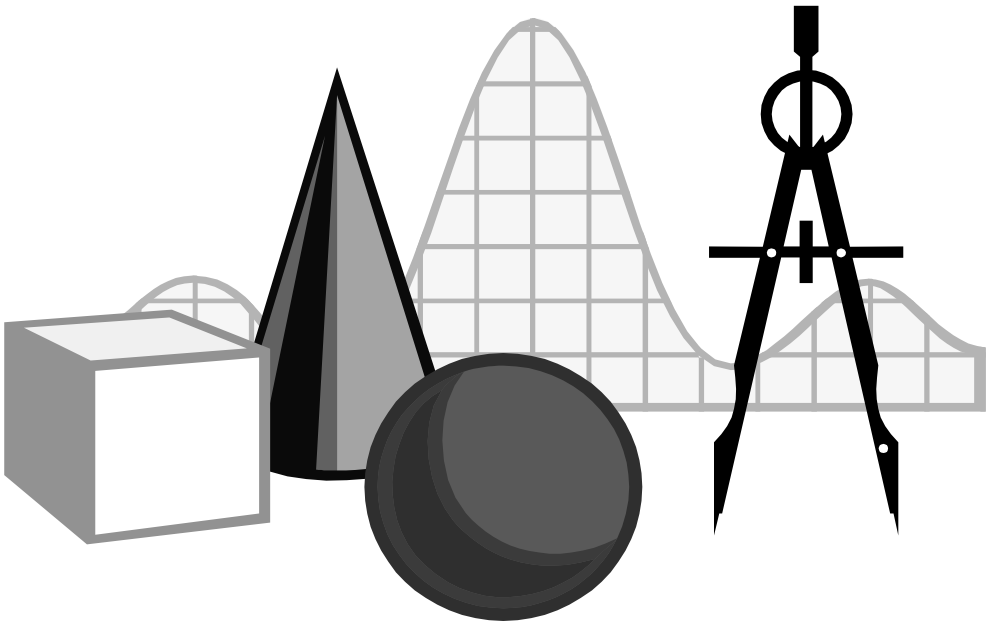
MATHEMATICS STANDARDS OF LEARNING ENHANCED SCOPE AND SEQUENCE

Grade 7



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Introduction

The *Mathematics Standards of Learning Enhanced Scope and Sequence* is a resource intended to help teachers align their classroom instruction with the Mathematics Standards of Learning that were adopted by the Board of Education in October 2001. The Mathematics Enhanced Scope and Sequence is organized by topics from the original Scope and Sequence document and includes the content of the Standards of Learning and the essential knowledge and skills from the Curriculum Framework. In addition, the Enhanced Scope and Sequence provides teachers with sample lesson plans that are aligned with the essential knowledge and skills in the Curriculum Framework.

School divisions and teachers can use the Enhanced Scope and Sequence as a resource for developing sound curricular and instructional programs. These materials are intended as examples of how the knowledge and skills might be presented to students in a sequence of lessons that has been aligned with the Standards of Learning. Teachers who use the Enhanced Scope and Sequence should correlate the essential knowledge and skills with available instructional resources as noted in the materials and determine the pacing of instruction as appropriate. This resource is not a complete curriculum and is neither required nor prescriptive, but it can be a valuable instructional tool.

The Enhanced Scope and Sequence contains the following:

- Units organized by topics from the original Mathematics Scope and Sequence
- Essential knowledge and skills from the Mathematics Standards of Learning Curriculum Framework
- Related Standards of Learning
- Sample lesson plans containing
    - Instructional activities
    - Sample assessments
    - Follow-up/extensions
    - Related resources
    - Related released SOL test items.

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Organizing Topic Rational Numbers

Standards of Learning

7.1 The student will compare, order, and determine equivalent relationships between fractions, decimals, and percents, including use of scientific notation for numbers greater than 10.

7.4 The student will
a) solve practical problems using rational numbers (whole numbers, fractions, decimals) and percents; and
b) solve consumer-application problems involving tips, discounts, sales tax, and simple interest.

7.5 The student will formulate rules for and solve practical problems involving basic operations (addition, subtraction, multiplication, and division) with integers.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Represent a number in fraction, decimal, and percent forms. Fractions will have denominators of 12 or less.	_____
• Compare, order, and determine equivalent relationships among fractions, decimals, and percents. Decimals are limited to the thousandth place, and percents are limited to the tenth place.	_____
• Write a number greater than 10 in scientific notation.	_____
• Compare, order, and determine equivalent relationships between numbers larger than 10 written in scientific notation.	_____
• Compare very large numbers, using scientific notation.	_____
• Order no more than five numbers written as fractions, decimals, percents, and numbers larger than 10 written in scientific notation in ascending (least to greatest) or descending (greatest to least) order.	_____
• Determine solutions to everyday problems involving whole numbers, decimals, fractions, and percents. Fractions are limited to having denominators no greater than 12.	_____
• Compute the tip and total bill for a meal.	_____
• Compute a discount and the resulting (sale) price for one discount.	_____
• Compute the sales tax for purchased items.	_____
• Compute the simple interest earned on an investment or a loan for a specified number of years and/or months.	_____
• Formulate rules for adding integers.	_____
• Formulate rules for subtracting integers.	_____

- Formulate rules for multiplying integers. _____
- Formulate rules for dividing integers. _____
- Solve practical problems involving addition, subtraction, multiplication, and division with integers. _____

Fractions, Decimals, and Percents

Reporting category Number and Number Sense

Overview Students show the relationship between fractions, decimals and percents using one hundred boards or graph paper.

Related Standard of Learning 7.1

Objectives

- The student will identify shaded areas using fractions, decimals and percents.
- The student will estimate shaded areas using fractions, decimals and percents.

Materials needed

- Graph paper
- Hundred board paper

Instructional activity

1. Review fractions, decimals, and percent recognition, before teaching this activity.
2. Using overhead transparency of the graph paper or hundreds board, shade a part of the board, and have students represent the shaded region as a fraction, decimal and percent. (For example, 4/5, .8, 80%.)
3. Progress using wholes and parts of a whole (Example: 1.50, 150%, 1 1/2).
4. Discuss these questions with students:
   - Why are these fractions, decimals and percents equivalent?
   - What is the pattern for writing the fraction, decimal, and percent?
5. Have students make their own equivalent relationships and share with a partner.

Sample assessment

- Give each student a shaded graph from a hundreds board. Have students write the relationship and share with the teacher and partner.
- Have students write in their math journals about the relationships between fractions, decimals, and percents.

Comparing Fractions, Decimals, and Percents

Reporting category Number and Number Sense
Overview Students compare fractions, decimals and percents.
Related Standard of Learning 7.1

Objective

- The student will compare, order and determine equivalent relationships among fractions, decimals, and percents.

Materials needed

- Graph paper
- Hundreds boards

Instructional activity

1. Review previous lesson, recognizing and determining the relationship between fractions, decimals and percent. A good Web site to reference is http://www.matti.usu.edu/ using the Percentages Activity in Grades 6–8 Number and Operations.
2. Using an overhead transparency of a hundreds board or graph paper, display three boards, and shade each board differently (Example: 22, 45, 76).
3. Have students represent the displayed numbers from the boards using a fraction, decimal, and a percent.
4. Model how to compare the amounts on the three boards, using the inequality symbols and ordering them from least to greatest or greatest to least.
5. Write each in a different form. Example: First board—display as a fraction, second board—display as a decimal, and the third board—display as a percent. Then change the arrangement of relationships.
6. Have students create their own displays to share with a partner.
7. Guide students into comparing and ordering the fractions, decimals, and percents without pictorial representation. This will require prior knowledge in converting fractions to decimals and vice versa. Then have students compare fractions to percents.

Sample assessment

- Provide different fraction, decimal, and percent equivalents and have students show the relationship using the inequality symbols.

Follow-up/extension

- Demonstrate, using an overhead calculator, how to convert fractions and decimals to percents.
- Have students find percents, fractions, and decimals in magazines and newspapers and display them in a comparison format.

Scientific Notation

Reporting category Number and Number Sense

Overview Students use a chart to determine distances from planets to the sun and compare and order these distances, using scientific notation.

Related Standard of Learning 7.1

Objectives

- The student will write numbers in scientific notation.
- The student will compare and order and determine equivalent relationships between numbers larger than 10 written in scientific notation.

Materials needed

- Internet Web site for distances between the planets and the sun http://www.smv.org/pubs/GiantSSexcel.htm
- Newspapers and magazines for each group
- A copy of the "Using Scientific Notation with The Solar System" activity sheet for each student

Instructional activity

1. Review the concept of exponents with students.
2. Have students use the chart in order to write the distances from the sun to the nine planets using scientific notation and to compare the distances using the inequality symbols. Have students order the distances of the five planets that are farthest from the sun. Have students rewrite the standard form from the scientific notation.
3. Have students (with partners) gather rational numbers (fractions, decimals, large numbers) from all types of resources, periodicals, media, and Internet. Display all numbers in a comparison form. Teacher may set up guidelines on the number of percents, fractions, decimals and scientific notations. Remind students to convert their data to meet the requirements. Also, have them show the relationship of the numbers if they have to convert.

Sample assessment

- Assess students' distance comparisons of the planets.

Follow-up/extension

- Have students create a scale model of the sun and nine planets. Have them use a different scale for planet diameters and distances.

Using Scientific Notation with the Solar System

We like to think Earth is huge, but it is actually one of the smaller planets in our solar system. Using scientific notation, complete the chart below.

Planet	Approx. diameter (miles)	Diameter, using scientific notation	Approx. distance from the sun (miles)	Distance, using scientific notation
Earth	**7,930**		**93,000,000**	
Jupiter	**88,900**		**484,000,000**	
Mars	**4,220**		**142,000,000**	
Mercury	**3,030**		**36,300,000**	
Neptune	**30,800**		**2,800,000,000**	
Pluto	**1,450**		**3,670,000,000**	
Saturn	**74,900**		**888,000,000**	
Uranus	**31,800**		**1,780,000,000**	
Venus	**7,520**		**67,200,000**	

List the planets in order from largest to smallest, using scientific notation.

Planet	Diameter, using scientific notation

Order the planets by distance from the sun, least to greatest, using scientific notation.

Planet	Distance from the sun, using scientific notation

Using the five planets that are farthest from the sun, compare the distances using inequality symbols.

Discounts, Tips, Sales Tax and Simple Interest

Reporting category Computation and Estimation
Overview Students calculate tips, discounts, sales tax, and simple interest.
Related Standard of Learning 7.4

Objectives

- The student will compute the tip for the total bill.
- The student will compute discounts.
- The student will compute simple interest.

Materials needed

- Menus from local restaurants
- Newspaper ads for sales
- Credit card bills, bank statements for loans (receive from local banks)

Instructional activity

Activity A

1. Have students order food from a menu, calculate the total bill, identify the sales tax, and calculate the tax using percent × number, then add back to net total.
2. Have students calculate the tip from the total of the bill, using percent × number.
3. Repeat this activity again, giving the students a budget to keep while ordering food and tip.

Activity B

1. Have students cut out newspaper ads for sale items. Mount the ads onto paper.
2. Emphasize the terms *sale price*, *discount*, and *original price*. For those ads showing a certain percentage off, have students calculate the discount amount (discount percent × original price = discount amount) and sale price (original price – discount amount = sale price) for each item.
3. Have the students add the sales tax to the sale price to get the total sale price.
4. Ask students to think of a different way to calculate sale price. Lead them to recognize that they can multiply the percentage of the original price to be *paid* after the discount has been deducted, e.g., for a $60 pair of shoes at a 20-percent-off sale, the sale price = 80% of $60 plus sales tax.
5. Then, give the students a budget for their spending. Have them calculate the discount and sale price of items in the newspaper ads, including the sales tax. Have them make a list of the items they could buy on sale within their budget, showing the total they would spend.

Activity C

1. Simple interest: Using a savings account amount, calculate the amount of simple interest that would be made during a set time. Emphasize substitution for the principal, rate and time. Also, have students find the total amount in savings account after interest.
2. Simple interest: Have students "buy" a car using a loan and keeping the price under $15,000. Try to use the current interest rate. Calculate the interest with 3 years and 5 years. Discuss with class the preferred time. Always show the total amount borrowed. Discuss results.

Sample assessment

- Assess student understanding of calculations as they work.

Integers

Reporting category	Computation and Estimation
Overview	Students use representations to understand how to solve for addition of integers.
Related Standard of Learning	7.5

Objective

- The student will formulate rules for adding integers using representations.

Materials needed

- Number line
- Two-sided counters or two different color squares (preferably red and yellow)

Instructional activities

Activity A

1. Review with students the concept of integers, negative and positive integers, and absolute value with opposites for zero pairs. The Virtual Manipulatives Web site, http://www.matti.usu.edu/, provides a dynamic visual representation under Grades 3–5, 6–8 Number and Operations. Try Color Chip Addition, Color Chip Subtraction, Number Line Bounce, Circle 99, Circle 0, and Circle 21.
2. Model: 2 red squares equals a –2 and 3 yellow markers equals a +3
   - Model that opposites are zero pairs. Then model this number sentence: 2 + (–2) = 0
   - Ask students the following questions. Suppose you have four red tiles. How many yellow tiles are needed to represent zero?
3. Model and write: 3 yellow tiles + 2 yellow tiles. Then, write the answer.
   - Continue to model both positive integers and then both negative integers. Write the number sentence that represents each set of tiles.
   - Have students formulate the rule for like signs.
4. Model and write: –2 tiles + 4 tiles.
   - Demonstrate to the students by making zero pairs until there are no more zero pairs.
   - Show that the remaining tiles represent the solution. Have students write –2 + (+4) = 2.
   - Continue with more examples of unlike signs. Model and write.
5. Have students formulate the rule for unlike signs.
6. Have students write the rule for adding with like signs, using a visual representation that uses the squares. Have them write the rule for adding unlike signs and give several examples, using the squares.
7. Have students pair up and create their own rule to share or to solve, using the tiles. Have students write word problems to match these situations.
8. Do a quick check to determine if students understand by displaying problems on a graphing calculator on an overhead. Have students number 1 through 10 on their papers. Enter an addition of two or three integers. Do not press ENTER. Ask students to put thumbs up for a positive sum and thumbs down for a negative sum. Ask students to explain their thumb position. Have students record the sum. Press ENTER for immediate feedback.

Activity B

1. Use the number line to model addition of integers, with students using their own number line.
2. Briefly review the absolute value of a number with the zero pairs.
3. Model/write adding like integers on the number line. Reinforce the rule described in the previous activity.
4. Model/write adding unlike integers on the number line, reinforcing the rule from the previous activity.
5. Model a problem on a number line, and have students write the numerical equation for the model. Look for the trouble spot of how the equation is written. Emphasize beginning with zero. The direction of the line would be the first number in the equation, then the second direction would be the second number in the equation followed by the answer.

Sample assessment

- Have students model addition of integers using the tiles or the number line and sharing answers.

Integers: Addition and Subtraction

Reporting category	Computation and Estimation
Overview	Students use pictorial representations to formulate the rule for subtracting like and unlike integers.
Related Standard of Learning	7.5

Objective

- The student will formulate rules for subtracting integers.

Materials needed

- Red and yellow tiles

Instructional activities

Activity A

1. Model/write subtracting with like signs: –4 – (–2)
2. Start with –4 tiles then, take away –2 tiles.
3. The remaining tiles represent the answer: –2.
4. Do several examples, modeling with like signs.
5. Model/write: 4 – 6
   - Start with 4 positive tiles.
   - There are not enough positive tiles to take away.
   - Make 2 zero pairs (emphasize that opposite integers make a zero pair and adding it to the pictorial gives the rule of "adding the opposite" meaning). Discuss how the resulting value has not changed.
   - Take away 6 positive tiles. The remaining tiles represent the answer: –2.
6. Model/write more of these examples. Emphasize to the students to make zero pairs.
7. Have students formulate a rule for these models.

Activity B

1. Continue with modeling/writing a number line. A virtual number line works better for most students (see http://www.matti.usu.edu/). Using a number line can be very difficult for students to understand. Students need to have a clear understanding of the tiles before a number line can be used.
2. Start out modeling simple subtraction: 6 – 4 and 5 – 7.
3. Progress to modeling 7 – (–2). This is where the concept of adding the opposites will have to be understood by students. Comparison subtraction can be used here. "What is the difference between 7 and –2? Seven is nine larger than –2."
4. Formulate rule for subtracting integers.

Sample assessment

- Have students model and solve subtracting integers on a number line.

Integers Multiplication and Division

Reporting category Computation and Estimation
Overview Students calculate the average gain or loss of their stock.
Related Standard of Learning 7.5

Objectives

- The student will formulate a rule to multiply integers.
- The student will formulate a rule to divide integers.

Materials needed

- Seven days of newspapers (Dow Jones Industrial stock market page)
- A pair of dice or a deck of cards for each group of game players

Instructional activities

Activity A

1. Display repeated addition to solve for answer.
2. Show how this repeated addition can be written as multiplication. Example:
   - $-3 + -3 + -3 = -9$
   - $3(-3) = -9$
3. Show many examples of such repeated addition written as multiplication.
4. Show a pattern sequence with products: $-2(3)$, $-2(2)$, $-2(1)$, $-2(0)$, $-2(-1)$, $-2(-2)$, $-2(-3)$. A graphing calculator may be used to view these products in order to test a new rule.
5. Have students formulate the rule from the pattern.
6. Have students practice solving multiplication of integers using the rules.

Activity B: The 100 Wins Game with dice

1. Provide each group of players with a pair of dice.
2. In this game, odd numbers are negative and even numbers are positive.
3. Each player takes turns rolling the dice, multiplying the two numbers displayed on the dice, and recording the product on paper.
4. Each player adds his/her answers to create a running score for each turn.
5. The first player to reach 100 is the winner.

Activity B (alternative): The 100 Wins Game with playing cards

1. Provide each group of players with a deck of cards.
2. In this game, red cards are negative and black are positive.
3. Each player takes turns drawing two cards from the deck (discard face cards), multiplying their values, and recording the product. Then he/she replaces the cards in the deck in random locations.
4. Each player adds his/her answers to create a running score for each turn.
5. The first player to reach 100 is the winner.

Activity C

1. Establish that multiplication and division are the inverse operations of one another because one undoes the other.
2. Display on board or with an overhead graphing calculator:
   - 3(4) = 12
   - 12/4 = 3
   - 12/3 = 4
   - 3(–4) = –12
   - –12/(–4) = 3
   - –12/3 = –4
3. These patterns demonstrate the above statement about inverse operations and reinforce that the rules would be the same for multiplication and division.

Sample assessment

- Have students solve multiplication and division problems with positive and negative integers.

Integers Game

Reporting category Computation and Estimation

Overview Students design a game board to incorporate all the operations of integers.

Related Standard of Learning 7.5

Objectives

- The student will design a game board to include the applications of integers.
- The student will evaluate each game, note the positive aspects, and suggest improvements.

Materials needed

- Big construction paper
- Markers
- Colored pencils
- Paper
- Glue

Instructional activity

1. Begin this activity after the lessons on addition and subtraction of integers. Prior to the activity, bring in some game boards to display in room.
2. Have students choose a setting or scene for the game board such as a cave, castle, underwater location, or other. Have them decide if they want the players to move by selecting cards, rolling number cubes (dice), or both. Ask, "Is the winner to be determined by the number of points a person has or by reaching an end square?"
3. Have students make a draft of their game board and add details to it.
4. Have students use integers to determine positive/negative points or positive/negative movement.
5. Encourage students to be creative with the adventures or misfortunes. For example, falling of a cliff = move –7 spaces, catching a starfish = move +5 spaces.
6. Have students add details to their board that would include multiplication/division of integers. Have them use illustrations on the board.
7. Have students write the rules for their game and organize all parts needed to play the game.
8. Hold a game day in class during which various student games are played by small groups or for a whole class activity.
9. Have students practice playing their games with family or friends.

Sample assessment

- Have students play the games while the teacher observes them solving problems. A rubric or point sheet may be given to students at the onset of the game to provide target expectations. Use the rubric for editing at each checkpoint.

Sample resources

Mathematics SOL Curriculum Framework.

Thinking Rationally about Fractions, Decimals, and Percent: Instructional Activities for Grades 4 through 8 – Lesson plans available from VDOE at http://www.pen.k12.va.us/VDOE/Instruction/Math/FractionsDecimalsPercent.pdf.

NCTM, *Principles and Standards for School Mathematics,* pages 214–221.

http://mathforum.org/paths/fractions/m.fraclessons.html – Middle School Lessons and Materials for Teachers – Fractions.

http://www.mathguide.com/lessons/Integers.html – Operations with Integers from Mathguide.com.

http://math.rice.edu/~lanius/fractions/ – Who Wants Pizza? A fun way to learn about fractions.

http://www.funbrain.com/linejump/index.html – Number-line activities.

http://www.col-ed.org/cur/math/math47.txt – Another lesson involving positive and negative integers.

http://www.matti.usu.edu/nlvm/nav/ – The National Library of Virtual manipulatives.

Sample released test items

Nelda needed a wrench to fix her bicycle. A $\frac{5}{8}$ inch wrench was too big, and a $\frac{1}{4}$ inch wrench was too small.

Which of the following would be bigger than the $\frac{1}{4}$ inch wrench and smaller than the $\frac{5}{8}$ inch wrench?

A $\frac{1}{8}$ inch

B $\frac{1}{2}$ inch

C $\frac{2}{3}$ inch

D $\frac{4}{5}$ inch

Which of the following is *not* true?

F $\frac{2}{5} = 0.4 = 40\%$

G $\frac{7}{10} = 0.7 = 70\%$

H $2\frac{1}{2} = 2.5 = 25\%$

J $3\frac{3}{8} = 3.375 = 337.5\%$

Which is an integer?

F $^{-}19$

G 1.5

H $\sqrt{2}$

J $\frac{1}{3}$

Organizing Topic Number Sense/Number Theory

Standards of Learning

7.2 The student will simplify expressions that contain rational numbers (whole numbers, fractions, and decimals) and positive exponents, using order of operations, mental mathematics, and appropriate tools.

7.3 The student will identify and apply the following properties of operations with real numbers:
a) the commutative and associative properties for addition and multiplication;
b) the distributive property;
c) the additive and multiplicative identity properties;
d) the additive and multiplicative inverse properties; and
e) the multiplicative property of zero.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Identify the real number equation that represents each property of operations with real numbers, when given several real number equations.	
• Explore the properties of real numbers, using diagrams and manipulatives.	
• Test the validity of properties by using examples of the properties of operations on real numbers.	
• Identify the property of operations with real numbers that is illustrated by a real number equation.	
• Simplify expressions by using the order of operations in a demonstrated step-by-step approach.	
• Find the value of numerical expressions, using order of operations, mental mathematics, and appropriate tools. Exponents are limited to positive values. Fractions are limited to having denominators of 12 or less. Decimals are limited to the thousandth place.	

Properties of Real Numbers

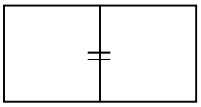
Reporting category Number and Number Sense
Overview Students explore the properties of real numbers.
Related Standard of Learning 7.3

Objectives

- The student will explore the properties of real numbers, using diagrams and manipulatives.
- The student will test the validity of operations by using examples of the properties of operations on real numbers.
- The student will identify the real number equation that represents each property of operations with real numbers, when given several real number equations.
- The student will identify the property of operations with real numbers that is illustrated by a real number equation.

Materials needed

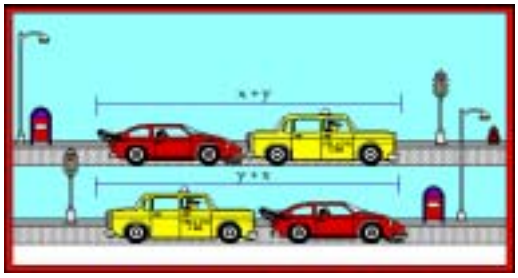
- Balance scale and weights
- Color chips (two-sided counters)
- Paper cups
- Equation mat (a sheet of paper folded in half with an = symbol on the centerfold)
- Grid paper



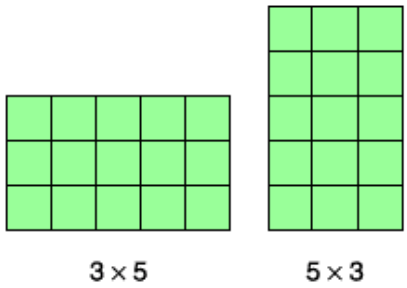
Instructional activity

Part I: Commutative and associative properties

1. Explain to students that they will be exploring the properties of operations with real numbers. Distribute supplies of color chips and paper cups to each group of students. Have students construct an equation mat on a blank sheet of paper.
2. Ask students to explain the principles of a balance scale or a seesaw. They can cite another example in which the force on the left is equal to the force on the right. Demonstrate this principle for students with the balance scale and the weights.
3. Model for students on an overhead equation mat a similar process, using the cups and the color chips to represent various quantities. Demonstrate that what is changed (added or subtracted) from one side must be changed on the other side as well for the equation to remain "balanced."
4. Demonstrate the commutative property for addition by placing five red chips and three yellow chips on one side of the equation mat. Arrange the chips on the other side by placing the three yellow chips first and then the five red chips. Ask students if the value of the chips has changed. (A real-life description would be to relate the distance students travel [commute] to school in terms of traveling from home to school in the morning, and from school to home in the afternoon. The distance is still the same. Another example uses the visual at right relating to parking two cars of different lengths on a street. No matter which order is used, the length of the parking area remains the same.)



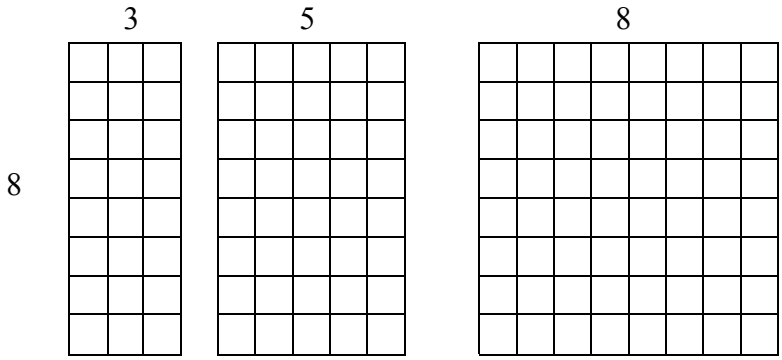
5. Using the color chips, have students create an example of the commutative property, and have their partners write down the equation represented.
6. Demonstrate an example of the associative property by placing 3 red chips, 2 yellow chips, and 1 red chip on one side of the mat. "Associate" two of the sets together, the red and the yellow. Ask students how many chips there are, then have them add the additional chip. On the right side of the equation mat, place the same groupings of chips, but this time "associate" the two yellow chips with the one red chip. Ask students for the total, and then add the 3 red chips. Explain that this is an example of the associative property for addition. Have students create an example and have their partners write down the equation represented.
7. Brainstorm with students how they could model the commutative property of multiplication. One example is to use arrays on grid paper to demonstrate this property using the areas of several rectangles.



8. Discuss with students the similarities of the two properties — commutative property of addition and commutative property of multiplication. Have students create an example of the commutative property of multiplication and have their partners write down the equation represented.

Part II: Distributive property

1. Using an overhead sheet of grid paper, create an outline of two rectangles. One should have a length of 8 units and a width of 3 units, and one should have a length of 8 units and a width of 5 units.



2. Ask students to determine the area of the first grid and second grid. Then ask students to place the two grids together (pictured in the third grid and determine the area of the total grid. Ask students to represent their solutions with diagrams. Have students record the different ways they might have come up with the solution. Some students may look at the problem as 8 (3 + 5) by adding the two widths together prior to multiplying by the length. Others may decide to multiply the dimensions of each grid separately and then add the two areas together.
3. Ask several students who have solved the problem in different ways to share their solutions, including their diagrams. Explain that those students who chose to multiply the dimensions of each individual grid and then add were actually using the distributive property. Explain that the distributive property lets you "spread out" numbers so that they are easier to work with. This property helps when doing mental math. Ask students to provide examples of other equations where the distributive property could be used.

Part III: Identity and inverse elements

1. Explain that the properties that are referred to as *identity* properties are really just common sense. These identity elements are numbers that when combined with other numbers do not change the numbers. Ask students what number this would be when they are using addition (0). Tell students that when zero is added to any number, the number remains the same. Ask students what number would be used in multiplication (1). When 1 is multiplied by any number, the number remains the same.
2. Tell students that *inverses* are numbers that combine with other numbers and result in identity elements. For the additive inverse, the two numbers would sum to zero. Review with students the concepts of negative and positive integers, reminding them of activities using the two-sided counters (red and yellow) to represent the negative and positive values. Using two color chips — one for negative integers and one for positive integers — and the equation mat, demonstrate for students the process of finding the additive inverse. Place five red chips on the right side of the equation mat. Ask students what needs to be added to the left side of the mat to have the total value of zero. Explain that the yellow chips represent negative values. Place the five yellow chips on the left side. Demonstrate how to make the zero pairs, leaving zero chips remaining on the mat. Record the process with the following equation: $5 + (-5) = 0$. Have students create an example of finding the additive inverse, using their two-color counters, and recording their results.
3. To demonstrate the multiplicative inverse, have students brainstorm what numbers would be used as factors with other numbers to have a product of 1. Have students determine with a few examples ($4 \times ? = 1$, $1/3 \times ? = 1$). Have students generate ideas as to how to find the multiplicative inverse of a number. Explain that this number is referred to as the reciprocal of the number.
4. Have students describe the similarities and differences in the multiplicative and additive inverse properties.
5. For the multiplicative property of zero, explain to students that this property is very simple; any number multiplied by zero is equal to zero. Remind students that division by zero is not a possible arithmetic operation.

Sample assessment

- Provide real equations that are examples of all the properties, and have students match the equations with the properties.
- Have students exhibit understanding of properties by writing a "Properties Poem" or creating a slide show using software to present to the class.
- Have students create a foldable flip chart of the properties using three sheets of paper. Each page contains a definition, picture of objects illustrating the property, and an example and a non-example.
- Have students play the game "Four Corners" with one property placed in each corner of the room. As an equation is revealed on the overhead, students walk to the corner that has the correct property.

Order of Operations

Reporting category Number and Number Sense
Overview Students simplify expressions, using order of operations.
Related Standard of Learning 7.2

Objectives

- The student will simplify expressions, using orders of operations.
- The student will find the value of numerical expressions, using order of operations with exponents, fractions, and decimals.

Materials needed

- Graph paper
- Dice
- Data paper
- Pre-cut triangles

Instructional activity

Activity A

1. Review with students the process of finding the area of rectangles.
2. Use an overhead transparency displaying three rectangles: 9 by 5, 9 by 8, and 9 by 4, shaded and labeled.
3. Write the dimensions under each rectangle.
4. Combine any two rectangles. Example: 9 by 4 and 9 by 5. Find the total area: $9 \times 4 + 9 \times 5$.
5. Have students solve in a variety of ways. Teacher will ask students to share answers. Then actually put the two rectangles together, and count the squares for the total area.
6. Lead activity back the original set up: $9 \times 4 + 9 \times 5$, and solve. Ask students how to solve in order to get the answer 81.
7. This will lead the students in completing the multiplication steps; then add.
8. Extend the activity by combining all three rectangles to solve for the total area.
9. Lead students into writing down the steps to find the area using order of operations.

Activity B

1. Have students use the dice to roll 6 times and record the numbers on a data sheet.
2. Once all blanks are filled, have students solve for the answer. Lead the students in following the order of operations: parenthesis (grouping symbols), add/subtract, or multiply/divide.
3. Once students have practiced this activity, incorporate all steps of the order of operations (except using exponents), using the dice and a data sheet. This will give you a quick assessment of their understanding of order of operations.

Activity C

1. Review with students the meaning of exponents.
1. Triangle Method: Give students expressions, and have them solve for each order by bringing the next step down and continuing to solve until the answer results. The triangle method consists of solving each step, with the answer as the tip of the triangle, as shown at right.

$(4 + 5)4 - 3^2 + 9(2)$
$9(4) - 3^2 + 9(2)$
$9(4) - 9 + 9(2)$
$36 - 9 + 18$
$27 + 18$
45

2. Put random rational numbers on the board, and have students make their own expressions, trying to make the longest triangle, the largest triangle number, or the smallest triangle number.

Sample assessment

- Have students use data sheets to fill in, making the longest triangle, largest triangle, or smallest triangle. Peer editing can allow for immediate feedback.
- Using only fours, operation symbols, and knowledge of order of operations, have students make each of the target numbers one through twelve. For example, $(4 + \frac{4}{4}) \bullet 4 = 20$.

Sample resources

NCTM, *Principles and Standards for School Mathematics,* pages 214–221.

Mathematics SOL Curriculum Framework.

Sample released test items

Mrs. Hansen asked Eli to apply the distributive property to the expression, 2(7 + 3). Which of the following should Eli have written?

F $2(10)$
G $2(7) + 2(3)$
H $2(3 + 7)$
J $(7 + 3) \cdot 2$

Which is an example of the associative property of multiplication?

F $7 \bullet 0 \bullet 9 = 0$

G $4 \bullet (7 \bullet {}^{-}3) = 4 \bullet ({}^{-}3 \bullet 7)$

H $\left(6 \bullet \frac{1}{6}\right) \bullet 3 = 3$

J $5 \bullet (3 \bullet {}^{-}8) = (5 \bullet 3) \bullet {}^{-}8$

If $a + b = a$, then b equals —

F $^{-}1$
G 0
H 1
J ^{-}a

What is the value of $64 - 4 \cdot 2^3 + 7$?

A 4
B 25
C 39
D 247

$72 - (7 + 8) \bullet 4$ is equivalent to —

A 292
B 260
C 87
D 12

The number of diagonals that can be drawn in a polygon with n sides can be determined by $\frac{n(n-3)}{2}$. How many diagonals can be drawn in a polygon with 10 sides?

F 130
G 70
H 65
J 35

Organizing Topic Proportional Reasoning

Standards of Learning

7.6 The student will use proportions to solve practical problems, which may include scale drawings, that contain rational numbers (whole numbers, fractions, and decimals), and percents.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Write proportions that represent equivalent relationships between two sets.	_____
• Solve a proportion to find a missing term.	_____
• Apply proportions to solve problems that involve percents.	_____
• Apply proportions to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and/or decimals no less than tenths.	_____

Ratio and Proportions

Reporting category Computation and Estimation
Overview Students solve proportions using practical situations.
Related Standard of Learning 7.6

Objectives

- The student will write proportions that represent equivalent relationships between two sets.
- The student will solve proportion to find the missing term.

Materials needed

- Books, magazines
- Rulers/meter sticks
- Construction paper
- Scissors
- U.S. maps
- String
- Graph paper

Instructional activity

Activity A

1. Have students read aloud to a partner for two minutes.
2. After the two minutes, have them count the number of words read.
3. Have students write it as a ratio of number of word to time.
4. Let the partners go through the same process in steps 1–3.
5. With the ratio of words to two minutes, find out how many words can be read in 60 minutes and how many minutes will it take to read 200 words.

Activity B

1. Have partners make shapes similar to each other's.
2. Have each student measure the length of the sides.
3. Set up a proportion to see if the shapes are truly similar.
4. Have students do this again, leaving out a measurement this time. Have students set up proportions to solve for missing length.

Activity C

1. Have students pair up and each measure the height of his/her partner.
2. Have students go outside and measure the length each other's shadow. Have them write the ratio of height to shadow length.
3. Have students find an object such as a tree, bush, or pole and measure the length of its shadow. Have them set up the proportion to find the height of the object.

Activity D

1. Using maps, have students find the scale for the map.
2. Have students plan a trip from a city in Virginia to another city in Virginia or another state.
3. Have students measure the distance from starting point to destination. Using string to place over the highways/roads will help in getting an accurate measurement. Then, have students measure the length of the string to set up the proportion.
4. To extend this activity: Have students calculate the length of time the trip will take. For example, traveling at 60 mph, how long will it take the student to travel to a particular city? Have them use conversions to calculate the hours, days, and weeks. Have students estimate the cost of money for food, gas, and lodging to get to the destination. Students may display this data using a spreadsheet. Use the spreadsheet to predict traveling times for different speeds and to track costs for different destinations.

Activity E

1. Scale drawings: This activity depends on your school/classroom setup. Have students measure the actual length and width of the hallway (if in pods or groupings) where their classroom is located, and record their data.
2. Give the students the scale of 1 in. to 5 ft.
3. Have the students draw a scale model of the hallway setup on graph paper. Have them show measurements and proportional setup used to draw the model. Drawing a scale model of the basketball court is another option.

Sample assessment

- Use accuracy of the scale drawings based on the given scale to assess student understanding.
- Ask students to find examples of proportions in their elective classes. Students might predict how many cups of water are needed to boil given amounts of rice, given the ratio of 1 cup of rice to 1.5 cups of water. Have them set up a table or spreadsheet to display.

Proportions and Percents

Reporting category Computation and Estimation

Overview Students use newspaper ads to find the amount of discount a customer will receive and the resulting sale price.

Related Standard of Learning 7.6

Objectives

- The student will determine the sale price given the original price and the percent discount.
- The student will determine the original price given the sale price and the percent discount.
- The student will determine the percent of discount given the original and sale price.

Materials needed

- Newspaper advertisements
- Calculators (optional)
- Scissors
- Glue
- Paper

Instructional activity

1. Have students find the discount of an item by multiplying the original cost by a rate (discount percent).
2. Have students find the sale price by subtracting the discount from the original price.
3. Have students find the discount percent, given the original and sale prices.

$$\frac{\text{Change}}{\text{Original}} = \frac{r}{100}$$

4. Have students work in pairs and look through the ads for 10 different items they would like to purchase. Be sure that either the sale price or original price is located in the ad. Have the students cut out each ad and paste it on the recording sheet.
5. Have the students fill out the recording sheet with the information collected so far.
6. Give the students the 10 different percent-discount coupons, let them determine which item they would use each coupon on, and have them put the coupon on that item on the recording sheet. Have students find the missing prices (either original or sale). One task might be to figure out how to save the most money, given 10 items and 10 different discount coupons.
7. Have the students determine the original or sale price from the information that is given.
8. Have the students find an item that gives an original price and the sale price but does not describe the discount taken. Have the students use the same proportion to determine the percent change. Have the students write a sentence describing what they did to find the percent change.
9. Have students turn in all work so you can determine their level of understanding.

Sample assessment

- Use the activity for an assessment.

- For homework, have students repeat this activity on their own with different items and with different percent discounts. For example, have them "buy" five holiday gifts for a given family, given a budgeted amount, five different discount coupons, and the stipulation to add 5% sales tax. Have them display this data in a table or spreadsheet.
- Ask students the following summary questions: How much money did you save using the discounts? Describe which strategy saved the most money. Have students explain how they calculated the sales tax for all five gifts.

Proportions and Percents Recording Page

Picture of Item	Original Price	New Price	Change in Price	Percent Discount	Work

Sample resources

Mathematics SOL Curriculum Framework.

Thinking Rationally about Fractions, Decimals, and Percent: Instructional Activities for Grades 4 through 8 – Lesson plans available from VDOE at http://www.pen.k12.va.us/VDOE/Instruction/Math/FractionsDecimalsPercent.pdf.

NCTM, Principles and Standards for School Mathematics, pages 214–221.

http://www.aaamath.com/g62a-ratios.html – A Web site that explores ratios.

http://illuminations.nctm.org/lessonplans/6-8/billion/index.html – Students use a simulation to predict population growth.

http://www.figurethis.org/challenges/c33/challenge.htm – A proportional reasoning challenge.

Sample released test items

Chris used a copy machine to enlarge a drawing to 150% of its original size. If the width of the original drawing was 37 centimeters, what is the width of the copy of the drawing?

F 37.0 cm

G 55.5 cm

H 92.5 cm

J 150.0 cm

An interior designer made a scale model of a room. Each 1 inch in the model represented 12 feet in the actual room. If the length of a hall was $2\frac{1}{4}$ inches in the model, what was the actual length of the hall?

A $8\frac{1}{4}$ ft

B $13\frac{1}{2}$ ft

C $16\frac{1}{2}$ ft

D 27 ft

Organizing Topic Measurement

Standards of Learning

7.7 The student, given appropriate dimensions, will
a) estimate and find the area of polygons by subdividing them into rectangles and right triangles; and
b) apply perimeter and area formulas in practical situations.

7.8 The student will investigate and solve problems involving the volume and surface area of rectangular prisms and cylinders, using concrete materials and practical situations to develop formulas.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Subdivide a polygon into rectangles and right triangles, estimate the area of the rectangles and/or right triangles to estimate the area of the polygon, and find the area of the rectangles and/or right triangles to determine the area of the polygon.	
• Apply perimeter formulas to solve real-life problems.	
• Apply area formulas to solve real-life problems.	
• Develop a procedure and formula for finding the surface area of a rectangular prism.	
• Solve practical problems that require finding the surface area of a rectangular prism.	
• Develop a procedure and formula for finding the surface area of a cylinder.	
• Solve practical problems that require finding the surface area of a cylinder	
• Develop a procedure and formula for finding the volume of a rectangular prism.	
• Solve practical problems that require finding the volume of a rectangular prism.	
• Develop a procedure and formula for finding the volume of a cylinder.	
• Solve practical problems that require finding the volume of a cylinder.	

Area/Perimeter of Rectangles and Triangles

Reporting category Measurement

Overview Students measure a room in their home; draw an outline of the floor plan, and label length of sides. Record the dimensions, perimeter, and area of room.

Related Standard of Learning 7.7

Objectives

- The student will estimate the area of rectangles and triangles.
- The student will apply perimeter and area formulas in practical situations.

Materials needed

- Magazines
- Rulers, meter sticks

Instructional activities (Use activities in Unit 20.)

Activity A

1. Have students estimate the perimeters of magazines and posters in the classroom. Then have students measure these items to find the actual measurement. Discuss reasonableness of estimates.
2. Have students make rectangles and fold them diagonally. Have students estimate the area of the whole rectangle, then estimate the area of the triangle, focusing on the relationship of one-half of the whole. Have students actually measure the rectangle and find the area then divide the area in half to find the triangle.

Activity B

1. Have the students find the perimeter and area of selected places within the school building.

Sample assessment

- Have students present a shape to a partner, describe how to find the perimeter of the rectangle or triangle, and find the perimeter.
- Have students measure a room in their home and find the perimeter and area of the room.

Perimeter and Area

Reporting category Measurement

Overview Students measure perimeter and area of various figures and determine the formula for area. Students design a two-dimensional city and determine perimeter and area of buildings. Students represent the city on grid paper.

Related Standard of Learning 7.7

Objectives

- The student will apply the perimeter formula to solve problems.
- The student will apply area formulas to solve problems.

Materials needed

- Grid paper
- Geoboards
- Construction paper
- Rulers
- Pre-made centimeter squares

Instructional activities

Activity A

1. Using 1-centimeter squares, have students arrange squares into 2 by 3, 4 by 5, etc., to find the perimeter of the squares and rectangles. Have students draw all models onto graph paper and label. Discuss with students how to find the perimeter of other polygons. Remind students of the characteristics of quadrilaterals. Lead students from repeated addition to multiplication. This sets the stage for formulas.
2. Give students a number that represents a perimeter, and have students make a polygon that represents that perimeter.
3. Have students work in groups to find the perimeter of polygon surfaces in the room and write a formula. Have students present their work to the class.

Activity B

1. Have students use the idea from Activity A to apply the area formulas for square and rectangles, using graph paper to draw what they made. Incorporate the parallelogram and rhombus; emphasizing characteristics to find the area. Introduce the terms *base* and *height*.
2. Using geoboards, have students make a rectangle or square and make a diagonal. The object is to have students discover a triangle and discuss how to find the area of a triangle. Relate the terms *base* and *height*. Also, have students make parallelograms and use a diagonal to make triangles. This is to compare/contrast between the right triangle and others. Students should always draw on graph paper or geoboard paper what they made on geoboards.
3. Give students a number that represents area. Students make a quadrilateral or triangle to represent the given area.

Activity C

1. Area of City/Town: Have students design a city on graph paper, using perimeter and area measurements to measure the two-dimensional form of the shapes. All shapes should be labeled, the formulas for deriving certain measurements presented, and the answers to the formulas stated on each shape.

Sample assessment

- Have students create their own polygon shapes without graph paper (use rulers). Have them write formulas for determining perimeter and area and solutions.
- Have students create their own shapes, combining rectangles and right triangles, state formulas, and calculate combined areas.

Surface Area: Rectangular Prisms and Cylinders

Reporting category Measurement

Overview Students wrap a rectangular prism. Students design a label for a cylindrical product.

Related Standard of Learning 7.8

Objectives

- The student will develop procedures and solve for surface area of rectangular prisms.
- The student will develop procedures and solve for surface area of a cylinder.

Materials needed

- Rulers
- Tissue boxes
- Wrapping paper or wallpaper (Most stores will give you samples of wallpaper free.)
- Tape
- Scissors
- Colored pencils, markers or crayons

Instructional activity

Activity A (requires prior knowledge of recognition of prisms and cylinders)

1. Give each student a tissue box, and have students identify the shape of the faces, spatial relationship (front, back, top, bottom, side, side), and the number of faces. Have students label the top, bottom, back, front, side, and side. Have them cut the box to make the box flat (a net). Again, identify shape of faces and the spatial relationships of top, bottom, etc. Have students measure and label the dimensions of each shape. Have them find the area of each face and indicate this on the face. Reinforce writing the formulas and showing answers. Then, have students recreate the 3-dimensional shape of their box and apply tape to hold it together. Discuss with students how they will find the total area, incorporating the term, *surface area* because it is 3-dimensional shape at this time. They will conclude by adding together all the face areas. Development of formula: Ask students to write a procedure for calculating surface area in words. Then, have them translate this word formula to symbols. How many different ways did they devise? Discuss which method is most efficient.
2. Have students unfold the shape, look closely at each face, and write out the formulas of each face on separate paper. Lead students into comparing the different faces — seeing the relationship between front and back: they have the same measurements. Why? (Opposite faces are equal.) Then, derive a shorter formula into $2(lw)$. (This is in two-dimensional form.) When students see the relationship and derive all the parts of the formula, have students fold the shape and tape it together again. Ask, "What happens to some of the lengths/widths?" (They become the height.) Then, have students see the formula of $2(lw) + 2(lh) + 2(hw)$ = surface area.

Activity B

1. Reinforce previous activity, and have students construct a rectangular prism. Have them measure and write the surface area on the prism.

2. Have students construct a net to cover the prism with wrapping paper or wallpaper. They will have to measure the paper to have the correct area needed to cover their prism.
3. Have students tape the paper onto the prism to check the fit. Ask students if they overestimated or if their fit was reasonable? Students who are still developing the formula can wrap a gelatin box with 1cm grid paper and calculate surface area using their student-developed formula.

Activity C

1. Have each student bring in a can with a wrap-around label on it.
2. Have students cut the label off the can and identify the shapes that make up the can (a cylinder).
3. Have the students draw the net form of the cylinder on graph paper. Have them estimate the surface area, using the grids.
4. Have students state the formula of the area of a circle. Ask how many circles are in a cylinder. State the formula of $2\pi r^2$. Then, ask students about the rectangle shape, "What is the area of this rectangle?" (bh). Lead students to understand that the actual shape the rectangle makes is a cylinder. Have students make the cylindrical shape with the label. The base is round, so the shape made is the circumference of the cylinder: $b = 2\pi r$. Restate the formula of $2\pi r^2 + 2\pi rh$.
5. Have students find the surface area of their cans, using the formula. Have them compare it to their estimate.

Activity D

1. Have students bring in a cylindrical object. (Have such objects in class in case students do not bring one.)
2. Have students follow the formula stated before. Have them draw the object in net form, labeling all measurements.
3. Have students find the surface area of their cylinders.

Activity E

1. Have students design their own label for a cylindrical product. They must state somewhere on the label the surface area of the label and the surface area of the uncovered part. Then have them indicate the total surface area of the can.
2. Remind students to be creative, colorful and neat.

Activity F

1. Display 5 to 7 cans and boxes, (grocery items). Have the students direct you in physically ordering the containers, least to greatest, by surface area. Discuss with the class how the dimensions helped them estimate.
2. Have the students calculate actual surface areas of the objects and reorder them, if necessary. Discuss the reasonableness of their estimates and any surprises.

Sample assessment

- Assess students through observations of developing the formula, accuracy of calculations, and their projects.

Surface Area and Volume

Reporting category Measurement

Overview Students create/use solid objects and find the surface area and volume of the objects.

Related Standard of Learning 7.8

Objectives

- The student will develop and solve for volume of rectangular prisms.
- The student will develop and solve for volume of cylinders.

Materials needed

- Centimeter cubes
- Handout
- Tissue boxes or boxes
- Different size cylinders
- Magazines
- Poster board, construction paper, scissors, markers

Instructional activity

Activity A

1. Using centimeter cubes, have pairs of students make a rectangular prism, using 24 cubes.
2. Have students fill in the blanks on the handout:

Base dimensions		Base Area (square units)	Height	No. of cubes used
____ × ____	=	_____	_____	_____

3. Students will have various base and height dimensions, but the number of cubes used will still be 24.
4. Have students show each other their alien creations.
5. Tell students they can use up to 36 cubes to make a rectangular prism, and ask them to record their data on the handout.
6. Discuss with students how to derive a formula for the volume of a rectangular prism.
7. The discussion should lead to *lwh*. Emphasize area of the base × height so students can generalize the formula to apply to prisms having different-size bases.

Activity B

1. Using the tissue boxes from the "Surface Area" activity, have students estimate, measure, and use the formula to find the volume of their tissue box.
2. Reinforce that the volume is the number of centimeter cubes that will fit into the prism.
3. Give students a volume, and have them create the rectangular prism. Use graph or plain paper to show model and dimensions. How many different prisms can be made with a given volume?

Activity C

1. Using the cans from the "Surface Area" activity, have students look at the base of the cans and conclude that the cubes must fill in the area of the can's lid, therefore the base area of the circle is πr^2. Maintaining the idea of stacking the bases up for the height, the formula can be derived as $V = \pi r^2 h$. Relate this concept to area of base × height from their work with rectangular prisms.
2. Have a model of a wedding cake, round and stacked three layers tall. Divide the class into three groups. Each group will be responsible for finding the volume for each layer. Conclude by ask students to find the total volume of wedding cake.

Activity D

1. Have students use magazines to pick out pictures of cylindrical objects and estimate the volume of each object. This will require them to estimate the height and radius of each actual object pictured.
2. Require students to find five actual objects to complete the activity described in #1. Have students draw each object and label measurements, showing the formula and answer.

Activity E: Alien Task

1. Have students design a space creature using only rectangular prisms and cylinders. Have students use poster board, construction paper, and other realistic objects.
2. Have students design a two-dimensional view on graph paper.
3. Have students create the 3-D model and show accurate measurements. Have students present calculations for surface area and volume.

Sample assessment

- Assess students as they investigate the formula.

Surface Area and Volume

Reporting category Measurement

Overview Students solve for surface area and volume; replacing variables with numbers.

Related Standard of Learning 7.8

Objectives

- The student will use the SOL formula sheet to solve for surface area of rectangular prisms and cylinders.
- The student will use the SOL formula sheets to solve for the volume of rectangular prisms and cylinders.

Materials needed

- SOL formula sheets

Instructional activity

1. Review prisms and cylinders with students.
2. Give students a handout with drawings of various rectangular prism and cylinders (measurements given).
3. Have students label each measurement with the corresponding variable (within formula) using correct units.
4. Have students substitute the variable and solve using orders of operations when necessary.
5. Give students a handout that states the volume, and have them solve for the base or height.
6. Have students draw shapes described in word problems and label measurements to solve.

Sample assessment

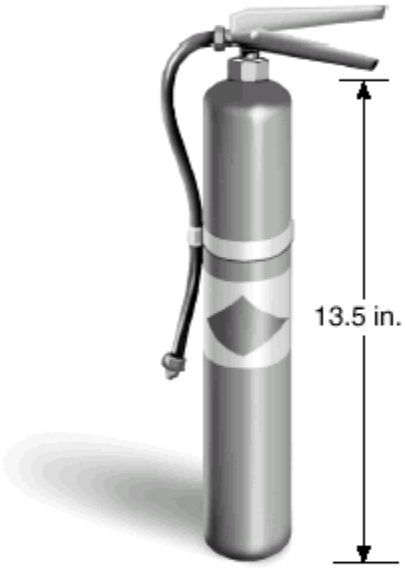
- Have students demonstrate understanding by showing their work on the board.

Follow-up/extension

- Assign homework for students to solve problems finding surface area or volume. Incorporate word problems.

Sample released test items

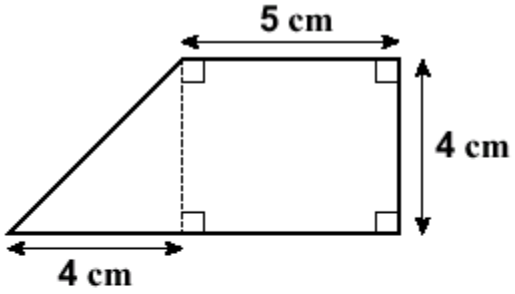
The cylindrical cannister of this fire extinguisher has a radius of 2.5 inches and is 13.5 inches high.



Which is *closest* to the number of cubic inches it will hold when filled?

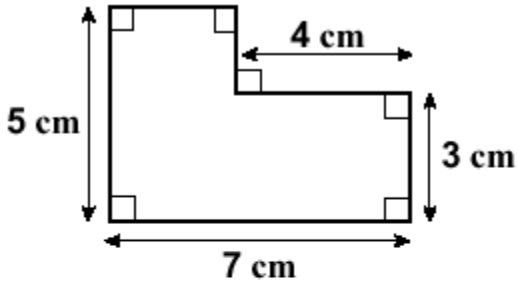
A 1,060
B 265
C 212
D 115

Sample released test items



What is the area of this figure?

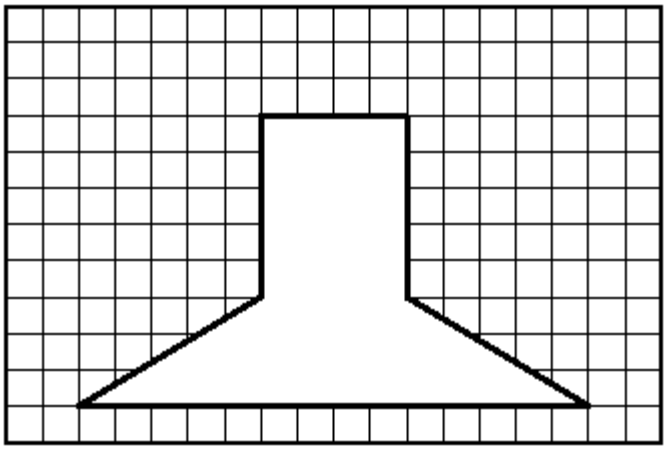
A 18 cm^2
B 26 cm^2
C 28 cm^2
D 36 cm^2



What is the area of this figure?

A 19 cm^2
B 24 cm^2
C 27 cm^2
D 35 cm^2

In this scale drawing, each square unit represents 1 square centimeter.



What is the area of the figure represented by the drawing?

F 15 cm^2
G 20 cm^2
H 32 cm^2
J 47 cm^2

If Gina measured the length of all 4 sides of the top of her desk and added them together, what would she have?

A The diameter
B The volume
C The perimeter
D The area

Su Li wants to place a protective covering over a rectangular flower bed that measures 3.2 meters by 4.3 meters. How many square meters of covering will she need?

F 7.5
G 13.76
H 15.0
J 27.52

Organizing Topic Geometry

Standards of Learning

7.9 The student will compare and contrast the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. Deductive reasoning and inference will be used to classify quadrilaterals.

7.10 The student will identify and draw the following polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon.

7.11 The student will determine if geometric figures — quadrilaterals or triangles — are similar and write proportions to express the relationships between corresponding parts of similar figures.

7.12 The student will identify and graph ordered pairs in the four quadrants of a coordinate plane.

7.13 The student, given a polygon in the coordinate plane, will represent transformations — rotation and translation — by graphing the coordinates of the vertices of the transformed polygon and sketching the resulting figure.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Identify the classification(s) to which a quadrilateral belongs.	_____
• Compare and contrast attributes of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid.	_____
• Classify quadrilaterals, using deductive reasoning and inference.	_____
• Identify by the number of sides or number of angles the following polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon.	_____
• Draw a pentagon, hexagon, heptagon, octagon, nonagon, and decagon, using a variety of tools.	_____
• Identify corresponding sides and corresponding angles of similar figures.	_____
• Write proportions to express the relationships between the lengths of corresponding sides of similar figures.	_____
• Examine congruence of corresponding angles and proportionality of corresponding sides to determine if quadrilaterals or triangles are similar.	_____
• Identify and label the axes of a coordinate plane.	_____
• Identify and label the quadrants of a coordinate plane.	_____
• Identify the quadrant in which an ordered pair is positioned by examining the ordered pair.	_____

- Graph ordered pairs in the four quadrants of a coordinate plane. _____
- Identify ordered pairs represented by points in the four quadrants of the coordinate plane. _____
- Identify the coordinates of the image of a polygon that has been translated either vertically or horizontally. _____
- Identify the coordinates of the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin. _____
- Sketch the image of a polygon translated vertically or horizontally. _____
- Sketch the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin. _____

Quadrilaterals

Reporting category Geometry

Overview Students design a Venn diagram to show the relationship between the quadrilaterals.

Related Standard of Learning 7.9

Objectives

- The student will identify quadrilaterals.
- The student will compare and contrast attributes of the quadrilaterals.

Materials needed

- Construction paper
- Scissors
- Markers

Instructional activity

Activity A

1. Have students describe the difference between a parallelogram and a rectangle by describing their opposite sides, their angles, and the parallel nature of their sides. Have students draw and cut out an example of each shape, writing its characteristics on it. Make sure the wording is correct.
2. Have the students compare the rhombus and square, following the same procedures.
3. Then have them compare the trapezoids (right and isosceles), using the same procedures.
4. Point out to the students that they have described each shape according to its sides and angles. This gives them their own visuals for understanding quadrilaterals.

Activity B

1. Have students compare and contrast quadrilaterals. Using the models from the previous activity, have students make duplicate copies of the quadrilaterals. Have students place a quadrilateral on top of the original set that matches the description. Emphasize the vocabulary. Emphasize that the square is a special type of rhombus. Emphasize the term *right angle* for square. This leads the students into making the Venn diagram.
2. Make the Venn diagram using a similar layout from the previous activity.
3. Make a mobile showing the quadrilateral relationships.

Sample assessment

- Observe student work as they work with quadrilaterals.

Follow-up/extension

- Have students make a polygon picture book, and have them label the characteristics of all polygons, quadrilaterals, and triangles.
- This picture book can be a family album or a design of a city. Encourage students to be creative.

- Have students write “Who Am I?” riddles based on attributes of quadrilaterals. Students can create quadrilaterals with manipulatives.

Identifying Polygons

Reporting category Geometry
Overview Students design a polygon city.
Related Standard of Learning 7.10

Objectives

- The student will identify and draw polygons: pentagon, hexagon, heptagon, octagon, nonagon, and decagon.
- The student will use geoboards and graph paper to draw the polygons.

Materials needed

- Geoboards
- Graph paper
- A variety of magazines
- Clothes hanger and/or construction paper

Instructional activity

Activity A

1. Have students model the polygons on geoboards and graph paper. Define the prefixes of pentagon, hexagon, heptagon, octagon, nonagon, and decagon. Also, use the terms *regular* and *irregular* polygons.
2. Have students work in pairs. Have them find pictures in the magazines of the various shapes to use in designing their polygon cities. Specifications can be set up to incorporate a school, city government building, grocery store, and houses.

Sample assessment

- Assess students based on final polygon city project.

Follow-up/extension

- Have students find examples of polygons other than from magazine pictures.

Similar Figures

Reporting category Geometry

Overview Students discover the relationship between similar figures by observing characteristics of a set of rectangles.

Related Standard of Learning 7.11

Objectives

- The student will sort a set of rectangles into three groups and determine the relationship between the members of a set.
- The student will discover the relationship between figures that are similar.

Materials needed

- For each pair of students, 14 rectangles of the following sizes cut from grid paper:
  A. 4-by-8
  B. 2-by-8
  C. 3-by-12
  D. 1-by-4
  E. 1-by-2
  F. 1-by-1
  G. 3-by-3
  H. 2-by-4
  I. 5-by-10
  J. 5-by-5
  K. 4-by-16
  L. 7-by-7
  M. 3-by-6
  N. 10-by-10

  Note: You may want to draw the diagonals on the rectangles for the students before they cut them out.
- Recording sheet
- Graph paper, first quadrant

Instructional activity

1. Have students cut out each of the 14 rectangles and divide them into three groups, where the members in each group are the same shape but different in size.
2. Students should find that there are 5 shapes in the first group that all are squares — the 1-by-1, 3-by-3, 5-by-5, 7-by-7, and 10-by-10 rectangles. Then they should form a second group that would consist of the 4-by-8, 1-by-2, 2-by-4, 5-by-10 and 3-by-6 rectangles. The third group will have the remaining 2-by-8, 3-by-12, 1-by-4, and 4-by-16 rectangles.
3. Once students have divided them into the three sets, have them arrange each group from smallest to largest and look at the relationships between members of the same group. You can have them stack them on top of each other and tell them to put the largest rectangle on the bottom and each larger one on top, sharing the same corner. This will allow them to see that the diagonals line up.
4. Once students have them arranged, discuss the following questions with the class:

- What patterns do you see within each family? Have them focus on length and width.
- What do you notice about the diagonals in each group of rectangles?
- How can you determine another member of the group?

5. Have students complete the chart with the information from the rectangles in each group.
6. Discuss the following questions once the charts are completed:
   - In a group, what patterns do you notice going down the chart?
   - In a group, what patterns do you notice going across, from width to length?
7. Have students place a fraction bar between the width and length. Ask them what they notice about each of the fractions in each set.
8. Based on this discussion, have students define what constitutes similar figures.
9. To extend this activity to look at the graphical side of the data, have students graph the length and width of each rectangle. It works best if they do each set in its own color. They should see that the coordinates associated with a given set form a line.
10. Have students discuss the following questions from the graphs:
    - How can you use the graph to find another member of a group?
    - Given a rectangle, how can you tell if it is similar to the ones already graphed?
11. Have the students explore the slope of each line. If you have not talked about slope, have them count up and over between points on the line. What is the relationship between this number and what the students discovered on the chart?

Sample assessment

- As students follow the instructions, observe their reactions. Observe how students group the rectangles. Pay close attention to who is answering the questions you ask and what they say. You might want to have each pair write down answers on small whiteboards before they share as a class so that everyone is involved in the discussion and accountable. Further assessment can be done when the class formalizes the definition for similar figures. Have all students write their definitions after the discussion and hand them in for your review.
- For homework or more practice, give students another set of rectangles to work with, but this time do not have them copied on grid paper. Have the students measure the sides and determine their length and width and which ones are similar.

Follow-up/extension

- Steps 9, 10 and 11 could be done as a follow up or as an extension.

Similar Figures Recording Sheet

Group One

Rectangle	Width	Length
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

Group Two

Rectangle	Width	Length
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

Group Three

Rectangle	Width	Length
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

Similar Figures and Proportion

Overview Students measure the height of objects and the length of their corresponding shadow to determine the height of an object that cannot be measured.

Related Standard of Learning 7.11

Objectives

- The student will measure the heights and corresponding shadows of several predetermined objects outside.
- The student will find the height of an object from the length of its shadow using proportions.

Materials needed

- Paper for recording heights and lengths of shadows
- Graph paper
- Measuring tapes

Instructional activity

1. As preparation for this activity, find 8 to 10 objects outside on school grounds that students can easily measure.
2. Have students work in groups to determine the heights of each object and the length of its corresponding shadow. Be sure that you determine units for measuring before beginning. You could have each group measure one object and then have all groups share the data back in the classroom. Before students begin measuring, set a standard for how each object should be measured to ensure uniformity.
3. Have one object, such as a flagpole, basketball hoop, or tree that can only be measured its shadow.
4. Have the students complete the chart in the classroom.
5. Discuss the following questions:
   - What patterns do you see in the ratios formed by each height and its shadow?
   - What would happen if we did this activity at a different time of the day? Would the same pattern appear?
   - What would happen if we did this activity at a different time of the year? Would the same pattern appear?
6. Have students graph the shadow's length on the *x*-axis and the corresponding height of the object of the *y*-axis.
7. Ask students what they notice about the points?
8. Discuss the following questions:
   - What is the relationship between the average ratio and the graph?
   - How can you use the graph to find the height of an unknown object if you know the length of its shadow?
   - Will this method work no matter when you measured the length of the shadow?
9. Complete the activity by using the average ratio and the graph to determine the height of the tall object.

Sample assessment

- Informal assessment can be done while students are answering discussion questions. For more formal assessment, give them the shadow length of several other objects and ask them to independently find the height of the objects.

Follow-up/extension

- To extend this activity, have students draw scale drawings using proportions.

Practical Proportions Recording Sheet

Object	Height	Shadow height	Ratio $\frac{\text{Height}}{\text{Shadow}}$

Calculate the average ratio: _____

Too tall object: Shadow length _____

Graphing

Reporting category Geometry
Overview Students play "Sink Your Ship"
Related Standard of Learning 7.12

Objectives

- The student will graph ordered pairs in the four quadrants.
- The student will identify the four quadrants.

Materials needed

- Geoboards
- Centimeter graph paper

Instructional activity

Activity A

1. Have students draw a coordinate plane and label *x*- and *y*-axes. Label lines and identify the quadrants. Establish how ordered pairs are plotted through the *x*- and *y*-axes.
2. Have students plot several ordered pairs. Identify the quadrant in which the point is placed. Have students find the pattern of determining — without plotting, based on the coordinates — the quadrant in which the point would be placed.

Activity B

1. Have students draw a simple picture and label ordered pairs. The ordered pairs should placed so students can connect the dots to make the picture.
2. Have students trade drawings with a partner, then connect the ordered pairs on their partner's drawing to make and color the picture.

Activity C

1. Using the geoboards, make the *x*- and *y*-axes. With partners, have students mark points on the geoboard with a sticker (small round). Taking turns, have students ask for particular ordered pairs to locate the partner's points. The first person who locates all the points is the winner. When guessing ordered pairs, have students write down their guesses.

Sample assessment

- Use classroom activities for assessment.

Follow-up/extension

- Have students design a picture made up of polygons.
- Have students use the graphing calculator to enter the *x* values into List 1, and *y* values into List 2. Have them enter the first ordered pair again at the end of the list. Have students set up an *xy* line from the stat plot menu, then press ZoomStat to view. Note that transformations of this polygon are a natural extension. Ask students how they could translate the polygon two units left and three units down.

Translation and Rotation

Reporting category Geometry
Overview Students translate polygons and identify coordinates.
Related Standard of Learning 7.13

Objectives

- The student will identify/sketch coordinates of a polygon that has been translated over the *x*- or *y*-axis.
- The student will identify/sketch coordinates of a triangle or square that has been rotated 90 or 180 degrees.

Materials needed

- Geoboards
- Graph paper
- Patty paper
- Braids
- Pre-cut circles

Instructional activity

Activity A

1. Using the geoboard, have students make a square or triangle in quadrant I and translate it to quadrant II, III, IV, each time recording the coordinates (Use different colored bands.).
2. Have students transfer what is on the geoboard to graph paper, using colored pencils to illustrate each translation.

Activity B

1. Have students make an original square on a geoboard. Give students a rule to translate the object and record the coordinates, for example, right 2 spaces, up 1 space. Have them use the rule on each coordinate.
2. Have students transfer the geoboard sketch to graph paper.
3. Have students make their own rules with partners and sketch the image. Emphasize that they need to record the coordinates.

Activity C

1. Show students the following setup. Cut a circle into equal fourths, and attach in the center with a brad the quarters occupying the four quadrants. Number the quarters/quadrants I, II, III, and IV, and then move one of the quarters to overlap another of the quarters (move one of the quarters from quadrant I to quadrant II). Discuss what the figure looks like now.
2. Have students draw the original image on paper and then draw the image of the circle with one quarter moved to the second quadrant, using an arrow to indicate the movement. Use students' prior knowledge of a circle having 360 degrees to reinforce the fact that one quarter has moved 90 degrees

3. Have students move the quarters now in quadrant II into quadrant III. Discuss that this additional 90-degrees of movement now represents a total movement of 180 degrees. Have the students draw this image near the original shape, again using an arrow. Relate the circle to the coordinate plane.

Activity D

1. Have students use a coordinate plane, place a rectangle in quadrant I, and label the vertices of the rectangle. Have them rotate the rectangle 90 degrees about the origin, draw this rotation, and label vertices.
2. From the original position, have students rotate the rectangle 180 degrees, draw this rotation, and label vertices.
3. Have them repeat this process with a right triangle.
4. Then have students trace on patty paper the rectangle with vertices and axes. Have them place a pencil point on the origin and rotate the patty paper 90 degrees. Have them repeat with an 80-degree rotation. Have students compare the coordinates of the image to the original.

Activity E

1. Using pre-determined shapes on a coordinate plane, have students identify the rotation and list the coordinates.
2. Have students make their own rotations starting at any quadrant and rotating the shape 90 degrees or 180 degrees.

Sample assessment

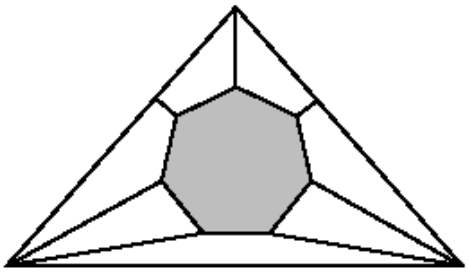
- Use activities for student assessment.

Follow-up/extension

- Have students practice translating and rotating shapes.

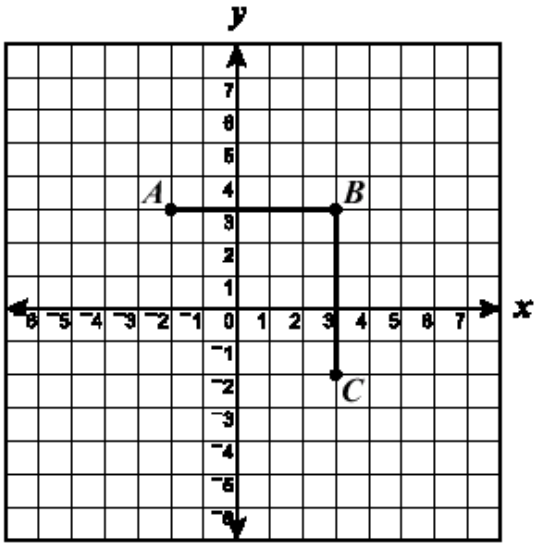
Sample released test items

Look at the design below.



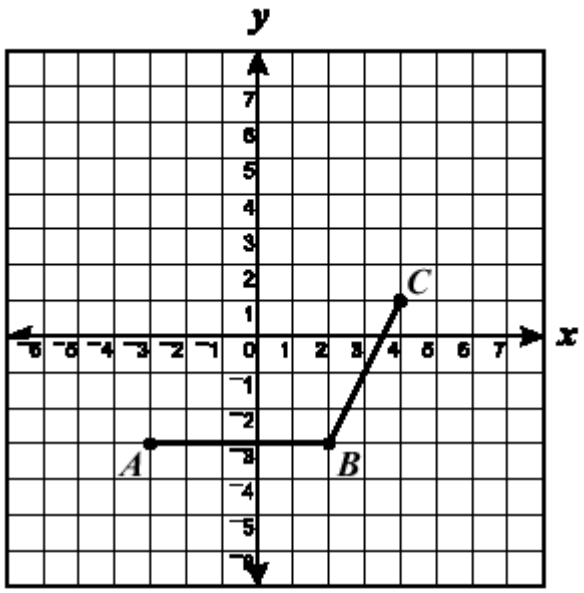
Which term identifies the shaded part in the center of the design?

A Heptagon
B Pentagon
C Nonagon
D Decagon



Points *A*, *B*, and *C* are vertices of a square. Which must be the coordinates of the fourth vertex?

F $(^{-}2, 2)$
G $(^{-}2, ^{-}2)$
H $(0, ^{-}2)$
J $(^{-}3, ^{-}2)$



Points *A*, *B*, and *C* are vertices of a parallelogram. What are the coordinates of the fourth vertex?

A (0, 0)

B (0, 1)

C ($^{-}$1, 1)

D ($^{-}$1, $^{-}$1)

Sample resources

Navigating through Geometry in Grade 6 through Grade 8 – available from NCTM. Contains additional lessons for data analysis activities.

Geometry for Middle School Teachers Professional Development Module available from VDOE.

Van Hiele Levels of Geometric Thought CD – available through the Virginia Department of Education – Contains assessments to determine children's level of geometric thinking.

http://www.learnnc.org/LearnNC/lessonp.nsf/docunid/272DFA – A lesson plan on constructing three-dimensional figures.

http://artsedge.kennedy-center.org/teaching_materials/curricula/curriculum.cfm?curriculum_id=213&mode=full – Using the context of lighthouses, this lesson plan focuses on the geometric figures required to construct structures.

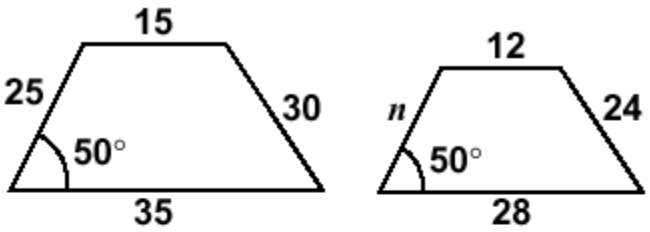
http://www.learner.org/teacherslab/math/geometry/space/ – Lessons plans that focus on visualization of three-dimensional objects.

www.mathcats.com – Creative, interactive site for students with geometry activities.

www.matti.usu.edu – A library of virtual manipulatives.

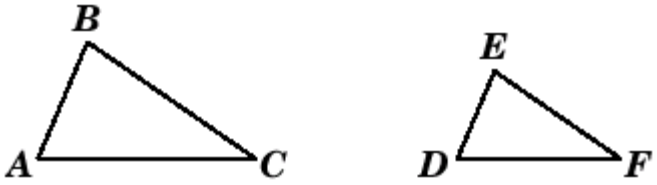
www.standards.nctm.org/document/eexamples/chap4/4.2/ – An interactive geoboard.

Sample released test items



These figures are similar. What is the length of side *n*?

F 20
G 18
H 16
J 15



If ΔABC is similar to ΔDEF, which of the following must be true?

A $\frac{AB}{AC} = \frac{DE}{EF}$

B $\frac{AB}{DF} = \frac{AC}{EF}$

C $\frac{AB}{BC} = \frac{DE}{DF}$

D $\frac{AB}{DE} = \frac{AC}{DF}$

Organizing Topic Statistics

Standards of Learning

7.16 The student will create and solve problems involving the measures of central tendency (mean, median, mode) and the range of a set of data.

7.17 The student, given a problem situation, will collect, analyze, display, and interpret data, using a variety of graphical methods, including
a) frequency distributions;
b) line plots;
c) histograms;
d) stem-and-leaf plots;
e) box-and-whisker plots; and
f) scattergrams.

7.18 The student will make inferences, conjectures, and predictions based on analysis of a set of data.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Examine the range to understand spread or dispersion of the data.	
• Describe the three measures of central tendency (mean, median, or mode) and situations in which each would best represent the data for data sets with no more than 20 data points.	
• Solve problems by finding the mean of a set of no more than 20 numbers.	
• Solve problems by finding the median of a set of data of no more than 20 numbers when the numbers are arranged from least to greatest, including data sets that have one middle number and data sets that have two middle numbers.	
• Solve problems by finding the mode of a set of data of no more than 20 numbers.	
• Identify the mode in a set of data, given that there may be one, more than one, or no mode.	
• Solve problems by finding the range of a set of data of no more than 20 numbers.	
• Solve problems with multiple identical data points for which the mode is the best descriptor. Two modes are sufficient for data sets with no more than 20 data points.	

- Solve problems in which some data points are outliers and thus, for which the median is the best descriptor. The data sets will contain no more than 20 data points. _____
- Solve problems in which the data points are similar and thus, for which the mean is the best descriptor. The data sets will contain no more than 20 data points. _____
- Collect, analyze, display, and interpret a data set of no more than 20 items, using frequency distributions. _____
- Collect, analyze, display, and interpret a data set of no more than 20 items, using line plots. _____
- Collect, analyze, display, and interpret a data set of no more than 20 items, using histograms. _____
- Collect, analyze, display, and interpret a data set of no more than 20 items, using stem-and-leaf plots where the stem is listed in ascending order and the leaves are in ascending order with or without commas between leaves. _____
- Collect, analyze, display, and interpret a data set of no more than 20 items, using box-and-whisker plots that identify the minimum, maximum, median, the upper- and lower-extremes, range, and interquartile range. _____
- Collect, analyze, display, and interpret a data set of no more than 20 items, using scattergrams. _____
- Interpret data points in scattergrams as having positive, negative, or no relationship. _____
- Organize data not exceeding 20 items into tables and/or graphs that provide a clear representation of dispersion or convergence of the data. _____
- Determine patterns and relationships within data sets (e.g., trends). _____
- Make inferences, conjectures, and predictions based on analysis of a set of data not exceeding 20 items. _____

Grab a Handful

Reporting category Probability and Statistics

Overview Students collect data about the handful of linking cubes and use the data to explore the mean, median and mode.

Related Standards of Learning 7.16, 7.18

Objectives

- The student will examine and find the range to understand spread or dispersion of data.
- The student will describe the three measures of central tendency (mean, median or mode) and situations in which each would best represent the data.
- The student will find the mean, median and mode.

Materials needed

- Linking cubes
- Sticky notes
- Recording sheet

Instructional activity

Note: This activity can be found in the *Probability and Statistics Teachers Resource Guide,* developed by the Virginia Department of Education.

1. Students grab a handful of cubes from a bucket or bag. The handful should be as large as possible.
2. Each student connects all of his cubes and writes the number of cubes in his train on a sticky note.
3. Students then take their sticky notes to the chalkboard and construct a bar graph or a line plot.
4. Students then line up at the front of the room according to the number of cubes each student has. They should line up in front of each other when there is more than one student with the same number of cubes. This human line plot should be similar to the line plot on the board with sticky notes.
5. The instructor leads the group through a discussion of the measures of central tendency using the vocabulary words. While discussing the range, students learn about outliers.
6. Before discussing the words, *mode*, *median*, and *mean*, students should be asked what they know. Assessing prior knowledge is key. They also should be encouraged to predict the median and the mean. The mode is easily illustrated on the human line plot and the line plot on the board.
7. Before the median discussion begins, ask students to spread out in one line.
8. Ask students to move out from the line in pairs — one from each end at the same time. The idea with this is that students will see how, as you approach the center, there will be one or two students remaining. They experience what the center really is. The movement away from center makes this very clear to students. Discussion should follow regarding the median and what it represents.
9. Before illustrating the mean, students should again discuss what they are really trying to find. Encourage students to share ways they could find the mean using the cubes.
10. Then have students share linking cubes with each other until all students have a similar number of cubes. They share until sharing anymore isn't helpful to reach the same number of cubes for each student. More than likely, two groups will exist. Some students will have trains of one number and

the other group will have trains of another number. At this time, students discuss the mean and also use the term *average*.

11. End the session by reviewing the measures of central tendency students have learned and stating again the actual answers for this collection of data. Students should be encouraged to discuss how changes in the data would affect the measures of central tendency. Ask students to complete the Recording Sheet.

Sample assessment

- The class can be assessed informally throughout the discussion part of the activity. The recording sheet may also be used as a form of assessment. To further check students' understanding of mean, median and mode, have the students look up statistics on the Internet. Possible examples would be weather data, sports statistics, and population data. Use this data to create problems asking students to determine which measure of central tendency would be best to use to describe the data set, and have them find it.

Grab a Handful Recording Sheet

Name: _____ Date: _____

Number of cubes I grabbed in one handful: _____

Number of students participating in this activity: _____

Total number of cubes grabbed by the students: _____

Using the data, I discovered the following:

Range of the data: _____

Identify any outliers, and tell why they are outliers.

Mode(s) of the data: _____

How did you know?

Median of the data: _____

What is the meaning of median?

Mean of the data: _____

Why is the mean also called the average?

Which measure of central tendency best represents the data and why?

Drops on a Penny

Reporting category Probability and Statistics

Overview Students collect data to determine how many drops of water fit on a penny. Students construct a stem-and-leaf plot to display the number of drops that were required and one to display their guess.

Related Standards of Learning 7.16, 7.17, 7.18

Objectives

- The student will collect and analyze data, using a penny and a water dropper.
- The student will use the data that has been collected to draw a stem-and-leaf plot.
- The student will use the stem-and-leaf plot to determine the range, the median, mode, and mean of the data.

Materials needed

- Pennies
- Water
- Cups
- Eye dropper
- Paper towels
- Sticky notes

Instructional activity

1. Begin the activity by asking the students to estimate the number of drops of water that will fit on the head of a penny.
2. Have each student write down his or her guess. Collect the estimates, and place them on the board. Discuss with the class ways to organize the data. Lead them to the idea of putting numbers with the same tens digit or hundreds digit together.
3. Show students how to display the data in a stem-and-leaf plot with the stem being either the hundreds digit or tens digit depending on the guesses given by the students.
4. Have the students work in groups to collect the data. Put the penny on a paper towel and drop one drop of water at a time while another member of the group counts the drops. Tell the students to count all the drops including the drop that makes the water overflow onto the paper towel.
5. Have each group record their data in a chart. Have the students record the ones digit on a sticky note for each group.
6. Create a stem-and-leaf plot of the estimate vs. the actual number of drops, with each group contributing a stem from the data they collected using the sticky notes.
7. Use the stem-and-leaf plot that was created to determine the mean, median, mode, and range of data. Discuss the distribution of the data on each side. What is the general shape — J, U, mound, etc. of the distribution? Compare the distribution of the estimates versus the actual data. Explain any differences. Are there any clusters or outliers? Then discuss range and central tendency.
8. Compare the mean, median, mode, and range for the two stem-and-leaf plots. Have students make conclusions based on the data.

9. Have the students discuss and determine which measure of central tendency would be the best descriptor for the data.

Sample assessment

- Students can be assessed informally throughout the activity as they gather data, work in their groups, and a stem-and-leaf plot as a class.
- After the class has constructed a stem-and-leaf plot, have the students create another plot with data you give them. Possible data sets include number of letters in students' first and last names, students' heights in inches, or students' ages in days or months.

Follow-up/extension

- This lesson could be extended by creating back-to-back stem-and-leaf plots with two set of data.

Box It Up

Reporting category Probability and Statistics

Overview Students create a human box-and-whisker plot using the number of letters in their first and last name as data points.

Related Standards of Learning 7.16, 7.17, 7.18

Objectives

- The student will create a box-and-whisker plot.
- The student will use the box-and-whisker plot to analyze a set of data.
- The student will use determine the mean, median, mode, and range for a set of data.

Materials needed

- Yarn
- Paper plates
- Signs labeled *median, lower extreme, upper extreme, lower quartile*, and *upper quartile.*
- Sticky notes

Instructional activity

2. Begin the activity by having each student determine the number of letters in his/her first and last name.
3. Have the students line up in order from least to greatest across the front of the room. Students with the same number stand one behind the other.
4. Have the students pair off and walk down the sides of the room as the highest number will wave and stand across from the lowest number and the next highest will pair off and stand across from the next lowest number until everyone is paired off or there is one remaining in the middle. You may want to make the data set be an odd number to start with and then go back and look at what happens when there is an even number of points.
5. Explain to the students that they have found the middle of the data, called the *median*, and give that person the sign labeled *median*.
6. Repeat the pairing off exercise for the lower half of the data and then for the upper half. Do not count the median when locating the quartiles. This is a common point of confusion. The students have now found the upper and lower quartiles.
7. Give these students the appropriate signs or have two people hold the sign if the number is between two data points.
8. Take the yarn and form a box around the students that are from the lower quartile to the upper quartile. Have the students hold the yarn so that it looks like they are boxed in.
9. Now give the sign that says Lower Extreme to the smallest data point and the sign that says Upper Extreme to the highest data point.
10. Have students each hold a piece of string that will extend from them to either the upper quartile or the lower quartile as appropriate. This will form the whiskers.
11. To allow everyone to see the box-and-whiskers plot, have each person replace himself or herself with a sticky note on the wall, creating a number line. Tape the string and signs to the wall where appropriate.

12. Discuss with the students the meanings of the various data points. Use the following questions:
    - What do each of the five summary points describe?
    - What does the range in this data set tell us?
    - Are there outliers? What does it mean to be an outlier? What does an outlier do to a box-and-whisker plot? What happens to the plot if we modify it or take out the outlier?
    - What does it mean to be "in the box"?
    - If a new seventh grader entered our class, where is it likely that he/she would land on this plot? What is the range of letters we would expect in the name of the new student?
13. Repeat the exercise with an even number of data points and discuss the different ways it will be necessary to find the points necessary to draw the box and whiskers.

Sample assessment

- Informally assess student understanding through a discussion of the box-and-whisker plot and the meaning of each of the points graphed in it. Have students discuss what type of information can be determined from the plot. Question them about finding the mode and mean from a box-and-whisker plot.

Follow-up/extension

- Have students create two box-and-whisker plots that can be compared side by side. An example would be to put the boys' average for a chosen item in one plot and the girls' average for a chosen item in another and then compare the two plots. Have students name the advantages of using box-and-whisker plots to compare data sets as compared to another display.
- Have students use a graphing calculator to compare two box-and-whisker plots. If there are outliers, have them compare the modified plot to the plot containing the outlier(s).

Marketing with Scattergrams

Reporting category	Probability and Statistics
Overview	Students create and conduct a survey to collect data. They use the data collected to create a scattergram that will be used to make inferences about the data and to answer a marketing question.
Related Standards of Learning	7.16, 7.17, 7.18

Objectives

- The student will develop a survey to answer questions about a marketing decision that needs to be made.
- The student will conduct the survey and collect data on people's preferences in radio stations.
- The student will take the data that has been collected and create a scattergram, which they will use to help convince others which station should be used to advertise their product.

Materials needed

- Copies of surveys created by students
- Graph paper
- Overheads or chart paper

Instructional activity

1. Tell the students they are to pretend they work for a marketing firm. Their customers/clients are a pet store, a clothing store with trendy clothes for males and females, a weight loss and exercise facility, and a toy store. The students must determine on what radio station their clients should advertise.
2. Have students design a survey to collect data from various radio stations to determine on what radio station their clients should advertise.
3. Remind the students they will need to collect some sort of quantitative data in order to create a scattergram. Discuss with the class which two data sets will be most relevant in determining which station to advertise on. The most likely will probably be *age* and *number of listening hours per day/week.*
4. As a class, agree on the survey questions everyone will use. Specify to students the number of radio stations to survey. Discuss a sample pool of radio stations and the ways a survey can be biased if the sample does not represent the population as a whole.
5. Have the students survey the sample radio stations. When students bring their surveys to class, sort the surveys from each radio station. Group students, and give each group the surveys from one station.
6. Have students in each group work together to create a scattergram of the data and develop a presentation that will convince their client which radio station they should advertise on. What is the relationship between the two data sets — positive, negative, or none. Have groups present their finding to the class.

Sample assessment

- The presentations should include the method of organizing the data, an appropriate display, a summary statement including data analysis involving critical measures (central tendency, range), and inferences/ conjectures/predictions based on data.

Follow-up/extension

- Have the students write a letter to their clients explaining their decision and the mathematical thinking that was involved in making that decision.

Sample resources

http://illuminations.nctm.org/lessonplans/3-5/airplanes/ – In this lesson, students make paper airplanes and explore attributes related to increasing flight distances. Each student collects data from three flights of the airplane and finds the median distance. Students then collect, organize, display, and interpret the median distances for the class in a stem-and-leaf plot.

http://score.kings.k12.ca.us/lessons/mandm.html – A lesson plan that uses small individual bags of M&M's candies to review students' understanding of estimating, sorting, graphing, mean, median, mode, fractions, percentage, and averaging.

www.manatee.k12.fl.us/sites/elementary/palmasola/mathlabtutstat1.htm – A tutorial for students involving mean, median, and mode.

http://www.brainpop.com/math/dataprobability/meanmodemedianrange/index.weml – An interactive Web site for students using probability concepts.

http://illuminations.nctm.org/lessonplans/6-8/baseball/index.html – Having Fun with Baseball Statistics. These activities allow students to explore statistics surrounding baseball. They are exposed to connections between various mathematical concepts and see where this mathematics is used in areas with which they are familiar.

Navigating through Data Analysis and Probability in Grade 6 through Grade 8 – Available from NCTM. Contains additional lessons for data analysis activities.

Probability and Statistics Professional Development Module available from VDOE.

Sample released test items

Jorge has test scores of 82, 79, 72, and 78. What is the least score he can make on the next test if he wishes to have an average (mean) score of 80?

F 92

G 90

H 89

J 88

This is a list of the number of minutes each member of a study group spent studying for a test.

75 40 15 0 60 50 120 90 90 20

What was the median number of minutes spent studying for the test by the group?

A 50
B 55
C 56
D 60

Soccer is the world's most popular sport. The table lists the records of 5 World Cup winners.

Country	Games Won	Games Lost	Ties	Total Points
Argentina	24	15	9	57
Brazil	44	11	11	99
England	18	11	12	48
Italy	31	11	12	74
West Germany	39	14	15	93

What was the mean number of total points scored by these teams?

A 51
B 74
C 74.2
D 99

The list shows the scores made by each member of Jaime's discussion group on the last test.

71 80 62 93 68 87 73 78

Which frequency table correctly displays the information?

F

Interval	Frequency
60–70	2
70–80	4
80–90	2
90–100	1

G

Interval	Frequency
60–69	2
70–79	3
80–89	2
90–99	1

H

Interval	Frequency
65	2
75	3
85	2
95	1

J

Interval	Frequency
60	2
70	4
80	2
90	1

This is a list of the number of miles driven by each committee member to a regional meeting of the Clean Air Society.

53 57 78 56 72 60 73 94 92 87

Which stem-and-leaf plot correctly displays the information?

A

Stem	Leaf
5	3,6,7
6	0
7	2,3,8
8	7
9	2, 4

B

Stem	Leaf
5	3,6,7
6	
7	2,3,8
8	7
9	2, 4

C

Stem	Leaf
5	3
6	1
7	3
8	1
9	2

D

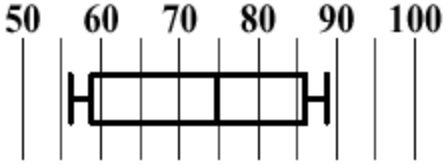
Stem	Leaf
5	3
6	0
7	3
8	1
9	2

This is a list of the number of miles driven by each committee member to a regional meeting of the Clean Air Society.

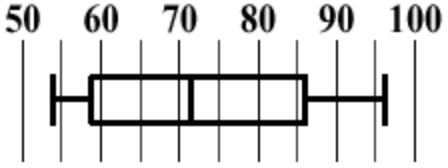
53 57 78 56 72 60 73 94 92 87

Which box-and-whisker plot correctly displays the information?

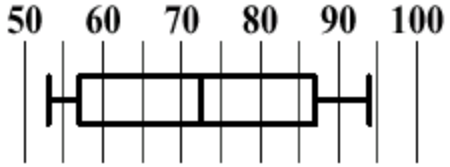
F



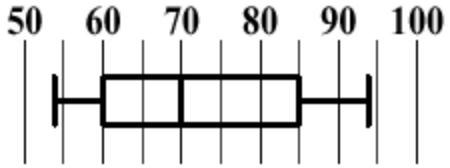
G



H

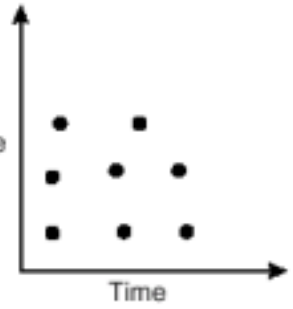


J

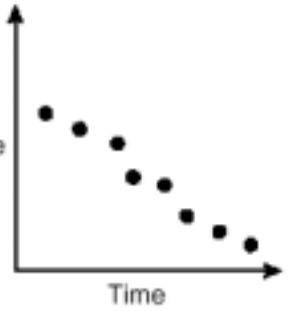


Which scatterplot would most likely show the correct relationship between time traveled and distance traveled during an airline flight?

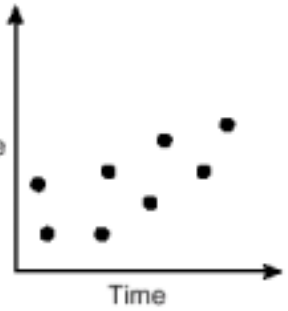
A



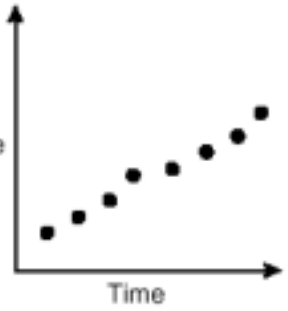
B



C



D

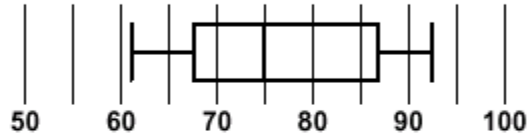


The list shows the scores made by each member of Jaime's discussion group on the last test.

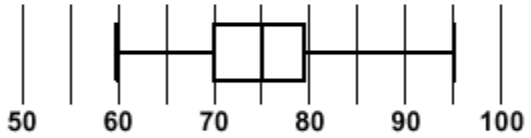
69 79 62 93 73 81 73 78

Which box-and-whiskers plot correctly displays the information?

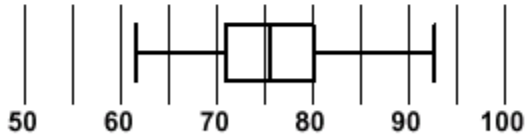
F



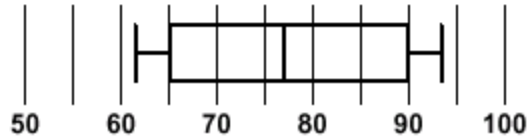
G



H



J

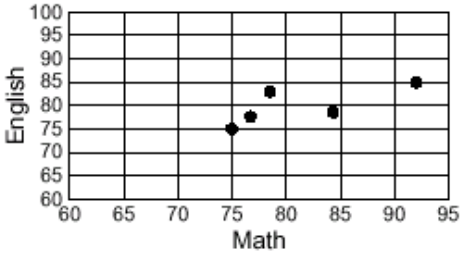


The table shows the Math and English scores of Art and 4 of his friends.

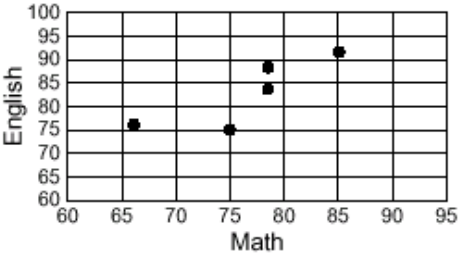
Student	Math	English
Art	84	78
Bonnie	67	76
Cathy	92	85
Don	75	75
Ellie	78	88

Which scattergram correctly shows the relationship between Math and English scores for the group of friends?

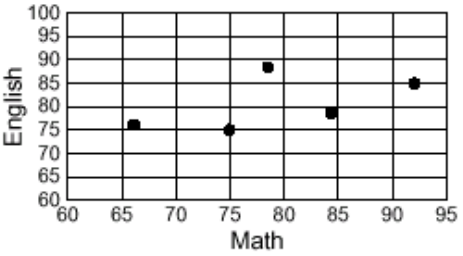
A



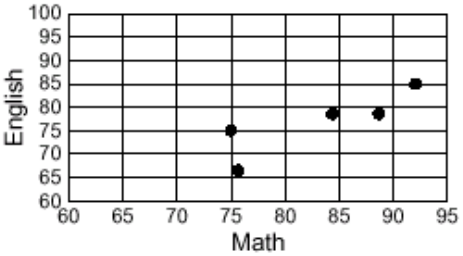
B



C



D



This is a list of Beth's English homework scores for the grading period.

93, 83, 64, 84, 76, 83, 78, 76, 60, 81

Which stem-and-leaf plot correctly displays the information?

F

Stem	Leaf
6	II
7	III
8	IIII
9	I

G

Stem	Leaf
6	4
7	6, 8
8	1, 3, 4
9	3

H

Stem	Leaf
6	4
7	6, 6, 8
8	1, 3, 3, 4
9	3

J

Stem	Leaf
6	0, 4
7	6, 6, 8
8	1, 3, 3, 4
9	3

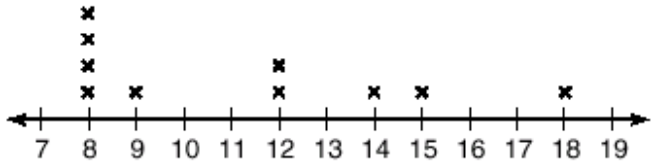
The table shows the number of birthday cards a shop sold each day in a two-week period.

Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
15	8	9	8	10	12	18

Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
7	9	8	12	8	15	14

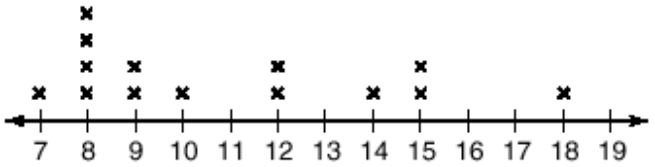
Which line plot displays this information?

F



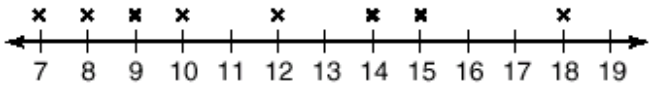
Number of Birthday Cards Sold

G



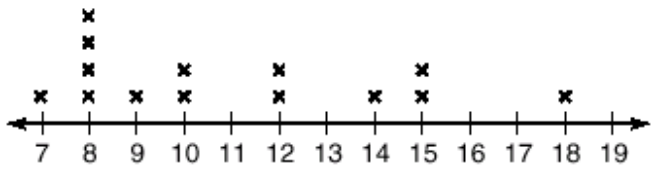
Number of Birthday Cards Sold

H



Number of Birthday Cards Sold

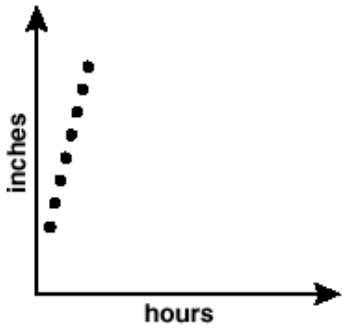
J



Number of Birthday Cards Sold

Which scatterplot best shows the relationship between a person's height and the time that person spends watching television?

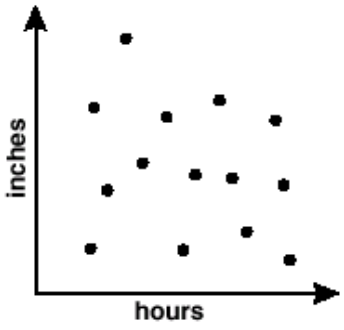
F



G

H

J



Organizing Topic Probability

Standards of Learning

7.14 The student will investigate and describe the difference between the probability of an event found through simulation versus the theoretical probability of that same event.

7.15 The student will identify and describe the number of possible arrangements of several objects, using a tree diagram or the Fundamental (Basic) Counting Principle.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Determine the theoretical probability of an event.	
• Describe changes in the experimental probability as the number of trials increases.	
• Investigate and describe the difference between the probability of an event found through simulation versus the theoretical probability of that same event.	
• Describe the number of possible arrangements of no more than three types of objects, using a tree diagram.	
• Compute the number of possible arrangements of no more than three types of objects by using the Fundamental (Basic) Counting Principle.	

Probable Families

Reporting category Probability and Statistics

Overview Students determine the theoretical probability of having different combinations of five children and relate it to Pascal's triangle. A coin will be used to simulate the birth of a child and results will be organized for students to look for patterns and determine theoretical probabilities.

Related Standard of Learning 7.14

Objectives

- The student will generate data and analyze the data for patterns.
- The student will determine the theoretical probability of an event.
- The student will relate the theoretical probability to Pascal's triangle.
- The student will discuss the difference between the theoretical probability of an event and the experimental probability of an event.

Materials needed

- Pascal triangle handout
- Jones family handout
- Coins
- Summary worksheet

Instructional activity

1. Begin class with a brief discussion of probability and how the probability of an event can be determined.
2. Introduce Pascal's Triangle, and explain that it can be used to assist one in solving probability problems. Ask students to look for patterns.
3. As a class, determine the first four rows of the Triangle, and then have groups determine the next four rows. As a class, verify the groups' answers. Encourage a discussion of how the groups found each row in the Triangle.
4. Tell the students they will be helping Mr. and Mrs. Jones, who want to have five children, determine the probability of having all girls or all boys. They would name boys 1) Bob, 2) Bill, 3) Berry, 4) Brian, and 5) Benny. Girls would be named 1) Gina, 2) Grace, 3) Gill, 4) Gerry, and 5) Gwenyth. For example, if the third child born is a girl, then her name will be Gill. If the fourth child born is a boy, his name will be Brian.
5. Toss a coin to simulate the Jones' births (a graphing calculator can also run a simulation). A head indicates the birth of a girl, and a tail will mean a boy. Do several examples with the class so that they understand the procedure. For example if HHTHT is tossed, then the children will be named Gina, Grace, Berry, Gerry and Benny.
6. Have students work in pairs to complete the chart. At the bottom of the worksheet, have them guess how many possible combinations of five children the Jones could have.
7. Reassemble the class, and look at groups' results. The following are possible questions to ask to collect the data needed to complete the summary chart.

- Did any group get an all-girl family? What are their names?
- How can we record an all-boy family without writing all of their names. (BBBBB)
- Did anyone get a combination of four boys and one girl? What were their names? Make a list of all the combinations that were found, and determine if there are any other possibilities.

8. Have students determine the possible combinations of having 5 boys, 4 boys, 3 boys, 2 boys, 1 boy, or 0 boys. They should recognize this would be the same as having 1 girl, 2 girls, 3 girls, 4 girls, or 5 girls.
9. Have students determine the total number of possible combinations there could be. (32)
10. Now have them determine the theoretical probability of having the following:
    - What is the theoretical probability of the Jones having all girls? (1/32)
    - What is the theoretical probability of the Jones having all boys? (1/32)
    - What is the theoretical probability of the Jones having 3 girls and 2 boys? (10/32)
    - What is the theoretical probability of the Jones having 1 boy and 4 girls? (5/32)
11. Have students relate their answers to Pascal's Triangle. You can repeat the procedure for a family of six by using Pascal's triangle and have them answer questions.
12. Discuss with students how the results of this experiment might be different if the data had been collected by surveying families that have five children.

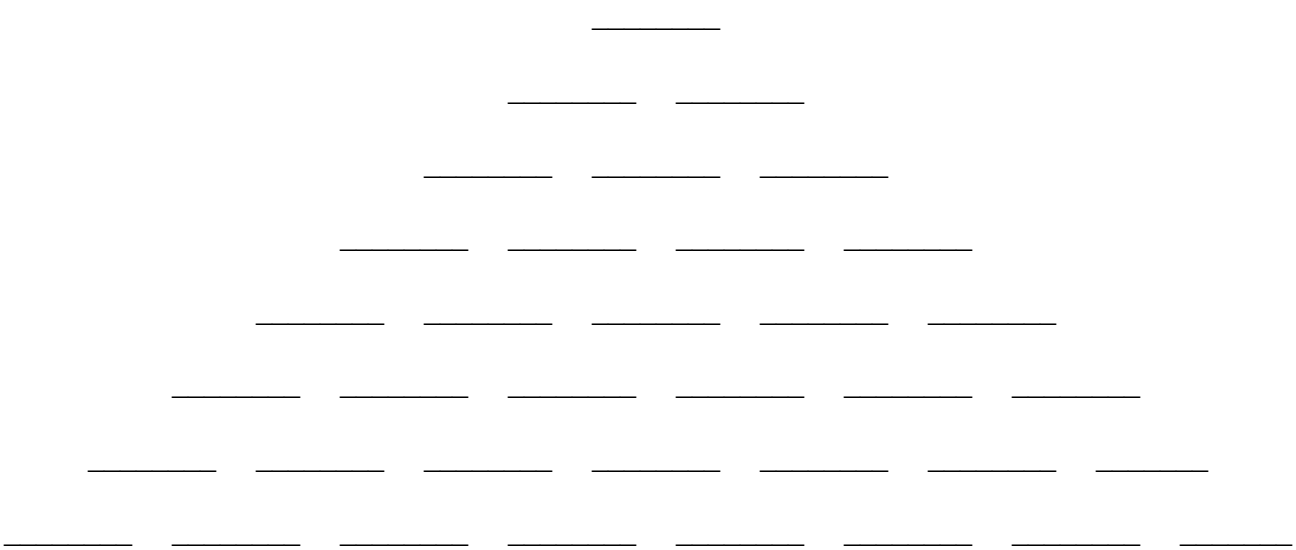
Sample assessment

- Assess students informally as they perform the activity and engage in a discussion of the results of the simulation.
- Assess students formally by having them write a paragraph describing the results of the activity.

Follow-up/extension

- Have students determine the following and relate their answers to Pascal's Triangle:
    - How many possible ways are there for a family to have a two-child family? (4)
    - How many possible ways are there for a family to have a three-child family? (8)
    - How many possible ways are there for a family to have a four-child family? (16)

Pascal's Triangle



Jones family names **1. Bob** **1. Gina** **2. Bill** **2. Grace** **3. Berry** **3. Gill** **4. Brian** **4. Gerry** **5. Benny** **5. Gwenyth**	1. 2. 3. 4. 5.
1. 2. 3. 4. 5.	1. 2. 3. 4. 5.
1. 2. 3. 4. 5.	1. 2. 3. 4. 5.
1. 2. 3. 4. 5.	1. 2. 3. 4. 5.

Summary of Jones Family Possible Boy/Girl Combinations

5 Boys	4 Boys	3 Boys	2 Boys	1 Boy	0 Boys

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5 Boys	4 Boys	3 Boys	2 Boys	1 Boy	0 Boys
BBBBB	BBBBG	BBBGG	BBGGG	BGGGG	GGGGG
	BBBGB	BBGBG	BGBGG	GBGGG	
	BBGBB	BGBBG	BGGBG	GGBGG	
	BGBBB	GBBBG	BGGGB	GGGBG	
	GBBBB	BBGGB	GBBGG	GGGGB	
		BGBGB	GBGBG		
		GBBGB	GBGGB		
		BGGBB	GGBBG		
		GBGBB	GGBGB		
		GGBBB	GGGBB		

Regatta Probability

Reporting category Probability and Statistics

Overview Students determine the probability of an event occurring by examining both the theoretical and experimental probability of the event. Sample space and tree diagrams will be used.

Related Standards of Learning 7.14, 7.15, 7.18

Objectives

- The student will generate data by rolling a number cube and playing a game.
- The student will analyze the data and determine the experimental probability of an event occurring.
- The student will generate a log of all the possible combinations of sums of 2 dice and use it to determine the theoretical probability of an event.
- The student will use a tree diagram to analyze data and determine the probability of an even occurring.

Materials needed

- Two number cubes for each group
- Regatta game sheet
- Log sheet
- "The Regatta" transparency
- 12 beans or small counters per pair
- "The Log" recording sheet
- Large graph paper
- Colored pencils or markers

Instructional activity

Note: This activity is found in the Virginia Department of Education's *Teacher Resource Guide, Probability and Statistics.*

1. Format: Pairs or small group
2. Background: Students will be conducting an experiment — any activity that has two or more clearly discernible results or outcomes. As a result of the experiment, students should be able to list the sample space for that experiment. Sample space is a collection of all possible outcomes. Based on this sample space, students should be able to determine the probability of an event occurring. Probability is defined as the ratio of the number of favorable outcomes to all outcomes of an experiment. An event is defined as any subset of the outcomes or any subset of the sample space – usually the outcome we are looking for, a favorable outcome. When all possible outcomes of a simple experiment are equally likely (each result is as likely to occur as every other), the theoretical probability of an event is number of outcomes in the event divided by the number of possible outcomes. Experimental probability is based on the results of an experiment rather than a theoretical analysis of the experiment. Theoretical probability is based on a logical analysis of an experiment, not on experimental results.
3. The probability of an event is always 0, 1, or any number between 0 and 1.
4. An impossible event has a probability of 0.

5. A certain event has a probability of 1.
6. Probabilities between 0 and 1 may be expressed as a ration, a decimal, or a percent.

Part I

1. Explain to the students that they will play a game using two number cubes or dice. Place "The Regatta" game board transparency on the overhead and place 12 counters on the starting line, one to represent each yacht.
2. Explain how the yachts move across the course. After a player rolls the cubes, he moves the yacht whose number is the sum of cubes ahead ONE space. For example, if a three and a five are rolled, the player moves yacht #8 one space forward.
3. Students should predict which yacht will reach the finish line first. Record several responses. Have all students predict, by a show of hands, which yacht they think will win.
4. Demonstrate the game on the overhead. Select two students to model taking turns rolling the cubes and moving the appropriate yacht.
5. Pause frequently during the demonstration to look at the results. Ask students if they notice any patterns in the way the yachts are moving. Ask students to comment on why some of the yachts have not moved yet. Students may want to predict again as the race progresses, changing their favorite as they watch the race. As new information is received, refinements in predictions are allowed.
6. Post the class graph and record the winning yacht.
7. Remind students about accurate recording of their results.
8. Distribute "The Regatta" game boards, counters, and number cubes.
9. Have the students begin playing "The Regatta." Each race should be recorded on the class graph as soon as it is completed.
10. Students should play the game at least four times. All results should be recorded on the class graph so that a large amount of data can be collected for analysis in Part II.
11. Provide time for discussion of "The Regatta" game. Ask students to discuss any surprises they found in the results. Ask for "true statements" about the graph.

Part II

1. Review the results displayed on the class graph. Ask students to explain the results shown.
2. Lead the group into a discussion of the mathematics involved in "The Regatta." Ask, "How many ways could you make a sum of six with two dice?" Remind them this can include reversals, as 4 + 2 and 2 + 4. For example, a green four and a white two is a different arrangement of the dice than a white four and a green two. On the overhead, record all the possible ways to make six using two dice.

Die 1		Die 2
1	+	5
2	+	4
3	+	3
4	+	2
5	+	1

3. List the ways to make several other numbers such as the number that won the most, or numbers that won few races.
4. On the overhead, show "The Log" recording sheet.

5. To help students understand "The Log" recording sheet, color the dice on the transparency. For example, if students used a red die and a green die, shade the dice in the horizontal row red and the dice in the vertical row green. Demonstrate how to fill in part of the chart by finding the sum and placing the sum in the appropriate box.
6. Distribute "The Log" recording sheet to students. Have them color the dice on "The Log" to match the ones they used when they played "The Regatta." Have them complete "The Log."
7. Check to see that students have completed "The Log" correctly. Display a completed Log on the overhead.
8. Ask the students to discuss any patterns they notice on "The Log." Ask if "The Log" shows all of the ways to make sum six, as well as other sums. Ask the students to discuss the frequency of obtaining each sum. How does this frequency relate to the winners of "The Regatta"?
9. Ask the students how many ways a seven can be made. From a review of "The Log," they should determine that there are six ways to make a seven. How many possible sums can be obtained by rolling two dice? From "The Log", there are 36 possible outcomes when you roll two dice.
10. Have the students use the chart to show numerical ways to probabilities. One way of representing the probability of each number 1 through 12 is shown below:
    - a sum of 7: 6 out of 36 or 1 out of 6 or 1/6
    - a sum of 6 or 8: 5 out of 36 or 1 out of 7 1/5
    - a sum of 5 or 9: 4 out of 36 or 1 out of 9 or 1/9
    - a sum of 4 or 10: 3 out of 36 or 1 out of 12 or 1/12
    - a sum of 3 or 11: 2 out of 36 or 1 out of 18 or 1/18
    - a sum of 2 or 12: 1 out of 36 or 1/36
    - a sum of 1: 0 out of 36 or 0
11. These fractional probabilities can be used to remind students of the relationship of a written number to the real world.

Part III

1. Give each pair of students a calculator, and have them calculate the experimental probability of each winner in "The Regatta." Then have them calculate the theoretical probability of each sum listed on "The Log." A comparison of the two probabilities should be made.

Sample assessment

- Assess students informally throughout the activity through questions and discussion. Their recording sheets can serve as an informal observation of their understanding of the activity as well.
- Assess students formally by having them calculate the probability of a given event. Possible situations could be determined by putting different color shapes in a bag and asking them to find a theoretical and experimental probability of picking a red square or a blue triangle.

Variation for Part II

1. Lead the group into the development of the *sample space* — all possible outcomes. Record all the sums previously identified on the chalkboard or overhead.

2	3	4	5	6	7	8	9	10	11	12

2. Ask, "How many ways can I get a sum of 2? Of 3?" and so on. Continue until all sum combinations are found. Record each response.

2	3	4	5	6	7	8	9	10	11	12
(1,1)	(1,2)	(1.3)	(1,4)	(1,5)						
	(2,1)	(3,1)	(4,1)	(5,1)						
		(2,2)	(2,3)	(2,4)						
			(3,2)	(4,2)						
				(3,3)						

3. At this point, ask the students if they see a pattern. If so, ask them to predict how many combinations there are to get a sum of 7. Record the predictions. Then verify the number of combinations.

2	3	4	5	6	7	8	9	10	11	12
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)					
	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)					
		(2,2)	(2,3)	(2,4)	(2,5)					
			(3,2)	(4,2)	(5,2)					
				(3,3)	(3,4)					
					(4,3)					

4. Ask, "Can you predict how many combinations there are to get a sum of 8?" Record the prediction. Verify the number of combinations.

2	3	4	5	6	7	8	9	10	11	12
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(2,6)				
	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)	(6,2)				
		(2,2)	(2,3)	(2,4)	(2,5)	(3,5)				
			(3,2)	(4,2)	(5,2)	5,3)				
				3,3)	(3,4)	(4,4)				
					(4,3)					

(Students may suggest (1,7), which is fine. Remind them that number cubes have only 1 to 6 to work with.)

5. Ask, "What happened to our prediction? What went wrong? Do you want to predict how many combinations there are for 9?" Record any predictions. Verify the combinations.

2	3	4	5	6	7	8	9	10	11	12
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(2,6)	(3,6)			
	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)	(6,2)	(6,3)			
		(2,2)	(2,3)	(2,4)	(2,5)	(3,5)	(4,5)			
			(3,2)	(4,2)	(5,2)	(5,3)	(5,4)			
				(3,3)	(3,4)	(4,4)				
					(4,3)					

6. Continue in a similar fashion until the entire sample space is listed.

2	3	4	5	6	7	8	9	10	11	12
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)
	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	
		(2,2)	(2,3)	(2,4)	(2,5)	(3,5)	(4,5)	(5,5)		
			(3,2)	(4,2)	(5,2)	(5,3)	(5,4)			
				(3,3)	(3,4)	(4,4)				
					(4,3)					

7. How many possible outcomes are there?
8. Note: This is a good time to construct a tree diagram and show students how to determine the sample space with a tree diagram.
9. Ask, "Which sum do you think should occur most often? Does this match your prediction or the results shown on our graph? Why or why not?"

Regatta Game Board

1							**1**
2							**2**
3							**3**
4							**4**
5							**5**
6							**6**
7							**7**
8							**8**
9							**9**
10							**10**
11							**11**
12							**12**

The Log

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Creating Tree Diagrams

Reporting category Probability and Statistics

Overview Students construct tree diagrams and determine sample space using the tree diagram and the Fundamental (Basic) Counting Principle.

Related Standard of Learning 7.15

Objectives

- The student will solve problems involving sample space by creating a tree diagram.
- The student will use the Fundamental (Basic) Counting Principle to determine the sample space for a problem.

Materials needed

- Recording sheet for pizza shop activity
- Real Meal Restaurant handout

Instructional activity

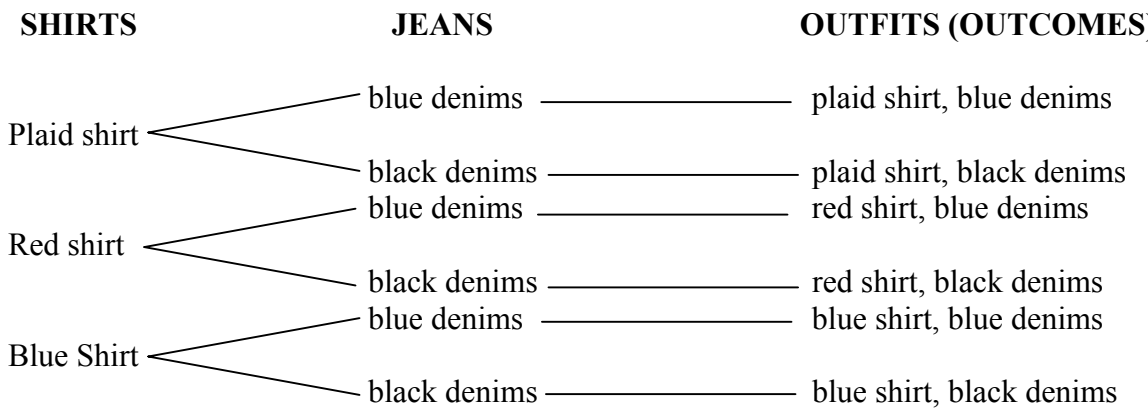
Part I

Note: This activity can be found in the Virginia Department of Education *Probability and Statistics Teacher's Resource Guide.*

1. Choose three students to come to the front of the room. Try to choose people who are wearing different types of outfits.
2. Construct a tree diagram (as a total group) of all the possible combinations of outfits that can be made from the clothes the students are wearing. For example: (blue shirt (person 1), jeans (person 2), tennis shoes (person 3).
3. Continue with the "Tree Diagrams" handout on constructing tree diagrams from pizza choices.
4. Discuss how the sample space changes when you add additional choices.

Part II

1. *Background:* The Fundamental Counting Principle is a method for finding the number of ways that two or more events can occur by multiplying the number of ways that each event can occur. The Principle states that if successive choices are made, and then the total number of choice is the product of the number of choices at each stage.
2. For example, if you have 3 shirts and 2 pairs of jeans, then you have a total of 6 different outfits to wear. Each shirt may be worn with each pair of jeans. There are 3 shirts times 2 pairs of jeans for a total of 6 outfits.



3. Based on the menu of the Real Meal Restaurant, have students use the Fundamental Counting Principle to determine the number of different meals that can be served.
4. Based on customer wishes, have students determine and display the choices using a tree diagram.

Sample assessment

- Have students bring in a menu from a local restaurant, and repeat the activity. You may want to limit the menu to certain items or even find a small menu that would not create too many choices for the students.

Sample resources

http://rec-puzzles.org/probability.html – A Web site containing many word problems involving probability and their solutions.

http://nces.ed.gov/nceskids/probability – National Center for Education Statistics Web site with probability activities for students and many other resources.

http://mathforum.org/dr.math/faq/faq.boy.girl.html – Probability activities and information from the Math Forum.

http://standards.nctm.org/document/chapter5/data.htm#bp4 – NCTM's Principles and Standards information about probability at the 3–5 grade levels.

http://www.pbs.org/teachersource/mathline/lessonplans/esmp/chances/chances_procedure.shtm – PBS lesson plan on probability.

http://mathforum.org/probstat/probstat.lessons.html – Lesson plans from the Math Forum on probability.

http://www.brainpop.com/math/dataprobability/meanmodemedianrange/index.weml – An interactive Web site for students using probability concepts.

Tree Diagrams

You are trying to decide which pizza to order for dinner. Your choices for crust are: regular, thin, and deep dish. You want only one topping and will choose either pepperoni or sausage.

Construct a tree diagram to show the possibilities you have from which to choose one crust with one topping.

How would your sample space change if you added bacon as a third topping choice?

Real Meal Restaurant

SANDWICHES: **Ham and Turkey Club** **Hamburger**
Rachael on Rye **Deli Cold Cut Special**
Sliced BBQ **Pork BLT**

FRENCH FRIES: **small** **medium** **large**

SALADS: **Garden Salad** **DRESSINGS:** **Ranch**
Chef Salad **French**
Cobb Salad **Creamy Italian**

BEVERAGES:

Coke	**small**	**medium**	**large**
Pepsi	**small**	**medium**	**large**
Sprite	**small**	**medium**	**large**
Tea:	**medium**	**large**	
Coffee:	**medium**	**large**	
Milk:	**regular**	**low-fat**	

1. How many possible meals can be served at the Real Meal?

2. How many choices are there if a customer wants the following:

   a. A soft drink, sandwich and fries? Display the choices with a tree diagram.

   b. A sandwich, fries and milk? Display the choices with a tree diagram.

   c. A salad with dressing and tea? Display the choices with a tree diagram.

   d. A sandwich, salad with dressing, and coffee? Display the choices with a tree diagram.

Sample released test items

On your first draw, what is the probability of drawing a red card, without looking, from a shuffled deck containing 6 red cards, 6 blue cards, and 8 black cards?

A 10%

B 20%

C 30%

D 40%

A package contains 7 bags of tortilla chips, 3 bags of cheese puffs, 4 bags of potato chips, and 6 bags of corn chips. If Steve reaches into the package and selects one bag without looking, what is the probability he will choose potato chips?

F $\frac{2}{20}$

G $\frac{1}{5}$

H $\frac{3}{10}$

J $\frac{7}{20}$

Dave is a member of a bowling league. The table below shows his record for the last 10 games (100 frames) he bowled. Based on his previous record, what is the probability that Dave will bowl a strike in the next frame?

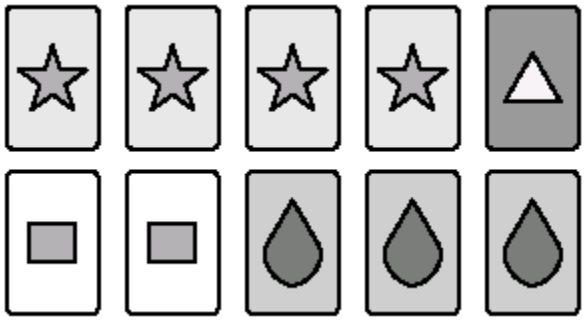
A $\frac{3}{100}$

B $\frac{3}{50}$

C $\frac{3}{25}$

D $\frac{6}{25}$

These cards are used to play a game between two players.



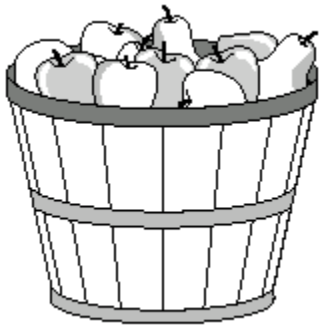
If Sally draws one card at random, what is the probability it will be a card with a star?

F $\frac{2}{5}$

G $\frac{3}{10}$

H $\frac{1}{5}$

J $\frac{1}{10}$



The basket contains 30 apples, 20 pears, and 10 peaches. What is the probability that the first piece of fruit taken from the basket will be a peach?

A $\frac{1}{2}$

B $\frac{1}{3}$

C $\frac{1}{5}$

D $\frac{1}{6}$

The cafeteria offers 3 different meats, 4 different vegetables, 5 different desserts and 2 different drinks for lunch. How many different meals consisting of 1 meat, 1 vegetable, 1 dessert, and 1 drink are offered?

A 14
B 49
C 70
D 120

Joan and Barry are candidates for class president. Orville, Sally, Consuela, Harry, and Rebecca are candidates for vice president. Sam, William, and Frederica are candidates for secretary. How many different combinations of president, vice president, and secretary are possible?

A 11
B 18
C 30
D 40

The chart shows the pizza menu for the local pizza parlor.

Pizza

Size	Crust	Toppings
Small Medium Large Extra Large	Thin Original Thick	Pepperoni Hamburger Cheese Sausage

Which of the following shows the total number of ways Andy can buy a pizza with one topping?

A $4 + 3 \times 4$

B $4 \times 3 \times 4$

C $(4 \times 3) + (4 \times 3)$

D $(4 \times 4) + 3$

Organizing Topic Patterns, Functions, and Algebra

Standards of Learning

7.19 The student will represent, analyze, and generalize a variety of patterns, including arithmetic sequences and geometric sequences, with tables, graphs, rules, and words in order to investigate and describe functional relationships.

7.20 The student will write verbal expressions as algebraic expressions and sentences as equations.

7.21 The student will use the following algebraic terms appropriately: equation, inequality, and expression.

7.22 The student will
a) solve one-step linear equations and inequalities in one variable with strategies involving inverse operations and integers, using concrete materials, pictorial representations, and paper and pencil; and
b) solve practical problems requiring the solution of a one-step linear equation.

Essential understandings, knowledge, and skills	**Correlation to textbooks and other instructional materials**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to analyze situations to discover a variety of patterns.	
• Analyze numeric and geometric sequences to discover a variety of patterns.	
• Determine patterns and relationships within data sets (e.g., trends).	
• Make inferences, conjectures, and predictions based on analysis of a set of data not exceeding 20 items.	
• Represent a variety of patterns, using tables, graphs, rules, and words, in order to investigate and describe functional relationships.	
• Identify the real number equation that represents each property of operations with real numbers, when given several real number equations.	
• Explore the properties of real numbers, using diagrams and manipulatives.	
• Test the validity of properties by using examples of the properties of operations on real numbers.	
• Identify the property of operations with real numbers that is illustrated by a real number equation.	
• Simplify expressions by using the order of operations in a demonstrated step-by-step approach.	

- Find the value of numerical expressions, using order of operations, mental mathematics, and appropriate tools. Exponents are limited to positive values. Fractions are limited to having denominators of 12 or less. Decimals are limited to the thousandth place. _____
- Write verbal expressions as algebraic expressions. _____
- Write verbal sentences as algebraic equations. _____
- Apply the following algebraic terms appropriately: equation, inequality, and expression. _____
- Identify examples of equations, inequalities, and expressions. _____
- Represent and demonstrate steps in solving equations in one variable, using concrete materials, pictorial representation, and algebraic sentences. _____
- Represent and demonstrate steps in solving inequalities in one variable, using concrete materials, pictorial representation, and algebraic sentences. _____
- Translate one-step word problems and practical problems into algebraic equations and solve them. _____
- Generalize a variety of patterns. _____

Patterns, Functions, and Graphing

Reporting category Patterns, Functions, and Algebra
Overview Students graph functions.
Related Standards of Learning 7.18, 7.19

Objective

- The student will analyze patterns, including arithmetic sequences and geometric sequences using tables, graphs, rules, and words.

Materials needed

- Cardboard box with opening on the side

Instructional activity

Activity A

1. Display a geometric pattern on the overhead. Ask, "What would be the next 5 in the pattern?"
2. Give students several of these geometric patterns, and have them draw the next *x* number of patterns.
3. Continue this reasoning with arithmetic patterns. Have students look at what is happening between the numbers in a pattern. Point out the pattern among the numbers to explain what the next numbers would be.
4. Have students select one of the arithmetic patterns and set it up into a table form. Have them write the rule above the box containing the first number and then write the next numbers, and so on.

Activity B

1. Display a function box. Label the top opening as the Input and the side opening as the Output. Display the word *RULE* on the front of the box.
2. Put a number into the box, and pull out an answer. Repeat several times. Display the numbers in a table format as the numbers are put in and pulled out. Students will discover the relationship of the numbers as a pattern. Have students state the pattern as a rule on the table.
3. Show the relationship with the Input as the *x* and the Output as the *y*.
4. Practice solving for missing parts of a table by using the rule.
5. With students working in pairs, have one student create the rule and the other develop the table.

Activity C

1. Display several tables showing patterns, and several pattern rules. Have students match the rule to the table. Reinforce replacing the *x* with the number and solving for the *y*.
2. Have students practice this by creating their own tables and rules and sharing with their partners.
3. Review with students the coordinate plane and the *x*- and *y*-axes. Select one of the tables, and ask students how they would graph the table.
4. Show the students that the coordinates are the Input and Output. Students will see the relationship and understand why the rule is a linear equation. The straight line will show the linear part.
5. Have students graph their tables on a coordinate plane and indicate the rule on the line.

Sample assessment

- Assess students during classroom activities.

Expressions and Equations

Reporting category Patterns, Functions, and Algebra

Overview Students show representations and relationships with rational numbers.

Related Standards of Learning 7.2, 7.20, 7.21, 7.22

Objectives

- The student will substitute rational numbers for variables and solve using orders of operations.
- The student will write word phrases as algebraic expressions and sentences as equations.
- The student will use the algebraic terms: equation, inequality, and expression.
- The student will solve one-step equations and inequalities using pictorial representations and paper and pencil.
- The student will solve practical problems using one-step equations.

Materials needed

- Paper
- Pre-made model of a balanced scale

Instructional activity

Activity A: Prior knowledge of variables

1. Have students take their pulse for 15 seconds and record number.
2. Ask students, "How would you calculate the number of heartbeats in 1 minute?"
3. Lead students in setting up the expression of number of heartbeats × 4. This would represent the total heartbeats in 1 minute.
4. Have students set up an expression of the heartbeats times m (e.g., $72m$), where m represents the minutes.
5. Ask students, "How would you calculate the number of heartbeats in 5 minutes?" Students will show set up of $72m$; $m = 5$; $72 \times 5 = 360$
6. Give students this word problem: "Suppose you want to buy two CDs and a $9 video. What is the cost? Write an expression to represent this situation. Let c represent the cost of the CDs." Ask, "What type of operation will be performed between the CDs and video?"
7. Students should state: $2c + 9$.
8. Have students translate other expressions from verbal situations, or evaluate expressions if you are giving them replacement values.

Activity B

2. Show students the triangle method for using order of operations to simplify numerical expressions. The triangle method consists of solving each step, with the answer as the tip of the triangle, as shown at right.
3. Have students simplify many expressions, using the triangle method.

$$(4 + 5)4 - 3^2 + 9(2)$$
$$9(4) - 3^2 + 9(2)$$
$$9(4) - 9 + 9(2)$$
$$36 - 9 + 18$$
$$27 + 18$$
$$45$$

Activity C

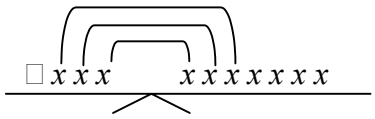
1. Write the words *expression, equation,* and *inequality* on the board.
2. Have students give examples of each and distinguish the difference. Examples:
   - Expressions can be numerical or variable: Substitute for variables to find the value, using order of operations.
   - Equations have an equal sign to solve and a variable.
   - Inequalities use the < and > signs, and the solution is a set of numbers that satisfy inequality.
3. The word *inequality* will need to broken down into equal or "not equal." Then, lead students into what other signs can be used that do not mean equal.

Activity D

1. Have students write words that mean add, subtract, multiply, or divide.
2. Have students practice writing simple expressions using these words. Write the algebraic expressions as verbal expressions.
3. Lead students into complex expressions as in the example mentioned in Activity A.
4. Have pairs of students create their own expressions. Have one student write the expression (verbal or algebraic). Have the other student evaluate if value is given to the variable.

Activity E

1. Use the scale mat, model $N + 3 = 7$. Using colored chips, establish which chip would represent the variable, and have the other color represent the numbers. The scale represents a balance. Keep the scale balanced by performing the same procedure on both sides.



2. Show the removal of pairs of *x*'s.
3. Have students write the answer out: $N = 4$
4. Have students check answer by substituting into the equation: $4 + 3 = 7$
5. Practice adding equations until the students understand they are doing the opposite operation.
6. Demonstrate subtraction equation. Emphasize zero pairs are added to both sides. Explain use of zero pairs in some subtraction equations. State the answer, then check equation.

7. Once the students understand keeping a balanced scale, have students draw the scale under the equation to show all work:

$$\begin{array}{r|l} 3x = 12 & \\ \hline 3 & 3 \end{array}$$

$x = 4$

Check: $3(4) = 12$

Activity F

1. Review the list of terms that mean *add, subtract, multiply* and *divide*. Have students compare a verbal expression to an equation written as a word problem. Have the students find the word *is* in the verbal form of the equation. Show how an answer (number) follows this word.
2. Have students write an algebraic equation for an equation presented in verbal form, and vice versa.
3. Pair students, and have one student in each pair write on an index card an algebraic equation while the other writes on a separate index card the same equation in verbal form.
4. Collect all cards, and separate the equations into verbal and written.
5. Display the equation cards on the board. Divide the class into two teams. Have teams match the equations in verbal form to the equations in algebraic form. Have the teams take turns, imposing a ten-second time limit for finding matches. When a player makes a match, give his/her team a point.

Activity G

1. Review the inequality skill: $s > 5$. Ask students, "What numbers would make this statement true?" Show these number in a set form (6, 7, 8, …).
2. Draw a number line. Ask students, "How would you display this set on this number line?"
3. Demonstrate the line from the 5 to the end. (Display with an open circle).
4. Lead into the inequality equation: $s + 3 < 10$.
5. Explain that it is still an equation, and solve the same way, ending with the $s < 7$. Write the set of numbers, then display on a number line.
6. As a follow up, have number lines with answers displayed, and have the students match the inequality equation.

Sample assessment

- During student practice of evaluating expressions, and solving equations and inequalities equations, have students model on board or dry erase boards or share with partners.

Sample resources

Navigating through Algebra in Grades 6–8 – Available from NCTM. Contains additional lessons for algebraic activities.

www.brainpop.com/math/algebra/equationswvar/index.weml – Interactive Web site for students using algebraic concepts.

www.mathgoodies.com/lessons/vol7/equations.html – Lessons on writing algebraic equations.

http://math.rice.edu/~lanius/Lessons/calen.html – An algebraic activity that uses the calendar.

www.matti.usu.edu – A library of virtual manipulatives and lesson plans.

http://illuminations.nctm.org/lessonplans/6-8/PoolTable/index.html – Analyzing numeric and geometric patterns of Paper Pool .

Sample released test items

Which means "6 times a number minus 7 is 5 more than 4 times that number"?

F $6n - 7 = 4n + 5$
G $6(n - 7) = 4(n + 5)$
H $7 - 6n = 5 + 4n$
J $6(4n + 5) = 7$

Which means "6 less than 5 times a number is 4 more than 3 times that number"?

A $5n - 6 = 3n + 4$
B $5(n - 6) = 3(n + 4)$
C $6 - 5n = 4 + 3n$
D $5(3n + 4) = 6$

Working together, Joy and Steve collected 39 pounds of aluminum cans for recycling. If Joy collected j pounds, which of the following shows the number of pounds collected by Steve?

F $j + 39$
G $j - 39$
H $39 - j$
J $39\,j$

Hilary has $9 less than Barbara. Together they have $21. If x represents Barbara's money, which of the following expresses this relationship?

A $(x + 9) + x = 21$
B $(x - 9) + x = 21$
C $x - 9 = 21 + x$
D $x = 21 + x - 9$

twice the number of books divided by fifteen

Which represents the phrase in the box?

A $\frac{15n}{2}$

B $\frac{2n}{15}$

C $\frac{2 + n}{15}$

D $\frac{n}{2(15)}$

Which of the following is *not* true?

A $5x - 7 = 21$ is an expression.

B $4a + 16$ is an expression with 2 terms.

C In the expression $7a + 2b$, the coefficient of b is 2.

D $5x + 7y$ is an expression with two variables.

Which of the following is *not* true?

A $3x - 8$ is an expression with one variable.

B $6x + 2y - 7$ is an expression with 3 terms.

C In the expression, $4x + 6y$, the coefficient of x is 4.

D $5x + 4 = 39$ is an expression.

$$3y = 7x - 9$$

Which best describes the circled portion of the equation?

A expression
B variable
C term
D coefficient

Which of these is an inequality?

F $4x = 5y$

G $3x - 6 = 12$

H $x^2 - 3x + 4$

J $3x < x - 2$

What is the solution to $2x + 5 < 17$?

F $x < 6$
G $x < 11$
H $x < 12$
J $x < 24$

What value of x satisfies the following?

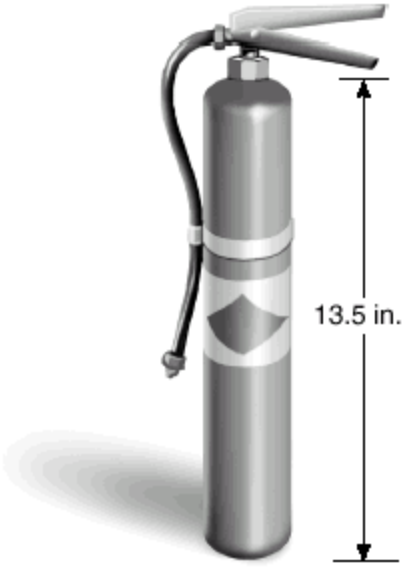
$$4x + 12 = 100$$

F 13
G 22
H 28
J 37

Janna has driven 200 miles. If she averages 50 miles per hour, how many *more* hours must she drive before she has driven a total of 500 miles?

A 3
B 6
C 12
D 14

The cylindrical cannister of this fire extinguisher has a radius of 2.5 inches and is 13.5 inches high.



Which is *closest* to the number of cubic inches it will hold when filled?

A 1,060
B 265
C 212
D 115