lids, measuring tapes or string and rulers, calculator
- Alternate strategy: Set up stations with a round object at each station and have groups rotate through the stations.

Instructional activity

Activity A

1. Divide class into small groups and distribute materials.
2. Have student teams make a table or chart that shows columns for Name or Number of Object, Circumference, Diameter, and *?*
3. Have students measure and record each object's circumference and diameter. Then divide the circumference by the diameter and record result in the *?* column.
4. Have students find the average for the *?* column and compare to other groups in the class to determine a pattern. Students can then find the average number for the class.
5. Explain to the students that they have just discovered pi, which is very important in finding the circumference of an object. (You may wish to give some historical information about pi at this time, or have students research the information.)
6. Have student groups come up with a formula to find the circumference of an object, knowing only the diameter of that object and pi. Students must prove their formula works by demonstration and measuring to check their results.

Sample assessment

- Have students write their conclusions for the activities they have just done. Students may also share what they have learned with other members of the class.
- Give students three problems listing only the diameter of each object and have them find the circumference.
- Encourage students to share learned knowledge with parents.

Alternate Strategy: If you have access to a dynamic geometry program (i.e., Geometer's Sketchpad™ or Cabri™ Geometry):

- Give students the handout that describes the procedure for completing the investigation and have them work in pairs to complete it.
- How could you verify that the constant value, 3.14, was not just a bug in the software — that circumference divided by diameter of any circle is always a constant?
- Investigation Procedure:
  - Using the dynamic geometry software, draw a circle.
  - Measure the radius, diameter, and circumference of the circle and record your results rounded to thousandths in the table below.
  - Calculate the ratio of the circumference and the diameter. Record your results.

	Radius	**Diameter**	**Circumference**	$\frac{C}{D}$
1				
2				
3				
4				
5				

  - Repeat your procedure with four more circles. Record your results carefully.

Activity B: Calculating Area and Circumference

1. Students should be familiar with the formulas for area and circumference of a circle. Have students use dynamic geometry software (or the teacher uses a demonstration station with software) and draw a circle with a known radius.
2. Have students calculate the area and circumference of the circle and verify their results, using the software.
3. Repeat several times.

Exploring Angles

Reporting categories	Measurement, Geometry
Overview	Students explore angles and identify types of angles by visual estimation. They learn to draw and measure angles of specific measures by working with a partner.
Related Standard of Learning	6.13

Objectives

- The student will estimate angle measures, using 45º, 90º, 180º as referents, and use the appropriate tools to measure the given angles.
- The student will measure and draw right, acute, and obtuse angles.

Materials needed

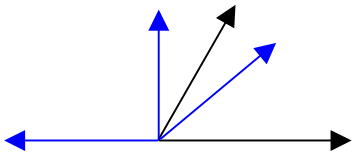
- Protractors (including large one for use on chalk or dry erase board)
- "Exploring Angles," one copy for each student
- Board or overhead

Instructional activity

1. Lead a discussion on angles. Ask students to point out examples of angles in the classroom. Have students define *angles* in their own words. Ensure students' understanding of the definition of angles.
2. Draw a variety of angles (acute, obtuse, right) on the board or overhead. Ensure that the angles are facing different directions (this will be useful in helping them to understand the use of protractors and choosing the appropriate scale). Ask students to describe the differences among the angles (their size).
3. Write the following terms on the overhead: *acute, obtuse, right.* Instruct students to define them in their own words (encourage students to take risks by reassuring them that they will have an opportunity to correct their definitions).
4. Discuss student definitions.
5. Draw and measure examples of each type of angle on the board or overhead, and label with appropriate term. Lead students to realize the proper definition for each angle type. Encourage them to revisit their previous definitions and make any corrections necessary.
6. Distribute a protractor to each student and discuss its use. Ask students why the protractor has 2 scales (to measure all types of angles facing either direction).
7. Ask students how they would know which scale to use (depends on the type of angle they are measuring). Lead students to understand the importance of determining the type of angle prior to measuring the angle.
8. Draw more angles on the board or overhead to ensure students' understanding of the types of angles.
9. Distribute "Exploring Angles" worksheet. Complete the first two problems of part 1 with students, and allow students to complete the remaining on their own. Check student responses.

10. Draw an angle on the board or overhead and ask students to estimate the size of the angle. Allow students to explain their methods for estimation. Lead students to realize that by visualizing what 90º, 45º, and 180º angles look like, they can make more accurate estimations.
11. Demonstrate the use of referents by drawing dotted or different colored rays where those referent points are.

    Example:



12. Work several more examples, having students verbalize what they were thinking and visualizing to arrive at an estimate. Stress to students that their estimates do not need to be exact.
13. Refer back to "Exploring Angles" worksheet and instruct students to complete part 2. Check student responses.
14. Demonstrate to students how to use the protractor to draw an angle of a specific size. Remind students to first think about the type of angle they are drawing in order to ensure they choose the appropriate scale. Once the angle is drawn, demonstrate to students how to measure the angle.
15. Instruct students to draw an angle that measures 75º. Walk them through the process. Have students trade papers with a partner to measure each other's angles.
16. Repeat #15 with several more examples. (You may want to encourage students to construct their angles facing various directions so students will have the opportunity to practice choosing the appropriate scale).
17. Have students choose their own angle measure to draw and then trade with a partner to first estimate the measure and then find the actual measure.
18. Allow students to work several more examples with their partners.
19. Assign part 3 of "Exploring Angles" worksheet. Check student responses.

Sample assessment

- Students can be assessed informally throughout the class as they work independently or with their partners. Once students have had enough practice, have them complete a quiz on identifying types of angles, estimating angle measure, drawing angles of a given measure, and measuring angles.
- Have students create a journal entry that explains the process of drawing and measuring angles and how first identifying the type and estimating the measure can ensure success when finding actual measures.

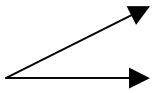
Follow-up/extension

- This lesson could be extended by having students measure the examples of angles identified early in the lesson.

Exploring Angles

Name: _____ Date: _____

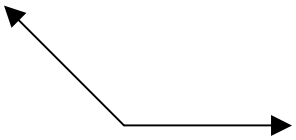
Identify the type of each angle. Estimate the measure of each angle. Measure each angle. Then review parts 1 & 2 to ensure they are correct.

1. 

Type _____

Estimate _____

Actual _____

2. 

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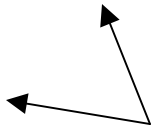
3. 

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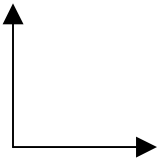
Estimate _____

Actual _____

4. 

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Estimate _____

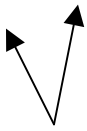
Actual _____

5. 

Type _____

Estimate _____

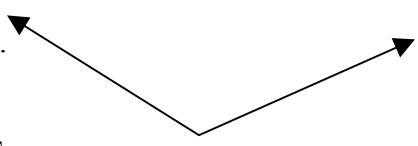
Actual _____

6. 

Type _____

Estimate _____

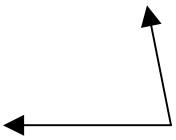
Actual _____

7. 

Type _____

Estimate _____

Actual _____

8. 

Type _____

Estimate _____

Actual _____

Exploring Triangles

Reporting categories	Measurement, Geometry
Overview	Students explore and identify characteristics of triangles and types of triangles while also discovering that the sum of the angles of a triangle total 180º.
Related Standards of Learning	6.13, 6.14

Objectives

- The student will measure right, obtuse, and acute triangles.
- The student will identify the sum of the measures of the angles of any triangle.
- The student will determine that the sum of the measures of the angles of a triangle is 180º.

Materials needed

- "Exploring Triangles" (two pages), one copy for each student
- Protractors

Instructional activity

1. Review the definitions of *acute, obtuse*, and *right angles*.
2. Draw an acute triangle on the board or overhead. Label the measure of each of the interior angles. Ask students to name the shape (triangle) and to describe its characteristics (3 sides, 3 angles).
3. Draw an obtuse triangle on the board or overhead, with the angle measures labeled. Instruct students to compare and contrast the 2 triangles. Allow students to share their ideas.
4. Draw a right triangle on the board or overhead, with the angle measures labeled. Instruct students to compare and contrast with the other 2 triangles. Allow students to share their ideas.
5. Explain to students that one way to classify triangles is by their angles. Have students use prior knowledge of angle types to predict the name of each triangle based on their angle measures. Allow students to share their ideas in naming. Ensure that all students properly named each triangle as acute, right, or obtuse. Lead students to realize that triangles are named based on the size of their **largest** angle.
6. Distribute "Exploring Triangles" worksheet to students and have them complete (it may be necessary to review proper procedure for measuring angles prior to assigning this worksheet). You may find it helpful to complete the first problem with students. Check student responses on worksheet.
7. Draw two triangles (one acute, one obtuse) on the board or overhead with the measure of the interior angles labeled. Ask students to identify the type of each triangle.
8. Instruct students to make a list of the similarities of the triangles. Have students share what they found. Lead students to realize that the sum of the angles in both triangles is 180º.
9. Ask students to predict if all triangles' interior angles will have a sum of 180º and to justify their predictions. Ask students if they know a way to find out if their predictions are correct or incorrect (check more examples).
10. Refer students to the "Exploring Triangles" worksheet they just completed. Have students check their theory by finding the sum of the measures of the angles in each triangle. Instruct students to record findings in a chart. Work additional examples if necessary.

11. Have students write an explanation of what they discovered, and allow students to share their ideas. Ensure that all students understand that the sum of the measures of the interior angles of a triangle totals 180º.

Sample assessment

- Informally assess as students work to complete activity.
- Follow up with an additional independent activity that requires students to find a missing angle measure when two of the angles are given.

Follow-up/extension

- Have the students complete the activity, "Sum Angles of a Triangle," which is found at http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/midgeo3.PDF

Sample resources

Learning and Teaching Measurement: 2003 Yearbook with Classroom Activities Companion Booklet, NCTM, 2003. Information related to measurement and activities that go beyond the content.

http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/midgeo3.PDF – VDOE mathematics module, "Geometry for Middle School Teachers," lesson title: "Sum Angles of a Triangle."

http://mathforum.org/paths/measurement/e.measlessons.html – Ideas and resources for teaching measurement that includes lesson plans, materials, common questions, and software.

http://standards.nctm.org/document/chapter5/meas.htm – Information on measurement from *Principles and Standards for School Mathematics.*

www.teachnet.com/lesson/math/geometry/circlesingeo.html – Activities to introduce the concept of circumference.

Exploring Triangles

Name: _____ Date: _____

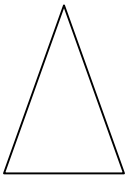
Record triangle findings in the chart below.

Triangle	Measure ∠1	Measure ∠2	Measure ∠3	Sum
1				
2				
3				
4				
5				
6				

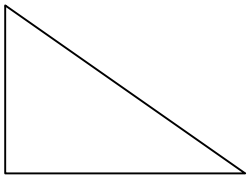
Exploring Triangles

Name: _____ Date: _____

Use a protractor to measure all interior angles of each triangle. Label the measure of each angle. Determine the type of triangle based on the angle measures.

1. 

Type _____

2. 

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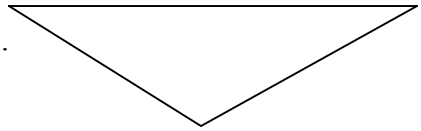
3. 

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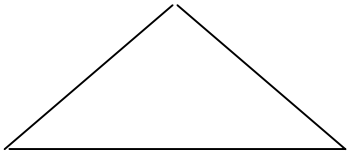
4. 

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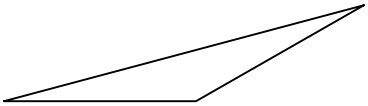
5. 

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Sample released test items

The largest angle in Δ *ABC* measures 104°. What kind of triangle is Δ *ABC*?

A Equiangular
B Obtuse
C Right
D Acute

The angles in Δ *ABC* measure 27°, 73°, and 80°. What kind of triangle is Δ *ABC*?

F Equiangular
G Acute
H Obtuse
J Right

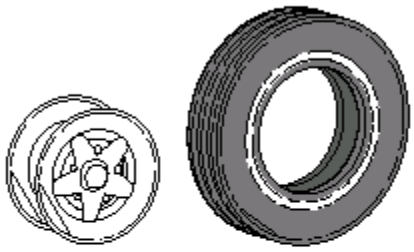
Which of the following indicates the greatest weight?

A 600 pounds
B 0.25 tons
C 9,000 ounces
D 0.5 tons

Su Li wants to place a protective covering over a rectangular flower bed that measures 3.2 meters by 4.3 meters. How many square meters of covering will she need?

F 7.5
G 13.76
H 15.0
J 27.52

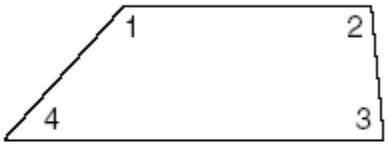
24 The wheel rim has a diameter of 15 inches.



Which is *closest* to the inside circumference of the tire designed to fit on the rim?

F 47.1 in.
G 94.2 in.
H 176.6 in.
J 706.5 in.

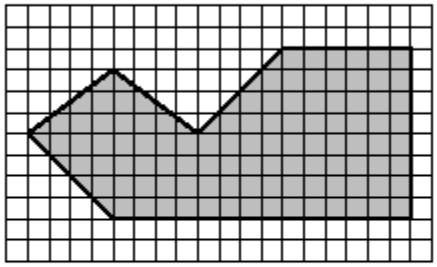
25 Look at the angles in this quadrilateral.



Which angle measure is closest to 48°?

A ∠1
B ∠2
C ∠3
D ∠4

26 A grid is placed over a cross-sectional drawing of a molding. Each square of the grid represents one square centimeter.



How many square centimeters are contained in the area of the cross section?

F 44

G 96

H 108

J 208

Organizing Topic Geometry

Standards of Learning

6.14 The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties.

6.15 The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes**.** Examples of noncongruent and congruent figures will be included.

6.16 The student will construct the perpendicular bisector of a line segment and an angle bisector.

6.17 The student will sketch, construct models of, and classify solid figures (rectangular prism, cone, cylinder, and pyramid).

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Classify triangles, quadrilaterals, pentagons, and hexagons.	
• Classify a triangle based on the size of its angles and/or its sides.	
• Identify the sum of the measures of the angles of any triangle or quadrilateral.	
• Determine that the sum of the measures of the angles of a triangle is 180 degrees.	
• Classify a triangle by its angles.	
• Classify and describe the similarities and differences in sets of triangles by sorting.	
• Classify quadrilaterals by pairs of parallel sides by sorting.	
• Identify and describe the similarities and differences in sets of quadrilaterals by sorting.	
• Characterize polygons as congruent and noncongruent according to the measures of their sides and angles.	
• Determine the congruence of segments, angles, and polygons by direct comparison, given their attributes.	
• Construct the perpendicular bisector of a line segment by using a variety of tools.	
• Construct the bisector of an angle by using a variety of tools.	
• Construct models for rectangular prisms, cones, cylinders, and pyramids.	
• Sketch rectangular prisms, cones, cylinders, and pyramids from three-dimensional models.	

- Identify a three-dimensional model of a prism, cone, cylinder, or pyramid from its two-dimensional representation.
- Classify rectangular prisms, cones, cylinders, and pyramids by their two-dimensional representations.

Triangle Sort

Reporting category Geometry

Overview Students explore types of triangles by sorting a variety of triangles and classifying them according to their characteristics. They then justify their groups based on the characteristics.

Related Standard of Learning 6.14

Objective

- The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties.

Materials needed

- Baggies of triangles from "Student Triangle Sorting Pieces" (two pages), one baggie for each group
- "Triangle Sort Activity Recording Form," one copy for each student

Instructional activity

1. Place students into groups of four. Have students work together to review the characteristics of triangles.
2. Distribute to students a plastic baggie with triangles from the "Triangle Sorting Activity" handouts. Allow a few minutes of free exploration.
3. Explain to students that they will be working in their groups to sort their triangles, first in one group based on a certain attribute and then into a second group based on a different attribute. (The two attributes will be measure of sides and measure of angles; however, do not reveal that to students at this time). Tell students they must justify their reason for placing their triangles into the groups they choose.
4. Instruct groups to first choose one attribute and sort triangles into groups using that attribute.
5. Distribute the "Triangle Sort Activity Recording Form," and instruct students to record their first set of groups onto the chart.
6. Instruct groups to now choose a second attribute and resort their triangles according to that attribute. Stress to students that it is not necessary for them to put the same triangles into the same group.
7. Again, instruct students to record their new set of groups onto their chart. Stress that they need to justify their sets by giving reasons for their grouping.
8. Write the following terms on the board or overhead: *acute, right*, and *obtuse.* Ask students if there are any groups who used these ideas to sort their triangles. Allow a group who used this as one method of sorting to share which triangles were acute, which were right, and which were obtuse.
9. Instruct all groups to resort their triangles into those groups and add to their recording form.
10. Write the following terms on the board or overhead: *isosceles, scalene, equilateral*. Ask students if they are familiar with these terms. If no one is, explain the meaning of each. Ask students if any sorted their triangles into groups based on the length of sides. Allow a group who used this as a method of sorting to share which triangles were isosceles, which were scalene, and which were equilateral.
11. Instruct all groups to resort their triangles into those groups and add to their recording chart.

12. Explain to students that typically, triangles are named for both their angle measure and length of sides, for example, equilateral/acute triangles have all sides congruent and the largest angle is acute.
13. Instruct students to classify each triangle in their set by both methods of classification. Check student results.
14. Have students write a journal entry that explains methods for classifying triangles and describes attributes of a triangle.

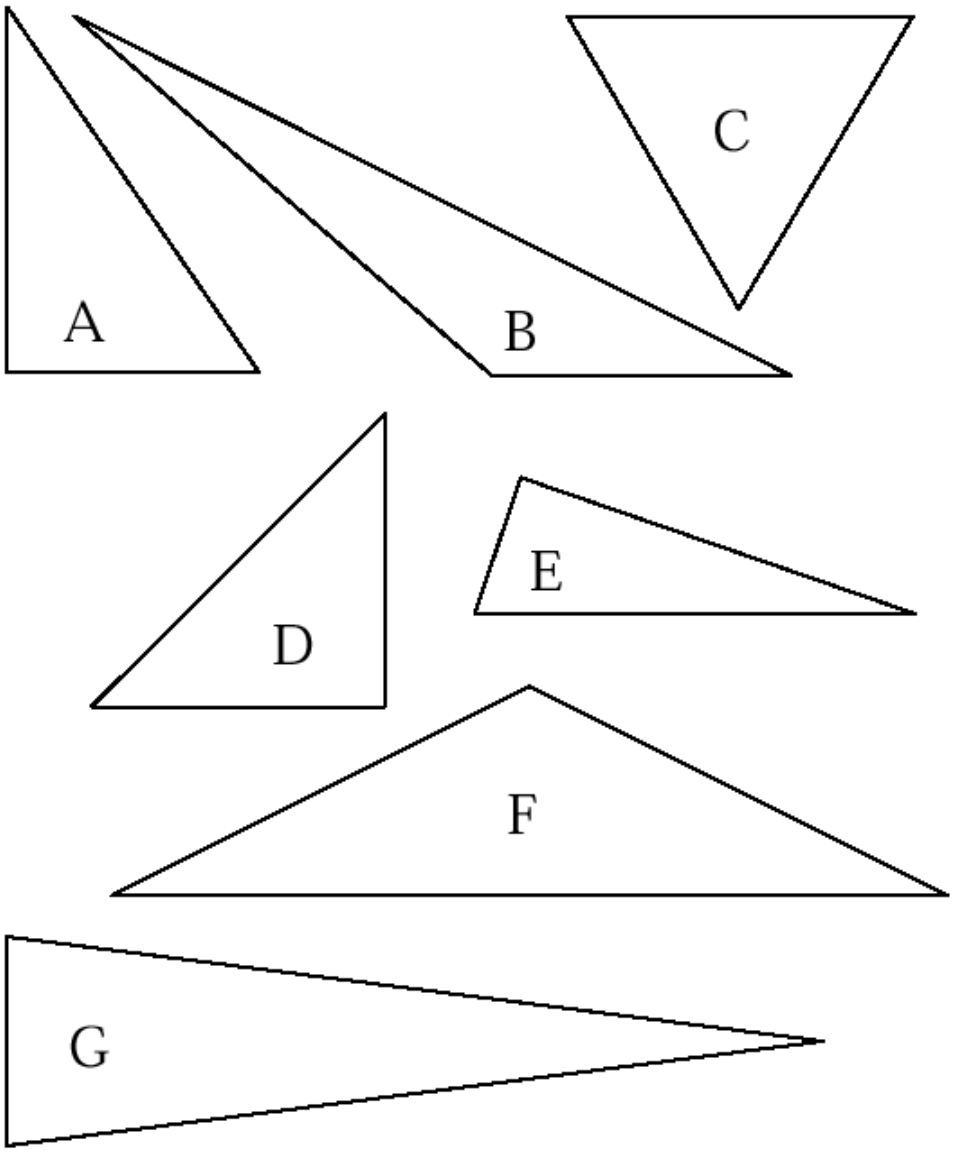
Sample assessment

- Assess students informally as they work through the activity in their groups.
- Use item 13 as a formal assessment for each individual student or the group.
- The journal entry can become part of a student portfolio.

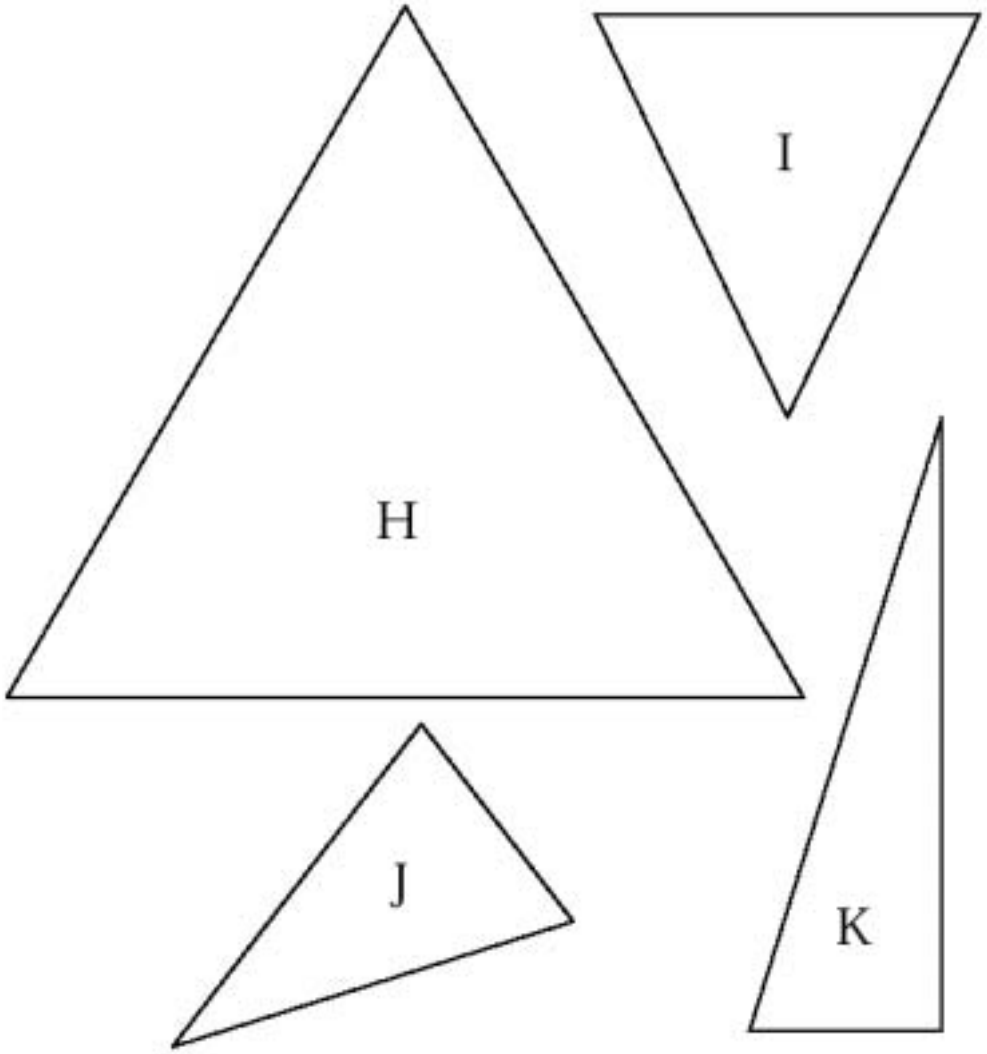
Follow-up/extension

- Have higher-level students research the use of triangles in construction and draw conclusions as to which type of triangle, if any, is best suited for building strong structures.
- Require students to locate examples of triangles in the classroom and classify according to both methods learned in class.

Student Triangle Sorting Pieces



Student Triangle Sorting Pieces
page 2



Triangle Sort Activity Recording Form

Name: _____ Date: _____

Classification 1

Reason for this classification: _____

Number of sub-groups in this classification: _____

List each sub group, the letters of the triangles that belong in each sub group, and the reason for placing in that sub group.

Classification 2

Reason for this classification: _____

Number of sub-groups in this classification: _____

List each sub group, the letters of the triangles that belong in each sub group, and the reason for placing in that sub group.

Congruence

Reporting category Geometry

Overview Students create a definition for congruence based on examples shown in class. They then devise a plan of determining congruence and test their plan to see if it works.

Related Standard of Learning 6.15

Objective

- The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes. Examples of noncongruent and congruent figures will be included.

Materials needed

- Rulers
- Protractors
- Clean overhead transparencies
- "Congruence Worksheet" (three pages), one copy for each student

Instructional activity

1. Begin a discussion about congruent figures. Show students a variety of line segments, angles, and polygons. Label the congruent pairs and introduce the term *congruent.* Label the other pairs as *noncongruent.*
2. Ask students to create a definition for the term *congruent* based on the examples shown. Discuss student definitions, and ensure that all students understand the concept of congruence.
3. Ask students to explain what *noncongruent* means. Allow students to share their responses.
4. Explain to students that they will need to be able to compare geometric figures and determine if they are congruent.
5. Place students with a partner and have them create a procedure for determining if lines, angles, and polygons are congruent. Encourage the students to be creative and very specific with their methods. (They do not need to test their theories at this time, just put them in writing.)
6. Allow students to share the methods they created.
7. Distribute the "Congruence Worksheet" to students, and instruct them to experiment with their chosen method to determine if effective (at this point, do not discourage any methods). Be sure to provide students with any materials they may need to test their theories (rulers, protractors, transparencies, dot paper, etc.)
8. Have students explain in writing whether or not their method worked and why they think it did or did not.
9. Allow students who found effective methods to explain and demonstrate to classmates.
10. Have students write a journal entry on what congruence is and how to determine the congruence of any geometric figure. Encourage them to include examples.

Sample assessment

- Informally assess students as they work to understand congruence, and create methods for determining congruence.

- Use their journal entry as part of a student portfolio or a quiz grade.

Follow-up/extension

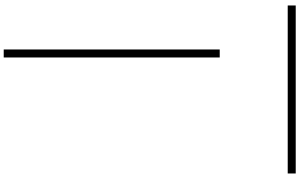
- Have students locate congruent geometric figures throughout the classroom. Complete Similarity or Congruence activity at http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/midgeo1.PDF.

Congruence Worksheet

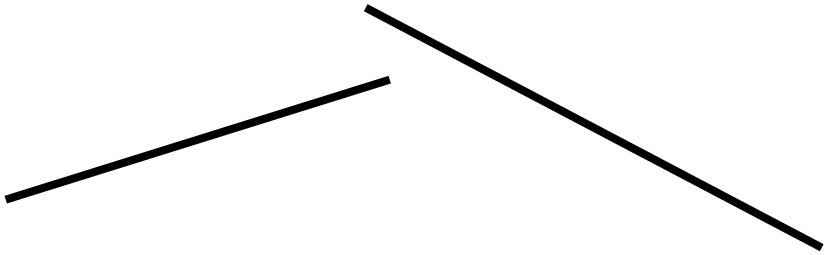
Name: _____ Date: _____

A. Determine if each pair of lines are congruent or noncongruent. Write your answer on the space by the problem number.

1. _____

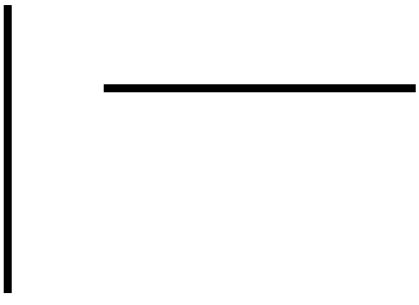


2. _____

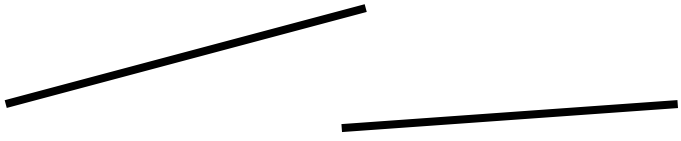


3. _____

4. _____

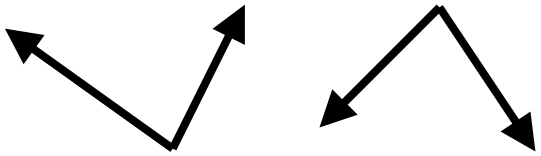


5. _____

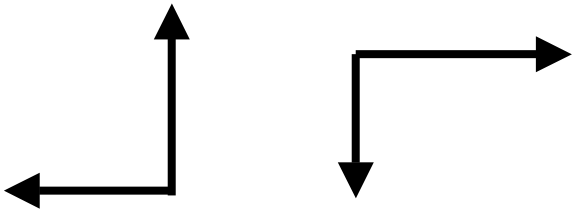


B. Determine if each pair of angles is congruent or noncongruent. Write your answer on the space by the problem number.

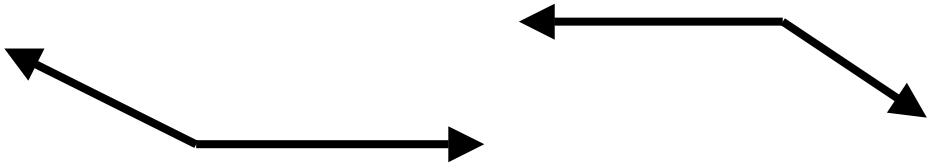
1. _____



2. _____



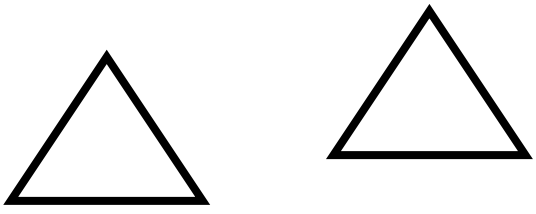
3. _____



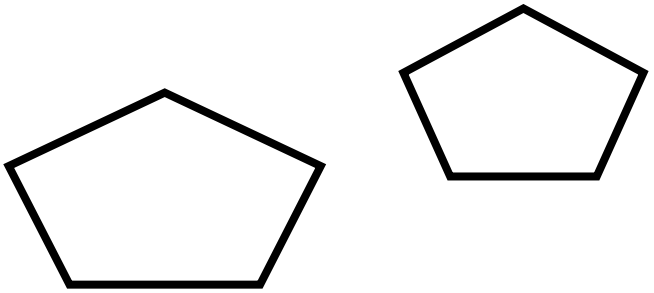
C. Determine if each pair of polygons are congruent or noncongruent. Write your answer on the space by the problem number.

1. _____

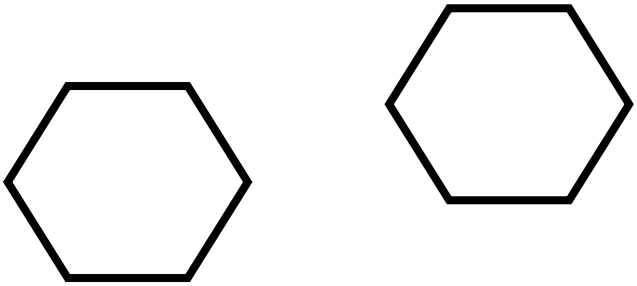
2. _____



3. _____



4. _____



Bisectors

Reporting category	Geometry
Overview	Students create a definition for *bisect* and then use appropriate steps to bisect line segments and angles.
Related Standard of Learning	6.16

Objective

- The student will construct the perpendicular bisector of a line segment and an angle bisector.

Materials needed

- Compass
- Ruler
- "Bisecting Worksheet" (two pages), one copy for each student

Instructional activity

1. Write the word *bisect* on the overhead and have students create a definition, using their own prior knowledge. Explain to students that by the end of class they will know what *bisect* means and will have the opportunity to correct wrong definitions.
2. Draw a line segment on the board or overhead and demonstrate the method for bisecting it, using a compass. (Don't use the term *bisect* at this time).
3. Ask students what you have done to the line segment. (Expected responses include "cut in half," "made perpendicular lines," "created two segments.")
4. Repeat step 2, using an angle.
5. Explain to students that they have seen 2 geometric figures *bisected.* Have students revisit their definitions and see if their idea of a definition is correct. Encourage them to change their original definition if they feel they need to.
6. Have students share their definitions, and ensure that all students understand what it means to bisect a line segment or angle.
7. Demonstrate again the method for bisecting a line segment and an angle. Instruct students to copy steps as you demonstrate them.
8. Place students with a partner. Instruct students to draw a line segment of a length of their choice. They should not tell their partners the length of their segment.
9. Have students trade segments with their partner and use the steps they were just shown to bisect the segment.
10. Have partners trade back and check the bisections by measuring the newly created segments. Ensure that they understand that each new segment should be congruent.
11. Repeat the process with an angle.
12. Monitor students to ensure they are following proper procedures.
13. Distribute "Bisecting Worksheet" to students, and instruct them to complete it.

Sample assessment

- Informally assess students as they explore a definition for *bisect* and practice the steps for bisecting segments and angles.
- You can use "Bisecting Worksheet" as a graded quiz.

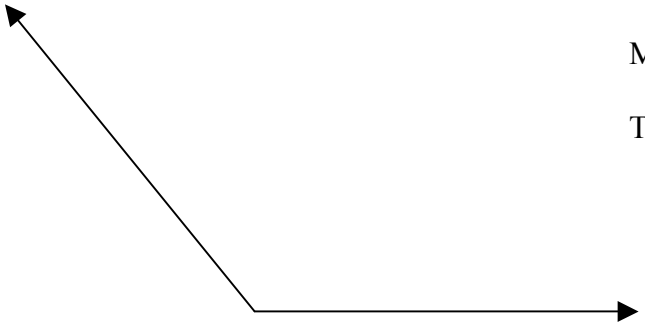
Follow-up/extension

- Have students complete a journal entry on bisecting and explain when it is used in real life.

Bisecting Worksheet

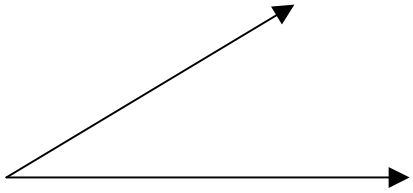
Name: _____ Date: _____

A. Measure each angle below and label the angle type (acute, obtuse, right, or straight). Then construct the bisector of each angle.

1. 

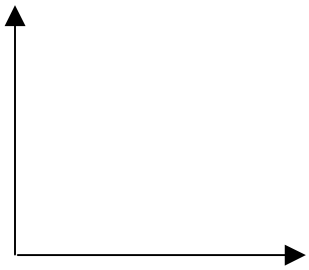
Measure _____

Type _____

2. 

Measure _____

Type _____

3. 

Measure _____

Type _____

4. 

Measure _____

Type _____

5. 

Measure _____

Type _____

B. Construct the perpendicular bisector of each line segment below.

1. _____

2. _____

Exploring Polygons

Reporting category	Geometry
Overview	Students explore polygons through sorting, identify polygons based on their attributes, and draw polygons.
Related Standard of Learning	6.14

Objective

- The student will classify triangles, quadrilaterals, pentagons, and hexagons.

Materials needed

- Pattern blocks
- A copy of the "Polygons Notes" handout for each student
- A copy of the "Polygons Semantic Map" handout for each student

Instructional activity

1. Review the definition of *polygon*. (Polygon means *many sides*. A polygon is a closed plane figure made up of line segments.)
2. Review the meaning of *congruence*.
3. Place students into groups of four and distribute pattern blocks to each group.
4. Instruct the groups to sort the pattern blocks according to similarities. Have students record why they sorted the shapes as they did.
5. Allow students to share their discoveries.
6. Ask students to describe each polygon and to name it if they can. Help them with any figures that students are unable to name.
7. Distribute notes on polygons, and instruct students to work in their groups to complete them. Check to see that all students complete correctly.
8. Distribute copies of the "Polygons Semantic Map," and explain to students this shows another method of demonstrating classification of polygons. Explain to students how to complete the map.
9. Have students complete the map independently.

Sample assessment

- Informally assess students as they work in their groups to sort polygons and complete notes
- Use the semantic map as a formal assessment.

Follow-up/extension

- Have students complete selected activities from "Plane Figures and Their Properties," found at http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/Geom_middle.pdf.

Polygon Notes

Name: _____ Date: _____

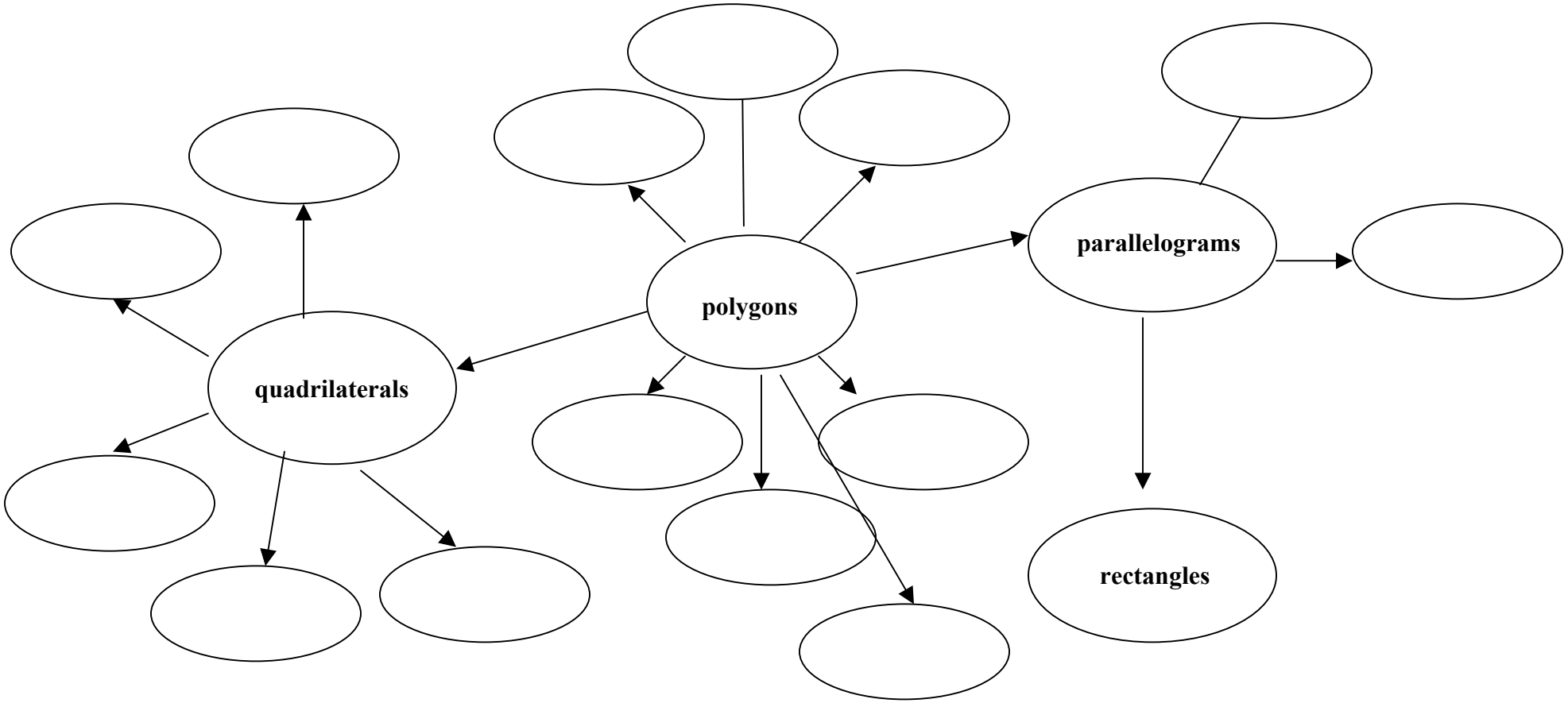
Name	Definition/Description	Drawing
Polygon		
Quadrilateral		
Parallelogram		
Rectangle		
Square		
Rhombus		
Trapezoid		
Triangle		
Pentagon		
Hexagon		

Polygons Semantic Map

Name: _____ Date: _____

Use the word bank below to complete the map below. You may use a term more than once. Place each term with every category in which it belongs. The main headings for each group have already been listed.

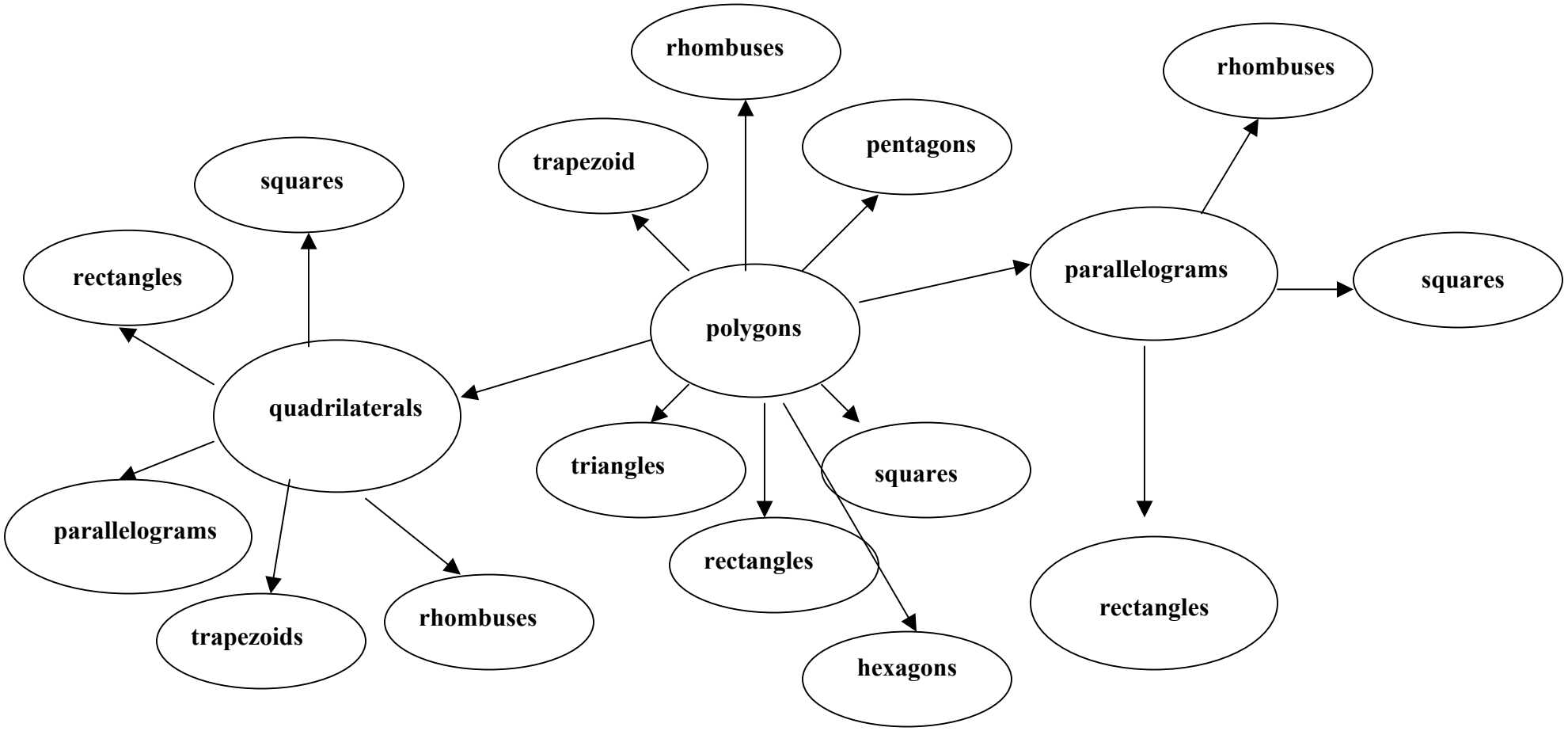
Word Bank: **squares, rectangles, parallelograms, trapezoids, rhombuses, pentagons, triangles, hexagons**.



Polygons Semantic Map Key

Use the word bank below to complete the map below. You may use a term more than once. Place each term with every category in which it belongs. The main headings for each group have already been listed.

Word Bank: **squares, rectangles, parallelograms, trapezoids, rhombuses, pentagons, triangles, hexagons**.



Exploring Quadrilaterals

Reporting category Geometry

Overview Students explore characteristics of quadrilaterals and sort various quadrilaterals according to those characteristics.

Related Standard of Learning 6.14

Objectives

- The student will classify quadrilaterals by pairs of parallel sides by sorting.
- The student will identify and describe the similarities and differences in sets of quadrilaterals by sorting.

Materials needed

- Quadrilateral sorting pieces (see below), cut apart and placed into baggies.
- A copy of the "Quadrilateral Notes" handout for each student

Instructional activity

1. Review the types of polygons by displaying a triangle, quadrilateral, pentagon, and hexagon and having students name and describe each shape.
2. Distribute to partners baggies containing the following quadrilateral sorting pieces: trapezoid, square, rectangle, parallelogram, and rhombus.
3. Ask students to name the type of polygon all of these shapes could be classified as. (quadrilateral)
4. Ask students what else the shapes could be classified as. Ensure that all students know the name of each shape.
5. Instruct students to sort the shapes according to their similarities. Allow students to describe the groups they created.
6. Distribute copies of "Quadrilateral Notes" to students, and instruct them to complete.
7. Discuss results. Ensure that all students correctly completed the notes.
8. Have students use what they have learned about quadrilaterals to write riddles about each shape. The riddles should describe the characteristics of each quadrilateral without actually naming it.
9. Place students with a partner, and allow them to share their riddles and solve each other's set of riddles.

Sample assessment

- Informally assess students on sorting activity and notes.
- Formally assess student-created riddles.
- You may also want to create a quiz to check student understanding of polygons.

Follow-up/extension

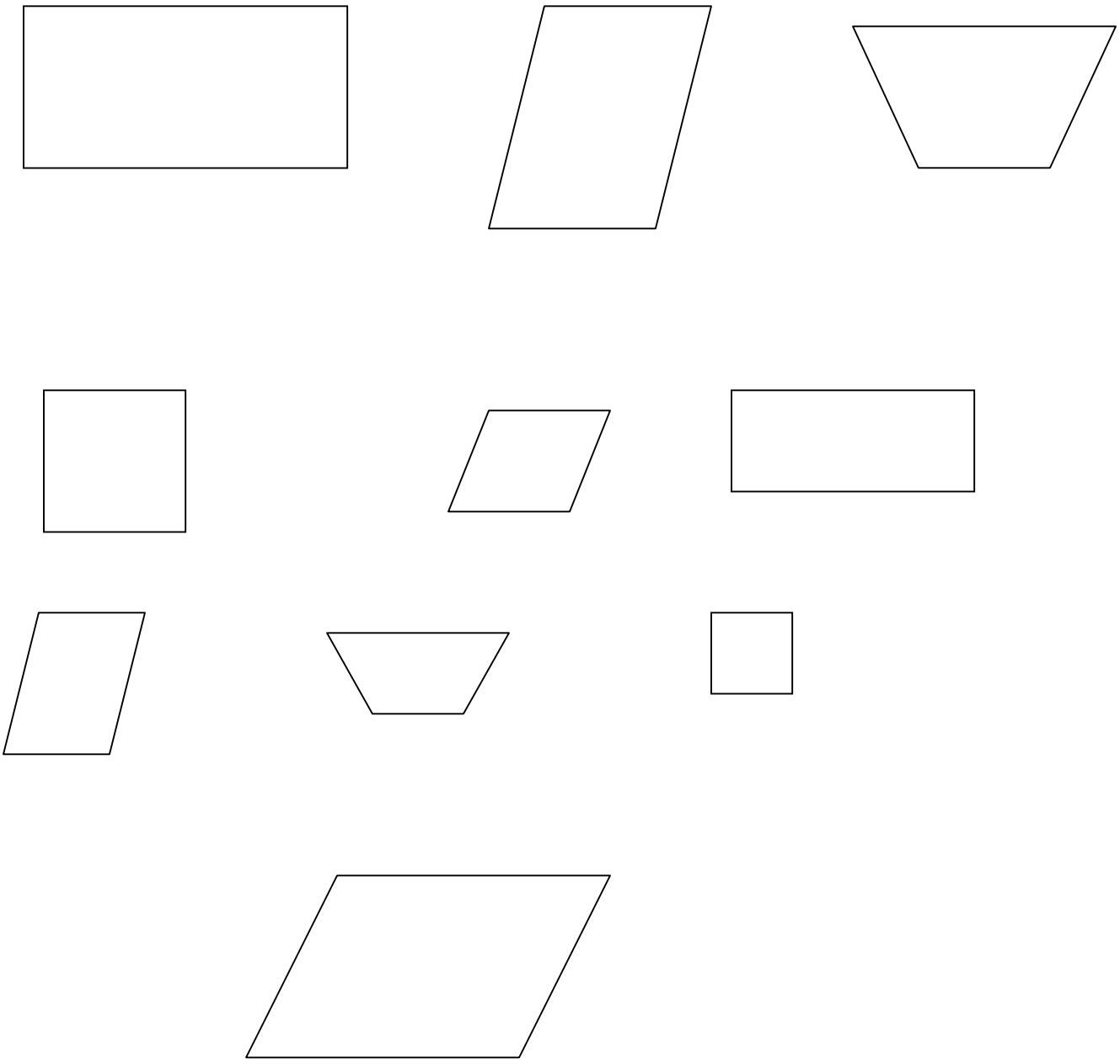
- Have students complete selected activities from "Quadrilaterals and Their Properties," found at http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/Geom_middle.pdf

Quadrilateral Notes

Name: _____ Date: _____

Name of Quadrilateral	Definition/Description	Number of Parallel Sides	Drawing

Quadrilateral Sorting Pieces



Polygons

Reporting category Geometry

Overview Students examine congruence of polygons by first sorting pattern blocks into groups of congruent and noncongruent figures. They then trace the figures on dot paper and measure the sides and angles to prove congruence and noncongruence.

Related Standard of Learning 6.15

Objectives

- The student will characterize polygons as congruent and noncongruent according to the measures of their sides and angles.
- The student will determine the congruence of segments, angles, and polygons by direct comparison, given their attributes.

Materials needed

- Dot paper
- Pattern blocks
- Rulers
- Protractors

Instructional activity

1. Review the definition of *congruence*. Ask students to explain how to determine congruence of geometric figures.
2. Distribute pattern blocks to students, and have students sort them according to congruent and noncongruent polygons.
3. Allow students to share their results.
4. Have students trace the congruent figures on dot paper and then measure their sides and angles to prove congruence. Have students record their results on their dot paper.
5. Have students trace the noncongruent figures on dot paper and measure their sides and angles to prove noncongruence.
6. Encourage students to make any corrections to their original sort.
7. Allow students to share their findings with the class.
8. Have students create drawings of two congruent polygons and two noncongruent polygons. Once they have drawn their figures, have them trade with a partner and check each other's drawings.
9. Have students write a journal entry that explains congruence and describes how to determine if polygons are congruent or not.

Sample assessment

- Informally assess students as they sort and prove congruence or noncongruence.
- Journal entries provide a good record of student understanding and can be used as a formal assessment tool.

Follow-up/extension

- Have the students complete the "Similarity and Congruence with Geostrips" activity found at http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/Geom_middle.pdf.

Quadrilaterals

Reporting category	Geometry
Overview	Students explore characteristics of triangles and quadrilaterals and discover the sum of the measures of the angles of triangles and quadrilaterals.
Related Standard of Learning	6.14

Objective

- The student will identify the sum of the measures of the angles of any triangle or quadrilateral.

Materials needed

- A copy of the "Quadrilaterals" worksheet for each student

Instructional activity

1. Review the proper procedure for measuring angles.
2. Have students draw any type of triangle. Ask students to measure the interior angles of their triangle and to determine the sum. (180°)
3. Allow students to share their results, leading them to understand that the sum of the angles of any triangle is 180°.
4. Draw a picture of a square on the board or overhead. Ask students to identify the shape.
5. Demonstrate to students how to divide the square into two triangles.
6. Ask students whether they could use what they know about the sum of the angles of a triangle to figure out the sum of the angles of a square. Allow students to share their ideas, leading them to realize that the sum of the angles of a square is 360°.
7. Ask students to predict if all quadrilaterals would have the same angles sum. Have students write their predictions along with an explanation.
8. Distribute the "Quadrilaterals" worksheet. to students, and have them identify each shape based on its attributes.
9. Explain to students they will use the sheet to find the measures of the angles.
10. Allow students to work with a partner to complete the worksheet.
11. Once the worksheet is completed, have students revisit their predictions to determine if they were correct.
12. Allow students to share their results.
13. Have students complete a journal entry on quadrilaterals, including types, descriptions of each type, and finding the sum of the measures of the angles of a quadrilateral. Ask them to include in their entry an easy way to always remember that the sum of the measures of the angles of a quadrilateral is 360°.

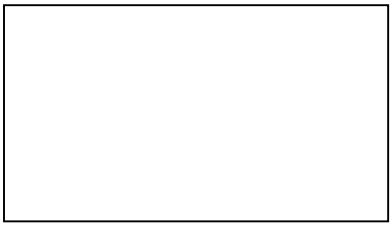
Sample assessment

- Informally assess students as they complete the activity.
- Use the journal entry as a formal assessment.

Quadrilaterals

Name: _____ Date: _____

Divide each quadrilateral into triangles and determine the sum of the measures of the angles of each quadrilateral. Record the results on the space provided.

1. 

Name of quadrilateral: _____

Sum of measures of angles: _____

2. 

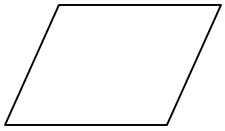
Name of quadrilateral: _____

Sum of measures of angles: _____

3. 

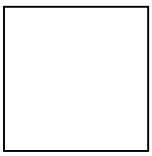
Name of quadrilateral: _____

Sum of measures of angles: _____

4. 

Name of quadrilateral: _____

Sum of measures of angles: _____

5. 

Name of quadrilateral: _____

Sum of measures of angles: _____

Types of Triangles

Note: This lesson was adapted from AIMS Education Foundation: Pieces and Patterns — "Trying Triangles" activity. The Types of Triangles Recording Sheet is a modified version of that included in the AIMS activity.

Reporting category	Measurement / Geometry
Overview	Students conduct experiments using three number cubes to find combinations that will create triangles. They then analyze the created triangle and classify according to attributes.
Related Standard of Learning	6.14

Objectives

- The student will classify a triangle based on the sizes of its angles and/or its sides.
- The student will classify and describe the similarities and differences in sets of triangles by sorting.

Materials needed

- Three numbered cubes per group of students
- 18 plastic straws for each group — 3 straws of each of the following 6 lengths: 1-in., 2-in., 3-in., 4-in., 5-in., and 6-in.
- Yarn or string for threading straws
- A copy of the "Types of Triangles Recording Sheet" for each student

Instructional activity

1. Place students into groups of three to four, and distribute straws, yarn, and number cubes to each group.
2. Demonstrate to students how the ends of straws must meet to form a triangle. Thread a sample together and tie to create a triangle.
3. Have the students toss three dice and determine whether they can form a triangle with straws whose lengths match the numbers thrown. If so, have them thread the three straws together with yarn or string and tie to form the triangle.
4. Distribute the recording sheet to students, and explain how to record results as they conduct the experiment. Tell them to wait to complete triangle-type section.
5. Instruct students to create as many possible triangles that they can in the allotted time.
6. After students have had time to conduct several trials, lead a classroom discussion on results.
7. Possible questions to pose students include: "Which combinations of numbers will not result in the construction of a triangle?" (The sum of the two shorter sides must be greater than the longer side.) "Which pattern of numbers results in the formation of a triangle? How many triangles can be formed from the numbers rolled?"
8. Ask students if they can use what they learned to classify their triangles by the types. If necessary, review the method for classifying triangles by length of sides and measures of angles.
9. Have students return to their recording sheet and complete the type of triangle based on the lengths of the sides.
10. Allow students to share their results, and ensure that all students understand how to classify triangles based on their attributes.

Sample assessment

- Informally assess students as they complete the lab activity.
- Use the recording chart, and specifically types of triangles, as a formal assessment tool.

Follow-up/extension

- Have students sketch the triangles as they create them and also identify the type based on angle measures.

Types of Triangles Recording Sheet

Name: _____ Date: _____

Length of Side A	Length of Side B	Length of Side C	Creates a triangle?	Type of triangle?

3-D Nets

Reporting category Geometry

Overview Students create nets from three-dimensional geometric figures and then identify the figure those nets would create.

Related Standard of Learning 6.17

Objectives

- The student will construct models for rectangular prisms, cones, cylinders, and pyramids.
- The student will sketch rectangular prisms, cones, cylinders, and pyramids.

Materials needed

- Various containers in shapes of prisms, cones, cylinders, and pyramids
- Wrapping paper
- Scissors
- Tape

Instructional activity

1. Display a rectangular prism, and ask students to identify the polygons that create the shape.
2. Have students explain the difference between a rectangle and a rectangular prism.
3. Explain to students that they will be exploring how prisms and pyramids are created.
4. Display a variety of containers, and identify each shape.
5. Place students into groups of four. Give each group a set of two or three containers to wrap. Include as many prisms, pyramids, cones, cylinders as possible. Also give each group a roll of wrapping paper.
6. Instruct students to wrap their containers as neatly as possible, being careful to crease well along all edges and using as little paper as possible. Caution them not to let the paper overlap itself at any point.
7. Have students carefully unwrap their containers, making sure not to tear the paper.
8. Once students have wrapped and unwrapped their containers, they should cut any excess paper from their nets and then visualize the resulting paper as the original container. Explain to students that they have created nets for their shapes.
9. Allow groups to discuss the results of the activity. Have students write an explanation of how they can examine a net and determine what shape would result when the net is constructed.
10. Collect all containers and nets from the groups, and randomly arrange them. Do not place a net with its 3-D representation.
11. Have students match the net with the appropriate container.
12. Allow students to share their results and explain their answers.
13. Review the names of the geometric solids used in the activity.
14. Have students create a journal entry that explains how to create a net from a geometric solid or a geometric solid from a given net. Note: Keep containers and nets for next lesson.

Sample assessment

- Informally assess students as they work in their groups to create nets.
- You can formally assess students as they match all containers with the nets or as they complete a teacher-created assessment that requires them to match nets with geometric solids.
- Journal entries can become part of a portfolio, or they can be collected for formal assessment of understanding.

More Nets

Reporting category Geometry

Overview Students use the nets from the previous lesson to create a model of the original container. They then create nets from prisms, pyramids, cones, and cylinders, this time by simply looking at the shape and not taking it apart.

Related Standard of Learning 6.17

Objectives

- The student will construct models for rectangular prisms, cones, cylinders, and pyramids.
- The student will sketch rectangular prisms, cones, cylinders, and pyramids from three-dimensional models.

Materials needed

- Nets and containers from previous lesson
- Poster board
- Scissors
- Tape
- Glue
- Dot paper or graph paper

Instructional activity

1. Review prisms, pyramids, cones, and cylinders.
2. Display completed nets from the previous lesson, and explain that to students that they will be creating three-dimensional shapes from the nets. Discuss what "three-dimensional" or "3-D" means.
3. Place students into same groups used in the previous activity. Distribute poster board, scissors, tape, and glue to the groups.
4. Distribute a set of nets to each group, ensuring that they do not get one of the nets that they made during the previous activity.
5. Explain that their task is first to identify the 3-D shape that is represented by the net and then to construct the shape, using the net.
6. Have students first glue the net to their poster board, being very careful to glue only one flat surface area or face, which is defined by the creases. If necessary, they can use a marker to trace the creases to better identify the face they should glue.
7. Once the net is secure on the poster, instruct students to fold along the creases and construct the model of the container. Monitor students and assist where needed.
8. Have students repeat the process for each net.
9. Allow groups to display their constructed shapes and share their experiences and what they learned as they worked through the activity.
10. Display all constructions, and discuss.
11. Distribute dot paper or graph paper to students and have them choose 1 pyramid, 1 prism, 1 cone, and 1 cylinder from the displayed shape first to sketch in its 3-D form and then to sketch a net that

would create the form. Check student results. Ensure that all students can properly sketch nets and 3-D figures.

Sample assessment

- Informally assess students as they create their models from nets and as they share what they learned during the activity.
- Use their final sketches of 3-D figures and nets to formally assess.

Nets to 3-D

Reporting category Geometry

Overview Students create nets from given three-dimensional models. They then label the nets and sketch models of their nets.

Related Standard of Learning 6.17

Objectives

- The student will construct models for rectangular prisms, cones, cylinders, and pyramids.
- The student will sketch rectangular prisms, cones, cylinders, and pyramids from three-dimensional models.

Materials needed

- Geometric solids
- Index cards
- A copy of the "Three-Dimensional Figures" worksheet for each student
- Teacher-created worksheet on geometric solids

Instructional activity

1. Review three-dimensional shapes by displaying geometric solids and having students name each. Review the shapes of all faces.
2. Ask students to imagine what the shapes would look like if they were opened/unfolded.
3. Group students into pairs and give each pair two or three geometric solids and index cards.
4. Instruct students to measure each face and then draw the faces on their index cards.
5. Have students cut out the faces and then tape them together to create a net of the three-dimensional shape. Caution students that when taping faces together, they must position them properly, as not all arrangements will work so that when the net is folded, it will create the original shape.
6. Distribute the "Three-Dimensional Figures" worksheet to students, and instruct them to complete it with data for each of their solids. Tell students once they have sketched their net, they can then construct their model to make sure they can create the correct solid from the net.
7. Once all groups of students have completed their worksheet, allow students to present results to the class. As groups are presenting, other students should add that group's results to their own worksheet. Ensure that all students have a completed worksheet that includes all solids.
8. Distribute a worksheet on geometric solids (created by teacher or take from an available workbook) to students, and have them complete independently.

Sample assessment

- Informally assess as students work to complete the activity.
- Use the Three-Dimensional Figures worksheet to formally assess students, or use the second worksheet.

Three-Dimensional Figures

Name: _____ **Date:** _____

Name of Solid	Shape of Face	Sketch of Net	Sketch of Solid

Geometric Solid Sort

Reporting category Geometry

Overview Students work in groups to sort geometric solids according to attributes. They explain the criteria used when sorting and then compare with methods used by other groups.

Related Standard of Learning 6.17

Objectives

- The student will classify rectangular prisms, cones, cylinders, and pyramids by their two-dimensional representation.

Materials needed

- Geometric solids
- Chart paper

Instructional activity

Note: This activity comes from "Geometry for Middle School Teachers — What's My Shape" activity, found at *http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/Geom_middle.pdf*.

1. Review geometric solid ideas, such as lateral face, base, edges, and vertices.
2. Place students into groups of four, and provide each group with a set of geometric solids.
3. Instruct students to sort according to attributes and explain their reason for each grouping.
4. Have students sketch on chart paper the figures in each group they created and explain in writing the criteria used for each set.
5. Display all completed charts around the room and conduct a walk-about, allowing each group to move from chart to chart to analyze other groups' sorting methods. Instruct them to compare these methods with their own methods.
6. Discuss results, and allow students to share their discoveries.
7. Sort all solids into the following groups: prisms, pyramids, cones, and cylinders. Identify each classification. Lead students to understand that each shape is identified using the two-dimensional shapes that are put together to create the three-dimensional object. For example, a rectangular prism has two bases that are rectangular and four lateral faces that are rectangular.
8. Instruct students to work in their groups to determine what two-dimensional attributes each group of geometric solids has. Allow students to share their ideas, ensuring that they understand the differences in each group.
9. Have students write a journal entry that explains how to classify geometric solids as prisms, pyramids, cones, and cylinders. Tell them to include the two-dimensional shapes that can be used to create each 3-D figure.

Sample assessment

- Informally assess students based on their group activity.
- Formally assess their journal entries to ensure that individuals understand the concepts.
- You may want to provide students with a written quiz or test to complete at this time.

Follow-up/extension

- Have student complete selected "Solid Geometry" activities, found at http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/Geom_middle.pdf.

What Is It?

Reporting category Geometry

Overview Students use their understanding of solid figures to pose questions about an unknown solid figure to determine its shape. They also have an opportunity to identify a geometric solid by touching it but not seeing it.

Related Standard of Learning 6.17

Objective

- The student will identify a three-dimensional model of a prism, cone, cylinder, or pyramid from its two-dimensional representation.

Materials needed

- A set of geometric solids
- A box or bag

Instructional activity

Part 1

1. Review the definitions of *prism*, *pyramid*, *cone*, and *cylinder*.
2. Hold up a box or bag and explain to students that it contains a geometric solid. Their job is to determine what is in the bag by asking a series of yes/no questions about the object, but they may not ask the name of the object.
3. Allow students about five minutes to develop a list of questions to ask.
4. Solicit and have students respond to the questions until someone is able to identify the shape.
5. Repeat the procedure with a variety of geometric solids.

Part 2

6. Empty the bag or box, and begin the process with new shapes, this time telling students they will not be able to ask questions, but rather they may simply touch the shape to figure out what it is.
7. Allow students to come up one at a time and reach into the container to feel the shape.
8. Once all students have had an opportunity to touch the shape, allow them to share what they think the shape is.
9. Continue accepting responses until you receive a correct one.
10. Repeat the procedure with a variety of geometric solids.

Sample assessment

- Informally assess students as they participate in the activities.
- Use the follow-up/extension as a formal assessment, if you wish.

Follow-up/extension

- After completing several rounds of the above activities, have the students sketch the shapes they believe are in the bag based on the answers to the yes/no questions or on what they felt when they touched the object.

- Have student complete the "Ask Me About It" and "What's My Shape? Touch Me" activities found at http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/Geom_middle.pdf.

Polyhedra

Note: This activity comes from "Geometry for Middle School Teachers — Creating Nets" activity, found at http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/Geom_middle.pdf

Reporting category	Geometry
Overview	Students cut apart various polyhedra to form nets.
Related Standard of Learning	6.17

Objectives

- The student will identify a three-dimensional model of a prism, cone, cylinder, or pyramid from its two-dimensional representation.
- The student will classify prisms, cones, cylinders, and pyramids by their two-dimensional representations.

Materials needed

- Cardboard cereal boxes, canisters, milk cartons, etc. — several examples of each shape.
- Scissors

Instructional activity

1. Review geometric solids by displaying a variety of containers and having students identify the two-dimensional shapes that are put together to create the containers and by naming the 3-D figures.
2. Demonstrate the activity by carefully cutting apart one of the containers and flattening it. Keep all pieces connected and display the net that is created.
3. Explain to students that there are a variety of ways to create a net from an object and that the one you just created is only one example. Challenge students to find another way to create a net from the same object.
4. Allow students to work with a partner to create a different net. Share results when students have finished.
5. Repeat the process with a variety of containers.
6. Distribute "Creating Nets" worksheet found on page 211 in Geometry Module 4: Creating Nets Activity at http://www.pen.k12.va.us/VDOE/Instruction/Elem_M/Geom_middle.pdf

Sample assessment

- Informally assess students on the activity.
- Use the completed worksheet for a formal assessment.

More Polyhedra

Reporting category Geometry

Overview Students construct models of polyhedra, using toothpicks and clay. They then explore the two-dimensional shapes that create the faces of the three-dimensional solid.

Related Standard of Learning 6.17

Objectives

- The student will construct models of rectangular prisms, cones, cylinders, and pyramids.
- The student will identify a three-dimensional model of a prism, cone, cylinder, or pyramid from its two-dimensional representation.

Materials needed

- Bubble solution in large buckets
- Clay
- Toothpicks

Instructional activity

1. Review various polyhedra by first displaying a net of a solid and having students name the solid it creates. Then display the solid represented by the net. Also review edges and vertices of the three-dimensional shape.
2. Explain to students they will be creating models of polyhedra using clay and toothpicks.
3. Place students in pairs, and give each pair a model of various prisms and pyramids.
4. Demonstrate the activity by constructing a cube, using toothpicks and clay. Explain that the toothpicks form the edges of the shape and the clay holds the edges together at the vertices.
5. Allow partners to work together to create models of all of their shapes. Monitor students as they work to create their models. Have students share their creations with the class and describe the attributes of their figures.
6. Provide students with buckets of bubble solution (outside if weather permits). Explain that they will dip their models into the solution. Ask students to predict what will happen. Allow them to share their predictions.
7. Demonstrate the procedure with the cube made previously. Ensure that students realize that the bubble solution will create the faces of the geometric solids.
8. Have students dip each of their models into the solution and then sketch the geometric solid that they see by first sketching each face and then sketch the three-dimensional figure.
9. Have students write a journal entry about the likenesses and differences of pyramids and prisms.

Sample assessment

- Informally assess students as they create their models and complete the activity.
- Use the journal as a formal assessment.

Sample resources

Navigating through Geometry in Grade 6 through Grade 8 – Available from NCTM. Contains additional lessons for data analysis activities.

Geometry for Middle School Teachers Professional Development Module available from VDOE.

Van Hiele Levels of Geometric Thought CD – Available through the Virginia Department of Education. Contains assessments to determine children's level of geometric thinking.

http://www.learnnc.org/LearnNC/lessonp.nsf/docunid/272DFA – A lesson plan on constructing three-dimensional figures.

http://artsedge.kennedy-center.org/teaching_materials/curricula/curriculum.cfm?curriculum_id=213&mode=full – Using the context of lighthouses, this lesson plan focuses on the geometric figures required to construct structures.

http://www.learner.org/teacherslab/math/geometry/space/ – Lessons plans that focus on visualization of three-dimensional objects.

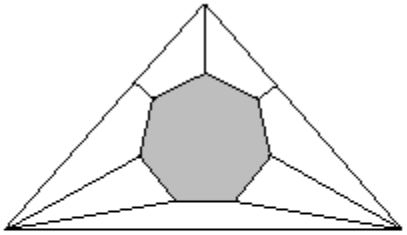
www.mathcats.com – Creative, interactive site for students with geometry activities.

www.matti.usu.edu – A library of virtual manipulatives.

www.standards.nctm.org/document/eexamples/chap4/4.2/ – An interactive geoboard.

Sample released test items

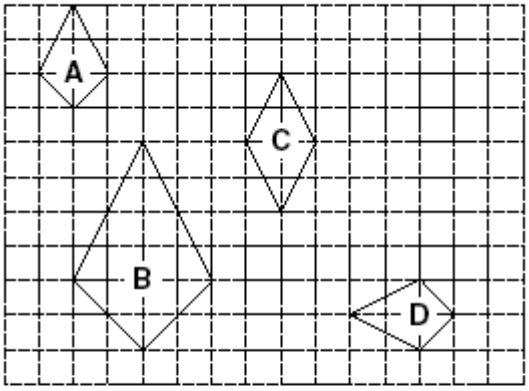
19 Look at the design below.



Which term identifies the shaded part in the center of the design?

A Heptagon
B Pentagon
C Nonagon
D Decagon

23 Which figures appear to be congruent?



A A and B
B B and D
C C and D
D D and A

Organizing Topic Data Analysis

Standards of Learning

6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including
a) line, bar, and circle graphs;
b) stem-and-leaf plots; and
c) box-and-whisker plots.
Circle graphs will be limited to halves, fourths, and eighths.

6.19 The student will describe the mean, median, and mode as measures of central tendency, describe the range, and determine their meaning for a set of data.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Determine essential information necessary to solve consumer application problems.	
• Collect data sets of no more than 20 items by using tally sheets, surveys, observations, questionnaires, interviews, and polls.	
• Organize data by using lists, charts, and tables.	
• Organize and display data in bar and line graphs, displaying the information as clearly as possible by using increments of whole numbers, fractions, and decimals rounded to the nearest tenth.	
• Organize and display data in circle graphs by depicting information as fractional parts that are limited to halves, fourths, and eighths.	
• Organize and display data sets of no more than 20 numbers in stem-and-leaf plots where the stem is listed in ascending order and the leaves are in ascending order, with or without commas between leaves.	
• Organize and display data sets of no more than 20 numbers in box-and-whisker plots, identifying the lower extreme (minimum), lower quartile, median, upper quartile, and upper extreme (maximum). Use the critical points in a box-and-whisker plot to determine the range and the interquartile range.	
• Decide which type of graph is appropriate for a given situation. ° Bar graphs are used to display categorical (discrete) data. ° Line graphs are used to display continuous data. ° Circle graphs are used to show a relationship of the parts to a whole.	
• Interpret data from line, bar, and circle graphs and from stem-and-leaf and box-and-whisker plots.	

- Find the mean for a set of data. _____
- Find the median for a set of data. _____
- Find the mode for a set of data. _____
- Find the range for a set of data. _____
- Describe the three measures of central tendency and a situation in which each would best represent a set of data. _____

Movie Data

Reporting category Probability and Statistics

Overview Students collect data from the current movie section of a newspaper and represent the data in various graphical representations, i.e., line, bar, and circle graphs.

Related Standard of Learning 6.18

Objectives

- The student will collect data by using tally sheets and surveys.
- The student will organize the data by using a chart.
- The student will display the data in a line, bar, and circle graph.

Materials needed

- A movie page from a recent newspaper, one for each group of four students

Instructional activity

1. Distribute the movie section from a recent newspaper, and ask students to examine the information displayed. Ask students what kind of information could be collected from this section. Have students generate a list of five-to-ten questions that they might want to research about the data shown in the movie section. Sample questions could include:
   - At which theaters are new movies opening this weekend?
   - What are the latest and earliest time that movies begin?
   - How many of each type of theater (Regal, AMC, etc.) are there?
   - What are the most popular movies?
   - How many different movies can you see at each theater?
2. Once students have generated questions, have each group select one of the questions to research. Students will organize the data into a chart, decide on a type of graph that would correctly represent the data, and graph the data in a line or bar graph. Upon completion, students will present their correctly labeled graphs of the data to the class. Assess graphs based on correct representation of data and inclusion of a title, labels for the data categories, an appropriate scale, and a key.
3. For the second part of the lesson, ask students how they might use the data in the movie section to answer the question, "What will be the most popular movie this weekend?" Take a survey among the students of which movie will be the most popular, and tally the results. On the following Monday, find the movie-attendance figures in the newspaper and determine which movie actually was "most popular" based on attendance over the weekend. Have students use the information from their survey and Monday's actual attendance information to create circle graphs. Remind students that the circle graph for the *predicted* popular movie is created by drawing a circle and dividing the circle into wedges that represent the percentage of students who voted for each movie. The circle graph for the *actual* most popular movie will require the use of the attendance figures in the newspaper.

Sample assessment

- Use bar and circle graphs created by students as an assessment.

- Use journal entries related to the process of collecting, organizing, and displaying data as additional assessment.
- Have students create additional questions that can be answered using the data in their graphs.

Circle Graphs

Reporting category	Probability and Statistics
Overview	Students construct a circle graph based on data collected in the classroom.
Related Standard of Learning	6.18

Objective

- The student will organize and display data in circle graphs by depicting information as fractional parts that are limited to halves, fourths, and eighths.

Materials needed

- Ball of yarn
- A copy of the three worksheets ("Favorite Ice Creams," "Favorite Amusement Park Rides," and "Favorite Chocolate Treat") for each student
- Compasses, rulers, protractors

Instructional activity

1. Introduce the concept of a circle graph by creating a human circle graph. Explain to the students that they will create a circle graph based on the types of shoes they are wearing. Brainstorm how they could organize themselves by shoe types. Once categories (e.g., tie shoes, slip-ons, sandals) have been determined, have students move outside or into the gym where they can create a large circle. Help students organize themselves into the categories associated with their type of shoe.
2. Select one student to be at the center of the circle. Start the yarn ball at the center student, extend it to the student who begins each category of shoe, then around the circle to the end of that category, and finally back to the center. Continuing on, next extend it to the person at the beginning of the next category, around the circle to the end of that category, back to the center, and so on until the circle is completed. Have the students place the yarn carefully on the floor and move away from the created circle.
3. Ask students to estimate the fractional parts of the circle that are represented by each category of shoe, estimate the central angles created by the wedges, and record these data. Upon returning to the classroom, review the data by recreating the circle on an overhead. Have students determine the percentage of each category. Using a protractor, check to see if their angles estimates were correct.
4. Model using the "Favorite Ice Cream" graph on the overhead. Explain that a circle graph is a graph of data in which parts of a whole are represented as sectors of a circle. A *sector* is a section of the circle bound by two radii and an arc of the circle. Each sector usually contains the actual number or percent of the whole and a label identifying what the sector represents. Some circle graphs use a legend to label the sectors of the graph. The whole is represented by the area of the circle. The parts are represented by the areas of the sectors of the circle. The graph has a descriptive title.
5. Distribute the "Favorite Amusement Park Rides" worksheet, and have the students generate the circle graph, working with a partner. Have the students share their results with another set of partners and assess whether they have included all the attributes of a well-constructed circle graph, as described in step 4.

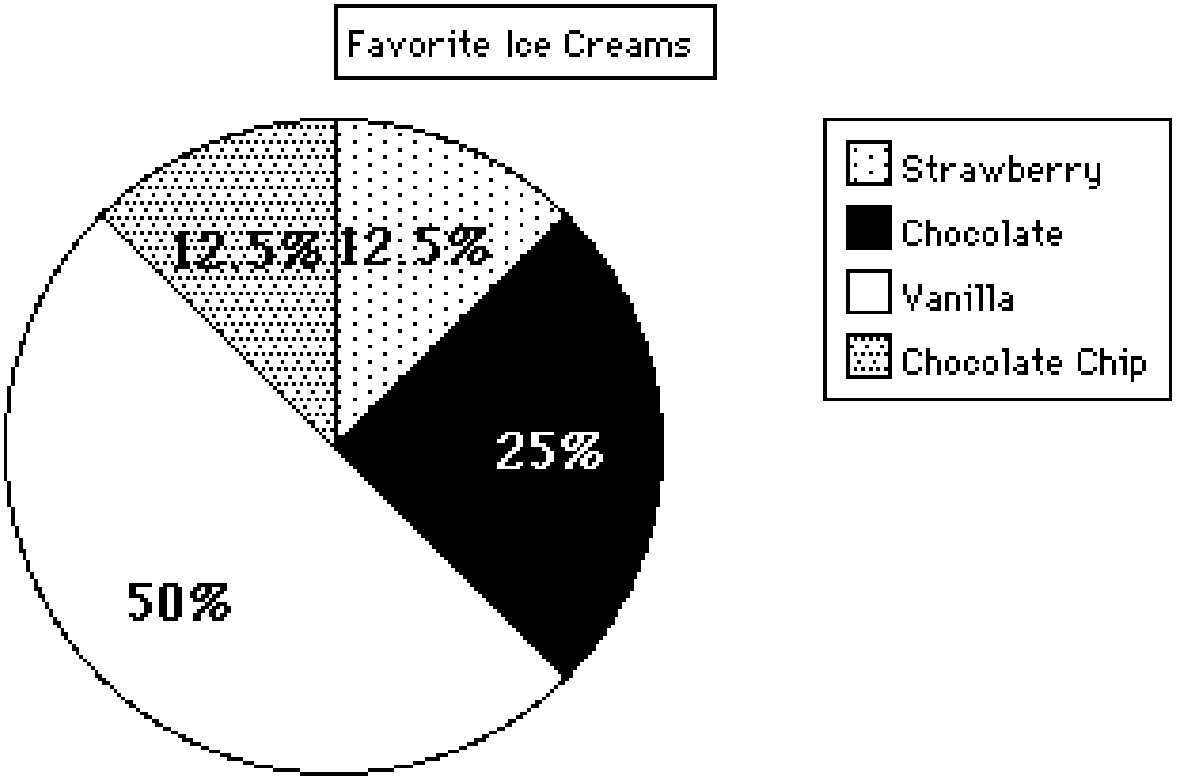
Sample assessment

- Use the "Favorite Chocolate Treat" worksheet as an assessment tool.

Favorite Ice Creams

Flavor	# of Students	Fraction	Central Angle
Strawberry	3	3/24 or 1/8	45°
Chocolate	6	6/24 or 1/4	90°
Vanilla	12	12/24 or 1/2	180°
Chocolate Chip	3	3/24 or 1/8	45°
Total	24	24/24	360°

Students	Fraction	Central Angle



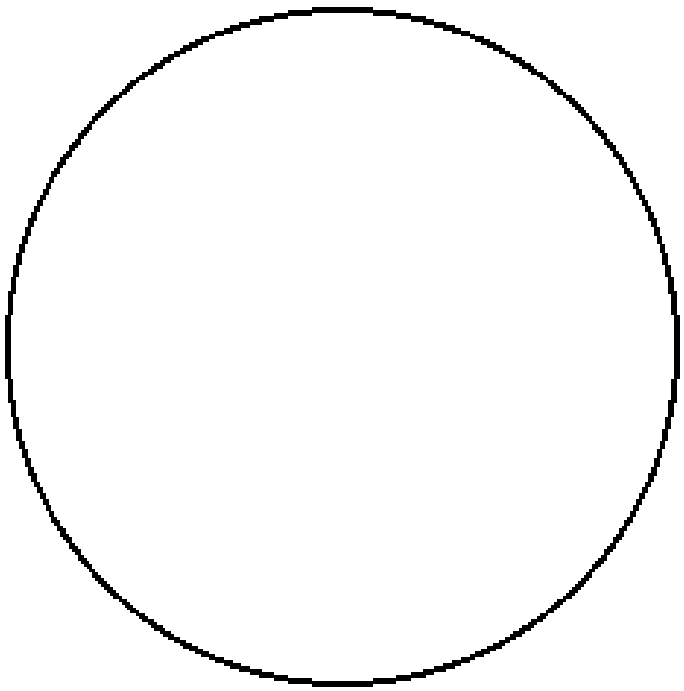
Name: _____ **Date:** _____

Favorite Amusement Park Rides

Use the information in the chart to make a circle graph of the favorite amusement park rides of the students surveyed.

Favorite Ride	Number of Students	Fraction	Central Angle
Sea Monster	16		
Twizzler	12		
Super Spin	8		
Water Log	6		
Wall Climber	6		
Total			
	Students	Fraction	Central Angle

Circle Graph



Explain what the graph tells you about the students' preferences for amusement park rides.

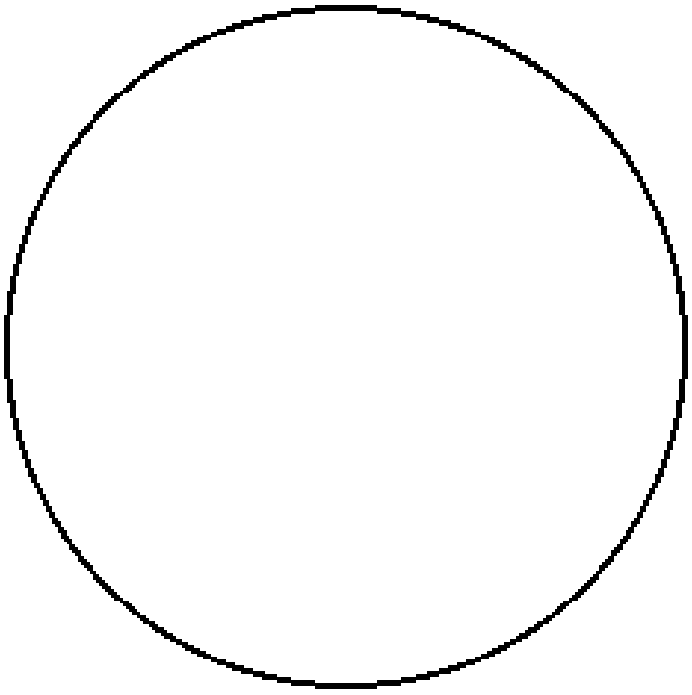
Name: _____ **Date:** _____

Favorite Chocolate Treat

Use the information in the chart to make a circle graph of the favorite chocolate treat of the students surveyed.

Favorite Chocolate Treat	Number of Students	Fraction	Central Angle
Chocolate Cake	6		
Chocolate Ice Cream	12		
Chocolate Chip Cookie	20		
Chocolate Candy	16		
Chocolate Milk	6		
Total	Students	Fraction	Central Angle

Circle Graph



Explain what the graph tells you about the students' preferences for chocolate treats.

Box-and-Whisker Plots

Reporting category Probability and Statistics

Overview Following a brief discussion of the term *median*, the vocabulary listed below, and box-and whisker plots, students gather data from the whole class. A human box-and-whisker plot is constructed from the data. Small groups work together to compare two box-and-whisker plots.

Related Standard of Learning 6.18

Objectives

- The student will organize and display data in box-and-whisker plots, identifying the lower extreme (minimum), lower quartile, median, upper quartile, and upper extreme (maximum).
- The student will use the critical points in a box-and-whisker plot to determine the range and the interquartile range.

Materials needed

- Large picture of a cat or tiger (optional)
- Transparency of the "Vocabulary" worksheet
- Overhead marking pens
- One 3-by-5 card for each student
- Thick craft yarn
- Scissors
- Six signs with the following labels: "median," "lower extreme," "upper extreme," "lower quartile," "upper quartile," and "interquartile range" and with string attached to hang around the necks of students
- Line on the ground (e.g., tape on the floor, chalk on concrete, or line in the tile)
- Polaroid camera or video camera (optional)
- Blank transparency
- A copy of the "Assessing Box-and-Whisker Plots" worksheet for each student

Instructional activity

1. *Background Information:* A box-and-whisker plot is a type of graph used to represent data. It is most appropriate when you want to show the median, first and third quartiles, and least and greatest of a set of data. The "box" is like a cat's face, and the "whiskers" are formed by the data that extends out from the box.
2. Review the vocabulary for this topic, using the vocabulary transparency and writing in the definitions. If you have a picture of a cat or tiger, briefly show how the face forms a box with the whiskers coming out the side.
3. Set the context by telling the students that they need to find the "average" number of letters in the first and last names of the students in the class in order to decide what to charge for class T-shirts. The manufacturer charges per letter to personalize them, but we want to charge everyone the same amount, so we'll look at a way to find that average amount by using a box-and-whisker plot. Have each student write the sum of the letters in their first and last name on the 3-by-5 index card.

4. Have the students stand and organize themselves in a line (using the line on the floor) from the least sum to the greatest. Check for proper placement and discuss any discrepancies. Have the students hold their cards in front of them.
5. Locate the pertinent points in this data set, as follows:
   - Locate the **lower extreme** (the smallest sum) and hang that sign around the neck of the student representing that point. Repeat by hanging the **upper extreme** sign around the neck of the student with the largest sum.
   - Locate the **median** by having the lower extreme wave to the upper extreme. Continue to move to the center of the line by having each successive student from both ends of the line wave to each other. The last student to wave should be the median with an equal number of students on both sides. (Note: If two students are in the middle, you need to discuss the fact that the median would be the average of these two numbers. For a first experience, it is better to plan ahead and have an odd number of students in the line, leaving only one person to be the median; an extra student could be the designated photographer or verifier.) Hang the median sign around the neck of the person at that point.
   - Find the median of the upper half of the data by repeating the waving process this time having the wave start with the median and the upper extreme. Label this student as the **upper quartile,** and discuss the fact that this student divides the upper half of the data in two equal parts.
   - Repeat this process to find the **lower quartile,** using the median and the lower extreme. Hang the sign, and discuss the fact that this student divides the lower half of the data in two equal parts.
6. Build the box to include the interquartile range, using the thick craft yarn. Start with the end of the yarn at the lower quartile and have that student hold the end of the yarn shoulder-high. Run the yarn past the median, who may hold on to the yarn to stabilize it, and on to the upper quartile's shoulder. Drop the yarn down to the waist of the upper quartile and have it held there. Run the yarn past the median and back to the waist of the lower quartile, and then back up to the shoulder where you started. Cut the yarn. Thus, the yarn outlines the box. Have a student on either side of the median hold the card that says **interquartile range**. Discuss the fact that the box encloses 50% or half of the group. It contains the middle half of all the data.
7. To create the whiskers, tie a piece of yarn to the center of one side of the lower quartile end of the box, and run it out to the lower extreme. Cut the yarn, and repeat this process from the upper quartile to the upper extreme.
8. If a camera is available, take a picture of the final box-and-whisker plot so the students can have the complete picture of what they have made.
9. Use the blank transparency and the data collected to construct an accurate box-and-whisker plot. Use equal intervals for each sum along a line on the transparency. Have the students help you reconstruct on the transparency what was done to create the human box-and-whisker plot.
10. Compare the human box plot with the one on the transparency. How do they compare? How do they differ? Elicit that each student representing a piece of data had his or her own space in the human box plot, while in the actual box plot, data with the same sum shares a point on a line, thus compacting the data. All the quartiles of the human box plot appear to be the same size, while on the actual plot their sizes may differ even though each quartile contains the same number of sums.
11. Brainstorm with the class other scenarios for collecting data and constructing box-and-whisker plots. Some suggestions might include cost of CDs or some other item students this age purchase, or scores on a set of test papers.

12. Have students form pairs, and give each person a copy of the "Assessing Box-and-Whisker Plots" worksheet. Have them discuss the two box-and-whisker plots on the sheet. Then have each student complete the paragraph as directed.
13. Form small groups by combining two pairs. Have students share their paragraphs in their small groups.

Vocabulary

median –

lower extreme –

upper extreme –

lower quartile –

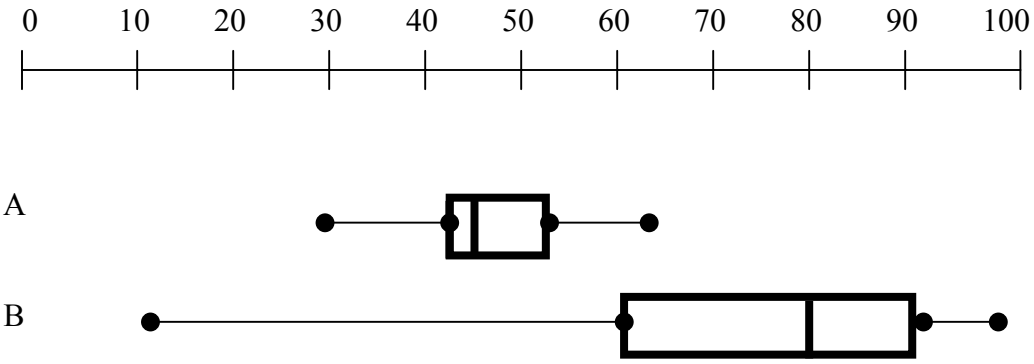
upper quartile –

interquartile range –

Assessing Box-and-Whisker Plots

Name: _____ **Date:** _____

The following box-and-whisker plot represents the test scores for students in two different classes:



Write a paragraph comparing how these two classes did on this test. Give as much information as you can. Refer to vocabulary used in the beginning of the lesson.

Measures of Central Tendency

Reporting category Probability and Statistics

Overview Students grab a handful of linking cubes from a bag of cubes. The number of cubes that each student grabs is recorded and then used to illustrate measures of central tendency and range.

Related Standard of Learning 6.19

Objectives

- The student will find the mean for a set of data.
- The student will find the median for a set of data.
- The student will find the mode for a set of data.
- The student will find the range for a set of data.
- The student will describe the three measures of central tendency and a situation in which each would represent a set of data.

Materials needed

- Linking cubes
- Sticky notes
- A copy of the "Grab a Handful Recording Sheet" for each student

Instructional activity

1. Introduce the activity by explaining to students that they will be investigating measures of central tendency. Ask students to brainstorm what they think these measures are. Have students suggest possible definitions based on learning about these terms in fifth grade. Tell students that they will be determining these measures today without using pencil or paper.
2. Have students grab as large a handful of cubes as possible from a bucket or bag. Tell students to connect all of the cubes and write the number of cubes in their "train" on a sticky note.
3. Explain to the students that they will use their sticky notes to construct a line plot on the chalkboard. Have students line up across the front of the room according to the number of cubes each student has. They should line up in front of each other when there is more than one student with the same number of cubes. This human line plot should be similar to the line plot made on the board with sticky notes. Have them face the board and place their sticky notes on the board directly in front of them.
4. Lead the group through a discussion of the measures of central tendency, using the vocabulary words. Before discussing the words *mode, median* and *mean,* students should be asked what they know. Assessing prior knowledge is key. They also should be encouraged to predict the median and the mean. The mode is easily illustrated on the human line plot and the line plot on the board. (Definitions listed below are from the Curriculum Framework.)
    - The *mean* is the numerical average of the data set. It is found by adding the numbers in the data set together and dividing the sum by the number of data pieces in the set.
    - The *median* is the middle value of a data set in ranked order. If there is an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values.

- The *mode* is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal. Examples:
  - For 2, 3, 4, 5, 5, 6, 7, 8, 8, 8, 9, 11, the mode is 8.
  - For 2, 3, 4, 5, 5, 5, 7, 8, 8, 8, 9, 11, the modes are 5 and 8 (bimodal).
  - For 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 17, there is no mode.
- The *range* is the difference between the greatest and least values in a set of data. It shows the spread in a set of data.

5. Before discussion of the median begins, ask students to spread out in one line in order to form a representation of the data from the smallest handful to the largest handful. Ask students to move out from the line in pairs — one from each end at the same time. This models the concept of what "center" really is. The movement away from center makes this very clear to students: as the center is approached, one or two students will remain — the center. Discussion should follow regarding the median and what it represents.
6. Before illustrating the mean, students should discuss again what they are really trying to find. Encourage students to share ways they could find the mean, using the cubes. Then have students share linking cubes with each other until all students have a similar number of cubes. Have them share until further sharing is not helpful for reaching the same number of cubes for each student. More than likely two groups will exist.: some students will have trains of one number of cubes, and the other group will have trains of another number. At this time, have students discuss the mean, and also use the term *average*.
7. End the session by reviewing the measures of central tendency and stating again the actual answers for this collection of data. Students should be encouraged to discuss how changes in the data would affect the measures of central tendency. Questions such as What would happen if we added more students to our sample? What if the largest handful was 10 cubes larger than our current largest handful? and What if there were fewer students in our sample? should be addressed by students in a problem-solving approach.

Sample assessment

- Ask the students to complete the Recording Sheet.
- Have the students write about situations in which each measure of central tendency best represents the data.

Sample resources

www.nces.ed.gov/nceskids/Graphing/ – An interactive Web site for students that allows them to create several types of graphs.

www.aaamath.com/B/sta.htm – An interactive Web site for students with activities to practice finding mean, median, and mode.

http://illuminations.nctm.org/lessonplans/3-5/airplanes/ – In this lesson, students make paper airplanes and explore attributes related to increasing flight distances. Each student collects data from three flights of the airplane and finds the median distance. Students then collect, organize, display, and interpret the median distances for the class in a stem-and-leaf plot.

http://score.kings.k12.ca.us/lessons/mandm.html – A lesson plan that uses small individual bags of "M&M's"® Candies to review students' understanding of estimating, sorting, graphing, mean, median, mode, fractions, percentage, and averaging.

www.manatee.k12.fl.us/sites/elementary/palmasola/mathlabtutstat1.htm – A tutorial for students involving mean, median, and mode.

http://www.brainpop.com/math/dataprobability/meanmodemedianrange/index.weml – An interactive Web site for students using probability concepts.

http://illuminations.nctm.org/lessonplans/6-8/baseball/index.html – Having Fun with Baseball Statistics. These activities allow students to explore statistics surrounding baseball. They are exposed to connections between various mathematical concepts and see where this mathematics is used in areas with which they are familiar.

Navigating through Data Analysis and Probability in Grade 6 through Grade 8 – Available from NCTM. Contains additional lessons for data analysis activities.

Probability and Statistics Professional Development Module – Available from VDOE.

Statistics Professional Development Module – Available from VDOE.

Grab a Handful Recording Sheet

Name: _____ Date: _____

Number of cubes I grabbed in one handful: _____

Number of students participating in this activity: _____

Total number of cubes grabbed by all of the students: _____

Using the data, I discovered the following:

Range of the data: _____

Mode(s) of the data: _____

How did you know? _____

_____

Median of the data: _____

What is the meaning of median? _____

_____

Mean of the data: _____

Why is the mean also called the average? _____

_____

Which measure of central tendency best represents the data and why? _____

_____

For each of the following measures of central tendency, describe a situation in which the measure would be the best representation for the data.

Mean: _____

Median: _____

Mode: _____

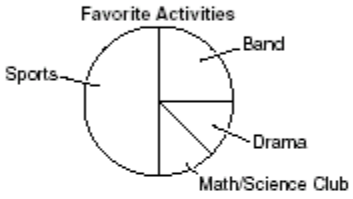
Sample released test items

41 Harry asked several classmates to name their favorite after-school activity and showed the results in this table.

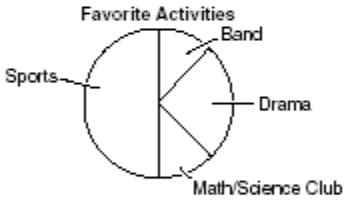
Activity	Number of Students
Band	12
Drama	6
Math/ Science Club	6
Sports	24

Which graph correctly displays this information?

A



B



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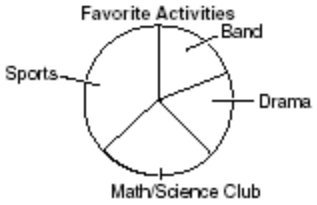


Organizing Topic Probability

Standards of Learning

6.20 The student will
- a) make a sample space for selected experiments and represent it in the form of a list, chart, picture, or tree diagram; and
- b) determine and interpret the probability of an event occurring from a given sample space and represent the probability as a ratio, decimal or percent, as appropriate for the given situation.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Plan and carry out experiments that use concrete materials to find a sample space.	
• Determine the sample space for selected experiments and represent the sample space for up to 20 possibilities as a list, chart, picture, and/or tree diagram.	
• Given a sample space, determine the probability of a simple event. Represent the probability as a ratio, fraction, decimal, or percent where the fraction's denominator does not exceed 20, decimals are rounded to tenths, and percent is rounded to $\frac{1}{10}$ of a percent.	

Fair or Not Fair

Reporting category	Probability and Statistics
Overview	Using one die, students will investigate probability, sample space, and fairness.
Related Standard of Learning	6.20

Objectives

- The student will determine sample space.
- The student will determine fairness of a game.
- The student will represent probability as a ratio, fraction, decimal, or percent.

Materials needed

- One die for each pair of students
- A copy of the "Fair or Not Fair? Game/Recording Sheet" for each pair of students

Instructional activity

Background Note: The *sample space* of an experiment is nothing more than the collection of all the possible outcomes for that experiment. In this case, all of the possible outcomes — the sample space — of the roll of a die are: 1, 2, 3, 4, 5, and 6. A game is considered fair if the likelihood of winning is the same as the likelihood of losing.

1. Explain to students that they will be investigating probability by playing dice games with different ways of winning. After they have played a game and examined the possibilities, they will determine if the game is fair or unfair. Tell them that they will need to know all the possible outcomes of the game, which is referred to as the *sample space*.
2. Model the activity with the first game, with the teacher rolling for player A and members of the class rolling for player B. Each player first decides which numeral(s) will score a point for him/her (see Game Sheet for examples), and then he/she makes a prediction about how many times this numeral(s) will show up in 20 rolls of the die. Show students how to record the prediction — the probability — as a ratio, a fraction, a decimal, and a percent. For example, if the numerals 1, 2, or 3, are predicted to show up 12 out of 20 times, this can be written as 12 to 20, 12:20, 12/20, .6, or 60%. This prediction also indicates that the numerals 4, 5, or 6 would show up 8 out of 20 times, and this should be recorded as 8 to 20, 8:20, 8/20, .4, or 40%. Then, play the game, recording each roll with tally marks, to see *which player's prediction is closest to the actual result*.
3. Each game should be played with 20 rolls, and the result of each game recorded.
4. Ask the students whether their results matched their prediction? Were the games fair? Why or why not? What could be done to make the game fair if it was not?
5. Have the students construct the sample space to help in making the decision about ways to make the game fair.

Sample assessment

- Have students record in their math journals about their investigation of probability, explaining the process of creating the sample space and how they determined the fairness of each game.

Sample resources

http://rec-puzzles.org/probability.html – A Web site containing many word problems involving probability and their solutions.

http://nces.ed.gov/nceskids/probability – National Center for Education Statistics Web site with probability activities for students and many other resources.

http://mathforum.org/dr.math/faq/faq.boy.girl.html – Probability activities and information from the Math Forum.

http://standards.nctm.org/document/chapter5/data.htm#bp4 – NCTM's Principles and Standards information about probability at the 3–5 grade levels.

http://www.pbs.org/teachersource/mathline/lessonplans/esmp/chances/chances_procedure.shtm – PBS lesson plan on probability.

http://mathforum.org/probstat/probstat.lessons.html – Lesson plans from the Math Forum on probability.

Probability and Statistics Professional Development Module – Available from VDOE Web site. Contains activities related to this strand that can be modified for student use.

Navigating through Data Analysis and Probability in Grades 6–8 – Available from NCTM. Contains additional lessons for probability activities.

Sample released test items

36 **A package contains 7 bags of tortilla chips, 3 bags of cheese puffs, 4 bags of potato chips, and 6 bags of corn chips. If Steve reaches into the package and selects one bag without looking, what is the probability he will choose potato chips?**

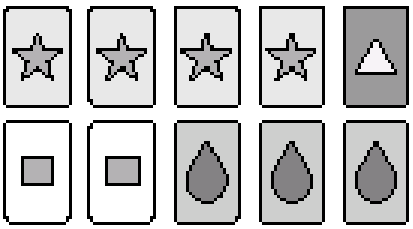
F $\frac{2}{20}$

G $\frac{1}{5}$

H $\frac{3}{10}$

J $\frac{7}{20}$

38 **These cards are used to play a game between two players.**



If Sally draws one card at random, what is the probability it will be a card with a star?

F $\frac{2}{5}$

G $\frac{3}{10}$

H $\frac{1}{5}$

J $\frac{1}{10}$

Fair or Not Fair? Game Sheet

Name: _____ **Date:** _____

Game Number	Player	Scoring Numerals
1	A B	the numbers 1, 2, or 3 the numbers 4, 5, or 6
2	A B	any odd number any even number
3	A B	any number less than 4 any number greater than 4 (a 4 does not win)
4	A B	any prime number any composite number

Fair or Not Fair? Recording Sheet

Game Number	Player: Scoring Numerals	Tally	Total
1	A: 1, 2, 3		
	B: 4, 5, 6		
2	A: odd numbers		
	B: even numbers		
3	A: numbers < 4		
	B: numbers > 4		
4	A: prime numbers		
	B: composite numbers		

Organizing Topic Patterns, Functions, and Algebra

Standards of Learning

6.21 The student will investigate, describe, and extend numerical and geometric patterns, including triangular numbers, patterns formed by powers of 10, and arithmetic sequences.

6.22 The student will investigate and describe concepts of positive exponents, perfect squares, square roots, and, for numbers greater than 10, scientific notation. Calculators will be used to develop exponential patterns.

6.23 The student will
a) model and solve algebraic equations, using concrete materials;
b) solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions; and
c) use the following algebraic terms appropriately: *variable, coefficient, term*, and *equation*.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Investigate and apply strategies to recognize and describe the change between terms in numerical patterns.	_____
• Investigate and apply strategies to recognize and describe geometric patterns.	_____
• Describe verbally and in writing the relationships between consecutive terms in a numerical or geometric pattern.	_____
• Extend and apply numerical and geometric patterns to similar situations.	_____
• Create numerical and geometric patterns by using a given rule or mathematical relationship.	_____
• Describe numerical and geometric patterns, including triangular numbers.	_____
• Recognize and describe patterns with exponents by using a calculator.	_____
• Recognize and describe patterns of perfect squares.	_____
• Recognize and describe patterns with square roots and squares by using squares, grid paper, and calculators.	_____
• Recognize powers of 10 by examining patterns in a place-value chart: $10^4 = 10{,}000$, $10^3 = 1000$, $10^2 = 100$, and $10^1 = 10$.	_____
• Represent a one-step equation, using a variety of concrete materials such as colored chips on an equation mat, algebra tiles, or weights on a balance scale.	_____

- Solve a one-step equation by demonstrating the steps algebraically.
- Use the following algebraic terms appropriately: *equation*, v*ariable, term,* and *coefficient*.
- Identify examples of equations, variables, terms, and coefficients.

Toothpick Patterns

Lesson adapted from materials developed by Education Development Center, Inc. (EDC)

Reporting category Patterns, Functions, and Algebra

Overview Students explore patterns created with toothpicks as they create squares, rectangles, and triangles.

Related Standards of Learning 6.21, 6.22, 6.23

Objectives

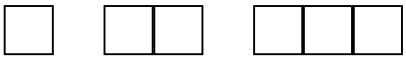
- The student will investigate strategies as he/she describes the change in a growing pattern and describes the relationships between consecutive terms in a growing pattern.
- The student will extend patterns.
- The student will recognize and describe patterns of perfect squares.
- The student will model and solve algebraic equations, using concrete materials.

Materials needed

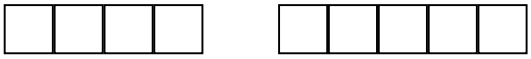
- 100 toothpicks per group of four students, or grid paper on which students can draw the growing figures.

Instructional activity

1. Introduce the lesson to the students by explaining that they will be discovering growing patterns in rectangles, squares, and triangles that they will create with toothpicks.
2. Once the toothpicks are distributed to each group, model for the students how to construct the first three rectangles in a sequence using the toothpicks.



3. Have students construct the next two rectangles in the sequence.



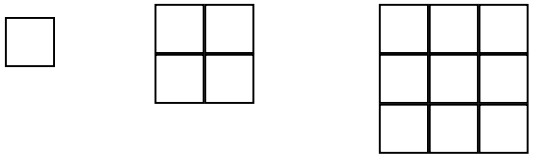
4. Have students describe the pattern that they see in the rectangle constructions. For example, students may notice that for each new rectangle, three toothpicks are added, while the beginning square had one extra toothpick.
5. Create an input/output table to record the number of toothpicks needed to construct each rectangle.

Input (total length of rectangle in toothpicks)	**Output (number of toothpicks needed)**
1	4
2	7
3	10
4	13
5	16
6	
7	

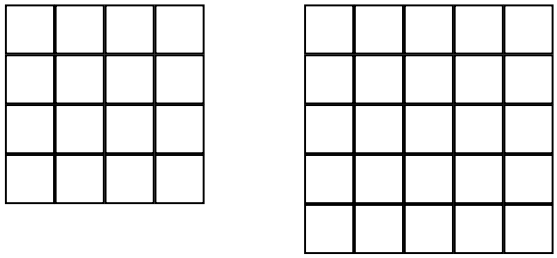
6. Have students look for patterns in the data. Ask if they can predict the number of toothpicks needed to make a rectangle with a side that is six toothpicks long. Seven toothpicks long.
7. If necessary, have students continue constructing rectangles until they notice a pattern and are able to describe it.
8. Ask students to describe a rule for predicting the number of toothpicks needed to make a rectangle of any length. Students may have observed that the length of the rectangle (l) multiplied by 3 plus 1 equals the number of toothpicks (t) needed to construct the rectangle. If students are ready, you may want to introduce how to translate the verbal rule into a mathematical expression. For example, toothpicks = length × 3 + 1; or $t = 3l + 1$.

Part II: Growing Squares

1. In this part of the lesson, students will construct squares that create a growing pattern, different from the rectangle pattern created in the previous lesson.
2. Model for students how to construct the first three squares in this pattern, using toothpicks.



3. Have students construct the next two squares in the sequence, using toothpicks or drawing on grid paper.



4. Have students create an input/output table to record the number of small squares inside each large square, as shown below:

Input (length of side of square in toothpicks)	**Output (number of small squares inside large square)**
1	1
2	4
3	9
4	16
5	25
6	
7	

5. Encourage students to look for patterns in the data. Ask the students if they can predict the number of small squares inside a large square with a side that is six toothpicks long. Seven toothpicks long. If necessary, have students continue constructing squares until they notice a pattern and are able to describe it.

6. Have students describe a rule for predicting the number of small squares inside a large square with a side of any length. Students may have observed that the length of the square (l) multiplied by itself equals the number of small squares (s) inside. If students are ready, you may want to introduce how to translate the verbal rule into a mathematical expression. For example, small squares = length of side × length of side; or $s = l^2$.

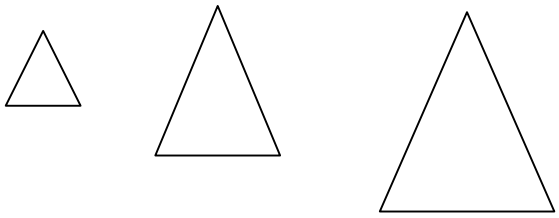
7. Have students create an additional input/output table to record the number of toothpicks needed to construct each square in part II, as shown below:

Input (length of side of square in toothpicks)	**Output (number of toothpicks needed)**
1	4
2	12
3	24
4	40
5	60
6	
7	

8. Encourage students to look for patterns in the data. Ask the students if they can predict the number of toothpicks needed to make a square with sides that are six toothpicks long. Seven toothpicks long.
9. If necessary, have students continue constructing squares until they notice a pattern and are able to describe it.
10. Have students describe a rule for predicting the number of toothpicks needed to make a square of any length. Some students will refer back to the toothpick constructions. These students may observe that the length (l) of the square multiplied by 1 more than itself and then multiplied by 2 equals the number of toothpicks (t) needed; or $[2l(l + 1)] = t$. Other students will refer to the number patterns in the table. These students may notice that the outputs increase first by 8, then 12, then 16, then 20, etc.

Follow-up/extension

- Have students construct the first five toothpick triangles. Then have students create an input/output table (input = length of side, and output = number of toothpicks needed) and describe a general rule for finding the number of toothpicks needed to construct any size triangle.



- Have students construct the first five toothpick hexagons. Then have students create an input/output table (input = length of side, and output = number of toothpicks needed) and describe a general rule for finding the number of toothpicks needed to construct any size hexagon.
- Have students choose any polygon (e.g., pentagon, heptagon, octagon). Have students construct the first five toothpick polygons. Then have student create an input/output table (input = length of side, and output = number of toothpicks needed) and describe a general rule for finding the number of toothpicks needed to construct any size polygon.

Squares and Square Roots

Reporting category Patterns, Functions, and Algebra

Overview Students explore square roots, using color tiles, grid paper, and the area of a square.

Related Standard of Learning 6.22

Objectives

- The student will review the patterns of perfect squares.
- The student will recognize and describe patterns with square roots and squares by using squares, grid paper, and calculators.

Materials needed

- 100 color tiles per group of four students
- Grid paper
- Calculators

Instructional activity

1. Distribute the color tiles to each group of students. Ask them to take 9 tiles and arrange them in a square. Ask students to describe the dimensions of the square by the number of tiles in a row and the number of tiles in a column. Explain that for a square, the measure of the lengths of the sides are all the same, so the number of tiles on any side are the same. Model how to write the dimensions as part of the Area formula: $l \times w$, or 3×3. Indicate to students that since the dimensions are the same, we can simply write 3^2. The area formula for the square is simplified to s^2, s being the length of any side. Record the information in a chart. Have students represent the square on grid paper, recording the total number of tiles in the center of the grid and the dimensions on one side of the grid.

Number of tiles (area of square)	**Dimensions of square**	**Number of tiles on a side (length of a side)** (Add this symbol at the end of the lesson: $\sqrt{area\ of\ square}$)
9	3×3	3
16	4×4	4

2. Model the same process for students, using 16 tiles.
3. At this point, ask students if they can determine what the area of the next larger square would be, either by looking at the pattern in the dimensions of the square (the dimensions appear to be increasing by 1) or by previous knowledge of square numbers.
4. Have students continue to work in groups to find the rest of the squares that can be created with 100 tiles. Have them record their findings in the chart and represent the squares created on the grid paper.

5. After the chart has been completed, explain to students that the dimension of one side of the square is also the square root of that square. When you find a square root of a number, it is the number which, when multiplied by itself, produces the given number. Explain that *squaring a number* and *taking a square root of a number* are inverse operations. At this point, add the radical sign to the last column, and explain that the symbol indicates one is to find the square root of the number. Also explain to the students that the numbers in their first column are perfect squares — the result of multiplying any whole number by itself.

Sample assessment

- Have students describe in their math journals the process of finding the square root of a number, representing the process with examples from their grids. Students should also describe perfect squares.

How Far Is It to the Sun?

Reporting category Patterns, Functions, and Algebra

Overview Students estimate distances from the various planets to the sun and research actual distances. Students learn to express these large numbers, using scientific notation.

Related Standard of Learning 6.22

Objectives

- The student will recognize powers of 10 by examining patterns in a place-value chart.
- The student will write scientific notation for a number greater than 10.

Materials needed

- Internet Web site for distances between the planets and the sun http://www.smv.org/pubs/GiantSSexcel.htm

Instructional activity

Note: This activity may be correlated with earth/space concepts in science.

1. Draw a circle on the board to represent Earth, and place another larger circle at the far end of the board to represent the sun. Connect the two circles with a line segment. Ask students to estimate the approximate distance between the Earth and the sun, and record several responses. Tell students that the actual distance from the Earth to the sun is 150,000,000 kilometers. Note any student responses that were close to the actual distance. Also, remind students that the distance from a planet to the sun varies because of the planets' elliptical orbits. For the purpose of this lesson, the distances used will be considered average distances.
2. Continue by placing another circle to represent the planet Venus about one-third of the distance between the Earth and the sun. Ask students to estimate the approximate distance between the sun and Venus. Record responses. Tell students the average distance from Venus to the sun is 108,000,000 kilometers. Note any responses close to the actual distance.
3. Place another circle to represent Mercury about two-thirds of the distance between the Earth and the sun. Ask students to estimate the approximate distance between the sun and Mercury. Record responses. Tell students the average distance between the sun and Mercury is 58,000,000 kilometers, and note close responses.
4. Discuss with students the problems with keeping track of all the zeros in these large numbers and that scientists prefer a "short-hand" method called "scientific notation." Explain that scientific notation consists of a positive number greater than or equal to 1 but less than 10, multiplied by some power of 10.

	Place Value											
	Hundred billion	**Ten billion**	**Billion**	**Hundred million**	**Ten million**	**Million**	**Hundred thousand**	**Ten thousand**	**Thousand**	**Hundred**	**Ten**	**One**
Power of Ten									10^3 ($10 \times 10 \times 10$)	10^2 (10×10)	10^1	
									1,000	100	10	

5. Using a chart, begin to record the powers of 10, starting with 10^1. Ask students to explain what number 10^1 could represent. Remind students that when using exponential notation, the base is the number multiplied (10) and the exponent represents the number of times the base is used as a factor, so 10^1 would be only 10. Continue with 10^2, asking students to represent this as 10×10. Then 10^3 as $10 \times 10 \times 10$.
6. Ask students to describe the patterns that they see. They should note the increasing exponent's relation to the increasing number of zeros in the product and the number of 10s being multiplied.
7. Have students complete the chart to use as a reference when working with large planetary distances.
8. Refer back to the distances recorded earlier in the lesson. Model for students how to write the distance from Earth to the sun in scientific notation. Explain that 150,000,000 can be written as the product of two factors. One factor has to be greater than or equal to 1 but less than 10. The second factor has to be a power of 10. Demonstrate how to determine the first factor by moving the decimal point 8 places to the left to make a number greater than one but less than 10. The power of 10 is also determined by the number of decimal places moved, in this case 10^8. Have the students refer back to the place-value chart to check whether 10^8 is equal to hundred million, the place value of the original distance.

Planet	**Average distance from the sun in kilometers**	**First factor greater than or equal to 1 but less than 10.**	**Second factor as a power of 10**	**Expressed in scientific notation**
Earth	150,000,000	1.5	10^8	1.5×10^8
Venus				
Mercury				

9. Have students complete the chart for the distances from the sun to Venus and to Mercury. Students can access the following Web site http://www.smv.org/pubs/GiantSSexcel.htm to find the distances for the other six planets and complete the chart for all the planets.

Sample assessment

- The completed chart can be used as an assessment.
- Have students describe in their math journals the process of expressing numbers in scientific notation and examples of situations in which this would be useful.

Balancing Equations

Reporting category Patterns, Functions, and Algebra

Overview Students use unit blocks, variable blocks, and a visual scale to illustrate the two sides of an equation and its equality. "Balance" will be maintained by keeping the sides of the scale equal in value.

Related Standard of Learning 6.23

Objectives

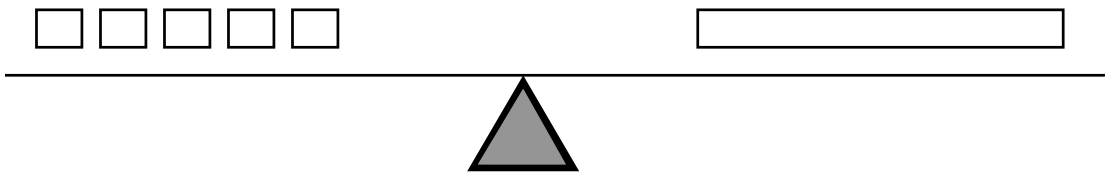
- The student will represent a one-step equation, using a visual of a balance scale and unit blocks.
- The student will use the following terms appropriately: *equation, variable, coefficient,* and *term*.
- The student will identify examples of equations, variables, coefficients, and terms.

Materials needed

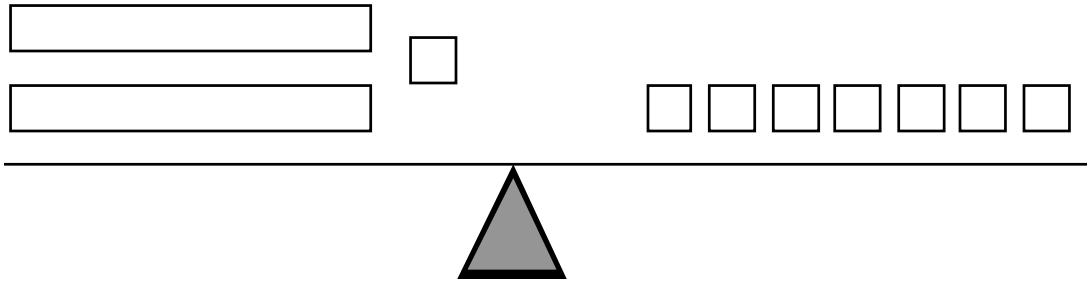
- Pan balance
- 34 pennies
- A copy of the "Balance Scale Mat" handout for each student
- Rods and unit blocks from base-10 materials or other manipulatives
- A copy of the "Algebra Vocabulary" handout for each student

Instructional activity

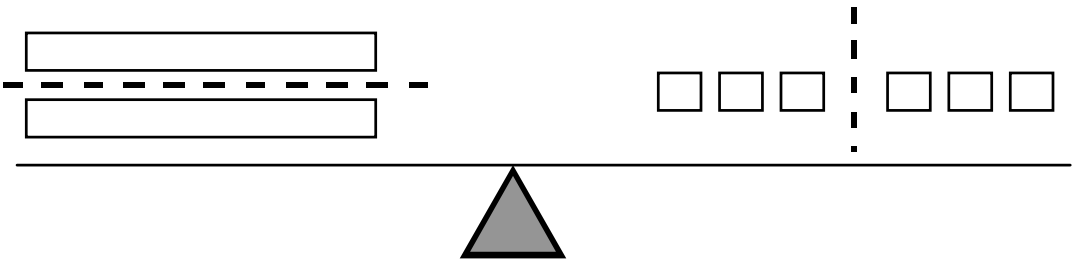
1. Begin the lesson by demonstrating the concept of balance with an actual pan balance, using pennies. Place 10 pennies on each pan. The pans should balance. Remove five pennies from one pan. The balance should tip toward the heavier pan. Remove five pennies from the other pan so the pans balance. Now add 12 pennies to one pan (for a total of 17 in one and 5 in the other and ask the students how many pennies must be added to the other pan to get the pans to balance. Be sure the students understand that for the pans to balance you must always have the same number of pennies in each pan.
2. Using the "Balance Scale Mat" handout and the rod and unit blocks pictured below, explain that this balance scale shows that 5 unit blocks balances 1 long rod. Ask students how they can determine the value of the long rod? Students should relate that the long rod is valued at 5 in order to balance the 5 unit blocks in the model of the equation.



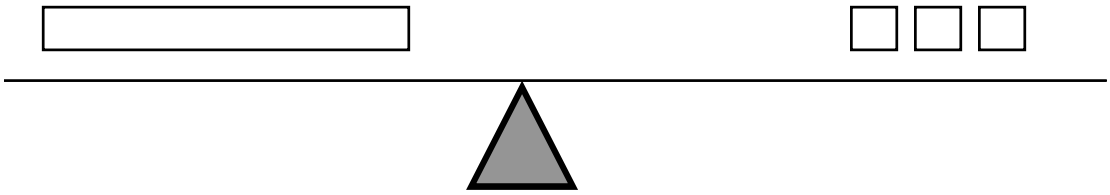
3. Present students with the following balance problem on the "Balance Scale Mat" handout.



4. Encourage students to develop a plan or strategy for finding the value of the long rod based on the unit blocks. One strategy would be to remove one unit block from each side of the balance scale. Introduce the method for recording the balance picture as an equation. For example, the first step would be written as $2x + 1 = 7$. After removing the one unit block the equation would be $2x + 1 - 1 = 7 - 1$. This becomes $2x = 6$. Now the balance scale has two groups of blocks on each side.



In order to balance the scale, each side needs to be divided into two groups. $\frac{2x}{2} = \frac{6}{2}$. The value for a rod is indicated by the value of one of the groups of unit blocks. Students should now recognize that in order for the scale balance, the long rod must be valued at three unit blocks, or $x = 3$.



5. Assign student partners to work together, using their Balance Scale Mats to solve the following equations:
   - $3x + 1 = 13$
   - $x + 4 = 2x + 3$.
6. Discuss solutions with the class.
7. Review correct algebra terminology in the context of the equations presented above. Distribute the "Algebra Vocabulary" sheet to students, and have them match the vocabulary words to the appropriate examples in the chart, using the context clues in the paragraph.

Sample assessment

- Have students describe in their math journals the process of solving equations, using the balance scale to represent the process. Give students balance-scale pictures, and have them record the steps taken together with the corresponding equations.

Sample resources

www.brainpop.com/math/algebra/equationswvar/index.weml – Interactive Web site for students using algebraic concepts.

www.mathgoodies.com/lessons/vol7/equations.html – Lessons on writing algebraic equations.

http://math.rice.edu/~lanius/Lessons/calen.html – An algebraic activity that uses the calendar.

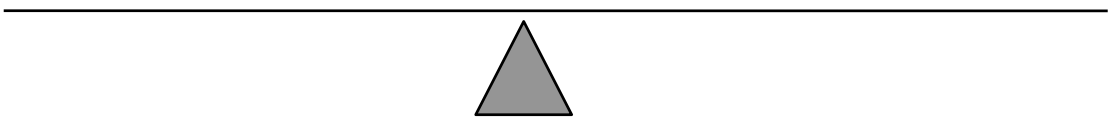
www.matti.usu.edu - A library of virtual manipulatives and lesson plans.

http://illuminations.nctm.org/lessonplans/6-8/PoolTable/index.html – Analyzing numeric and geometric patterns of Paper Pool.

Navigating through Algebra in Grades 6–8 – Available from NCTM. Contains additional lessons for algebraic activities.

Balance Scale Mat

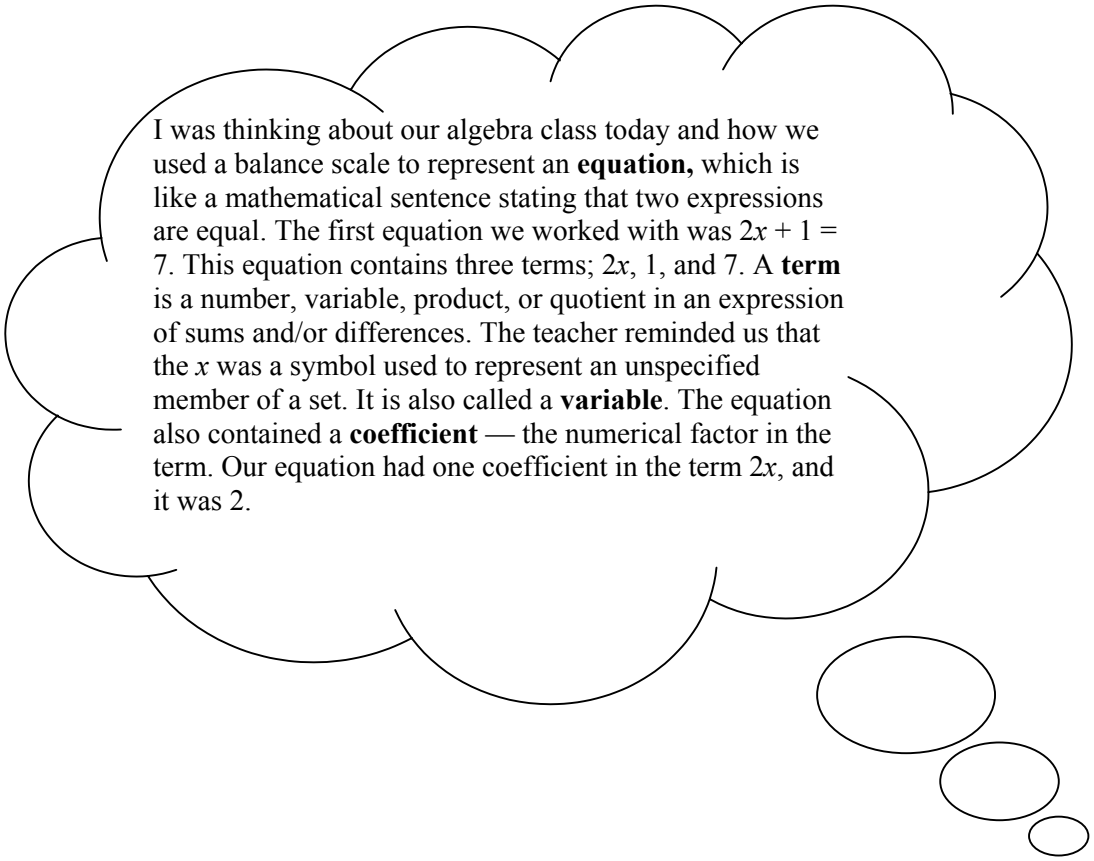
Name: _____ Date: _____



Algebra Vocabulary

Name: _____ Date: _____

Read the following paragraph. Pay special attention to the algebra terms printed in bold.

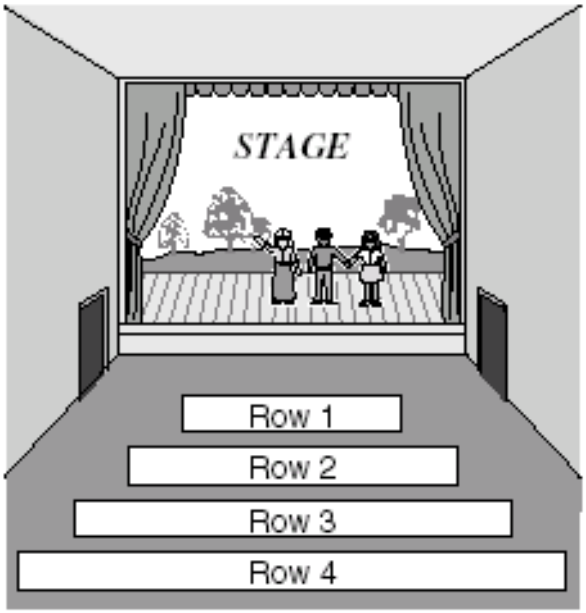


In the chart below, write a definition for each algebra vocabulary word and then write an example in the appropriate column.

Algebra Vocabulary	Definition	Example
Equation		
Term		
Variable		
Coefficient		

Sample released test items

14 **In the center section of an auditorium, each row has 2 more seats than the row in front of it.**



The front row of the section contains 23 seats. How many seats are in the 10^{th} row from the stage?

F 33
G 39
H 41
J 43

60

n	2	3	4	5	6
n^2	4	9	16	25	36

Which *best* describes the value of $(4.5)^2$?

F Between 4 and 9
G Between 9 and 16
H Between 16 and 25
J Greater than 25