FUNCTIONS
Session 4

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Function Overview		149			
Function Machines	I am a Wizard!	150	K.1, K.6, 1.8		Large cardboard box to create a magic box or screen, a variety of concrete materials (i.e., buttons, chalk, popsicle sticks)
	Fun with Function Machines	151	K.1, K.2, K.6,1.8	Fun with Function Machines Activity Sheet	Input/output cards, recording chart for the instructor, input/output mats and manipulatives for each participant
Functional Relationships	Build the Rule	155	1.4, 1.8, 2.26, 3.4, 3.25	Function Machine Activity Sheet, 10-Strip Activity Sheet	Sets of cards for the instructor, color tiles in two colors,
	Guess My Rule	157	2.26, 3.4,, 3.25 4.22	Function Machine Activity Sheet	Sets of cards for the instructor, index cards (1 per participant)
	Other Function Machines	161	2.25, 2.26, 3.24, 4.22, 5.19	Two-function Machines Activity Sheet, Fish Function Machines Activity Sheet	Calculator, overhead calculator when possible
	Toothpick Patterns	167	3.24, 3.25		Toothpicks
Graphing Functions	Graphing Patterns	169	4.18		Tennis shoes, graph paper
	Black plus White Equals Five	171	4.18		Two different colored counters, graph paper transparency
	Graph the Rule	172	4.18, 5.20	What's My Rule? Activity Sheets 1-6	Function machines, graph paper
	Figurate Numbers	180	4.21, 5.20	Square Numbers Activity Sheet, Triangular Numbers Activity Sheet, Tower of Cubes Problem Activity Sheet	Colored tiles and cubes,

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Reflections	Reflections on Functions	187	All of the Above		Blank sheet of paper

Key Idea: Function Overview

Description:

One of the most important concepts in mathematics is that of a function. A function is a relationship that pairs each number in a given set of numbers with exactly one number in a second set of numbers. While participants in the elementary grades are not exposed to the formal definition of function, participants begin to develop this pairing concept as they work with function machine models. In fact, participants become familiar with models of function machines in the real-world; for instance, when they insert a quarter in a gum machine which gives them two pieces of gum.

In the following activities, participants will explore instructional approaches to developing the concept of a function. They will start with a concrete representation of a function rule and gradually make the representations more abstract by going from physical objects to pictures to the standard representation followed by connections to their graphical representations. By initially exposing participants to concrete representations of function models, the use of the standard representation is usually an easy transition for participants.

Activity: I am a Wizard!

Format: Whole group

Objective: Participants will be able to understand the concept of an input and an output related by a rule.

Related SOL: K.1, K.6, 1.8

Materials: A magic box or a screen made out of a large cardboard box and a variety of concrete materials (i.e., buttons, chalk, popsicle sticks)

Time Required: 15 minutes

Directions:

1. Decorate a large backless cardboard box or a three-fold project screen so that it looks like a weird machine. There should be an IN slot and an OUT slot. The slots should be big enough for participants to put their hand filled with objects into the box.
2. The instructor whispers instructions to the participant such as "Take the objects that are given to you, add 1 more to them and then return the objects through the OUT slot." The participant then goes into the box to play the wizard in the Magic Box. They will need buttons, chalk, popsicle sticks, etc.
3. Another participant places some objects such as popsicle sticks into the IN slot announcing how many objects they are putting into the machine. If they put in 4 sticks, the wizard will return 5 sticks when the rule is add 1. Another classmate might put in 2 cubes and the wizard will return 3 cubes or input 3 pieces of chalk to get back 4 pieces of chalk. Continue letting participants take turns putting in various objects. When appropriate, have another participant record the inputs and outputs.

   When participants think that they know the rule, then they may not tell anyone in the class but must prove their knowledge by telling what the output will be when a certain number is input. The other participants continue taking turns until most of the participants have discovered the rule.

4. Let participants take turns being the wizard in the box. Each turn will be a different rule. The rule should be limited to adding or subtracting a small number such as 0, 1, 2, or 3. Participants, however, may wish to stump the others when used in the classroom by using more difficult rules.

Activity: Fun With Function Machines

Format: Whole group

Objective: Participants will use concrete objects to model the concept of input and output.

Related SOL: K.1, K.2, K.6,1.8

Materials: Fun with Function Machines Activity Sheet, input/output cards, recording chart for the instructor, input/output mats and manipulatives for each participant

Time Required: 20 minutes

Directions:

A function machine is a concrete approach to demonstrating relationships between numbers and serves as an initial exploration of the concept of a function. In this activity participants model the input and output numbers using manipulatives while trying to observe the rule that defines the relationship. Function machines always follow a rule.

Note: In these beginning explorations of function in the classroom, it is important to give the input numbers in sequential order. (i.e., 1, 2, 3…).

IN	OUT
1	2
2	3
3	4

1. Find the card with the number 1 on it. Show the card to the participants and ask what they see on the card. Record the number on a chart under "IN". Tell them to watch carefully as you put the number into the box (the 1 side up) and observe what number comes out.
2. Record that number on the chart under "OUT". Continue with the next card, showing the participants the card and recording the number on the chart under "IN.
3. Continue with the cards in sequence, recording the "IN" number. Ask the participants to predict what number they think will come out each time.
4. While the instructor is recording the inputs and outputs, the participants should be modeling the process with manipulatives, such as beans, junk, cubes, etc. Each participant works on a 2-column mat with labels IN and OUT. As the instructor shows the card with the 1, the participants put 1 object in the first box on their paper. When they see 2 come out, the participants put 2 objects in the square next to the first one on their mat and continue as each new card goes in and comes out.

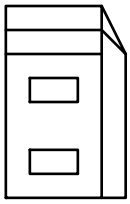
In the classroom, using this "hands on" experience will give the participants a better understanding of the concepts of more, less, before, and after as well as seeing a pattern develop. It gives meaning to the abstract numbers the instructor is recording.

5. Extension - Ask participants ahead of time to bring a half-gallon cardboard milk carton. Have participants create milk carton function machines during the staff development program.

FUN WITH FUNCTION MACHINES

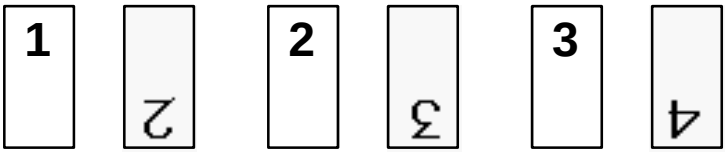
Instructions for Creating a Function Machine

Cut a slit across the top of a half-gallon cardboard milk carton about half an inch high and three inches wide. Cut another slit across the bottom. Cut a piece of tagboard 3.5 inches wide and eight inches long. Fold under about .5 inches at the top and bottom of the tagboard. This is the slide the card will travel along. The tag board should go inside the box with the folded flange taped to the outside of the top slit. The slide should curve from the top slit, touch the back of the carton and come out the bottom slit. The cards will be placed into the machine with the input number facing up. It is important that the card "flip" on its way down the slide so that the output number is facing up when the cards slides out of the box. Adjust the length of the slide if necessary before taping the bottom folded flange to the bottom slit. Decorate your function machine.



Construction of Cards

Prepare cards as shown. You will want to create about 10 cards per rule. The cards should have a notation about which is the input side. The size of the cards depends upon the size of the opening of slits of your function machine. The cards below would be appropriate for the classroom; however, cards representing more challenging functions should be made for the participants.



INPUT	OUTPUT

Activity: Build the Rule

Format: Whole group

Objective: Participants will use concrete objects to model the concept of input and output and will identify the function rule.

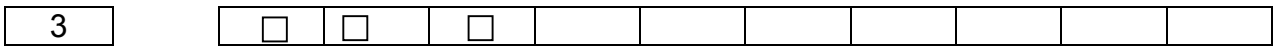
Related SOL: 1.4, 1.8, 2.26, 3.4, 3.25

Materials: Function Machine Activity Sheet, sets of cards for the instructor, color tiles in two colors, 10-Strip Activity Sheet

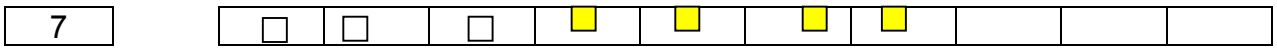
Time Required: 10 minutes

Directions:

1. The instructor shows the participants the first card. The participants build that number on their 10-strip using one color of tiles. For example, the input card is 3.
2. The instructor inserts the number into the Function Machine. When the card comes out, the instructor shows the participants the number and they build UP TO that number using the other color of tiles.



To continue the above example: The Output card says 7. So the participants add 4 color tiles in a color different to the original 3.



3. Discuss what you had to do to get the new number. This is the rule (i.e., input plus four equals output).
4. Record the input and output numbers on the chart.
5. Use more interesting rules to challenge the participants (i.e., □ X 2 + 1).
6. Discuss the value of using manipulatives to discover the rule.

IN	OUT

10-Strip to accompany Build the Rule activity

To make linear 10-strips, copy this page, and tape two strips together to create a strip of 10 squares.

Activity: Guess My Rule

Format: Whole group

Objective: Participants will solve problems involving functional relationships.

Related SOL: 2.26, 3.4, 3.25, 4.22

Materials: Function Machine Activity Sheet, sets of cards for the instructor, index cards (1 per participant)

Time Required: 15 minutes

Directions:

1. Show participants the function machine and cards.

   Point out the input slot and the output slot. Discuss the terms input and output. Explain to participants that this machine takes numbers in, performs operations on the number and then spits them out. For example, if the machine's rule is to add 3 then when the number 4 is input a 7 is output; if the number 6 is input a 9 is output; if the number 12 is input a 15 is output, etc.

2. Begin feeding cards into the machine. Record the input and output numbers.
3. Continue with additional cards until the rule is guessed.
4. When making cards you should have about 10 cards per rule. Extra cards can be put through the Function Machine to verify the rule. Make sets of cards for +2, +3, +4, +5, -2, -3, -4, -5, etc. As participants become comfortable with recording data in numerical order, shuffle the cards and use the cards out of order .
5. The Function Machines handout may be used for additional questions
6. Pass out one index card to each participant. Have participants make up a rule and write the Inputs, respective Outputs and the rule on their card. Break into partner pairs. Have partner pairs challenge another set of partner pairs to guess their rule.

Background Information:

How Partners Can Discover Rules

There are a variety of methods for participants to discover the rule in functional relationships. A few suggestions follow as to some of the ways in which participants can discover rules. Successful guessing of rules depends on experience and practice.

Looking at Differences:

If the participants cannot immediately identify a number pattern, have the participants order the numbers consecutively from least to greatest in the input box, placing the

respective numbers in the output box. Have the participants then look at the difference between the output numbers. Have the participants compare the difference between the input number and the output number. Have them do some detective work to determine what happens to a number after it goes into the function. If the number increases slightly, perhaps the participants are dealing with a "The number plus something? " situation. If the number differences are large, consider multiplication and division.

Recognizing Famous Numbers:
Participants should be familiar with various types of number such as odd number (2n-1) and the even numbers (2n). They should learn the rules for detecting whether a given number is a multiple of each of the whole numbers 3, 4, 5, 6, 8, 9, 11 (i.e., multiples of 9: the sum of the digits is always a multiple of 9, etc.).

Participants should be able to recognize the numbers related to the powers of numbers and their differences (i.e., powers of 2: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512), and that the differences between the powers of 2 are the same as the powers of 2.

Recognizing Figurate Numbers:
By fifth grade participants should be familiar with figurate numbers including triangular numbers, square numbers, etc.

Input - Output Charts

1.

Rule	
INPUT	OUTPUT

2.

Rule	
INPUT	OUTPUT

3.

Rule	
INPUT	OUTPUT

4.

Rule	
INPUT	OUTPUT

5.

Rule	
INPUT	OUTPUT

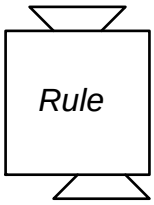
6.

Rule	
INPUT	OUTPUT

Function Machines

This is a function machine. It takes the number that you feed in, processes it and then creates the output. The processing takes place according to some rule.

Input



Output

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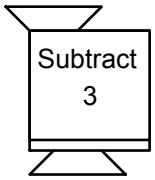
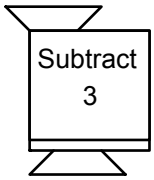
1. 4



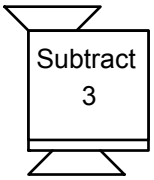
a.? =____

3



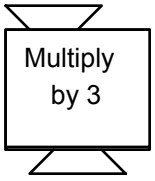
b. ? = ____

?= _____



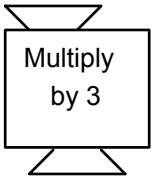
c. 9

2. 2



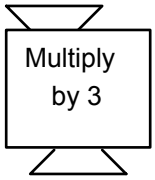
a.? =____

0



b. ? = ____

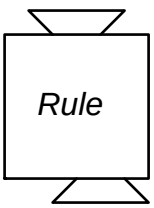
?



c. 9

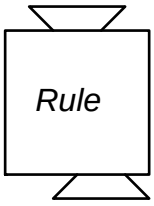
3. Find a rule that connects the inputs and outputs in problem 3. One rule fits all.

12



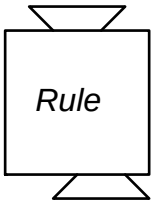
4

3



1

24



8

Rule: _____

Activity: Other Function Machines

Format: Small group

Objective: Participants will explore function machines using calculators and other function machine formats

Related SOL: 2.25, 2.26, 3.24, 4.22, 5.19

Materials: Calculator, overhead calculator when possible, Two-Function Machines Activity Sheet, Fishy Function Machines Activity Sheet

Time Required: 30 minutes

Directions:

PART I: Calculator activities (calculators with a built-in constant function can be used as a function machine).

1. For example, with the TI-108 clear the calculator, then press "+", "6", "=". Then, without touching any other keys, press "4", "=" and the display will show 10. The calculator is now a "+6" machine.
2. Hand the calculator to a participant with instructions not to touch any key until told to do so. Invite the participants to select a number. If the number is 7, instruct the participants to enter "7" and then press "=". The display will show 13.
3. The participant can continue to choose numbers to enter followed by "=" until the class predicts the rule that the calculator is using. Usually participants get the rule after only one or two examples as long as only addition and subtraction are allowed.
4. Have the participants explore functions in their groups using the calculator. Call upon participants to challenge the class to guess their function by placing the function in the calculator and then calling upon the participants for their inputs.

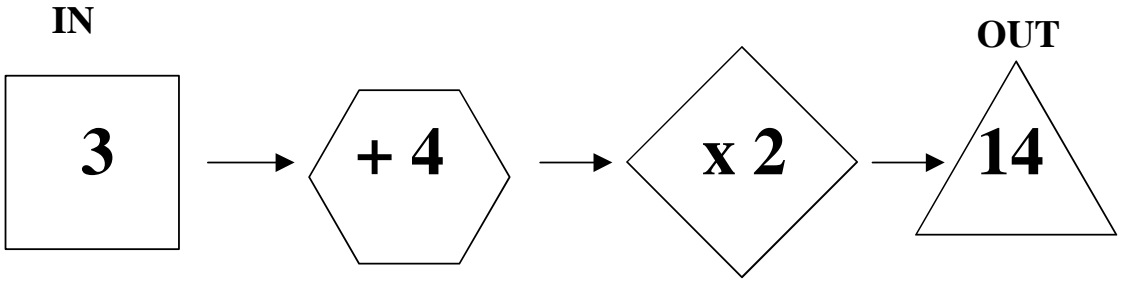
PART II: Two-Function Machines

1. Have the participants complete the Two-Function Machine Activity Sheet. They will be required to apply function rules to determine the output when given the input and to apply function rules to determine the input when given the output. Discuss.
2. Discuss what strategies to use to “undo” the operations from right to left to get the answers. (Guess and check, inverse operations, etc.)
3. Have participants develop a set of two-function machines appropriate for their students on the blank Two-Function Machine Activity Sheet. Include problems where students must “undo” the operations.

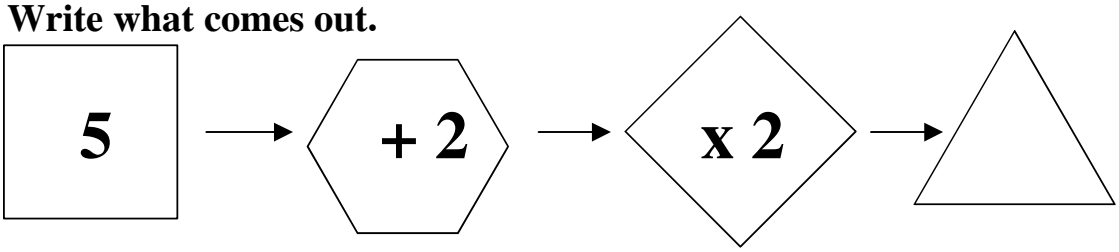
PART III: Fishy Functions

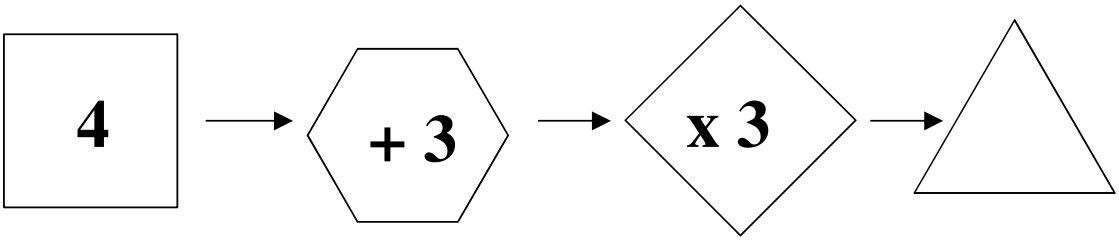
1. Have the participants deduce the function rule from the examples at the top of Fishy Function Machines Activity Sheet. Ask participants how they determine the rule and what strategies they would expect students to use to determine the function.
2. Ask participants to apply the function rule to determine the output when given the input and vice versa (determine the input when given the output).
3. Ask the participants to design a few problems on their own that follow the rule and then finally to write the rule, e.g., the fish multiplies the first number by 2 then adds the second number to that product.
4. Finally, have participants design their own “Fishy Functions” on the blank Fishy Function Machines Activity Sheet and challenge others to their “Fishy Function”.

Two-Function Machines

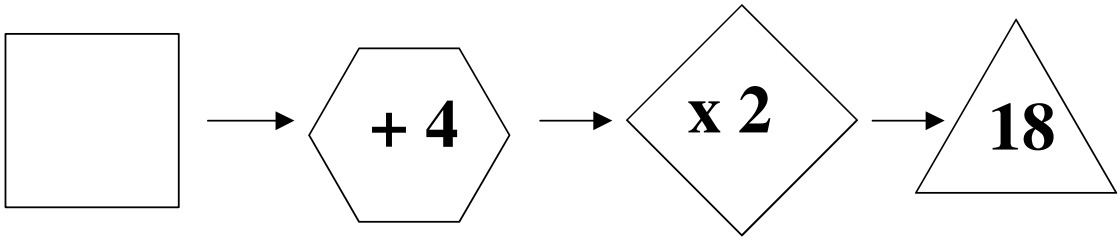


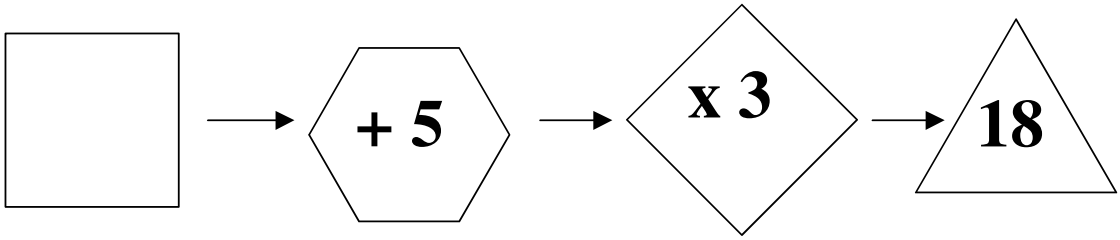
Write what comes out.



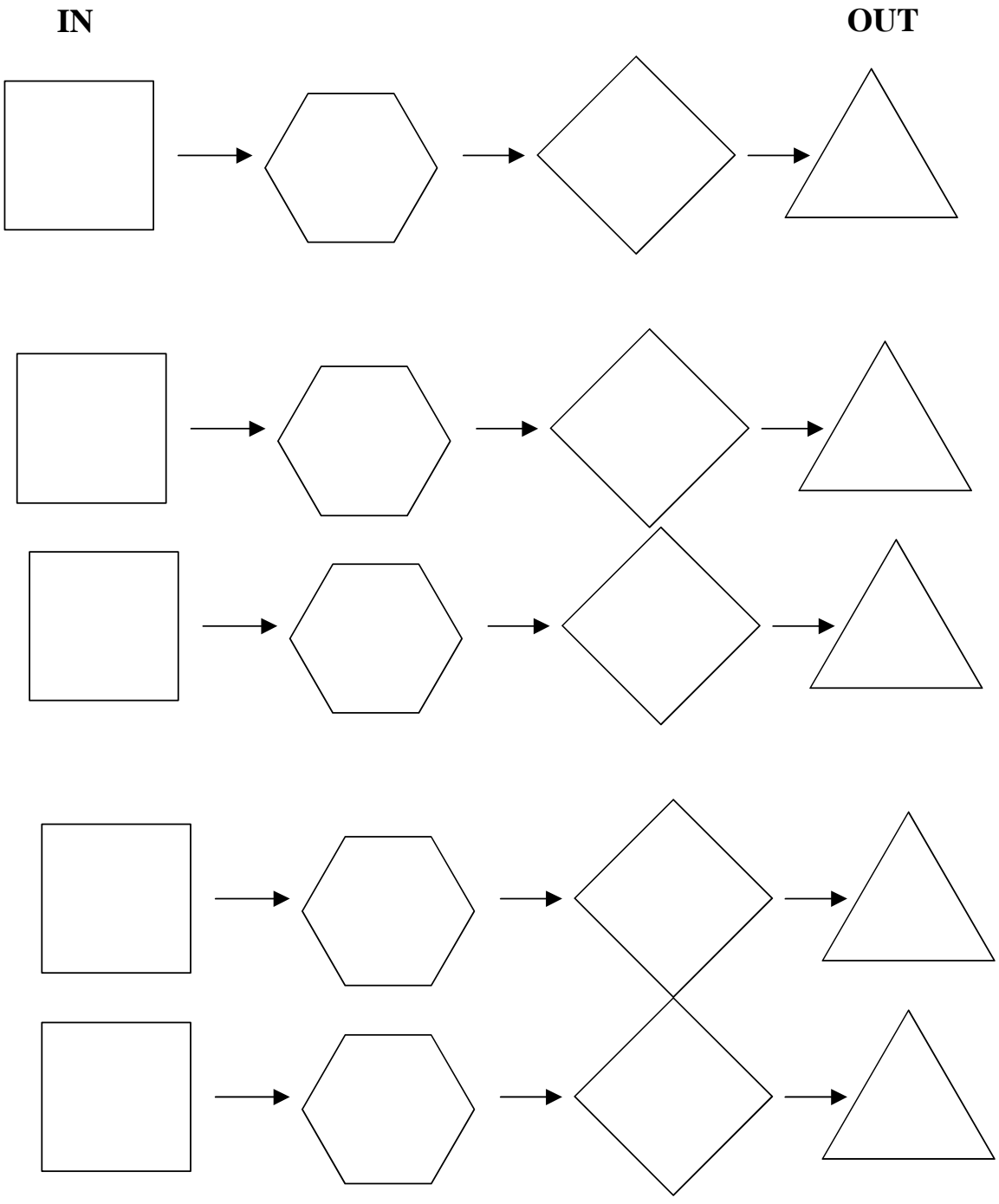


Write what goes in.

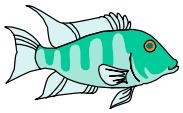




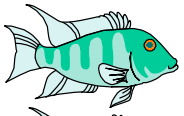
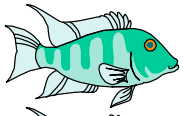
Two-Function Machines

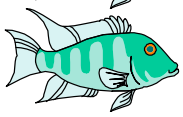


Fishy Function

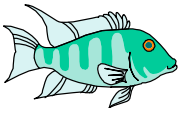
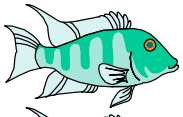
What is  doing?

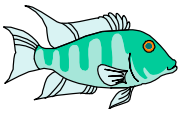
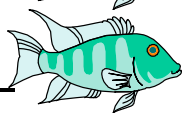
Study theses examples.

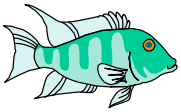
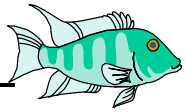
2  3 = 7 4  0 = 8

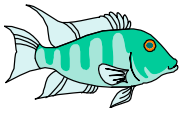
1  1 = 3 0  3 = 3

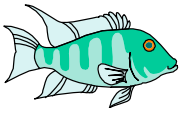
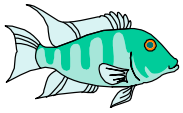
Fill in the numbers.

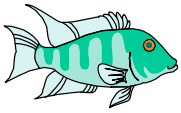
3  3 = ___ 5  1 = ___

4  3 = ___ ___  1 = 7

1  5 = ___ ___  5 = 13

Complete these  **problems using different numbers.**

_  _ = 8 _  _ = 8

Tell what  **is doing.** _____

_____

Fishy Function

What is  doing?

Study theses examples.




Fill in the numbers.






Complete these  **problems using different numbers.**




Tell what  **is doing.** _____

_____

Activity: Toothpick Patterns

Format: Whole group

Objective: Participants will be able to analyze and make predictions about patterns.

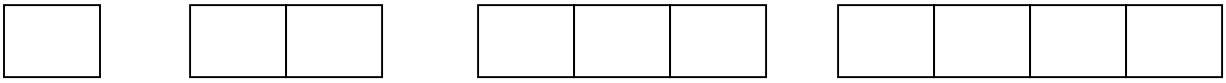
Related SOL: 3.24, 3.25

Materials: Toothpicks

Time Required: 10 minutes

Directions:

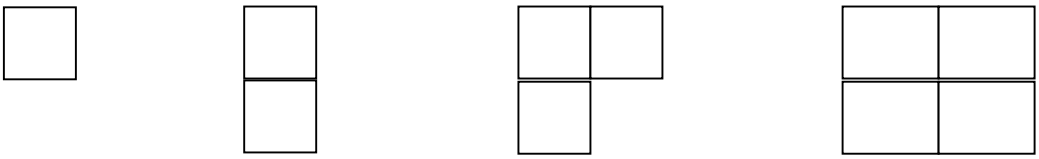
1. Distribute toothpicks so that participants may create the strip pattern.
2. The first term in the sequence is a square made with the toothpicks. The second term in the sequence is 2 connected squares. The third term is 3 connected squares and so on.



3. How many toothpicks does it take to make 5 connected squares? First organize the data in table form.

Number of squares	Number of toothpicks
1	4
2	7
3	10
4	13

4. How can you determine the number of toothpicks required to make 5 connected squares? What is the rule (e.g., (n x 3) + 1)?
5. Would the answer be the same if the squares were attached differently? What would the next term look like?



6. What would the answers be if the squares were attached to make a staircase (e.g., $n^2 + 3n$) ?
7. What about triangles? Hexagons?

Activity: Graphing Patterns

Format: Whole group

Objective: Participants will make patterns using concrete objects, represent the objects visually, organize data in a chart and then represent the pattern on a graph.

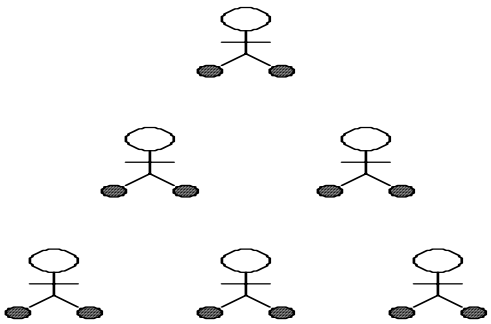
Related SOL: 4.18

Materials: Tennis shoes, graph paper

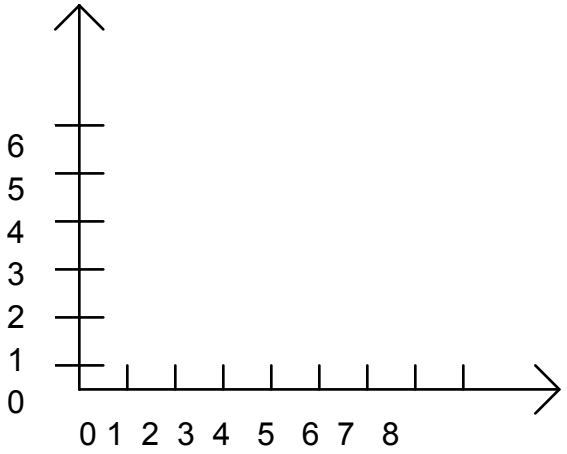
Time Required: 10 minutes

Directions:
Tennis Shoes Pattern

1. Have the participants that are wearing tennis shoes model this pattern while the instructor records the data



Participants	Tennis Shoes
1	2
2	4
3	6



2. Represent this data using ordered pair notation. (1,2) means 1 student and 2 shoes, (2,4) means 2 students and 4 shoes, etc.
3. Tell participants that in the classroom instructors should explain how to record both pieces of data by using one number line to record the number of participants and another number line to record the number of shoes. Instructors should show the participants how these number lines are connected together at right angles to give us the x-axis and the y-axis.
4. Have the participants plot the point on a coordinate graph. Label the horizontal line (x-axis) "Number of Participants" and the vertical line (y-axis) "Number of Tennis Shoes." Ask participants if they notice a pattern on the graph. Can they predict from the graph how many shoes there will be for 5 students? Is (6,16) in this pattern? Why or why not? (The second coordinate number is twice the first.) Is (5,11)? Why or why not? (All second coordinate numbers are even since they are multiples of two.)
5. If you have 20 tennis shoes, how many participants will you have? (10)
   If you have 32 tennis shoes, how many participants will you have? (16)
   If you have 8 participants, how many tennis shoes would there be? (16)
   Will the number of participants and tennis shoes ever be the same? (No, except for 0 participants, 0 tennis shoes)
   Will the number of tennis shoes ever be three times the number of participants? (No)
6. Have the participants identify relationships that will create linear functions such as participants, fingers; packs of gum, sticks of gum; buttons, number of holes in buttons. Create a list for participants to copy.

Activity: Black Plus White Equals Five

Format: Whole group

Objective: Participants will be able to graph ordered pairs.

Related SOL: 4.18

Materials: Two different colored counters, graph paper transparency

Time Required: 5 minutes

Directions:

1. Model all the possible combinations of five, using beans or cubes.

5 black 0 white	4 black 1 white	3 black 2 white	2 black 3 white	1 black 4 white	0 black 5 white

2. When this concrete information is put into a graph, the horizontal axis will represent white beans and the vertical axis will represent black beans.
3. The first situation, 0 white beans and 5 black beans, is represented by the point (0,5).

   The second situation, 1 white bean and 4 black beans, is represented by the point (1,4). The last point on the coordinate axis represents 5 white beans and 0 black beans (5,0).

4. A table could be made of this data.

number of white beans	number of black beans
0	5
1	4
2	3
3	2
4	1
5	0

5. Have participants model combinations for 6, 4, 8, 10, 9. Have them review each others work. Ask them to describe what they have learned.

Activity: Graph the Rule!

Format: Whole group

Objective: Participants will be able to represent functional relationships both graphically and in tables using ordered pairs in the first quadrant of the coordinate plane.

Related SOL: 4.18, 5.20

Materials: What's My Rule? Activity Sheets 1-6, function machines, graph paper

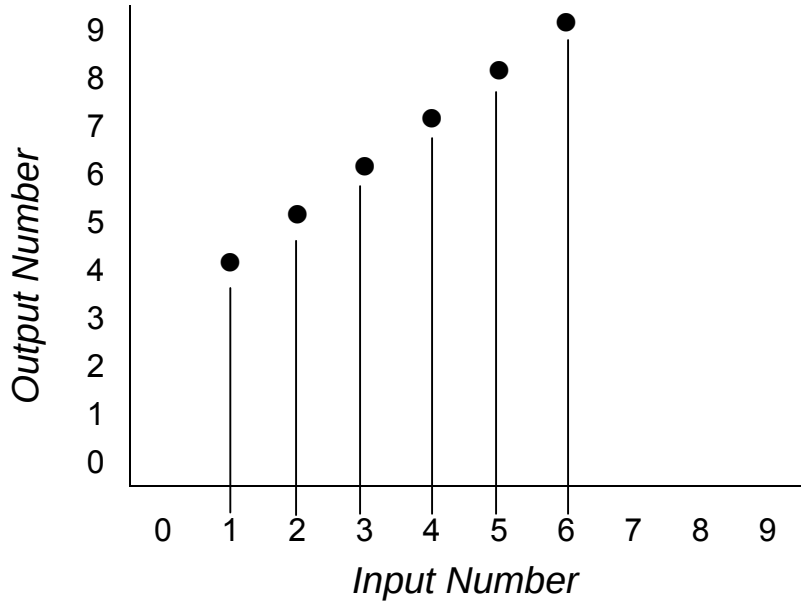
Time Required: 15 minutes

Directions:

1. Review activities with function machines, emphasizing the relationship between the input/output numbers and the rule.
2. Lead participants in an activity that graphs the rule.

   For example, if the rule is "Add 3", participants will locate the input number on the horizontal axis and then apply the rule to locate the output number vertically.

Graph of the Rule "Add Three"



3. Can you predict what the output number will be when 5 is the input number just by looking at the graph?

4. Inform participants that the Input and the Output numbers can be written as an ordered pair where the Input number is the first number and the Output number is the second number. These numbers are separated by a comma and enclosed in parentheses.

Record your findings in a table:

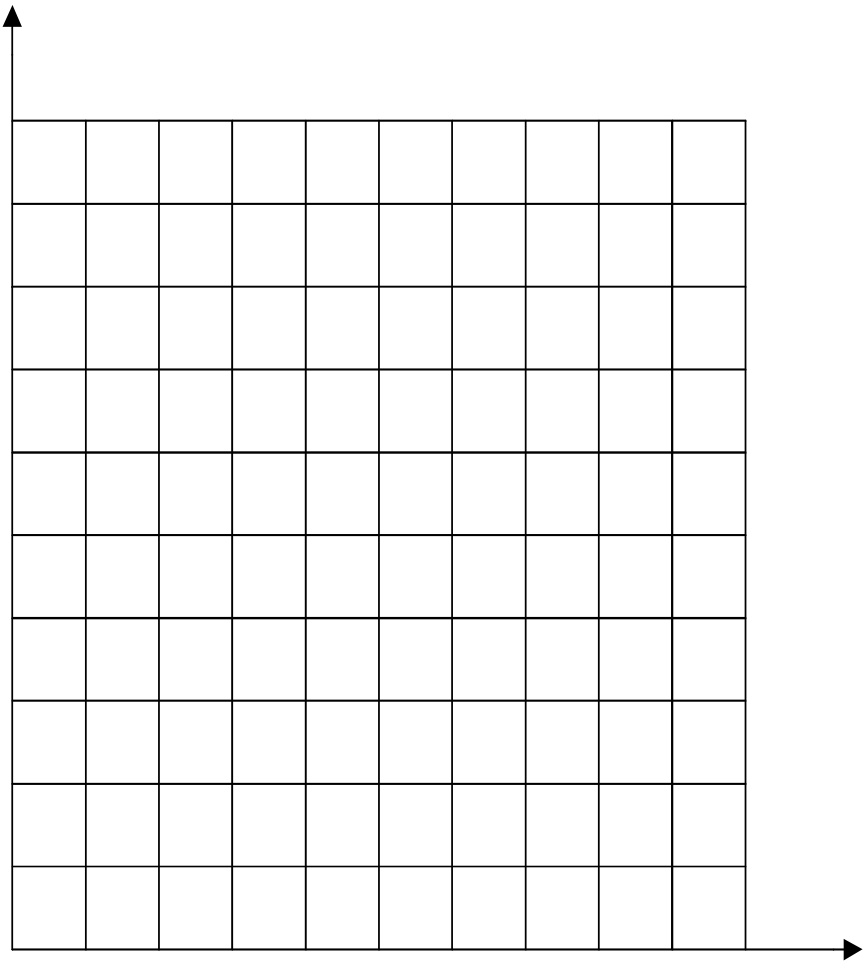
Rule: Add three to the input number to get the output number		
Input Number	Output Number	Ordered Pair (Input Number, Output Number)
1	4	(1,4)
2	5	(2,5)
3	6	(3,6)
4	7	(4,7)
5	8	(5,8)

5. Do the ordered pairs (3,5) and (5,3) follow the same rule? Are they located at the same point on a graph? Why or why not?
6. Provide several examples to participants where the instructor uses the Function Machine while the participants graph the corresponding ordered pairs. The instructor should use consecutive input numbers beginning with "1" .
7. Have participants record the input/output numbers in a table.
8. Have participants complete the tables in What's My Rule? problems #1 - #6 that follow and construct the graph for each rule. Additional problems similar to those presented can be made by participants.

WHAT'S MY RULE? #1

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule. Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	0
1	3
2	6
3	9
4	12
	21
9	
10	
12	
	42
	48
	51



WRITE THE RULE:

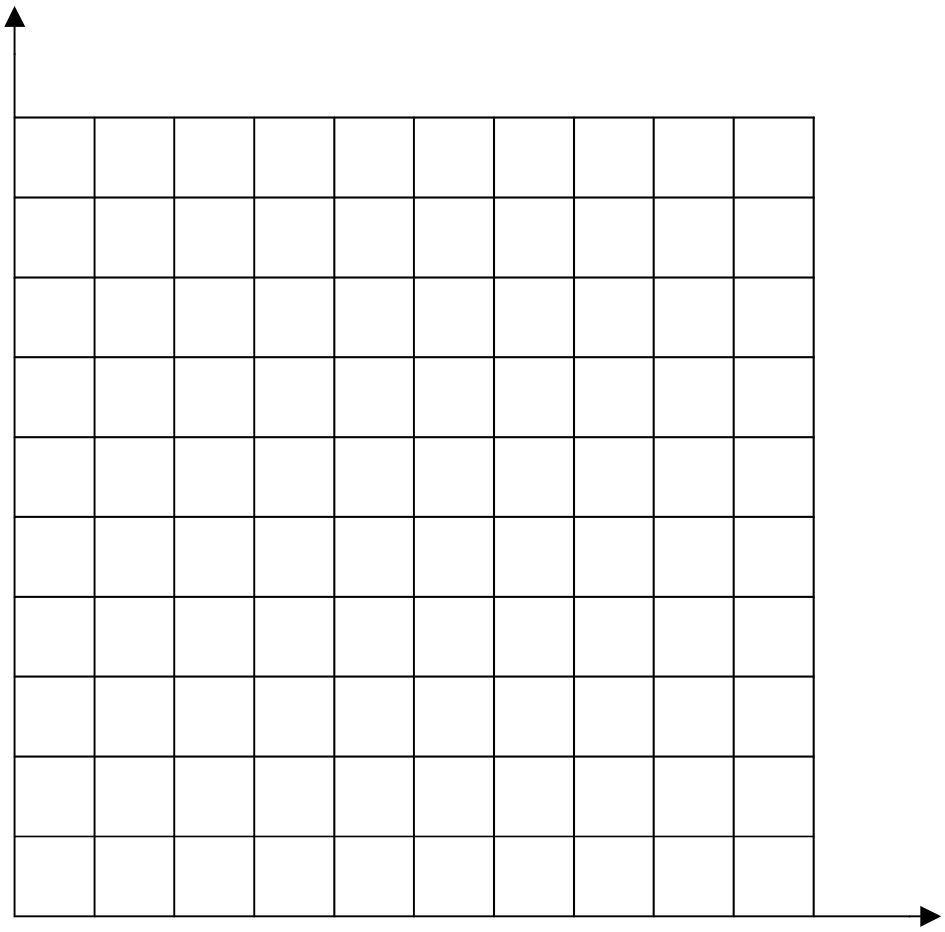
_____

_____

WHAT'S MY RULE #2

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule. Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	1
1	4
2	7
3	10
4	13
	19
9	
10	
15	
	52
	61
	151



WRITE THE RULE:

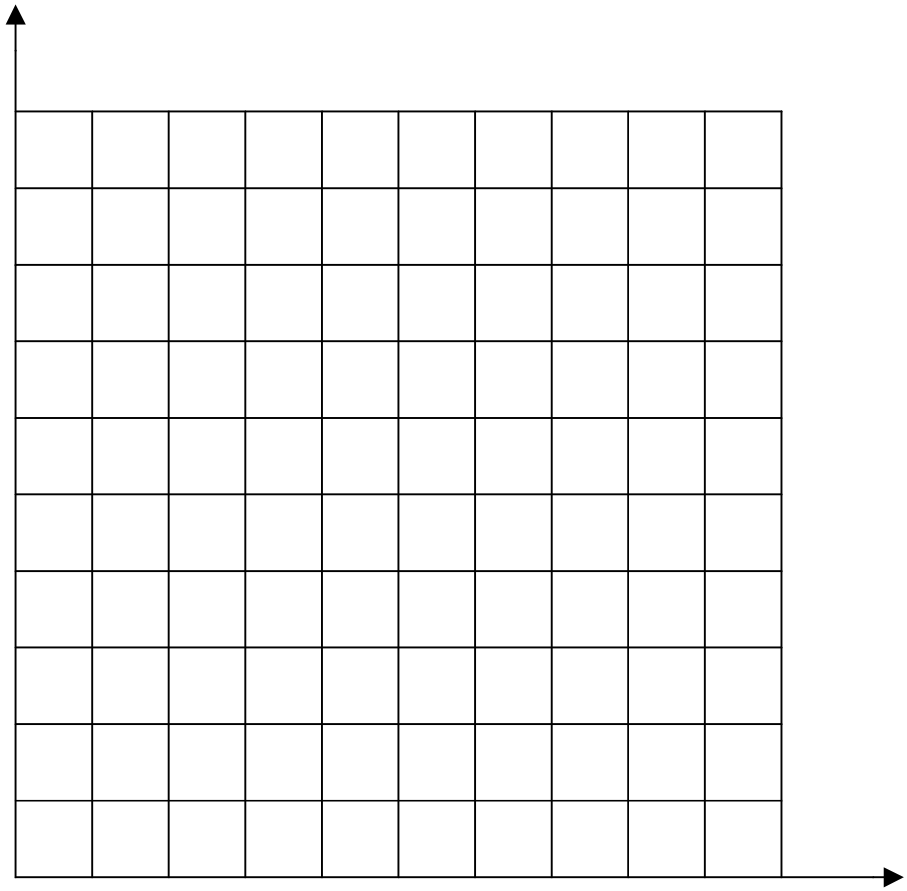
_____

_____

WHAT'S MY RULE? #3

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule. Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	2
1	6
2	10
3	14
4	18
	30
9	
10	
15	
0	94
1	126
2	162



WRITE THE RULE:

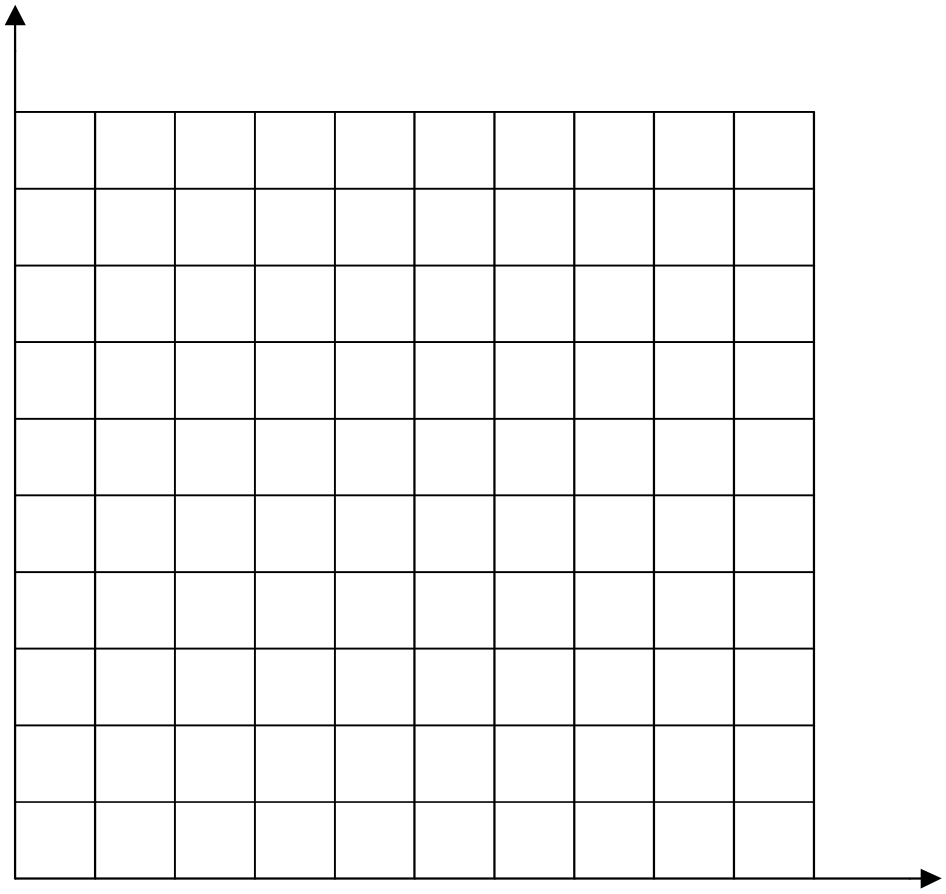
_____

_____

WHAT'S MY RULE? #4

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule. Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	0
1	4
2	8
3	12
4	18
	20
8	
9	
10	
0	80
	100



WRITE THE RULE:

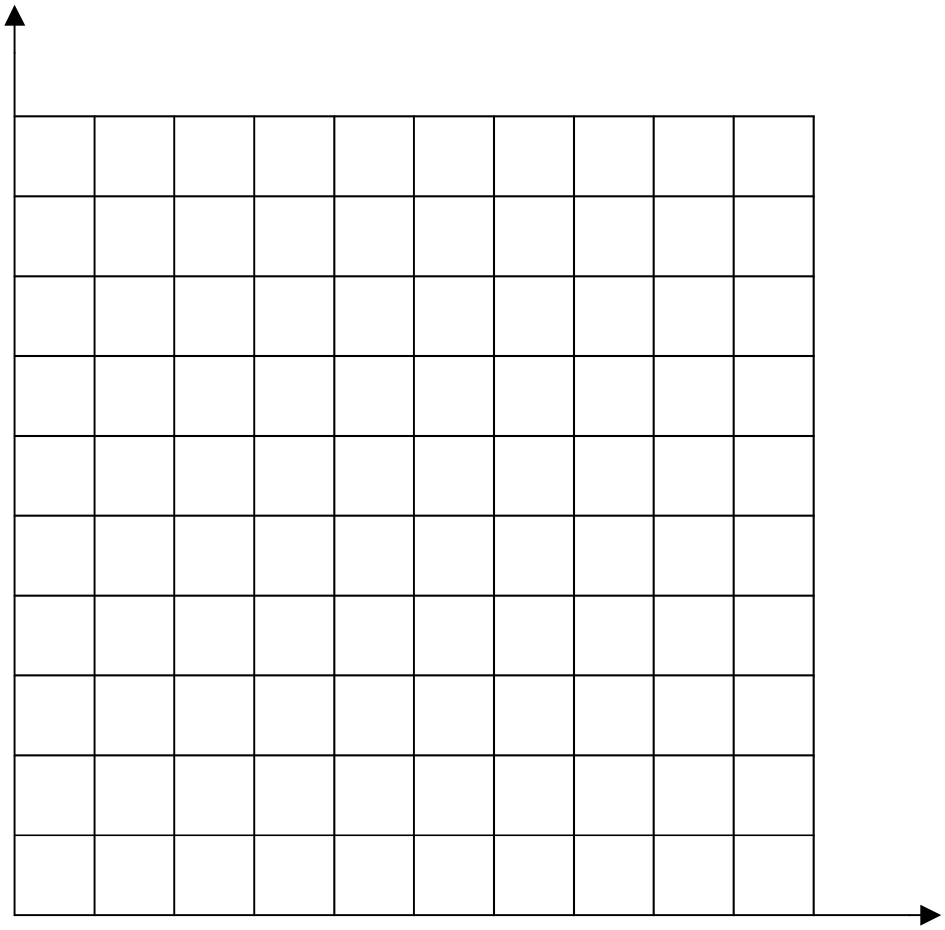
_____

_____

WHAT'S MY RULE? #5

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule.
   Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	0
1	4
2	8
3	12
4	18
	20
8	
9	
10	
0	80
	100



WRITE THE RULE:

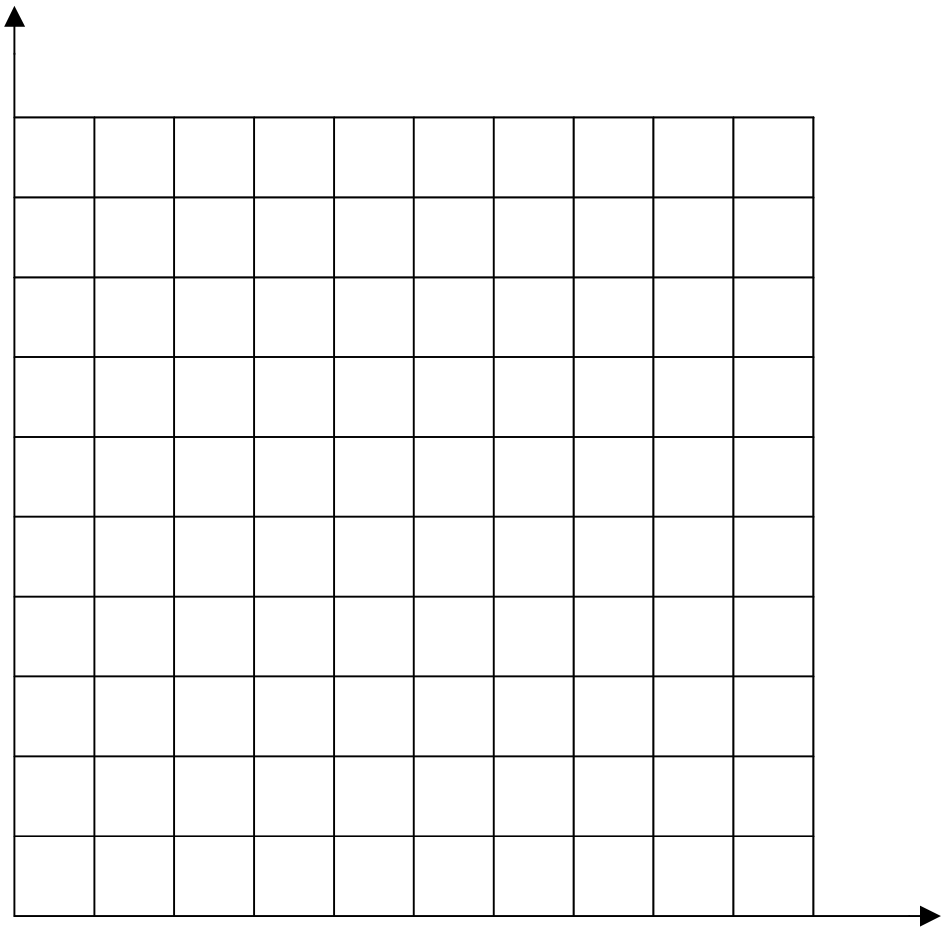
_____

_____

WHAT'S MY RULE? #6

1. The table shows some inputs and some outputs.
2. Complete the table.
3. Use words to write a rule for finding the output when you know the input.
4. Optional: Look at the graph of the rule.
   Use graph paper to graph the first few input/output numbers in the table.

Input	Output
0	
1	3
2	5
3	7
4	
	13
7	
10	
10	
	25
	81



WRITE THE RULE:

_____

_____

Activity: Figurate Numbers

Format: Partner Pair

Objectives: Participants will investigate figurate numbers, including square and triangular numbers.

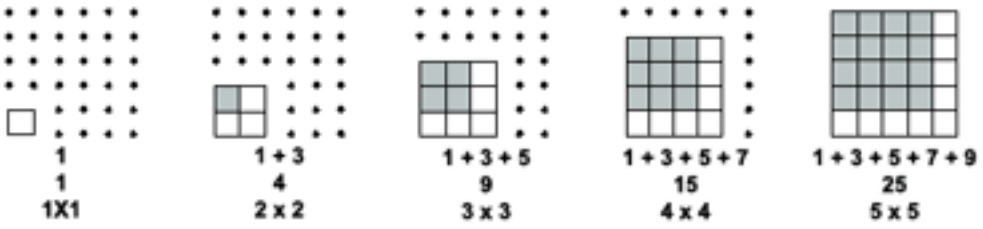
Related SOL: 4.21, 5.20

Materials: Colored tiles and cubes, Square Numbers Activity Sheet, Triangular Numbers Activity Sheet, Tower of Cubes Problem Activity Sheet

Time Required: 30 minutes

Directions:

1. For each problem, have participants work through the process of:
   a. Building the model of the pattern using manipulatives.
   b. Completing the function table.
   c. Explaining the pattern verbally and make predictions about what will happen later in the sequence.
   d. Justifying their explanations through reasoning about the structure of the relationship.
2. For example, with “Square Numbers,” participants might use color tiles or inch graph paper to show squares, beginning with a 1-by-1 square, then a 2-by-2 square, and so forth. With a set of five or six squares they can experiment with their relative size and with ways of representing the patterns they notice in the models.



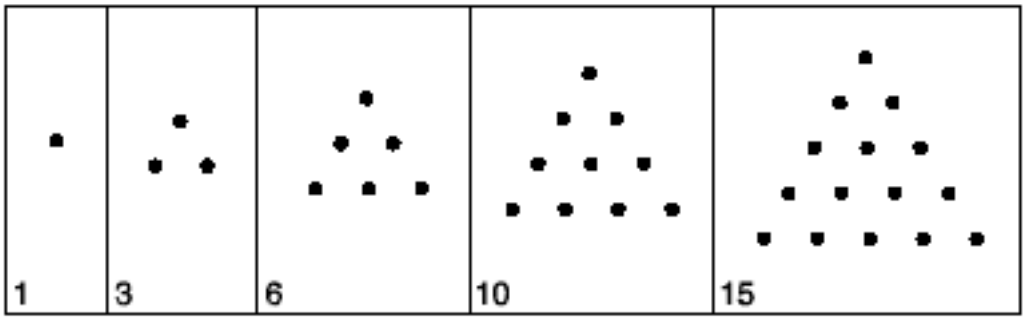
One participant might notice that a square always fits into the "corner" of the next larger square leaving an L-shaped empty space. They my verbalize this pattern and described the area of the larger square as equal to the area of the smaller plus "two sides" and one more. They may have represented their solution by writing “Area of a 5 x 5 square = area of a 4 x 4 square +4 +4 +1”.

Another participant may note that the area of the shaded square was increasing by an odd number with each new square. They may have generated the numbers for each square number by writing “Area of a 3 square = 1 + 3 + 5”

3. Discuss what participants may do for the “Triangular Numbers” problem. (Discussion from the NCTM Standards 2000)

   When students are asked to explain the relationships involved in the triangular numbers, they often pursue the development of mathematical concepts through the study of patterns. When searching for a generalization, students can consider or generate a set of specific instances, organize them, and look for a pattern. The identification of regularities underlying patterns or sequences is typically based on inductive reasoning. On the basis of observations made about portions of a pattern, conjectures can be made and then tested using inductive or deductive inference.

   Have students investigate patterns in figurate numbers such as the triangular numbers. First ask students to generate the first five numbers in the set of triangular numbers by using the visual structure of the numbers. Knowing that triangular numbers could be represented by rows of dots in the form of equilateral triangles, students progressing from the first to the fifth noticed that they added a row at the bottom of the triangle with one more dot than the bottom row in the previous triangle.



First five triangular numbers

   Secondly, students should be asked by their teacher to predict (without drawing) how many dots would be needed for the next triangular number. Reflecting on what they did to generate the sequence thus far, students will quickly concluded that the sixth triangular number would have six more dots than the fifth triangular number. Further, they see that this would seem to go on forever in the sequence of numbers. In fact, this led to a generalization about triangular numbers: Each triangular number is the sum of consecutive counting numbers. Thus, students can engage in recursive reasoning about the structure of this sequence of numbers, using the previous numbers to generate the next number. When the teacher asks the students to find the 100th term in the sequence, they knew that it would be 100

more than the 99th term. However, students will not usually know what that would be. If students are not using a computer, it is not expedient to list the first 99 terms and we would expect that students would need another way to think about the pattern.

To facilitate their work, the teacher might suggest that they make a chart to record their observations about the triangular numbers and the differences between numbers that they had detected thus far. After studying the table for a few minutes, students may see a relationship between the differences and the number. Perhaps they will note that the 2nd term is half of the product of 2 and 3, the 3rd is half of the product of 3 and 4, etc. For instance, the third term in the sequence is 6, the differences associated with 6 are 3 and 4, and 1/2 of the product (12) is 6.

Term	1st	2nd	3rd	4th	5th	6th
Number	1	3	6	10	15	21
Difference		2	3	4	5	6

The teacher may then ask the group to check to see if this method would work to find the next number. Using their original sequence and the recursive pattern noted, the students could test their conjecture for the 7th term as see if it is correct. Then the teacher may ask for a conjecture regarding the 100th triangular number. Using the conjecture, the 100th number should be (100)(101)/2. Finally the teacher may decide to press the students to think about how they might verify the generalized rule for the sequence-half the product of the term number and the next term number, or n(n + 1)/2.

To help the students understand the validity of their generalization, the teacher may have students look at a simpler, related problem. The teacher may ask students to use "little Gauss's method" to find the sum of the first ten counting numbers. That is, ask students to add matched pairs of numbers: 1 + 10, 2 + 9, 3 + 8, 4 + 7, 5 + 6, from which student can quickly see that the sum of each pair equals 11. Students would then conclude that the sum of the five pairs is (10)(11)/2 or 110/2 or 55. Next, the teacher can challenge the students to connect this problem to their work with the triangular numbers. After a few minutes, students should observe that the pattern they had first seen with the triangular numbers meant that each of these numbers was the sum of consecutive (counting) numbers. Therefore, they can see that 55 must be the 10th triangular number, since it is the sum of the first ten (counting) numbers. After some discussion, be sure that everyone sees the connections among the original recursive pattern and the triangular numbers.

4. Discuss what participants may do for the “Tower of Cubes” problem. (Discussion from the NCTM Standards 2000)

For example: “A fourth grader might make a table and decide from examining the table that multiplying the number of cubes by 4 and adding 2 yields the number of unit squares "Because it works every time."

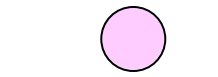
Number of cubes **n**	Surface area (square units) **f(n)**
1	6
2	10
3	14
4	18

Solution to the tower of cubes problem

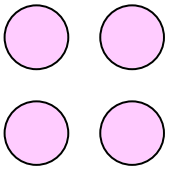
A fifth grader should be challenged to justify the rule with reference to the geometric model. For example, "It's 4 times the number of cubes plus 2 more because there are always 4 square units around each cube and one extra on each end of the tower," or "It's 5 for the end cubes, then 4 more for all the cubes in between, because the end cubes have 6 but one is covered up, and the cubes in between have 6 but two are covered up." Participants might also note the iterative nature of the pattern: "You add four to the previous number." Once a pattern is established, participants can use it to answer questions like, "What is the surface area of a tower with 50 cubes?" or "How many cubes would there be in a tower with a surface area of 242 square units?"

In this example, some participants may use a table to organize and order their data, whereas others use concrete objects (connecting cubes) to model the growth of an arithmetic sequence. Some participants may use words, whereas others use numbers and symbols to express the generalization they find. All these are important and appropriate ways of organizing and expressing ideas about function, and participants in grades 3-5 should feel comfortable moving among them.

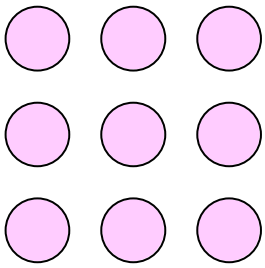
Square Numbers



n=1



n=2



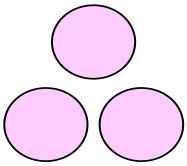
n=3

n	f(n)

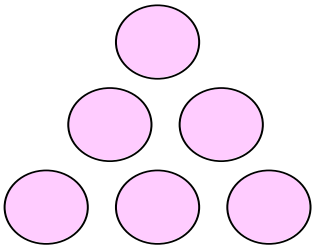
RULE:_____

Triangular Numbers

n=1



n=2



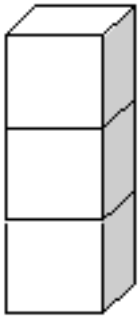
n=3

n	f(n)

RULE:_____

Tower of Cubes Problem

What is the surface area of the tower of cubes? As the line of cubes is extended, how does the surface area change?



N	f(n)

RULE:_____

Activity: Reflections on Functions

Format: Partner Pair

Objectives: Participants will leave class with an action plan for developing the concept of function with their students.

Related SOL: All Patterns, Functions, and Algebra Standards of Learning.

Materials: Blank sheet of paper

Time Required: 10 minutes

Directions:

1. Have teachers choose a partner who works at the same grade level as they work.
2. Have the teachers reflect on the activities in the module and develop a plan for teaching the concept of function appropriate to their grade level. The plan should include conceptual development using manipulative and activities. Ask them to try some of the activities, if appropriate, and report back to the class on the results at the next session.