



Patterns, Functions, and Algebra For Elementary School Teachers

**A Professional Development Training
Program to Implement the 2001
Virginia Standards of Learning**

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**Office of Elementary Instructional Services
Virginia Department of Education
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Introduction

The *Patterns, Functions and Algebra for Elementary School Teachers* is a staff development training program designed to assist teachers in implementing the Virginia Standards of Learning for mathematics. This staff development program provides a sample of meaningful and engaging activities correlated to the Patterns, Functions and Algebra strand of the grades K-5 Mathematics *Standards of Learning*.

The purpose of the staff development program is to enhance teachers' content knowledge and their use of instructional strategies for teaching in the patterns, functions and algebra strand of the K-5 Mathematics *Standards of Learning*. Teachers will receive intensive training in ways to develop student understanding of patterning, functional relationships and the foundations of algebraic thinking. Through explorations, problem solving, and hands-on experiences, teachers will engage in discussions and strategies to guide instruction and classroom assessment. Elementary teachers will work to develop techniques to help students:

- recognize, construct, extend, create, analyze, generalize, and describe patterns;
- use pattern-based thinking to understand and represent mathematical and real-world phenomena;
- develop categorization and classification skills;
- determine mathematical rules and develop an understanding of functional relationships;
- use tables, rules, variables, open sentences, and graphs to describe patterns and other relationships;
- model real world situations by representing data in tables, pictures, graphs, open sentences, equations or inequalities, rules, and functions;
- develop strategies for evaluating expressions and finding the solution to equations and inequalities; and
- form and verify generalizations based on observations of patterns and relationships.

Through these activities, it is anticipated that teachers will develop new techniques that are sure to enhance student achievement in their classroom.

Designed to be presented by teacher trainers, this staff development program includes directions for the trainer, as well as the black line masters for overhead transparencies and handouts. In some instances, related student activities are included. Trainers should adapt the materials to best fit the needs of their audience, adding materials that may be more appropriate for their audience and eliminating materials that have been used in previous training sessions. Trainers are encouraged to use technology, as appropriate. All materials in this document may be duplicated and distributed as desired for use in Virginia.

Patterns, Functions, and Algebra



The training programs are organized into five three-hour modules that may be offered by school divisions for recertification points or for a one-credit graduate course, when university credit can be arranged.

The *Patterns, Functions and Algebra for Elementary School Teachers* training program is being provided to school divisions through an appropriation from the General Assembly and in accordance with the Virginia Department of Education's responsibility to develop and pilot model teacher, principal, and superintendent training activities geared to the Standards of Learning content and assessments, and to technology applications.



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GLOSSARY

<i>Additive Inverse</i>	A number's opposite. When the number and it's opposite are added together the sum is zero. Example: $2 + (-2) = 0$.
<i>Arithmetic sequence</i>	A sequence where the difference between consecutive terms is always the same. Example: 3, 6, 9, ...
<i>Common factor</i>	If a number is a factor of two or more numbers, it is a common factor of that set of numbers.
<i>Common multiple</i>	A number that is a multiple of each of two or more given numbers. Example: 24 is a common multiple of 4 and 3.
<i>Composite number</i>	A whole number greater than 1 that has more than two factors.
<i>Constant</i>	A quantity whose value does not change.
<i>Cube (in numeration)</i>	The third power of a number. Example: 8^3 is read "Eight to the third power."
<i>Equation</i>	A mathematical statement in which two expressions are equal. Example: $x - 10 = 6$
<i>Exponent</i>	A number telling how many times the base is used as a factor. Example: $8^3 = 8 \times 8 \times 8$, where 3 is the exponent and 8 is the base.
<i>Expression</i>	A mathematical phrase made up of variables and/or numbers and symbols. Example: $3x + 4$
<i>Factor</i>	A whole number that divides another whole number without leaving a remainder. Example: 8 is a factor of 48.
<i>Formula</i>	A rule showing relationships among quantities. Example: $A=bh$
<i>Function</i>	A rule that matches two sets of numbers such that for each first number there is only one possible second number according to the rule.
<i>Geometric Sequence</i>	A sequence where the ratio between consecutive terms is always the same. Example: 3, 6, 12, ...
<i>Greatest Common Factor (GCF)</i>	The largest factor two numbers have in common. Example: 6 is the GCF of 24 and 18.
<i>Inequality</i>	A statement that two expressions are not equal. Example: $x + 2 \geq 6$

Patterns, Functions, and Algebra



<i>Inverse operations</i>	Operations that “undo” each other, such as addition and subtraction.
<i>Least Common Denominator (LCD)</i>	The least common multiple (LCM) of two or more denominators.
<i>Least Common Multiple (LCM)</i>	The smallest common multiple of two numbers. Example: 56 is the LCM of 8 and 14.
<i>Multiple</i>	The product of a given number and another whole number. Example: 21 is a multiple of 3 (and 7) because $3 \times 7 = 21$.
<i>Order of operations</i>	Rules describing the sequence to use in computation: 1) compute within grouping symbols; 2) compute exponents and/or roots; 3) multiply and divide from left to right; 4) add and subtract from left to right.
<i>Perfect square</i>	The square of a whole number.
<i>Positive numbers</i>	Numbers greater than zero.
<i>Power</i>	An exponent.
<i>Prime factorization</i>	Writing a number as a product of prime numbers. Example: $30 = 2 \times 3 \times 5$.
<i>Prime number</i>	A whole number greater than 1 whose only factors are 1 and itself. The first five primes are 2, 3, 5, 7, and 11.
<i>Proportion</i>	A statement showing two equal ratios.
<i>Radical sign</i>	$\sqrt{\quad}$ used to represent a square root.
<i>Reciprocals</i>	Two numbers whose product is 1. Example: $5/7$ and $7/5$ are reciprocals.
<i>Solutions of an equation or inequality</i>	Values of a variable that make an equation or inequality true.
<i>Solve</i>	To find the solutions of an equation or inequality.
<i>Square root</i>	The length of the side of a square with an area equal to a given number. Example: 6 is the square root of 36 because $6 \times 6 = 36$.
<i>Substitute</i>	To replace a variable with a known value.
<i>Term</i>	One number in a sequence.
<i>Variable</i>	A quantity whose values may vary.
<i>Whole number</i>	A number in the set $(0, 1, 2, 3, \dots)$