Variables
Session 3

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Variables	Patio Tiles	104	3.25, 4.22, 5.20, 5.21, 5.22	Patio Tiles Participant Sheet, Patio Tiles Possibilities Overhead Sheet	Patio Tiles overhead
	The Varying Uses of Variables`	108	5.21, 5.22	Varying Uses of Variables Activity Sheet, Concept of a Variable reprint, Sources of Confusion in Algebra	Blank overhead transparencies, overhead markers
	Open Sentences	116	1.8, 2.10, 2.26, 3.4, 5.20, 5.21	Variables in Open Sentences Activity Sheet	Scissors
	Number Magic	120	3.8, 3.10, 4.7, 4.8, 4.9		Overhead transparency, with large number 5 written in lemon juice, a large ashtray, matches
	Number Magic Explanation - Manipulatives	121	5.20		Manipulatives (raisin boxes/ color tiles, or place value rods/unit pieces
	Number Magic Explanation - Visual	123	5.20	Number Tricks Activity Sheets, Number Tricks Answer Sheet; Number Magic Answer Key, with Algebraic Column, Answer Key; Number Magic Problem #2, with Algebraic Column; Number Magic Problem #3, with Algebraic Column	Manipulatives

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Sample Assessments	Patterning and Variables	136		Patterning and Variables Activity Sheets	Sample Assessments
	Popping for Pennies	144	4.5, 4.22	Popping for Pennies Activity Sheet	Sample assessments, scoring rubric

Activity: Patio Tiles

Format: Whole group

Objective: Participants, given the visual representation of the patio problem, will generate equivalent number sentences to describe the method they used to calculate the total number of tiles needed to complete the border of a patio. The patio problem provides a meaningful context in which to generate equivalent numerical expressions and to lay the foundation for the concept of variables.

Related SOL: 3.25, 4.22, 5.20, 5.21, 5.22

Materials: Patio Tiles, Patio Tiles Participant and Possibilities Activity Sheets

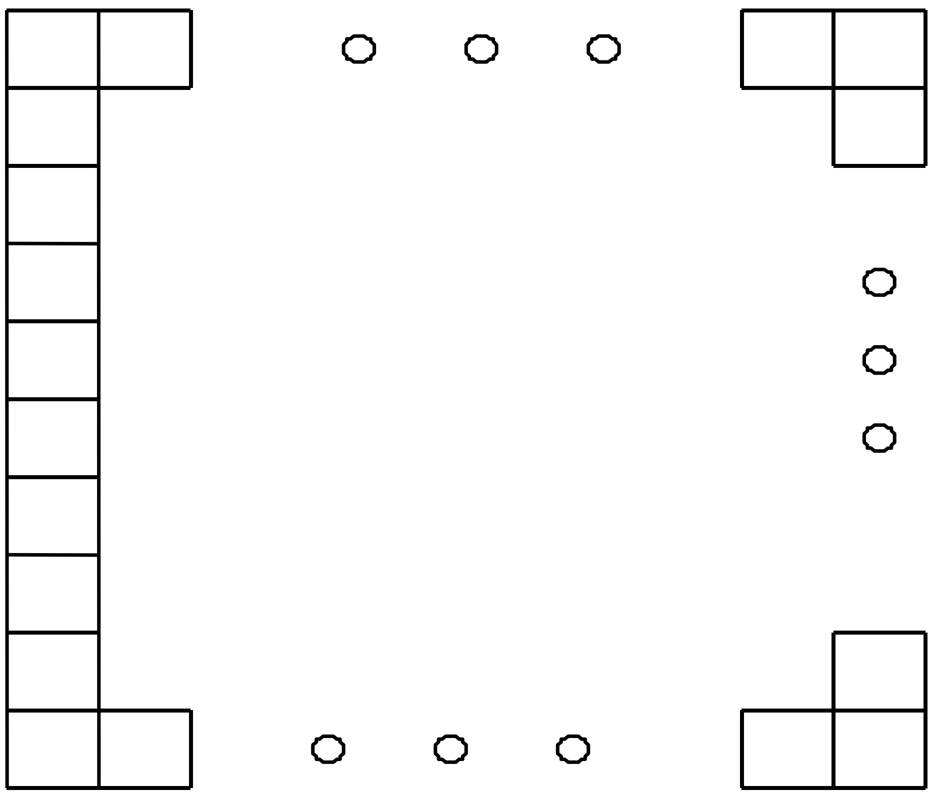
Time Required: 30 Minutes

Directions:

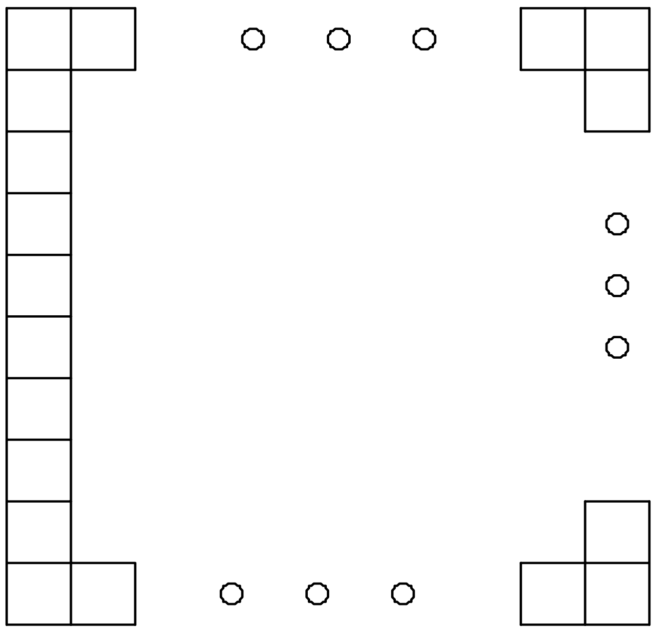
1. Distribute copies of the Patio Tiles Activity Sheet to the teachers.
2. Place the overhead of the Patio Tiles problem on the overhead and ask the teachers to write down the number sentence which represents the way in which they visualize the patio border as they work to determine the number of tiles needed to complete the border. Have teachers work individually on this problem, generating as many possible answers as possible.
3. Hand out transparencies with the printed image and select teachers to draw their picture solutions on the transparency and have them write the number sentence represented. Participants should present these to the other participants.
4. Display the overhead of the Possibilities Sharing equivalent expressions connecting the expressions of various participants' solutions. Translate some of the responses to algebraic expressions using variables.

PATIO TILES

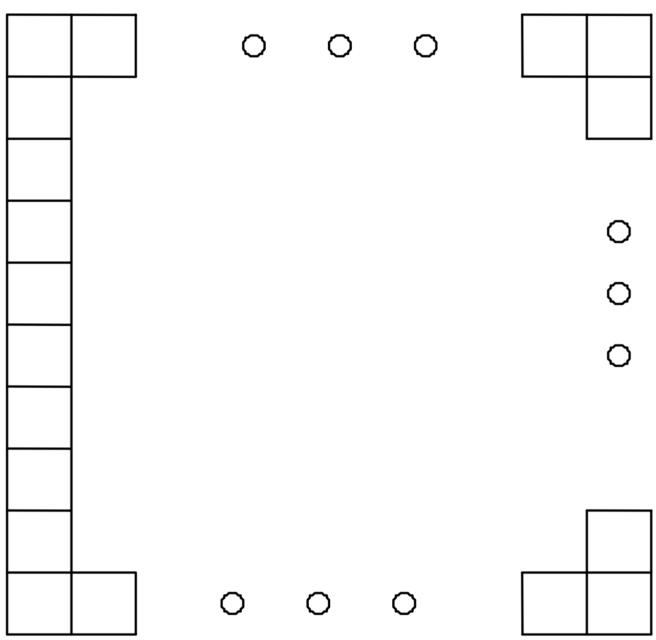
Tiles are placed to make the boundary of a square patio with 10 tiles on a side. How many tiles will be used to make the complete boundary of the square patio? Solve the problem in as many different ways as you can.



PATIO TILES



Numerical Expression: _________



Numerical Expression: _________

PATIO TILES

Answer: 36 tiles are needed to make the square

Number Sentences Representing the Approach Used to Calculate the Total Number of Tiles in the Square	Approaches to Calculating the Total Tiles in the Square
counting one-by-one to get 36	Counting
10 + 9 + 9 + 8 = 36	
(2 x 10) + (2 x 8) = 36	Grouping
(4 x 8) + 4 = 36	
(4 x 3) + (4 x 6) = 36	
2 x (10 + 8) = 36	Symmetry
(2 x 17) + 2 = 36 *...(17 = 9 + 8; plus two corners)*	
4 x { 8 + (2 x 1/2)} = 36 *...(half corners)*	
(4 x 10) - 4 = 36 *...(minus the corners)*	Generalization
4 x 9 = 36	

Activity: The Varying Uses of Variables

Format: Small groups

Objective: Participants will develop an understanding of the three ways in which a variable is used-- as specific unknowns, as generalizers, and as varying quantities. The first activity in this workshop provides an opportunity for participants to brainstorm about the various uses of variables and ways to address confusion that participants have about these uses.

Related SOL: 5.21, 5.22

Materials: Blank overhead transparencies, overhead markers, Varying Uses of Variables Activity Sheet, Concept of a Variable reprint, Sources of Confusion in Algebra

Time Required: 20 minutes

Directions:

1. Ask each group to generate examples of the different ways that a variable may be used. Have them write them on an overhead and a have group leader present them to the class.
2. Have participants read the reprint of Concept of the Variable (reprinted information from the Partners in Change Project at Boston University).
3. Use the ideas presented by the small groups to summarize the three types of varying uses of variables that are described in the article. Use the transparency of “The Varying Use of Variables” to guide the discussion.
4. Have participants discuss the points on a transparency of “Some Sources Of Confusion In Algebra”.

Concept of the Variable

One of the most important ideas in algebra is the concept of variable. Historically, algebra has progressed through three major stages, each defined by the concept of variable prevalent during that period. Algebra in its earliest stage did not include symbols but rather used ordinary language to describe solution processes for certain types of problems. The second stage (ca. AD 250-1600) included the use of symbols to represent unknown specific quantities with the goal of solving problems for the unknowns. In the third stage (1600-present), symbols have been used to express general relationships, to prove rules, and to describe functions.

Francoise Viete (1540-1603) was the first to use letters in formal mathematics notation. Shortly thereafter, Rene Descartes used letters in a more systematic way: *a, b, c* for constants and *x, y, z* for the unknowns. When Descartes went to have his manuscript, *La Geometrie*, published in 1637, the printer found that Descartes was using too many of certain letters, so much so that the printer was running out of some of the type-set letters. He communicated with Descartes, asking if it mattered which letters represented the unknowns. Descartes replied that the specific letter was unimportant, as long as the unknown was represented by *x, y, z*. The printer, Jan Maire of Leyden, Holland, had plenty of the letter *x* on hand, so he used *x* to represent the unknown. Thus, Maire's decision contributed to the fact that we now "solve for *x*" in algebra!

As the concept of variable developed historically, uses of letters expanded. Experts categorize variables in different ways, but there is general agreement that any particular use of a variable is determined by the mathematical context. The most common uses of letters are as specific unknowns, as generalizers, and as varying quantities.

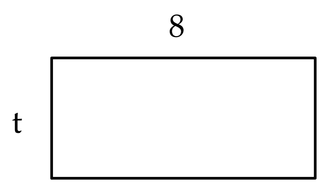
In recent decades, mathematics education researchers have begun investigating participants' usage of variables. Research by Dietmar Kuchemann (1978) on the use of letters in equations is presented for your information. Kuchemann reserved the term *variable* for letters that represent varying quantities.

- *Letter evaluated*. When a letter is evaluated, it is immediately (often mentally) replaced with a number. For example, in a + 5 = 8, participants would solve the equation by repeatedly substituting values for a (perhaps using trial and error) until a value of 8 was produced on the left side of the equation.

- *Letter ignored*. When a letter is ignored, computations are performed on the numbers in an equation as if avoiding the letter. For example, to solve 12a + 20 = 44, participants would use "undoing" or inverse operations (subtract 20 from 44 and divide the result by 12). Although the participants don't actually ignore the letter, they do "work around it." Participants who ignore the letter can solve equations such as $ax + b = c$ but cannot solve equations such as $ax + b = cx + d$ with letters on both sides.

- *Letter as object*. When a letter is used as an object, it is manipulated as a "thing". Participants often manipulate letters when solving equations by combining similar terms (e.g., $3x + 2x = 5x$). When using algebra tiles to model polynomial operations or equations solutions, participants manipulate x tiles and x^2 tiles as objects. The fact that the letter x represents a specific unknown number does not have to be considered consciously in order to manipulate the symbols and correctly solve an equation.

- *Letter as specific unknown*. In this case, letter refers to a specific but unknown number. Participants are able to work with letter in an equation to represent the relationships between quantities without first replacing the letter with a particular numerical value. For example, participants could represent the area of the rectangle below by 8t, with the knowledge that t represents some specific amount.



  Familiar responses from participants such as, "I can't do it because I don't know both sides" and "There isn't an answer because you don't know what it is" reflects participants' struggles with the variable in this situation.

- *Letter as a generalizer*. When a letter is used as a generalized number, it represents the values of a set of unknown numbers rather than a single value. Letters in identities or properties such as $1 \times a = a$ or $1 = n (1/n)$ convey relationships that are always true about multiplication of real numbers.

- *Letter as varying quantity*. In this case, the letter takes on a range of values and has a systematic relationship with another letter, as in a function of formula (e.g., $y = 2x$). Letters in a formula represent varying numbers that in the stated equation explain a real-world relationship. For example, the equation $F = 32 + (9/5)C$ represents the relationship between Fahrenheit and Centigrade measures of temperature.

Kuchemann identified his first three categories of usage of variable (evaluated, ignored, as an object) as being the most elementary. At this level, participants may be able to get the correct answer to a symbolic equation without really thinking of the variable as representing a number. Such limited understanding of variables may be the cause of participants' inability to identify or interpret the variable in problem solutions, or to generate equations to represent relationships presented in prose.

When participants can use variables as specific unknowns, generalized numbers or varying quantities, they are exhibiting what Kuchemann considers a higher understanding of the concept of variable. They are able to accept the lack of closure of not knowing what number(s) are represented by a specific variable. Participants reach Kuchemann's highest level of use of variables-as varying quantities-when they are able to treat variables in many ways (object, specific unknown, generalizer, and varying quantity) based on the context and complexity of the situation.

In the process of learning algebra, participants often are unaware of or confused by the different meanings for letters in equations, inequalities, and functions. For example, if participants understand variables only as representations of specific unknowns, it not surprising that they have difficulty interpreting inequalities where variables represent sets of numbers.

Reprinted information from the Partners in Change Project at Boston University

The Varying Uses of Variables

Variable As Specific Unknowns:

- When Variables Are Immediately Evaluated

When a variable refers to a fixed but unknown quantity with only one possible value that is often immediately evaluated and replaced with a number.

Example: $t + 2 = 6$

- When Variables Are Ignored

When computations are preformed on the numbers in an equation as if avoiding the letter.

Example: $12a + 20 = 44$

$12a = 44 - 20$

$a = 24/12 = 2$

- When Variables Are Used As Objects

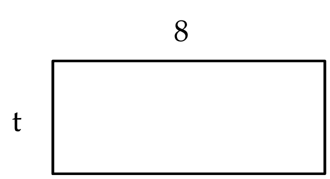
When a variable is used as an object, it is manipulated as a "thing".

Example: $3x + 2x = 5x$

- When Variables Are A Specific Unknown

When a variable refers to a specific but unknown number.

Example: area of the rectangle = $8t$



The Varying Uses of Variables

Variable As Generalizes

- When Variables Are Generalizes

When a variable represents the values of a set of unknown numbers rather than a single value.

Example: Letters in identities or properties

$1 \times a = a$

$1 = n\,(1/n)$

Variable As Varying Quantities

- When Variables Are Varying Quantities

When a variable takes on a range of values and has a systematic relationship with another letter, as in a function of formula.

Example: $y = 2x$

$F = 32 + (9/5)C$

Some Sources Of Confusion In Algebra

- Variables Often Depend Upon Their Context

- The most familiar use of variables in algebra is as a specific unknown in an equation.
  Example: 20 + 2x = 15

- A given variable represents the same number throughout an equation no matter how many times it appears.
  Example: 5x+ 2x = 14

- Sometimes variables are used as a constant.
  Example: c = 186,000 miles per second
  where c represent the speed of light

- Different variables do not always equate to different solutions. In these equations the solutions are not different.
  Example: 3m + 4 = 19
  3x + 4 = 19

Some Sources Of Confusion In Algebra

- Sometimes variables in a single equation can have different uses.

Example: the standard form of the equation of a line in coordinate geometry, $y = mx + b$.

Here, b and m represent constants specific to the equation, and y and x are covarying quantities, with x being the independent and y being the dependent variable.

- Sometimes there are multiple values for x that will make the equation true.

Example: 12 = Y * Y - Y

Solutions: Y = 4 and Y = -3 satisfy the equation

- Often in measurement, variables are used as labels.

Example: 3F = 1Y

Here, the letters serve as abbreviations for the words they represent, three feet equal one yard. The variables DO NOT mean three times the number of feet equals one times the number of yards.

Activity: Open Sentences

Format: Partner Pairs

Objective: By identifying a number to replace the variable in an "open sentence", participants create what is often referred to as a "true sentence". Participants will review various types of open sentences and identify a sequence of steps for developing the concept of open sentences in grades K - 5.

Related SOL: 1.8, 2.10, 2.26, 3.4, 5.20, 5.21

Materials: Variables in Open Sentences Activity Sheet, scissors

Time Required: 30 minutes

Directions:

1. On the Variables in Open Sentences Activity Sheet, have the participants each individually solve the simple "open sentences" to make them "true sentences".
2. Then, have the participants work in partner pairs. Ask one of the participants in their partner pairs to cut-up their handout into sixteen cards.
3. Ask the participants to determine a sequence they would use when teaching and developing the concept of "open sentences" in grades K-5. Have them group the cards into a sequence for those they would use in Grades K and 1, in Grades 2 and 3, and in Grades 4 and 5. Have participants write the letters of the cards in the sequence they would use to develop the concept of variables in open sentences for each of the grade level groupings.
4. Although there is no specific order to the set of cards, the sequence of cards should be considered in groupings of cards, earlier cards being developmentally appropriate for Grades K and 1, middle being appropriate for Grades 2 and 3, and the more difficult and complex for those in Grades 4 and 5.

One Possible Grouping:

Grades K and 1 Cards:
F, J, I, B

Grades 2 and 3 Cards:
E, A, C, H

Grades 4 and 5 Cards:
D, L, G, K, M, N, O, P

Variables in Open Sentences

A.

$\square + \square + \square = 9$

$\square + \triangle = 8$

$\square =$ ____

$\triangle =$ ____

D.

$\square + \square = 12$

$\square + \triangle + \triangle = 10$

$\square =$ ____

$\triangle =$ ____

B.

$5 + \square = 8$

$\square + \triangle = 7$

$\square =$ ____

$\triangle =$ ____

E.

$\square + \square = 8$

$\square + \triangle = 9$

$\square =$ ____

$\triangle =$ ____

C.

$\square + \square = 8$

$\square + \triangle + \triangle = 10$

$\square =$ ____

$\triangle =$ ____

F.

$\square + 3 = 5$

$\triangle + 2 = 6$

$\square =$ ____

$\triangle =$ ____

G.

$\square + \square + \square = 12$

$\square - \square + \triangle = 3$

$\square =$ ___

$\triangle =$ ___

H.

$\square + \square + \square = 15$

$\square - \triangle = 1$

$\square =$ ___

$\triangle =$ ___

I.

$\square + 4 = 5$

$\square + \triangle = 3$

$\square =$ ___

$\triangle =$ ___

J.

$\square + 3 = 8$

$8 - \square = 3$

$\square =$ ___

$\triangle =$ ___

K.

$\square + \square + \square = 15$

$\square - \triangle - \triangle = 1$

$\square =$ ___

$\triangle =$ ___

L.

$\square + \square = 14$

$\square - \square + \triangle = 5$

$\square =$ ___

$\triangle =$ ___

Variables in Open Sentences

M.

Which of these phrases describes the expression $3x - 12$?

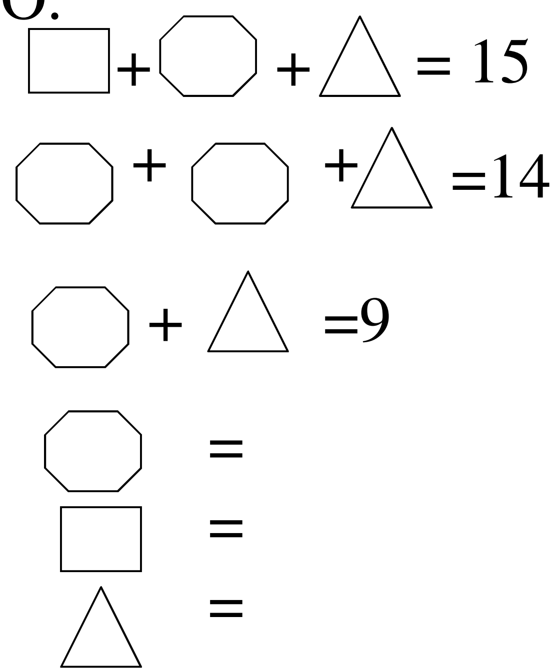
A. twelve less than three times a number

B. twelve times three less than a number

C. three less than twelve times a number

D. twelve more than three times a number

O.



N.

Solve $12 + n = 21$

A. 33

B. 12

C. 9

D. 4

P.

Evaluate $3s - t$ for $t = 6$ and $s = 4$

A. 18

B. 6

C. 14

D. 9

Activity: Number Magic

Format: Whole group

Objective: Participants will use numbers to represent mathematical expressions in a number trick.

Related SOL: 3.8, 3.10, 4.7, 4.8, 4.9

Materials: Overhead transparency with a large number 5 written in lemon juice (the overhead must be prepared ahead so that the lemon juice can dry), a large ashtray, matches

Time Required: 10 minutes

Directions:

1. Announce to the participants that you are able to read their minds and that you are going to do an activity to prove it. Have a participant go to the board or overhead projector while you position yourself so that you cannot see the participant's work. As you call out instructions, the participant at the board will work the problem.

   Here are the instructions:

   Choose any whole number.
   Add 7.
   Multiply by 2. (Double it.)
   Subtract 4.
   Divide the number by 2.
   Subtract the original number.
   Write your answer on a <u>small</u> piece of paper.
   Fold the paper so that the answer is not visible.

2. Explain that, while you know what the hidden number is (you can read their minds), you don't want to make it too easy on yourself! Take the paper and carefully set fire to it while holding it over a large ashtray. Place the burning paper into the ashtray. After the flames are out, dump the ashes onto the prepared overhead transparency and turn the overhead on. As you rub the ashes across the overhead transparency, the heat from the ashes causes the number 5 to appear.

The obvious question is "How did you do that?" The following activities lead participants through a visual and pictorial explanation.

Activity: Number Magic Explanation - Manipulatives

Format: Small group

Objectives: Participants will be able to use manipulatives to model a series of mathematical statements in the form of a number trick.

Related SOL: 5.20

Materials: Manipulatives of choice such as raisin boxes and color tiles, or place value rods and unit pieces

Time Required: 10 minutes

Directions:

1. After seeing the number trick performed, participants are eager to know how the trick works. Have the class follow the instructions of the number trick with a person choosing a number of their choice so that they see that the answer is always 5. Have participants make observations and discuss their ideas about why everyone got the same number.
2. Provide manipulatives so that they may follow along as the instructor uses manipulatives to model the instructions. The following example will use boxes and color tiles. Participants must understand that the box holds a certain number of tiles and the number of tiles in the box can vary from problem to problem. However, in a given problem, once a number of tiles has been assigned to a box, that number cannot be changed, and every box in that problem must contain the same number of tiles.

   When participants hear the instruction "Choose any whole number" they will put a number of color tiles in the box. The number of color tiles that they use is their choice. Every box used in this problem however, must have the same number of tiles in it. Tiles cannot be added to or taken away from inside the box during this problem.

   After participants are comfortable with the concept of the box taking on different values for different problems they should be encouraged to remember the value of the box rather than always putting the tiles inside. Eventually, we want them to think of the box as "some number" rather than a specific number.

3. The following is a re-enactment of the magic trick problem with a picture of the manipulatives that the participants will be using.

Choose any whole number.

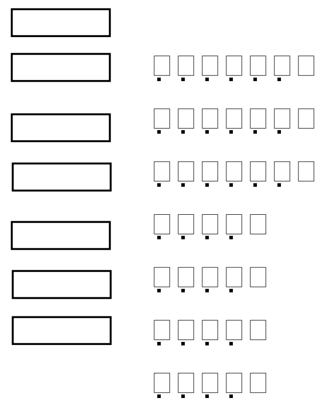
Add 7.

Multiply by 2. (Double it.)

Subtract 4.

Divide the number by 2.

Subtract the original number.



4. Participants are able to see that the box (the number they chose) is not the solution. They have a picture that shows that no matter what number you start with you will always end up with the number

Activity: Number Magic Explanation - Visual

Format: Individual

Objective: Participants will be able to draw a visual representation of a series of mathematical statements in the form of a number trick.

Related SOL: 5.20

Materials: Manipulatives; Number Tricks Activity Sheets, Number Tricks Answer Sheet; Number Magic Answer Key, Number Magic with Algebraic Column, Number Magic Answer Key; Number Magic Problem #2, Number Magic Problem #2 with Algebraic Column; Number Magic Problem #3, Number Magic Problem #3 with Algebraic Column

Time Required: 20 minutes

Directions:

1. Once participants are proficient with the modeling it is time to move to drawing visual representations of the modeling. This step is the link between the modeling with manipulatives that we did in the previous activity and the abstract algebraic method that we will use in the following activity. This is a very important step. Rectangles (▭) will be used to represent the boxes and squares (□) will be used to represent the color tiles.

2. Consider this number trick:
   Choose any whole number.
   Add the next consecutive number.
   Add 7.
   Divide the sum by 2.
   Subtract your first number.

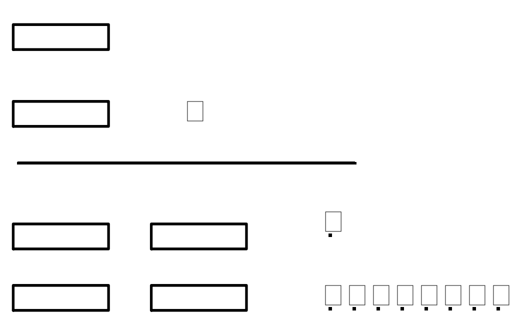
3. Have the participants model with their manipulative. Show how to draw a picture that will visually represent the actions that they have just completed. Here is the same example represented pictorially.

   Choose any whole number:

   Add the next consecutive number

   The result is:

   Add Seven



Divide by 2 □□□□

Subtract the Original Number: □□□□

4. Overhead transparencies can be made from the Number Magic charts provided. These can be useful as you show participants how to move between the visual, verbal, and numerical models.
5. Some discussion should take place concerning vocabulary. For example, consider the different ways that instructions could be given:

    Double the number and take away two.
    Take twice the number, then minus two.
    Multiple by two, subtract two.

    These activities offer teachers an interesting way to teach vocabulary. Have participants state the instructions as many different ways as they can.

6. The Number Tricks Activity Sheet gives some additional tricks to try.

Closure

Write the clues to a number trick. Show how your trick works with numbers, and picture.

Number Tricks

1. Write down any number.

   Add 4.

   Multiply by 2.

   Subtract 4.

   Divide by 2.

   Subtract your original number.

   Write down your answer.

2. Write down any number.

   Multiply by 3.

   Add 6.

   Subtract the number you originally wrote down.

   Divide by 2.

   Subtract 3.

   Write your answer.

3. Write down any number.

Add 10.

Multiply by 2.

Subtract 8.

Add your original number.

Divide by 3.

Subtract 4.

Write down your answer.

NUMBER TRICKS

1.	Write down any number.	n
	Add 4.	n + 4
	Multiply by 2.	2(n + 4) = 2n + 8
	Subtract 4.	2n + 8 - 4 = 2n + 4
	Divide by 2.	(2n + 4)/2 = n + 2
	Subtract your original number.	n + 2 - n = 2
	Write down your number.	2
2.	Write down any number.	n
	Multiply by 3.	3n
	Add 6.	3n + 6
	Subtract the number you originally wrote down.	3n + 6 - n = 2n + 6
	Divide by 2.	(2n + 6)/ 2 = n + 5
	Subtract 3.	n + 3 - 3
	Write your answer.	n
3.	Choose any number between 1 and 9.	n
	Multiply by 5.	5n
	Add 3.	5n + 3
	Multiply by 2.	10n + 6
	Choose another number between 1 and 9 and add to the product.	10n + 6 + x
	Subtract 6.	10n + x
	What is your answer?	(2 digit number where the first number chosen is the tens digit and the second number chosen is the ones digit.)
4.	Choose and number.	n.
	Add the number of days in Nov.	n + 30
	Multiply by the number of days in a school week	5(n + 30) = 5n + 150
	Subtract the number of years in a century.	5n + 150 - 100 = 5n + 50
	Double the answer.	10n + 100
	Delete the “0” in the ones place.	n + tens
	Subtract the original number.	n + tens - n = tens
	What is the result?	tens

NUMBER MAGIC for Mind Reading Trick

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
Think of a number from 1 to 10.		
Add 7.		
Multiply by 2 (double it).		
Subtract 4.		
Find ½ of the result.		
Subtract the original number.		
Write your answer.		

NUMBER MAGIC - Answer key for Mind Reading Trick

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
Think of a number from 1 to10.	7	
Add 7.	14	
Multiply by 2 (double it).	28	
Subtract 4.	24	
Find 1/2 of the result.	12	
Subtract the original number.	5	
Write your answer.	5	

NUMBER MAGIC - Mind Reading Trick with Algebraic Column

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)
Think of a number from 1 to 10.			
Add 7.			
Multiply by 2 (double it).			
Subtract 4.			
Find ½ of the result.			
Subtract the original number.			
Write your answer.			

NUMBER MAGIC - Answer key for Mind Reading Trick with Algebraic

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)
Think of a number from 1 to 10.	7		n
Add 7.	14		n + 7
Multiply by 2 (double it).	28		2n + 14
Subtract 4.	24		2n + tens
Find 1/2 of the result.	12		n + 5
Subtract the original number.	5		n + 5 - n
Write your answer.	5		5

NUMBER MAGIC PROBLEM #2

Fill in the other columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
Think of a number from 1 to 10.		
Multiply your number by 6.		
Add 12 to the result.		
Take half.		
Subtract 6.		
Divide by 3.		
Write your answer.		

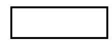
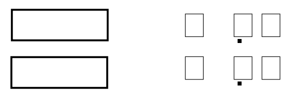
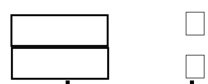
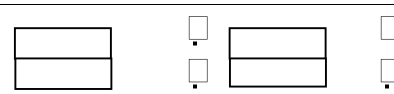
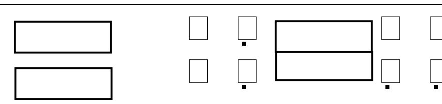
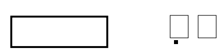
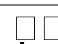
NUMBER MAGIC PROBLEM #2 WITH ALGEBRAIC COLUMN

Fill in the other columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
Think of a number from 1 to 10.		
Multiply your number by 6.		
Add 12 to the result.		
Take half.		
Subtract 6.		
Divide by 3.		
Write your answer.		

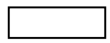
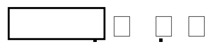
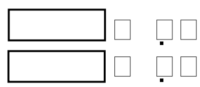
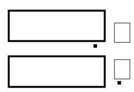
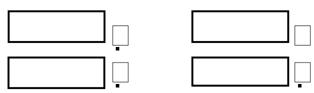
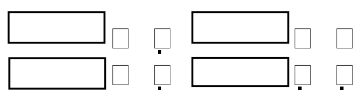
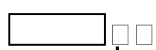
NUMBER MAGIC PROBLEM #3

Fill in the other columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)
		
		
		
		
		
		
		
		

NUMBER MAGIC PROBLEM #3 WITH ALGEBRAIC COLUMN

Fill in the other columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)
			
			
			
			
			
			
			
			

Activity: Patterning and Variables

Format: Partner Pairs

Objective: Participants will match assessments with SOL and describe the action verbs that the assessment is measuring.

Related SOL: All Standards of Learning in the Patterns, Functions and Algebra strand.

Materials: The sample assessments, Patterning and Variables Activity Sheets

Time Required: 20 minutes

Directions:

1. Distribute the handouts and have participants work in partner pairs to review the assessments.
2. Have participants determine what action is being required by the assessment - to extend a pattern, to create a pattern, etc.
3. Have participants identify the SOL that could be assessed with the assessment.
4. The following assessments have been adapted from TIMSS (Third International Mathematics and Science Study) Population 1 Item Pool.

Here is part of a wall chart that lists numbers from 1 to 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25					

Below is part of the same wall chart. What number should be in the box with the question mark inside?

43	
53	
	?

A. 34
B. 44
C. 54
D. 64

Which pair of numbers follows the rule, "Multiply the first number by 5 to get the second number?"

A.	15	3
B.	6	11
C.	11	6
D.	3	15

Here is the beginning of the pattern of tiles.

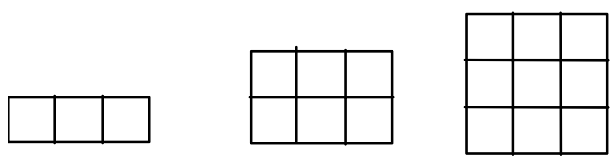


Figure 1 Figure 2 Figure 3

If the pattern continues, how many tiles will be in Figure 6?

A. 12
B. 15
C. 18
D. 21

These numbers are part of a pattern.

50, 46, 42, 38, 34, ...

What do you have to do to get the next number?

Answer: ______________________________

A. Add 4

B. Subtract 2

C. Add 2

D. Subtract 4

These shapes are arranged in a pattern.

O ∆ O O ∆ ∆ O O O ∆ ∆ ∆

Which set of shapes is arranged in the same pattern?

A. ★□★□★★□□★★□□

B. □★□□★□□□★□□□

C. ★□★★□□★★★□□□

D. □□★★□★□□★★□★

If $c - 4 = 7$, where c = the number of cars in the parking lot this morning, which of the following is true?

A. There were three cars in the parking lot this morning.

B. After 4 cars arrived, there were 7 cars left.

C. After 4 cars drove away, there were 7 cars left.

D. After 7 cars arrived, there were 4 cars left.

Tanya has read the first 78 pages in a book that is 130 pages long. Which number sentence could Tanya use to find the number of pages she must read to finish the book?

A. $130 + 78 = \square$

B. $\square - 78 = 130$

C. $130 \div 78 = \square$

E. $130 - 78 = \square$

What do you have to do to each number in Column A to get the number next to it in Column B?

Column A	Column B
10	2
15	3
25	5
50	10
10	2

A. Add 8 to the number in Column A.

B. Subtract 8 from the number in Column A.

C. Multiply the number in Column A by 5.

E. Divide the number in Column A by 5.

Activity: Popping For Pennies

Format: Small Group

Objective: The participant will discuss performance based assessments and their merits as a component of classroom assessments.

Related SOL: 4.5, 4.22

Materials: Sample assessments, scoring rubric, Popping for Pennies Activity Sheet

Time Required: 10 minutes

Directions:

1. Instruct the participants to read and complete the assessment.
2. Have the participants compare their results and discuss the scoring rubric.
3. Discuss the merits of using performance-based assessments as part of classroom assessment. How do these type of assessments support the mathematics goals of the Virginia Standards of Learning that include Mathematical Problem Solving, Mathematical Reasoning, Mathematical Communication, Mathematical Connections, and Mathematical Representation.

♦ Assessment Description:
The participant will construct a chart to compare two methods of computing salary: straight per hour rate versus a doubling method.

♦ Assessment Purpose:
To assess the participant's ability to:
- identify, describe, and continue numerical patterns;
- find decimal sums and products;
- create charts to represent the pattern; and
- write a letter of resignation in business letter format.

♦ Background Knowledge or Skills:
- recognizing and describing a pattern of adding and multiplying with money;
- creating charts;
- writing a business letter.

♦ Equipment or Materials:
- copy of task, 2 extra sheets of paper (one for charts and one for letter) and pencil
- calculator (optional)

♦ Directions for Administering Task:
- Distribute task copies and review task requirements with participants.

- Extension
  - Study the mathematical relationship between the day number and the salary as a power of 2 (use of calculator or spreadsheet optional) (e.g., Day 4= \$.08 = $2^{(4-1)} = 2^3 = 8$; Day 50 = $2^{(50-1)} = 2^{49}$).
    Participants can develop this on a spreadsheet.

- ## Scoring Rubric:

3+ Exceeds Expectations
- Salary chart organized and correctly computed for second, third, and fourth week.
- Pattern described correctly with elaboration.
- Total earnings correctly computed for both situations - doubling and minimum wage.
- Letter clearly explains resignation and follows business letter format.

3 Meets Expectations
- Salary chart organized but may contain minor errors in calculation.
- Pattern described incorrectly.
- Total earnings may contain minor errors.
- Letter follows business letter format and explains resignation.

2 Partially Meets Expectations
- Salary chart is poorly organized with frequent errors in calculations.
- Pattern is not described accurately.
- Total earnings may be inaccurate.
- Letter not in business letter format and does not explain resignation.

1 Inadequate Response
- Salary chart incomplete.
- Total earnings missing.
- Letter is only a short statement with no explanation.

0 No Response

Name:______________ Date:_______________

POPPING FOR PENNIES

You have applied for a job at a movie theater as the popcorn popper, but the owner does not want to hire you because you have never had a job. In desperation, because you need money, you offer to sell popcorn for just pennies a day. Instead of paying you the minimum wage of $4.35 per hour for the three hours you will work each day, the owner eagerly accepts your proposal and starts you at one cent the first day and agrees to pay you in the pattern established by the chart below. Here is a chart showing you how he calculated your paycheck of $1.27 for the first week.

Day	Salary
1	1¢
2	2¢
3	4¢
4	8¢
5	16¢
6	32¢
7	64¢
Week One	$1.27

A. The owner is very pleased with your work and believes he is getting a lot of popcorn popped for very little money. He quickly agrees to continue the payment pattern for another three weeks (28 days total). On a separate piece of paper, make a chart like the one above showing your wage for each day (days 8 - 28), and the total of your paycheck for the second, third, and fourth weeks. Label your work and describe the pattern.

B. What was the total of your earnings for four weeks using the pattern method of payment? ____________

C. What would the movie theater owner have paid you for four weeks of work if you had earned the minimum wage for three hours a day? __________

D. Which method of payment would have been better for the movie theater owner and why?

E. Over the past four weeks, you have made so much money you decide to resign from your job. On a separate piece of paper, write a letter to the theater owner telling him you no longer need your job and thanking him for hiring you.